JEE Main 2024 April 5 Shift 1 Mathematics Question Paper

Time Allowed :1 Hour	**Maximum Marks :**100	**Total Questions :**30

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The test is of 3 hours duration.
2. The question paper consists of 90 questions. The maximum marks are 300.
3. There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage.
4. Each part (subject) has two sections.
(i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and −1 mark for wrong answer.
(ii) Section-B: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and −1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

Mathematics

Question 1: Let d be the distance of the point of intersection of the lines:

$$\frac{x+6}{3} = \frac{y}{2} = \frac{z+1}{1} \quad \text{and} \quad \frac{x-7}{4} = \frac{y-9}{3} = \frac{z-4}{2}$$

from the point $(7, 8, 9)$. Then $d^2 + 6$ is equal to:

Options:
(1) 72
(2) 69
(3) 75
(4) 78

Question 2: Let a rectangle $ABCD$ of sides 2 and 4 be inscribed in another rectangle $PQRS$ such that the vertices of the rectangle $ABCD$ lie on the sides of the rectangle $PQRS$. Let a and b be the sides of the rectangle $PQRS$ when its area is maximum. Then $(a+b)^2$ is equal to:

Options:
(1) 72

(2) 60
(3) 80
(4) 64

Question 3: Let two straight lines drawn from the origin O intersect the line $3x + 4y = 12$ at the points P and Q such that $\triangle OPQ$ is an isosceles triangle and $\angle POQ = 90^\circ$. If $I = OP^2 + PQ^2 + OQ^2$, then the greatest integer less than or equal to I is:

Options:
(1) 44
(2) 48
(3) 46
(4) 42

Question 4: If $y = y(x)$ is the solution of the differential equation:

$$\frac{dy}{dx} + 2y = \sin(2x), \quad y(0) = \frac{3}{4},$$

then $y\left(\frac{\pi}{8}\right)$ is equal to:

Options:
(1) $e^{-\pi/8}$
(2) $e^{-\pi/4}$
(3) $e^{\pi/4}$
(4) $e^{\pi/8}$

Question 5: For the function:

$$f(x) = \sin x + 3x - \frac{2}{\pi}(x^2 + x), \quad \text{where } x \in \left[0, \frac{\pi}{2}\right],$$

consider the following two statements:
(I) f is increasing in $\left(0, \frac{\pi}{2}\right)$.
(II) f' is decreasing in $\left(0, \frac{\pi}{2}\right)$.
Between the above two statements,
(1) only (I) is true.
(2) only (II) is true.
(3) neither (I) nor (II) is true.
(4) both (I) and (II) are true.

Question 6: If the system of equations:

$$11x + y + \lambda z = -5,$$

$$2x + 3y + 5z = 3,$$

$$8x - 19y - 39z = \mu$$

has infinitely many solutions, then $\lambda^4 - \mu$ is equal to:

Options:
(1) 49
(2) 45
(3) 47
(4) 51

Question 7: Let $A = \{1, 3, 7, 9, 11\}$ and $B = \{2, 4, 5, 7, 8, 10, 12\}$. Then the total number of one-one maps $f : A \to B$, such that $f(1) + f(3) = 14$, is:

Options:
(1) 180
(2) 120
(3) 480
(4) 240

Question 8: If the function

$$f(x) = \frac{\sin 3x + \alpha \sin x - \beta \cos 3x}{x^3},$$

is continuous at $x = 0$, then $f(0)$ is equal to:

Options:
(1) 2
(2) -2
(3) 4
(4) -4

Question 9: The integral

$$\int_0^{\pi/4} \frac{136 \sin x}{3 \sin x + 5 \cos x} \, dx$$

is equal to:

Options:
(1) $3\pi - 50 \log_e 2 + 20 \log_e 5$
(2) $3\pi - 25 \log_e 2 + 10 \log_e 5$
(3) $3\pi - 10 \log_e (2\sqrt{2}) + 10 \log_e 5$
(4) $3\pi - 30 \log_e 2 + 20 \log_e 5$

Question 10: The coefficients a, b, c in the quadratic equation $ax^2 + bx + c = 0$ are chosen from the set $\{1, 2, 3, 4, 5, 6, 7, 8\}$. The probability of this equation having repeated roots is:

Options:
(1) $\frac{3}{256}$
(2) $\frac{1}{128}$
(3) $\frac{1}{64}$
(4) $\frac{3}{128}$

Question 11: Let A and B be two square matrices of order 3 such that $|A| = 3$ and $|B| = 2$. Then:

$$\left|A^T A(\text{adj}(2A))^{-1}(\text{adj}(4B))(\text{adj}(AB))^{-1}AA^T\right|$$

is equal to:

Options:
(1) 64
(2) 81
(3) 32
(4) 108

Question 12: Let a circle C of radius 1 and closer to the origin be such that the lines passing through the point $(3, 2)$ and parallel to the coordinate axes touch it. Then the shortest distance of the circle C from the point $(5, 5)$ is:

Options:
(1) $2\sqrt{2}$
(2) 5
(3) $4\sqrt{2}$
(4) 4

Question 13: Let the line $2x + 3y - k = 0$, $k > 0$, intersect the x-axis and y-axis at the points A and B, respectively. If the equation of the circle having the line segment AB as a diameter is:

$$x^2 + y^2 - 3x - 2y = 0,$$

and the length of the latus rectum of the ellipse:

$$x^2 + 9y^2 = k^2$$

is $\frac{m}{n}$, where m and n are coprime, then $2m + n$ is equal to:

Options:
(1) 10
(2) 11

(3) 13
(4) 12

Question 14: Consider the following two statements:
Statement I: For any two non-zero complex numbers z_1, z_2,

$$(|z_1| + |z_2|) \frac{|z_1|}{|z_1|} + \frac{|z_2|}{|z_2|} \leq 2 (|z_1| + |z_2|) .$$

Statement II: If x, y, z are three distinct complex numbers and a, b, c are three positive real numbers such that:

$$\frac{a}{|y-z|} = \frac{b}{|z-x|} = \frac{c}{|x-y|},$$

then:

$$\frac{a^2}{|y-z|} + \frac{b^2}{|z-x|} + \frac{c^2}{|x-y|} = 1.$$

Between the above two statements:

(1) Both Statement I and Statement II are incorrect.

