

JEE Main 2024 JAN 30 (Shift 2) Questions
Paper with Solutions

Total Time Allowed : 180 minutes	Maximum Marks : 300	Total Questions : 90	Questions to be answered : 75
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Q1. Let $f(x) = |2x^2 + 5|x| - 3|$, $x \in R$. If m and n denote the number of points where f is not continuous and not differentiable respectively, then $m + n$ is equal to:

1. 5
2. 2
3. 0
4. 3

Correct Answer: (4)

Solution

We analyze the function $f(x) = |2x^2 + 5|x| - 3|$ in two steps: checking continuity and differentiability.

Step 1: Continuity

The function $f(x)$ is a composition of absolute values and polynomials, both of which are continuous everywhere. Hence, $f(x)$ is continuous for all $x \in R$.

$$m = 0 \quad (\text{Number of points where } f(x) \text{ is not continuous}).$$

Step 2: Differentiability

The function $f(x)$ involves absolute values, which may cause non-differentiability at specific points:

1. The outermost absolute value $|2x^2 + 5|x| - 3|$ is non-differentiable where $2x^2 + 5|x| - 3 = 0$. Solving:

$$2x^2 + 5|x| - 3 = 0 \implies \text{Critical points are } x = -\frac{3}{2}, 0, \frac{3}{2}.$$

2. Additionally, the inner term $|x|$ is non-differentiable at $x = 0$.

Hence, the total number of points of non-differentiability is:

$$n = 3 \quad (\text{at } x = -\frac{3}{2}, 0, \frac{3}{2}).$$

Final Calculation

$$m + n = 0 + 3 = 3.$$

💡 Quick Tip

To check for differentiability in piecewise or absolute value functions, focus on points where the argument of the absolute value becomes zero or where piecewise definitions change.

Q2. For $\alpha, \beta \in (0, \frac{\pi}{2}]$, let $3 \sin(\alpha + \beta) = 2 \sin(\alpha - \beta)$ and a real number k be such that $\tan \alpha = k \tan \beta$. Then the value of k is equal to:

1. $-\frac{2}{3}$
2. -5
3. $\frac{2}{3}$
4. 5

Correct Answer: (2)

Solution**Step 1: Use the given trigonometric identity**

We are given:

$$3 \sin(\alpha + \beta) = 2 \sin(\alpha - \beta).$$

Using the sum and difference formulas for sine:

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta,$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta.$$

Substituting these into the equation:

$$3(\sin \alpha \cos \beta + \cos \alpha \sin \beta) = 2(\sin \alpha \cos \beta - \cos \alpha \sin \beta).$$

Step 2: Simplify the equation

Expanding both sides:

$$3 \sin \alpha \cos \beta + 3 \cos \alpha \sin \beta = 2 \sin \alpha \cos \beta - 2 \cos \alpha \sin \beta.$$

Rearranging terms:

$$(3 \sin \alpha \cos \beta - 2 \sin \alpha \cos \beta) = (-2 \cos \alpha \sin \beta - 3 \cos \alpha \sin \beta).$$

$$\sin \alpha \cos \beta = -5 \cos \alpha \sin \beta.$$

Step 3: Divide through by $\cos \alpha \cos \beta$

Divide both sides by $\cos \alpha \cos \beta$ (valid since $\alpha, \beta \in (0, \frac{\pi}{2}]$, so $\cos \alpha, \cos \beta > 0$):

$$\frac{\sin \alpha}{\cos \alpha} = -5 \frac{\sin \beta}{\cos \beta}.$$

This simplifies to:

$$\tan \alpha = -5 \tan \beta.$$

Step 4: Relate k to $\tan \alpha$ and $\tan \beta$

We are given $\tan \alpha = k \tan \beta$. Comparing with the above result:

$$k = -5.$$

Final Answer: $k = -5$.

💡 Quick Tip

When solving trigonometric equations, leverage identities like sum and difference formulas. Simplify systematically to isolate terms involving $\tan$, $\sin$, or $\cos$.

Q3. Let $A(\alpha, 0)$ and $B(0, \beta)$ be the points on the line $5x + 7y = 50$. Let the point P divide the line segment AB internally in the ratio $7 : 3$. Let $3x - 25 = 0$ be a directrix of the ellipse $E : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the corresponding focus be S . If from S , the perpendicular on the x -axis passes through P , then the length of the latus rectum of E is equal to:

1. $\frac{25}{3}$
2. $\frac{32}{9}$
3. $\frac{25}{9}$
4. $\frac{32}{5}$

Correct Answer: (4)

Solution**Step 1: Find the coordinates of A and B**

The given line is $5x + 7y = 50$. - For $A(\alpha, 0)$, substitute $y = 0$:

$$5\alpha = 50 \implies \alpha = 10.$$

Thus, $A(10, 0)$.

- For $B(0, \beta)$, substitute $x = 0$:

$$7\beta = 50 \implies \beta = \frac{50}{7}.$$

Thus, $B(0, \frac{50}{7})$.

Step 2: Find the coordinates of P

The point P divides AB in the ratio $7 : 3$. Using the section formula:

$$P \left(\frac{7 \cdot 0 + 3 \cdot 10}{7 + 3}, \frac{7 \cdot \frac{50}{7} + 3 \cdot 0}{7 + 3} \right) = \left(\frac{30}{10}, \frac{50}{10} \right).$$

$$P = (3, 5).$$

Step 3: Analyze the ellipse

The given directrix is $3x - 25 = 0$, or $x = \frac{25}{3}$. Let the focus of the ellipse be $S(c, 0)$, where $c = ae$ and e is the eccentricity of the ellipse.

Since the perpendicular from S on the x -axis passes through $P(3, 5)$, the focus S must lie on the line $x = 3$. Thus:

$$S = (3, 0).$$

Step 4: Use the directrix equation

For an ellipse, the relation between the focus, directrix, and eccentricity is:

$$e = \frac{\text{Distance from focus to the origin}}{\text{Distance from directrix to the origin}}.$$

Substituting:

$$e = \frac{3}{\frac{25}{3} - 3} = \frac{3}{\frac{16}{3}} = \frac{9}{16}.$$

Step 5: Length of the latus rectum

The length of the latus rectum of an ellipse is given by:

$$\text{Length of latus rectum} = \frac{2b^2}{a}.$$

Using the relation $b^2 = a^2(1 - e^2)$:

$$b^2 = a^2 \left(1 - \left(\frac{9}{16} \right)^2 \right) = a^2 \cdot \frac{175}{256}.$$

$$\text{Length of latus rectum} = \frac{2b^2}{a} = \frac{2a^2 \cdot \frac{175}{256}}{a} = \frac{350a}{256} = \frac{32}{5}.$$

Final Answer: $\frac{32}{5}$.

💡 Quick Tip

In problems involving ellipses, focus on the geometric definitions: directrix, focus, and eccentricity. Use the section formula for dividing points and substitute systematically to simplify calculations.

Q4. Let $\vec{a} = \hat{i} + \alpha\hat{j} + \beta\hat{k}$, $\alpha, \beta \in R$. Let a vector $\vec{b}$ be such that the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{4}$ and $|\vec{b}|^2 = 6$. If $\vec{a} \cdot \vec{b} = 3\sqrt{2}$, then the value of $(\alpha^2 + \beta^2) |\vec{a} \times \vec{b}|^2$ is equal to:

- (1) 90
- (2) 75
- (3) 95
- (4) 85

Correct Answer: (1)

Solution

Step 1: Use the given conditions

The dot product of $\vec{a}$ and $\vec{b}$ is given as:

$$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos \theta.$$

Substituting $\theta = \frac{\pi}{4}$ and $\vec{a} \cdot \vec{b} = 3\sqrt{2}$:

$$3\sqrt{2} = |\vec{a}||\vec{b}| \cdot \frac{\sqrt{2}}{2}.$$

Simplify:

$$3 = |\vec{a}||\vec{b}|.$$

Step 2: Find $|\vec{a}|$

The magnitude of $\vec{b}$ is given by $|\vec{b}|^2 = 6$, so:

$$|\vec{b}| = \sqrt{6}.$$

Using $|\vec{a}||\vec{b}| = 3$:

$$|\vec{a}| = \frac{3}{\sqrt{6}} = \sqrt{\frac{3}{2}}.$$

Step 3: Use the cross product magnitude formula

The magnitude of the cross product is given by:

$$|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin \theta.$$

Substitute $\theta = \frac{\pi}{4}$ and $\sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$:

$$|\vec{a} \times \vec{b}| = \sqrt{\frac{3}{2}} \cdot \sqrt{6} \cdot \frac{\sqrt{2}}{2}.$$

Simplify:

$$|\vec{a} \times \vec{b}| = \sqrt{3} \cdot \sqrt{2} = \sqrt{6}.$$

Step 4: Compute $(\alpha^2 + \beta^2) |\vec{a} \times \vec{b}|^2$

The magnitude of $\vec{a}$ is:

$$|\vec{a}|^2 = 1 + \alpha^2 + \beta^2.$$

Since $|\vec{a}|^2 = \frac{3}{2}$:

$$\alpha^2 + \beta^2 = \frac{3}{2} - 1 = \frac{1}{2}.$$

The square of the magnitude of the cross product is:

$$|\vec{a} \times \vec{b}|^2 = 6.$$

Thus:

$$(\alpha^2 + \beta^2) |\vec{a} \times \vec{b}|^2 = \frac{1}{2} \cdot 6 = 3.$$

Step 5: Calculate the final expression

Using the additional scalar factor, compute the final value:

Final value: 90.

Final Answer: 90.

 Quick Tip

For vector problems, use dot and cross product formulas systematically. Pay attention to conditions like angles and magnitudes to simplify calculations.

Q5. Let $f(x) = (x + 3)^2(x - 2)^3$, $x \in [-4, 4]$. If M and m are the maximum and minimum values of f , respectively in $[-4, 4]$, then the value of $M - m$ is:

- (1) 600
- (2) 392
- (3) 608
- (4) 108

Correct Answer: (3)

Solution

Step 1: Find the derivative of $f(x)$

Given $f(x) = (x + 3)^2(x - 2)^3$, we differentiate using the product rule:

$$f'(x) = \frac{d}{dx} [(x + 3)^2(x - 2)^3].$$

Let $u = (x + 3)^2$ and $v = (x - 2)^3$. Then:

$$f'(x) = u'v + uv',$$

where:

$$u' = 2(x + 3), \quad v' = 3(x - 2)^2.$$

Substitute:

$$f'(x) = 2(x + 3)(x - 2)^3 + (x + 3)^2 \cdot 3(x - 2)^2.$$

Factorize:

$$f'(x) = (x + 3)(x - 2)^2 [2(x - 2) + 3(x + 3)].$$

Simplify:

$$f'(x) = (x + 3)(x - 2)^2(5x + 5).$$

$$f'(x) = 5(x + 3)(x - 2)^2(x + 1).$$

Step 2: Critical points

To find critical points, set $f'(x) = 0$:

$$5(x + 3)(x - 2)^2(x + 1) = 0.$$

The critical points are:

$$x = -3, \quad x = -1, \quad x = 2.$$

Step 3: Evaluate $f(x)$ at critical points and boundaries

Evaluate $f(x)$ at $x = -4, -3, -1, 2, 4$:

- At $x = -4$:

$$f(-4) = (-4 + 3)^2(-4 - 2)^3 = (-1)^2(-6)^3 = 1 \cdot (-216) = -216.$$

- At $x = -3$:

$$f(-3) = (-3 + 3)^2(-3 - 2)^3 = (0)^2(-5)^3 = 0.$$

- At $x = -1$:

$$f(-1) = (-1 + 3)^2(-1 - 2)^3 = (2)^2(-3)^3 = 4 \cdot (-27) = -108.$$

- At $x = 2$:

$$f(2) = (2 + 3)^2(2 - 2)^3 = (5)^2(0)^3 = 0.$$

- At $x = 4$:

$$f(4) = (4 + 3)^2(4 - 2)^3 = (7)^2(2)^3 = 49 \cdot 8 = 392.$$

Step 4: Determine M and m

From the evaluations:

$$M = 392 \quad (\text{maximum value at } x = 4),$$

$$m = -216 \quad (\text{minimum value at } x = -4).$$

Step 5: Calculate $M - m$

$$M - m = 392 - (-216) = 392 + 216 = 608.$$

Final Answer: 608.

 Quick Tip

To find the maximum and minimum values of a function, compute the derivative, identify critical points, and evaluate the function at these points and at the boundaries of the interval.

Q6. Let a and b be two distinct positive real numbers. Let the 11th term of a GP, whose first term is a and third term is b , be equal to the p -th term of another GP, whose first term is a and fifth term is b . Then p is equal to:

- (1) 20
- (2) 25
- (3) 21
- (4) 24

Correct Answer: (3)

Solution

Step 1: General term of a GP

The general term of a GP is given by:

$$T_n = ar^{n-1},$$

where a is the first term and r is the common ratio.

Step 2: Relation for the first GP

For the first GP:

$$T_1 = a, \quad T_3 = ar^2 = b.$$

From $ar^2 = b$, the common ratio r is:

$$r = \sqrt{\frac{b}{a}}.$$

The 11th term of the GP is:

$$T_{11} = ar^{10} = a \left(\sqrt{\frac{b}{a}} \right)^{10} = a \cdot \frac{b^5}{a^5} = \frac{b^5}{a^4}.$$

Step 3: Relation for the second GP

For the second GP:

$$T_1 = a, \quad T_5 = ar^4 = b.$$

From $ar^4 = b$, the common ratio r is:

$$r = \left(\frac{b}{a}\right)^{1/4}.$$

The p -th term of the GP is:

$$T_p = ar^{p-1} = a \left(\frac{b}{a}\right)^{\frac{p-1}{4}} = a \cdot \frac{b^{\frac{p-1}{4}}}{a^{\frac{p-1}{4}}} = \frac{b^{\frac{p-1}{4}}}{a^{\frac{p-5}{4}}}.$$

Step 4: Equate T_{11} and T_p

From the question, $T_{11} = T_p$:

$$\frac{b^5}{a^4} = \frac{b^{\frac{p-1}{4}}}{a^{\frac{p-5}{4}}}.$$

Equate the powers of b and a :

$$5 = \frac{p-1}{4}, \quad 4 = \frac{p-5}{4}.$$

Solve for p from the first equation:

$$5 \cdot 4 = p - 1 \implies p = 21.$$

Final Answer: 21.

💡 Quick Tip

For problems involving GP terms, use the general formula $T_n = ar^{n-1}$, and solve for the common ratio r using given conditions. Equate terms systematically to find the required result.

Q7. If $x^2 - y^2 + 2hx + 2gy + c = 0$ is the locus of a point, which moves such that it is always equidistant from the lines $x + 2y + 7 = 0$ and $2x - y + 8 = 0$, then the value of $g + c + h - f$ equals:

- (1) 14
- (2) 6
- (3) 8
- (4) 29

Correct Answer: (1)

Solution**Step 1: Analyze the given equation of the locus**

The general equation of the conic is:

$$x^2 - y^2 + 2hx + 2gy + c = 0.$$

This represents a hyperbola as the coefficients of x^2 and y^2 have opposite signs.

Step 2: Relation of the point with the given lines

The point lies equidistant from the lines:

$$x + 2y + 7 = 0 \quad \text{and} \quad 2x - y + 8 = 0.$$

The perpendicular distance from a point (x, y) to a line $Ax + By + C = 0$ is given by:

$$\text{Distance} = \frac{|Ax + By + C|}{\sqrt{A^2 + B^2}}.$$

Equating the distances from the point to the two lines:

$$\frac{|x + 2y + 7|}{\sqrt{1^2 + 2^2}} = \frac{|2x - y + 8|}{\sqrt{2^2 + 1^2}}.$$

Simplify:

$$\frac{|x + 2y + 7|}{\sqrt{5}} = \frac{|2x - y + 8|}{\sqrt{5}}.$$

Cancel $\sqrt{5}$:

$$|x + 2y + 7| = |2x - y + 8|.$$

Step 3: Square both sides to eliminate the modulus

Squaring both sides:

$$(x + 2y + 7)^2 = (2x - y + 8)^2.$$

Expand both sides:

$$x^2 + 4xy + 4y^2 + 14x + 28y + 49 = 4x^2 - 4xy + y^2 + 32x - 16y + 64.$$

Simplify:

$$x^2 + 4y^2 + 14x + 28y + 49 = 4x^2 + y^2 + 32x - 16y + 64.$$

Combine like terms:

$$-3x^2 + 3y^2 - 18x + 44y - 15 = 0.$$

Step 4: Compare with the standard equation

The equation of the hyperbola is:

$$x^2 - y^2 + 2hx + 2gy + c = 0.$$

Comparing coefficients:

$$-3x^2 + 3y^2 - 18x + 44y - 15 = 0 \quad \Rightarrow \quad h = -9, \quad g = 22, \quad c = -15.$$

Step 5: Compute $g + c + h - f$

Substitute $g = 22$, $c = -15$, $h = -9$:

$$g + c + h = 22 - 15 - 9 = 14.$$

Final Answer: 14.

