KEAM 2026 Engineering April 19

Question Paper with Solutions

Conducted by CEE Kerala

General Instructions

(i) **Duration:** The total duration of the examination is 3 hours (180 minutes).

(ii) **Total Marks:** The complete paper carries a maximum of 600 marks.

(iii) **Structure:** The paper has 3 Sections:

- **Section A:** 45 Multiple Choice Questions (Physics).
- **Section B:** 30 Multiple Choice Questions (Chemistry).
- **Section B:** 75 Multiple Choice Questions (Mathematics).

(iv) **Compulsory Questions:** All 150 questions are compulsory.

(v) Each question has four options. Only **one** option is correct.

(vi) **Correct Answer:** +4 marks.

(vii) **Incorrect Answer:** −1 (Negative marking).

(viii) **Unanswered/Marked for Review:** 0 marks.

Physics

1. A spherical conductor contains 5×10^6 electrons. If the Radius of the sphere is 10cm, find the electricfield at its surface

Correct Answer: 0.72 N/C

Solution:

Step 1: Understanding the Concept:

The electric field at the surface of a spherical conductor is calculated by assuming all its excess charge is concentrated at the geometric center.

The total charge is due to the given number of electrons.

Step 2: Key Formula or Approach:

The formula for the electric field E at the surface of a sphere of radius R with charge Q is:

$$E = \frac{1}{4\pi\epsilon_0}\frac{Q}{R^2}$$

where $Q = n \cdot e$, with n being the number of electrons, and $e = 1.6 \times 10^{-19}$ C.

Step 3: Detailed Explanation:

The given number of electrons is $n = 5 \times 10^6$.

The magnitude of the total charge Q is:

$$Q = 5 \times 10^6 \times 1.6 \times 10^{-19}\ \text{C} = 8 \times 10^{-13}\ \text{C}$$

The radius of the sphere is $R = 10$ cm $= 0.1$ m.

Substitute these values into the electric field formula:

$$E = (9 \times 10^9)\frac{8 \times 10^{-13}}{(0.1)^2}$$

$$E = \frac{72 \times 10^{-4}}{0.01}$$

$$E = 72 \times 10^{-2}\ \text{N/C} = 0.72\ \text{N/C}$$

Step 4: Final Answer:

The electric field at its surface is 0.72 N/C.

Quick Tip: Always convert all given units to standard SI units (like cm to m) before substituting them into electromagnetic formulas to prevent scaling errors.

2. Find the total energy released when 235g of 235_U undergoes complete fission. Assume that the energy released per fission is about 200 MeV

Correct Answer: 1.93×10^{13} J

Solution:

Step 1: Understanding the Concept:

When 235_U undergoes fission, a massive amount of energy is released per individual nucleus. To find the total energy, we must calculate the total number of atoms in the given mass of uranium.

Step 2: Key Formula or Approach:

The number of atoms N is found using the mole concept:

$$N = \frac{m}{M} \times N_A$$

The total energy released is:

$$E_{\text{total}} = N \times E_f$$

where E_f is the energy released per fission event.

Step 3: Detailed Explanation:

The given mass of Uranium-235 is $m = 235$ g.

The molar mass M of 235_U is 235 g/mol.

Therefore, the number of moles is $n = \frac{235}{235} = 1$ mole.

The number of atoms in 1 mole is Avogadro's number, $N_A = 6.022 \times 10^{23}$.

The energy released per fission is $E_f = 200$ MeV.

Total energy in MeV is:

$$E_{\text{total}} = 6.022 \times 10^{23} \times 200 \text{ MeV} = 1.2044 \times 10^{26} \text{ MeV}$$

Convert this energy to Joules using the conversion factor $1 \text{ MeV} = 1.6 \times 10^{-13}$ J:

$$E_{\text{total}} = 1.2044 \times 10^{26} \times 1.6 \times 10^{-13} \text{ J} \approx 1.927 \times 10^{13} \text{ J}$$

Step 4: Final Answer:

The total energy released is approximately 1.93×10^{13} J.

Quick Tip: For rapid nuclear physics calculations, memorize that 1 mole of any fissile substance releasing 200 MeV per reaction will yield roughly 1.9×10^{13} J of energy.

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3. Transverse wave in a string is given by $y = 3\sin 2(25t + 0.4x)$m What is the velocity of wave?

Correct Answer: 62.5 m/s

Solution:

Step 1: Understanding the Concept:

The equation represents a traveling transverse wave on a string.

The velocity of the wave (phase velocity) is the speed at which the waveform propagates through the medium.

Step 2: Key Formula or Approach:

The standard wave equation is $y(x,t) = A\sin(\omega t + kx)$.

The wave velocity v is determined by the ratio of the angular frequency to the angular wave number:

$$v = \frac{\omega}{k}$$

Step 3: Detailed Explanation:

The given wave equation is:

$$y = 3\sin 2(25t + 0.4x)$$

Expand the argument of the sine function by multiplying the factor of 2:

$$y = 3\sin(50t + 0.8x)$$

By comparing this to the standard form $y = A\sin(\omega t + kx)$, we extract the coefficients:

$$\omega = 50 \text{ rad/s}$$

$$k = 0.8 \text{ m}^{-1}$$

Calculate the wave velocity:

$$v = \frac{\omega}{k} = \frac{50}{0.8}$$

$$v = \frac{500}{8} = 62.5 \text{ m/s}$$

Step 4: Final Answer:

The velocity of the wave is 62.5 m/s.

Quick Tip: To quickly find the wave velocity from an equation grouped as $f(at \pm bx)$, just take the ratio of the coefficient of t to the coefficient of x, which is a/b.

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4. A ball of mass 200g strikes a wall with a speed 5m/s and rebounds with same speed in the opposite direction. If the average force exerted on the —-is 5N, find the time of contact between the ball and wall

Correct Answer: 0.4 s

Solution:

Step 1: Understanding the Concept:

When the ball rebounds off the wall, it experiences a change in momentum due to an average force applied over a specific time duration.

This process is governed by the impulse-momentum theorem.

Step 2: Key Formula or Approach:

Impulse is defined as the average force multiplied by the time interval, and it equals the change in momentum:

$$J = F_{\text{avg}} \cdot \Delta t = \Delta p$$

where the change in momentum is $\Delta p = m(v_f - v_i)$.

Step 3: Detailed Explanation:

The mass of the ball is given as $m = 200 \text{ g} = 0.2 \text{ kg}$.

The initial velocity is $v_i = 5 \text{ m/s}$ (taking the direction towards the wall as positive).

The final velocity after rebounding is $v_f = -5 \text{ m/s}$.

Calculate the change in momentum:

$$\Delta p = m(v_f - v_i) = 0.2(-5-5) = 0.2(-10) = -2 \text{ kg m/s}$$

The magnitude of the change in momentum is $|\Delta p| = 2 \text{ kg m/s}$.

The average force F is given as 5 N.

Using the impulse equation to solve for the time of contact Δt:

$$F \cdot \Delta t = |\Delta p|$$

$$5 \cdot \Delta t = 2$$

$$\Delta t = \frac{2}{5} = 0.4 \text{ s}$$

Step 4: Final Answer:

The time of contact is 0.4 s.

Quick Tip: For perfectly elastic collisions where an object rebounds straight back with the exact same speed, the change in momentum simplifies directly to $2mv$.

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5. Two identical cells, each of emf 2V and internal resistance 0.1Ω, are connected in parallel. Find effective emf and the effective internal resistance of the combination.

Correct Answer: Effective emf $= 2\text{V}$, Effective internal resistance $= 0.05\Omega$

Solution:

Step 1: Understanding the Concept:

When identical cells are connected in parallel, the overall electromotive force (emf) is equal to the emf of a single cell.

However, their internal resistances act like standard resistors connected in parallel, reducing the total internal resistance.

Step 2: Key Formula or Approach:

For n identical cells connected in parallel, the effective emf is:

$$E_{\text{eff}} = E$$

The effective internal resistance is calculated using parallel resistance rules:

$$r_{\text{eff}} = \frac{r}{n}$$

Step 3: Detailed Explanation:

The problem states we have two identical cells, so $n = 2$.

The emf of each cell is $E = 2$ V.

The internal resistance of each cell is $r = 0.1\Omega$.

The effective emf of the entire parallel combination remains:

$$E_{\text{eff}} = 2 \text{ V}$$

The effective internal resistance is calculated as:

$$r_{\text{eff}} = \frac{0.1}{2} = 0.05\Omega$$

Step 4: Final Answer:

The effective emf is 2V and the internal resistance is 0.05Ω.

Quick Tip: Connecting identical cells in parallel does not increase the voltage output, but it significantly drops the internal resistance, allowing the circuit to deliver higher current efficiently.

6. A copper wire of cross sectional area 2mm^2 carries a current I and has drift velocity V$_1$. Another copper wire of cross- sectional area 1.5mm^2 carries a current 2I and has drift veloc-ity v$_2$. Find ratio $\frac{V_1}{V_2}$

Correct Answer: $\frac{3}{8}$

Solution:

Step 1: Understanding the Concept:

Drift velocity is the average velocity that charge carriers gain in a material due to an applied electric field.

It is directly proportional to the current and inversely proportional to the cross-sectional area of the conductor.

Step 2: Key Formula or Approach:

The relationship between current I and drift velocity v_d is given by:

$$I = neAv_d \implies v_d = \frac{I}{neA}$$

where n is the charge carrier density, e is the elementary charge, and A is the cross-sectional area.

Step 3: Detailed Explanation:

For the first copper wire, the drift velocity V_1 is:

$$V_1 = \frac{I}{neA_1}$$

For the second copper wire, the drift velocity V_2 is:

$$V_2 = \frac{2I}{neA_2}$$

Since both wires are made of copper, the electron density n is exactly the same for both.

Take the ratio of V_1 to V_2:

$$\frac{V_1}{V_2} = \frac{\left(\frac{I}{neA_1}\right)}{\left(\frac{2I}{neA_2}\right)}$$

$$\frac{V_1}{V_2} = \frac{I}{neA_1} \times \frac{neA_2}{2I} = \frac{A_2}{2A_1}$$

Substitute the given areas $A_1 = 2\ \text{mm}^2$ and $A_2 = 1.5\ \text{mm}^2$:

$$\frac{V_1}{V_2} = \frac{1.5}{2 \times 2} = \frac{1.5}{4} = \frac{3}{8}$$

Step 4: Final Answer:

The ratio $\frac{V_1}{V_2}$ is $3/8$.

Quick Tip: When taking ratios involving the same material, material-specific constants like n and e always cancel out. Jump straight to the proportionality $v_d \propto I/A$ to save time.

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7. A solid sphere of radius 20cm has the same mass as a solid cylinder. If their moments of inertia about their respective central axes are equal, find radius of the cylinder.

Correct Answer: $8\sqrt{5}$ cm (or ≈ 17.89 cm)

Solution:

Step 1: Understanding the Concept:

The moment of inertia depends on how a body's mass is distributed relative to its axis of rotation.

We are setting the moment of inertia of a solid sphere equal to that of a solid cylinder of identical mass.

Step 2: Key Formula or Approach:

The moment of inertia of a solid sphere about its central axis is:

$$I_s = \frac{2}{5}MR_s^2$$

The moment of inertia of a solid cylinder about its longitudinal central axis is:

$$I_c = \frac{1}{2}MR_c^2$$

Step 3: Detailed Explanation:

Let M be the mass of both the solid sphere and the solid cylinder.

The radius of the solid sphere is $R_s = 20$ cm.

Equating their moments of inertia:

$$I_s = I_c$$

$$\frac{2}{5}MR_s^2 = \frac{1}{2}MR_c^2$$

Cancel the mass M from both sides of the equation:

$$\frac{2}{5}R_s^2 = \frac{1}{2}R_c^2$$

Substitute the given radius of the sphere $R_s = 20$ cm:

$$\frac{2}{5}(20)^2 = \frac{1}{2}R_c^2$$

$$\frac{2}{5} \times 400 = \frac{1}{2}R_c^2$$

$$160 = \frac{1}{2}R_c^2$$

Multiply by 2 to solve for R_c^2:

$$R_c^2 = 320$$

Take the square root to find R_c:

$$R_c = \sqrt{320} = \sqrt{64 \times 5} = 8\sqrt{5} \text{ cm}$$

Numerically, this evaluates to $8 \times 2.236 \approx 17.89$ cm.

Step 4: Final Answer:

The radius of the cylinder is $8\sqrt{5}$ cm.

Quick Tip: To speed up calculations in exams, keep a list of common moments of inertia memorized (e.g., solid sphere: $\frac{2}{5}MR^2$, hollow sphere: $\frac{2}{3}MR^2$, solid cylinder: $\frac{1}{2}MR^2$).

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8. In which thermodynamic process does the internal energy of an ideal gas remain unchanged?

Correct Answer: Isothermal process

Solution:

Step 1: Understanding the Concept:

For an ideal gas, the internal energy is an exclusive function of its absolute temperature.
If the temperature of the gas does not change during a thermodynamic process, its internal energy will remain entirely constant.

Step 2: Key Formula or Approach:

The change in internal energy ΔU for an ideal gas is formulated as:

$$\Delta U = nC_v\Delta T$$

where n is the number of moles, C_v is the molar heat capacity at constant volume, and ΔT is the change in temperature.

Step 3: Detailed Explanation:

For the internal energy to remain unchanged, the change in internal energy must be zero ($\Delta U = 0$).
Looking at the formula $\Delta U = nC_v\Delta T$, since n and C_v are non-zero constants, $\Delta U = 0$ strictly requires $\Delta T = 0$.
A thermodynamic process where the temperature remains strictly constant ($\Delta T = 0$) is defined as an isothermal process.
Therefore, in an isothermal process, the internal energy of an ideal gas remains unchanged.

Step 4: Final Answer:

The process is an Isothermal process.

Quick Tip: Remember this fundamental rule for ideal gases: Internal Energy (U) $\propto$ Temperature (T). No change in temperature means zero change in internal energy.

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9. A gun fires 25 bullets per second. Each bullet has a mass of 10g and is fired with a velocity of 20m/s. Find the recoil force on gun.

Correct Answer: 5 N

Solution:

Step 1: Understanding the Concept:

When the gun fires bullets, it provides forward momentum to them.

According to Newton's Third Law, the gun experiences an equal and opposite rate of change of momentum, causing a recoil force.

Step 2: Key Formula or Approach:

By Newton's Second Law, the average force is the rate of change of momentum:

$$F = \frac{dp}{dt} = \frac{n \cdot m \cdot v}{t}$$

where n/t is the firing rate (bullets per second), m is the mass of a single bullet, and v is its muzzle velocity.

Step 3: Detailed Explanation:

The rate of firing bullets is $\frac{n}{t} = 25$ bullets/second.

The mass of each bullet is $m = 10\text{ g} = 0.01\text{ kg}$.

The velocity of the bullets is $v = 20\text{ m/s}$.

Substitute these values into the force equation:

$$F = \left(\frac{n}{t}\right) \cdot m \cdot v$$

$$F = 25 \times 0.01 \times 20$$

$$F = 25 \times 0.2$$

$$F = 5\text{ N}$$

Step 4: Final Answer:

The recoil force on the gun is 5 N.

Quick Tip: Force can be easily thought of as the product of mass flow rate and velocity ($F = \dot{m}v$). Here, $\dot{m} = 25$ bullets/s $\times$ 0.01 kg/bullet $= 0.25$ kg/s.

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10. A satellite moves in an elliptical orbit around planet such that its maximum distance and minimum distance from planet are in the ration 3:1. If its speed at the nearest point(perigee)is V, find its speed at the farthest point (Apogee)

Correct Answer: $V/3$

Solution:

Step 1: Understanding the Concept:

A satellite moving in an elliptical orbit is subject solely to a central gravitational force acting towards the planet.

Because central forces exert zero net torque, the angular momentum of the satellite around the planet remains strictly conserved.

Step 2: Key Formula or Approach:

The conservation of angular momentum states that:

$$L_{\text{perigee}} = L_{\text{apogee}}$$

$$mv_p r_p = mv_a r_a$$

where v_p, r_p are the speed and distance at the perigee (nearest point), and v_a, r_a are the speed and distance at the apogee (farthest point).

Step 3: Detailed Explanation:

The ratio of the maximum distance to the minimum distance is given as:

$$\frac{r_a}{r_p} = \frac{3}{1} \implies r_a = 3r_p$$

The speed at the nearest point (perigee) is given as $v_p = V$.

Applying the conservation of angular momentum:

$$mv_pr_p = mv_ar_a$$

Cancel the satellite mass m from both sides:

$$v_pr_p = v_ar_a$$

Substitute $v_p = V$ and $r_a = 3r_p$:

$$V \cdot r_p = v_a \cdot (3r_p)$$

Cancel r_p from both sides:

$$V = 3v_a$$

Solve for the speed at the apogee v_a:

$$v_a = \frac{V}{3}$$

Step 4: Final Answer:

The speed at the farthest point is $V/3$.

Quick Tip: For objects in elliptical orbits, velocity is inversely proportional to the radial distance ($v \propto 1/r$). Therefore, if the distance triples, the speed becomes exactly one-third.

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11. A particle moves such that its position is given by $y = t^2 + 2t + 3(m)$. Find the average acceleration of the particle between $t = 3s$ and $t = 6s$

Correct Answer: $2\ m/s^2$

Solution:

Step 1: Understanding the Concept:

Average acceleration represents the total change in velocity divided by the total time taken for that change.

To compute it, we first need to determine the velocity function by taking the derivative of the position function.

Step 2: Key Formula or Approach:

Velocity v is the derivative of position y with respect to time t:

$$v = \frac{dy}{dt}$$

Average acceleration a_{avg} over an interval is:

$$a_{\text{avg}} = \frac{v(t_f) - v(t_i)}{t_f - t_i}$$

Step 3: Detailed Explanation:

The given position function is:

$$y = t^2 + 2t + 3$$

Differentiate y with respect to t to find the velocity function:

$$v(t) = \frac{d}{dt}(t^2 + 2t + 3) = 2t + 2 \text{ m/s}$$

Calculate the velocity at the initial time $t_i = 3$ s:

$$v(3) = 2(3) + 2 = 6 + 2 = 8 \text{ m/s}$$

Calculate the velocity at the final time $t_f = 6$ s:

$$v(6) = 2(6) + 2 = 12 + 2 = 14 \text{ m/s}$$

Now, compute the average acceleration:

$$a_{\text{avg}} = \frac{v(6) - v(3)}{6 - 3}$$

$$a_{\text{avg}} = \frac{14-8}{3} = \frac{6}{3} = 2 \text{ m/s}^2$$

Notice that since velocity is a linear function of time, the acceleration is constant, meaning instantaneous and average acceleration are identical.

Step 4: Final Answer:

The average acceleration is 2 m/s^2.

Quick Tip: Whenever the position function is a simple quadratic equation $(at^2 + bt + c)$, the acceleration will always be constant and equal to twice the coefficient of the t^2 term $(2a)$.

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12. A = $\frac{B}{CD^2}$, If B, C and D have dimension of inductance reactance, capacitive reactance and angular frequency. then dimension of A

Correct Answer: $[M^0 L^0 T^2]$

Solution:

Step 1: Understanding the Concept:

To evaluate the dimensions of the quantity A, we must substitute the dimensional formulas of B, C, and D into the given relationship.

Inductive reactance and capacitive reactance both measure opposition to alternating current, meaning they share the exact same dimensions as resistance.

Step 2: Key Formula or Approach:

The given formula is:

$$A = \frac{B}{CD^2}$$

We evaluate the dimensions: - B is inductive reactance (X_L), dimension $[R]$ - C is capacitive reactance (X_C), dimension $[R]$ - D is angular frequency (ω), dimension $[T^{-1}]$

Step 3: Detailed Explanation:

Since both B and C represent reactance, their dimensional formulas are identical to that of electrical resistance $[R]$.

Thus, the ratio $\frac{B}{C}$ perfectly cancels out and becomes dimensionless:

$$\left[\frac{B}{C}\right] = \frac{[R]}{[R]} = [M^0L^0T^0]$$

The parameter D represents angular frequency, which has the dimension of inverse time:

$$[D] = [T^{-1}]$$

Now, substitute these dimensions into the expression for A:

$$[A] = \frac{[B]}{[C][D]^2} = \frac{[R]}{[R]\cdot[T^{-1}]^2}$$

$$[A] = \frac{1}{[T^{-2}]}$$

$$[A] = [T^2]$$

Expanding this into the standard MLT format:

$$[A] = [M^0L^0T^2]$$

Step 4: Final Answer:

The dimension of A is $[M^0L^0T^2]$.

Quick Tip: Reactance, resistance, and impedance all share the same physical dimensions. Grouping them simply as $[R]$ helps bypass complex MLTA expansions during dimensional analysis.

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13. A beam of unpolarized light of intensity I_0 is incident on a polarizer. A second polaroid is placed in the path such that its transmission axis makes an angle of 45° with the first polaroid. What is the intensity of light after it passes through the second polaroid.

Correct Answer: $I_0/4$

Solution:

Step 1: Understanding the Concept:

When a beam of unpolarized light passes through an initial polarizer, it becomes perfectly plane-polarized, and its intensity drops by exactly half.

When this newly polarized light passes through a second polarizer (often called an analyzer), its transmission follows Malus's Law.

Step 2: Key Formula or Approach:

Intensity after the first polarizer for unpolarized incident light:

$$I_1 = \frac{I_0}{2}$$

Malus's Law for the intensity after the second polarizer:

$$I_2 = I_1 \cos^2 \theta$$

where θ is the angle between the transmission axes of the two polarizers.

Step 3: Detailed Explanation:

The initial intensity of the unpolarized beam is given as I_0.

After passing through the first polarizer, the intensity becomes:

$$I_1 = \frac{I_0}{2}$$

The second polaroid is placed such that its axis is at an angle of $\theta = 45^\circ$ relative to the first.

Apply Malus's Law to determine the final intensity I_2:

$$I_2 = I_1 \cos^2(45^\circ)$$

Substitute $I_1 = \frac{I_0}{2}$ and the trigonometric value $\cos(45^\circ) = \frac{1}{\sqrt{2}}$:

$$I_2 = \left(\frac{I_0}{2}\right)\left(\frac{1}{\sqrt{2}}\right)^2$$

$$I_2 = \left(\frac{I_0}{2}\right)\left(\frac{1}{2}\right)$$

$$I_2 = \frac{I_0}{4}$$

Step 4: Final Answer:

The intensity of light after passing through the second polaroid is $I_0/4$.

Quick Tip: Always check the wording carefully to see if the incident light is unpolarized or already polarized. If unpolarized, the first polarizer always halves the intensity before Malus's law takes over.

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14. For an electron of mass m and charge 'e' ration of angular momentum to magnetic

Correct Answer: $2m/e$

Solution:

Step 1: Understanding the Concept:

The question asks for the ratio of the orbital angular momentum of a revolving electron to its generated magnetic dipole moment.

This property is derived from the classical model of an electron orbiting a nucleus, which defines the gyromagnetic ratio.

Step 2: Key Formula or Approach:

For an electron orbiting in a circular path, the relationship between the magnetic moment μ and orbital angular momentum L is:

$$\mu = \frac{e}{2m}L$$

where e is the elementary charge and m is the mass of the electron.

Step 3: Detailed Explanation:

The magnetic moment μ of a revolving electron is defined as $\mu = I \cdot A$.

The equivalent current is $I = \frac{e}{T} = \frac{ev}{2\pi r}$, and the area is $A = \pi r^2$.

So, the magnetic moment is:

$$\mu = \left(\frac{ev}{2\pi r}\right)(\pi r^2) = \frac{evr}{2}$$

The orbital angular momentum L of the electron is:

$$L = mvr$$

Dividing μ by L yields the gyromagnetic ratio:

$$\frac{\mu}{L} = \frac{\frac{evr}{2}}{mvr} = \frac{e}{2m}$$

The question specifically asks for the ratio of angular momentum (L) to the magnetic moment (μ).

Therefore, we invert the standard gyromagnetic ratio:

$$\frac{L}{\mu} = \frac{2m}{e}$$

Step 4: Final Answer:

The ratio of angular momentum to magnetic moment is $2m/e$.

Quick Tip: Read ratio questions carefully! The standard gyromagnetic ratio is $\mu/L = e/2m$. The inverse ratio is $L/\mu = 2m/e$. Paying close attention to the requested order is crucial.

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15. Which of the following have highest modulus of elasticity

(A) Steel

(B) Aluminium

(C) Brass

(D) Glass

Correct Answer: (A) Steel

Solution:

Step 1: Understanding the Concept:

The modulus of elasticity, commonly known as Young's modulus, is a metric of a material's inherent stiffness.

A higher modulus indicates that the material resists deformation more strongly and undergoes less strain for a given applied stress.

Step 2: Key Formula or Approach:

Young's modulus Y is defined by the formula:

$$Y = \frac{\text{Stress}}{\text{Strain}}$$

Materials that require immense force to stretch or compress incrementally have a high Y.

Step 3: Detailed Explanation:

Let's evaluate the approximate Young's modulus values for the materials listed in the options:

- Steel: ≈ 200 GPa
- Brass: ≈ 100 GPa
- Aluminium: ≈ 70 GPa
- Glass: ≈ 50 to 90 GPa (depending heavily on composition)

Comparing these values, steel requires significantly more force to produce the exact same amount of longitudinal deformation compared to the others.

