KEAM 2026 Engineering April 20

Question Paper with Solutions

Conducted by CEE Kerala

General Instructions

(i) **Duration:** The total duration of the examination is 3 hours (180 minutes).

(ii) **Total Marks:** The complete paper carries a maximum of 600 marks.

(iii) **Structure:** The paper has 3 Sections:

- **Section A:** 45 Multiple Choice Questions (Physics).
- **Section B:** 30 Multiple Choice Questions (Chemistry).
- **Section B:** 75 Multiple Choice Questions (Mathematics).

(iv) **Compulsory Questions:** All 150 questions are compulsory.

(v) Each question has four options. Only **one** option is correct.

(vi) **Correct Answer:** +4 marks.

(vii) **Incorrect Answer:** −1 (Negative marking).

(viii) **Unanswered/Marked for Review:** 0 marks.

PHYSICS

1. The slope of the tangent drawn on a position-time graph at any instant is equal to the instantaneous ______.

(A) acceleration

(B) force

(C) velocity

(D) momentum

(E) impulse

Correct Answer: (C) velocity

Solution:

Step 1: Understanding the Concept:

A position-time graph represents the variation of an object's position with respect to time.

The slope of a curve at a specific point indicates the instantaneous rate of change of the dependent variable (position) with respect to the independent variable (time).

Step 2: Key Formula or Approach:

The mathematical definition for instantaneous velocity is the first derivative of position x with respect to time t, represented as $v = \frac{dx}{dt}$.

Step 3: Detailed Explanation:

Geometrically, the derivative $\frac{dx}{dt}$ represents the slope of the tangent line drawn to the $x - t$ curve at any particular instant.

Because the rate of change of position over time defines velocity, this slope exactly equals the object's instantaneous velocity at that moment.

Step 4: Final Answer:

The slope of the tangent drawn on a position-time graph at any instant is equal to the instantaneous velocity.

Quick Tip: Always recall the basic graphical relationships in kinematics: the slope of a position-time graph gives velocity, while the slope of a velocity-time graph gives acceleration.

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2. The dimensions for the gravitational constant G are ____.

(A) $M^{-1}L^{2}T^{-2}$

(B) $M^0L^0T^0$

(C) MT^{-2}

(D) ML^2T^{-2}

(E) $M^{-1}L^3T^{-2}$

Correct Answer: (E) $M^{-1}L^3T^{-2}$

Solution:

Step 1: Understanding the Concept:

The dimensional formula for any universal constant can be derived by isolating it in a fundamental physics equation where the dimensions of all other quantities are known.

For the universal gravitational constant G, we use Newton's law of universal gravitation.

Step 2: Key Formula or Approach:

Newton's law of gravitation states that the gravitational force F is given by $F = G\frac{m_1 m_2}{r^2}$.

Rearranging this formula to solve for G gives $G = \frac{F \cdot r^2}{m_1 \cdot m_2}$.

Step 3: Detailed Explanation:

We substitute the known dimensional formulas into the rearranged equation:

The dimension of force F is $[MLT^{-2}]$.

The dimension of distance r is $[L]$, so r^2 is $[L^2]$.

The dimension of mass m_1 and m_2 is $[M]$.

Plugging these in, we get:

$$[G] = \frac{[MLT^{-2}] \cdot [L^2]}{[M] \cdot [M]}$$

Simplifying the terms:

$$[G] = \frac{[ML^3T^{-2}]}{[M^2]}$$

$$[G] = [M^{1-2}L^3T^{-2}]$$

$$[G] = [M^{-1}L^{3}T^{-2}]$$

Step 4: Final Answer:

The dimensions for the gravitational constant G are $M^{-1}L^{3}T^{-2}$.

Quick Tip: To avoid memorization errors, always quickly derive the dimensional formula of constants like G, Planck's constant h, or permittivity ϵ_0 from their standard defining equations during the exam.

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3. The de Broglie wavelength associated with an electron accelerated by a potential of 64 V is _____.

(A) 1.227 nm
(B) 0.613 nm
(C) 0.302 nm
(D) 0.153 nm
(E) 2.454 nm

Correct Answer: (D) 0.153 nm

Solution:

Step 1: Understanding the Concept:

According to de Broglie's hypothesis, a moving particle like an electron has a wave nature with an associated wavelength.

When an electron is accelerated from rest through a potential difference, it gains kinetic energy, which can be directly related to its de Broglie wavelength.

