JEE MAIN 2024 1 Feb Shift-I Question Paper

Total Time Allowed : 180 minutes	Maximum Marks : 300	Total Questions : 90	Number of questions to be answered : 75

SECTION-A

Question 1: A bag contains 8 balls, whose colours are either white or black. 4 balls are drawn at random without replacement and it was found that 2 balls are white and other 2 balls are black. The probability that the bag contains equal number of white and black balls is:

(1) $\frac{2}{5}$
(2) $\frac{2}{7}$
(3) $\frac{1}{7}$
(4) $\frac{1}{5}$

Question 2: The value of the integral

$$\int_0^{\frac{\pi}{4}} \frac{x\,dx}{\sin^4(2x)+\cos^4(2x)} \text{ equals:}$$

(1) $\frac{\sqrt{2}\pi^2}{8}$
(2) $\frac{\sqrt{2}\pi^2}{16}$
(3) $\frac{\sqrt{2}\pi^2}{32}$
(4) $\frac{\sqrt{2}\pi^2}{64}$

Question 3: If

$$A=\begin{bmatrix}\sqrt{2} & 1\\ -1 & \sqrt{2}\end{bmatrix}, \quad B=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}, \quad C=ABA^T \textbf{ and } X=A^TC^2A,$$

then $\det(X)$ **is equal to:**

(1) 243
(2) 729
(3) 27
(4) 891

Question 4: If $\tan A=\frac{1}{\sqrt{x(x^2+x+1)}}$, $\tan B=\frac{\sqrt{x}}{\sqrt{x^2+x+1}}$
and

$\tan C = \left(x^3 + x^2 + x\right)^{\frac{1}{2}}$, $0 < A, B, C < \frac{\pi}{2}$, **then**
$A + B$ is equal to:

$$(1)\ C (2)\ \pi - C (3)\ 2\pi - C (4)\ \frac{\pi}{2} - C$$

Q5: If n is the number of ways five different employees can sit into four indistinguishable offices where any office may have any number of persons including zero, then n is equal to:
(1) 47
(2) 53
(3) 51
(4) 43

Question 6: Let $S = \{z \in C : |z-1| = 1$ **and** $(\sqrt{2}-1)(z+\bar{z}) = (\bar{z}-z) = 2\sqrt{2}\}$**. Let** $z_1, z_2 \in S$ **be such that** $|z_1| = \max_{z\in S}|z|$ **and** $|z_2| = \min_{z\in S}|z|$**. Then** $\sqrt{|z_1 - z_2|^2}$ **equals:**
(1) 1
(2) 4
(3) 3
(4) 2

Question 7: Let the median and the mean deviation about the median of 7 observations 170, 125, 230, 190, 210, a, b be 170 and $\frac{205}{7}$ **respectively. Then the mean deviation about the mean of these 7 observations is:**
(1) 31
(2) 28
(3) 30
(4) 32

Question 8: Let $\vec{a} = -5\vec{i} + 3\vec{j} - 3\vec{k}$, $\vec{b} = \vec{i} + 2\vec{j} - 4\vec{k}$ **and**

$$\vec{c} = \left(\left((\vec{a} \times \vec{b}) \times \vec{i}\right) \times \vec{i}\right)$$

Then $\vec{c} \cdot \left(-\vec{i} + \vec{j} + \vec{k}\right)$ is equal to:
(1) -12
(2) -10
(3) -13
(4) -15

Question 9: Let

$$S = \{x \in R : (\sqrt{3}+\sqrt{2})^x + (\sqrt{3}-\sqrt{2})^x = 10\}$$

Then the number of elements in S is:
(1) 4
(2) 0
(3) 2
(4) 1

Question 10: The area enclosed by the curves $xy + 4y = 16$ and $x + y = 6$ is equal to:
(1) $28 - 30\log 2$
(2) $30 - 28\log 2$
(3) $30 - 32\log 2$
(4) $32 - 30\log 2$

Question 11: Let $f : R \to R$ and $g : R \to R$ be defined as

$$f(x) = \begin{cases} \log_e x, & x > 0 \\ e^{-x}, & x \le 0 \end{cases} \quad \text{and} \quad g(x) = \begin{cases} x, & x \ge 0 \\ e^{x}, & x < 0 \end{cases}$$