Therefore, steel possesses the highest modulus of elasticity among the choices provided.

Step 4: Final Answer:

Steel has the highest modulus of elasticity.

Quick Tip: Steel is remarkably stiffer than most other common engineering metals. It is roughly three times stiffer than aluminium and twice as stiff as brass.

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16. At which condition do the experimental P-V curve and predicted P - V curve closely match?

(A) Low temperature and high pressure

(B) low temperature and low pressure

Correct Answer: High temperature and low pressure

Solution:

Step 1: Understanding the Concept:

The predicted P-V curve is generated assuming ideal gas behavior.

Experimental P-V curves are derived from real gases.

Real gases behave like ideal gases only under specific conditions where the volume of the gas molecules and the intermolecular forces become negligible.

Step 2: Key Formula or Approach:

The ideal gas equation is:

$$PV = nRT$$

This model strictly assumes zero intermolecular attraction and zero individual volume for the gas particles themselves.

Step 3: Detailed Explanation:

Real gases deviate from ideal behavior predominantly because gas molecules possess finite physical volume and exert intermolecular attractive forces upon one another.

At high pressures, a gas is significantly compressed, meaning the volume occupied by the gas molecules themselves is no longer negligible compared to the total container volume.

At low temperatures, the kinetic energy of the molecules drops, which makes the intermolecular attractive forces much more dominant and significant.

Conversely, at high temperatures, the high kinetic energy completely overwhelms intermolecular forces.

At low pressures, the molecules are spaced very far apart, making their individual volumes negligible.

Therefore, the ideal gas equation (predicted curve) closely matches real gas behavior (experimental curve) exclusively at High Temperature and Low Pressure.

Step 4: Final Answer:

The curves match closely at High temperature and low pressure.

Quick Tip: To remember the ideal gas conditions, picture a "free" molecule: it desires a lot of space (low pressure) and a lot of energy to zip past others (high temperature).

17. Which law is the symmetrical counter part of Faradays law in electromagnetic induction

(A) Ampere Maxwell law

(B) Ampere Circuital law

(C) Gauss's law

(D) Coulombs law

Correct Answer: (A) Ampere Maxwell law

Solution:

Step 1: Understanding the Concept:

Maxwell's equations reveal a profound mathematical symmetry between electric and magnetic fields.

Faraday's law dictates that a time-varying magnetic field induces a circulating electric field.

The symmetrical counterpart must describe the reverse: a time-varying electric field inducing a circulating magnetic field.

Step 2: Key Formula or Approach:

Faraday's Law of induction in integral form:

$$\oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi_B}{dt}$$

The Ampere-Maxwell Law in integral form:

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{\text{enc}} + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

Step 3: Detailed Explanation:

Faraday's law shows that a changing magnetic flux ($d\Phi_B/dt$) serves as a source for an electric field.

James Clerk Maxwell theorized that for true symmetry in nature, a changing electric flux ($d\Phi_E/dt$) should identically serve as a source for a magnetic field.

He introduced the displacement current term ($\mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$) into Ampere's original circuital law to correct it for time-varying fields.

This modified equation, known universally as the Ampere-Maxwell law, successfully acts as the symmetrical counterpart to Faraday's law.

Step 4: Final Answer:

The Ampere Maxwell law is the symmetrical counterpart.

Quick Tip: Symmetry in electromagnetism is easy to trace: Changing B-field $\implies$ E-field (Faraday's Law). Changing E-field $\implies$ B-field (Ampere-Maxwell Law, via the displacement current).

---

18. A wire of fixed length is bent into a single circular turn, producing a magnetic field B at is center. If the same wire is bent into 3 circular turns(carrying the same current), What will be the magnetic field at the centre

Correct Answer: $9B$

Solution:

Step 1: Understanding the Concept:

The magnetic field generated at the center of a circular coil depends directly on the number of turns, the current flowing, and inversely on the radius of the coil.

When a wire of fixed length is bent into a higher number of turns, the radius of the coil must proportionately decrease.

Step 2: Key Formula or Approach:

The magnetic field B at the center of a coil with n turns and radius r is given by:

$$B = \frac{\mu_0 n I}{2r}$$

The total fixed length of the wire is dictated by $L = n(2\pi r)$.

Step 3: Detailed Explanation:

Let the total fixed length of the wire be L.

When bent into a single turn ($n_1 = 1$), let its radius be R.

$$L = 2\pi R \implies R = \frac{L}{2\pi}$$

The initial magnetic field is:

$$B = \frac{\mu_0(1)I}{2R}$$

When the exact same wire is bent into 3 turns ($n_2 = 3$), let the new radius be r.

$$L = 3 \times (2\pi r) \implies r = \frac{L}{6\pi} = \frac{R}{3}$$

The new magnetic field B' is:

$$B' = \frac{\mu_0 n_2 I}{2r}$$

Substitute $n_2 = 3$ and $r = \frac{R}{3}$ into the equation:

$$B' = \frac{\mu_0(3)I}{2(R/3)} = \frac{9\mu_0 I}{2R}$$

Since the original field is $B = \frac{\mu_0 I}{2R}$, we find:

$$B' = 9B$$

Step 4: Final Answer:

The new magnetic field at the center will be $9B$.

Quick Tip: If a wire of constant length is bent into an n-turn coil, the magnetic field at its center always increases by a factor of n^2 because the turns scale by n and the radius scales by $1/n$.

---

19. find rms current

$\mathbf{i} = 4\sqrt{2}\sin \mathbf{w} + 3\sqrt{2}\cos \mathbf{wt}$

Correct Answer: 5 A

Solution:

Step 1: Understanding the Concept:

The given equation represents a composite alternating current containing both a sine and a cosine component of the same angular frequency.

Because sine and cosine functions are perfectly orthogonal to each other, the total RMS current is found by taking the square root of the sum of the squares of their individual RMS values.

Step 2: Key Formula or Approach:

For a composite current of the form $i(t) = I_1 \sin(\omega t) + I_2 \cos(\omega t)$, the effective (RMS) value is:

$$I_{\text{rms}} = \sqrt{I_{1\text{rms}}^2 + I_{2\text{rms}}^2}$$

where the individual RMS values are $I_{1\text{rms}} = \frac{I_1}{\sqrt{2}}$ and $I_{2\text{rms}} = \frac{I_2}{\sqrt{2}}$.

Step 3: Detailed Explanation:

The instantaneous current is $i(t) = 4\sqrt{2}\sin(\omega t) + 3\sqrt{2}\cos(\omega t)$ (interpreting 'w' as 'wt').

The peak values of the two orthogonal components are:

$$I_1 = 4\sqrt{2}$$

$$I_2 = 3\sqrt{2}$$

Calculate the RMS value for each component individually:

$$I_{1\text{rms}} = \frac{4\sqrt{2}}{\sqrt{2}} = 4 \text{ A}$$

$$I_{2\text{rms}} = \frac{3\sqrt{2}}{\sqrt{2}} = 3 \text{ A}$$

Because the components are orthogonal, the total combined RMS current is the geometric sum:

$$I_{\text{rms}} = \sqrt{(I_{1\text{rms}})^2 + (I_{2\text{rms}})^2}$$

$$I_{\text{rms}} = \sqrt{4^2 + 3^2}$$

$$I_{\text{rms}} = \sqrt{16+9} = \sqrt{25} = 5 \text{ A}$$

Step 4: Final Answer:

The RMS current is 5 A.

Quick Tip: Alternatively, you can find the resultant peak amplitude first using phasors: $I_{\text{peak}} = \sqrt{(4\sqrt{2})^2 + (3\sqrt{2})^2} = \sqrt{32+18} = \sqrt{50} = 5\sqrt{2}$. Then divide this peak by $\sqrt{2}$ to yield 5.

---

20. Ratio of distance travelled by a freely falling body in successive intervals of time is

Correct Answer: 1 : 3 : 5 : 7...

Solution:

Step 1: Understanding the Concept:

This classic problem demonstrates Galileo's Law of Odd Numbers.

It posits that the sequential distances traversed by a body falling freely from rest in successive, equal intervals of time stand to one another in the same ratio as the odd numbers.

Step 2: Key Formula or Approach:

The total distance fallen from rest in time t under constant acceleration g is:

$$S = \frac{1}{2}gt^2$$

The specific distance traveled strictly in the n-th interval of time is:

$$d_n = S(n) - S(n-1)$$

Step 3: Detailed Explanation:

Let the time intervals be of equal duration, say t_0.

Total distance fallen in the first interval (time t_0):

$$S_1 = \frac{1}{2}g(t_0)^2$$

Total distance fallen by the end of the second interval (time $2t_0$):

$$S_2 = \frac{1}{2}g(2t_0)^2 = 4\left(\frac{1}{2}gt_0^2\right) = 4S_1$$

Distance fallen exclusively within the second interval:

$$d_2 = S_2 - S_1 = 4S_1 - S_1 = 3S_1$$

Total distance fallen by the end of the third interval (time $3t_0$):

$$S_3 = \frac{1}{2}g(3t_0)^2 = 9S_1$$

Distance fallen exclusively within the third interval:

$$d_3 = S_3 - S_2 = 9S_1 - 4S_1 = 5S_1$$

Comparing the distances fallen in the successive intervals (first, second, third, etc.):

$$d_1 : d_2 : d_3 : \cdots = S_1 : 3S_1 : 5S_1 : \ldots$$

$$\text{Ratio} = 1 : 3 : 5 : 7 : \ldots$$

Step 4: Final Answer:

The ratio is 1 : 3 : 5 : 7...

Quick Tip: Galileo's Law of Odd Numbers is valid for any kinematic scenario involving an object starting from rest with a constant acceleration, not exclusively for freely falling bodies.

---

21. A block of 10kg mass moving on a frictionless surface with 5m/s compresses the spring by 5cm and come to rest. What is the force constant of the spring.

Correct Answer: 10^5 N/m

Solution:

Step 1: Understanding the Concept:

When the block compresses the spring and comes to rest, its initial kinetic energy is entirely converted into the elastic potential energy of the spring.

This is a direct application of the law of conservation of mechanical energy.

Step 2: Key Formula or Approach:

The conservation of energy equation is given by equating kinetic energy to elastic potential energy:

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2$$

where m is the mass, v is the velocity, k is the spring constant, and x is the compression distance.

Step 3: Detailed Explanation:

The given mass of the block is $m = 10$ kg.

The initial velocity of the block is $v = 5$ m/s.

The compression of the spring is $x = 5$ cm $= 0.05$ m.

Substitute these values into the energy conservation equation:

$$\frac{1}{2}(10)(5)^2 = \frac{1}{2}k(0.05)^2$$

Cancel the $\frac{1}{2}$ from both sides:

$$10 \times 25 = k \times 0.0025$$

$$250 = k \times 2.5 \times 10^{-3}$$

Solve for the spring constant k:

$$k = \frac{250}{2.5 \times 10^{-3}}$$

$$k = 100 \times 10^3 = 10^5 \text{ N/m}$$

Step 4: Final Answer:

The force constant of the spring is 10^5 N/m.

Quick Tip: Always convert distances from centimeters to meters before applying energy conservation formulas to ensure your final answer is in standard SI units (N/m).

---

22. A uniform rod of mass m and length l rotating in a horizontal circle about a vertical axis passing through one of its ends with angular velocity v. What is the angular momentum of the rod

Correct Answer: $\frac{ml^2v}{3}$

Solution:

Step 1: Understanding the Concept:

The angular momentum of a rigid body rotating about a fixed axis depends on its moment of inertia about that axis and its angular velocity.

Here, the rod is rotating about an axis passing through its end.

Step 2: Key Formula or Approach:

The angular momentum L is given by:

$$L = I\omega$$

where I is the moment of inertia and ω is the angular velocity.

The moment of inertia of a uniform rod of mass m and length l about an axis through its end is:

$$I = \frac{ml^2}{3}$$

Step 3: Detailed Explanation:

The text defines the angular velocity with the symbol v instead of the conventional ω.

Therefore, we substitute $\omega = v$.

Using the moment of inertia for a rod rotating about its end:

$$I = \frac{ml^2}{3}$$

Substitute I and the given angular velocity into the angular momentum formula:

$$L = \left(\frac{ml^2}{3}\right) \times v$$

$$L = \frac{ml^2 v}{3}$$

Step 4: Final Answer:

The angular momentum of the rod is $\frac{ml^2 v}{3}$.

Quick Tip: Pay close attention to the notation used in the question text. Even if 'v' is conventionally used for linear velocity, if the text explicitly states "angular velocity v", you must use it as the angular parameter.

---

23. If threshold wavelengths of two metals are in the 3:1. What is the ratio of their work func-tion.

Correct Answer: 1 : 3

Solution:

Step 1: Understanding the Concept:

The work function of a metal is the minimum energy required to eject an electron from its surface.

It is inversely proportional to the threshold wavelength of the metal.

Step 2: Key Formula or Approach:

The work function Φ is related to the threshold wavelength λ_0 by the equation:

$$\Phi = \frac{hc}{\lambda_0}$$

where h is Planck's constant and c is the speed of light.

Step 3: Detailed Explanation:

From the formula, it is clear that the work function is inversely proportional to the threshold wavelength ($\Phi \propto \frac{1}{\lambda_0}$).

Let the threshold wavelengths of the two metals be λ_{01} and λ_{02}.

The given ratio is:

$$\frac{\lambda_{01}}{\lambda_{02}} = \frac{3}{1}$$

The ratio of their work functions Φ_1 to Φ_2 will be the inverse of their wavelength ratio:

$$\frac{\Phi_1}{\Phi_2} = \frac{\frac{hc}{\lambda_{01}}}{\frac{hc}{\lambda_{02}}} = \frac{\lambda_{02}}{\lambda_{01}}$$

Substitute the known ratio:

$$\frac{\Phi_1}{\Phi_2} = \frac{1}{3}$$

Step 4: Final Answer:

The ratio of their work functions is 1 : 3.

Quick Tip: Remember that energy and wavelength are always inversely related. If the wavelength ratio is $A : B$, the corresponding energy (or work function) ratio will always be $B : A$.

---

24. If the electric potential is V = 3x^2 + 4x, then magnitude of Electric field at x = 1m is

Correct Answer: 10 V/m

Solution:

Step 1: Understanding the Concept:

The electric field is defined as the negative gradient of the electric potential.

When potential is given as a function of position, taking its derivative with respect to position yields the electric field.

Step 2: Key Formula or Approach:

In one dimension, the electric field E is calculated using:

$$E = -\frac{dV}{dx}$$

Step 3: Detailed Explanation:

The given electric potential function is:

$$V(x) = 3x^2 + 4x$$

Differentiate the potential V with respect to x to find the electric field function:

$$E(x) = -\frac{d}{dx}(3x^2 + 4x)$$

$$E(x) = -(6x + 4)$$

We need to evaluate the electric field at the position $x = 1$ m:

$$E(1) = -(6(1) + 4)$$

$$E(1) = -(6 + 4) = -10 \text{ V/m}$$

The question specifically asks for the magnitude of the electric field:

$$|E| = |-10| = 10 \text{ V/m}$$

Step 4: Final Answer:

The magnitude of the electric field is 10 V/m.

Quick Tip: The negative sign in $E = -dV/dx$ indicates that the electric field points in the direction of decreasing potential. Always remember to take the absolute value if only the magnitude is requested.

---

25. Which of the following is not extensive variable.

A) total mass B) internal energy

C) Volume D) Density E) Work done

(A) total mass

(B) internal energy

(C) Volume

(D) Density

(E) Work done

Correct Answer: (D) Density

Solution:

Step 1: Understanding the Concept:

Thermodynamic variables are classified into two types: intensive and extensive.

Extensive variables depend on the size or the amount of matter in the system, whereas intensive variables are independent of the amount of matter.

Step 2: Key Formula or Approach:

If a system is divided into two equal parts, an extensive property is halved, while an intensive property remains the same.

For example, the ratio of two extensive variables always results in an intensive variable (e.g., Mass / Volume = Density).

Step 3: Detailed Explanation:

Let's analyze the given options:

- Total mass: Depends on the amount of matter (Extensive).
- Internal energy: Depends on the total amount of substance (Extensive).
- Volume: Depends on the size of the system (Extensive).
- Work done: Depends on the magnitude of the process/mass involved (Extensive).
- Density: Defined as mass per unit volume. No matter how much of the substance you have, its density remains constant at a given temperature and pressure (Intensive).

Therefore, Density is not an extensive variable.

Step 4: Final Answer:

Density is not an extensive variable.

Quick Tip: A simple test for extensive vs. intensive: Imagine cutting the system in half. If the property's value gets cut in half (like volume or mass), it's extensive. If it stays the same (like temperature or density), it's intensive.

---

26. What is the mass of one molecules of water in kg

Correct Answer: 2.99×10^{-26} kg

Solution:

Step 1: Understanding the Concept:

The molar mass of a substance represents the mass of one mole (Avogadro's number) of its molecules.

To find the mass of a single molecule, we divide the molar mass by Avogadro's number.

Step 2: Key Formula or Approach:

The mass of one molecule m is given by:

$$m = \frac{M}{N_A}$$

where M is the molar mass and N_A is Avogadro's number (6.022×10^{23} mol^{-1}).

Step 3: Detailed Explanation:

The chemical formula for water is H_2O.

The molar mass M of water is approximately 18 g/mol, which is 18×10^{-3} kg/mol.

Avogadro's number is $N_A = 6.022 \times 10^{23}$.

Substitute these values into the formula to find the mass of one molecule in kg:

$$m = \frac{18 \times 10^{-3} \text{ kg}}{6.022 \times 10^{23}}$$

$$m \approx 2.989 \times 10^{-26} \text{ kg}$$

Rounding to appropriate significant figures, we get 2.99×10^{-26} kg.

Step 4: Final Answer:

The mass of one molecule of water is 2.99×10^{-26} kg.

Quick Tip: Always ensure your molar mass is converted to kilograms (by multiplying by 10^{-3}) before dividing by Avogadro's number if the final answer is required in standard SI units (kg).

---

27. A capacitor as capacitance C = 9pF. If the energy stored in it is 18×10^{-8}J. Find charge stored in capacitor.

Correct Answer: 1.8×10^{-9} C (or 1.8 nC)

Solution:

Step 1: Understanding the Concept:

A capacitor stores electrical energy in the electric field created between its plates.

This stored energy is directly related to the capacitance and the total charge stored on the plates.

Step 2: Key Formula or Approach:

The energy U stored in a capacitor can be expressed in terms of charge Q and capacitance C:

$$U = \frac{Q^2}{2C}$$

Rearranging to solve for the charge Q:

$$Q = \sqrt{2CU}$$

Step 3: Detailed Explanation:

The given capacitance is $C = 9\text{ pF} = 9 \times 10^{-12}$ F.

The energy stored is $U = 18 \times 10^{-8}$ J.

Substitute these values into the rearranged energy formula:

$$Q = \sqrt{2 \times (9 \times 10^{-12}) \times (18 \times 10^{-8})}$$

Combine the numerical terms and the powers of 10:

$$Q = \sqrt{18 \times 18 \times 10^{-20}}$$

Taking the square root:

$$Q = 18 \times 10^{-10} \text{ C}$$

This can be rewritten in standard scientific notation as:

$$Q = 1.8 \times 10^{-9} \text{ C} = 1.8 \text{ nC}$$

Step 4: Final Answer:

The charge stored in the capacitor is 1.8×10^{-9} C.

Quick Tip: When dealing with squares and square roots in exponent arithmetic, try to make the exponent an even number before taking the square root to avoid calculation errors.

---

28. The radius of inner most orbit of Hydrogen is 0.53A°. What is the radius of 3rd orbit

Correct Answer: 4.77 Å

Solution:

Step 1: Understanding the Concept:

According to Bohr's model of the hydrogen atom, the radius of an electron's orbit is quantized. The radius scales directly with the square of the principal quantum number (orbit number).

Step 2: Key Formula or Approach:

The radius of the n-th orbit for a hydrogen atom is given by:

$$r_n = r_1 \times n^2$$

where r_1 is the Bohr radius (radius of the innermost orbit) and n is the principal quantum number.

Step 3: Detailed Explanation:

The radius of the innermost orbit (1st orbit) is given as $r_1 = 0.53$ Å.

We need to find the radius of the 3rd orbit, so $n = 3$.

Substitute these values into the formula:

$$r_3 = 0.53 \times (3)^2$$

$$r_3 = 0.53 \times 9$$

Calculate the product:

$$r_3 = 4.77 \text{ Å}$$

Step 4: Final Answer:

The radius of the 3rd orbit is 4.77 Å.

Quick Tip: Remember the proportionality $r \propto n^2/Z$. For Hydrogen ($Z = 1$), finding the radius of any higher orbit simply involves multiplying the ground state radius by a perfect square (4, 9, 16, etc.).

---

29. Order of the electric field required to pull out electrons from a metal by field emission is

Correct Answer: $\sim 10^8$ V/m

Solution:

Step 1: Understanding the Concept:

Field emission (or cold emission) is the process of extracting electrons from a solid surface by subjecting it to a very strong external electric field.

This strong field causes the potential barrier at the surface of the metal to become thin enough for electrons to tunnel through it.

Step 2: Key Formula or Approach:

This is a standard factual concept based on quantum tunneling principles. No direct formula calculation is needed for the order of magnitude.

Step 3: Detailed Explanation:

Under normal conditions, electrons are bound to a metal surface by a potential barrier (work function).

To pull them out without applying thermal energy (thermionic emission) or light energy (photoelectric emission), an exceptionally strong electric field must be applied.

This field strongly bends the potential barrier, allowing quantum tunneling.

Experimental data and quantum mechanical derivations show that the electric field must be on the order of 10^8 V/m to produce significant field emission.

Step 4: Final Answer:

The order of the required electric field is 10^8 V/m.

Quick Tip: Field emission is purely a quantum mechanical tunneling effect and requires fields near the breakdown strength of typical dielectrics, usually around 10^8 V/m.

---

30. Relative viscosity of blood $\frac{\eta}{\eta_{\text{water}}}$ is constant in which temperature range

Correct Answer: 0°C to 37°C

Solution:

Step 1: Understanding the Concept:

Viscosity generally decreases with an increase in temperature for liquids.

However, the relative viscosity (the ratio of the viscosity of a fluid to the viscosity of water at the same temperature) behaves differently.

Step 2: Key Formula or Approach:

Relative viscosity is defined as:

$$\eta_r = \frac{\eta_{\text{blood}}}{\eta_{\text{water}}}$$

Both blood and water experience a decrease in absolute viscosity as temperature rises.

Step 3: Detailed Explanation:
Because blood is primarily composed of water (plasma), the temperature dependence of its viscosity is quite similar to that of pure water across typical biological and ambient ranges.
As temperature changes, both η_{blood} and η_{water} decrease at approximately the same proportional rate.
As a result, their ratio (relative viscosity) remains largely unaffected and constant.
Experimental studies establish that this relative viscosity of blood remains remarkably constant in the temperature range from 0°C up to normal body temperature, 37°C.

Step 4: Final Answer:
The relative viscosity is constant in the range of 0°C to 37°C.

Quick Tip: While absolute viscosity is highly temperature-dependent, relative viscosity compares two similar fluids (blood and water), meaning their temperature dependencies effectively cancel each other out within biological ranges.

---

31. Magnifying power of simple microscope can be increased by using.

Correct Answer: A convex lens of shorter focal length.

Solution:
Step 1: Understanding the Concept:
A simple microscope consists of a single converging (convex) lens.
It forms a virtual, erect, and magnified image of an object placed between its optical center and principal focus.

Step 2: Key Formula or Approach:
The magnifying power M of a simple microscope when the final image is formed at the least distance of distinct vision (D) is given by:

$$M = 1 + \frac{D}{f}$$

where f is the focal length of the lens and D is typically 25 cm.

Step 3: Detailed Explanation:

Looking at the formula $M = 1 + \frac{D}{f}$, the term D is a biological constant for a normal eye.

Therefore, the magnifying power M is inversely proportional to the focal length f.

To maximize the magnifying power M, the fraction $\frac{D}{f}$ must be as large as possible.

This is achieved by minimizing the denominator, which means using a convex lens with a smaller (shorter) focal length.

Step 4: Final Answer:

It can be increased by using a lens of shorter focal length.

Quick Tip: Higher power lenses have shorter focal lengths ($P = 1/f$). Therefore, increasing the optical power of the lens directly increases the magnifying power of the microscope.

---

32. In a Si crystal contaning N atoms at absolute zero, energy state of

Correct Answer: The valence band is completely full and the conduction band is completely empty.

Solution:

Step 1: Understanding the Concept:

Silicon (Si) is an intrinsic semiconductor with an energy band gap of about 1.1 eV between its valence band and conduction band.

At absolute zero (0 K), thermal energy is zero.

Step 2: Key Formula or Approach:

The distribution of electrons across energy states is governed by Fermi-Dirac statistics, but conceptually, at 0 K, electrons occupy the lowest possible energy states available.

Step 3: Detailed Explanation:

In a Silicon crystal containing N atoms, there are $4N$ valence electrons.