Step 2: Key Formula or Approach:

The specific formula for the de Broglie wavelength λ of an electron accelerated through a potential difference V (in volts) is given by $\lambda = \frac{1.227}{\sqrt{V}}$ nm.

Step 3: Detailed Explanation:

We are given the accelerating potential $V = 64$ V.

Substitute this value directly into the empirical formula:

$$\lambda = \frac{1.227}{\sqrt{64}} \text{ nm}$$

Calculate the square root of the denominator:

$$\lambda = \frac{1.227}{8} \text{ nm}$$

Perform the final division:

$$\lambda = 0.153375 \text{ nm}$$

Rounding to three decimal places to match the given options gives 0.153 nm.

Step 4: Final Answer:

The de Broglie wavelength associated with the electron is 0.153 nm.

Quick Tip: Remembering the simplified formula $\lambda = \frac{1.227}{\sqrt{V}}$ nm or $\lambda = \frac{12.27}{\sqrt{V}}$ Å specifically for electrons saves valuable time over using the full $\lambda = \frac{h}{\sqrt{2m_e eV}}$ expression.

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4. The horizontal range of a projectile is maximum when the angle of projection is _____.

(A) $0°$

(B) $30°$

(C) 45°

(D) 60°

(E) 90°

Correct Answer: (C) 45°

Solution:

Step 1: Understanding the Concept:

The horizontal range of a projectile is the total horizontal distance it travels before hitting the ground.

This distance depends on the initial launch velocity and the angle of projection.

Step 2: Key Formula or Approach:

The formula for the horizontal range R of a projectile is $R = \frac{u^2 \sin(2\theta)}{g}$, where u is initial velocity, θ is the angle of projection, and g is acceleration due to gravity.

Step 3: Detailed Explanation:

For a given constant initial velocity u, the range R is directly proportional to $\sin(2\theta)$.

To achieve the maximum range R_{max}, the value of $\sin(2\theta)$ must be at its maximum possible value.

The maximum value of the sine function is 1, which occurs when its argument is 90°.

Therefore, we set up the equation:

$$\sin(2\theta) = 1$$

$$2\theta = 90^\circ$$

Solving for the angle θ:

$$\theta = \frac{90^\circ}{2} = 45^\circ$$

Step 4: Final Answer:

The horizontal range of a projectile becomes maximum when the angle of projection is $45°$.

Quick Tip: For any angle θ, the horizontal range is the same as for the complementary angle $(90° - \theta)$, but the peak of the maximum range always occurs exactly at $45°$.

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5. The graph between displacement and time for a particle moving with uniform acceleration is a _____.

(A) straight line with a positive slope

(B) parabola

(C) ellipse

(D) straight line parallel to time axis

(E) straight line perpendicular to time axis

Correct Answer: (B) parabola

Solution:

Step 1: Understanding the Concept:

Uniform acceleration implies that the object's velocity changes by equal amounts in equal time intervals.

We need to determine the mathematical relationship between displacement and time to identify the shape of its graph.

Step 2: Key Formula or Approach:

The kinematic equation that connects displacement s, initial velocity u, uniform acceleration a, and time t is $s = ut + \frac{1}{2}at^2$.

Step 3: Detailed Explanation:

In this equation, u and a are constants.

The equation $s = ut + \frac{1}{2}at^2$ shows that displacement s is a quadratic function of time t (due to the t^2 term).

In mathematics, any equation of the form $y = Ax^2 + Bx + C$ (with $A \neq 0$) graphically represents a parabola.

Mapping our equation to this standard form, the y-axis represents s and the x-axis represents t.

Because of the quadratic dependence, the resulting curve plotted on a displacement-time graph is a parabola.

Step 4: Final Answer:

The graph between displacement and time for a particle moving with uniform acceleration is a parabola.

Quick Tip: When analyzing graphs, look at the degree of the independent variable in the equation. A linear relationship (degree 1) gives a straight line, while a quadratic relationship (degree 2) gives a parabola.

CHEMISTRY

1. Zr (Z = 40) and Hf (Z = 72) have similar atomic and ionic radii because of:

(A) Diagonal relationship

(B) Lanthanoid contraction

(C) Having similar chemical properties

(D) Belonging to same group

Correct Answer: (B) Lanthanoid contraction

Solution:

Step 1: Understanding the Concept:

The elements Zirconium (Zr) and Hafnium (Hf) belong to Group 4 of the periodic table, located in the 4d and 5d transition series, respectively.

Normally, atomic and ionic radii increase significantly as we move down a group due to the addition of a new principal electron shell.