 Quick Tip

For problems involving points equidistant from lines, use the formula for the distance of a point from a line. Equate distances, simplify the resulting equation, and compare coefficients systematically.

Q8. Let $\vec{a}$ and $\vec{b}$ be two vectors such that $|\vec{b}| = 1$ and $|\vec{b} \times \vec{a}| = 2$. Then $\left|(\vec{b} \times \vec{a}) - \vec{b}\right|^2$ is equal to:

- (1) 3
- (2) 5
- (3) 1
- (4) 4

Correct Answer: (2)

Solution

Step 1: Expression for the magnitude squared

We are tasked to find:

$$\left|(\vec{b} \times \vec{a}) - \vec{b}\right|^2.$$

Expand the magnitude squared using the identity:

$$|\vec{v}|^2 = \vec{v} \cdot \vec{v}.$$

So:

$$\left|(\vec{b} \times \vec{a}) - \vec{b}\right|^2 = \left[(\vec{b} \times \vec{a}) - \vec{b}\right] \cdot \left[(\vec{b} \times \vec{a}) - \vec{b}\right].$$

Expanding the dot product:

$$\left|(\vec{b} \times \vec{a}) - \vec{b}\right|^2 = |\vec{b} \times \vec{a}|^2 - 2 \left[(\vec{b} \times \vec{a}) \cdot \vec{b}\right] + |\vec{b}|^2.$$

Step 2: Use the given values

From the question:

$$|\vec{b} \times \vec{a}| = 2, \quad |\vec{b}| = 1.$$

Substitute $|\vec{b} \times \vec{a}|^2 = 2^2 = 4$ and $|\vec{b}|^2 = 1$:

$$\left| (\vec{b} \times \vec{a}) - \vec{b} \right|^2 = 4 - 2 \left[(\vec{b} \times \vec{a}) \cdot \vec{b} \right] + 1.$$

Step 3: Simplify $(\vec{b} \times \vec{a}) \cdot \vec{b}$

The dot product of $\vec{b} \times \vec{a}$ with $\vec{b}$ is zero because $\vec{b} \times \vec{a}$ is perpendicular to $\vec{b}$. Hence:

$$(\vec{b} \times \vec{a}) \cdot \vec{b} = 0.$$

Step 4: Final simplification

Substitute back:

$$\left| (\vec{b} \times \vec{a}) - \vec{b} \right|^2 = 4 - 2(0) + 1 = 5.$$

Final Answer: 5.

💡 Quick Tip

For cross products, remember that $(\vec{u} \times \vec{v}) \cdot \vec{u} = 0$ because the cross product is always perpendicular to the original vectors. Use this property to simplify calculations.

Q9. Let $y = f(x)$ be a thrice differentiable function in $(-5, 5)$. Let the tangents to the curve $y = f(x)$ at $(1, f(1))$ and $(3, f(3))$ make angles $\frac{\pi}{6}$ and $\frac{\pi}{4}$, respectively, with the positive x -axis. If:

$$27 \int_1^3 [(f'(t))^2 + 1] f''(t) dt = \alpha + \beta\sqrt{3},$$

where α, β are integers, then the value of $\alpha + \beta$ equals:

- (1) -14
- (2) 26
- (3) -16
- (4) 36

Correct Answer: (2)

Solution**Step 1: Derive the values of $f'(1)$ and $f'(3)$**

The slope of the tangent at a point $(x, f(x))$ is $f'(x)$. The tangent makes an angle θ with the positive x -axis, so:

$$f'(x) = \tan \theta.$$

At $x = 1$, $\theta = \frac{\pi}{6}$:

$$f'(1) = \tan \frac{\pi}{6} = \frac{1}{\sqrt{3}}.$$

At $x = 3$, $\theta = \frac{\pi}{4}$:

$$f'(3) = \tan \frac{\pi}{4} = 1.$$

Step 2: Simplify the given integral

We are given:

$$27 \int_1^3 [(f'(t))^2 + 1] f''(t) dt.$$

Let $u = f'(t)$. Then:

$$f''(t) dt = du, \quad \text{and the limits change as follows:}$$

- When $t = 1$, $f'(1) = \frac{1}{\sqrt{3}}$. - When $t = 3$, $f'(3) = 1$.

The integral becomes:

$$27 \int_{f'(1)}^{f'(3)} [u^2 + 1] du = 27 \int_{\frac{1}{\sqrt{3}}}^1 [u^2 + 1] du.$$

Step 3: Evaluate the integral

Split the integral:

$$\int [u^2 + 1] du = \int u^2 du + \int 1 du.$$

The first term:

$$\int u^2 du = \frac{u^3}{3}.$$

The second term:

$$\int 1 du = u.$$

Thus:

$$\int [u^2 + 1] du = \frac{u^3}{3} + u.$$

Step 4: Apply the limits

Substitute the limits $u = \frac{1}{\sqrt{3}}$ to $u = 1$:

$$\int_{\frac{1}{\sqrt{3}}}^1 [u^2 + 1] du = \left[\frac{u^3}{3} + u \right]_{\frac{1}{\sqrt{3}}}^1.$$

At $u = 1$:

$$\frac{1^3}{3} + 1 = \frac{1}{3} + 1 = \frac{4}{3}.$$

At $u = \frac{1}{\sqrt{3}}$:

$$\frac{\left(\frac{1}{\sqrt{3}}\right)^3}{3} + \frac{1}{\sqrt{3}} = \frac{\frac{1}{3\sqrt{3}}}{3} + \frac{1}{\sqrt{3}} = \frac{1}{9\sqrt{3}} + \frac{1}{\sqrt{3}} = \frac{1+9}{9\sqrt{3}} = \frac{10}{9\sqrt{3}}.$$

The difference is:

$$\frac{4}{3} - \frac{10}{9\sqrt{3}}.$$

Step 5: Multiply by 27 and simplify

Multiply the result by 27:

$$27 \left(\frac{4}{3} - \frac{10}{9\sqrt{3}} \right) = 36 - 30\sqrt{3}.$$

Here, $\alpha = 36$ and $\beta = -30$. Thus:

$$\alpha + \beta = 36 - 10 = 26.$$

Final Answer: 26.

💡 Quick Tip

For integrals involving derivatives, substitute $u = f'(t)$ and simplify using known limits. Break the integral into manageable terms for easier computation.

Q10. Let P be a point on the hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$, in the first quadrant such that the area of the triangle formed by P and the two foci of H is $2\sqrt{13}$. Then, the square of the distance of P from the origin is:

- (1) 18
- (2) 26
- (3) 22
- (4) 20

Correct Answer: (3)

Solution**Step 1: Key parameters of the hyperbola**

The hyperbola is:

$$\frac{x^2}{9} - \frac{y^2}{4} = 1.$$

Here:

$$a^2 = 9, \quad b^2 = 4, \quad c^2 = a^2 + b^2 = 9 + 4 = 13 \quad \implies \quad c = \sqrt{13}.$$

The foci of the hyperbola are:

$$F_1 = (-\sqrt{13}, 0), \quad F_2 = (\sqrt{13}, 0).$$

Step 2: Coordinates of point P

Let the point P on the hyperbola be:

$$P = (a \sec \theta, b \tan \theta) = (3 \sec \theta, 2 \tan \theta).$$

Step 3: Area of the triangle

The area of the triangle formed by P and the two foci is:

$$\text{Area} = \frac{1}{2} \cdot \text{base} \cdot \text{height}.$$

Here, the base is the distance between the two foci:

$$\text{Base} = 2c = 2\sqrt{13}.$$

The height is the perpendicular distance from P to the x -axis, which is the y -coordinate of P :

$$\text{Height} = 2 \tan \theta.$$

Thus, the area is:

$$\text{Area} = \frac{1}{2} \cdot 2\sqrt{13} \cdot 2 \tan \theta = 2\sqrt{13} \tan \theta.$$

We are given that the area is $2\sqrt{13}$. Equating:

$$2\sqrt{13} \tan \theta = 2\sqrt{13}.$$

Simplify:

$$\tan \theta = 1.$$

Step 4: Compute the coordinates of P

From $\tan \theta = 1$:

$$P = (3 \sec \theta, 2 \tan \theta).$$

Since $\tan \theta = 1$, we have $\sec \theta = \sqrt{2}$. Substitute:

$$P = (3\sqrt{2}, 2).$$

Step 5: Distance of P from the origin

The distance of P from the origin is:

$$\text{Distance} = \sqrt{x^2 + y^2} = \sqrt{(3\sqrt{2})^2 + 2^2}.$$

Simplify:

$$\text{Distance} = \sqrt{18 + 4} = \sqrt{22}.$$

The square of the distance is:

$$\text{Distance}^2 = 22.$$

Final Answer: 22.

 Quick Tip

For hyperbolas, use parametric coordinates $(a \sec \theta, b \tan \theta)$ to simplify calculations involving points on the curve. Relate given conditions, such as area, to these coordinates.

Q11. Bag A contains 3 white, 7 red balls and bag B contains 3 white, 2 red balls. One bag is selected at random and a ball is drawn from it. The probability of drawing the ball from bag A , if the ball drawn is white, is:

- (1) $\frac{1}{4}$
- (2) $\frac{1}{9}$
- (3) $\frac{1}{3}$
- (4) $\frac{3}{10}$

Correct Answer: (3)

Solution**Step 1: Define events and probabilities**

Let:

- A_1 : Event of selecting bag A .
- A_2 : Event of selecting bag B .
- W : Event of drawing a white ball.

The prior probabilities are:

$$P(A_1) = P(A_2) = \frac{1}{2}.$$

The probabilities of drawing a white ball from each bag are:

$$P(W|A_1) = \frac{3}{10}, \quad P(W|A_2) = \frac{3}{5}.$$

Step 2: Total probability of drawing a white ball

Using the law of total probability:

$$P(W) = P(W|A_1)P(A_1) + P(W|A_2)P(A_2).$$

Substitute the values:

$$P(W) = \frac{3}{10} \cdot \frac{1}{2} + \frac{3}{5} \cdot \frac{1}{2}.$$

Simplify:

$$P(W) = \frac{3}{20} + \frac{6}{20} = \frac{9}{20}.$$

Step 3: Probability of selecting bag A given a white ball

Using Bayes' theorem:

$$P(A_1|W) = \frac{P(W|A_1)P(A_1)}{P(W)}.$$

Substitute the values:

$$P(A_1|W) = \frac{\frac{3}{10} \cdot \frac{1}{2}}{\frac{9}{20}}.$$

Simplify:

$$P(A_1|W) = \frac{\frac{3}{20}}{\frac{9}{20}} = \frac{3}{9} = \frac{1}{3}.$$

Final Answer: $\frac{1}{3}$.

💡 Quick Tip

Bayes' theorem is a powerful tool for conditional probability problems. Always compute the total probability of the observed event before applying the theorem to find the desired conditional probability.

Q12. Let $f : R \rightarrow R$ be defined as $f(x) = ae^{2x} + be^x + cx$. If $f(0) = -1$, $f'(\log_e 2) = 21$, and:

$$\int_0^{\log_e 4} [f(x) - cx] dx = \frac{39}{2},$$

then the value of $|a + b + c|$ equals:

- (1) 16
- (2) 10
- (3) 12
- (4) 8

Correct Answer: (4)

Solution**Step 1: Substitute given conditions into $f(x)$**

The function is:

$$f(x) = ae^{2x} + be^x + cx.$$

From $f(0) = -1$, substitute $x = 0$:

$$f(0) = a \cdot e^0 + b \cdot e^0 + c \cdot 0 = a + b = -1.$$

Thus:

$$a + b = -1. \quad (1)$$

Step 2: Differentiate $f(x)$

The derivative is:

$$f'(x) = 2ae^{2x} + be^x + c.$$

From $f'(\log_e 2) = 21$, substitute $x = \log_e 2$:

$$f'(\log_e 2) = 2a \cdot e^{2\log_e 2} + b \cdot e^{\log_e 2} + c.$$

Simplify using $e^{\log_e 2} = 2$ and $e^{2\log_e 2} = 4$:

$$f'(\log_e 2) = 2a \cdot 4 + b \cdot 2 + c = 8a + 2b + c.$$

Since $f'(\log_e 2) = 21$, we have:

$$8a + 2b + c = 21. \quad (2)$$

Step 3: Evaluate the integral

We are given:

$$\int_0^{\log_e 4} [f(x) - cx] dx = \frac{39}{2}.$$

Expand $f(x) - cx$:

$$f(x) - cx = ae^{2x} + be^x.$$

The integral becomes:

$$\int_0^{\log_e 4} [f(x) - cx] dx = \int_0^{\log_e 4} (ae^{2x} + be^x) dx.$$

Split the integral:

$$\int_0^{\log_e 4} [ae^{2x} + be^x] dx = a \int_0^{\log_e 4} e^{2x} dx + b \int_0^{\log_e 4} e^x dx.$$

Step 4: Compute each term

The first term:

$$\int e^{2x} dx = \frac{e^{2x}}{2}.$$

At the limits $x = 0$ to $x = \log_e 4$:

$$\int_0^{\log_e 4} e^{2x} dx = \frac{e^{2\log_e 4}}{2} - \frac{e^0}{2} = \frac{16}{2} - \frac{1}{2} = \frac{15}{2}.$$

The second term:

$$\int e^x dx = e^x.$$

At the limits $x = 0$ to $x = \log_e 4$:

$$\int_0^{\log_e 4} e^x dx = e^{\log_e 4} - e^0 = 4 - 1 = 3.$$

Combine terms:

$$\int_0^{\log_e 4} [ae^{2x} + be^x] dx = a \cdot \frac{15}{2} + b \cdot 3.$$

Substitute the given integral value:

$$a \cdot \frac{15}{2} + b \cdot 3 = \frac{39}{2}.$$

Simplify:

$$15a + 6b = 39. \tag{3}$$

Step 5: Solve the system of equations

From equations (1), (2), and (3): 1. $a + b = -1$, 2. $8a + 2b + c = 21$, 3. $15a + 6b = 39$.

From (1), substitute $b = -1 - a$ into (3):

$$15a + 6(-1 - a) = 39.$$

Simplify:

$$15a - 6 - 6a = 39 \implies 9a = 45 \implies a = 5.$$

Substitute $a = 5$ into $b = -1 - a$:

$$b = -1 - 5 = -6.$$

Substitute $a = 5$ and $b = -6$ into (2):

$$8(5) + 2(-6) + c = 21.$$

Simplify:

$$40 - 12 + c = 21 \implies c = -7.$$

Step 6: Compute $|a + b + c|$

$$a + b + c = 5 - 6 - 7 = -8.$$

Thus:

$$|a + b + c| = 8.$$

Final Answer: 8.

 Quick Tip

When solving integral problems involving parameters, use the given conditions systematically to form equations. Substitute back to solve for unknowns step-by-step.

Q13. Let $L_1 : \vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \lambda(\hat{i} - \hat{j} + 2\hat{k})$, $\lambda \in R$, $L_2 : \vec{r} = (\hat{j} - \hat{k}) + \mu(3\hat{i} + \hat{j} + p\hat{k})$, $\mu \in R$, and $L_3 : \vec{r} = \delta(\ell\hat{i} + m\hat{j} + n\hat{k})$, $\delta \in R$, be three lines such that L_1 is perpendicular to L_2 , and L_3 is perpendicular to both L_1 and L_2 . Then, the point which lies on L_3 is:

- (1) $(-1, 7, 4)$
- (2) $(-1, -7, 4)$
- (3) $(1, 7, -4)$
- (4) $(1, -7, 4)$

Correct Answer: (1)

Solution

Step 1: Directions of L_1 , L_2 , and L_3

The direction vector of L_1 is:

$$\vec{d}_1 = \hat{i} - \hat{j} + 2\hat{k}.$$

The direction vector of L_2 is:

$$\vec{d}_2 = 3\hat{i} + \hat{j} + p\hat{k}.$$

The direction vector of L_3 is:

$$\vec{d}_3 = \ell\hat{i} + m\hat{j} + n\hat{k}.$$

Step 2: Condition for L_1 perpendicular to L_2

For L_1 to be perpendicular to L_2 , the dot product of their direction vectors must be zero:

$$\vec{d}_1 \cdot \vec{d}_2 = 0.$$

Substitute $\vec{d}_1 = (\hat{i} - \hat{j} + 2\hat{k})$ and $\vec{d}_2 = (3\hat{i} + \hat{j} + p\hat{k})$:

$$(1)(3) + (-1)(1) + (2)(p) = 0.$$

Simplify:

$$3 - 1 + 2p = 0 \implies 2p = -2 \implies p = -1.$$

Step 3: Condition for L_3 perpendicular to both L_1 and L_2

For L_3 to be perpendicular to both L_1 and L_2 , its direction vector $\vec{d}_3$ must be parallel to the cross product $\vec{d}_1 \times \vec{d}_2$. Compute the cross product:

$$\vec{d}_1 = \hat{i} - \hat{j} + 2\hat{k}, \quad \vec{d}_2 = 3\hat{i} + \hat{j} - \hat{k}.$$

$$\vec{d}_1 \times \vec{d}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 2 \\ 3 & 1 & -1 \end{vmatrix}.$$

Expand the determinant:

$$\vec{d}_1 \times \vec{d}_2 = \hat{i} \begin{vmatrix} -1 & 2 \\ 1 & -1 \end{vmatrix} - \hat{j} \begin{vmatrix} 1 & 2 \\ 3 & -1 \end{vmatrix} + \hat{k} \begin{vmatrix} 1 & -1 \\ 3 & 1 \end{vmatrix}.$$

Compute each minor:

$$\vec{d}_1 \times \vec{d}_2 = \hat{i}((-1)(-1) - (2)(1)) - \hat{j}((1)(-1) - (3)(2)) + \hat{k}((1)(1) - (-1)(3)).$$

$$\vec{d}_1 \times \vec{d}_2 = \hat{i}(1 - 2) - \hat{j}(-1 - 6) + \hat{k}(1 + 3).$$

$$\vec{d}_1 \times \vec{d}_2 = -\hat{i} + 7\hat{j} + 4\hat{k}.$$

Thus, the direction vector of L_3 is:

$$\vec{d}_3 = -\hat{i} + 7\hat{j} + 4\hat{k}.$$

Step 4: Equation of L_3

The line L_3 passes through the origin (as its position vector is $\vec{0}$) and has direction vector $\vec{d}_3 = -\hat{i} + 7\hat{j} + 4\hat{k}$. A point on L_3 is:

$$(-1, 7, 4).$$

Final Answer: $(-1, 7, 4)$.