(2) Statement I is incorrect but Statement II is correct.

(3) Statement I is correct but Statement II is incorrect.

(4) Both Statement I and Statement II are correct.

Question 15: Suppose $\theta \in \left[0, \frac{\pi}{4}\right]$ is a solution of $4\cos\theta - 3\sin\theta = 1$. Then $\cos\theta$ is equal to:

Options:
(1) $\frac{4}{3\sqrt{6}-2}$
(2) $\frac{6-\sqrt{6}}{3\sqrt{6}-2}$
(3) $\frac{6+\sqrt{6}}{3\sqrt{6}+2}$
(4) $\frac{4}{3\sqrt{6}+2}$

Question 16: If

$$\frac{1}{\sqrt{1}+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \cdots + \frac{1}{\sqrt{99}+\sqrt{100}} = m,$$

and

$$\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \cdots + \frac{1}{99\cdot 100} = n,$$

then the point (m, n) lies on the line:
Options:

(1) $11(x-1) - 100(y-2) = 0$

(2) $11(x-2) - 100(y-1) = 0$

(3) $11(x-1) - 100y = 0$

(4) $11x - 100y = 0$

Question 17: Let $f(x) = x^5 + 2x^3 + 3x + 1$, $x \in R$, and $g(x)$ be a function such that $g(f(x)) = x$ for all $x \in R$. Then:

$$\frac{g(7)}{g'(7)}$$

is equal to:

Options:
(1) 7
(2) 42
(3) 1
(4) 14

Question 18: If $A(1, -1, 2)$, $B(5, 7, -6)$, $C(3, 4, -10)$, and $D(-1, -4, -2)$ are the vertices of a quadrilateral $ABCD$, then its area is:

Options:
(1) $12\sqrt{29}$
(2) $24\sqrt{29}$
(3) $24\sqrt{7}$
(4) $48\sqrt{7}$

Question 19: The value of the integral:

$$\int_{-\pi}^{\pi} \frac{2y(1+\sin y)}{1+\cos^2 y}\, dy$$

is:

Options:
(1) π^2
(2) $\frac{\pi^2}{2}$
(3) $\frac{\pi}{2}$
(4) $2\pi^2$

Question 20: If the line

$$\frac{2-x}{3} = \frac{3y-2}{4\lambda+1} = 4 - z$$

makes a right angle with the line:

$$\frac{x+3}{3\mu} = \frac{1-2y}{6} = \frac{5-z}{7},$$

then $4\lambda + 9\mu$ is equal to:

Options:
(1) 13
(2) 4
(3) 5
(4) 6

Question 21: From a lot of 10 items, which include 3 defective items, a sample of 5 items is drawn at random. Let the random variable X denote the number of defective items in the sample. If the variance of X is σ^2, then $96\sigma^2$ is equal to __________.

Question 22: If the constant term in the expansion of

$$(1 + 2x - 3x^3)\left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9$$

is p, then $108p$ is equal to __________.

Question 23: The area of the region enclosed by the parabolas

$$y = x^2 - 5x \quad \textbf{and} \quad y = 7x - x^2$$

is __________.

Question 24: The number of ways of getting a sum 16 on throwing a dice four times is __________.

Question 25: If

$$S = \{a \in R : |2a - 1| = 3\lfloor a \rfloor + 2\{a\}\},$$

where $\lfloor t \rfloor$ denotes the greatest integer less than or equal to t, and $\{t\}$ represents the fractional part of t, then

$$72\sum_{a\in S} a \quad \textbf{is equal to} \quad \_\_\_\_\_\_\_\_\_\_.$$

Question 26: Let f be a differentiable function in the interval $(0, \infty)$ such that $f(1) = 1$ and

$$\lim_{t\to x} \frac{t^2 f(x) - x^2 f(t)}{t - x} = 1$$

for each $x > 0$. Then $2f(2) + 3f(3)$ is equal to __________.

Question 27: Let $a_1, a_2, a_3, \ldots$ be in an arithmetic progression of positive terms. Let

$$A_k = a_1^2 - a_2^2 + a_3^2 - a_4^2 + \cdots + a_{2k-1}^2 - a_{2k}^2.$$

If $A_3 = -153$, $A_5 = -435$, and $a_1^2 + a_2^2 + a_3^2 = 66$, then $a_{17} - A_7$ is equal to ___________.

Question 29: The number of distinct real roots of the equation

$$|x||x+2| - 5|x+1| - 1 = 0$$

is ___________.

Question 30: Suppose AB is a focal chord of the parabola $y^2 = 12x$ of length l and slope $m < \sqrt{3}$. If the distance of the chord AB from the origin is d, then ld^2 is equal to _________.

Correct Answer: 108