However, the atomic radius of Zr (160 pm) is almost identical to that of Hf (159 pm).

Step 2: Key Formula or Approach:

The approach here is to correlate the anomalous atomic size trend with the electronic configuration and the shielding effect of inner electrons.

We analyze the effect of filling the 4f subshell before the 5d subshell in the periodic table.

Step 3: Detailed Explanation:

Before the 5d transition series begins, the 4f orbitals are progressively filled with 14 electrons, forming the lanthanide series.

The shape of f-orbitals is highly diffused, which makes f-electrons very poor at shielding the outer valence electrons from the pull of the nucleus.

As the nuclear charge increases across the lanthanide series, the poor shielding by the 4f electrons causes the effective nuclear charge (Z_{eff}) to increase significantly.

This results in a steady and significant decrease in the atomic and ionic radii along the lanthanide series, a phenomenon termed "Lanthanoid contraction."

By the time we reach Hafnium ($Z = 72$), this contraction has exactly offset the expected increase in size due to the addition of the new 6s and 5d shells.

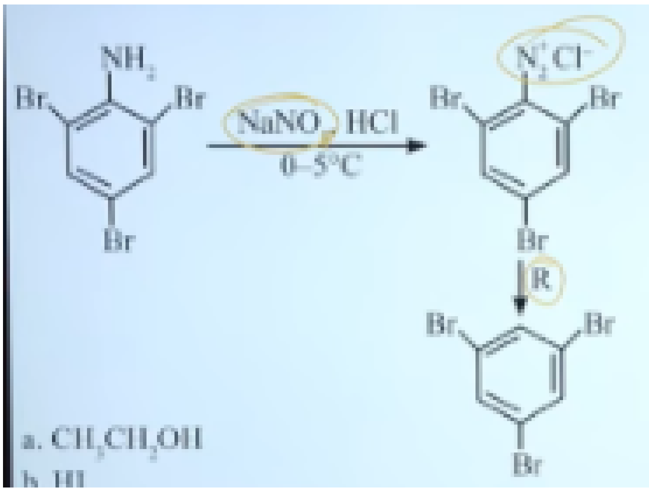
Consequently, Hf ends up having a size very similar to its upper group neighbor, Zr.

Step 4: Final Answer:

The similar radii are caused by the Lanthanoid contraction.

Quick Tip: Lanthanoid contraction is the standard explanation for the virtually identical atomic radii of 4d and 5d transition elements in the same group (e.g., Zr-Hf, Nb-Ta, Mo-W), which also causes them to have very similar chemical properties and makes them difficult to separate.

2. The reagent 'R' in the given sequence of chemical reaction is:



(A) CH_3CH_2OH

(B) HI

(C) CuCN/KCN

(D) H_2O

Correct Answer: (A) CH_3CH_2OH

Solution:

Step 1: Understanding the Concept:

The given reaction sequence outlines the removal of an amino group ($-NH_2$) from an aromatic ring.

This process is called deamination. It is a two-step procedure involving the formation of a diazonium salt intermediate followed by its reduction.

Step 2: Key Formula or Approach:

The general approach for deamination involves two key reactions:

1. Diazotization: $\text{Ar-NH}_2 + \text{NaNO}_2 + \text{HCl} \xrightarrow{0\text{–}5^\circ\text{C}} \text{Ar-N}_2^+\text{Cl}^-$

2. Reduction (replacement by H): $\text{Ar-N}_2^+\text{Cl}^- \xrightarrow{\text{Reducing Agent (R)}} \text{Ar-H} + \text{N}_2 \uparrow + \text{HCl}$

Step 3: Detailed Explanation:

The first step shows 2,4,6-tribromoaniline reacting with $NaNO_2$ and HCl at cold temperatures.

This successfully forms the diazonium salt, 2,4,6-tribromobenzenediazonium chloride.
In the second step, the diazonium group ($-N_2^+Cl^-$) needs to be replaced by a hydrogen atom to yield 1,3,5-tribromobenzene.
This is a reduction reaction. The mild reducing agents typically employed for this specific transformation are either hypophosphorous acid (H_3PO_2) in the presence of water, or ethanol (CH_3CH_2OH).
When ethanol is used, it acts as a hydride donor, reducing the diazonium salt while it gets oxidized to acetaldehyde (CH_3CHO).
The reaction is:
$\text{Ar-}N_2^+Cl^- + CH_3CH_2OH \rightarrow \text{Ar-H} + N_2 + HCl + CH_3CHO$
Evaluating the options, CH_3CH_2OH is the correct mild reducing agent required for this conversion.