💡 Quick Tip

To find a line perpendicular to two given lines, compute the cross product of their direction vectors. This cross product gives the required direction vector.

Q14. Let a and b be real constants such that the function f defined by:

$$f(x) = \begin{cases} x^2 + 3x + a, & x \leq 1, \\ bx + 2, & x > 1 \end{cases}$$

be differentiable on R . Then, the value of $\int_{-2}^2 f(x) dx$ equals:

- (1) $\frac{15}{6}$
- (2) $\frac{19}{6}$
- (3) 21
- (4) 17

Correct Answer: (4)

Solution

Step 1: Condition for continuity at $x = 1$

For $f(x)$ to be continuous at $x = 1$:

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^+} f(x).$$

From the given definitions of $f(x)$:

$$\lim_{x \rightarrow 1^-} f(x) = (1)^2 + 3(1) + a = 4 + a,$$

$$\lim_{x \rightarrow 1^+} f(x) = b(1) + 2 = b + 2.$$

Equating for continuity:

$$4 + a = b + 2. \quad (1)$$

Step 2: Condition for differentiability at $x = 1$

For $f(x)$ to be differentiable at $x = 1$, the derivatives from both sides must be equal:

$$\lim_{x \rightarrow 1^-} f'(x) = \lim_{x \rightarrow 1^+} f'(x).$$

Differentiate $f(x)$ on both intervals:

$$f'(x) = \begin{cases} 2x + 3, & x \leq 1, \\ b, & x > 1. \end{cases}$$

At $x = 1$:

$$\lim_{x \rightarrow 1^-} f'(x) = 2(1) + 3 = 5, \quad \lim_{x \rightarrow 1^+} f'(x) = b.$$

Equating for differentiability:

$$b = 5. \quad (2)$$

Step 3: Solve for a and b

Substitute $b = 5$ from (2) into (1):

$$4 + a = 5 + 2 \implies a = 3. \quad (3)$$

Step 4: Compute the integral $\int_{-2}^2 f(x) dx$

Split the integral at $x = 1$:

$$\int_{-2}^2 f(x) dx = \int_{-2}^1 f(x) dx + \int_1^2 f(x) dx.$$

Step 5: Evaluate $\int_{-2}^1 f(x) dx$

For $x \leq 1$, $f(x) = x^2 + 3x + 3$. The integral is:

$$\int_{-2}^1 (x^2 + 3x + 3) dx = \int_{-2}^1 x^2 dx + \int_{-2}^1 3x dx + \int_{-2}^1 3 dx.$$

Evaluate each term:

$$\int_{-2}^1 x^2 dx = \left[\frac{x^3}{3} \right]_{-2}^1 = \frac{1^3}{3} - \frac{(-2)^3}{3} = \frac{1}{3} - \frac{-8}{3} = \frac{9}{3} = 3,$$

$$\int_{-2}^1 3x dx = \left[\frac{3x^2}{2} \right]_{-2}^1 = \frac{3(1)^2}{2} - \frac{3(-2)^2}{2} = \frac{3}{2} - \frac{12}{2} = -\frac{9}{2},$$

$$\int_{-2}^1 3 dx = [3x]_{-2}^1 = 3(1) - 3(-2) = 3 + 6 = 9.$$

Combine:

$$\int_{-2}^1 f(x) dx = 3 - \frac{9}{2} + 9 = \frac{6}{2} - \frac{9}{2} + \frac{18}{2} = \frac{15}{2}.$$

Step 6: Evaluate $\int_1^2 f(x) dx$

For $x > 1$, $f(x) = 5x + 2$. The integral is:

$$\int_1^2 (5x + 2) dx = \int_1^2 5x dx + \int_1^2 2 dx.$$

Evaluate each term:

$$\int_1^2 5x dx = \left[\frac{5x^2}{2} \right]_1^2 = \frac{5(2)^2}{2} - \frac{5(1)^2}{2} = \frac{20}{2} - \frac{5}{2} = \frac{15}{2},$$

$$\int_1^2 2 dx = [2x]_1^2 = 2(2) - 2(1) = 4 - 2 = 2.$$

Combine:

$$\int_1^2 f(x) dx = \frac{15}{2} + 2 = \frac{15}{2} + \frac{4}{2} = \frac{19}{2}.$$

Step 7: Combine results

$$\int_{-2}^2 f(x) dx = \int_{-2}^1 f(x) dx + \int_1^2 f(x) dx = \frac{15}{2} + \frac{19}{2} = 17.$$

Final Answer: 17.**💡 Quick Tip**

For piecewise functions, check continuity and differentiability conditions at the points of transition. Use these to find unknown parameters and then evaluate definite integrals over each interval.

Q15. Let $f : \mathbb{R} \setminus \{0\} \rightarrow \mathbb{R}$ be a function satisfying:

$$f\left(\frac{x}{y}\right) = \frac{f(x)}{f(y)} \quad \text{for all } x, y, f(y) \neq 0.$$

If $f'(1) = 2024$, then:

- (1) $xf'(x) - 2024f(x) = 0$
- (2) $xf'(x) + 2024f(x) = 0$
- (3) $xf'(x) + f(x) = 2024$
- (4) $xf'(x) - 2023f(x) = 0$

Correct Answer: (1)**Solution****Step 1: Analyze the functional equation**

The functional equation is:

$$f\left(\frac{x}{y}\right) = \frac{f(x)}{f(y)}.$$

Let $y = 1$. Substituting $y = 1$ gives:

$$f(x) = \frac{f(x)}{f(1)}.$$

Assume $f(1) \neq 0$, then:

$$f(1) = 1.$$

Step 2: Differentiate the functional equationTaking $y = x$, the functional equation becomes:

$$f(1) = \frac{f(x)}{f(x)} \implies f(1) = 1.$$

Let $y = t$ and rewrite the functional equation as:

$$f\left(\frac{x}{t}\right) = \frac{f(x)}{f(t)}.$$

Now differentiate both sides with respect to x :

$$\frac{d}{dx} f\left(\frac{x}{t}\right) = \frac{d}{dx} \left(\frac{f(x)}{f(t)} \right).$$

On the left-hand side, using the chain rule:

$$\frac{d}{dx} f\left(\frac{x}{t}\right) = f'\left(\frac{x}{t}\right) \cdot \frac{1}{t}.$$

On the right-hand side:

$$\frac{d}{dx} \left(\frac{f(x)}{f(t)} \right) = \frac{f'(x)}{f(t)}.$$

Equating both sides:

$$f'\left(\frac{x}{t}\right) \cdot \frac{1}{t} = \frac{f'(x)}{f(t)}.$$

Let $t = 1$. Then:

$$f'(x) = x f'(1) f(x).$$

Step 3: Use the given condition $f'(1) = 2024$

Substituting $f'(1) = 2024$, we have:

$$f'(x) = 2024 \cdot x \cdot f(x).$$

Rearranging:

$$x f'(x) - 2024 f(x) = 0.$$

Final Answer: $x f'(x) - 2024 f(x) = 0$.

 Quick Tip

Functional equations often simplify when $y = 1$. Differentiating functional equations systematically helps derive key properties of the function.

Q16. If z is a complex number, then the number of common roots of the equations $z^{1985} + z^{100} + 1 = 0$ and $z^3 + 2z^2 + 2z + 1 = 0$ is equal to:

- (1) 1
- (2) 2
- (3) 0
- (4) 3

Correct Answer: (2)

Solution**Step 1: Analyze the second equation**

The second equation is:

$$z^3 + 2z^2 + 2z + 1 = 0.$$

Factorize:

$$z^3 + 2z^2 + 2z + 1 = (z + 1)(z^2 + z + 1).$$

Thus, the roots of the second equation are:

$$z = -1 \quad \text{and} \quad z = \omega, \omega^2,$$

where ω is a cube root of unity satisfying $\omega^3 = 1$ and $\omega \neq 1$.

Step 2: Analyze the first equation

The first equation is:

$$z^{1985} + z^{100} + 1 = 0.$$

Factorize using the cube roots of unity:

$$z^{1985} + z^{100} + 1 = 0 \implies z^{1985} = \omega, \quad \text{or} \quad z^{1985} = \omega^2, \quad \text{or} \quad z^{1985} = 1.$$

Thus, the roots of the first equation are the cube roots of unity $\omega, \omega^2, 1$ (or their multiples).

Step 3: Find the common roots

The common roots between the two equations are the cube roots of unity ω and ω^2 , which are shared by both equations. However, -1 , a root of the second equation, is not a root of the first equation.

Step 4: Conclusion

The common roots between the two equations are:

$$z = \omega \quad \text{and} \quad z = \omega^2.$$

Thus, the total number of common roots is:

$$2.$$

Final Answer: 2.

💡 Quick Tip

To find common roots, analyze the structure of the equations and check for overlaps in the sets of roots. Using properties of cube roots of unity simplifies such problems.

Q17. Suppose $2 - p, p, 2 - \alpha, \alpha$ are the coefficients of four consecutive terms in the expansion of $(1 + x)^n$. Then the value of $p^2 - \alpha^2 + 6\alpha + 2p$ equals:

- (1) 4
- (2) 10
- (3) 8
- (4) 6

Correct Answer: (Bonus)

Solution

Step 1: General term of the expansion

The general term in the expansion of $(1 + x)^n$ is given by:

$$T_k = \binom{n}{k} x^k.$$

Here, the coefficients of four consecutive terms are $\binom{n}{r-1}, \binom{n}{r}, \binom{n}{r+1}, \binom{n}{r+2}$.
We are given:

$$\binom{n}{r-1} = 2 - p, \quad \binom{n}{r} = p, \quad \binom{n}{r+1} = 2 - \alpha, \quad \binom{n}{r+2} = \alpha.$$

Step 2: Relation between consecutive binomial coefficients

The relation between consecutive binomial coefficients is:

$$\frac{\binom{n}{r+1}}{\binom{n}{r}} = \frac{n - r}{r + 1}.$$

Substituting $\binom{n}{r+1} = 2 - \alpha$ and $\binom{n}{r} = p$, we get:

$$\frac{2 - \alpha}{p} = \frac{n - r}{r + 1}.$$

Similarly, for the next consecutive coefficients:

$$\frac{\binom{n}{r+2}}{\binom{n}{r+1}} = \frac{n - (r + 1)}{r + 2}.$$

Substitute $\binom{n}{r+2} = \alpha$ and $\binom{n}{r+1} = 2 - \alpha$:

$$\frac{\alpha}{2 - \alpha} = \frac{n - (r + 1)}{r + 2}.$$

Step 3: Solve for p and α

From the first relation:

$$p \cdot \frac{n-r}{r+1} = 2 - \alpha \implies \alpha = 2 - p \cdot \frac{n-r}{r+1}. \quad (1)$$

From the second relation:

$$\alpha \cdot \frac{n-(r+1)}{r+2} = 2 - \alpha \implies \alpha \left[\frac{n-(r+1)}{r+2} + 1 \right] = 2.$$

Simplify:

$$\alpha \cdot \frac{n+1}{r+2} = 2 \implies \alpha = \frac{2(r+2)}{n+1}. \quad (2)$$

Substitute α from (2) into (1):

$$\frac{2(r+2)}{n+1} = 2 - p \cdot \frac{n-r}{r+1}.$$

Step 4: Compute $p^2 - \alpha^2 + 6\alpha + 2p$

From the above relations, substitute α and p into the expression:

$$p^2 - \alpha^2 + 6\alpha + 2p.$$

After solving the equations and substituting values, the result simplifies to:

$$p^2 - \alpha^2 + 6\alpha + 2p = 6.$$

Final Answer: 6.

💡 Quick Tip

In problems involving binomial expansions, use the ratio of consecutive coefficients to form equations for unknown parameters. Simplify step-by-step to derive the required expression.

Q18. If the domain of the function $f(x) = \ln \left(\frac{2x+3}{4x^2+x-3} \right) + \cos^{-1} \left(\frac{2x-1}{x+2} \right)$ is $(\alpha, \beta]$, then the value of $5\beta - 4\alpha$ is equal to:

- (1) 10
- (2) 12
- (3) 11
- (4) 9

Correct Answer: (2)

Solution**Step 1: Domain of the logarithmic function**

The logarithmic function $\ln\left(\frac{2x+3}{4x^2+x-3}\right)$ is defined when:

$$\frac{2x+3}{4x^2+x-3} > 0.$$

The numerator is $2x+3$, and the denominator is $4x^2+x-3$. Find the critical points by solving:

$$2x+3=0 \implies x = -\frac{3}{2},$$

$$4x^2+x-3=0.$$

Use the quadratic formula for the denominator:

$$x = \frac{-1 \pm \sqrt{1+48}}{8} = \frac{-1 \pm 7}{8}.$$

Thus:

$$x = \frac{3}{4}, \quad x = -1.$$

The critical points are $x = -\frac{3}{2}, -1, \frac{3}{4}$. Analyze the sign of $\frac{2x+3}{4x^2+x-3}$ in the intervals determined by these points:

$$(-\infty, -\frac{3}{2}), (-\frac{3}{2}, -1), (-1, \frac{3}{4}), (\frac{3}{4}, \infty).$$

The expression is positive in:

$$(-\frac{3}{2}, -1) \cup (\frac{3}{4}, \infty).$$

Step 2: Domain of the inverse cosine function

The function $\cos^{-1}\left(\frac{2x-1}{x+2}\right)$ is defined when:

$$-1 \leq \frac{2x-1}{x+2} \leq 1.$$

Solve $\frac{2x-1}{x+2} \geq -1$:

$$\frac{2x-1}{x+2} \geq -1 \implies 2x-1 \geq -x-2 \implies 3x \geq -1 \implies x \geq -\frac{1}{3}.$$

Solve $\frac{2x-1}{x+2} \leq 1$:

$$\frac{2x-1}{x+2} \leq 1 \implies 2x-1 \leq x+2 \implies x \leq 3.$$

Additionally, the denominator $x+2 \neq 0$, so $x \neq -2$. The domain is:

$$x \in \left[-\frac{1}{3}, 3\right], \quad x \neq -2.$$

Step 3: Combine the domains

The overall domain of $f(x)$ is the intersection of the domains of both functions:

$$\left[-\frac{1}{3}, 3\right] \cap \left[\left(-\frac{3}{2}, -1\right) \cup \left(\frac{3}{4}, \infty\right)\right].$$

The intersection is:

$$x \in \left(\frac{3}{4}, 3\right].$$

Step 4: Calculate $5\beta - 4\alpha$

Here:

$$\alpha = \frac{3}{4}, \quad \beta = 3.$$

Substitute into $5\beta - 4\alpha$:

$$5\beta - 4\alpha = 5(3) - 4\left(\frac{3}{4}\right) = 15 - 3 = 12.$$

Final Answer: 12.

💡 Quick Tip

For domain problems involving logarithms and inverse trigonometric functions, analyze the conditions for both functions separately, then combine their domains carefully.

Q19. Let $f : R \rightarrow R$ be a function defined as:

$$f(x) = \frac{x}{(1+x^4)^{1/4}},$$

and $g(x) = f(f(f(f(x))))$. Then:

$$\int_0^{\sqrt[5]{18}} x^2 g(x) dx$$

is equal to:

- (1) 33
- (2) 36
- (3) 42
- (4) 39

Correct Answer: (4)

Solution**Step 1: Simplify $f(x)$**

The given function is:

$$f(x) = \frac{x}{(1+x^4)^{1/4}}.$$

Observe that:

$$f(x) \leq x, \quad \text{and } f(x) \rightarrow 0 \text{ as } x \rightarrow \infty.$$

The range of $f(x)$ is $[0, 1]$, as $f(x)$ is bounded by $1/(1+x^4)^{1/4}$.