The valence band contains exactly enough quantum states to hold these $4N$ electrons.
At absolute zero temperature ($T = 0$ K), there is absolutely no thermal energy available to excite any electron across the band gap.
Therefore, all electrons settle into the lowest energy configurations.
As a result, all $4N$ states in the valence band are completely occupied (full), and there are zero electrons in the higher-energy conduction band (empty).
This makes the pure silicon crystal behave as a perfect insulator at absolute zero.

Step 4: Final Answer:
The valence band is completely full, and the conduction band is completely empty.

Quick Tip: Remember that semiconductors behave as perfect insulators at 0 K because there are no free charge carriers available in the conduction band.

---

33. 2q and q having equal momentum enter uniform magnetic field in a direction perpendicular to magnetic field. Find ratio of radii

Correct Answer: 1 : 2

Solution:
Step 1: Understanding the Concept:
When a charged particle enters a uniform magnetic field perpendicular to its direction of motion, it experiences a magnetic Lorentz force that acts as a centripetal force.
This causes the particle to move in a circular path.

Step 2: Key Formula or Approach:
Equating the magnetic force to the centripetal force:

$$qvB = \frac{mv^2}{r}$$

Rearranging for the radius r:

$$r = \frac{mv}{qB} = \frac{p}{qB}$$

where $p = mv$ is the momentum of the particle.

Step 3: Detailed Explanation:

Let the two particles be designated as 1 and 2, with charges $q_1 = 2q$ and $q_2 = q$.

They have the exact same momentum, so $p_1 = p_2 = p$.

They enter the identical uniform magnetic field, so $B_1 = B_2 = B$.

Using the radius formula $r = \frac{p}{qB}$, it is clear that for constant p and B, the radius is inversely proportional to the charge ($r \propto \frac{1}{q}$).

Now, calculate the ratio of their radii:

$$\frac{r_1}{r_2} = \frac{\frac{p}{q_1 B}}{\frac{p}{q_2 B}} = \frac{q_2}{q_1}$$

Substitute the given charge values:

$$\frac{r_1}{r_2} = \frac{q}{2q} = \frac{1}{2}$$

Step 4: Final Answer:

The ratio of their radii is 1 : 2.

Quick Tip: When momentum is constant, a particle with double the charge experiences double the magnetic force, which bends its path much tighter, resulting in exactly half the radius.

---

34. Instantaneous displacement of a wave is $y = 2\left(\sin pt + \sqrt{3}\cos pt\right)$ cm . Find amplitude of wave in cm.

Correct Answer: 4 cm

Solution:

Step 1: Understanding the Concept:

The given wave equation is a superposition of two simple harmonic motions (a sine and a cosine function) that share the same angular frequency.

These components are orthogonal (phase difference of 90°), so their resultant amplitude is found using vector addition.

Step 2: Key Formula or Approach:

For a wave equation of the form $y = A\sin(\omega t) + B\cos(\omega t)$, the resultant amplitude R is:

$$R = \sqrt{A^2 + B^2}$$

Step 3: Detailed Explanation:

The given wave equation is:

$$y = 2\left(\sin pt + \sqrt{3}\cos pt\right)$$

Expand the equation by distributing the 2:

$$y = 2\sin pt + 2\sqrt{3}\cos pt$$

Comparing this to the standard superposition form $y = A\sin(pt) + B\cos(pt)$: The amplitude of the sine component is $A = 2$ cm.

The amplitude of the cosine component is $B = 2\sqrt{3}$ cm.

Calculate the resultant amplitude R:

$$R = \sqrt{A^2 + B^2}$$

$$R = \sqrt{(2)^2 + (2\sqrt{3})^2}$$

$$R = \sqrt{4 + 4(3)}$$

$$R = \sqrt{4 + 12} = \sqrt{16}$$

$$R = 4 \text{ cm}$$

Step 4: Final Answer:

The amplitude of the wave is 4 cm.

Quick Tip: You can also factor out the resultant directly: $2(\sin pt + \sqrt{3}\cos pt) = 4(\frac{1}{2}\sin pt + \frac{\sqrt{3}}{2}\cos pt) = 4\sin(pt + \pi/3)$, immediately showing the amplitude is 4.

---

35. In an air cure solenoid with L = 0.5mH filled with soft iron of relative permeability 1500. Find new L

Correct Answer: 0.75 H (or 750 mH)

Solution:

Step 1: Understanding the Concept:

The self-inductance of a solenoid depends strongly on the magnetic permeability of the material inside its core.

Introducing a ferromagnetic material like soft iron drastically amplifies the magnetic field and thus the inductance.

Step 2: Key Formula or Approach:

The inductance L of a solenoid filled with a core material is related to its air-core inductance L_0 by:

$$L = \mu_r \cdot L_0$$

where μ_r is the relative permeability of the core material.

Step 3: Detailed Explanation:

The initial inductance of the air-core solenoid is given as $L_0 = 0.5$ mH.

The relative permeability of the soft iron core is $\mu_r = 1500$.

Substitute these values into the formula to find the new inductance:

$$L = 1500 \times 0.5 \text{ mH}$$

$$L = 750 \text{ mH}$$

To convert this to standard units (Henries):

$$L = 750 \times 10^{-3} \text{ H} = 0.75 \text{ H}$$

Step 4: Final Answer:

The new inductance is 0.75 H.

Quick Tip: Permeability acts exactly like a multiplier for magnetic field strength inside a solenoid. If you know the air-core parameter, just multiply it by μ_r to get the new core parameter.

---

36. Bar magnet is rotated from parallel position to 45° position work done is 2.07J. Find workdone to rotate from 45° to antiparallel position.

Correct Answer: ≈ 12.06 J

Solution:

Step 1: Understanding the Concept:

The work done in rotating a magnetic dipole (bar magnet) in a uniform magnetic field is equal to the change in its potential energy.

Step 2: Key Formula or Approach:

The work done W in rotating a magnet from an initial angle θ_1 to a final angle θ_2 is:

$$W = MB(\cos\theta_1 - \cos\theta_2)$$

where M is the magnetic moment and B is the magnetic field strength.

Step 3: Detailed Explanation:

Case 1: Rotating from parallel ($\theta_1 = 0^\circ$) to 45° ($\theta_2 = 45^\circ$).

The work done is $W_1 = 2.07$ J.

$$W_1 = MB(\cos 0^\circ - \cos 45^\circ) = MB\left(1 - \frac{1}{\sqrt{2}}\right)$$

From this, we can solve for the constant term MB:

$$MB = \frac{2.07}{1 - \frac{1}{\sqrt{2}}}$$

Case 2: Rotating from 45° ($\theta_1 = 45°$) to antiparallel ($\theta_2 = 180°$).

The new work done W_2 is:

$$W_2 = MB(\cos 45° - \cos 180°) = MB\left(\frac{1}{\sqrt{2}} - (-1)\right) = MB\left(\frac{1}{\sqrt{2}} + 1\right)$$

Substitute the expression for MB from Case 1 into the equation for W_2:

$$W_2 = 2.07 \times \frac{1 + \frac{1}{\sqrt{2}}}{1 - \frac{1}{\sqrt{2}}} = 2.07 \times \frac{\sqrt{2}+1}{\sqrt{2}-1}$$

Rationalize the fraction:

$$\frac{\sqrt{2}+1}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1} = \frac{(\sqrt{2}+1)^2}{2-1} = (\sqrt{2}+1)^2 = 2 + 1 + 2\sqrt{2} = 3 + 2\sqrt{2}$$

Since $\sqrt{2} \approx 1.414$:

$$3 + 2(1.414) = 3 + 2.828 = 5.828$$

Calculate W_2:

$$W_2 = 2.07 \times 5.828 \approx 12.064 \text{ J}$$

Step 4: Final Answer:

The work done is approximately 12.06 J.

Quick Tip: Whenever evaluating ratios of work done in rotating fields, keep the MB parameter as a grouped constant rather than trying to evaluate M and B separately.

---

37. Real object is placed at focus infront of a convex mirror of focal length f. Find the distance to the image formed.

Correct Answer: $f/2$ (behind the mirror)

Solution:

Step 1: Understanding the Concept:

A convex mirror always forms a virtual, erect, and diminished image behind the mirror, regardless of the object's position.

We use standard Cartesian sign convention where distances measured in the direction of incident light are positive.

Step 2: Key Formula or Approach:

The mirror formula is given by:

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

where v is the image distance, u is the object distance, and f is the focal length.

Step 3: Detailed Explanation:

For a convex mirror, the focal length is positive $(+f)$ because the focus lies behind the mirror.

The real object is placed in front of the mirror at the distance of the focus, so its coordinate is $u=-f$.

Substitute these values into the mirror formula:

$$\frac{1}{v}+\frac{1}{-f}=\frac{1}{f}$$

Rearrange to solve for the image position v:

$$\frac{1}{v}-\frac{1}{f}=\frac{1}{f}$$

$$\frac{1}{v}=\frac{1}{f}+\frac{1}{f}=\frac{2}{f}$$

Invert both sides to find v:

$$v=+\frac{f}{2}$$

The positive sign indicates that the image is formed behind the mirror.

Step 4: Final Answer:

The distance to the image formed is $f/2$.

Quick Tip: Do not confuse convex mirrors with concave mirrors. An object at the focus of a concave mirror forms an image at infinity, but for a convex mirror, it forms strictly at $f/2$.

---

38. Ratio of magnitude of gravitational potential energy to that of kinetic energy of with atellite of mass m

Correct Answer: 2 : 1

Solution:

Step 1: Understanding the Concept:

For a satellite in a stable circular orbit around a planet, its orbital speed is determined by the balance of gravitational force and required centripetal force.

This precise dynamic dictates fixed ratios between its kinetic and potential energies.

Step 2: Key Formula or Approach:

The gravitational potential energy (U) of a satellite of mass m orbiting a planet of mass M at distance r is:

$$U = -\frac{GMm}{r}$$

The kinetic energy (K) of the satellite is:

$$K = \frac{GMm}{2r}$$

Step 3: Detailed Explanation:

The question asks for the ratio of the magnitude of gravitational potential energy to the magnitude of kinetic energy.

The magnitude of the potential energy is:

$$|U| = \left|-\frac{GMm}{r}\right| = \frac{GMm}{r}$$

The kinetic energy is already strictly positive:

$$K = \frac{GMm}{2r}$$

Now, calculate the ratio $|U|/K$:

$$\text{Ratio} = \frac{\frac{GMm}{r}}{\frac{GMm}{2r}}$$

$$\text{Ratio} = \frac{1}{\frac{1}{2}} = 2$$

Therefore, the ratio of their magnitudes is 2 to 1.

Step 4: Final Answer:

The ratio is 2.

Quick Tip: A useful shortcut for orbiting systems governed by inverse-square forces: $|U| = 2K$ and total energy $E = -K$. Memorizing this saves time deriving formulas.

---

39. Uniform metallic wire of radius 'r' and length 'ℓ' is heated by passing constant current. The heat produced can be made 1/4 times if

(A) 2ℓ

(B) $\frac{\ell}{2}, \frac{r}{2}$

(C) $2\ell, \frac{r}{2}$

(D) $2r$

(E) $2\ell, 2r$

Correct Answer: (D) $2r$

Solution:

Step 1: Understanding the Concept:

Joule heating is produced when a current flows through a wire.

The amount of heat generated depends on the resistance of the wire, which is a function of its physical dimensions (length and radius).

Step 2: Key Formula or Approach:

The heat produced H by a constant current I in time t is:

$$H = I^2Rt$$

The resistance R of a wire of resistivity ρ, length ℓ, and radius r is:

$$R = \rho\frac{\ell}{A} = \rho\frac{\ell}{\pi r^2}$$

Thus, $H \propto R \propto \frac{\ell}{r^2}$.

Step 3: Detailed Explanation:

Assuming constant current, to make the heat H become $\frac{1}{4}$ of its original value, the resistance R must become $\frac{1}{4}$ of its original value.

Let's test the options to see which gives $R' = \frac{R}{4}$:

- Option A (2ℓ): $R' \propto \frac{2\ell}{r^2} = 2R$
- Option B ($\ell/2, r/2$): $R' \propto \frac{\ell/2}{(r/2)^2} = \frac{\ell/2}{r^2/4} = 2R$
- Option C ($2\ell, r/2$): $R' \propto \frac{2\ell}{(r/2)^2} = \frac{2\ell}{r^2/4} = 8R$
- Option D ($2r$): $R' \propto \frac{\ell}{(2r)^2} = \frac{\ell}{4r^2} = \frac{1}{4}R$
- Option E ($2\ell, 2r$): $R' \propto \frac{2\ell}{(2r)^2} = \frac{2\ell}{4r^2} = \frac{1}{2}R$

Clearly, making the radius $2r$ perfectly reduces the resistance to a quarter, thereby producing $1/4$ the heat under constant current.

Step 4: Final Answer:

The heat produced is made $1/4$ times if the radius is $2r$.

Quick Tip: Always identify whether "constant current" ($H \propto R$) or "constant voltage" ($H \propto 1/R$) is given. This defines whether you need to increase or decrease resistance to achieve the desired heat output.

---

40. Capillary tubes of radii 1 : 2 are dipped is same solution. Then the ratio of height of liquid risp

Correct Answer: 2 : 1

Solution:

Step 1: Understanding the Concept:

When a capillary tube is dipped in a liquid, the liquid rises (or falls) inside the tube due to surface tension.

Jurin's law dictates that the height of this capillary rise is inversely proportional to the radius of the tube.

Step 2: Key Formula or Approach:

The capillary rise h is given by Jurin's Law:

$$h = \frac{2T\cos\theta}{\rho g r}$$

where T is surface tension, θ is the contact angle, ρ is the liquid density, g is gravity, and r is the tube radius.

Since both tubes are dipped in the same liquid, all parameters except h and r are constant, yielding:

$$h \propto \frac{1}{r}$$

Step 3: Detailed Explanation:

Let the radii of the two capillary tubes be r_1 and r_2, and their corresponding capillary rise heights be h_1 and h_2.

The given ratio of their radii is:

$$\frac{r_1}{r_2} = \frac{1}{2}$$

Because the height is inversely proportional to the radius, the ratio of their heights will be the direct inverse of the ratio of their radii:

$$\frac{h_1}{h_2} = \frac{r_2}{r_1}$$

Substitute the given radius ratio:

$$\frac{h_1}{h_2} = \frac{2}{1}$$

Step 4: Final Answer:

The ratio of the height of liquid rise is 2 : 1.

Quick Tip: Remember the simple rule of thumb for capillarity: The narrower the tube, the higher the liquid rises. A tube half as wide will always draw liquid exactly twice as high.

Chemistry

1. Reagent used to convert decanol to decanoic acid

(A) Tollen's reagent

(B) Jones reagent

(C) Grignard reagent

(D) Fehling's reagent

(E) DIBAC—H

Correct Answer: (B) Jones reagent

Solution:

Step 1: Understanding the Concept:

The oxidation of a primary alcohol to a carboxylic acid requires the use of a strong oxidizing agent. Mild oxidizing agents will stop at the aldehyde stage.

Step 2: Key Formula or Approach:

The approach is to identify the functional group transformation (primary alcohol to carboxylic acid) and select the corresponding strong oxidizing agent from the given options.

Step 3: Detailed Explanation:

Decanol is a primary alcohol with a 10-carbon chain.

To convert it into decanoic acid, complete oxidation must occur.

Jones reagent, which is a mixture of chromium trioxide (CrO_3) in aqueous sulfuric acid and acetone, is a strong oxidizing agent.

It oxidizes primary alcohols first to aldehydes, and then rapidly further to carboxylic acids.

Tollen's reagent and Fehling's reagent are mild oxidizing agents that only oxidize aldehydes to carboxylic acids, but do not oxidize alcohols.

Grignard reagents are used for nucleophilic addition to carbonyls, not for oxidation.

DIBAC—H is a reducing agent, typically used to reduce esters or nitriles to aldehydes.

Therefore, Jones reagent is the correct choice.

Step 4: Final Answer:

The correct reagent is Jones reagent.

Quick Tip: Remember that weak oxidizing agents like PCC or PDC will stop the oxidation of a primary alcohol at the aldehyde stage.

Only strong oxidizing agents like Jones reagent or acidified $KMnO_4$ will take it all the way to a carboxylic acid.

---

2. IUPAC name of $(CH_3)_3C-CH_2Br$

Correct Answer: 1-Bromo-2,2-dimethylpropane

Solution:

Step 1: Understanding the Concept:

To find the IUPAC name, identify the longest carbon chain containing the principal functional group (here, the halogen atom).

Step 2: Key Formula or Approach:

The approach is to expand the condensed structural formula, number the longest parent chain starting from the end closer to the substituent, and name the substituents alphabetically.

Step 3: Detailed Explanation:

The given compound is $(CH_3)_3C-CH_2Br$.

Expanding the structure, we see a central carbon attached to three methyl groups and one $-CH_2Br$ group.

The longest continuous carbon chain that includes the carbon attached to the bromine atom is 3 carbons long (a propane chain).

We number the chain starting from the carbon directly attached to the bromine atom to give it the lowest locant.

Carbon-1 is the $-CH_2Br$ carbon.

Carbon-2 is the central quaternary carbon, which has two remaining methyl groups attached as substituents.

Carbon-3 is one of the terminal methyl groups.

The substituents are one 'bromo' group at C-1 and two 'methyl' groups at C-2.

When writing the name, substituents are listed in alphabetical order.

Thus, 'bromo' comes before 'methyl'.

Combining these parts, the IUPAC name is 1-Bromo-2,2-dimethylpropane.

Step 4: Final Answer:

The IUPAC name is 1-Bromo-2,2-dimethylpropane.

Quick Tip: Always identify the longest continuous carbon chain first.

Do not be confused by the condensed formula; drawing out the expanded structure helps in accurately locating the parent chain and substituents.

---

3. Geometry of a molecule AB_3E_2 with 3 bond pairs and 2 lone pairs

Correct Answer: T-shaped

Solution:

Step 1: Understanding the Concept:

According to the Valence Shell Electron Pair Repulsion (VSEPR) theory, the geometry of a molecule depends on the total number of electron domains around the central atom.

Step 2: Key Formula or Approach:

The approach uses VSEPR rules where Steric Number = (Number of Bond Pairs) + (Number of Lone Pairs). Minimizing lone-pair repulsions defines the final molecular shape.

Step 3: Detailed Explanation:

The molecule has the generic formula AB_3E_2.

It possesses 3 bond pairs (B) and 2 lone pairs (E), making a total of 5 electron domains.

With 5 electron domains, the central atom undergoes sp^3d hybridization.

The basic electron domain geometry for 5 domains is trigonal bipyramidal.

To minimize repulsion (especially lone pair-lone pair and lone pair-bond pair repulsions), the lone pairs occupy the more spacious equatorial positions.

The three bond pairs occupy the remaining two axial positions and one equatorial position.

The resulting molecular geometry, observing only the atoms and ignoring the lone pairs visually, takes the shape of the letter 'T'.

Therefore, the molecular geometry is T-shaped.

Step 4: Final Answer:

The molecular geometry is T-shaped.

Quick Tip: In a trigonal bipyramidal electron geometry (sp^3d), lone pairs always occupy equatorial positions first because the bond angles are 120°, which provides more space and minimizes repulsion compared to axial positions (90°).

---

4. Which donot form carbylamine

(A) Ethanamines

(B) Benzamine

(C) Porp-2-amine

(D) Propan-1-amine

(E) N-methylethanamine

Correct Answer: (E) N-methylethanamine

Solution:

Step 1: Understanding the Concept:

The carbylamine reaction, also known as the isocyanide test, is a chemical test exclusively used to detect the presence of primary (1°) amines.

Step 2: Key Formula or Approach:

The approach is to identify the degree of amine for each option; secondary (2°) and tertiary (3°) amines do not undergo this reaction.

Step 3: Detailed Explanation:

In the carbylamine test, a primary amine is heated with chloroform and alcoholic potassium hydroxide to form a foul-smelling isocyanide (carbylamine).

Let us analyze the given options to determine their amine class.

(A) Ethanamines (e.g., $CH_3CH_2NH_2$) are aliphatic primary amines, so they give the test.

(B) Benzamine (aniline, $C_6H_5NH_2$) is an aromatic primary amine, so it gives the test.

(C) Porp-2-amine ($CH_3CH(NH_2)CH_3$) is an aliphatic primary amine, so it gives the test.

(D) Propan-1-amine ($CH_3CH_2CH_2NH_2$) is an aliphatic primary amine, so it gives the test.

(E) N-methylethanamine ($CH_3CH_2NHCH_3$) is a secondary amine.

Since it is a secondary amine, it does not form carbylamine.

Step 4: Final Answer:

N-methylethanamine does not form carbylamine.

Quick Tip: The Carbylamine test is a reliable qualitative test for both aliphatic and aromatic primary amines.

If you see any prefix like "N-methyl" or "N,N-dimethyl", it indicates a secondary or tertiary amine, which will definitely fail this test.

---

5. Which transition metal has more than one metallic structure at normal temperature?

(A) Cr

(B) Ni

(C) Mn

(D) V

(E) Cu

Correct Answer: (C) Mn

Solution:

Step 1: Understanding the Concept:

Certain transition metals exhibit polymorphism (or allotropy), meaning they can exist in more than one crystalline structure.

Step 2: Key Formula or Approach:

The approach is to recall the allotropic forms of 3d transition metals and identify which one has highly complex or multiple structures.

Step 3: Detailed Explanation:

Manganese (Mn) has a highly stable half-filled $3d^5$ electron configuration.

Due to this stability, it exhibits complex bonding characteristics and exists in four different distinct polymorphic forms (alpha, beta, gamma, and delta).

At normal (room) temperature, it primarily exists in the α-manganese form, which is a highly complex body-centered cubic structure with 58 atoms per unit cell.

Its tendency to form multiple distinct metallic structures sets it apart from typical transition metals like Cr, Ni, V, and Cu, which generally maintain a single standard bcc or fcc structure at normal conditions.

Therefore, Manganese is known for having more than one complex metallic structural form.

Step 4: Final Answer:

Mn has more than one metallic structure.

Quick Tip: Manganese is an anomaly in the 3d series; its $3d^5$ configuration makes its metallic bonding weaker and its crystal lattice highly complex compared to neighboring metals.

---

6. An organic compound $C_5H_{10}O$ does not reduces Tollen's reagent but forms addition compound with sodium hydrogen sulphite and gives the Iodoform test. On vigorous oxidation, it gives ethanoic acid and propanoic acid.

Correct Answer: Pentan-2-one

Solution:

Step 1: Understanding the Concept:

By identifying the functional groups corresponding to specific chemical tests, the exact structure of an unknown compound can be deduced.

Step 2: Key Formula or Approach:

The approach is to interpret each qualitative test: $NaHSO_3$ test for carbonyls, Tollen's test for distinguishing aldehydes from ketones, and the Iodoform test for methyl ketones, then use Popoff's rule for oxidation cleavage.

Step 3: Detailed Explanation:

The given molecular formula is $C_5H_{10}O$, which has a degree of unsaturation of 1, pointing to either a double bond or a ring.

Since the compound forms an addition compound with sodium hydrogen sulphite ($NaHSO_3$), it must contain a carbonyl group (making it an aldehyde or a ketone).

The compound does not reduce Tollen's reagent, which confirms it is a ketone and not an aldehyde.

Furthermore, it gives a positive Iodoform test, indicating the presence of a methyl ketone group (CH_3-CO-).

A 5-carbon ketone containing a methyl ketone group must be Pentan-2-one ($CH_3-CO-CH_2-CH_2-CH_3$).

To verify, let's look at the oxidation products.

Upon vigorous oxidation of ketones, the carbon-carbon bonds break according to Popoff's rule, where the carbonyl group tends to stay with the smaller alkyl fragment.

Oxidative cleavage of Pentan-2-one between C-2 and C-3 gives a 2-carbon fragment and a 3-carbon fragment.

These fragments oxidize to ethanoic acid (CH_3COOH) and propanoic acid (CH_3CH_2COOH). This perfectly matches the given products.

Step 4: Final Answer:

The organic compound is Pentan-2-one.

Quick Tip: Use chemical tests as "clues" to puzzle out organic structures.

$NaHSO_3$ test = Carbonyl group.

Negative Tollen's test = Ketone.

Positive Iodoform test = Terminal methyl group attached to the carbonyl.

Popoff's Rule helps in predicting the cleavage products of unsymmetrical ketones.

---

7. Which are complex reaction

(i) Oxidation of ethane

(ii) Thermal decomposition of HI on gold surface

(iii) Saponification of methyl acetate

(iv) Nitration of phenol

(v) Decomposition of NH_3 on hot Pt surface

Correct Answer: (i) and (iv)

Solution:

Step 1: Understanding the Concept:

In chemical kinetics, reactions that occur in a single step are called elementary reactions. Reactions that occur in a sequence of elementary steps (a mechanism) to give the final products are called complex reactions.

Step 2: Key Formula or Approach:

The approach is to classify reactions based on their known mechanisms; multi-step mechanisms involving intermediates are complex.

Step 3: Detailed Explanation:

According to standard Chemical Kinetics theory, complex reactions involve multiple intermediate steps or yield mixed side products.

(i) Oxidation of ethane: It passes through a series of intermediate steps (forming alcohols, aldehydes, and acids) before final conversion to CO_2 and H_2O.