Step 4: Final Answer:
The reagent 'R' is ethanol (CH_3CH_2OH).

Quick Tip: For the direct substitution of a diazonium group with a hydrogen atom, always look for either Ethanol (CH_3CH_2OH) or Hypophosphorous acid ($H_3PO_2 + H_2O$) among the options.

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3. Reaction of butanone with methylmagnesium bromide followed by hydrolysis gives:

(A) 2-methyl-2-butanol
(B) 2-butanol
(C) 3-methyl-2-butanol
(D) 2,2-dimethyl-1-butanol
(E) 2-pentanol

Correct Answer: (A) 2-methyl-2-butanol

Solution:

Step 1: Understanding the Concept:

The question asks for the product of a Grignard reaction.

A Grignard reagent (RMgX) acts as a strong nucleophile and attacks the electrophilic carbonyl carbon of a ketone or aldehyde.

Reacting a ketone with a Grignard reagent followed by acid hydrolysis always yields a tertiary alcohol.

Step 2: Key Formula or Approach:

The general mechanism involves:

1. Nucleophilic Addition: $R_1 - C(=O) - R_2 + R_3MgX \rightarrow R_1 - C(O^-MgX^+)(R_3) - R_2$

2. Hydrolysis: $R_1 - C(O^-MgX^+)(R_3) - R_2 \xrightarrow{H_3O^+} R_1 - C(OH)(R_3) - R_2 + Mg(OH)X$

Step 3: Detailed Explanation:

The starting material is butanone, a 4-carbon ketone with the structure $CH_3 - C(=O) - CH_2 - CH_3$.

The Grignard reagent is methylmagnesium bromide (CH_3MgBr). The methyl group (CH_3^-) acts as the nucleophile.

The methyl carbanion attacks the carbonyl carbon of butanone, breaking the π-bond and pushing electrons onto the oxygen to form an alkoxide intermediate:

$$CH_3 - C(=O) - CH_2 - CH_3 + CH_3MgBr \rightarrow CH_3 - C(O^-MgBr^+)(CH_3) - CH_2 - CH_3$$

Subsequent hydrolysis with water and an acid catalyst protonates the alkoxide oxygen:

$$CH_3 - C(O^-MgBr^+)(CH_3) - CH_2 - CH_3 \xrightarrow{H_2O/H^+} CH_3 - C(OH)(CH_3) - CH_2 - CH_3$$

Now, we determine the IUPAC name of the product.

The longest carbon chain containing the –OH group has 4 carbons (butane).

Numbering from the left gives the lowest locant for the –OH group, placing it at position 2.

There is also a methyl substituent at position 2.

Therefore, the name is 2-methyl-2-butanol.

Step 4: Final Answer:

The final product is 2-methyl-2-butanol.

Quick Tip: To predict the product of a Grignard reaction with a carbonyl, simply attach the alkyl group from the Grignard reagent directly to the carbonyl carbon, and change the double-bonded oxygen ($=O$) to a hydroxyl group ($-OH$).

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4. X and Y in the below reaction are _____ and _____ respectively

$$C_6H_5 - CO_2H + X \xrightarrow{\text{Heat}} C_6H_5 - COCl \xrightarrow{H_2/Pd/BaSO_4,\ \text{Quinoline}} Y$$

(A) $SOCl_2$ and C_6H_5CHO

(B) $(COCl)_2$ and $C_6H_5CH_3$

(C) $SOCl_2$ and $C_6H_5CH_3$

(D) $(COCl)_2$ and $C_6H_5CH_2OH$

(E) $SOCl_2$ and $C_6H_5CH_2Cl$

Correct Answer: (A) $SOCl_2$ and C_6H_5CHO

Solution:

Step 1: Understanding the Concept:

The sequence involves two distinct organic transformations: the conversion of a carboxylic acid to an acid chloride, and the subsequent partial reduction of the acid chloride to an aldehyde.

Step 2: Key Formula or Approach:

Identify the specific reagents required for each transformation:

Reaction 1: $\text{R-COOH} \xrightarrow{\text{Chlorinating Agent (X)}} \text{R-COCl}$

Reaction 2: R-COCl $\xrightarrow[\text{Rosenmund Reduction}]{H_2,\ Pd/BaSO_4,\ \text{Quinoline}}$ R-CHO (Y)

Step 3: Detailed Explanation:

In the first step, benzoic acid ($C_6H_5 - CO_2H$) is converted to benzoyl chloride ($C_6H_5 - COCl$). This conversion requires a chlorinating agent. Common reagents for this are Thionyl chloride ($SOCl_2$), Phosphorus pentachloride (PCl_5), or Phosphorus trichloride (PCl_3).