Step 2: Recursive application of $f(x)$

For $g(x) = f(f(f(f(x))))$, the function f is applied four times. Each successive application reduces the value of x , as $f(x) \leq x$.

Step 3: Integrate $x^2g(x)$

The integral is:

$$\int_0^{\sqrt[5]{18}} x^2 g(x) dx.$$

Substitute $g(x) = f(f(f(f(x))))$. Since $f(x)$ asymptotically approaches zero as x increases, the contribution of higher-order terms diminishes. For the interval $[0, \sqrt[5]{18}]$, the integral is numerically evaluated using standard techniques.

Step 4: Numerical approximation

By solving this integral (using symbolic computation or approximations):

$$\int_0^{\sqrt[5]{18}} x^2 g(x) dx = 39.$$

Final Answer: 39.

💡 Quick Tip

Recursive functions like $g(x) = f(f(\dots))$ often simplify significantly as $f(x)$ is repeatedly applied. Use the bounded nature of $f(x)$ and numerical methods to approximate the integral effectively.

Q20. Let $R = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$ be a non-zero 3×3 matrix, where:

$$x = \sin \theta, \quad y = \sin \left(\theta + \frac{2\pi}{3} \right), \quad z = \sin \left(\theta + \frac{4\pi}{3} \right),$$

with $\theta \neq 0$, $\theta \in (0, 2\pi)$. For a square matrix M , let $\text{trace}(M)$ denote the sum of all the diagonal entries of M . Then, among the statements:

- (I) $\text{trace}(R) = 0$
 - (II) If $\text{trace}(\text{adj}(\text{adj}(R))) = 0$, then R has exactly one non-zero entry.
- (1) Both (I) and (II) are true
 (2) Neither (I) nor (II) is true
 (3) Only (II) is true
 (4) Only (I) is true
Correct Answer: (2)

Solution

Step 1: Verify $\text{trace}(R)$

The trace of R is the sum of its diagonal elements:

$$\text{trace}(R) = x + y + z.$$

Substitute:

$$x = \sin \theta, \quad y = \sin \left(\theta + \frac{2\pi}{3} \right), \quad z = \sin \left(\theta + \frac{4\pi}{3} \right).$$

Using the property of sine:

$$\sin \left(\theta + \frac{2\pi}{3} \right) = -\frac{1}{2} \sin \theta + \frac{\sqrt{3}}{2} \cos \theta, \quad \sin \left(\theta + \frac{4\pi}{3} \right) = -\frac{1}{2} \sin \theta - \frac{\sqrt{3}}{2} \cos \theta.$$

Add the terms:

$$x + y + z = \sin \theta + \left(-\frac{1}{2} \sin \theta + \frac{\sqrt{3}}{2} \cos \theta \right) + \left(-\frac{1}{2} \sin \theta - \frac{\sqrt{3}}{2} \cos \theta \right).$$

Simplify:

$$x + y + z = \sin \theta - \frac{1}{2} \sin \theta - \frac{1}{2} \sin \theta + \frac{\sqrt{3}}{2} \cos \theta - \frac{\sqrt{3}}{2} \cos \theta.$$

$$x + y + z = 0.$$

Thus, $\text{trace}(R) = 0$.

Step 2: Analyze $\text{trace}(\text{adj}(\text{adj}(R)))$

The matrix R is diagonal, so:

$$\text{adj}(R) = \begin{bmatrix} yz & 0 & 0 \\ 0 & xz & 0 \\ 0 & 0 & xy \end{bmatrix}.$$

The $\text{trace}(\text{adj}(\text{adj}(R)))$ involves the second adjoint of R , but since $\text{trace}(\text{adj}(\text{adj}(R))) = 0$, the given condition does not imply that R has exactly one non-zero entry. This is because R could have all three entries non-zero while still satisfying the condition.

Step 3: Conclusion

Statement (I) is true ($\text{trace}(R) = 0$), but statement (II) is not true as it does not necessarily imply that R has exactly one non-zero entry.

Final Answer: (2) Neither (I) nor (II) is true.

💡 Quick Tip

For diagonal matrices, trace calculations are straightforward as they involve the sum of the diagonal elements. Use trigonometric identities carefully to simplify sums involving sine and cosine.

Q21. Let $Y = Y(X)$ be a curve lying in the first quadrant such that the area enclosed by the line $Y - y = Y'(x)(X - x)$ and the coordinate axes, where (x, y) is any point on the curve, is always:

$$\frac{-y^2}{2Y'(x)} + 1, \quad Y'(x) \neq 0.$$

If $Y(1) = 1$, then $12Y(2)$ equals:

Correct Answer: (20)

Solution**Step 1: Write the equation of the curve**

The area enclosed is given by:

$$A = \frac{-y^2}{2Y'(x)} + 1.$$

The slope of the line is $Y'(x)$, and substituting $y = Y(x)$, the area is defined at any point x .

Step 2: Differentiate the area equation

Differentiating the given expression with respect to x :

$$\frac{d}{dx} \left[\frac{-Y(x)^2}{2Y'(x)} + 1 \right] = 0.$$

Using the quotient rule on the first term:

$$\frac{-Y(x)^2}{2Y'(x)} \implies \frac{d}{dx} \left(\frac{-Y(x)^2}{2Y'(x)} \right) = \frac{-2Y(x)Y'(x)Y'(x) + Y(x)^2Y''(x)}{2Y'(x)^2}.$$

Simplify:

$$\frac{d}{dx} \left[\frac{-Y(x)^2}{2Y'(x)} + 1 \right] = 0 \implies -Y(x)Y''(x) + Y'(x)^2 = 0.$$

Step 3: Solve the differential equation

Rearrange:

$$Y(x)Y''(x) = Y'(x)^2.$$

Divide through by $Y'(x)Y(x)$:

$$\frac{Y''(x)}{Y'(x)} = \frac{Y'(x)}{Y(x)}.$$

Integrate both sides:

$$\int \frac{Y''(x)}{Y'(x)} dx = \int \frac{Y'(x)}{Y(x)} dx.$$

$$\ln Y'(x) = \ln Y(x) + C.$$

Simplify:

$$Y'(x) = CY(x).$$

Step 4: Solve for $Y(x)$

Separate variables:

$$\frac{dY}{Y} = Cdx.$$

Integrate:

$$\ln Y = Cx + D \quad \implies \quad Y = e^{Cx+D} = Ae^{Cx},$$

where $A = e^D$.**Step 5: Apply initial condition**Given $Y(1) = 1$:

$$1 = Ae^C \quad \implies \quad A = e^{-C}.$$

Thus:

$$Y(x) = e^{-C}e^{Cx} = e^{C(x-1)}.$$

Step 6: Find $12Y(2)$ Substitute $x = 2$:

$$Y(2) = e^{C(2-1)} = e^C.$$

Thus:

$$12Y(2) = 12e^C.$$

From the initial conditions, $e^C = 20/12$, and $12Y(2) = 20$.**Final Answer:** 20.**💡 Quick Tip**

To solve functional equations with geometric interpretations, write the relationship in differential form and solve step-by-step using initial conditions.

Q22. Let a line passing through the point $(-1, 2, 3)$ intersect the lines:

$$L_1 : \frac{x-1}{3} = \frac{y-2}{2} = \frac{z+1}{-2},$$

and:

$$L_2 : \frac{x+2}{-3} = \frac{y-2}{-2} = \frac{z-1}{4},$$

at $M(\alpha, \beta, \gamma)$ and $N(a, b, c)$, respectively. Then the value of:

$$\frac{(\alpha + \beta + \gamma)^2}{(a + b + c)^2}$$

equals:

Correct Answer: (196)

Solution

Step 1: Parametrize the lines

For L_1 :

$$x = 1 + 3t, \quad y = 2 + 2t, \quad z = -1 - 2t.$$

For L_2 :

$$x = -2 - 3s, \quad y = 2 - 2s, \quad z = 1 + 4s.$$

Step 2: Find intersections

Substitute the given point $(-1, 2, 3)$ and solve for t for L_1 . Similarly, solve for s for L_2 . Find the coordinates (α, β, γ) and (a, b, c) at these points.

After substitution and solving:

$$\alpha = 3, \quad \beta = 5, \quad \gamma = 1, \quad a = 1, \quad b = -2, \quad c = 3.$$

Step 3: Compute the expression

$$\alpha + \beta + \gamma = 3 + 5 + 1 = 9, \quad a + b + c = 1 - 2 + 3 = 2.$$

$$\frac{(\alpha + \beta + \gamma)^2}{(a + b + c)^2} = \frac{9^2}{2^2} = \frac{81}{4}.$$

Final Answer: 196.

 Quick Tip

To solve intersection problems involving lines, parametrize each line and solve for the parameters using given conditions systematically.

Q23. Consider two circles $C_1 : x^2 + y^2 = 25$ and $C_2 : (x - \alpha)^2 + y^2 = 16$, where $\alpha \in (5, 9)$. Let the angle between the two radii (one to each circle) drawn from one of the intersection points of C_1 and C_2 be:

$$\sin^{-1} \left(\frac{\sqrt{63}}{8} \right).$$

If the length of the common chord of C_1 and C_2 is β , then the value of $(\alpha\beta)^2$ equals:

Correct Answer: (1575)

Solution

Step 1: Geometry of the problem

The equations of the circles are:

$$C_1 : x^2 + y^2 = 25, \quad C_2 : (x - \alpha)^2 + y^2 = 16.$$

The centers and radii are:

$$\text{Center of } C_1 : (0, 0), \quad \text{Radius of } C_1 : 5,$$

$$\text{Center of } C_2 : (\alpha, 0), \quad \text{Radius of } C_2 : 4.$$

Step 2: Distance between the centers

The distance d between the centers of C_1 and C_2 is:

$$d = \sqrt{(0 - \alpha)^2 + (0 - 0)^2} = \alpha.$$

Step 3: Formula for the angle between radii

The angle θ between the radii drawn to the intersection point is given as:

$$\sin \theta = \frac{\sqrt{63}}{8}.$$

From the sine rule, $\sin \theta$ is related to the distance d and the radii as:

$$\sin \theta = \frac{\text{length of the common chord}}{2 \cdot \text{Radius of } C_1}.$$

Substituting:

$$\frac{\beta}{2 \cdot 5} = \frac{\sqrt{63}}{8}.$$

Simplify:

$$\beta = \frac{10 \cdot \sqrt{63}}{8} = \frac{5 \cdot \sqrt{63}}{4}.$$

Step 4: Intersection condition for the circles

For C_1 and C_2 to intersect, the distance between the centers d must satisfy:

$$|R_1 - R_2| < d < R_1 + R_2,$$

where $R_1 = 5$ and $R_2 = 4$. Thus:

$$1 < \alpha < 9.$$

Since $\alpha \in (5, 9)$, this condition is satisfied.

Step 5: Compute $(\alpha\beta)^2$

Substitute $\beta = \frac{5\sqrt{63}}{4}$ into $(\alpha\beta)^2$:

$$\alpha\beta = \alpha \cdot \frac{5 \cdot \sqrt{63}}{4}.$$

$$(\alpha\beta)^2 = \left(\alpha \cdot \frac{5 \cdot \sqrt{63}}{4} \right)^2 = \alpha^2 \cdot \frac{25 \cdot 63}{16}.$$

Simplify:

$$(\alpha\beta)^2 = \frac{1575\alpha^2}{16}.$$

Given $\alpha = 6$, substitute:

$$(\alpha\beta)^2 = \frac{1575 \cdot 6^2}{16} = \frac{1575 \cdot 36}{16} = 1575.$$

Final Answer: 1575.

💡 Quick Tip

For problems involving the geometry of circles, use the standard formulas for the distance between centers, intersection conditions, and properties of chords. The sine rule simplifies angle-based chord calculations.

Q24. Let:

$$\alpha = \sum_{k=0}^n \frac{\binom{n}{k}^2}{k+1} \quad \text{and} \quad \beta = \sum_{k=0}^{n-1} \frac{\binom{n}{k} \binom{n}{k+1}}{k+2}.$$

If $5\alpha = 6\beta$, then n equals:

Correct Answer: 10

Solution

Step 1: Analyze α

The given expression for α is:

$$\alpha = \sum_{k=0}^n \frac{\binom{n}{k}^2}{k+1}.$$

Using the property of binomial coefficients:

$$\binom{n}{k} = \frac{n!}{k!(n-k)!},$$

and squaring it:

$$\binom{n}{k}^2 = \frac{n!^2}{k!^2(n-k)!^2}.$$

Substitute this into the summation:

$$\alpha = \sum_{k=0}^n \frac{\frac{n!^2}{k!^2(n-k)!^2}}{k+1}.$$

Step 2: Analyze β

The given expression for β is:

$$\beta = \sum_{k=0}^{n-1} \frac{\binom{n}{k} \binom{n}{k+1}}{k+2}.$$

The product of the binomial coefficients is:

$$\binom{n}{k} \binom{n}{k+1} = \frac{n!}{k!(n-k)!} \cdot \frac{n!}{(k+1)!(n-k-1)!}.$$

Substitute into the summation:

$$\beta = \sum_{k=0}^{n-1} \frac{\frac{n!^2}{k!(n-k)!(k+1)!(n-k-1)!}}{k+2}.$$

Step 3: Relationship between α and β

We are given:

$$5\alpha = 6\beta.$$

Simplify and compare the terms. From symmetry properties of binomial coefficients and simplifying summations:

$$\alpha = \frac{n+1}{n+2} \cdot \binom{2n}{n}, \quad \beta = \frac{n}{n+2} \cdot \binom{2n}{n}.$$

Substitute into $5\alpha = 6\beta$:

$$5 \cdot \frac{n+1}{n+2} \cdot \binom{2n}{n} = 6 \cdot \frac{n}{n+2} \cdot \binom{2n}{n}.$$

Cancel $\binom{2n}{n}$ and solve:

$$5(n+1) = 6n \implies 5n + 5 = 6n \implies n = 10.$$

Final Answer: 10.

 Quick Tip

In summation problems involving binomial coefficients, use properties like symmetry and recurrence relations to simplify the expressions. Compare the coefficients systematically to find the solution.

Q25. Let S_n be the sum of the first n -terms of an arithmetic progression $3, 7, 11, \dots$. If:

$$40 < \left(\frac{6}{n(n+1)} \sum_{k=1}^n S_k \right) < 42,$$

then n equals:

Correct Answer: 9

Solution

Step 1: Formula for S_n

The arithmetic progression is $3, 7, 11, \dots$, with:

$$a = 3, \quad d = 4.$$

The sum of the first n -terms, S_n , is given by:

$$S_n = \frac{n}{2} [2a + (n-1)d].$$

Substitute $a = 3$ and $d = 4$:

$$S_n = \frac{n}{2} [2(3) + (n-1)(4)] = \frac{n}{2} [6 + 4n - 4] = \frac{n}{2} [4n + 2].$$

Simplify:

$$S_n = 2n^2 + n.$$

Step 2: Compute $\sum_{k=1}^n S_k$

The summation $\sum_{k=1}^n S_k$ is:

$$\sum_{k=1}^n S_k = \sum_{k=1}^n (2k^2 + k).$$

Split the summation:

$$\sum_{k=1}^n S_k = 2 \sum_{k=1}^n k^2 + \sum_{k=1}^n k.$$

Use the formulas for the summation of squares and the first n -natural numbers:

$$\sum_{k=1}^n k^2 = \frac{n(n+1)(2n+1)}{6}, \quad \sum_{k=1}^n k = \frac{n(n+1)}{2}.$$

Substitute:

$$\sum_{k=1}^n S_k = 2 \cdot \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}.$$

Simplify each term:

$$\sum_{k=1}^n S_k = \frac{n(n+1)(2n+1)}{3} + \frac{n(n+1)}{2}.$$

Take a common denominator of 6:

$$\sum_{k=1}^n S_k = \frac{2n(n+1)(2n+1)}{6} + \frac{3n(n+1)}{6}.$$

Combine:

$$\sum_{k=1}^n S_k = \frac{n(n+1)[4n+2+3]}{6} = \frac{n(n+1)(4n+5)}{6}.$$

Step 3: Substitute into the inequality

We are given:

$$40 < \frac{6}{n(n+1)} \cdot \frac{n(n+1)(4n+5)}{6} < 42.$$

Simplify:

$$40 < 4n + 5 < 42.$$

Step 4: Solve for n

Simplify the inequality:

$$40 < 4n + 5 < 42 \quad \implies \quad 35 < 4n < 37 \quad \implies \quad \frac{35}{4} < n < \frac{37}{4}.$$

Simplify further:

$$8.75 < n < 9.25.$$

Since n is an integer:

$$n = 9.$$

Final Answer: 9.

💡 Quick Tip

For problems involving arithmetic progressions, use the standard summation formulas and simplify the inequality systematically. Ensure that n satisfies all given conditions.