This is a classic complex chain reaction.

(ii) Thermal decomposition of HI on a gold surface: This is a classic example of a zero-order reaction on a solid catalyst surface, often treated as elementary at high concentrations.

(iii) Saponification of methyl acetate: This is a bimolecular second-order elementary reaction in basic kinetics discussions.

(iv) Nitration of phenol: It occurs via electrophilic aromatic substitution, involving intermediate sigma complexes, and yields a mixture of ortho and para products.

This multi-step pathway makes it a typical complex reaction.

(v) Decomposition of NH_3 on hot Pt surface: This is another zero-order elementary-like catalytic surface reaction.

Therefore, based on standard textbook classifications, (i) and (iv) represent complex consecutive and parallel reaction mechanisms respectively.

Step 4: Final Answer:

(i) Oxidation of ethane and (iv) Nitration of phenol are complex reactions.

Quick Tip: Reactions such as complete combustion, or those that generate multiple intermediate side products like nitration, are always complex because they cannot happen via a single simultaneous molecular collision.

---

8. Conc. of H ions in HCl solution is 3×10^{-3}M then pH = ——-?

Correct Answer: 2.52

Solution:

Step 1: Understanding the Concept:

The pH of a solution is a measure of its acidity, mathematically defined as the negative base-10 logarithm of the hydrogen ion concentration.

Step 2: Key Formula or Approach:

The formula used is:

$$\text{pH} = -\log_{10}[\text{H}^+]$$

Step 3: Detailed Explanation:

We are given that the concentration of hydrogen ions $[\text{H}^+]$ is 3×10^{-3} M.

Substitute the given value directly into the pH formula:

$$\text{pH} = -\log_{10}(3 \times 10^{-3})$$

Using the logarithmic property $\log(a \times b) = \log(a) + \log(b)$:

$$\text{pH} = -(\log_{10}(3) + \log_{10}(10^{-3}))$$

We know that $\log_{10}(10^{-3}) = -3$ and the standard accepted value for $\log_{10}(3) \approx 0.4771$.

$$\text{pH} = -(0.4771 - 3)$$

$$\text{pH} = 3 - 0.4771$$

$$\text{pH} = 2.5229$$

Rounding to two decimal places, the calculated pH is 2.52.

Step 4: Final Answer:

The pH of the solution is 2.52.

Quick Tip: A quick way to estimate pH for an ion concentration of $A \times 10^{-B}$ is to use the formula $\text{pH} = B - \log(A)$.

Memorizing basic log values like $\log(2) \approx 0.30$ and $\log(3) \approx 0.48$ will save you calculation time during the exam.

---

9. Mass of ethanoic acid required to prepare 0.5m solution containing 100g of water?

Correct Answer: 3 g

Solution:

Step 1: Understanding the Concept:

Molality (m) is a concentration term defined as the number of moles of solute dissolved per kilogram of the solvent.

Step 2: Key Formula or Approach:

The relevant formulas are:

$$\text{Molality } (m) = \frac{\text{Moles of solute } (n)}{\text{Mass of solvent in kg}}$$

$$\text{Moles of solute } (n) = \frac{\text{Mass } (W)}{\text{Molar Mass } (M)}$$

Step 3: Detailed Explanation:

The solvent here is water, and its mass is given as 100 g.

First, convert this mass into kilograms:

$$\text{Mass of solvent} = \frac{100}{1000} = 0.1 \text{ kg}$$

The target molality (m) of the solution is given as 0.5 m (mol/kg).

Now, calculate the required number of moles (n) of the solute (ethanoic acid):

$$0.5 = \frac{n}{0.1}$$

$$n = 0.5 \times 0.1 = 0.05 \text{ moles}$$

Next, calculate the molar mass of ethanoic acid (CH_3COOH).

Using the atomic masses: Carbon (C) = 12, Hydrogen (H) = 1, Oxygen (O) = 16.

$$M = 12 + (3 \times 1) + 12 + (2 \times 16) + 1$$

$$M = 12 + 3 + 12 + 32 + 1 = 60 \text{ g/mol}$$

Now, calculate the mass (W) of ethanoic acid corresponding to 0.05 moles:

$$W = n \times M$$

$$W = 0.05 \text{ mol} \times 60 \text{ g/mol}$$

$$W = 3 \text{ g}$$

Step 4: Final Answer:

The mass of ethanoic acid required is 3 g.

Quick Tip: Always double-check that your solvent mass is converted to kilograms when working with molality equations.

A common mistake is using grams directly, which offsets the final answer by a factor of 1000.

---

10. Spin only magnetic moement given not correct is

(A) Ni^{2+} (4.73)

(B) Fe^{2+} (4.90)

(C) Ti^{2+} (2.84)

(D) CO^{2+} (3.89)

(E) Mg^{2+} (5.92)

Correct Answer: (A) Ni^{2+} (4.73)

Solution:

Step 1: Understanding the Concept:

The spin-only magnetic moment (μ) is dependent on the number of unpaired electrons (n) present in the outermost d-orbitals of the transition metal ion.

Step 2: Key Formula or Approach:

The formula for spin-only magnetic moment is:

$$\mu = \sqrt{n(n+2)} \text{ B.M.}$$

Step 3: Detailed Explanation:

Let's evaluate the number of unpaired electrons and the theoretical magnetic moment for each given ion.

For option (A), Ni^{2+}:

The electronic configuration of Ni is $[Ar]4s^2 3d^8$.

The Ni^{2+} ion has an outer configuration of $3d^8$.

According to Hund's rule, filling 8 electrons in 5 d-orbitals leaves 2 unpaired electrons ($n = 2$).

$$\mu = \sqrt{2(2+2)} = \sqrt{8} \approx 2.83 \text{ B.M.}$$

The value given in the option is 4.73, which is drastically incorrect.

For option (B), Fe^{2+}:

Configuration is $3d^6$, which has 4 unpaired electrons ($n = 4$).

$$\mu = \sqrt{4(4+2)} = \sqrt{24} \approx 4.90 \text{ B.M.}$$

(This is correct).

For option (C), Ti^{2+}:

Configuration is $3d^2$, which has 2 unpaired electrons ($n = 2$).

$$\mu = \sqrt{2(4)} = \sqrt{8} \approx 2.83 \text{ B.M.}$$

(This is correct).

For option (D), CO^{2+} (Cobalt ion Co^{2+}):

Configuration is $3d^7$, which has 3 unpaired electrons ($n = 3$).

$$\mu = \sqrt{3(5)} = \sqrt{15} \approx 3.87 \text{ B.M.}$$

(This is approximately 3.89, so it is considered correct).

For option (E), the text says Mg^{2+} (5.92). This is a clear typographical error in the paper for Mn^{2+} (Manganese).

Mn^{2+} has a $3d^5$ configuration with 5 unpaired electrons ($n = 5$).

$$\mu = \sqrt{5(7)} = \sqrt{35} \approx 5.92 \text{ B.M.}$$

Assuming the intended ion was Mn^{2+}, this value is correct.

Thus, the unequivocally incorrect value matched to the wrong ion is in option (A).

Step 4: Final Answer:

The given magnetic moment for Ni^{2+} is not correct.

Quick Tip: To quickly estimate the spin-only magnetic moment, note that the value is always slightly greater than the number of unpaired electrons (e.g., $n = 2$ gives ~ 2.8, $n = 3$ gives ~ 3.9). Since 4.73 starts with a 4, it corresponds to an ion with 4 unpaired electrons, not 2.

---

11. Minimum energy required to remove an atom from sodium is 3.313×10^{-19}g. Maximum wavelength of radiation that will get photoelectron

Correct Answer: 600 nm

Solution:

Step 1: Understanding the Concept:

The minimum energy required to remove an electron from a metal surface is called its work function (Φ).

The maximum wavelength (threshold wavelength, λ_0) capable of ejecting a photoelectron corresponds exactly to this minimum energy.

Step 2: Key Formula or Approach:

The energy of a photon relates to its wavelength by the Planck-Einstein relation:

$$E = \frac{hc}{\lambda}$$

Where h is Planck's constant (6.626×10^{-34} J s) and c is the speed of light (3×10^8 m/s).

Note: The unit in the question is mistakenly typed as "g.", but it implies Joules (J).

Step 3: Detailed Explanation:

Given the work function (minimum energy) $E = 3.313 \times 10^{-19}$ J.

We rearrange the formula to solve for the maximum wavelength λ:

$$\lambda = \frac{hc}{E}$$

Substitute the known constants into the equation:

$$\lambda = \frac{6.626 \times 10^{-34} \text{ J s} \times 3 \times 10^8 \text{ m/s}}{3.313 \times 10^{-19} \text{ J}}$$

First, multiply the numerator:

$$hc \approx 19.878 \times 10^{-26} \text{ J m}$$

Now, divide by the energy:

$$\lambda = \frac{19.878 \times 10^{-26}}{3.313 \times 10^{-19}}$$

Notice that $\frac{19.878}{3.313} \approx 6.00$.

$$\lambda = 6.00 \times 10^{-7} \text{ m}$$

To convert meters to nanometers ($1 \text{ nm} = 10^{-9}$ m):

$$\lambda = 600 \times 10^{-9} \text{ m} = 600 \text{ nm}$$

Step 4: Final Answer:

The maximum wavelength of radiation is 600 nm.

Quick Tip: To save calculation time in exams, memorize the value of $hc \approx 1240\ \text{eV} \cdot \text{nm}$ or $hc \approx 19.89 \times 10^{-26}\ \text{J} \cdot \text{m}$.

Also, observe that 3.313 is exactly half of 6.626, making the division trivial without a calculator!

---

12. Find the emf of the reaction at 298K

$\mathbf{Ni_{(s)} + 2Ag^{+}_{2\times10^{-3}M} \rightarrow Ni^{+2}_{4\times10^{-2}M} + 2Ag_{(s)}}$

$\mathbf{(E^{0}_{cell} = 1.5}$ **at 298K)**

Correct Answer: 1.38 V

Solution:

Step 1: Understanding the Concept:

The electromotive force (EMF) of a cell under non-standard conditions is determined using the Nernst equation, which correlates the cell potential with the standard potential and the reaction quotient.

Step 2: Key Formula or Approach:

The Nernst equation at 298 K is:

$$E_{\text{cell}} = E^{0}_{\text{cell}} - \frac{0.0591}{n} \log_{10} Q$$

Where n is the number of moles of electrons transferred, and Q is the reaction quotient.

Step 3: Detailed Explanation:

From the given cell reaction:

$$\text{Ni}_{(s)} + 2\text{Ag}^{+}_{(aq)} \rightarrow \text{Ni}^{+2}_{(aq)} + 2\text{Ag}_{(s)}$$

The number of electrons transferred $n = 2$.

The reaction quotient Q is given by:

$$Q = \frac{[\text{Ni}^{+2}]}{[\text{Ag}^{+}]^2}$$

Substitute the given concentrations into the expression for Q:

$$Q = \frac{4 \times 10^{-2}}{(2 \times 10^{-3})^2}$$

Calculate the denominator:

$$(2 \times 10^{-3})^2 = 4 \times 10^{-6}$$

Now, calculate Q:

$$Q = \frac{4 \times 10^{-2}}{4 \times 10^{-6}} = 10^4$$

Now, substitute $E^0_{\text{cell}} = 1.5$ V, $n = 2$, and $Q = 10^4$ into the Nernst equation:

$$E_{\text{cell}} = 1.5 - \frac{0.0591}{2} \log_{10}(10^4)$$

Since $\log_{10}(10^4) = 4$:

$$E_{\text{cell}} = 1.5 - \frac{0.0591}{2} \times 4$$

$$E_{\text{cell}} = 1.5 - (0.0591 \times 2)$$

$$E_{\text{cell}} = 1.5 - 0.1182 = 1.3818 \text{ V}$$

Rounding to two decimal places, we get 1.38 V.

Step 4: Final Answer:

The emf of the cell is 1.38 V.

Quick Tip: Always remember to square the concentration of the silver ion $[Ag^+]$ in the reaction quotient Q, as its stoichiometric coefficient in the balanced equation is 2. Missing this power is the most common error in Nernst equation problems.

---

13. $X_2 + O_2 \rightleftharpoons 2XO$

Concenttration of X_2 and O_2 are 4×10^{-3} and 3×10^{-3} respectively. Equilibrium concentration of XO (Kc = 0.5)

Correct Answer: 2.45×10^{-3} M

Solution:

Step 1: Understanding the Concept:

The equilibrium constant K_c for a reversible reaction is defined as the ratio of the product of the equilibrium concentrations of the products to the product of the equilibrium concentrations of the reactants, each raised to the power of their stoichiometric coefficients.

Step 2: Key Formula or Approach:

For the reaction $X_2 + O_2 \rightleftharpoons 2XO$, the equilibrium expression is:

$$K_c = \frac{[XO]^2}{[X_2][O_2]}$$

Step 3: Detailed Explanation:

We are given:

$K_c = 0.5$

$[X_2] = 4 \times 10^{-3}$ M

$[O_2] = 3 \times 10^{-3}$ M

We need to find the concentration of XO, let's denote it as [XO].

Substitute the given values into the K_c expression:

$$0.5 = \frac{[XO]^2}{(4 \times 10^{-3}) \times (3 \times 10^{-3})}$$

Calculate the denominator:

$$(4 \times 10^{-3}) \times (3 \times 10^{-3}) = 12 \times 10^{-6}$$

Now the equation becomes:

$$0.5 = \frac{[XO]^2}{12 \times 10^{-6}}$$

Multiply both sides by 12×10^{-6}:

$$[XO]^2 = 0.5 \times 12 \times 10^{-6}$$

$$[XO]^2 = 6 \times 10^{-6}$$

Take the square root of both sides to find [XO]:

$$[XO] = \sqrt{6 \times 10^{-6}}$$

$$[XO] = \sqrt{6} \times 10^{-3}$$

We know that $\sqrt{4} = 2$ and $\sqrt{9} = 3$, so $\sqrt{6}$ is approximately 2.45.

$$[XO] \approx 2.45 \times 10^{-3} \text{ M}$$

Step 4: Final Answer:

The equilibrium concentration of XO is 2.45×10^{-3} M.

Quick Tip: When dealing with equilibrium constant expressions, pay strict attention to the stoichiometric coefficients. The coefficient '2' for XO means its concentration must be squared in the numerator.

---

14. Which pais have ability to form p$\pi - p\pi$ multiple bonds

(A) C and O

(B) B and N

(C) N and P

(D) F and Cl

(E) C and Si

Correct Answer: (A) C and O

Solution:

Step 1: Understanding the Concept:

The ability of elements to form strong $p\pi - p\pi$ multiple bonds (like double or triple bonds) depends heavily on their atomic size.

Small atomic size allows for effective lateral overlap of the p-orbitals.

Step 2: Key Formula or Approach:

The approach is to identify elements belonging to the 2nd period of the periodic table, as they possess small sizes and compact $2p$ orbitals, making $p\pi - p\pi$ overlap highly effective.

Step 3: Detailed Explanation:

Let's analyze the given pairs:

(A) C and O: Both are 2nd-period elements with small atomic radii. They effectively overlap their $2p$ orbitals to form stable $p\pi - p\pi$ double bonds, as seen in molecules like CO_2, ketones, and aldehydes.

(B) B and N: Although both are 2nd period, typical multiple bonding is less dominant compared to Carbon and Oxygen, though they do form $p\pi - p\pi$ in specific aromatic-like rings (e.g., borazine). However, C and O is the classic and most prolific pair for this.

(C) N and P: Phosphorus is a 3rd-period element. Its $3p$ orbitals are larger and more diffuse, resulting in weak $p\pi - p\pi$ overlap.

(D) F and Cl: Halogens typically form single bonds. Chlorine is a 3rd-period element and does not favor $p\pi - p\pi$ bonding.

(E) C and Si: Silicon is a 3rd-period element. Its larger size makes $p\pi - p\pi$ overlap with carbon very weak (which is why CO_2 is a gas with double bonds, while SiO_2 forms a solid network of single bonds).

Thus, Carbon and Oxygen are the best and most universally recognized pair for forming robust $p\pi - p\pi$ multiple bonds.

Step 4: Final Answer:

C and O have the ability to form $p\pi - p\pi$ multiple bonds.

Quick Tip: Elements of the second period (C, N, O) uniquely form stable $p\pi - p\pi$ multiple bonds. Elements in the third period and below generally avoid $p\pi - p\pi$ bonding due to poor lateral overlap of their larger, more diffuse orbitals.

---

15. Pyridinium chlorochromate is a complex of ....

Correct Answer: Chromium trioxide (CrO_3), pyridine, and HCl

Solution:

Step 1: Understanding the Concept:

Pyridinium chlorochromate (PCC) is a common, mild oxidizing agent used extensively in organic chemistry to convert primary alcohols to aldehydes without over-oxidizing them to carboxylic acids.

Step 2: Key Formula or Approach:

The approach is to recall the preparation and chemical composition of the PCC reagent based on its name and standard textbook definitions.

Step 3: Detailed Explanation:

The name "Pyridinium chlorochromate" hints directly at its components.

It is synthesized by dissolving chromium trioxide (CrO_3) in hydrochloric acid (HCl), and then adding pyridine (C_5H_5N).

The reaction forms a complex salt with the formula $[C_5H_5NH]^+[CrO_3Cl]^-$.

The components of this complex are therefore: Pyridine (C_5H_5N), Hydrogen chloride (HCl), and Chromium trioxide (CrO_3).

Step 4: Final Answer:

It is a complex of Pyridine, CrO_3, and HCl.

Quick Tip: To remember the composition of PCC, just break down the name: "Pyridinium" comes from Pyridine + HCl, and "chlorochromate" comes from CrO_3 + HCl.

---

16. In Chemotherapy, liqannd used to remove excess of Cu

Correct Answer: D-Penicillamine

Solution:

Step 1: Understanding the Concept:

Chelation therapy is a medical procedure that involves the administration of chelating agents to remove heavy metals from the body.

Step 2: Key Formula or Approach:

The approach is to identify the specific chelating ligand structurally and medically suited to bind and safely excrete copper ions from the human body.

Step 3: Detailed Explanation:

Wilson's disease is a genetic disorder where excess copper accumulates in the body tissues, particularly in the liver and brain.

To treat this copper toxicity, specific chelating ligands are administered.

D-Penicillamine is a well-known therapeutic chelating agent (ligand) that binds tightly to copper ions, forming a stable, water-soluble complex.

This complex is then safely filtered by the kidneys and excreted in the urine.

Other chelators like EDTA are primarily used for lead poisoning, while Desferrioxamine is used for iron overload.

For copper specifically, D-Penicillamine is the standard choice.

Step 4: Final Answer:

D-Penicillamine is the ligand used to remove excess Cu.

Quick Tip: Questions linking coordination chemistry to medicine are common. Remember these classic matches:

Copper toxicity $\longrightarrow$ D-Penicillamine.

Lead toxicity $\longrightarrow$ EDTA.

Iron toxicity $\longrightarrow$ Desferrioxamine.

Anti-cancer $\longrightarrow$ Cisplatin.

---

17. IUPAC name of

$\mathbf{CH_3 - CH(OH) - CH_2 - CH(OH) - CH(C_2H_5)}$

|

$\mathbf{CH_3 - CH_2}$

Correct Answer: 5-Ethylheptane-2,4-diol

Solution:

Step 1: Understanding the Concept:

To name the organic compound properly, we must determine the longest continuous carbon chain containing the principal functional groups (hydroxyl groups, $-OH$) and assign the lowest possible numbers to these groups.

Step 2: Key Formula or Approach:

The approach involves expanding the condensed formula, tracing the longest chain containing both $-OH$ groups, numbering it to give the $-OH$ groups the lowest locants, and naming the substituents alphabetically.

Step 3: Detailed Explanation:

Let's analyze the structure given:

It is a continuous chain from the left: $CH_3 - CH(OH) - CH_2 - CH(OH) - CH$ ...

Attached to this last CH carbon is a (C_2H_5) group.

The vertical line | pointing down to $CH_3 - CH_2$ indicates that there is another ethyl group attached to the same CH carbon.

Starting from the left (methyl end), we trace through the carbons:

C-1: CH_3

C-2: CH(OH)

C-3: CH_2

C-4: CH(OH)

C-5: CH

From C-5, we have two identical ethyl groups ($-CH_2CH_3$). One of these groups will form part of the main parent chain, making it C-6 and C-7.

The parent chain is therefore 7 carbons long (Heptane).

The remaining ethyl group on C-5 acts as a substituent.

Now, let's establish the numbering direction.

Numbering from left to right gives the principal functional groups (the two $-OH$ groups) positions 2 and 4.

Numbering from right to left would give the $-OH$ groups positions 4 and 6.

The lowest locant set is (2,4), so we number from left to right.

Substituent: An "ethyl" group at position 5.

Principal functional groups: Two hydroxyl groups "diol" at positions 2 and 4.

Combining these parts: 5-ethyl + heptane + 2,4-diol = 5-Ethylheptane-2,4-diol.

Step 4: Final Answer:

The IUPAC name is 5-Ethylheptane-2,4-diol.

Quick Tip: Always be careful with terminal C_2H_5 or C_3H_7 groups in condensed structures. They often hide the true longest chain. Expanding them explicitly helps avoid selecting a shorter parent chain.

---

18. Which are carcinogenic hydrocarbon

(i) 1,2 - Benzanthracene

(ii) pent-1-yne

(iii) 1,2 - Benpyrene

(iv) cyclohexane

(v) 3-methyl cholanthrene

Correct Answer: (i), (iii) and (v)

Solution:

Step 1: Understanding the Concept:

Carcinogenic hydrocarbons are toxic compounds capable of causing cancer in living tissues. They are typically Polynuclear Aromatic Hydrocarbons (PAHs) containing multiple fused benzene rings.

Step 2: Key Formula or Approach:

The approach is to identify which of the given compounds are complex, fused-ring aromatic structures known for their toxicity and carcinogenicity.

Step 3: Detailed Explanation:

Polynuclear aromatic hydrocarbons containing more than two benzene rings fused together are generally toxic and possess cancer-producing (carcinogenic) properties. They are formed during the incomplete combustion of organic materials like tobacco, coal, and petroleum.

Let's evaluate the options:

(i) 1,2-Benzanthracene: This is a well-known PAH consisting of four fused aromatic rings. It is highly carcinogenic.

(ii) Pent-1-yne: This is a simple aliphatic alkyne. It is not carcinogenic.

(iii) 1,2-Benpyrene (Benzopyrene): This is a five-ring PAH, famously found in coal tar and cigarette smoke, and is one of the most potent known carcinogens.

(iv) Cyclohexane: This is a simple non-aromatic cyclic alkane. It is relatively non-toxic and not carcinogenic.

(v) 3-methyl cholanthrene: This is a highly potent carcinogenic PAH widely used in laboratory research to induce tumors in study models.

Therefore, the carcinogenic hydrocarbons are (i), (iii), and (v).

Step 4: Final Answer:

(i) 1,2-Benzanthracene, (iii) 1,2-Benpyrene, and (v) 3-methyl cholanthrene are carcinogenic.

Quick Tip: As a general rule in organic environmental chemistry, hydrocarbons with 3 or more fused aromatic rings (PAHs) should immediately be flagged as potentially carcinogenic and toxic.

---

19. Metals used in preparation of dihydrogen in lab

Correct Answer: Granulated Zinc (Zn)

Solution:

Step 1: Understanding the Concept:

In the laboratory, dihydrogen (H_2) gas is typically prepared by reacting an active metal with a dilute mineral acid or an aqueous alkali.

Step 2: Key Formula or Approach:

The approach is to identify the standard metal utilized in school and research laboratories for safely and efficiently generating hydrogen gas.

Step 3: Detailed Explanation:

The most common and standard method for preparing dihydrogen in the laboratory involves the reaction of granulated zinc with dilute hydrochloric acid (HCl) or dilute sulfuric acid (H_2SO_4).

The chemical reaction is:

$$Zn_{(s)} + 2H^+_{(aq)} \rightarrow Zn^{2+}_{(aq)} + H_{2(g)}$$

Granulated zinc is preferred over pure block zinc because the impurities present in it act as a local catalyst (forming micro-galvanic cells) which speeds up the reaction.

Alternatively, zinc can also react with aqueous alkali (like NaOH) to produce hydrogen and

sodium zincate, though acid is more commonly used.
Highly reactive metals like Sodium or Potassium are not used in the lab because their reaction with acids or water is violently explosive.

Step 4: Final Answer:

Granulated Zinc is the metal used.

Quick Tip: Always specify "granulated" zinc rather than just "zinc", as the granular form provides a larger surface area and its inherent impurities facilitate a steady, manageable rate of hydrogen evolution.

---

20. Volume ofmethanol to make 2L of 0.4M solution
$(\textbf{density} = 0.64\textbf{KgL}^{-1},\ \textbf{MM} = 32\ \textbf{g mol}^{-1})$

Correct Answer: 40 mL (or 0.04 L)

Solution:

Step 1: Understanding the Concept:

To find the required volume of a pure liquid solute to prepare a solution, we must first calculate the required mass of the solute using the molarity equation, and then convert that mass to volume using the given density.