Among the options provided, $SOCl_2$ and $(COCl)_2$ are capable of this transformation. Let's evaluate the second step to confirm the correct pair.

The second step involves treating benzoyl chloride with hydrogen gas (H_2) in the presence of Palladium on Barium sulfate ($Pd/BaSO_4$) and a poison like Quinoline.

This is the classic Rosenmund reduction. The $BaSO_4$ and quinoline intentionally "poison" or partially deactivate the palladium catalyst.

This deactivation prevents the resulting aldehyde from being further reduced into a primary alcohol.

Therefore, the acid chloride (C_6H_5-COCl) is selectively reduced to an aldehyde (C_6H_5-CHO), which is benzaldehyde.

Looking at the options, only (A) has $SOCl_2$ for X and the correct aldehyde C_6H_5CHO for Y.

Step 4: Final Answer:

The reagent X is $SOCl_2$ and product Y is benzaldehyde (C_6H_5CHO).

Quick Tip: The combination of H_2, Pd over $BaSO_4$ is the signature of the Rosenmund reduction, which specifically halts the reduction of an acid chloride at the aldehyde stage.

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5. The correct increasing order of the acid strength of benzoic acid (I), 4-nitrobenzoic acid (II), 3,4-dinitrobenzoic acid (III) and 4-methoxybenzoic acid (IV) is

(A) I < II < III < IV

(B) II < I < IV < III

(C) IV < I < II < III

(D) IV < II < I < III

(E) I < IV < II < III

Correct Answer: (C) IV < I < II < III

Solution:

Step 1: Understanding the Concept:

The acidic strength of substituted benzoic acids depends on the stability of their conjugate base, the benzoate anion.

Substituents on the benzene ring can stabilize or destabilize this anion through Inductive (I) and Mesomeric/Resonance (M) effects.

Step 2: Key Formula or Approach:

The approach is to evaluate the nature of each substituent:

- Electron Withdrawing Groups (EWGs): Disperse negative charge, stabilize the anion → Increase acidity.

- Electron Donating Groups (EDGs): Intensify negative charge, destabilize the anion → Decrease acidity.

Step 3: Detailed Explanation:

Let's analyze the given compounds and their substituents:

(I) Benzoic acid: The standard reference molecule with no substituents on the ring.

(II) 4-nitrobenzoic acid: The $-NO_2$ group is strongly electron-withdrawing through both the -I and -M effects. It pulls electron density away from the ring, highly stabilizing the carboxylate anion. Thus, it is more acidic than (I).

(III) 3,4-dinitrobenzoic acid: Contains two strongly electron-withdrawing $-NO_2$ groups. The combined -I and -M effects of two nitro groups stabilize the conjugate base to a much greater extent than a single nitro group. This will be the most acidic compound in the set.

(IV) 4-methoxybenzoic acid: The $-OCH_3$ group is an electron-donating group. While it has a weak -I effect, its strong +M effect (due to the lone pair on oxygen resonating into the ring) dominates. It donates electron density into the ring, destabilizing the negative charge on the carboxylate anion. Therefore, it is less acidic than unsubstituted benzoic acid (I).

Arranging them in increasing order of acidity:

Least acidic (destabilized by EDG) < Standard < More acidic (stabilized by one EWG) < Most acidic (stabilized by two EWGs).

Therefore: 4-methoxybenzoic acid (IV) < benzoic acid (I) < 4-nitrobenzoic acid (II) < 3,4-dinitrobenzoic acid (III).

Step 4: Final Answer:

The correct increasing order is IV < I < II < III.

Quick Tip: To quickly rank acid strengths: Identify EDGs (like $-OH, -OCH_3, -NH_2$) which decrease acidity, and EWGs (like $-NO_2, -CN, -COOH, -X$) which increase acidity. Multiple EWGs magnify the acidic strength further.

MATHEMATICS

1. The value of the integral $\int_0^{\pi} \frac{\cos x}{1+\sin^2 x} dx$ is

(A) 0

(B) 1

(C) $\frac{\pi}{2}$

(D) π

(E) 2π

Correct Answer: (A) 0

Solution:

Step 1: Understanding the Concept:

We are tasked with evaluating a definite integral involving trigonometric functions.