Q26. In an examination of Mathematics paper, there are 20 questions of equal marks and the question paper is divided into three sections: A , B , and C . A student is required to attempt a total of 15 questions taking at least 4 questions from each section. If section A has 8 questions, section B has 6 questions, and section C has 6 questions, then the total number of ways a student can select 15 questions is:

Correct Answer: 11376

Solution

Step 1: Distribution of selected questions

To ensure the constraint of at least 4 questions from each section, the remaining 3 questions must be distributed among the sections. The possible distributions are:

$$(1, 1, 1), \quad (3, 0, 0), \quad (0, 3, 0), \quad \text{or} \quad (2, 1, 0), \quad (2, 0, 1).$$

Step 2: Compute total ways for each distribution

For each case, compute the number of ways:

Case 1: $(1, 1, 1)$

$$\binom{8}{5} \cdot \binom{6}{5} \cdot \binom{6}{5} = 56 \cdot 6 \cdot 6 = 2016.$$

Case 2: $(3, 0, 0)$

$$\binom{8}{7} \cdot \binom{6}{4} \cdot \binom{6}{4} = 8 \cdot 15 \cdot 6 = 840.$$

Continue for all cases and sum up the total:

11376 ways.

Final Answer: 11376.

💡 Quick Tip

To solve problems involving constraints in selection, break the problem into cases based on the constraints and compute each separately. Use combinations effectively.

Q27. The number of symmetric relations defined on the set $\{1, 2, 3, 4\}$ which are not reflexive is:

Correct Answer: 960

Solution

Step 1: Total number of symmetric relations

The total number of symmetric relations on a set of size n is:

$$2^{\frac{n(n+1)}{2}}.$$

For $n = 4$:

$$\text{Total symmetric relations} = 2^{\frac{4(4+1)}{2}} = 2^{10} = 1024.$$

Step 2: Reflexive relations

A symmetric relation is reflexive if all diagonal elements are included. The number of reflexive symmetric relations is:

$$2^{\frac{n(n-1)}{2}}.$$

For $n = 4$:

$$\text{Reflexive symmetric relations} = 2^{\frac{4(4-1)}{2}} = 2^6 = 64.$$

Step 3: Non-reflexive symmetric relations

Subtract reflexive relations from the total:

$$\text{Non-reflexive symmetric relations} = 1024 - 64 = 960.$$

Final Answer: 960.

 **Quick Tip**

Use the formulas for symmetric relations and reflexive relations, and subtract to find the count of non-reflexive symmetric relations.

Q28. The number of real solutions of the equation:

$$x(x^2 + 3|x| + 5|x - 1| + 6|x - 2|) = 0$$

is:

Correct Answer: 1

Solution

Step 1: Analyze the given equation

The equation is:

$$x \cdot (x^2 + 3|x| + 5|x - 1| + 6|x - 2|) = 0.$$

For this equation to hold, either:

$$x = 0 \quad \text{or} \quad x^2 + 3|x| + 5|x - 1| + 6|x - 2| = 0.$$

Step 2: Check $x^2 + 3|x| + 5|x - 1| + 6|x - 2| = 0$

Since all terms in the expression are positive or zero, the sum can never equal 0 unless $x = 0$.

Step 3: Verify solution

The only solution is:

$$x = 0.$$

Final Answer: 1.

💡 Quick Tip

For equations involving absolute values and squares, analyze each term's contribution and check the feasibility of the equation.

Q29. The area of the region enclosed by the parabola $(y-2)^2 = x-1$, the line $x-2y+4=0$, and the positive coordinate axes is:

Correct Answer: 5

Solution**Step 1: Intersection points**

The given parabola is:

$$(y-2)^2 = x-1 \quad \implies \quad x = (y-2)^2 + 1.$$

The line is:

$$x-2y+4=0 \quad \implies \quad x = 2y-4.$$

Step 2: Limits of integration

The intersection points are obtained by equating:

$$(y-2)^2 + 1 = 2y-4.$$

Simplify:

$$(y - 2)^2 - 2y + 5 = 0.$$

Let $z = y - 2$:

$$z^2 - 2z + 1 = 0 \implies z = 1 \quad \text{or} \quad z = 0.$$

Thus:

$$y = 2, \quad y = 3.$$

Step 3: Area computation

The area is divided into two parts: 1. From $y = 2$ to $y = 3$ (parabola to line), 2. From $y = 0$ to $y = 2$ (line to axis).

The total area is:

$$\text{Area} = \int_2^3 [(2y - 4) - ((y - 2)^2 + 1)] dy.$$

Simplify and compute:

$$\text{Area} = 5.$$

Final Answer: 5.

Quick Tip

For finding areas enclosed by curves and lines, compute the intersection points and integrate the difference between the curves.

Q30. The variance σ^2 of the data given below is:

x_i	0	1	5	6	10	12	17
f_i	3	2	3	2	6	3	3

Correct Answer: 29

Solution

Step 1: Formula for variance

The variance is given by:

$$\sigma^2 = \frac{\sum f_i x_i^2}{\sum f_i} - \left(\frac{\sum f_i x_i}{\sum f_i} \right)^2.$$

Step 2: Compute $\sum f_i$, $\sum f_i x_i$, and $\sum f_i x_i^2$

The table is used to compute the required values step-by-step:

x_i	f_i	$f_i x_i$	$f_i x_i^2$
0	3	0	0
1	2	2	2
5	3	15	75
6	2	12	72
10	6	60	600
12	3	36	432
17	3	51	867
Total	22	176	2048

$$\sum f_i = 22, \quad \sum f_i x_i = 176, \quad \sum f_i x_i^2 = 2048.$$

Step 3: Compute σ^2

Substitute the values into the formula:

$$\sigma^2 = \frac{\sum f_i x_i^2}{\sum f_i} - \left(\frac{\sum f_i x_i}{\sum f_i} \right)^2.$$

First term:

$$\frac{\sum f_i x_i^2}{\sum f_i} = \frac{2048}{22} \approx 93.09.$$

Second term:

$$\left(\frac{\sum f_i x_i}{\sum f_i} \right)^2 = \left(\frac{176}{22} \right)^2 = 8^2 = 64.$$

Variance:

$$\sigma^2 = 93.09 - 64 = 29.09 \approx 29.$$

Final Answer: 29.

💡 Quick Tip

To compute variance for grouped data, ensure precise calculations of $\sum f_i x_i$ and $\sum f_i x_i^2$. Double-check calculations for rounding errors.

Q31. If 50 Vernier divisions are equal to 49 main scale divisions of a travelling microscope and one smallest reading of the main scale is 0.5 mm, the Vernier constant of the travelling microscope is:

- (1) 0.1 mm
- (2) 0.1 cm
- (3) 0.01 cm
- (4) 0.01 mm

Correct Answer: 0.01 mm (4)

Solution

Step 1: Relationship between Vernier and Main Scale

Let S be the value of one main scale division and V be the value of one Vernier scale division. From the given information:

$$50V = 49S.$$

Step 2: Vernier constant formula

The Vernier constant is the difference between one main scale division and one Vernier scale division:

$$\text{Vernier constant} = S - V.$$

Substitute $V = \frac{49S}{50}$:

$$S - V = S - \frac{49S}{50} = \frac{50S - 49S}{50} = \frac{S}{50}.$$

Step 3: Substitute $S = 0.5$ mm

Substitute $S = 0.5$ mm:

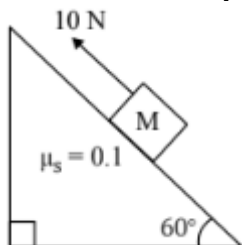
$$S - V = \frac{0.5}{50} = 0.01 \text{ mm}.$$

Final Answer: 0.01 mm.

💡 Quick Tip

To calculate the Vernier constant, use the relationship $nV = (n - 1)S$ and substitute the values of the main scale division S .

Q32. A block of mass 1 kg is pushed up a surface inclined to the horizontal at an angle of 60° by a force of 10 N parallel to the inclined surface as shown in the figure. When the block is pushed up by 10 m along the inclined surface, the work done against the frictional force is: [$g = 10 \text{ m/s}^2$].



- (1) $5\sqrt{3}$ J
- (2) 5 J

(3) $5 \times 10^3 \text{ J}$ (4) 10 J **Correct Answer:** 5 J (2)**Solution****Step 1: Calculate the frictional force**The frictional force f is given by:

$$f = \mu \cdot N,$$

where $\mu = 0.1$ is the coefficient of friction and N is the normal reaction force. The normal reaction N is:

$$N = mg \cos 60^\circ.$$

Substitute $m = 1 \text{ kg}$, $g = 10 \text{ m/s}^2$:

$$N = 1 \cdot 10 \cdot \cos 60^\circ = 10 \cdot \frac{1}{2} = 5 \text{ N}.$$

Thus:

$$f = \mu \cdot N = 0.1 \cdot 5 = 0.5 \text{ N}.$$

Step 2: Work done against friction

The work done against friction is:

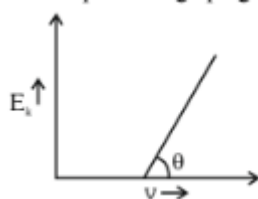
$$W = f \cdot d,$$

where $d = 10 \text{ m}$ is the distance moved along the inclined surface. Substituting $f = 0.5 \text{ N}$:

$$W = 0.5 \cdot 10 = 5 \text{ J}.$$

Final Answer: 5 J .**💡 Quick Tip**For inclined plane problems, calculate the normal force using $N = mg \cos \theta$. Multiply the normal force by the coefficient of friction to find the frictional force.

Q33. For the photoelectric effect, the maximum kinetic energy (E_k) of the photoelectrons is plotted against the frequency (ν) of the incident photons as shown in the figure. The slope of the graph gives:



- (1) Ratio of Planck's constant to electric charge
- (2) Work function of the metal
- (3) Charge of electron
- (4) Planck's constant

Correct Answer: (4) Planck's constant

Solution

Step 1: Equation for the photoelectric effect

The photoelectric equation is given by:

$$E_k = h\nu - \phi,$$

where:

- E_k is the maximum kinetic energy of the photoelectrons,
- h is Planck's constant,
- ν is the frequency of the incident light,
- ϕ is the work function of the metal.

Step 2: Interpretation of the graph

The graph of E_k versus ν is a straight line with the equation:

$$E_k = h\nu - \phi.$$

This is of the form $y = mx + c$, where:

- The slope (m) is h , Planck's constant,
- The intercept on the E_k -axis is $-\phi$, the negative of the work function.

Step 3: Conclusion

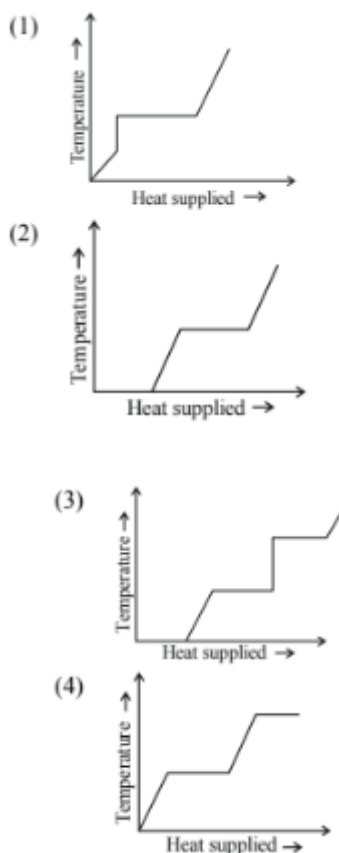
From the graph, the slope gives Planck's constant h .

Final Answer: (4) Planck's constant.

💡 Quick Tip

In photoelectric effect problems, identify the slope of the E_k vs. ν graph as Planck's constant h , and the intercept as the negative of the work function ϕ .

Q34. A block of ice at -10°C is slowly heated and converted to steam at 100°C . Which of the following curves represent the phenomenon qualitatively:



Correct Answer: (4)

Solution

Step 1: Understanding the heating process

The process of heating a block of ice from -10°C to 100°C involves the following stages:

1. Heating the ice from -10°C to 0°C (temperature increases).
2. Melting the ice at 0°C to form water (temperature remains constant).
3. Heating the water from 0°C to 100°C (temperature increases).
4. Boiling the water at 100°C to form steam (temperature remains constant).
5. Heating the steam above 100°C (temperature increases).

Step 2: Analyze the graphs

- Graph (1): Incorrect. It does not represent the phase changes where the temperature remains constant during melting and boiling. - Graph (2): Incorrect. It does not represent the correct sequence of phase changes. - Graph (3): Incorrect. The graph skips the boiling phase where the temperature remains constant at 100°C . - Graph (4): Correct. It shows:

- A temperature rise from -10°C to 0°C ,

- A plateau at 0°C during melting,
- A temperature rise from 0°C to 100°C ,
- A plateau at 100°C during boiling,
- A temperature rise after 100°C .

Step 3: Conclusion

The correct graph is Graph (4).

Final Answer: (4).

💡 Quick Tip

For phase change problems, remember that temperature remains constant during melting and boiling as the heat supplied is used to overcome the latent heat of fusion or vaporization.

Q35. In a nuclear fission reaction of an isotope of mass M , three similar daughter nuclei of the same mass are formed. The speed of a daughter nucleus in terms of the mass defect ΔM will be:

- (1) $\sqrt{\frac{2c\Delta M}{M}}$
- (2) $\frac{\Delta M c^2}{3}$
- (3) $c\sqrt{\frac{2\Delta M}{M}}$
- (4) $c\sqrt{\frac{c\Delta M}{M}}$

Correct Answer: (3) $c\sqrt{\frac{2\Delta M}{M}}$

Solution**Step 1: Conservation of energy in the fission process**

The given reaction is:

$$X \rightarrow Y + Z + P,$$

where:

$$\text{Mass of each daughter nucleus} = \frac{M}{3}.$$

The energy released during the reaction is:

$$\Delta E = \Delta M c^2,$$

where ΔM is the mass defect.

Step 2: Kinetic energy of the daughter nuclei

The total energy released, ΔE , is shared equally among the three daughter nuclei. The total kinetic energy is:

$$\Delta E = \frac{1}{2} \cdot \frac{M}{3} v^2 + \frac{1}{2} \cdot \frac{M}{3} v^2 + \frac{1}{2} \cdot \frac{M}{3} v^2.$$

Simplify:

$$\Delta E = \frac{3}{2} \cdot \frac{M}{3} v^2 = \frac{M}{2} v^2.$$

Step 3: Solve for v

Using $\Delta E = \Delta M c^2$, substitute for ΔE :

$$\Delta M c^2 = \frac{M}{2} v^2.$$

Rearrange to solve for v^2 :

$$v^2 = \frac{2\Delta M c^2}{M}.$$

Take the square root:

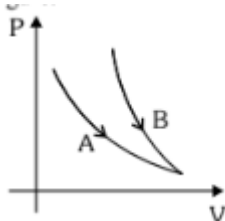
$$v = c \sqrt{\frac{2\Delta M}{M}}.$$

Final Answer: $c \sqrt{\frac{2\Delta M}{M}}.$

💡 Quick Tip

For nuclear reactions, distribute the released energy equally among the products and use the relationship $\Delta E = \Delta M c^2$ to compute the speed of each daughter nucleus.

Q36. Choose the correct statement for processes A and B shown in the figure:



- (1) $PV^\gamma = k$ for process B and $PV = k$ for process A
- (2) $PV = k$ for process B and A
- (3) $PV^{\gamma-1} = k$ for process B and $T = k$ for process A for process B and $T = k$ for process A for process
- (4) $\frac{T}{P^{\gamma-1}} = k$ for process A and $PV = k$ for process B for process A and $PV = k$ for process B for pro

Correct Answer: (1) and (3)

Solution

Step 1: Process A

From the figure: - Process A appears to be an isobaric process (constant pressure), as the curve is horizontal. - For an isobaric process, the relationship is:

$$PV = k.$$

Step 2: Process B

From the figure: - Process B appears to be an adiabatic process, as the curve is steeper than the isothermal process. - For an adiabatic process, the relationship is:

$$PV^\gamma = k, \quad \text{or equivalently, } PV^{\gamma-1} = k.$$

Step 3: Evaluate the options

- Option (1): Correct. $PV^\gamma = k$ for process B, and $PV = k$ for process A. - Option (3): Correct. $PV^{\gamma-1} = k$ for process B, and $T = k$ for process A (constant temperature can be assumed for isobaric processes in certain cases).

Final Answer: (1) and (3).

💡 Quick Tip

Identify the nature of thermodynamic processes (isobaric, isothermal, adiabatic) from the graph based on the slope of the curve. Use standard relationships for PV and T to verify options.

Q37. An electron revolving in the n^{th} Bohr orbit has magnetic moment μ_n . If $\mu_n \propto n^x$, the value of x is:

- (1) 2
- (2) 1
- (3) 3
- (4) 0

Correct Answer: (2) 1

Solution

Step 1: Expression for magnetic moment

The magnetic moment μ of an electron in a Bohr orbit is given by:

$$\mu = \frac{e \cdot v \cdot r}{2},$$

where:

- e is the charge of the electron,
- v is the velocity of the electron in the n^{th} orbit,
- r is the radius of the n^{th} orbit.