Step 2: Key Formula or Approach:

The required formulas are:

$$\text{Moles of solute } (n) = \text{Molarity } (M) \times \text{Volume of solution in L } (V)$$

$$\text{Mass } (W) = n \times \text{Molar Mass (MM)}$$

$$\text{Volume of pure liquid} = \frac{\text{Mass}}{\text{Density}}$$

Step 3: Detailed Explanation:

Given:

Molarity $(M) = 0.4$ M (mol/L)

Volume of solution $(V) = 2$ L

Molar Mass (MM) $= 32$ g/mol

Density $= 0.64$ Kg/L $= 640$ g/L (since 1 Kg $= 1000$ g).

First, calculate the required number of moles (n) of methanol:

$$n = M \times V = 0.4 \text{ mol/L} \times 2 \text{ L} = 0.8 \text{ moles}$$

Next, calculate the mass (W) of methanol required:

$$W = n \times \text{MM} = 0.8 \text{ moles} \times 32 \text{ g/mol} = 25.6 \text{ g}$$

Now, use the density to find the volume of pure methanol needed:

$$\text{Volume} = \frac{\text{Mass}}{\text{Density}}$$

$$\text{Volume} = \frac{25.6 \text{ g}}{640 \text{ g/L}}$$

$$\text{Volume} = 0.04 \text{ L}$$

Convert the volume into milliliters (mL):

$$0.04 \text{ L} \times 1000 \text{ mL/L} = 40 \text{ mL}$$

Step 4: Final Answer:

The required volume of methanol is 40 mL.

Quick Tip: Always ensure unit consistency. The density is given in Kg/L, but molar mass is usually in g/mol. Convert density to g/L (0.64 Kg/L $= 640$ g/L) before dividing the mass in grams to prevent major magnitude errors.

---

21. Enthalpy of combustion of benzene, graphite, dihydrogen are -3260, -390 and -290 kJ/mol. Find the enthalpy of formation of benzene

Correct Answer: 50 kJ/mol

Solution:

Step 1: Understanding the Concept:

Hess's Law states that the total enthalpy change for a chemical reaction is independent of the pathway. The enthalpy of formation can be calculated using the enthalpies of combustion of the reactants and products.

Step 2: Key Formula or Approach:

The mathematical relationship is:

$$\Delta H_f^\circ(\text{compound}) = \sum \Delta H_c^\circ(\text{reactants}) - \sum \Delta H_c^\circ(\text{products})$$

Alternatively, construct the formation reaction and use the constituent combustion equations.

Step 3: Detailed Explanation:

The target reaction is the formation of benzene (C_6H_6) from its standard state elements:

$$6C_{(graphite)} + 3H_{2(g)} \rightarrow C_6H_{6(l)}$$

We are given the standard enthalpies of combustion (ΔH_c):

1) $\Delta H_c(C_{graphite}) = -390$ kJ/mol

2) $\Delta H_c(H_2) = -290$ kJ/mol

3) $\Delta H_c(C_6H_6) = -3260$ kJ/mol

Using the formula derived from Hess's Law for formation from combustion data:

$$\Delta H_f(C_6H_6) = [6 \times \Delta H_c(C) + 3 \times \Delta H_c(H_2)] - [\Delta H_c(C_6H_6)]$$

Substitute the given values into the equation:

$$\Delta H_f(C_6H_6) = [6(-390) + 3(-290)] - (-3260)$$

Perform the multiplications:

$$6 \times (-390) = -2340 \text{ kJ}$$

$$3 \times (-290) = -870 \text{ kJ}$$

Add the reactant combustion enthalpies:

$$-2340 + (-870) = -3210 \text{ kJ}$$

Now, subtract the combustion enthalpy of the product (benzene):

$$\Delta H_f(C_6H_6) = -3210 - (-3260)$$

$$\Delta H_f(C_6H_6) = -3210 + 3260$$

$$\Delta H_f(C_6H_6) = +50 \text{ kJ/mol}$$

Step 4: Final Answer:

The enthalpy of formation of benzene is +50 kJ/mol.

Quick Tip: A common trap is forgetting to multiply the combustion enthalpies of the elements by their stoichiometric coefficients from the balanced formation equation (6 for Carbon, 3 for Hydrogen). Always write out the balanced formation equation first!

Maths

1. If $9\mathbf{P}_5 = 504(6\mathbf{P}_r)$**. Find** λ

Correct Answer: 2

Solution:

Step 1: Understanding the Concept:

The problem requires us to find the unknown value r (written as λ in the question text) in a permutations equation. We evaluate the numerical permutation first to simplify the equation.

Step 2: Key Formula or Approach:

The formula for permutations of n distinct objects taken r at a time is $nP_r = \frac{n!}{(n-r)!}$.

Step 3: Detailed Explanation:

Given the equation:

$$9P_5 = 504 \times (6P_r)$$

Calculate the value of $9P_5$:

$$9P_5 = 9 \times 8 \times 7 \times 6 \times 5 = 15120$$

Substitute this back into the original equation:

$$15120 = 504 \times (6P_r)$$

Solving for $6P_r$:

$$6P_r = \frac{15120}{504} = 30$$

We know that $6P_r$ represents the product of r consecutive integers starting downwards from 6.

$$6 \times 5 = 30$$

Since this consists of exactly 2 terms, we have $6P_2 = 30$.

Step 4: Final Answer:

Comparing $6P_r$ with $6P_2$, we obtain $r = 2$ (or $\lambda = 2$).

Quick Tip: Instead of expanding the factorials algebraically, computing the numerical value of known permutations directly is much faster for simple integer constants.

---

2. If $\vec{a} = 2\hat{i} - 2\hat{j} + 4\hat{k}$ $\vec{b} = -5\hat{i} - \hat{j} + 8\hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j} - \lambda\hat{k}$. If $\vec{a} + \vec{b} + \vec{c}$ is perpendicular to $\vec{a} - \vec{b} + \vec{c}$ Find λ

Correct Answer: 12 or -4

Solution:

Step 1: Understanding the Concept:
We are given three vectors. A condition states that the sum of these vectors and a specific difference of these vectors are orthogonal (perpendicular).

Step 2: Key Formula or Approach:
Two non-zero vectors $\vec{u}$ and $\vec{v}$ are perpendicular if and only if their dot product equals zero: $\vec{u} \cdot \vec{v} = 0$.

Step 3: Detailed Explanation:
Let $\vec{u} = \vec{a} + \vec{b} + \vec{c}$.

$$\vec{u} = (2-5+3)\hat{i} + (-2-1+1)\hat{j} + (4+8-\lambda)\hat{k} = 0\hat{i} - 2\hat{j} + (12-\lambda)\hat{k}$$

Let $\vec{v} = \vec{a} - \vec{b} + \vec{c}$.

$$\vec{v} = (2-(-5)+3)\hat{i} + (-2-(-1)+1)\hat{j} + (4-8-\lambda)\hat{k} = 10\hat{i} + 0\hat{j} + (-4-\lambda)\hat{k}$$

Since $\vec{u}$ and $\vec{v}$ are perpendicular, $\vec{u} \cdot \vec{v} = 0$:

$$(0)(10) + (-2)(0) + (12-\lambda)(-4-\lambda) = 0$$

$$(12-\lambda)(-4-\lambda) = 0$$

Factoring out a negative sign:

$$-(12-\lambda)(4+\lambda) = 0$$

This results in two values for λ.

Step 4: Final Answer:
Setting each factor to zero yields $\lambda = 12$ or $\lambda = -4$.

Quick Tip: Always simplify components neatly before taking the dot product. Writing missing components as zeros $(0\hat{i}, 0\hat{j})$ prevents misalignment errors.

---

3. Find $\begin{vmatrix} 11 & 1 & 1 \\ 1 & 21 & 1 \\ 1 & 1 & 31 \end{vmatrix}$

Correct Answer: 7100

Solution:

Step 1: Understanding the Concept:

The question requires evaluating a 3×3 numerical determinant. Given the small values, it can be expanded directly.

Step 2: Key Formula or Approach:

The determinant of a 3×3 matrix is calculated by expanding along any row or column. Expanding along the first row:

$$|A| = a(ei - fh) - b(di - fg) + c(dh - eg)$$

Step 3: Detailed Explanation:

Let the determinant be Δ.

$$\Delta = \begin{vmatrix} 11 & 1 & 1 \\ 1 & 21 & 1 \\ 1 & 1 & 31 \end{vmatrix}$$

Expand along the first row:

$$\Delta = 11(21 \times 31 - 1 \times 1) - 1(1 \times 31 - 1 \times 1) + 1(1 \times 1 - 21 \times 1)$$

Evaluate terms:

$$\Delta = 11(651 - 1) - 1(31 - 1) + 1(1 - 21)$$

$$\Delta = 11(650) - 1(30) + 1(-20)$$

$$\Delta = 7150 - 30 - 20$$

$$\Delta = 7150 - 50 = 7100$$

Step 4: Final Answer:

The value of the determinant is 7100.

Quick Tip: When a matrix has many 1s, direct expansion is often faster than using row operations, provided the diagonal multiplications are straightforward.

---

4. Find the eqn of the line passing through $(-1, 2, -4)$ and parallel to $\frac{-x-1}{4} = \frac{2y+1}{-1} = \frac{-z+4}{3}$

Correct Answer: $\frac{x+1}{8} = \frac{y-2}{1} = \frac{z+4}{6}$

Solution:

Step 1: Understanding the Concept:

To construct the equation of a line, we need a given point and its direction ratios. A parallel line will share the same or proportional direction ratios.

Step 2: Key Formula or Approach:

The standard symmetric equation of a line through (x_1, y_1, z_1) with direction ratios a, b, c is:

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

Step 3: Detailed Explanation:

The given parallel line equation is:

$$\frac{-x-1}{4} = \frac{2y+1}{-1} = \frac{-z+4}{3}$$

Convert it to the standard form by making the coefficients of x, y, z positive 1. For x, multiply numerator and denominator by -1: $\frac{x+1}{-4}$. For y, divide numerator and denominator by 2: $\frac{y+1/2}{-1/2}$. For z, multiply numerator and denominator by -1: $\frac{z-4}{-3}$. The standard form is:

$$\frac{x+1}{-4} = \frac{y+1/2}{-1/2} = \frac{z-4}{-3}$$

The direction ratios are proportional to $(-4, -1/2, -3)$. Multiplying by -2 to clear fractions gives $(8, 1, 6)$. The required line passes through $(-1, 2, -4)$ with direction ratios $(8, 1, 6)$. Substituting into the line equation format:

$$\frac{x-(-1)}{8} = \frac{y-2}{1} = \frac{z-(-4)}{6}$$

$$\frac{x+1}{8} = \frac{y-2}{1} = \frac{z+4}{6}$$

Step 4: Final Answer:

The equation of the line is $\frac{x+1}{8} = \frac{y-2}{1} = \frac{z+4}{6}$.

Quick Tip: Always ensure the coefficients of x, y, z are exactly $+1$ before identifying the direction ratios from the denominators.

---

5. Find the minimum value of $f(x) = \frac{x^{100}-1}{x^{100}+1}$

Correct Answer: -1

Solution:

Step 1: Understanding the Concept:

We need to find the absolute minimum value of a rational function that involves a high even power of x.

Step 2: Key Formula or Approach:

For any real number x, an even power yields a non-negative result, meaning $x^{100} \geq 0$. Algebraic manipulation simplifies finding the extreme values.

Step 3: Detailed Explanation:

Let $y = x^{100}$. Since x is real, $y \geq 0$. Rewrite the function in terms of y:

$$f(x) = \frac{y-1}{y+1}$$

We can rewrite this fraction by splitting the numerator:

$$f(x) = \frac{y+1-2}{y+1} = 1 - \frac{2}{y+1}$$

To find the minimum value of $f(x)$, we need to maximize the subtracted term, $\frac{2}{y+1}$. A fraction with a constant positive numerator is maximized when its positive denominator is minimized. The minimum value for the denominator $y+1$ occurs when y is at its minimum. Since $y \geq 0$, the minimum is $y = 0$. Substitute $y = 0$:

$$f_{min} = 1 - \frac{2}{0+1} = 1 - 2 = -1$$

Step 4: Final Answer:

The minimum value of the function is -1.

Quick Tip: When encountering functions of the form $\frac{g(x)-c}{g(x)+c}$, splitting the numerator is a fast way to determine boundaries based on the domain of $g(x)$.

6. If $y = \frac{3x^3 - 2x^2 + x}{|x|}$, $x \neq 0$ find $\frac{dy}{dx}$ at $x = -2$

Correct Answer: 14

Solution:

Step 1: Understanding the Concept:

We are required to differentiate a function containing an absolute value at a specific negative x-coordinate. We must resolve the absolute value definition before differentiating.

Step 2: Key Formula or Approach:

The definition of the absolute value function is $|x| = x$ for $x > 0$ and $|x| = -x$ for $x < 0$. We use the power rule $\frac{d}{dx}(x^n) = nx^{n-1}$ for differentiation.

Step 3: Detailed Explanation:

We need to calculate the derivative at $x = -2$. Since $-2 < 0$, in the neighborhood of this point, $|x| = -x$. Substitute this into the given function:

$$y = \frac{3x^3 - 2x^2 + x}{-x}$$

Since $x \neq 0$, we divide each term by $-x$:

$$y = -3x^2 + 2x - 1$$

Now, differentiate with respect to x:

$$\frac{dy}{dx} = -6x + 2$$

Substitute $x = -2$ into the derivative:

$$\left.\frac{dy}{dx}\right|_{x=-2} = -6(-2) + 2$$

$$\left.\frac{dy}{dx}\right|_{x=-2} = 12 + 2 = 14$$

Step 4: Final Answer:

The value of the derivative at $x = -2$ is 14.

Quick Tip: Always apply the piecewise definition of an absolute value function to remove the modulus signs locally before attempting to compute a derivative.

---

7. $\int \frac{\sin(\cot^{-1} x)}{1+x^2}$

Correct Answer: $\frac{x}{\sqrt{1+x^2}} + C$

Solution:

Step 1: Understanding the Concept:

The integral involves an inverse trigonometric function nested inside a standard trigonometric function. Integration by substitution is ideal since the derivative of the inner function is present in the integrand.

Step 2: Key Formula or Approach:

Use the substitution $t = \cot^{-1} x$, knowing that its derivative is $\frac{dt}{dx} = -\frac{1}{1+x^2}$. The standard integral is $\int \sin(t)dt = -\cos(t) + C$.

Step 3: Detailed Explanation:

Let $t = \cot^{-1} x$. Differentiating both sides with respect to x:

$$dt = -\frac{1}{1+x^2}dx \implies -dt = \frac{dx}{1+x^2}$$

Substitute these into the integral (assuming dx is implied in the problem text):

$$\int \sin(\cot^{-1} x)\frac{dx}{1+x^2} = \int \sin(t)(-dt) = -\int \sin(t)dt$$

Compute the integral:

$$= -(-\cos(t)) + C = \cos(t) + C$$

Now substitute back $t = \cot^{-1} x$:

$$= \cos(\cot^{-1} x) + C$$

To express this algebraically, let $\theta = \cot^{-1} x$, so $\cot\theta = \frac{x}{1}$. In a right triangle with angle θ, the adjacent side is x and opposite side is 1. The hypotenuse is $\sqrt{x^2+1^2} = \sqrt{1+x^2}$. Then $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{x}{\sqrt{1+x^2}}$. Substituting this back gives the final expression.

Step 4: Final Answer:

The evaluated integral is $\frac{x}{\sqrt{1+x^2}} + C$.

Quick Tip: Drawing a quick right-angled triangle mapping out "opposite, adjacent, hypotenuse" is the safest way to convert nested inverse trig expressions into algebraic ones.

---

8. Find the number of terms in $2, 6, 18 \ldots 1458$

Correct Answer: 7

Solution:

Step 1: Understanding the Concept:

We are given a sequence of numbers. We first verify its type by analyzing the ratio or difference between consecutive terms to apply the correct sequence formula.

Step 2: Key Formula or Approach:

For a Geometric Progression (GP), the n-th term is given by $T_n = ar^{n-1}$, where a is the first term and r is the common ratio.

Step 3: Detailed Explanation:

The given sequence is $2, 6, 18, \ldots, 1458$. Calculate the ratio of consecutive terms:

$$\frac{6}{2} = 3 \quad \text{and} \quad \frac{18}{6} = 3$$

Since the ratio is constant, it is a GP with $a = 2$ and $r = 3$. Let the total number of terms be n, so the last term is $T_n = 1458$. Using the n-th term formula:

$$1458 = 2 \cdot 3^{n-1}$$

Divide both sides by 2:

$$729 = 3^{n-1}$$

We know that $3^4 = 81$, $3^5 = 243$, and $3^6 = 729$. So, rewriting the equation gives:

$$3^6 = 3^{n-1}$$

Equating the exponents:

$$n - 1 = 6 \implies n = 7$$

Step 4: Final Answer:

The number of terms in the sequence is 7.

Quick Tip: Committing the first six powers of 2, 3, 4, and 5 to memory can significantly save time and avoid manual repeated multiplication.

---

9. Find the domain of $\frac{\log(x-5)}{x^2+3x-4}$

Correct Answer: $(5, \infty)$

Solution:

Step 1: Understanding the Concept:

The domain of a real-valued function is the set of x-values that yield valid real outputs. We must satisfy restrictions for both the logarithm function and the rational denominator.

Step 2: Key Formula or Approach:

Two mathematical conditions must be simultaneously satisfied: 1. The argument of the logarithm must be strictly positive: $g(x) > 0$. 2. The denominator cannot be zero: $h(x) \neq 0$.

Step 3: Detailed Explanation:

Apply the first condition for the numerator:

$$x - 5 > 0 \implies x > 5$$

Apply the second condition for the denominator:

$$x^2 + 3x - 4 \neq 0$$

Factorize the quadratic polynomial:

$$(x + 4)(x - 1) \neq 0$$

This indicates that $x \neq -4$ and $x \neq 1$. The overall domain is the intersection of these two conditions. We need $x > 5$ AND $x \notin \{-4, 1\}$. Since all numbers greater than 5 inherently exclude -4 and 1, the condition $x > 5$ alone is sufficient to satisfy all constraints. Thus, the domain is all real numbers strictly greater than 5.

Step 4: Final Answer:

The domain of the function is $(5, \infty)$.

Quick Tip: Always list out every individual domain constraint (logs, roots, denominators) before taking the intersection to ensure no restrictive conditions are missed.

10. $\lim_{x\to 0}\left[\frac{\sin^2 x}{1-\cos x}\right]$

Correct Answer: 2

Solution:

Step 1: Understanding the Concept:

We need to evaluate a trigonometric limit as $x \to 0$. Direct substitution gives a $\frac{0}{0}$ indeterminate form, so algebraic manipulation is needed using identities.

Step 2: Key Formula or Approach:

We use the fundamental Pythagorean trigonometric identity $\sin^2 x = 1 - \cos^2 x$, and the algebraic difference of squares formula $a^2 - b^2 = (a-b)(a+b)$.

Step 3: Detailed Explanation:

Substitute $x = 0$ to check the limit form:

$$\frac{\sin^2 0}{1-\cos 0} = \frac{0}{1-1} = \frac{0}{0}$$

Rewrite $\sin^2 x$ in the numerator using the identity:

$$\lim_{x\to 0}\frac{1-\cos^2 x}{1-\cos x}$$

Factor the numerator as a difference of squares:

$$1-\cos^2 x = (1-\cos x)(1+\cos x)$$

Substitute back into the expression:

$$\lim_{x\to 0}\frac{(1-\cos x)(1+\cos x)}{1-\cos x}$$

As $x \to 0$, $\cos x \to 1$ but for $x \neq 0$ near the limit, $\cos x \neq 1$. Thus, we can divide out the non-zero common factor $(1-\cos x)$:

$$\lim_{x\to 0}(1+\cos x)$$

Perform direct substitution:

$$1+\cos(0)=1+1=2$$

Step 4: Final Answer:

The evaluated limit is 2.

Quick Tip: For limits involving $1-\cos x$ in the denominator, replacing $\sin^2 x$ with $1-\cos^2 x$ is almost always the cleanest path compared to applying L'Hôpital's Rule.

---

11. If $\tan\alpha=\frac{5}{6}$, $\tan\beta=\frac{1}{11}\left(0<\alpha,\beta<\frac{\pi}{2}\right)$

Correct Answer: $\alpha+\beta=\frac{\pi}{4}$

Solution:

Step 1: Understanding the Concept:

We are given the tangent values of two acute angles. The logical implied goal (despite the missing explicit instruction in the text) is to find the sum of the angles, $\alpha+\beta$, using the tangent addition formula.

Step 2: Key Formula or Approach:

The compound angle formula for tangent is:

$$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$

Step 3: Detailed Explanation:

Substitute the given values $\tan\alpha = \frac{5}{6}$ and $\tan\beta = \frac{1}{11}$ into the formula:

$$\tan(\alpha+\beta) = \frac{\frac{5}{6}+\frac{1}{11}}{1-\left(\frac{5}{6}\right)\left(\frac{1}{11}\right)}$$

To simplify the numerator and denominator, find a common denominator of 66: Numerator: $\frac{5\times11+1\times6}{66} = \frac{55+6}{66} = \frac{61}{66}$ Denominator: $1-\frac{5}{66} = \frac{66-5}{66} = \frac{61}{66}$ Divide the simplified fractions:

$$\tan(\alpha+\beta) = \frac{\frac{61}{66}}{\frac{61}{66}} = 1$$

Since $0 < \alpha, \beta < \frac{\pi}{2}$, their sum $\alpha+\beta$ must be strictly within $(0,\pi)$. The unique angle within this range whose tangent is 1 is $\frac{\pi}{4}$.

$$\alpha+\beta = \frac{\pi}{4}$$

Step 4: Final Answer:

The value of $\alpha+\beta$ is $\frac{\pi}{4}$.

Quick Tip: When combining fractions in compound angle formulas, keeping the unreduced common denominator helps clear the fractions cleanly in the final step.

---

12. Find the sum of all 3 digit numbers using the digit 1, 2, 3, 4 without repeatation

Correct Answer: 6660

Solution:

Step 1: Understanding the Concept:

We are tasked with forming 3-digit numbers from 4 distinct non-zero digits without repetition and finding their total sum. We do this by calculating the sum of digits in each place value column.

Step 2: Key Formula or Approach:

The total number of numbers formed by taking r digits from n distinct digits is ${}^nP_r$. Each digit appears exactly $\frac{{}^nP_r}{n}$ times in any given place (units, tens, hundreds).

Step 3: Detailed Explanation:

The total number of 3-digit numbers is:

$$^4P_3 = 4 \times 3 \times 2 = 24 \text{ numbers}$$

Since all 4 digits are used equally without bias, each digit will appear in the units place exactly $\frac{24}{4} = 6$ times. The sum of the digits in the units place is:

$$S_{\text{units}} = 6 \times (1 + 2 + 3 + 4) = 6 \times 10 = 60$$

Similarly, each digit appears 6 times in the tens place and 6 times in the hundreds place. Thus, the column sums are: Sum of hundreds place $= 60$ Sum of tens place $= 60$ Sum of units place $= 60$ The total sum is computed by factoring in place values:

$$\text{Total Sum} = 60 \times 100 + 60 \times 10 + 60 \times 1$$

$$\text{Total Sum} = 60(100 + 10 + 1) = 60(111) = 6660$$

Step 4: Final Answer:

The sum of all such numbers is 6660.

Quick Tip: For r-digit numbers made from n distinct non-zero digits, the sum formula is: $\frac{{}^nP_r}{n} \times$ (Sum of given digits) $\times$ (111...r times).

---

13. $\int_0^1 \left[\tan^{-1}\left(\frac{1}{1+x+x^2+x^3}\right) + \tan^{-1}(1 + x + x^2 + x^3)\right] dx$

Correct Answer: $\frac{\pi}{2}$

Solution:

Step 1: Understanding the Concept:

The definite integral involves a sum of two inverse tangent functions where the arguments are reciprocals of each other.

Step 2: Key Formula or Approach:

An important identity for inverse trigonometric functions is:

$$\tan^{-1}(y) + \tan^{-1}\left(\frac{1}{y}\right) = \frac{\pi}{2} \quad \text{for } y > 0$$

Step 3: Detailed Explanation:

Let $y(x) = 1 + x + x^2 + x^3$. We must evaluate the integral over the interval $x \in [0, 1]$. For any x in $[0, 1]$, all terms in $y(x)$ are non-negative, meaning $y(x) \geq 1$, which strictly satisfies the condition $y > 0$. The integrand is:

$$I(x) = \tan^{-1}\left(\frac{1}{y(x)}\right) + \tan^{-1}(y(x))$$

Using the identity from Step 2, this entire expression evaluates to a constant:

$$I(x) = \frac{\pi}{2}$$

Substitute this back into the integral:

$$\int_0^1 \frac{\pi}{2}\, dx$$

Factor out the constant:

$$\frac{\pi}{2} \int_0^1 1\, dx = \frac{\pi}{2}[x]_0^1 = \frac{\pi}{2}(1 - 0) = \frac{\pi}{2}$$

Step 4: Final Answer:

The value of the integral is $\frac{\pi}{2}$.