This can be approached either by using standard properties of definite integrals or through a suitable substitution.

Step 2: Key Formula or Approach:

Method 1 relies on the property: $\int_0^{2a} f(x)dx = 0$ if $f(2a-x) = -f(x)$.

Method 2 utilizes substitution: Let $t = \sin x$, which implies $dt = \cos x dx$.

Step 3: Detailed Explanation:

Let the integral be $I = \int_0^{\pi} \frac{\cos x}{1+\sin^2 x} dx$.

Method 1: Using Definite Integral Properties

Let the integrand be denoted as $f(x) = \frac{\cos x}{1+\sin^2 x}$.

We evaluate the function at the upper limit minus x, which is $f(\pi - x)$:

$$f(\pi - x) = \frac{\cos(\pi - x)}{1 + \sin^2(\pi - x)}$$

Recall the trigonometric identities: $\cos(\pi - x) = -\cos x$ and $\sin(\pi - x) = \sin x$.

Substituting these identities back, we get:

$$f(\pi - x) = \frac{-\cos x}{1 + (\sin x)^2} = -\frac{\cos x}{1 + \sin^2 x} = -f(x)$$

Since $f(\pi - x) = -f(x)$, it directly follows from the properties of definite integrals that the integral over the interval $[0, \pi]$ is zero.

$$\int_0^{\pi} f(x) dx = 0$$

Method 2: Using Substitution

Let's employ the substitution $t = \sin x$.

Differentiating both sides with respect to x gives $dt = \cos x dx$.

Next, we must change the limits of integration to match the new variable t.

When the lower limit $x = 0$, $t = \sin(0) = 0$.

When the upper limit $x = \pi$, $t = \sin(\pi) = 0$.

Substituting these into the original integral transforms it to:

$$I = \int_0^0 \frac{1}{1 + t^2} dt$$

Because the upper and lower limits of integration are identical, the definite integral evaluates to 0.

Step 4: Final Answer:

The value of the integral is 0.

Quick Tip: Before performing complex substitutions, always check if the integrand satisfies the symmetry condition $f(2a-x) = -f(x)$. If it does, the integral is simply zero, saving significant calculation time.

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2. The value of k, if the circles $2x^2+2y^2-4x+6y=3$ and $x^2+y^2+kx+y=0$ cut orthogonally is

(A) 2

(B) 3

(C) 4

(D) 5

(E) 1

Correct Answer: (B) 3

Solution:

Step 1: Understanding the Concept:

Two circles intersect orthogonally if the angle between their tangents at the points of intersection is 90°.

This geometric condition translates to a specific algebraic relationship involving the coefficients of their general equations.

Step 2: Key Formula or Approach:

The general equation of a circle is written as $x^2+y^2+2gx+2fy+c=0$.

For two circles with equations $x^2+y^2+2g_1x+2f_1y+c_1=0$ and $x^2+y^2+2g_2x+2f_2y+c_2=0$, the condition for orthogonal intersection is:

$$2g_1g_2+2f_1f_2=c_1+c_2$$

Step 3: Detailed Explanation:

First, we must express the given equations in the standard general form where the coefficients of x^2 and y^2 are exactly 1.

First Circle: The given equation is $2x^2 + 2y^2 - 4x + 6y = 3$.

We divide the entire equation by 2 to normalize it:

$$x^2 + y^2 - 2x + 3y - \frac{3}{2} = 0$$

Comparing this with the general form $x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$, we extract the constants:

$2g_1 = -2 \Rightarrow g_1 = -1$

$2f_1 = 3 \Rightarrow f_1 = \frac{3}{2}$

$c_1 = -\frac{3}{2}$

Second Circle: The given equation is already in the general form: $x^2 + y^2 + kx + y = 0$.

Comparing this with $x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0$, we get:

$2g_2 = k \Rightarrow g_2 = \frac{k}{2}$

$2f_2 = 1 \Rightarrow f_2 = \frac{1}{2}$

$c_2 = 0$

Now, we apply the condition for orthogonality:

$$2g_1g_2 + 2f_1f_2 = c_1 + c_2$$

Substitute the values we have identified:

$$2(-1)\left(\frac{k}{2}\right) + 2\left(\frac{3}{2}\right)\left(\frac{1}{2}\right) = -\frac{3}{2} + 0$$

Simplifying the terms:

$$-k + \frac{3}{2} = -\frac{3}{2}$$

To solve for k, isolate the variable:

$$-k = -\frac{3}{2} - \frac{3}{2}$$

$$-k = -\frac{6}{2}$$

$$-k = -3$$

Therefore, multiplying by -1 yields $k = 3$.