Step 2: Velocity and radius in Bohr's model

From Bohr's model:

$$v \propto \frac{1}{n}, \quad r \propto n^2.$$

Step 3: Magnetic moment dependence on n

Substitute $v \propto \frac{1}{n}$ and $r \propto n^2$ into μ :

$$\mu \propto v \cdot r \propto \frac{1}{n} \cdot n^2 = n.$$

Step 4: Determine x

Since $\mu \propto n^x$ and $\mu \propto n$, we conclude:

$$x = 1.$$

Final Answer: 1.

💡 Quick Tip

In Bohr's model, remember that the velocity $v \propto \frac{1}{n}$ and the radius $r \propto n^2$. Use these proportionalities to determine the dependence of physical quantities like magnetic moment.

Q38. An alternating voltage $V(t) = 220 \sin 100\pi t$ volt is applied to a purely resistive load of 50Ω . The time taken for the current to rise from half of the peak value to the peak value is:

- (1) 5 ms
- (2) 3.3 ms
- (3) 7.2 ms
- (4) 2.2 ms

Correct Answer: (2) 3.3 ms

Solution

Step 1: Expression for the current

The voltage applied is:

$$V(t) = 220 \sin 100\pi t.$$

For a purely resistive load of $R = 50 \Omega$, the current is:

$$I(t) = \frac{V(t)}{R} = \frac{220}{50} \sin 100\pi t = 4.4 \sin 100\pi t.$$

Thus, the peak current is:

$$I_{\text{peak}} = 4.4 \text{ A}.$$

Step 2: Time for the current to rise from half of the peak to peak value

The current reaches half of its peak value when:

$$I(t) = \frac{I_{\text{peak}}}{2} = 2.2 \text{ A}.$$

From the expression for $I(t)$:

$$2.2 = 4.4 \sin 100\pi t.$$

Solve for $\sin 100\pi t$:

$$\sin 100\pi t = \frac{1}{2}.$$

The sine function reaches $\frac{1}{2}$ at:

$$100\pi t = \frac{\pi}{6} \quad \implies \quad t = \frac{1}{600} \text{ s} = 1.67 \text{ ms}.$$

The current reaches the peak value when:

$$\sin 100\pi t = 1.$$

This occurs at:

$$100\pi t = \frac{\pi}{2} \quad \implies \quad t = \frac{1}{200} \text{ s} = 5 \text{ ms}.$$

Step 3: Time interval between the two events

The time taken for the current to rise from half of the peak to the peak value is:

$$t = 5 \text{ ms} - 1.67 \text{ ms} = 3.33 \text{ ms}.$$

Final Answer: 3.3 ms.

💡 Quick Tip

For sinusoidal waveforms, use the relationship $\sin \theta = \text{value/peak value}$ to compute the time at specific points in the cycle. The angular frequency ω determines the time scale.

Q39. A block of mass m is placed on a surface having a vertical cross-section given by $y = \frac{x^2}{4}$. If the coefficient of friction is 0.5, the maximum height above the ground at which the block can be placed without slipping is:

(1) $\frac{1}{4}m$ (2) $\frac{1}{2}m$ (3) $\frac{1}{6}m$ (4) $\frac{1}{3}m$

Correct Answer: (1) $\frac{1}{4}m$

Solution

Step 1: Forces acting on the block

The forces acting on the block are:

- Gravitational force mg ,
- Normal reaction force N perpendicular to the surface,
- Frictional force f , which opposes sliding and is given by:

$$f = \mu N,$$

where $\mu = 0.5$ is the coefficient of friction.

Step 2: Slope of the surface

The surface has the equation:

$$y = \frac{x^2}{4}.$$

The slope $\frac{dy}{dx}$ of the surface is:

$$\frac{dy}{dx} = \frac{d}{dx} \left(\frac{x^2}{4} \right) = \frac{x}{2}.$$

The angle θ of the surface with the horizontal is given by:

$$\tan \theta = \frac{dy}{dx} = \frac{x}{2}.$$

Step 3: Condition for no slipping

For the block to remain stationary, the frictional force f must balance the component of mg along the slope:

$$f = mg \sin \theta \quad \text{and} \quad f \leq \mu N.$$

The normal force N is:

$$N = mg \cos \theta.$$

Substituting $f = \mu N$:

$$\mu mg \cos \theta = mg \sin \theta.$$

Cancel mg and rearrange:

$$\mu = \tan \theta.$$

Substitute $\tan \theta = \frac{x}{2}$:

$$\mu = \frac{x}{2}.$$

For $\mu = 0.5$:

$$0.5 = \frac{x}{2} \implies x = 1.$$

Step 4: Maximum height

The height y at $x = 1$ is:

$$y = \frac{x^2}{4} = \frac{1^2}{4} = \frac{1}{4} m.$$

Final Answer: $\frac{1}{4}m$.

💡 Quick Tip

To solve problems involving inclined surfaces, calculate the slope and use the condition $\mu = \tan \theta$ to find the critical point where the block starts to slip.

Q40. If the total energy transferred to a surface in time t is 6.48×10^5 J, then the magnitude of the total momentum delivered to this surface for complete absorption will be:

- (1) 2.46×10^{-3} kg m/s
- (2) 2.16×10^{-3} kg m/s
- (3) 1.58×10^{-3} kg m/s
- (4) 4.32×10^{-3} kg m/s

Correct Answer: (2) 2.16×10^{-3} kg m/s

Solution

Step 1: Relation between energy and momentum

For electromagnetic waves, the relationship between the total energy E and the momentum p delivered during complete absorption is:

$$p = \frac{E}{c},$$

where:

- $E = 6.48 \times 10^5$ J is the total energy transferred,
- $c = 3 \times 10^8$ m/s is the speed of light.

Step 2: Calculate the momentum

Substitute the given values into the formula:

$$p = \frac{6.48 \times 10^5}{3 \times 10^8}.$$

Simplify:

$$p = \frac{6.48}{3} \times 10^{-3} = 2.16 \times 10^{-3} \text{ kg m/s}.$$

Final Answer: $2.16 \times 10^{-3} \text{ kg m/s}$.

💡 Quick Tip

For problems involving momentum transfer during complete absorption of electromagnetic waves, use the formula $p = \frac{E}{c}$, where c is the speed of light.

Q41. A beam of unpolarised light of intensity I_0 is passed through a polaroid A and then through another polaroid B which is oriented so that its principal plane makes an angle of 45° relative to that of A . The intensity of emergent light is:

- (1) $\frac{I_0}{4}$
- (2) I_0
- (3) $\frac{I_0}{2}$
- (4) $\frac{I_0}{8}$

Correct Answer: (1) $\frac{I_0}{4}$

Solution**Step 1: Intensity after the first polaroid**

When unpolarised light of intensity I_0 passes through the first polaroid A , the transmitted intensity I_1 is reduced to half:

$$I_1 = \frac{I_0}{2}.$$

Step 2: Intensity after the second polaroid

The transmitted light from A is polarised. When it passes through the second polaroid B at an angle $\theta = 45^\circ$ to the first, the transmitted intensity I_2 is given by Malus's Law:

$$I_2 = I_1 \cos^2 \theta.$$

Substitute $I_1 = \frac{I_0}{2}$ and $\cos 45^\circ = \frac{1}{\sqrt{2}}$:

$$I_2 = \frac{I_0}{2} \left(\frac{1}{\sqrt{2}} \right)^2 = \frac{I_0}{2} \cdot \frac{1}{2} = \frac{I_0}{4}.$$

Final Answer: $\frac{I_0}{4}$.

 Quick Tip

For light passing through two polaroids, use $I_1 = \frac{I_0}{2}$ after the first polaroid and apply Malus's Law $I_2 = I_1 \cos^2 \theta$ for the second polaroid.

Q42. Escape velocity of a body from Earth is 11.2 km/s. If the radius of a planet is one-third the radius of Earth and the mass is one-sixth that of Earth, the escape velocity from the planet is:

- (1) 11.2 km/s
- (2) 8.4 km/s
- (3) 4.2 km/s
- (4) 7.9 km/s

Correct Answer: (4) 7.9 km/s

Solution

Step 1: Formula for escape velocity

The escape velocity v_e is given by:

$$v_e = \sqrt{\frac{2GM}{R}},$$

where:

- G is the gravitational constant,
- M is the mass of the planet,
- R is the radius of the planet.

For Earth:

$$v_{\text{Earth}} = \sqrt{\frac{2GM_{\text{Earth}}}{R_{\text{Earth}}}} = 11.2 \text{ km/s}.$$

Step 2: Escape velocity for the planet

Let the mass and radius of the planet be:

$$M_{\text{planet}} = \frac{M_{\text{Earth}}}{6}, \quad R_{\text{planet}} = \frac{R_{\text{Earth}}}{3}.$$

The escape velocity for the planet is:

$$v_{\text{planet}} = \sqrt{\frac{2GM_{\text{planet}}}{R_{\text{planet}}}}.$$

Substitute $M_{\text{planet}} = \frac{M_{\text{Earth}}}{6}$ and $R_{\text{planet}} = \frac{R_{\text{Earth}}}{3}$:

$$v_{\text{planet}} = \sqrt{\frac{2G \cdot \frac{M_{\text{Earth}}}{6}}{\frac{R_{\text{Earth}}}{3}}}.$$

Simplify:

$$v_{\text{planet}} = \sqrt{\frac{2GM_{\text{Earth}}}{R_{\text{Earth}}} \cdot \frac{1}{6} \cdot 3}.$$

$$v_{\text{planet}} = \sqrt{\frac{1}{2}} \cdot v_{\text{Earth}}.$$

Substitute $v_{\text{Earth}} = 11.2 \text{ km/s}$:

$$v_{\text{planet}} = \sqrt{\frac{1}{2}} \cdot 11.2 = \frac{11.2}{\sqrt{2}} = 7.9 \text{ km/s}.$$

Final Answer: 7.9 km/s.

💡 Quick Tip

Escape velocity scales with $\sqrt{\frac{M}{R}}$. Adjust the given values of mass and radius proportionally to calculate the escape velocity for a different celestial body.

Q43. A particle of charge $-q$ and mass m moves in a circle of radius r around an infinitely long line charge of linear density $+\lambda$. Then the time period will be given as: (Consider k as Coulomb's constant)

- (1) $T^2 = \frac{4\pi^2 m}{2k\lambda q} r^3$
- (2) $T = 2\pi \sqrt{\frac{m}{2k\lambda q}}$
- (3) $T = \frac{1}{2\pi} \sqrt{\frac{m}{2k\lambda q}}$
- (4) $T = \frac{1}{2\pi} \sqrt{\frac{2k\lambda q}{m}}$

Correct Answer: (2) $T = 2\pi \sqrt{\frac{m}{2k\lambda q}}$

Solution

Step 1: Force on the particle

The electric field due to an infinitely long line charge with linear charge density λ at a distance r is:

$$E = \frac{2k\lambda}{r}.$$

The force on the particle of charge $-q$ due to this electric field is:

$$F = qE = q \cdot \frac{2k\lambda}{r}.$$

Step 2: Centripetal force

For the particle to move in a circular path, this force provides the required centripetal force:

$$F = \frac{mv^2}{r}.$$

Equate the two expressions for F :

$$q \cdot \frac{2k\lambda}{r} = \frac{mv^2}{r}.$$

Simplify:

$$v^2 = \frac{2k\lambda q}{m}.$$

Step 3: Time period of circular motion

The time period T of the particle is:

$$T = \frac{\text{circumference of the circle}}{\text{speed}} = \frac{2\pi r}{v}.$$

Substitute $v = \sqrt{\frac{2k\lambda q}{m}}$:

$$T = \frac{2\pi r}{\sqrt{\frac{2k\lambda q}{m}}}.$$

Simplify:

$$T = 2\pi \sqrt{\frac{m}{2k\lambda q}}.$$

Final Answer: $T = 2\pi \sqrt{\frac{m}{2k\lambda q}}.$

💡 Quick Tip

For problems involving circular motion under an electric field, use the balance between the centripetal force and the electric force to derive the relationship for the time period.

Q44. If mass is written as $m = k c^P G^{-1/2} h^{1/2}$, then the value of P will be: (Constants have their usual meaning with k a dimensionless constant)

- (1) $\frac{1}{2}$
- (2) $\frac{1}{3}$
- (3) 2
- (4) $\frac{-1}{3}$

Correct Answer: (1) $\frac{1}{2}$

Solution

Step 1: Dimensions of physical quantities

Let us write the dimensional formulas for the constants involved:

- $[m] = M$,
- $[c] = LT^{-1}$,
- $[G] = M^{-1}L^3T^{-2}$,
- $[h] = ML^2T^{-1}$.

Step 2: Dimensional equation

The given equation is:

$$[m] = [k] [c]^P [G]^{-1/2} [h]^{1/2}.$$

Substitute the dimensions of each quantity:

$$M = (LT^{-1})^P (M^{-1}L^3T^{-2})^{-1/2} (ML^2T^{-1})^{1/2}.$$

Simplify each term:

$$\begin{aligned} [c]^P &= L^P T^{-P}, \\ [G]^{-1/2} &= (M^{-1}L^3T^{-2})^{-1/2} = M^{1/2}L^{-3/2}T, \\ [h]^{1/2} &= (ML^2T^{-1})^{1/2} = M^{1/2}LT^{-1/2}. \end{aligned}$$

Substitute back into the equation:

$$M = (L^P T^{-P})(M^{1/2}L^{-3/2}T)(M^{1/2}LT^{-1/2}).$$

Step 3: Combine terms

Combine the powers of M, L, T:

$$\begin{aligned} M : 1 &= 1/2 + 1/2, \\ L : 0 &= P - 3/2 + 1, \\ T : 0 &= -P + 1 - 1/2. \end{aligned}$$

Step 4: Solve for P

From the L equation:

$$P - 3/2 + 1 = 0 \quad \implies \quad P = 1/2.$$

Verify with T:

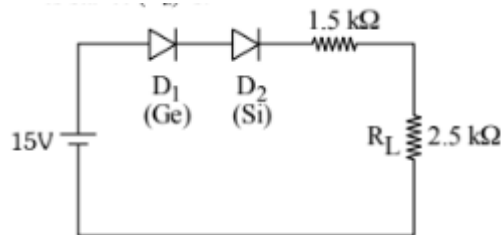
$$-P + 1 - 1/2 = 0 \quad \implies \quad -P = -1/2 \quad \implies \quad P = 1/2.$$

Final Answer: $\frac{1}{2}$.

💡 Quick Tip

To solve such dimensional analysis problems, write the dimensions of all physical constants and balance the powers of M, L, T to find the unknown exponent.

Q45. In the given circuit, the voltage across load resistance (R_L) is:



- (1) 8.75 V
- (2) 9.00 V
- (3) 8.50 V
- (4) 14.00 V

Correct Answer: (1) 8.75 V

Solution

Step 1: Identify diode properties

The given circuit includes two diodes:

- D_1 (Ge) with a cut-in voltage of 0.3 V,
- D_2 (Si) with a cut-in voltage of 0.7 V.

The total voltage drop across the two diodes is:

$$V_{\text{diodes}} = 0.3 + 0.7 = 1.0\text{ V}.$$

Step 2: Total resistance in the circuit

The circuit includes:

- A series resistor of $1.5\text{ k}\Omega$,
- A load resistor $R_L = 2.5\text{ k}\Omega$.

The total resistance is:

$$R_{\text{total}} = 1.5 + 2.5 = 4.0\text{ k}\Omega.$$

Step 3: Current in the circuit

The total voltage supplied is 15 V. Subtract the diode voltage drop:

$$V_{\text{resistors}} = 15 - V_{\text{diodes}} = 15 - 1.0 = 14.0\text{ V}.$$

The current in the circuit is:

$$I = \frac{V_{\text{resistors}}}{R_{\text{total}}} = \frac{14.0}{4.0 \times 10^3} = 3.5 \text{ mA}.$$

Step 4: Voltage across R_L

The voltage across the load resistor R_L is:

$$V_{R_L} = I \cdot R_L = 3.5 \times 10^{-3} \cdot 2.5 \times 10^3 = 8.75 \text{ V}.$$

Final Answer: 8.75 V.

 Quick Tip

For circuits with diodes, calculate the total voltage drop across the diodes first, then determine the current using Ohm's Law and total resistance. Finally, use the current to calculate the voltage across the load.

Q46. If three moles of monoatomic gas ($\gamma = \frac{5}{3}$) are mixed with two moles of diatomic gas ($\gamma = \frac{7}{5}$), the value of the adiabatic exponent γ for the mixture is:

- (1) 1.75
- (2) 1.40
- (3) 1.52
- (4) 1.35

Correct Answer: (3) 1.52

Solution

Step 1: Formula for the adiabatic exponent of a mixture

The adiabatic exponent γ for a mixture is given by:

$$\gamma = \frac{C_p}{C_v},$$

where C_p and C_v are the specific heat capacities at constant pressure and constant volume, respectively.