Quick Tip: Always check if the argument inside $\tan^{-1}$ is positive before applying $\tan^{-1}(x) + \tan^{-1}(1/x) = \pi/2$, since the sum is $-\pi/2$ if $x < 0$.

---

14. Find the differential equation of $y = Ae^x + Be^{-2x}$

Correct Answer: $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 0$

Solution:

Step 1: Understanding the Concept:

We are given a general solution of a linear differential equation with two arbitrary constants A and B. Because there are two constants, the corresponding differential equation will be a second-order linear homogeneous equation.

Step 2: Key Formula or Approach:

For a solution of the form $y = c_1e^{m_1x} + c_2e^{m_2x}$, the values m_1 and m_2 are roots of the characteristic (auxiliary) quadratic equation. Constructing the quadratic equation $(m-m_1)(m-m_2) = 0$ yields the differential equation structure.

Step 3: Detailed Explanation:

From the given expression $y = Ae^x + Be^{-2x}$, the coefficients in the exponential powers are $m_1 = 1$ and $m_2 = -2$. The characteristic equation having these roots is:

$$(m-1)(m-(-2)) = 0$$

$$(m-1)(m+2) = 0$$

Expand the equation:

$$m^2 + 2m - m - 2 = 0$$

$$m^2 + m - 2 = 0$$

To form the corresponding linear homogeneous differential equation with constant coefficients, we replace m^2 with y'' (or $\frac{d^2y}{dx^2}$), m with y' (or $\frac{dy}{dx}$), and the constant term with y:

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 0$$

Step 4: Final Answer:

The differential equation is $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 0$.

Quick Tip: Using the characteristic equation method skips the tedious algebraic elimination of constants A and B via repeated differentiation.

---

15. Solve $5 < |x - 1| < 15$

Correct Answer: $(-14, -4) \cup (6, 16)$

Solution:

Step 1: Understanding the Concept:

We have a compound absolute value inequality. This represents the set of points x whose distance from 1 on the number line is strictly between 5 and 15.

Step 2: Key Formula or Approach:

The inequality $a < |y| < b$ splits into two separate intervals corresponding to the positive and negative definitions of absolute value: $a < y < b$ OR $-b < y < -a$.

Step 3: Detailed Explanation:

Let $y = x - 1$. The inequality is $5 < |y| < 15$. This splits into two cases: Case 1 (Positive branch):

$$5 < x - 1 < 15$$

Add 1 to all parts of the inequality:

$$5 + 1 < x < 15 + 1 \implies 6 < x < 16$$

This yields the interval $(6, 16)$.

Case 2 (Negative branch):

$$-15 < x - 1 < -5$$

Add 1 to all parts:

$$-15 + 1 < x < -5 + 1 \implies -14 < x < -4$$

This yields the interval $(-14, -4)$. The final solution is the union of the two intervals representing both cases.

Step 4: Final Answer:

The solution set is $(-14, -4) \cup (6, 16)$.

Quick Tip: Visualizing absolute value as "distance from the center point" (here, distance from 1 is between 5 and 15) allows you to just add/subtract the bounds from the center.

---

16. Find the value of $\sin\left(2\sin^{-1}\frac{3}{5}\right)$

Correct Answer: 24/25

Solution:

Step 1: Understanding the Concept:

The problem requires evaluating a composite function involving sine and double an inverse sine. We can simplify this by setting the inverse sine equal to an angle variable.

Step 2: Key Formula or Approach:

Use the double-angle identity for sine: $\sin(2\theta) = 2\sin\theta\cos\theta$. Use Pythagorean relations to find $\cos\theta$ when $\sin\theta$ is known.

Step 3: Detailed Explanation:

Let $\theta = \sin^{-1}\left(\frac{3}{5}\right)$. This implies $\sin\theta = \frac{3}{5}$, and since $\frac{3}{5} > 0$, θ is in the first quadrant where all trigonometric functions are positive. Using the Pythagorean identity $\sin^2\theta + \cos^2\theta = 1$:

$$\left(\frac{3}{5}\right)^2 + \cos^2\theta = 1$$

$$\frac{9}{25} + \cos^2\theta = 1 \implies \cos^2\theta = 1 - \frac{9}{25} = \frac{16}{25}$$

$$\cos\theta = \frac{4}{5}$$

We are asked to evaluate $\sin(2\theta)$. Apply the double angle formula:

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

Substitute the calculated values:

$$\sin(2\theta) = 2\left(\frac{3}{5}\right)\left(\frac{4}{5}\right) = \frac{24}{25}$$

Step 4: Final Answer:

The value of the expression is $\frac{24}{25}$.

Quick Tip: Recognizing Pythagorean triples like (3, 4, 5) immediately gives you the cosine value as 4/5 without manual squaring and square-rooting.

---

17. If $f(x) = x^2 + 4x + 4; x \leq -2$. Find $f^{-1}(x)$

Correct Answer: $-2-\sqrt{x}$

Solution:

Step 1: Understanding the Concept:

We are given a quadratic function restricted to a specific domain to make it one-to-one (bijective). Finding the inverse involves solving for x in terms of y and handling signs correctly according to the domain restriction.

Step 2: Key Formula or Approach:

To find an inverse function $f^{-1}(y)$: 1. Let $y=f(x)$. 2. Complete the square to solve for x. 3. Choose the appropriate sign for the square root based on the domain of x.

Step 3: Detailed Explanation:

Let $y=x^2+4x+4$. Notice that the right side is a perfect square trinomial:

$$y=(x+2)^2$$

Take the square root of both sides. This introduces a $\pm$ symbol:

$$\sqrt{y}=|x+2| \implies x+2=\pm\sqrt{y}$$

We are given the domain restriction $x\leq -2$. This means $x+2\leq 0$. Therefore, the quantity $(x+2)$ is negative or zero. To reflect this, we must choose the negative root branch:

$$x+2=-\sqrt{y}$$

Solve for x:

$$x=-2-\sqrt{y}$$

To express it as a function of x, replace y with x:

$$f^{-1}(x)=-2-\sqrt{x}$$

Step 4: Final Answer:

The inverse function is $f^{-1}(x) = -2 - \sqrt{x}$.

Quick Tip: Always analyze the domain of the original function to decide whether to take the positive or negative branch when taking the square root. The domain of f becomes the range of f^{-1}.

---

18. Find the length of latus rectum of $y^2 + 8x + 4y + 12 = 0$

Correct Answer: 8

Solution:

Step 1: Understanding the Concept:

The equation represents a parabola. To find its defining properties, we must rewrite the general second-degree equation into the standard vertex form.

Step 2: Key Formula or Approach:

The standard forms of a horizontal parabola are $(y-k)^2 = 4a(x-h)$ or $(y-k)^2 = -4a(x-h)$. The length of the latus rectum for any parabola is the absolute value of the coefficient of the non-squared linear term, which is $4a$.

Step 3: Detailed Explanation:

The given equation is:

$$y^2 + 8x + 4y + 12 = 0$$

Keep terms containing y on one side and move the rest to the other side:

$$y^2 + 4y = -8x - 12$$

Complete the square on the left side by adding $(\frac{4}{2})^2 = 4$ to both sides:

$$y^2 + 4y + 4 = -8x - 12 + 4$$

$$(y+2)^2 = -8x - 8$$

Factor out the coefficient of x on the right side:

$$(y+2)^2 = -8(x+1)$$

This is in the standard form $(y-k)^2 = -4a(x-h)$, which opens to the left. Comparing this with the standard equation, the multiplier of the linear term $(x-h)$ is $-4a = -8$. The length of the latus rectum is the absolute value of this coefficient:

$$\text{Length} = |-8| = 8$$

Step 4: Final Answer:

The length of the latus rectum is 8.

Quick Tip: To quickly find the latus rectum of an equation $y^2+Ax+By+C=0$, it's simply the absolute value of the coefficient of x (which is $|A|$).

---

19. Find $\sin^{-1}\left(\sin\frac{5\pi}{9}\cdot\cos\frac{\pi}{9}+\sin\frac{\pi}{9}\cdot\cos\frac{5\pi}{9}\right)$

Correct Answer: $\pi/3$

Solution:

Step 1: Understanding the Concept:

The expression inside the inverse sine is an expanded trigonometric identity. After condensing it, we evaluate the inverse trigonometric function by ensuring the angle is within the principal branch.

Step 2: Key Formula or Approach:

Use the compound angle formula for sine: $\sin(A+B) = \sin A \cos B + \cos A \sin B$. The principal value branch of $\sin^{-1}(x)$ is $[-\frac{\pi}{2}, \frac{\pi}{2}]$. If an angle falls outside this range, use $\sin(\pi-\theta) = \sin\theta$ to find an equivalent angle inside the branch.

Step 3: Detailed Explanation:

Consider the inner expression:

$$\sin\left(\frac{5\pi}{9}\right)\cos\left(\frac{\pi}{9}\right) + \cos\left(\frac{5\pi}{9}\right)\sin\left(\frac{\pi}{9}\right)$$

This matches the $\sin(A+B)$ formula with $A = \frac{5\pi}{9}$ and $B = \frac{\pi}{9}$. Substitute and condense the sum:

$$\sin\left(\frac{5\pi}{9} + \frac{\pi}{9}\right) = \sin\left(\frac{6\pi}{9}\right) = \sin\left(\frac{2\pi}{3}\right)$$

Now we need to evaluate:

$$\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$$

We cannot simply cancel $\sin^{-1}$ and sin because $\frac{2\pi}{3}$ is outside the principal range $[-\frac{\pi}{2}, \frac{\pi}{2}]$. Find an equivalent angle in the principal range:

$$\sin\left(\frac{2\pi}{3}\right) = \sin\left(\pi - \frac{\pi}{3}\right) = \sin\left(\frac{\pi}{3}\right)$$

Now the expression is:

$$\sin^{-1}\left(\sin\frac{\pi}{3}\right)$$

Since $\frac{\pi}{3}$ is in $[-\frac{\pi}{2}, \frac{\pi}{2}]$, the functions cancel each other:

$$= \frac{\pi}{3}$$

Step 4: Final Answer:

The value is $\frac{\pi}{3}$.

Quick Tip: Always check if the argument of $\sin^{-1}(\sin\theta)$ lies strictly within $[-90^\circ, 90^\circ]$ before canceling.

---

20. If $Z_1 = \frac{5+7i}{7-5i}, Z_2 = \frac{3+2i}{3-2i}, Z_3 = \frac{1+11i}{11-i}$. Find the value of $Z_1 \times \overline{Z_1} + Z_2\overline{Z_2} + Z_3\overline{Z_3}$

Correct Answer: 3

Solution:

Step 1: Understanding the Concept:

The problem asks for a sum of products of complex numbers and their conjugates. We use complex modulus properties instead of painful algebraic expansion.

Step 2: Key Formula or Approach:

For any complex number Z, the product with its conjugate is equal to the square of its modulus: $Z\overline{Z} = |Z|^2$. Also, the modulus of a quotient is the quotient of the moduli: $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$.

Step 3: Detailed Explanation:

The expression is $|Z_1|^2 + |Z_2|^2 + |Z_3|^2$. Let's calculate the modulus squared for each fraction.

For $Z_1 = \frac{5+7i}{7-5i}$:

$$|Z_1|^2 = \frac{|5+7i|^2}{|7-5i|^2} = \frac{5^2+7^2}{7^2+(-5)^2} = \frac{25+49}{49+25} = \frac{74}{74} = 1$$

For $Z_2 = \frac{3+2i}{3-2i}$:

$$|Z_2|^2 = \frac{|3+2i|^2}{|3-2i|^2} = \frac{3^2+2^2}{3^2+(-2)^2} = \frac{9+4}{9+4} = \frac{13}{13} = 1$$

For $Z_3 = \frac{1+11i}{11-i}$:

$$|Z_3|^2 = \frac{|1+11i|^2}{|11-i|^2} = \frac{1^2+11^2}{11^2+(-1)^2} = \frac{1+121}{121+1} = \frac{122}{122} = 1$$

Now, substitute these back into the target expression:

$$Z_1\overline{Z_1} + Z_2\overline{Z_2} + Z_3\overline{Z_3} = |Z_1|^2 + |Z_2|^2 + |Z_3|^2$$

$$= 1 + 1 + 1 = 3$$

Step 4: Final Answer:

The evaluated sum is 3.

Quick Tip: Any complex fraction of the form $\frac{a+ib}{b-ia}$ or $\frac{a+ib}{a-ib}$ will always have a modulus of 1, saving time on calculation.

---

21. Find the eqn of the parabola having vertex $(2, -5)$ and focus $(5, -5)$

Correct Answer: $(y+5)^2 = 12(x-2)$

Solution:

Step 1: Understanding the Concept:

We are required to find the standard equation of a parabola.

Since the vertex and focus share the same y-coordinate, the axis of symmetry is horizontal, meaning the parabola opens to the right or left.

Step 2: Key Formula or Approach:

For a horizontal parabola with vertex (h, k), the standard equation is $(y-k)^2 = 4a(x-h)$, where a is the directed distance from the vertex to the focus.

Step 3: Detailed Explanation:

Given the vertex $(h, k) = (2, -5)$.

Given the focus is $(5, -5)$, which is of the form $(h+a, k)$.

Equating the x-coordinates to find a:

$$h + a = 5 \implies 2 + a = 5 \implies a = 3$$

Since $a > 0$, the parabola opens to the right.

Substitute $h = 2$, $k = -5$, and $a = 3$ into the standard formula:

$$(y-(-5))^2 = 4(3)(x-2)$$

$$(y+5)^2 = 12(x-2)$$

Step 4: Final Answer:

The equation of the parabola is $(y+5)^2 = 12(x-2)$.

Quick Tip: Always sketch the coordinates mentally: a focus to the right of the vertex means a horizontal parabola opening right, so the squared term is y and the multiplier is positive.

---

22. If $(3+i)x+(1-i)y+(3i-4) = (2x+1)+(x-y+2)i$. Find (x, y)

Correct Answer: Inconsistent (No solution)

Solution:

Step 1: Concept

Two complex numbers are equal if and only if their real parts and imaginary parts are equal separately.

Step 2: Expand LHS

$$(3+i)x+(1-i)y+(3i-4)$$

$$= 3x+ix+y-iy+3i-4$$

Group real and imaginary parts:

$$= (3x + y - 4) + i(x - y + 3)$$

Step 3: Write RHS

$$(2x + 1) + i(x - y + 2)$$

Step 4: Compare Real and Imaginary Parts

Real parts:

$$3x + y - 4 = 2x + 1 \Rightarrow x + y = 5 \quad (1)$$

Imaginary parts:

$$x - y + 3 = x - y + 2 \Rightarrow 3 = 2$$

Step 5: Conclusion

The equation $3 = 2$ is a contradiction, hence no solution exists.

Step 6: Final Answer

$$\boxed{\text{No solution (Inconsistent system)}}$$

Quick Tip: If equating imaginary and real parts yields a false absolute statement like $3 = 2$, confidently mark it as inconsistent rather than endlessly searching for an algebraic error.

---

23. Find the shortest distance b/w the line $\vec{r} = -\hat{i} + t\hat{k}$ and $\vec{r} = -\hat{j} + S\hat{i}$; $t, S \in R$

Correct Answer: 1

Solution:

Step 1: Understanding the Concept:

We are tasked with finding the shortest distance between two skew lines given in vector form.

Step 2: Key Formula or Approach:

The shortest distance d between two lines $\vec{r} = \vec{a}_1 + t\vec{b}_1$ and $\vec{r} = \vec{a}_2 + s\vec{b}_2$ is calculated using the scalar triple product:

$$d = \left| \frac{(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|} \right|$$

Step 3: Detailed Explanation:

Extract the position vectors and direction vectors from the given lines:

For Line 1: $\vec{a}_1 = -\hat{i}$ and $\vec{b}_1 = \hat{k}$.

For Line 2: $\vec{a}_2 = -\hat{j}$ and $\vec{b}_2 = \hat{i}$.

Find the difference between the position vectors:

$$\vec{a}_2 - \vec{a}_1 = -\hat{j} - (-\hat{i}) = \hat{i} - \hat{j}$$

Find the cross product of the direction vectors:

$$\vec{b}_1 \times \vec{b}_2 = \hat{k} \times \hat{i} = \hat{j}$$

Calculate the magnitude of the cross product:

$$|\vec{b}_1 \times \vec{b}_2| = |\hat{j}| = 1$$

Compute the dot product for the numerator:

$$(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = (\hat{i} - \hat{j}) \cdot \hat{j}$$

$$= (\hat{i} \cdot \hat{j}) - (\hat{j} \cdot \hat{j}) = 0 - 1 = -1$$

Apply the distance formula using the absolute value:

$$d = \left|\frac{-1}{1}\right| = 1$$

Step 4: Final Answer:

The shortest distance is 1.

Quick Tip: When standard unit vectors $(\hat{i}, \hat{j}, \hat{k})$ are used directly as directions, computing the cross product mentally (like $\hat{k} \times \hat{i} = \hat{j}$) saves a lot of matrix calculation time.

---

24. $\lim_{x\to 0} \frac{\sqrt{1-\cos(x^2)}}{1-\cos x}$

Correct Answer: $\sqrt{2}$

Solution:

Step 1: Understanding the Concept:

We must evaluate a trigonometric limit as $x \to 0$.

Direct substitution yields a $0/0$ form, indicating we should use trigonometric identities to simplify the expression.

Step 2: Key Formula or Approach:

The most useful half-angle trigonometric identity here is $1 - \cos\theta = 2\sin^2\left(\frac{\theta}{2}\right)$.

We also rely on the standard limit $\lim_{u\to 0} \frac{\sin u}{u} = 1$.

Step 3: Detailed Explanation:

Apply the half-angle identity to the expression inside the square root in the numerator:

$$1 - \cos(x^2) = 2\sin^2\left(\frac{x^2}{2}\right)$$

Apply the same identity to the denominator:

$$1 - \cos x = 2\sin^2\left(\frac{x}{2}\right)$$

Substitute these back into the limit expression:

$$L = \lim_{x\to 0} \frac{\sqrt{2\sin^2\left(\frac{x^2}{2}\right)}}{2\sin^2\left(\frac{x}{2}\right)}$$

As $x \to 0$, $\frac{x^2}{2}$ is positive, so the square root of the sine squared simplifies directly to the positive sine function:

$$L = \lim_{x\to 0} \frac{\sqrt{2}\sin\left(\frac{x^2}{2}\right)}{2\sin^2\left(\frac{x}{2}\right)}$$

To use the standard sine limit, divide the numerator and denominator by appropriate powers of x:

$$L = \frac{\sqrt{2}}{2} \lim_{x\to 0} \frac{\left(\frac{\sin(x^2/2)}{x^2/2}\right)\cdot \frac{x^2}{2}}{\left(\frac{\sin(x/2)}{x/2}\right)^2 \cdot \frac{x^2}{4}}$$

Since $\lim_{u\to 0} \frac{\sin u}{u} = 1$, the sine terms go to 1:

$$L = \frac{1}{\sqrt{2}} \cdot \frac{1\cdot \frac{x^2}{2}}{1\cdot \frac{x^2}{4}}$$

Cancel x^2 and simplify the constants:

$$L = \frac{1}{\sqrt{2}} \cdot \frac{1/2}{1/4} = \frac{1}{\sqrt{2}} \cdot 2 = \sqrt{2}$$

Step 4: Final Answer:

The value of the limit is $\sqrt{2}$.

Quick Tip: Maclaurin series approximations are highly effective here: $1-\cos\theta \approx \theta^2/2$. Thus $\sqrt{1-\cos(x^2)} \approx \sqrt{x^4/2} = x^2/\sqrt{2}$, and $1-\cos x \approx x^2/2$. Their ratio immediately yields $\sqrt{2}$.

---

25. $\int \frac{\sin t+\cos t}{13+36\sin 2t}dt$

Correct Answer: $\frac{1}{84}\ln\left|\frac{7+6(\sin t-\cos t)}{7-6(\sin t-\cos t)}\right|+C$

Solution:

Step 1: Understanding the Concept:

We are dealing with an integral where the numerator is the exact derivative of a trigonometric sum or difference.

We can express the denominator $\sin 2t$ in terms of this new variable to facilitate substitution.

Step 2: Key Formula or Approach:

Use the identity $(\sin t-\cos t)^2 = \sin^2 t+\cos^2 t-2\sin t\cos t = 1-\sin 2t$.

The standard integral formula used is $\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left|\frac{a+x}{a-x}\right|+C$.

Step 3: Detailed Explanation:

Let $u = \sin t-\cos t$.

Differentiating both sides with respect to t:

$$du = (\cos t+\sin t)dt$$

From the identity, we solve for $\sin 2t$:

$$u^2 = 1-\sin 2t \implies \sin 2t = 1-u^2$$

Substitute $\sin 2t$ in the denominator:

$$13 + 36 \sin 2t = 13 + 36(1 - u^2)$$

$$= 13 + 36 - 36u^2 = 49 - 36u^2$$

The integral transforms into:

$$\int \frac{du}{49 - 36u^2}$$

Factor out 36 in the denominator to match the standard form:

$$= \frac{1}{36} \int \frac{du}{\frac{49}{36} - u^2}$$

This is of the form $\int \frac{dx}{a^2 - x^2}$ with $a = \frac{7}{6}$.
Apply the integration formula:

$$= \frac{1}{36} \left[\frac{1}{2(7/6)} \ln \left| \frac{7/6 + u}{7/6 - u} \right| \right] + C$$

Simplify the constants and the fraction inside the natural log:

$$= \frac{1}{36} \left[\frac{3}{7} \ln \left| \frac{7 + 6u}{7 - 6u} \right| \right] + C$$

$$= \frac{1}{84} \ln \left| \frac{7 + 6u}{7 - 6u} \right| + C$$

Substitute back $u = \sin t - \cos t$:

$$= \frac{1}{84} \ln \left| \frac{7 + 6(\sin t - \cos t)}{7 - 6(\sin t - \cos t)} \right| + C$$

Step 4: Final Answer:

The value of the integral is $\frac{1}{84}\ln\left|\frac{7+6(\sin t-\cos t)}{7-6(\sin t-\cos t)}\right| + C$.

Quick Tip: Whenever the integrand has $\sin x \pm \cos x$ in the numerator and $\sin 2x$ in the denominator, always substitute $u = \sin x \mp \cos x$ (note the opposite sign) to easily eliminate dt.

---

26. $\lim_{x\to 1} \frac{x-1}{\sqrt[3]{x}-1}$

Correct Answer: 3

Solution:

Step 1: Understanding the Concept:

Direct substitution of $x = 1$ leads to an indeterminate $\frac{0}{0}$ form.

We can resolve this using algebraic substitution to remove the fractional powers before factoring.

Step 2: Key Formula or Approach:

Let $t = \sqrt[3]{x}$, which transforms the limit into evaluating a simple polynomial division.

We use the difference of cubes factorization: $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$.

Step 3: Detailed Explanation:

Let $t = \sqrt[3]{x}$. As $x \to 1$, $t \to 1$.

Since $t = x^{1/3}$, cubing both sides gives $x = t^3$.

Substitute x and $\sqrt[3]{x}$ into the limit expression:

$$\lim_{t\to 1} \frac{t^3 - 1}{t - 1}$$

Factor the numerator using the difference of cubes formula:

$$\lim_{t\to 1}\frac{(t-1)(t^2+t+1)}{t-1}$$

Cancel the common non-zero factor $(t-1)$:

$$\lim_{t\to 1}(t^2+t+1)$$

Evaluate the limit by direct substitution of $t=1$:

$$1^2+1+1=3$$

Step 4: Final Answer:

The limit evaluates to 3.

Quick Tip: For limits with fractional powers like $\sqrt[n]{x}$, substituting $t^n = x$ cleanly converts the entire expression into an easily factorable polynomial without needing L'Hôpital's rule.

---

27. If $f(x)=\frac{2x+3}{x-2}$; $x \neq 2$ and $x \in R$

Correct Answer: $f^{-1}(x)=f(x)$

Solution:

Step 1: Concept

To find the inverse of a function, replace $f(x)$ with y, solve for x in terms of y, and then replace y with x.

Step 2: Let

$$y = \frac{2x+3}{x-2}$$

Step 3: Solve for x

Multiply both sides by $(x-2)$:

$$y(x-2) = 2x+3$$

Expand:

$$yx - 2y = 2x + 3$$

Rearrange terms:

$$yx - 2x = 2y + 3$$

Factor x:

$$x(y-2) = 2y+3$$

Solve for x:

$$x = \frac{2y+3}{y-2}$$

Step 4: Replace y with x

$$f^{-1}(x) = \frac{2x+3}{x-2}$$

Step 5: Final Answer

$$\boxed{f^{-1}(x) = \frac{2x+3}{x-2}}$$

Quick Tip: A linear fractional transformation $f(x) = \frac{ax+b}{cx+d}$ is its own inverse if and only if $a = -d$. Here, $a = 2$ and $d = -2$, confirming immediately that $f(x)$ is self-invertible.

---

28. If $P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 10 & 100 & -1 \end{bmatrix}$ **Find** P^{4052}

Correct Answer: I

Solution:

Step 1: Concept

To evaluate a large power of a matrix, we check if the matrix follows a pattern such as $P^2 = I$.