Step 4: Final Answer:

The value of k is 3.

Quick Tip: A common mistake is applying the orthogonality formula without first ensuring the coefficients of x^2 and y^2 are 1. Always normalize the circle equations first.

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3. The axis of the parabola $x^2 + 6x + 4y + 5 = 0$ is

(A) $x = 0$

(B) $y = 1$

(C) $x + 3 = 0$

(D) $y = 4$

(E) $y + 2 = 0$

Correct Answer: (C) $x + 3 = 0$

Solution:

Step 1: Understanding the Concept:

The axis of symmetry of a parabola is a straight line that divides the parabola into two identical, mirrored halves.

For a parabola defined by a quadratic equation in x, the axis of symmetry is a vertical line passing through its vertex.

Step 2: Key Formula or Approach:

The approach is to convert the general equation into the standard vertex form by completing the square.

For an equation of the form $x^2 + Dx + Ey + F = 0$, completing the square yields $(x-h)^2 = 4a(y-k)$.

The vertex is (h, k), and the axis of symmetry is the vertical line $x = h$.

Step 3: Detailed Explanation:

We start with the given equation:

$$x^2 + 6x + 4y + 5 = 0$$

First, isolate the terms containing x on one side of the equation:

$$x^2 + 6x = -4y - 5$$

Next, we complete the square for the quadratic expression in x. We take half of the coefficient of x (which is $6/2 = 3$) and square it ($3^2 = 9$).

Add 9 to both sides of the equation to maintain balance:

$$x^2 + 6x + 9 = -4y - 5 + 9$$

The left side can now be factored as a perfect square:

$$(x+3)^2 = -4y + 4$$

Factor out the coefficient of y on the right side:

$$(x+3)^2 = -4(y-1)$$

This equation is now in the standard form $(x-h)^2 = 4a(y-k)$.

From this form, we can identify the vertex (h, k) of the parabola as $(-3, 1)$.

Because the squared term involves x, the parabola opens vertically (upwards or downwards).

The axis of symmetry for such a parabola is the vertical line that passes through its vertex.

Therefore, the equation of the axis is $x = h$, which means $x = -3$.

Rearranging this to match the given options gives $x + 3 = 0$.

Step 4: Final Answer:

The axis of the parabola is $x + 3 = 0$.

Quick Tip: To rapidly find the axis of symmetry of a parabola given in general form, take the partial derivative of the equation with respect to the squared variable and set it to zero. Here, $\frac{\partial}{\partial x}(x^2 + 6x + 4y + 5) = 2x + 6 = 0$, yielding $x = -3$ or $x + 3 = 0$.

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4. The distance between the foci of the ellipse $\frac{(x+2)^2}{9} + \frac{(y-1)^2}{4} = 1$ is

(A) $\sqrt{5}$

(B) $2\sqrt{5}$

(C) $3\sqrt{5}$

(D) $9\sqrt{5}$

(E) $7\sqrt{5}$

Correct Answer: (B) $2\sqrt{5}$

Solution:

Step 1: Understanding the Concept:

The foci are two fixed points on the major axis of an ellipse that define its shape.

The problem requires finding the straight-line distance between these two points based on the given equation of the ellipse.

Step 2: Key Formula or Approach:

The standard equation of an ellipse is $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$, assuming $a^2 > b^2$.

The lengths of the semi-major and semi-minor axes are a and b, respectively.

The eccentricity e is related to a and b by the formula $b^2 = a^2(1-e^2)$.

The distance from the center to each focus is ae, so the total distance between the two foci is $2ae$.

Step 3: Detailed Explanation:

The given equation of the ellipse is:

$$\frac{(x+2)^2}{9} + \frac{(y-1)^2}{4} = 1$$

By comparing this to the standard form $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$, we can deduce the squared lengths of the semi-axes:

The denominator of the x-term gives $a^2 = 9$, so $a = 3$.

The denominator of the y-term gives $b^2 = 4$, so $b = 2$.

Since $a > b$, the major axis is horizontal.

Next, we calculate the eccentricity e using the defining relationship for an ellipse:

$$b^2 = a^2(1-e^2)$$

Substitute the values of a^2 and b^2:

$$4 = 9(1-e^2)$$

Divide by 9:

$$\frac{4}{9} = 1 - e^2$$

Rearrange to solve for e^2:

$$e^2 = 1 - \frac{4}{9}$$

$$e^2 = \frac{9-4}{9} = \frac{5}{9}$$

Taking the square root (eccentricity is positive):

$$e = \frac{\sqrt{5}}{3}$$

The formula for the distance between the foci is $2ae$.