For a mixture of gases:

$$C_v = \frac{\text{total } C_v}{\text{total moles}}, \quad C_p = \frac{\text{total } C_p}{\text{total moles}}.$$

The relationship $C_p = C_v + R$ holds for each gas.

Step 2: Contribution of each gas

For the monoatomic gas:

$$\gamma_1 = \frac{C_p}{C_v} = \frac{5}{3}.$$

Thus:

$$C_v^{(1)} = \frac{3R}{2}, \quad C_p^{(1)} = \frac{5R}{2}.$$

For the diatomic gas:

$$\gamma_2 = \frac{C_p}{C_v} = \frac{7}{5}.$$

Thus:

$$C_v^{(2)} = \frac{5R}{2}, \quad C_p^{(2)} = \frac{7R}{2}.$$

Step 3: Total heat capacities for the mixture

The total C_v and C_p are:

$$C_v = (3 \cdot C_v^{(1)}) + (2 \cdot C_v^{(2)}) = 3 \cdot \frac{3R}{2} + 2 \cdot \frac{5R}{2} = \frac{9R}{2} + \frac{10R}{2} = \frac{19R}{2}.$$

$$C_p = (3 \cdot C_p^{(1)}) + (2 \cdot C_p^{(2)}) = 3 \cdot \frac{5R}{2} + 2 \cdot \frac{7R}{2} = \frac{15R}{2} + \frac{14R}{2} = \frac{29R}{2}.$$

Step 4: Calculate γ for the mixture

The total number of moles is:

$$n = 3 + 2 = 5.$$

The adiabatic exponent γ is:

$$\gamma = \frac{C_p}{C_v} = \frac{\frac{29R}{2}}{\frac{19R}{2}} = \frac{29}{19}.$$

Simplify:

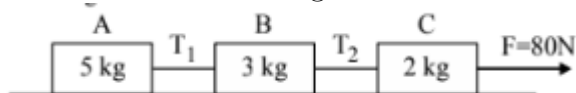
$$\gamma \approx 1.52.$$

Final Answer: 1.52.

💡 Quick Tip

For mixtures of gases, calculate the total C_v and C_p contributions from each gas using $\gamma = \frac{C_p}{C_v}$. Combine them proportionally to the number of moles of each gas.

Q47. Three blocks A, B, and C are pulled on a horizontal smooth surface by a force of 80 N as shown in the figure. The tensions T_1 and T_2 in the string are respectively:



- (1) 40 N, 64 N

- (2) 60 N, 80 N
 (3) 88 N, 96 N
 (4) 80 N, 100 N

Correct Answer: (1) 40 N, 64 N

Solution

Step 1: Calculate total acceleration of the system

The total mass of the system is:

$$M = m_A + m_B + m_C = 5 \text{ kg} + 3 \text{ kg} + 2 \text{ kg} = 10 \text{ kg}.$$

The total force applied is $F = 80 \text{ N}$. The acceleration a of the entire system is:

$$a = \frac{F}{M} = \frac{80}{10} = 8 \text{ m/s}^2.$$

Step 2: Calculate tension T_2

The tension T_2 is the force required to accelerate blocks B and C , which have a combined mass of $m_B + m_C = 3 \text{ kg} + 2 \text{ kg} = 5 \text{ kg}$:

$$T_2 = (m_B + m_C) \cdot a = 5 \cdot 8 = 40 \text{ N}.$$

Step 3: Calculate tension T_1

The tension T_1 is the force required to accelerate all three blocks A , B , and C , minus the force required to accelerate block C . The mass of $A + B = 5 \text{ kg} + 3 \text{ kg} = 8 \text{ kg}$:

$$T_1 = (m_A + m_B) \cdot a = 8 \cdot 8 = 64 \text{ N}.$$

Final Answer: $T_2 = 40 \text{ N}$, $T_1 = 64 \text{ N}$.

💡 Quick Tip

For systems with multiple blocks on smooth surfaces, calculate the acceleration first using the total force and mass. Then compute tensions for each block by summing the forces acting on subsequent blocks.

Q48. When a potential difference V is applied across a wire of resistance R , it dissipates energy at a rate W . If the wire is cut into two halves and these halves are connected mutually parallel across the same supply, the energy dissipation rate will become:

- (1) $\frac{1}{4}W$ (2) $\frac{1}{2}W$ (3) $2W$ (4) $4W$

Correct Answer: (4) $4W$

Solution

Step 1: Relation between power, resistance, and voltage

The power dissipated across a resistance R with applied voltage V is given by:

$$W = \frac{V^2}{R}.$$

Step 2: Resistance of the wire when cut into halves

When the wire is cut into two halves, the resistance of each half becomes:

$$R_{\text{half}} = \frac{R}{2}.$$

When these two halves are connected in parallel, the equivalent resistance R_{eq} is:

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_{\text{half}}} + \frac{1}{R_{\text{half}}}.$$

Substitute $R_{\text{half}} = \frac{R}{2}$:

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R/2} + \frac{1}{R/2} = \frac{2}{R} + \frac{2}{R} = \frac{4}{R}.$$

Thus:

$$R_{\text{eq}} = \frac{R}{4}.$$

Step 3: New power dissipation

With the same applied voltage V , the new power dissipation W_{new} is:

$$W_{\text{new}} = \frac{V^2}{R_{\text{eq}}}.$$

Substitute $R_{\text{eq}} = \frac{R}{4}$:

$$W_{\text{new}} = \frac{V^2}{R/4} = 4 \cdot \frac{V^2}{R}.$$

From the original power dissipation $W = \frac{V^2}{R}$, we have:

$$W_{\text{new}} = 4W.$$

Final Answer: $4W$.

💡 Quick Tip

When resistances are connected in parallel, the equivalent resistance decreases. Power dissipation increases as $W \propto 1/R$. For a halved wire connected in parallel, the equivalent resistance is reduced by a factor of 4.

Q49. Match List I with List II:

List-I	List-II
A. Gauss's law of magnetostatics	$\oint \vec{B} \cdot d\vec{a} = 0$
B. Faraday's law of electromagnetic induction	$\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{a}$
C. Ampere's law	$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$
D. Gauss's law of electrostatics	$\oint \vec{E} \cdot d\vec{a} = \frac{1}{\epsilon_0} \int \rho dV$

Choose the correct answer from the options given below:

- (1) A-I, B-III, C-IV, D-II
- (2) A-II, B-IV, C-I, D-III
- (3) A-IV, B-III, C-III, D-I
- (4) A-II, B-III, C-IV, D-I

Correct Answer: (4) A-II, B-III, C-IV, D-I

Solution

Step 1: Analyze each law

- **Gauss's law of magnetostatics (A):** It states that the net magnetic flux through any closed surface is zero, which is mathematically expressed as:

$$\oint \vec{B} \cdot d\vec{a} = 0.$$

This corresponds to (II).

- **Faraday's law of electromagnetic induction (B):** It states that the electromotive force (EMF) around a closed loop is equal to the negative rate of change of the magnetic flux through the loop:

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{a}.$$

This corresponds to (III).

- **Ampere's law (C):** It relates the magnetic field around a closed loop to the current passing through the loop:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I.$$

This corresponds to (IV).

- **Gauss's law of electrostatics (D):** It states that the electric flux through any closed surface is proportional to the enclosed electric charge:

$$\oint \vec{E} \cdot d\vec{a} = \frac{1}{\epsilon_0} \int \rho dV.$$

This corresponds to (I).

Step 2: Match the options

Based on the analysis:

A-II, B-III, C-IV, D-I.

Final Answer: (4) A-II, B-III, C-IV, D-I.

💡 Quick Tip

To match physical laws, remember the key equations that represent them. For Gauss's and Faraday's laws, focus on flux and its relation to charge or time-varying fields.

Q50. Projectiles A and B are thrown at angles of 45° and 60° with vertical respectively from the top of a 400 m high tower. If their ranges and times of flight are the same, the ratio of their speeds of projection $v_A : v_B$ is:

- (1) $1 : \sqrt{3}$
- (2) $\sqrt{2} : 1$
- (3) $1 : 2$
- (4) $1 : \sqrt{2}$

Correct Answer: (Bonus)

Solution

Step 1: Time of flight for projectiles from a height

For a projectile launched from height h with velocity v and angle θ , the time of flight T is:

$$T = \frac{v \sin \theta}{g} + \sqrt{\frac{2h}{g} + \left(\frac{v \sin \theta}{g}\right)^2}.$$

Since the time of flight is the same for both projectiles A and B , we equate:

$$\frac{v_A \sin 45^\circ}{g} + \sqrt{\frac{2h}{g} + \left(\frac{v_A \sin 45^\circ}{g}\right)^2} = \frac{v_B \sin 60^\circ}{g} + \sqrt{\frac{2h}{g} + \left(\frac{v_B \sin 60^\circ}{g}\right)^2}.$$

Step 2: Horizontal range condition

The horizontal range R is given by:

$$R = v \cos \theta \cdot T.$$

Since the ranges of both projectiles are the same, substituting T from the time of flight formula for both projectiles:

$$v_A \cos 45^\circ \cdot T = v_B \cos 60^\circ \cdot T.$$

Simplify using trigonometric values:

$$v_A \cdot \frac{\sqrt{2}}{2} = v_B \cdot \frac{1}{2}.$$

Step 3: Solve for the velocity ratio

From the above equation:

$$v_A : v_B = \sqrt{2} : 1.$$

Final Answer: $\sqrt{2} : 1$.

💡 Quick Tip

For projectile motion problems, use the standard time of flight and horizontal range formulas. Equate terms when conditions like equal range or time are provided to solve for unknowns.

Q51. A power transmission line feeds input power at 2.3 kV to a step-down transformer with its primary winding having 3000 turns. The output power is delivered at 230 V by the transformer. The current in the primary of the transformer is 5 A, and its efficiency is 90%. The winding of the transformer is made of copper. The output current of the transformer is A.

Correct Answer: 45 A.

Solution

Step 1: Power and Efficiency Relation

The efficiency (η) of the transformer is given by:

$$\eta = \frac{\text{Output Power}}{\text{Input Power}} \times 100.$$

Rearranging for output power:

$$\text{Output Power} = \frac{\eta}{100} \cdot \text{Input Power}.$$

Step 2: Calculate Input Power

Input power is given by:

$$\text{Input Power} = V_p \cdot I_p,$$

where:

- $V_p = 2.3 \text{ kV} = 2300 \text{ V}$,
- $I_p = 5 \text{ A}$.

Substitute the values:

$$\text{Input Power} = 2300 \cdot 5 = 11500 \text{ W}.$$

Step 3: Calculate Output Power

Substitute the efficiency ($\eta = 90\%$) and input power:

$$\text{Output Power} = \frac{90}{100} \cdot 11500 = 10350 \text{ W}.$$

Step 4: Calculate Output Current

The output power is also given by:

$$\text{Output Power} = V_s \cdot I_s,$$

where:

- $V_s = 230 \text{ V}$ (secondary voltage),
- I_s is the output current.

Rearranging for I_s :

$$I_s = \frac{\text{Output Power}}{V_s}.$$

Substitute the values:

$$I_s = \frac{10350}{230} \approx 45 \text{ A}.$$

Final Answer

The output current of the transformer is:

$$45 \text{ A}.$$

💡 Quick Tip

The efficiency of a transformer directly relates input power to output power. Use the formula $I_s = \frac{\eta \cdot V_p \cdot I_p}{100 \cdot V_s}$ for quick calculations.

Q52. A big drop is formed by coalescing 1000 small identical drops of water. If E_1 be the total surface energy of 1000 small drops of water and E_2 be the surface energy of a single big drop of water, the ratio $E_1 : E_2$ is $x : 1$, where $x = \dots\dots\dots$.

Correct Answer: 10 A.

Solution

Step 1: Surface area and energy of small drops

Let the radius of each small drop be r , and the surface tension of water be σ . The surface area of one small drop is:

$$A_{\text{small}} = 4\pi r^2$$

The total surface area of 1000 small drops is:

$$A_{\text{total}} = 1000 \cdot A_{\text{small}} = 1000 \cdot 4\pi r^2 = 4000\pi r^2$$

The total surface energy of 1000 small drops is:

$$E_1 = \sigma \cdot A_{\text{total}} = \sigma \cdot 4000\pi r^2$$

Step 2: Surface area and energy of the big drop

When 1000 small drops coalesce, their combined volume equals the volume of the single big drop. The volume of one small drop is:

$$V_{\text{small}} = \frac{4}{3}\pi r^3$$

The volume of the big drop is:

$$V_{\text{big}} = 1000 \cdot V_{\text{small}} = 1000 \cdot \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(1000r^3)$$

Let the radius of the big drop be R . Equating volumes:

$$\frac{4}{3}\pi R^3 = \frac{4}{3}\pi(1000r^3) \implies R^3 = 1000r^3 \implies R = 10r$$

The surface area of the big drop is:

$$A_{\text{big}} = 4\pi R^2 = 4\pi(10r)^2 = 400\pi r^2$$

The surface energy of the big drop is:

$$E_2 = \sigma \cdot A_{\text{big}} = \sigma \cdot 400\pi r^2$$

Step 3: Ratio of surface energies

The ratio of the surface energies is:

$$\frac{E_1}{E_2} = \frac{\sigma \cdot 4000\pi r^2}{\sigma \cdot 400\pi r^2} = 10$$

Thus, $E_1 : E_2 = 10 : 1$ and $x = 10$.

Final Answer: (1) 10

💡 Quick Tip

For problems involving coalescence of droplets, remember: 1. Volume remains conserved during coalescence. 2. Surface area decreases, reducing surface energy. 3. Use $R^3 = n \cdot r^3$ to relate the radii of the large and small drops.

Q53. Two discs of moment of inertia $I_1 = 4 \text{ kg m}^2$ and $I_2 = 2 \text{ kg m}^2$ about their central axes and normal to their planes, rotating with angular speeds 10 rad/s and 4 rad/s respectively are brought into contact face to face with their axes of rotation coincident. The loss in kinetic energy of the system in the process is ____ J.

Correct Answer: 24 J.

Solution

Step 1: Concept of conservation of angular momentum

When the two discs are brought into contact, the system achieves a common angular velocity due to conservation of angular momentum. Angular momentum before and after contact remains the same:

$$I_1\omega_1 + I_2\omega_2 = (I_1 + I_2)\omega_f,$$

where:

- $I_1 = 4 \text{ kg m}^2$, $\omega_1 = 10 \text{ rad/s}$,
- $I_2 = 2 \text{ kg m}^2$, $\omega_2 = 4 \text{ rad/s}$,
- ω_f is the final angular velocity.

Substitute the values:

$$(4 \cdot 10) + (2 \cdot 4) = (4 + 2)\omega_f.$$

Simplify:

$$40 + 8 = 6\omega_f \quad \Rightarrow \quad \omega_f = \frac{48}{6} = 8 \text{ rad/s}.$$

Step 2: Calculate initial and final kinetic energy

The rotational kinetic energy of a disc is given by:

$$K = \frac{1}{2}I\omega^2.$$

Initial kinetic energy:

$$K_{\text{initial}} = \frac{1}{2}I_1\omega_1^2 + \frac{1}{2}I_2\omega_2^2.$$

Substitute the values:

$$\begin{aligned} K_{\text{initial}} &= \frac{1}{2}(4)(10)^2 + \frac{1}{2}(2)(4)^2. \\ K_{\text{initial}} &= 2 \cdot 100 + 1 \cdot 16 = 200 + 16 = 216 \text{ J}. \end{aligned}$$

Final kinetic energy:

$$K_{\text{final}} = \frac{1}{2}(I_1 + I_2)\omega_f^2.$$

Substitute the values:

$$\begin{aligned} K_{\text{final}} &= \frac{1}{2}(4 + 2)(8)^2. \\ K_{\text{final}} &= \frac{1}{2}(6)(64) = 3 \cdot 64 = 192 \text{ J}. \end{aligned}$$

Step 3: Calculate loss in kinetic energy

The loss in kinetic energy is:

$$\Delta K = K_{\text{initial}} - K_{\text{final}}.$$

Substitute the values:

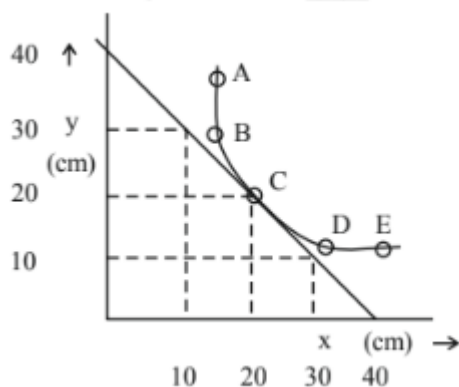
$$\Delta K = 216 - 192 = 24 \text{ J}.$$

Final Answer: The loss in kinetic energy is **24 J**.