Step 2: Compute P^2

$$P^2 = P \times P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 10 & 100 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 10 & 100 & -1 \end{bmatrix}$$

Step 3: Matrix Multiplication

Row 1:

$$(1,0,0) \cdot \text{columns} \Rightarrow (1,0,0)$$

Row 2:

$$(0,1,0) \cdot \text{columns} \Rightarrow (0,1,0)$$

Row 3:

$$\text{Column 1: } 10(1) + 100(0) + (-1)(10) = 10 - 10 = 0$$

$$\text{Column 2: } 10(0) + 100(1) + (-1)(100) = 100 - 100 = 0$$

$$\text{Column 3: } 10(0) + 100(0) + (-1)(-1) = 1$$

$$\Rightarrow P^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

Step 4: Use Property

Since $P^2 = I$, we write:

$$P^{4052} = (P^2)^{2026} = I^{2026} = I$$

Step 5: Final Answer

$$\boxed{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}$$

Quick Tip: For triangular matrices with ±1 on the main diagonal, always test P^2 first. They often turn out to be involutory matrices (matrices that are their own inverse, where $P^2 = I$).

---

29. If $|\vec{a}| = \sqrt{26}$, $|\vec{b}| = \sqrt{3}$ $\quad \vec{a} \times \vec{b} = 5\hat{i} + \hat{j} - 4\hat{k}$. Find $\vec{a} \cdot \vec{b}$

Correct Answer: ±6

Solution:

Step 1: Understanding the Concept:

We are given the magnitudes of two individual vectors and their full cross product vector. We need to find their dot product. These three vector properties are intrinsically linked via an algebraic identity.

Step 2: Key Formula or Approach:

Lagrange's Identity connects the dot product and cross product magnitudes of two vectors without needing the angle between them:

$$|\vec{a} \times \vec{b}|^2 + (\vec{a} \cdot \vec{b})^2 = |\vec{a}|^2 |\vec{b}|^2$$

Step 3: Detailed Explanation:

First, find the magnitude squared of the given cross product vector:

$$|\vec{a} \times \vec{b}| = \sqrt{5^2 + 1^2 + (-4)^2}$$

$$|\vec{a} \times \vec{b}| = \sqrt{25 + 1 + 16} = \sqrt{42}$$

Squaring this magnitude gives:

$$|\vec{a} \times \vec{b}|^2 = 42$$

We are given the squared magnitudes of the original vectors:

$$|\vec{a}|^2 = (\sqrt{26})^2 = 26$$

$$|\vec{b}|^2 = (\sqrt{3})^2 = 3$$

Substitute these values directly into Lagrange's Identity:

$$42 + (\vec{a} \cdot \vec{b})^2 = 26 \times 3$$

$$42 + (\vec{a} \cdot \vec{b})^2 = 78$$

Subtract 42 from both sides to isolate the dot product term:

$$(\vec{a} \cdot \vec{b})^2 = 78 - 42 = 36$$

Take the square root of both sides (accounting for both positive and negative possibilities):

$$\vec{a} \cdot \vec{b} = \pm 6$$

Step 4: Final Answer:

The possible values for $\vec{a} \cdot \vec{b}$ are 6 or -6.

Quick Tip: Lagrange's identity avoids dealing with trigonometric angles entirely. Remember it simply as $\text{Cross}^2 + \text{Dot}^2 = \text{Product of Magnitudes}^2$.

---

30. In a GP $a_1 = 7$, $a_n = 448$ and $S_n = 889$. Find the common ratio of the G.P

Correct Answer: 2

Solution:

Step 1: Understanding the Concept:

We are given the first term, the nth term, and the total sum of n terms of a Geometric Progression (GP). We must find the common ratio r.

Step 2: Key Formula or Approach:

For a GP, the sum of n terms can be written uniquely in terms of the last term a_n without knowing the value of n:

$$S_n = \frac{a_n r - a_1}{r-1} \quad \text{for } r \neq 1$$

Step 3: Detailed Explanation:

List the given values from the problem:

$a_1 = 7$

$a_n = 448$

$S_n = 889$

Substitute these values directly into the sum formula:

$$889 = \frac{448r - 7}{r-1}$$

Multiply both sides by $(r-1)$ to remove the fraction:

$$889(r-1) = 448r - 7$$

Distribute 889 on the left side:

$$889r - 889 = 448r - 7$$

Rearrange the terms to group r on one side and constants on the other:

$$889r - 448r = 889 - 7$$

$$441r = 882$$

Divide by 441:

$$r = \frac{882}{441} = 2$$

Step 4: Final Answer:

The common ratio of the Geometric Progression is 2.

Quick Tip: Always memorize the alternate GP sum formula $S_n = \frac{lr-a}{r-1}$ (where l is the last term). It drastically speeds up calculation in problems where the number of terms n is unknown.

---

31. If R $(-2, 2)$ is a point on the ellipse $\frac{(x-3)^2}{25} + \frac{(y+2)^2}{16} = 1$. If S and T are the focii of an ellipse find RS + RT

Correct Answer: 10

Solution:

Step 1: Understanding the Concept:

The problem asks for the sum of the distances from a given point on an ellipse to its two foci. This directly tests the fundamental geometric definition of an ellipse.

Step 2: Key Formula or Approach:

By the definition of an ellipse, the sum of the distances from any point P on the ellipse to the two foci F_1 and F_2 is always constant and strictly equal to the length of the major axis, $2a$.

Step 3: Detailed Explanation:

The given equation of the ellipse is:

$$\frac{(x-3)^2}{25} + \frac{(y+2)^2}{16} = 1$$

Compare this with the standard equation of a shifted ellipse $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$.

Since $25 > 16$, the major axis is horizontal, and we have:

$$a^2 = 25 \implies a = 5$$

$$b^2 = 16$$

Point R $(-2, 2)$ lies on the ellipse.

Let S and T be the two foci of this ellipse.

According to the foundational property of the ellipse:

$$\mathrm{RS} + \mathrm{RT} = 2a$$

Substitute the calculated value of $a = 5$:

$$\mathrm{RS} + \mathrm{RT} = 2(5) = 10$$

Step 4: Final Answer:

The sum RS + RT is 10.

Quick Tip: Recognizing the core geometric definitions of conic sections helps bypass long, unnecessary coordinate distance calculations. For any point P on an ellipse, $PF_1 + PF_2 = 2a$ always.

---

32. Find the coefficient of x^{-2} in $\left(3x - \frac{1}{3x}\right)^4$

Correct Answer: -4/9

Solution:

Step 1: Understanding the Concept:

We need to find the coefficient of a specific power of x within a binomial expansion. We solve this by writing the general term of the expansion and finding the index that produces the requested power.

Step 2: Key Formula or Approach:

The general term T_{k+1} in the binomial expansion of $(A+B)^n$ is given by:

$$T_{k+1} = \binom{n}{k} A^{n-k} B^k$$

Step 3: Detailed Explanation:

For the given expression, $A = 3x$, $B = -\frac{1}{3x}$, and $n = 4$.

Write out the general term:

$$T_{k+1} = \binom{4}{k}(3x)^{4-k}\left(-\frac{1}{3x}\right)^k$$

Separate the numerical constants from the variable x:

$$T_{k+1} = \binom{4}{k}3^{4-k}x^{4-k}(-1)^k\left(\frac{1}{3}\right)^k x^{-k}$$

Combine the exponents of base 3 and base x:

$$T_{k+1} = \binom{4}{k}(-1)^k 3^{4-2k}x^{4-2k}$$

We specifically want the coefficient of x^{-2}, so we set the exponent of x to -2:

$$4-2k=-2 \implies 2k=6 \implies k=3$$

Substitute $k=3$ back into the coefficient portion of the general term:

$$\text{Coefficient} = \binom{4}{3}(-1)^3 3^{4-2(3)}$$

$$\text{Coefficient} = 4 \times (-1) \times 3^{4-6}$$

$$\text{Coefficient} = -4 \times 3^{-2} = -4 \times \frac{1}{9} = \frac{-4}{9}$$

Step 4: Final Answer:

The coefficient is $-4/9$.

Quick Tip: Always carefully bracket and carry the negative signs from terms like $-1/3x$ into your general term equation, as sign errors are the most frequent mistake in binomial coefficient problems.

---

33. Find he solution set of $\frac{x-3}{x-2} \geq 1$

Correct Answer: $(-\infty, 2)$

Solution:

Step 1: Understanding the Concept:

We are evaluating a rational inequality. The standard, safe mathematical protocol is to bring all terms to one side so the inequality is compared to zero, rather than cross-multiplying directly which can destroy vital sign information.

Step 2: Key Formula or Approach:

Rewrite the inequality as $\frac{A}{B} - C \geq 0$, combine everything into a single fraction, and use sign analysis to find the valid intervals for x.

Step 3: Detailed Explanation:

Start with the inequality:

$$\frac{x-3}{x-2} \geq 1$$

Subtract 1 from both sides to compare to zero:

$$\frac{x-3}{x-2} - 1 \geq 0$$

Find a common denominator to merge the terms:

$$\frac{(x-3)-(x-2)}{x-2} \geq 0$$

Simplify the numerator:

$$\frac{x-3-x+2}{x-2} \geq 0$$

$$\frac{-1}{x-2} \geq 0$$

For a fraction with a negative numerator to be greater than or equal to zero, its denominator must be strictly negative. (It cannot be zero because that makes the fraction undefined).

$$x-2<0 \implies x<2$$

This inequality corresponds to all real numbers strictly less than 2.

Step 4: Final Answer:

The solution set is $(-\infty, 2)$.

Quick Tip: Never cross-multiply variables across an inequality symbol (like multiplying by $x-2$) unless you are absolutely certain the algebraic quantity is strictly positive.

---

34. $\int_0^1 x(1-x)^4 dx$

Correct Answer: 1/30

Solution:

Step 1: Understanding the Concept:

We need to evaluate a definite integral containing a binomial raised to a power. Applying a fundamental property of definite integrals will swap the complexity from the binomial to the single monomial, allowing for easy expansion.

Step 2: Key Formula or Approach:

Use the King's Rule property for definite integrals:

$$\int_0^a f(x)dx = \int_0^a f(a-x)dx$$

This property cleanly shifts the heavy power to a single term.

Step 3: Detailed Explanation:

Let the given integral be $I = \int_0^1 x(1-x)^4 dx$.

Apply the property by replacing x with $(1-x)$:

$$I = \int_0^1 (1-x)(1-(1-x))^4 dx$$

Simplify the expression inside the power:

$$I = \int_0^1 (1-x)(x)^4 dx$$

Distribute the x^4 into the binomial:

$$I = \int_0^1 (x^4 - x^5) dx$$

Now, integrate each term individually using the power rule:

$$I = \left[\frac{x^5}{5} - \frac{x^6}{6}\right]_0^1$$

Apply the upper limit (the lower limit of 0 evaluates to 0):

$$I = \left(\frac{1^5}{5} - \frac{1^6}{6}\right) - 0$$

$$I = \frac{1}{5} - \frac{1}{6}$$

Find a common denominator of 30 to subtract:

$$I = \frac{6-5}{30} = \frac{1}{30}$$

Step 4: Final Answer:

The evaluated integral is $1/30$.

Quick Tip: For integrals exactly of the form $\int_0^1 x^m(1-x)^n dx$, you can instantly use the Beta function formula: value is $\frac{m!n!}{(m+n+1)!}$. Here $m = 1, n = 4$, giving $\frac{1!4!}{6!} = \frac{24}{720} = \frac{1}{30}$.

---

35. Find the domain of $f(x) = 2[\sin^{-1}(2x-1)] - \frac{\pi}{4}$

Correct Answer: $[0, 1]$

Solution:

Step 1: Understanding the Concept:

The domain of a composite function depends completely on the restrictions of its inner functions. The outer operations (multiplication by 2 and subtraction) only affect the range, not the domain.

Step 2: Key Formula or Approach:

The standard valid domain for the real-valued inverse sine function $\sin^{-1}(u)$ is $-1 \le u \le 1$. We bind the inner argument of the function to this interval and solve for x.

Step 3: Detailed Explanation:

The argument fed into the inverse sine function is $2x-1$.
Set up the domain inequality based on the restriction:

$$-1 \le 2x-1 \le 1$$

Add 1 to all parts of the compound inequality:

$$-1+1 \le 2x \le 1+1$$

$$0 \le 2x \le 2$$

Divide all parts by 2 to isolate x:

$$0 \le x \le 1$$

Thus, the function produces valid real numbers for all x bounded between 0 and 1, inclusive.

Step 4: Final Answer:
The domain of the function is $[0,1]$.

Quick Tip: Constants added or multiplied outside an inverse trigonometric function alter the *range* of the function, but they have absolutely zero mathematical effect on the *domain*.

---

36. Find the value of $i^{13}+i^{14}+\cdots+i^{226}$

Correct Answer: $i-1$

Solution:

Step 1: Understanding the Concept:

We are tasked with finding the sum of consecutive integer powers of the imaginary unit i. The powers of i cycle repeatedly every 4 terms, and their sum over any full 4-term cycle equals zero.

Step 2: Key Formula or Approach:

The cyclic zero-sum property is: $i^n + i^{n+1} + i^{n+2} + i^{n+3} = 0$.

We count the total number of terms and find the remainder when divided by 4 to determine how many terms remain after all zero-sum cycles cancel out.

Step 3: Detailed Explanation:

The sequence of exponents runs consecutively from 13 to 226 inclusive.

Calculate the total number of terms N:

$$N = \text{Last Term} - \text{First Term} + 1 = 226 - 13 + 1 = 214$$

Now, divide N by 4 to determine the number of complete zero-sum cycles:

$$214 = 4 \times 53 + 2$$

This means there are exactly 53 complete groups of 4 consecutive powers that sum to zero, leaving exactly 2 terms uncancelled.

Since any sequence of 4 consecutive powers wraps symmetrically, we can evaluate the remaining 2 terms starting from the very beginning of the series:

$$\text{Sum} = i^{13} + i^{14}$$

Simplify these powers by dividing the exponents by 4 and keeping the remainder:

$$13 = 4 \times 3 + 1 \implies i^{13} = i^1 = i$$

$$14 = 4 \times 3 + 2 \implies i^{14} = i^2 = -1$$

Therefore, the final evaluated sum is:

$$\text{Sum} = i - 1$$

Step 4: Final Answer:

The value of the series is $i - 1$.

Quick Tip: To minimize calculation, when a remainder of r terms is left, simply compute the sum of the *first* r terms of the series, as it yields the exact identical result.

---

37. $\frac{4^{n+1}+16^{n+1}}{4^n+16^n}$ **= G.M of** 4 **and** 16 **find** n?

Correct Answer: -1/2

Solution:

Step 1: Understanding the Concept:

We are given a generalized expression for mathematical means and told it equals the specific Geometric Mean (GM) of two numbers. We need to identify the exponent parameter n.

Step 2: Key Formula or Approach:

The generalized mean expression $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ represents standard mathematical means for specific values of n:

Arithmetic Mean (AM) when $n = 0$

Geometric Mean (GM) when $n = -1/2$

Harmonic Mean (HM) when $n = -1$

Step 3: Detailed Explanation:

Let's verify the standard analytical result algebraically. Let the numbers be $a = 4$ and $b = 16$. The true Geometric Mean of a and b is calculated as:

$$GM = \sqrt{ab} = \sqrt{4 \times 16} = \sqrt{64} = 8$$

We are given the equation:

$$\frac{4^{n+1} + 16^{n+1}}{4^n + 16^n} = 8$$

Let's test the known GM parameter $n = -1/2$:

Numerator:

$$4^{-1/2+1} + 16^{-1/2+1} = 4^{1/2} + 16^{1/2} = \sqrt{4} + \sqrt{16} = 2 + 4 = 6$$

Denominator:

$$4^{-1/2} + 16^{-1/2} = \frac{1}{4^{1/2}} + \frac{1}{16^{1/2}} = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$$

Calculate the final value of the fraction:

$$\frac{6}{3/4} = 6 \times \frac{4}{3} = 8$$

This perfectly matches the required Geometric Mean.

Step 4: Final Answer:

The value of n is -1/2.

Quick Tip: Commit this universal identity to memory: For the generalized mean formula $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$, $n=0$ is AM, $n=-0.5$ is GM, and $n=-1$ is HM.

---

38. $(3+5x)e^y = x$, **find** $\frac{dy}{dx} =$

Correct Answer: $\frac{3}{3x+5x^2}$

Solution:

Step 1: Understanding the Concept:

We have an implicit equation involving an exponential function of y. To find the derivative, it is algebraically simpler to isolate y by taking the natural logarithm before differentiating.

Step 2: Key Formula or Approach:

Isolate the e^y term, take the natural logarithm using the property $\ln(a/b) = \ln a - \ln b$, and then apply the standard chain rule: $\frac{d}{dx}\ln(u) = \frac{1}{u}\cdot\frac{du}{dx}$.

Step 3: Detailed Explanation:

Given the equation:

$$(3+5x)e^y = x$$

Divide by $(3+5x)$ to isolate the exponential term:

$$e^y = \frac{x}{3+5x}$$

Take the natural logarithm (ln) on both sides of the equation:

$$y = \ln\left(\frac{x}{3+5x}\right)$$

Using fundamental logarithm properties to expand:

$$y = \ln x - \ln(3+5x)$$

Now, differentiate cleanly with respect to x:

$$\frac{dy}{dx} = \frac{1}{x} - \frac{1}{3+5x} \cdot \frac{d}{dx}(3+5x)$$

$$\frac{dy}{dx} = \frac{1}{x} - \frac{5}{3+5x}$$

Combine the two terms by finding a common denominator:

$$\frac{dy}{dx} = \frac{(3+5x)-5x}{x(3+5x)}$$

$$\frac{dy}{dx} = \frac{3}{3x+5x^2}$$

Step 4: Final Answer:

The derivative $\frac{dy}{dx}$ is $\frac{3}{3x+5x^2}$.

Quick Tip: Applying logarithms first converts complex quotient and product rule derivatives into much simpler addition/subtraction derivatives, saving time and massively reducing algebra errors.

---

39. If the end of a diameter of the circle is $(-4,-2)$, find the other end of the diameter

Correct Answer: $(4,2)$

Solution:

Step 1: Concept

The center of a circle is the midpoint of the diameter.

Step 2: Formula Used

Midpoint formula:

$$(h,k) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Step 3: Assume Center

Since the center is not given, assume the circle is centered at origin:

$$(h,k) = (0,0)$$

Step 4: Substitute Values

Let the other end be (x, y). Given point is $(-4,-2)$.

For x-coordinate:

$$\frac{-4+x}{2} = 0 \Rightarrow -4 + x = 0 \Rightarrow x = 4$$

For y-coordinate:

$$\frac{-2+y}{2} = 0 \Rightarrow -2 + y = 0 \Rightarrow y = 2$$

Step 5: Final Answer

$$\boxed{(4,2)}$$

Quick Tip: For standard origin-centered circles, the ends of a diameter are always symmetric reflections directly across the origin, found by simply negating both coordinates: $(-x_1, -y_1)$.

---

40. Find x in $4\sin^2 x - 2(1+\sqrt{3})\sin x + \sqrt{3} = 0, 15^\circ < x < 150^\circ$

Correct Answer: $30^\circ, 60^\circ, 120^\circ$

Solution:

Step 1: Understanding the Concept:

The given trigonometric equation is a quadratic equation expressed in terms of $\sin x$. We must solve for $\sin x$ using factorization and then map those values back to corresponding angles within the bounded interval.

Step 2: Key Formula or Approach:

Let $y = \sin x$ to form a standard quadratic equation $ay^2 + by + c = 0$. Once $\sin x$ values are found, use the unit circle to extract angles, carefully checking the explicit restriction $15° < x < 150°$.

Step 3: Detailed Explanation:

Let $y = \sin x$. The equation becomes:

$$4y^2 - 2(1+\sqrt{3})y + \sqrt{3} = 0$$

Expand the middle linear term:

$$4y^2 - 2y - 2\sqrt{3}y + \sqrt{3} = 0$$

Factor by grouping. Group the first two terms and the last two terms:

$$2y(2y-1) - \sqrt{3}(2y-1) = 0$$

Factor out the common binomial term $(2y-1)$:

$$(2y-1)(2y-\sqrt{3}) = 0$$

This gives two possible values for y:

$$y = \frac{1}{2} \quad \text{or} \quad y = \frac{\sqrt{3}}{2}$$

Case 1: $\sin x = \frac{1}{2}$

For angles between 0° and 180°, $x = 30^\circ$ or $x = 150^\circ$.

Case 2: $\sin x = \frac{\sqrt{3}}{2}$

For angles between 0° and 180°, $x = 60^\circ$ or $x = 120^\circ$.

Now, apply the strict given interval constraint: $15^\circ < x < 150^\circ$.

Note that the inequality is strictly less than, meaning 150° is explicitly excluded.

The valid angles satisfying all conditions are $30^\circ, 60^\circ, 120^\circ$.

Step 4: Final Answer:

The valid values of x are $30^\circ, 60^\circ, 120^\circ$.

Quick Tip: Pay close attention to strict vs. non-strict inequalities in domain restrictions (like $<$ vs $\leq$). Here, 150° is a valid solution to the root equation but gets rejected purely due to the strict inequality bound.

---

41. Find mean deviation about mean of 5, 6, 14, 15

Correct Answer: 4.5

Solution:

Step 1: Understanding the Concept:

The mean deviation about the mean is a measure of dispersion. It calculates the average absolute distance of each data point from the mean of the dataset.

Step 2: Key Formula or Approach:

1. Find the mean $(\bar{x})$ of the data: $\bar{x} = \frac{\sum x_i}{n}$.

2. Calculate the absolute deviation of each observation from the mean: $|x_i - \bar{x}|$.

3. Find the average of these absolute deviations: Mean Deviation $= \frac{\sum |x_i - \bar{x}|}{n}$.

Step 3: Detailed Explanation:

Given data points: 5, 6, 14, 15.

Number of observations $(n) = 4$.

First, calculate the mean $(\bar{x})$:

$$\bar{x} = \frac{5+6+14+15}{4} = \frac{40}{4} = 10$$

Next, find the absolute deviations $|x_i - \bar{x}|$ for each data point:

For 5: $|5-10| = |-5| = 5$

For 6: $|6-10| = |-4| = 4$

For 14: $|14-10| = |4| = 4$

For 15: $|15-10| = |5| = 5$

Now, sum these absolute deviations:

$$\sum |x_i - \bar{x}| = 5+4+4+5 = 18$$

Finally, calculate the mean deviation:

$$\text{Mean Deviation} = \frac{18}{4} = 4.5$$

Step 4: Final Answer:

The mean deviation about the mean is 4.5.

Quick Tip: Always remember to take the absolute value (ignore negative signs) when calculating deviations for Mean Deviation. If you don't, the sum of deviations around the mean will always mathematically equal zero!

42. $\sin 6^0 \times \sin 36^0 \times \sin 60^0 + \cos 12^0 \times \sin 42^0 \times \sin 18^0 =$

Correct Answer: $3\frac{}{8-\frac{\sqrt{3}}{4}\cos 42^\circ+\frac{\sqrt{5}+1}{16}}$

Solution:

Step 1: Concept

Use standard trigonometric identities:

$$\sin A \sin B = \frac{1}{2}[\cos(A-B)-\cos(A+B)]$$

$$\cos\theta\cos(60^\circ-\theta)\cos(60^\circ+\theta)=\frac{1}{4}\cos 3\theta$$

Step 2: Evaluate Second Term

$$\cos 12^\circ \sin 42^\circ \sin 18^\circ$$

Convert sine to cosine:

$$\sin 42^\circ = \cos 48^\circ, \quad \sin 18^\circ = \cos 72^\circ$$

$$= \cos 12^\circ \cos 48^\circ \cos 72^\circ$$

Using identity:

$$= \frac{1}{4}\cos 36^\circ$$

$$\cos 36^\circ = \frac{\sqrt{5}+1}{4}$$

$$\Rightarrow \text{Second term} = \frac{\sqrt{5}+1}{16}$$

Step 3: Evaluate First Term

$$\sin 6^\circ \sin 36^\circ \sin 60^\circ$$

$$\sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$= \frac{\sqrt{3}}{2} \sin 6^\circ \sin 36^\circ$$

Using identity:

$$\sin A \sin B = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$$

$$= \frac{\sqrt{3}}{2} \cdot \frac{1}{2}[\cos 30^\circ - \cos 42^\circ]$$

$$= \frac{\sqrt{3}}{4}\left(\frac{\sqrt{3}}{2} - \cos 42^\circ\right)$$

$$= \frac{3}{8} - \frac{\sqrt{3}}{4}\cos 42^\circ$$

Step 4: Add Both Terms

$$= \left(\frac{3}{8} - \frac{\sqrt{3}}{4}\cos 42^\circ\right) + \frac{\sqrt{5}+1}{16}$$

Step 5: Final Answer

$$\boxed{\frac{3}{8} - \frac{\sqrt{3}}{4}\cos 42^\circ + \frac{\sqrt{5}+1}{16}}$$

Quick Tip: Always look for the $\theta, 60^\circ - \theta, 60^\circ + \theta$ pattern in products of three sine or cosine terms. Using $\sin x = \cos(90^\circ - x)$ often reveals the hidden pattern.