Substitute the values of a and e into this formula:

$$\text{Distance} = 2 \cdot 3 \cdot \left(\frac{\sqrt{5}}{3}\right)$$

The 3 in the numerator and denominator cancel out:

$$\text{Distance} = 2\sqrt{5}$$

Step 4: Final Answer:

The distance between the foci is $2\sqrt{5}$.

Quick Tip: For an ellipse equation in the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, you can bypass finding the eccentricity explicitly. The distance between foci is simply $2\sqrt{a^2 - b^2}$ (if $a > b$). Here, $2\sqrt{9-4} = 2\sqrt{5}$.

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5. The circle passing through the point $(1,-2)$ and touching the x-axis at $(3,0)$ also passes through the point

(A) $(2,-5)$

(B) $(-5,-2)$

(C) $(-2,5)$

(D) $(-5,2)$

(E) $(5,-2)$

Correct Answer: (E) $(5,-2)$

Solution:

Step 1: Understanding the Concept:

We are tasked with finding an unknown point that lies on a specific circle.

To do this, we first need to determine the full equation of the circle using the geometric constraints provided: a point it passes through and a point where it is tangent to an axis.

Step 2: Key Formula or Approach:

The standard equation of a circle is $(x-h)^2+(y-k)^2=r^2$, where (h,k) is the center and r is the radius.

If a circle is tangent to the x-axis at a point $(x_1,0)$, its center lies directly above or below this point, so $h=x_1$.

Furthermore, the distance from the center to the tangent x-axis is the radius, meaning $r=|k|$, or $r^2=k^2$.

Step 3: Detailed Explanation:

The problem states the circle touches the x-axis at the point $(3,0)$.

This geometric fact gives us two pieces of information:

1. The x-coordinate of the center (h,k) must be $h=3$.

2. The radius r is the perpendicular distance to the x-axis, so $r=|k|$, which implies $r^2=k^2$.

Incorporating these into the standard circle equation, we get a simplified form:

$$(x-3)^2+(y-k)^2=k^2$$

We are also given that the circle passes through the point $(1,-2)$. This point must satisfy the circle's equation.

Substitute $x = 1$ and $y = -2$ into our simplified equation:

$$(1-3)^2 + (-2-k)^2 = k^2$$

Simplify the terms inside the parentheses:

$$(-2)^2 + (-(2+k))^2 = k^2$$

Since $(-(2+k))^2$ is equivalent to $(2+k)^2$, we expand the squares:

$$4 + (4 + 4k + k^2) = k^2$$

Combine the constant terms:

$$8 + 4k + k^2 = k^2$$

Subtract k^2 from both sides of the equation to solve for k:

$$8 + 4k = 0$$

$$4k = -8$$

$$k = -2$$

Now we know the center is $(3, -2)$ and the radius squared is $r^2 = (-2)^2 = 4$.

The complete equation of the circle is therefore:

$$(x-3)^2 + (y+2)^2 = 4$$

Finally, we must check which of the given options satisfies this equation.

Testing Option (A) $(2,-5)$: $(2-3)^2+(-5+2)^2 = (-1)^2+(-3)^2 = 1+9 = 10 \neq 4$.

Testing Option (B) $(-5,-2)$: $(-5-3)^2+(-2+2)^2 = (-8)^2+0^2 = 64 \neq 4$.

Testing Option (C) $(-2,5)$: $(-2-3)^2+(5+2)^2 = (-5)^2+7^2 = 25+49 = 74 \neq 4$.

Testing Option (D) $(-5,2)$: $(-5-3)^2+(2+2)^2 = (-8)^2+4^2 = 64+16 = 80 \neq 4$.

Testing Option (E) $(5,-2)$: $(5-3)^2+(-2+2)^2 = (2)^2+(0)^2 = 4+0 = 4$.

Since $4 = 4$, the point $(5,-2)$ perfectly satisfies the equation of the circle.

Step 4: Final Answer:

The circle also passes through the point $(5,-2)$.

Quick Tip: Recognizing that a circle touching the x-axis at $(h,0)$ has the equation $(x-h)^2+(y-k)^2 = k^2$ immediately reduces the problem to solving for a single unknown (k) using the other given point.