Then $g \circ f : R \to R$ is:
(1) one-one but not onto
(2) neither one-one nor onto
(3) onto but not one-one
(4) both one-one and onto

Question 12: If the system of equations

$$2x + 3y - z = 5$$
$$x + \alpha y + 3z = -4$$
$$3x - y + \beta z = 7$$

has infinitely many solutions, then $13\alpha\beta$ is equal to:
(1) 1110
(2) 1120
(3) 1210

(4) 1220

Question 13: For $0 < \theta < \frac{\pi}{2}$, if the eccentricity of the hyperbola $x^2 - y^2 cosec^2\theta = 5$ is $\sqrt{7}$ times the eccentricity of the ellipse $x^2 cosec^2\theta + y^2 = 5$, then the value of θ is:

(1) $\frac{\pi}{6}$
(2) $\frac{5\pi}{12}$
(3) $\frac{\pi}{3}$
(4) $\frac{\pi}{4}$

Question 14: Let $y = y(x)$ be the solution of the differential equation

$$\frac{dy}{dx} = 2x(x+y)^3 - x(x+y) - 1, \quad y(0) = 1$$

Then, $\left(\frac{1}{\sqrt{2}} + y\left(\frac{1}{\sqrt{2}}\right)\right)^2$ equals:

(1) $\frac{4}{4+\sqrt{e}}$
(2) $\frac{3}{3-\sqrt{e}}$
(3) $\frac{2}{1+\sqrt{e}}$
(4) $\frac{1}{2-\sqrt{e}}$

Question 15: Let $f : R \to R$ be defined as

$$f(x) = \begin{cases} \frac{a - b\cos 2x}{x^2}, & x < 0 \\ x^2 + cx + 2, & 0 \leq x \leq 1 \\ 2x + 1, & x > 1 \end{cases}$$

If f is continuous everywhere in R and m is the number of points where f is NOT differentiable, then $m + a + b + c$ equals:

(1) 1
(2) 4
(3) 3
(4) 2

Question 16: Let $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where $a > b$ is an ellipse, whose eccentricity is $\frac{1}{\sqrt{2}}$ and the length of the latus rectum is $\sqrt{14}$. Then the square of the eccentricity of $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is:

(1) $\frac{3}{2}$
(2) $\frac{7}{2}$
(3) $\frac{3}{2}$
(4) $\frac{5}{2}$

Question 17: Let $3, a, b, c$ be in A.P. and $3, a-1, b+1, c+9$ be in G.P. Then, the arithmetic mean of a, b, c is:
(1) -4
(2) -1
(3) 13
(4) 11

Question 18: Let $C_1 : x^2+y^2=4$ and $C_2 : x^2+y^2-4x+9=0$ be two circles. If the set of all values of x so that the circles C_1 and C_2 intersect at two distinct points lies in the interval $R=[a,b]$, then the point $(8a+12, 16b-20)$ lies on the curve:
(1) $x^2+2y^2-5x+6y=3$
(2) $5x^2-y=-11$
(3) $x^2-4y^2=7$
(4) $6x^2+y^2=42$

Question 19: If $5f(x)+4\left(\frac{1}{x}\right)=x^2-2$, $x \neq 0$ and $y=9x^2f(x)$, then y is strictly increasing in:
(1) $\left(0, \frac{1}{\sqrt{5}}\right) \cup \left(\frac{1}{\sqrt{5}}, \infty\right)$
(2) $\left(-\frac{1}{\sqrt{5}}, 0\right) \cup \left(\frac{1}{\sqrt{5}}, \infty\right)$
(3) $\left(0, \frac{1}{\sqrt{5}}\right) \cup \left(0, \frac{1}{\sqrt{5}}\right)$
(4) $\left(-\infty, \frac{1}{\sqrt{5}}\right) \cup \left(0, \frac{1}{\sqrt{5}}\right)$

Question 20: If the shortest distance between the lines

$$\frac{x-\lambda}{2}=\frac{y-2}{1}=\frac{z-1}{1}$$

and

$$\frac{x-\frac{1}{\sqrt{3}}}{1}=\frac{y-1}{-2}=\frac{z-2}{1}$$

is 1, then the sum of all possible values of λ is:
(1) 0
(2) $2\sqrt{3}$
(3) $3\sqrt{3}$
(4) $-2\sqrt{3}$

Question 21: If $x=x(t)$ is the solution of the differential equation

$$(t+1)dx = (2x + (t+1)^4)dt, \quad x(0) = 2$$

then, $x(1)$ equals:

Question 22: The number of elements in the set
$S = \{(x, y, z) : x, y, z \in Z, x + 2y + 3z = 42, x, y, z \geq 0\}$ equals __________.