---

43. If $y = e^{-x^2}$, find $\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} =$

Correct Answer: $-2y$

Solution:

Step 1: Understanding the Concept:

We are given an exponential function and need to evaluate a differential expression involving its first and second derivatives. We will find these derivatives using the chain and product rules.

Step 2: Key Formula or Approach:

The chain rule: $\frac{d}{dx}(e^{f(x)}) = e^{f(x)} \cdot f'(x)$.

The product rule: $\frac{d}{dx}(uv) = u'v + uv'$.

Step 3: Detailed Explanation:

Given the function:

$$y = e^{-x^2}$$

Find the first derivative $\frac{dy}{dx}$ using the chain rule:

$$\frac{dy}{dx} = e^{-x^2} \cdot \frac{d}{dx}(-x^2) = -2xe^{-x^2}$$

Find the second derivative $\frac{d^2y}{dx^2}$ by applying the product rule to $-2x \cdot e^{-x^2}$:

Let $u = -2x$ and $v = e^{-x^2}$.

$$\frac{d^2y}{dx^2} = (-2)(e^{-x^2}) + (-2x)(-2xe^{-x^2})$$

$$\frac{d^2y}{dx^2} = -2e^{-x^2} + 4x^2e^{-x^2}$$

Now substitute $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ into the target expression:

$$\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} = \left(-2e^{-x^2} + 4x^2e^{-x^2}\right) + 2x\left(-2xe^{-x^2}\right)$$

Expand the second term:

$$= -2e^{-x^2} + 4x^2e^{-x^2} - 4x^2e^{-x^2}$$

The $4x^2e^{-x^2}$ terms cancel out:

$$= -2e^{-x^2}$$

Since $y = e^{-x^2}$, we can substitute y back into the result:

$$= -2y$$

Step 4: Final Answer:

The evaluated expression is $-2y$ (or $-2e^{-x^2}$).

Quick Tip: To save time computing the second derivative, notice from the first derivative that $\frac{dy}{dx} = -2xy$. Differentiating this implicitly using the product rule gives $\frac{d^2y}{dx^2} = -2y - 2x\frac{dy}{dx}$, which rearranges instantly to the answer.

---

44. If $I = \int_{-1}^{1} \frac{x^4}{1-x^4} \cos^{-1}\left(\frac{2x}{1+x^2}\right) dx$ find 2I

Correct Answer: Diverges

Solution:

Step 1: Understanding the Concept:

For definite integrals with symmetric limits:

$$\int_{-a}^{a} f(x)\,dx = \int_{0}^{a} [f(x) + f(-x)]\,dx$$

Step 2: Key Formula or Property:

$$\cos^{-1}(-y) = \pi - \cos^{-1}(y)$$

Step 3: Simplification:

Let

$$f(x) = \frac{x^4}{1-x^4}\cos^{-1}\left(\frac{2x}{1+x^2}\right)$$

Then,

$$f(-x) = \frac{x^4}{1-x^4}\cos^{-1}\left(\frac{-2x}{1+x^2}\right)$$

Using identity:

$$\cos^{-1}(-y) = \pi - \cos^{-1}(y)$$

So,

$$f(-x) = \frac{x^4}{1-x^4}\left[\pi - \cos^{-1}\left(\frac{2x}{1+x^2}\right)\right]$$

Adding:

$$f(x) + f(-x) = \frac{x^4}{1-x^4}\cdot\pi$$

Thus,

$$I = \int_0^1 \pi\frac{x^4}{1-x^4}\,dx$$

$$\Rightarrow 2I = 2\pi\int_0^1 \frac{x^4}{1-x^4}\,dx$$

Step 4: Convergence Check:

The function $\frac{x^4}{1-x^4}$ has a vertical asymptote at $x = 1$.

Near $x \to 1$, the integrand behaves like:

$$\frac{1}{1-x}$$

which is not integrable.

Step 5: Final Conclusion:

$$\int_0^1 \frac{x^4}{1-x^4}dx \text{ diverges}$$

$$\Rightarrow I \text{ diverges} \quad \text{and hence} \quad 2I \text{ diverges}$$

Final Answer: Diverges

Quick Tip: When you see $\int_{-a}^{a} \text{Even}(x) \cdot \cos^{-1}(\text{Odd}(x))dx$, the entire integral immediately simplifies to $\frac{\pi}{2}\int_{-a}^{a} \text{Even}(x)dx$ or $\pi\int_{0}^{a} \text{Even}(x)dx$.

---

45. If $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ and $(\alpha I + \beta A)^2 = A$ find $\alpha^2 - \beta^2$?

Correct Answer: 0

Solution:

Step 1: Understanding the Concept:

We are dealing with a matrix polynomial equation. We will expand the squared expression algebraically, simplify it using the specific properties of matrix A, and equate components to solve for the constants.

Step 2: Key Formula or Approach:

Because the identity matrix I commutes with any matrix A (i.e., $IA = AI = A$), we can expand $(\alpha I + \beta A)^2$ just like regular binomials:
$(\alpha I + \beta A)^2 = \alpha^2 I^2 + 2\alpha\beta A + \beta^2 A^2$

Step 3: Detailed Explanation:

First, calculate A^2:

$$A^2 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} (0)(0)+(1)(-1) & (0)(1)+(1)(0) \\ (-1)(0)+(0)(-1) & (-1)(1)+(0)(0) \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

Observe that $A^2 = -I$.

Now expand the given equation $(\alpha I + \beta A)^2 = A$:

$$\alpha^2 I^2 + 2\alpha\beta A + \beta^2 A^2 = A$$

Substitute $I^2 = I$ and $A^2 = -I$:

$$\alpha^2 I + 2\alpha\beta A - \beta^2 I = A$$

Group the identity matrix terms together:

$$(\alpha^2 - \beta^2)I + 2\alpha\beta A = 0 \cdot I + 1 \cdot A$$

For this equation to hold, the corresponding matrix coefficients on both sides must be equal:
1) Coefficient of I: $\alpha^2 - \beta^2 = 0$
2) Coefficient of A: $2\alpha\beta = 1$
The question asks specifically for the value of $\alpha^2 - \beta^2$.
From equation (1), we directly see that it equals 0.

Step 4: Final Answer:
The value of $\alpha^2 - \beta^2$ is 0.

Quick Tip: The matrix $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ acts exactly like the imaginary unit i in complex numbers ($i^2 = -1$).
The expansion maps directly to $(\alpha + \beta i)^2 = \alpha^2 - \beta^2 + 2\alpha\beta i$.

---

46. If $\sin\theta \times \cos\theta > 0$ then θ lies in

Correct Answer: 1st or 3rd quadrant

Solution:

Step 1: Understanding the Concept:
We must determine the quadrants in a Cartesian plane where the product of the sine and

cosine functions is strictly positive.

Step 2: Key Formula or Approach:

The product of two real numbers is positive if and only if both numbers have the same sign (both positive or both negative). We evaluate the signs of trigonometric functions across the four quadrants using the ASTC (All, Sin, Tan, Cos) rule.

Step 3: Detailed Explanation:

The condition is $\sin\theta \cdot \cos\theta > 0$.

Case 1: Both functions are positive.

$\sin\theta > 0$ and $\cos\theta > 0$.

Sine is positive in Quadrants I and II.

Cosine is positive in Quadrants I and IV.

The overlap where both are positive is Quadrant I.

Case 2: Both functions are negative.

$\sin\theta < 0$ and $\cos\theta < 0$.

Sine is negative in Quadrants III and IV.

Cosine is negative in Quadrants II and III.

The overlap where both are negative is Quadrant III.

Alternatively, multiplying the inequality by 2 gives:

$2\sin\theta\cos\theta > 0 \implies \sin(2\theta) > 0$

This confirms the geometric logic.

Step 4: Final Answer:

The angle θ lies in the 1st or 3rd quadrant.

Quick Tip: A quick way to remember where $\sin\theta\cos\theta > 0$ is to realize it's equivalent to asking where $\tan\theta > 0$ (since $\tan\theta = \frac{\sin\theta\cos\theta}{\cos^2\theta}$), which is positive in Q1 and Q3.

47. $\begin{bmatrix} x & 3 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 2 \\ -1 & 0 & 1 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ 2 \\ 1 \end{bmatrix} = 0$

Correct Answer: $\pm 2i$

Solution:

Step 1: Understanding the Concept:

We must solve a matrix equation by performing sequential matrix multiplications (a row vector times a matrix, then the resulting row vector times a column vector) to generate a scalar algebraic equation in terms of x.

Step 2: Key Formula or Approach:

Multiply matrices strictly from left to right.

For a 1×3 row vector R and a 3×3 matrix M, the product RM is a new 1×3 row vector where each element is the dot product of R with a column of M.

Step 3: Detailed Explanation:

Let $R = \begin{bmatrix} x & 3 & -1 \end{bmatrix}$, $M = \begin{bmatrix} 1 & 1 & 2 \\ -1 & 0 & 1 \\ 1 & -1 & 1 \end{bmatrix}$, and $C = \begin{bmatrix} x \\ 2 \\ 1 \end{bmatrix}$.

First, calculate the intermediate row vector RM:

Column 1 calculation: $x(1) + 3(-1) + (-1)(1) = x - 3 - 1 = x - 4$

Column 2 calculation: $x(1) + 3(0) + (-1)(-1) = x + 0 + 1 = x + 1$

Column 3 calculation: $x(2) + 3(1) + (-1)(1) = 2x + 3 - 1 = 2x + 2$

So, $RM = \begin{bmatrix} x-4 & x+1 & 2x+2 \end{bmatrix}$.

Now, multiply this resulting row vector by the column vector C:

$$\begin{bmatrix} x-4 & x+1 & 2x+2 \end{bmatrix} \begin{bmatrix} x \\ 2 \\ 1 \end{bmatrix} = 0$$

Perform the dot product:

$$(x-4)(x)+(x+1)(2)+(2x+2)(1)=0$$

Expand all terms:

$$x^2-4x+2x+2+2x+2=0$$

Combine the x terms:

$$-4x+2x+2x=0x$$

The equation simplifies nicely to:

$$x^2+4=0$$

Subtract 4 from both sides:

$$x^2=-4$$

Taking the square root yields complex roots:

$$x=\pm 2i$$

Step 4: Final Answer:

The values of x are $2i$ and $-2i$.

Quick Tip: Always multiply the row vector and the 3×3 matrix first. It creates a simpler 1×3 vector, preventing you from having to hold a bulky 3×1 variable matrix in your working memory.

48. Solve $(x+2y)dx+(2x-y)dy=0$

Correct Answer: $x^2+4xy-y^2=C$

Solution:

Step 1: Understanding the Concept:

The given differential equation is of the form $M(x,y)dx+N(x,y)dy=0$.

We first need to check if the differential equation is exact by calculating its partial derivatives.

Step 2: Key Formula or Approach:

An equation $M\,dx+N\,dy=0$ is exact if:

$$\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$$

If exact, the general solution is given by:

$$\int M\,dx \text{ (treating y as constant)} + \int (\text{terms in } N \text{ free from } x)\,dy = C$$

Step 3: Detailed Explanation:

Identify M and N from the equation:

$$M=x+2y$$

$$N=2x-y$$

Calculate the partial derivatives:

$$\frac{\partial M}{\partial y}=\frac{\partial}{\partial y}(x+2y)=2$$

$$\frac{\partial N}{\partial x}=\frac{\partial}{\partial x}(2x-y)=2$$

Since $\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}=2$, the differential equation is exact.

Now, integrate M with respect to x (treating y as a constant):

$$\int M\,dx=\int (x+2y)\,dx=\frac{x^2}{2}+2xy$$

Next, integrate the terms in N that do not contain x with respect to y: The only term in $N = 2x - y$ free from x is $-y$.

$$\int (-y)\,dy = -\frac{y^2}{2}$$

Combine the two integrated parts to form the solution:

$$\frac{x^2}{2} + 2xy - \frac{y^2}{2} = C_1$$

To simplify, multiply the entire equation by 2:

$$x^2 + 4xy - y^2 = 2C_1$$

Since $2C_1$ is just another constant, we can denote it as C.

Step 4: Final Answer:

The solution is $x^2 + 4xy - y^2 = C$.

Quick Tip: Always test for exactness first! It is usually much faster and less prone to integration errors than using homogeneous substitutions ($y = vx$), even though both methods work for this specific problem.

---

49. $f(x) = 1 + x\log(x + \sqrt{x^2+1}) - \sqrt{x^2+1}, x \geq 0$ **strictly increasing in**

Correct Answer: $(0, \infty)$

Solution:

Step 1: Understanding the Concept:

A function $f(x)$ is strictly increasing in an interval if its first derivative is strictly positive ($f'(x) > 0$) for all points in that interval.

We need to differentiate the given function and analyze its sign.

Step 2: Key Formula or Approach:

Use the product rule and chain rule for differentiation.

$$\frac{d}{dx}[u \cdot v] = u'v + uv'$$

The condition for strictly increasing is $f'(x) > 0$.

Step 3: Detailed Explanation:

The given function is:

$$f(x) = 1 + x\log(x + \sqrt{x^2+1}) - \sqrt{x^2+1}$$

Differentiate $f(x)$ with respect to x:

$$f'(x) = 0 + \frac{d}{dx}\left[x\log(x + \sqrt{x^2+1})\right] - \frac{d}{dx}\left[\sqrt{x^2+1}\right]$$

Apply the product rule to the second term:

$$f'(x) = \left[1 \cdot \log(x + \sqrt{x^2+1}) + x \cdot \frac{d}{dx}\left(\log(x + \sqrt{x^2+1})\right)\right] - \frac{1}{2\sqrt{x^2+1}} \cdot 2x$$

Evaluate the derivative of the logarithmic term using the chain rule:

$$\frac{d}{dx}\left(\log(x + \sqrt{x^2+1})\right) = \frac{1}{x + \sqrt{x^2+1}} \cdot \left(1 + \frac{1}{2\sqrt{x^2+1}} \cdot 2x\right)$$

$$= \frac{1}{x + \sqrt{x^2+1}} \cdot \left(1 + \frac{x}{\sqrt{x^2+1}}\right)$$

$$= \frac{1}{x + \sqrt{x^2+1}} \cdot \left(\frac{\sqrt{x^2+1} + x}{\sqrt{x^2+1}}\right)$$

Notice that the $(x + \sqrt{x^2+1})$ terms cancel out perfectly:

$$= \frac{1}{\sqrt{x^2+1}}$$

Now, substitute this back into the main $f'(x)$ equation:

$$f'(x) = \log(x + \sqrt{x^2+1}) + x \cdot \left(\frac{1}{\sqrt{x^2+1}}\right) - \frac{x}{\sqrt{x^2+1}}$$

The last two terms cancel each other out:

$$f'(x) = \log(x + \sqrt{x^2+1})$$

For the function to be strictly increasing, we require $f'(x) > 0$:

$$\log(x + \sqrt{x^2+1}) > 0$$

This implies the argument of the logarithm must be greater than 1:

$$x + \sqrt{x^2+1} > 1$$

We are given the domain $x \geq 0$. If $x = 0$, $0 + \sqrt{0+1} = 1$, so $f'(0) = 0$. If $x > 0$, it is clear that $\sqrt{x^2+1} > \sqrt{1} = 1$. Therefore, $x + \sqrt{x^2+1}$ is strictly greater than 1 for all $x > 0$. Thus, $f'(x) > 0$ for all $x > 0$.

Step 4: Final Answer:

The function is strictly increasing in $(0, \infty)$.

Quick Tip: The derivative $\frac{d}{dx}\log(x + \sqrt{x^2+a^2}) = \frac{1}{\sqrt{x^2+a^2}}$ is a standard integration result reversed. Memorizing it saves substantial time when dealing with chain rules in calculus problems.

---

50. $\int \left[\frac{1}{(1+x)^2} - \frac{2}{(1+x)^3}\right] e^x\, dx$

Correct Answer: $\frac{e^x}{(1+x)^2} + C$

Solution:

Step 1: Understanding the Concept:

The integral involves the exponential function e^x multiplied by an algebraic expression consisting of two terms.

This is a classic signature of a specific integration by parts formula where one term is the derivative of the other.

Step 2: Key Formula or Approach:

The standard integral form is:

$$\int e^x[f(x)+f'(x)]dx = e^x f(x)+C$$

We need to identify which part of the bracketed expression is $f(x)$ and verify if the other part is its exact derivative $f'(x)$.

Step 3: Detailed Explanation:

The given integral is:

$$\int e^x\left[\frac{1}{(1+x)^2}-\frac{2}{(1+x)^3}\right]dx$$

Let's choose the first term as our function $f(x)$:

$$f(x)=\frac{1}{(1+x)^2}=(1+x)^{-2}$$

Now, differentiate $f(x)$ with respect to x using the power rule:

$$f'(x)=\frac{d}{dx}\left[(1+x)^{-2}\right]$$

$$f'(x)=-2(1+x)^{-3}\cdot\frac{d}{dx}(1+x)$$

$$f'(x)=-2(1+x)^{-3}\cdot 1=-\frac{2}{(1+x)^3}$$

We can observe that the second term inside the bracket is exactly $f'(x)$.

Therefore, the integral perfectly matches the standard form:

$$\int e^x[f(x)+f'(x)]dx$$

Using the formula, the result is simply $e^x f(x)+C$.

Substitute $f(x)$ back into the result:

$$=e^x\cdot\frac{1}{(1+x)^2}+C$$

$$= \frac{e^x}{(1+x)^2} + C$$

Step 4: Final Answer:

The evaluated integral is $\frac{e^x}{(1+x)^2} + C$.

Quick Tip: Whenever you see an integral of the form $\int e^x[A(x)+B(x)]dx$, almost always assume it falls under the $e^x[f(x)+f'(x)]$ rule. Let the simpler or lower-power term be $f(x)$ and differentiate it to check.

---

51. Perpendicular drawn from origin to the straight line $\sqrt{3}x + y - 24 = 0$ makes an angle α with positive direction of X - axis then $\alpha =$ ______

Correct Answer: $30°$ or $\pi/6$

Solution:

Step 1: Understanding the Concept:

To find the angle α that the perpendicular from the origin makes with the positive x-axis, we need to convert the general equation of the line into its normal form.

Step 2: Key Formula or Approach:

The normal form of a straight line is:

$$x\cos\alpha + y\sin\alpha = p$$

where p is the perpendicular distance from the origin (must be positive), and α is the angle this perpendicular makes with the positive x-axis.

To convert a general equation $Ax + By + C = 0$ to normal form, isolate the constant to make it positive, then divide the entire equation by $\sqrt{A^2+B^2}$.

Step 3: Detailed Explanation:

The given equation of the line is:

$$\sqrt{3}x + y - 24 = 0$$

Move the constant term to the right side to ensure it is positive (which represents distance p):

$$\sqrt{3}x + y = 24$$

Here, $A = \sqrt{3}$ and $B = 1$.

Calculate the dividing factor $\sqrt{A^2 + B^2}$:

$$\sqrt{(\sqrt{3})^2 + (1)^2} = \sqrt{3+1} = \sqrt{4} = 2$$

Divide the entire equation by 2:

$$\frac{\sqrt{3}}{2}x + \frac{1}{2}y = \frac{24}{2}$$

$$\frac{\sqrt{3}}{2}x + \frac{1}{2}y = 12$$

Compare this equation directly with the standard normal form $x\cos\alpha + y\sin\alpha = p$:

$$\cos\alpha = \frac{\sqrt{3}}{2}$$

$$\sin\alpha = \frac{1}{2}$$

$$p = 12$$

Since both $\cos\alpha$ and $\sin\alpha$ are positive, the angle α must lie in the first quadrant.

The angle in the first quadrant that satisfies both conditions is 30°.

In radians, this is $\pi/6$.

Step 4: Final Answer:

The angle α is 30°.

Quick Tip: Always make sure the constant term on the right side of the equals sign is positive before matching $\cos\alpha$ and $\sin\alpha$. If it's negative, multiply the whole equation by -1 first. This ensures you determine the correct quadrant for α.

---

52. Which of the following is not true

1) $f(x) = x|x|$ **differentiable in** $(-1, 1)$

2) $g(x) = \sqrt{|x|}$ **differentiable in** $(4, 5)$

3) $h(x) = |x-2| + |x-3|$ **differentiable in**

4) $k(x) = |x+1| + |x-6|$ **differentiable in**$(-1, 6)$

5) $f(x) = x + [x]$ **differentiable in x =**

Correct Answer: 3 and 5 due to incomplete text, but evaluating provided logic, 1, 2, and 4 are definitively True.

Solution:

Step 1: Understanding the Concept:

We need to analyze the differentiability of functions involving absolute values and the greatest integer function $[x]$ over specific intervals.

A function is not differentiable at points where it has a sharp corner (like the vertex of an absolute value function) or a discontinuity (like step functions).

Step 2: Key Formula or Approach:

For $|x-a|$, the function is continuous everywhere but not differentiable at the critical point $x = a$.

If the interval provided does not include the critical point, the function behaves as a smooth polynomial and is differentiable.

Step 3: Detailed Explanation:

Let us evaluate each option:

- **Option 1:** $f(x) = x|x|$.

  We can rewrite this piecewise: $f(x) = x^2$ for $x \geq 0$, and $f(x) = -x^2$ for $x < 0$.

  The derivative is $f'(x) = 2x$ for $x \geq 0$ and $f'(x) = -2x$ for $x < 0$.

At $x = 0$, the Left Hand Derivative (LHD) is $-2(0) = 0$, and the Right Hand Derivative (RHD) is $2(0) = 0$. Since LHD = RHD, it is differentiable at $x = 0$. It is therefore differentiable everywhere in $(-1, 1)$. **(True)**

- **Option 2:** $g(x) = \sqrt{|x|}$.
  In the given interval $(4, 5)$, x is strictly positive.
  Therefore, $|x| = x$, and the function simplifies to $g(x) = \sqrt{x}$.
  The derivative is $\frac{1}{2\sqrt{x}}$, which exists for all x in $(4, 5)$. **(True)**

- **Option 4:** $k(x) = |x+1| + |x-6|$ in the interval $(-1, 6)$.
  The critical points are $x = -1$ and $x = 6$, but the open interval $(-1, 6)$ strictly excludes these points.
  For any $x \in (-1, 6)$, $x+1 > 0$ (so $|x+1| = x+1$) and $x-6 < 0$ (so $|x-6| = -(x-6) = 6-x$).
  Substitute these into the function: $k(x) = (x+1) + (6-x) = 7$.
  Since $k(x)$ is a constant function in this interval, its derivative is exactly 0 everywhere in $(-1, 6)$. It is highly differentiable. **(True)**

Step 4: Final Answer:

Options 1, 2, and 4 are mathematically true. The false statement is among the incompletely printed options 3 or 5.

Quick Tip: For sum of absolute values like $|x-a| + |x-b|$, the function forms a flat "bucket" shape between a and b. It is constant and therefore perfectly differentiable strictly *inside* the interval (a, b), but non-differentiable exactly *at* the corners a and b.

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53. $f(x) = \begin{cases} \frac{2x^2+3x-5}{x-1}, & x \neq 1 \\ k, & x = 1 \end{cases}$ **is continuous at** $x = 1$**, then** $k =$

Correct Answer: 7

Solution:

Step 1: Understanding the Concept:

For a piecewise function to be continuous at a specific point $x = a$, the limit of the function as x approaches a must exactly equal the defined value of the function at that point. Mathematically, $\lim_{x \to a} f(x) = f(a)$.

Step 2: Key Formula or Approach:

We must calculate the limit of the upper piece as x approaches 1 and equate it to the lower piece.

$$\lim_{x \to 1} \frac{2x^2 + 3x - 5}{x - 1} = k$$

Since direct substitution yields a $\frac{0}{0}$ indeterminate form, we factorize the numerator to cancel the problematic denominator.

Step 3: Detailed Explanation:

Set up the limit equation for continuity at $x = 1$:

$$k = \lim_{x \to 1} \frac{2x^2 + 3x - 5}{x - 1}$$

Let's factor the quadratic numerator: $2x^2 + 3x - 5$.

We are looking for two numbers that multiply to $(2)(-5) = -10$ and add to 3. These numbers are 5 and -2.

Rewrite the middle term:

$$2x^2 - 2x + 5x - 5$$

Factor by grouping:

$$2x(x-1) + 5(x-1) = (2x+5)(x-1)$$

Now, substitute the factored form back into the limit:

$$k = \lim_{x \to 1} \frac{(2x+5)(x-1)}{x-1}$$

Since $x \to 1$ implies x is getting arbitrarily close to 1 but is not exactly equal to 1 $(x \neq 1)$, we can safely cancel the $(x - 1)$ terms from the numerator and denominator:

$$k = \lim_{x \to 1}(2x + 5)$$

Now evaluate the limit by direct substitution of $x = 1$:

$$k = 2(1) + 5$$

$$k = 7$$

Step 4: Final Answer:

The value of k is 7.

Quick Tip: Alternatively, you can use L'Hopital's Rule for $\frac{0}{0}$ limits. Differentiate the top and bottom separately: $\lim_{x\to 1} \frac{\frac{d}{dx}(2x^2+3x-5)}{\frac{d}{dx}(x-1)} = \lim_{x\to 1} \frac{4x+3}{1} = 4(1) + 3 = 7$. It's incredibly fast for polynomials.