Question 23: If the coefficient of x^{30} in the expansion of

$$\left(1 + \frac{1}{x}\right)^6 \left(1 + x^2\right)^7 \left(1 - x^3\right)^8$$

is α, then $|\alpha|$ equals:

Question 24: Let $3, 7, 11, 15, \ldots, 403$ and $2, 5, 8, 11, \ldots, 404$ be two arithmetic progressions. Then the sum of the common terms in them is equal to _____.

Question 25: Let $\{x\}$ denote the fractional part of x and

$$f(x) = \cos^{-1}(1 - \{x\}^2)\sin^{-1}(1 - \{x\}), \quad x \neq 0.$$

If L and R respectively denote the left-hand limit and the right-hand limit of $f(x)$ at $x = 0$, then

$$\frac{32}{\pi^2}(L^2 + R^2)$$

is equal to __.

Question 26: Let the line $L : \sqrt{2}x + y = \alpha$ pass through the point of the intersection P (in the first quadrant) of the circle $x^2 + y^2 = 3$ and the parabola $x^2 = 2y$. Let the line L touch two circles C_1 and C_2 of equal radius $2\sqrt{5}$. If the centers Q_1 and Q_2 of the circles C_1 and C_2 lie on the y-axis, then the square of the area of the triangle PQ_1Q_2 is equal to ______.

Question 27: Let $P = \{z \in C : |z + 2 - 3i| \leq 1\}$ and $Q = \{z \in C : |z - (1 - 5i)| < 8\}$. Let in $P \cap Q, |z - 3 + 2i|$ be maximum and minimum at z_1 and z_2 respectively. If $|z_1|^2 + |z_2|^2 = \alpha + \beta\sqrt{5}$, where α, β are integers, then $\alpha + \beta$ equals ____.

Q28: If

$$\int_0^{\frac{\pi}{2}} \frac{8\sqrt{2}\cos x\,dx}{(1+e^{\sin x})(1+\sin^4 x)} = \alpha x + \beta \log(3+2\sqrt{2}),$$

where α, β are integers, then $\alpha^2 + \beta^2$ equals __.

Question 29: Let the line of the shortest distance between the lines

$$L_1 : \mathbf{r} = (1 + 2j + 3k) + \lambda(i - j + k) \quad \textbf{and} \quad L_2 : \mathbf{r} = (-4i + 5j + 6k) + \mu(i + j - k)$$

intersect L_1 and L_2 at P and Q respectively. If α, β, γ is the midpoint of the line segment PQ, then $2(\alpha + \beta + \gamma)$ is equal to _____.

Question 30: Let $A = \{1, 2, 3, \ldots, 20\}$. Let R_1 and R_2 be two relations on A such that

$$R_1 = \{(a, b) : b \textbf{ is divisible by } a\}$$

$$R_2 = \{(a, b) : a \textbf{ is an integral multiple of } b\}$$

Then, the number of elements in $R_1 - R_2$ is equal to:

Question 31: How does the Young's modulus of elasticity change with an increase in temperature?
(1) Varies unpredictably
(2) Decreases
(3) Increases
(4) Remains constant

Question 32: If the radius of the Earth is R and the acceleration due to gravity on the Earth's surface is $g = \pi^2\,\mathrm{m/s^2}$, what will be the length of the second's pendulum at a height $h = 2R$ from the surface?
(1) $\frac{2}{9}$ m
(2) $\frac{1}{9}$ m
(3) $\frac{4}{9}$ m
(4) $\frac{8}{9}$ m

Question 33: In the circuit shown, if the power rating of the Zener diode is 10 mW, what is the value of the series resistance R_s required to regulate the input unregulated supply?