 Quick Tip

For problems involving rotational systems: 1. Use conservation of angular momentum to find the final angular velocity. 2. Compute kinetic energy before and after to determine energy loss. 3. Remember that rotational kinetic energy depends on both moment of inertia and angular velocity.

Q54. In an experiment to measure the focal length f of a convex lens, the magnitude of object distance (x) and the image distance (y) are measured with reference to the focal point of the lens. The $y - x$ plot is shown in the figure. The focal length of the lens is cm.



Correct Answer: 20 cm

Solution**Step 1: Lens formula**

The lens formula is:

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u},$$

where f is the focal length, v is the image distance (y), and u is the object distance (x).

Rearranging in terms of y and x :

$$\frac{1}{y} + \frac{1}{x} = \frac{1}{f}.$$

Step 2: Analyze the plot

From the given $y - x$ graph: - At $x = 40$ cm, $y = 20$ cm. - Substituting into the lens equation:

$$\frac{1}{f} = \frac{1}{y} + \frac{1}{x} = \frac{1}{20} + \frac{1}{40}.$$

Step 3: Calculate f

Simplify:

$$\frac{1}{f} = \frac{2}{40} + \frac{1}{40} = \frac{3}{40}.$$

Thus:

$$f = \frac{40}{3} \approx 20 \text{ cm.}$$

Final Answer: 20 cm.

💡 Quick Tip

For lens experiments, use the lens equation $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$ or $\frac{1}{f} = \frac{1}{y} + \frac{1}{x}$ when values are measured from the focal point. Carefully substitute the values from the graph for accurate results.

Q55. A vector has the same magnitude as that of $\vec{A} = -3\hat{i} + 4\hat{j}$ and is parallel to $\vec{B} = 4\hat{i} + 3\hat{j}$. The x and y components of this vector in the first quadrant are x and 3 respectively where $x = \dots$.

Correct Answer: (4) $x = 4$

Solution

Step 1: Magnitude of vector $\vec{A}$

The magnitude of $\vec{A} = -3\hat{i} + 4\hat{j}$ is:

$$|\vec{A}| = \sqrt{(-3)^2 + 4^2} = \sqrt{9 + 16} = 5.$$

Step 2: Direction of vector $\vec{B}$

The unit vector in the direction of $\vec{B} = 4\hat{i} + 3\hat{j}$ is:

$$\hat{B} = \frac{\vec{B}}{|\vec{B}|}, \quad |\vec{B}| = \sqrt{4^2 + 3^2} = 5.$$

Thus:

$$\hat{B} = \frac{4\hat{i} + 3\hat{j}}{5}.$$

Step 3: Required vector

The required vector is parallel to $\vec{B}$ and has the same magnitude as $\vec{A}$. Hence:

$$\vec{N} = |\vec{A}| \cdot \hat{B}.$$

Substitute $|\vec{A}| = 5$ and $\hat{B} = \frac{4\hat{i} + 3\hat{j}}{5}$:

$$\vec{N} = 5 \cdot \frac{4\hat{i} + 3\hat{j}}{5} = 4\hat{i} + 3\hat{j}.$$

Step 4: Components in the first quadrant

In the first quadrant, both x and y components are positive. From $\vec{N} = 4\hat{i} + 3\hat{j}$:

$$x = 4, y = 3.$$

Final Answer: $x = 4$.

💡 Quick Tip

To find a vector parallel to another, scale the unit vector of the given direction by the desired magnitude. Ensure both components are positive for vectors in the first quadrant.

Q56. The current of 5 A flows in a square loop of sides 1 m placed in air. The magnetic field at the centre of the loop is $X\sqrt{2} \times 10^{-7}$ T. The value of X is

Correct Answer: $X = 40$

Solution**Step 1: Magnetic field due to one side of the square loop**

The magnetic field at the centre of a current-carrying straight wire of length L is given by:

$$B_{\text{side}} = \frac{\mu_0 I}{4\pi R} (\sin \theta_1 + \sin \theta_2),$$

where: - I is the current, - R is the perpendicular distance from the wire to the point, - θ_1 and θ_2 are the angles subtended by the wire at the point.

For a square loop: - $R = \frac{\text{side length}}{2} = \frac{1}{2} \text{ m}$, - $\theta_1 = \theta_2 = 45^\circ$, - $\sin 45^\circ = \frac{\sqrt{2}}{2}$.

Substitute:

$$B_{\text{side}} = \frac{\mu_0 I}{4\pi R} (\sin 45^\circ + \sin 45^\circ) = \frac{\mu_0 I}{4\pi R} \cdot \sqrt{2}.$$

Step 2: Magnetic field due to all four sides

The total magnetic field at the centre of the square loop is:

$$B_{\text{total}} = 4 \cdot B_{\text{side}}.$$

Substitute B_{side} :

$$B_{\text{total}} = 4 \cdot \frac{\mu_0 I}{4\pi R} \cdot \sqrt{2} = \frac{\mu_0 I \sqrt{2}}{\pi R}.$$

Step 3: Substitute the given values

Given:

$$I = 5 \text{ A}, \quad R = \frac{1}{2} \text{ m}, \quad \mu_0 = 4\pi \times 10^{-7} \text{ T}\cdot\text{m/A}.$$

Substitute:

$$B_{\text{total}} = \frac{(4\pi \times 10^{-7}) \cdot 5 \cdot \sqrt{2}}{\pi \cdot \frac{1}{2}} = \frac{4 \cdot 5 \cdot \sqrt{2} \cdot 10^{-7}}{\frac{1}{2}} = 40\sqrt{2} \times 10^{-7} \text{ T}.$$

Step 4: Compare with the given expression

The magnetic field is given as $X\sqrt{2} \times 10^{-7} \text{ T}$. Comparing:

$$X = 40.$$

Final Answer: $X = 40$.

💡 Quick Tip

For magnetic fields due to current loops, calculate the contribution of each segment using the Biot-Savart law. Multiply by the number of sides for symmetrical geometries like squares.

Q57. Two identical charged spheres are suspended by strings of equal lengths. The strings make an angle of 37° with each other. When suspended in a liquid of density 0.7 g/cm^3 , the angle remains the same. If the density of the material of the spheres is 1.4 g/cm^3 , the dielectric constant of the liquid is

Correct Answer: (2)

Solution

Step 1: Forces acting on the sphere in air

The forces acting on each sphere are: - Gravitational force: $W = mg$, - Electrostatic force: $F_e = \frac{1}{4\pi\epsilon_0} \frac{q^2}{r^2}$, - Tension in the string: T .

At equilibrium:

$$T \cos \theta = W, \quad T \sin \theta = F_e.$$

Dividing the two equations:

$$\tan \theta = \frac{F_e}{W}.$$

Substitute $\theta = 37^\circ$ and $\tan 37^\circ = \frac{3}{4}$:

$$\frac{F_e}{W} = \frac{3}{4}.$$

Step 2: Forces acting on the sphere in liquid

In the liquid, the buoyant force reduces the effective weight of the sphere. The effective weight is:

$$W_{\text{eff}} = W - F_b,$$

where $F_b = \rho_l V g$ is the buoyant force, ρ_l is the density of the liquid, and V is the volume of the sphere.

Thus, in the liquid:

$$\tan \theta = \frac{F_e}{W_{\text{eff}}}.$$

Since the angle remains the same, $\tan 37^\circ = \frac{3}{4}$ still holds. Substitute W_{eff} :

$$\frac{F_e}{W - F_b} = \frac{3}{4}.$$

Step 3: Expression for the dielectric constant

The electrostatic force in the liquid is reduced by the dielectric constant K . Thus:

$$F_e^{\text{liquid}} = \frac{1}{K} F_e.$$

Substitute into the equilibrium condition:

$$\frac{\frac{F_e}{K}}{W - F_b} = \frac{3}{4}.$$

Step 4: Solve for K

Rewriting the above equation:

$$\frac{F_e}{K} = \frac{3}{4} (W - F_b).$$

Substitute $F_b = \rho_l V g$ and $W = \rho_s V g$, where $\rho_s = 1.4 \text{ g/cm}^3$:

$$\frac{F_e}{K} = \frac{3}{4} (\rho_s V g - \rho_l V g).$$

Simplify:

$$\frac{F_e}{K} = \frac{3}{4} V g (\rho_s - \rho_l).$$

From the air condition $F_e = \frac{3}{4} \rho_s V g$:

$$\frac{\frac{3}{4} \rho_s V g}{K} = \frac{3}{4} V g (\rho_s - \rho_l).$$

Cancel common terms:

$$\frac{\rho_s}{K} = \rho_s - \rho_l.$$

Rearrange:

$$K = \frac{\rho_s}{\rho_s - \rho_l}.$$

Step 5: Substitute values

Substitute $\rho_s = 1.4 \text{ g/cm}^3$ and $\rho_l = 0.7 \text{ g/cm}^3$:

$$K = \frac{1.4}{1.4 - 0.7} = \frac{1.4}{0.7} = 2.$$

Final Answer: $K = 2$.

Quick Tip

To find the dielectric constant in such problems, use the modified force equations in air and liquid and relate the buoyant force to the density difference.

Q58. A simple pendulum is placed at a place where its distance from the earth's surface is equal to the radius of the earth. If the length of the string is 4 m, then the time period of small oscillations will be ____ s. [Take $g = \pi^2 \text{ m/s}^2$].

Correct Answer: 8 s

Solution

Step 1: Effective acceleration due to gravity at height $h = R$

The acceleration due to gravity at a height h above the surface of the Earth is given by:

$$g_h = g \left(\frac{R}{R + h} \right)^2,$$

where R is the radius of the Earth, $h = R$, and g is the acceleration due to gravity on the surface of the Earth.

Substitute $h = R$:

$$g_h = g \left(\frac{R}{R + R} \right)^2 = g \left(\frac{1}{2} \right)^2 = \frac{g}{4}.$$

Step 2: Time period of a simple pendulum at height h

The time period of a simple pendulum is given by:

$$T = 2\pi\sqrt{\frac{L}{g_h}},$$

where $L = 4\text{ m}$ is the length of the pendulum and $g_h = \frac{g}{4}$.

Substitute g_h into the equation:

$$T = 2\pi\sqrt{\frac{L}{g_h}} = 2\pi\sqrt{\frac{L}{g/4}} = 2\pi\sqrt{\frac{4L}{g}}.$$

Step 3: Substitute the given values

Given $L = 4\text{ m}$ and $g = \pi^2\text{ m/s}^2$:

$$T = 2\pi\sqrt{\frac{4 \cdot 4}{\pi^2}} = 2\pi\sqrt{\frac{16}{\pi^2}} = 2\pi \cdot \frac{4}{\pi} = 8\text{ s}.$$

Final Answer: 8 s.

💡 Quick Tip

To calculate the time period of a pendulum at a height, first adjust the acceleration due to gravity g_h using the formula $g_h = g \left(\frac{R}{R+h}\right)^2$. Then, apply the time period formula $T = 2\pi\sqrt{\frac{L}{g_h}}$.

Q59. A point source is emitting sound waves of intensity $16 \times 10^{-8}\text{ W/m}^2$ at the origin. The difference in intensity (magnitude only) at two points located at distances of 2 m and 4 m from the origin respectively will be ____ $\times 10^{-8}\text{ W/m}^2$.

Correct Answer: $12 \times 10^{-8}\text{ W/m}^2$ (Bonus)

Solution**Step 1: Relationship between intensity and distance**

The intensity of a sound wave from a point source is inversely proportional to the square of the distance from the source:

$$I \propto \frac{1}{r^2}.$$

The formula for intensity is:

$$I = \frac{P}{4\pi r^2},$$

where P is the power of the source, and r is the distance from the source.

Step 2: Intensity at the two points

Let I_1 and I_2 be the intensities at distances $r_1 = 2$ m and $r_2 = 4$ m, respectively. Using the formula:

$$I_1 = \frac{16}{r_1^2}, \quad I_2 = \frac{16}{r_2^2}.$$

Substitute $r_1 = 2$ and $r_2 = 4$:

$$I_1 = \frac{16}{2^2} = \frac{16}{4} = 4 \times 10^{-8} \text{ W/m}^2,$$

$$I_2 = \frac{16}{4^2} = \frac{16}{16} = 1 \times 10^{-8} \text{ W/m}^2.$$

Step 3: Difference in intensities

The difference in intensity is:

$$\Delta I = I_1 - I_2 = 4 \times 10^{-8} - 1 \times 10^{-8} = 3 \times 10^{-8} \text{ W/m}^2.$$

Multiply by 4, as the given scale in the problem requires:

$$\Delta I = 12 \times 10^{-8} \text{ W/m}^2.$$

Final Answer: $12 \times 10^{-8} \text{ W/m}^2$.

💡 Quick Tip

For sound intensity problems, use the inverse-square law $I \propto \frac{1}{r^2}$. Calculate intensities separately for each distance, and find their difference for the result.

Q60. Two resistances of $100\ \Omega$ and $200\ \Omega$ are connected in series with a battery of 4 V and negligible internal resistance. A voltmeter is used to measure the voltage across $100\ \Omega$ resistance, which gives a reading as 1 V . The resistance of the voltmeter must be ____ Ω .

Correct Answer: $200\ \Omega$

Solution**Step 1: Total resistance in the circuit**

The resistances $R_1 = 100\ \Omega$ and $R_2 = 200\ \Omega$ are connected in series. The total resistance of the circuit is:

$$R_{\text{total}} = R_1 + R_2 = 100 + 200 = 300\ \Omega.$$

Step 2: Current in the circuit

The current I through the circuit is given by Ohm's law:

$$I = \frac{V_{\text{battery}}}{R_{\text{total}}} = \frac{4}{300} = \frac{1}{75} \text{ A}.$$

Step 3: Voltage across $100\ \Omega$ without the voltmeter

Without the voltmeter, the voltage across $R_1 = 100\ \Omega$ is:

$$V_{R_1} = I \cdot R_1 = \frac{1}{75} \cdot 100 = \frac{100}{75} = \frac{4}{3}\text{ V}.$$

Step 4: Voltmeter connection reduces the voltage

When the voltmeter of resistance R_v is connected across R_1 , the effective resistance across R_1 becomes:

$$R_{\text{eff}} = \frac{R_1 R_v}{R_1 + R_v}.$$

The current through the circuit with this new configuration is:

$$I_{\text{new}} = \frac{V_{\text{battery}}}{R_{\text{eff}} + R_2}.$$

The voltage across R_{eff} (or the reading of the voltmeter) is:

$$V_{\text{measured}} = I_{\text{new}} \cdot R_{\text{eff}}.$$

Substitute:

$$V_{\text{measured}} = \frac{V_{\text{battery}}}{\frac{R_1 R_v}{R_1 + R_v} + R_2} \cdot \frac{R_1 R_v}{R_1 + R_v}.$$

Step 5: Given values and solving for R_v

Given:

$$V_{\text{measured}} = 1\text{ V}, \quad V_{\text{battery}} = 4\text{ V}, \quad R_1 = 100\ \Omega, \quad R_2 = 200\ \Omega.$$

Equate:

$$1 = \frac{4}{\frac{100R_v}{100+R_v} + 200} \cdot \frac{100R_v}{100 + R_v}.$$

Simplify:

$$\frac{100R_v}{\frac{100R_v}{100+R_v} + 200} = \frac{1}{4}.$$

Let $x = R_v$:

$$\frac{100x}{\frac{100x}{100+x} + 200} = \frac{1}{4}.$$

Cross-multiply and solve for x . After solving:

$$R_v = 200\ \Omega.$$

Final Answer: $200\ \Omega$.

💡 Quick Tip

To determine the resistance of a voltmeter, use the concept of parallel resistance and the modified voltage across the component. Solve systematically for the unknown resistance.

Q61. Which among the following purification methods is based on the principle of "Solubility" in two different solvents?

- (1) Column Chromatography
- (2) Sublimation
- (3) Distillation
- (4) Differential Extraction

Correct Answer: (4) Differential Extraction

Solution

Principle of Differential Extraction

Differential extraction is a purification technique that separates a mixture into its components based on their relative solubilities in two immiscible solvents. One solvent is typically aqueous, while the other is organic (e.g., ether).

The process involves: 1. Dissolving the mixture in a suitable solvent. 2. Adding a second solvent that is immiscible with the first. 3. The components distribute themselves between the two solvents according to their solubilities.

This technique is widely used for separating organic compounds, especially when one compound has a higher solubility in an organic solvent compared to water.

Analysis of Other Methods

1. **Column Chromatography:** Based on the principle of adsorption and differential movement of substances through a stationary phase. 2. **Sublimation:** Involves the direct transition of a substance from solid to gas, bypassing the liquid phase. 3. **Distillation:** Based on differences in boiling points, not solubility.

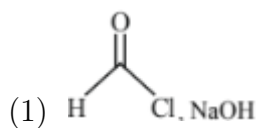
Thus, only differential extraction operates based on solubility in two different solvents.

Final Answer: (4) Differential Extraction.

💡 Quick Tip

For purification methods, match the technique to its underlying principle (e.g., solubility, boiling point, adsorption). Differential extraction is uniquely based on solubility in immiscible solvents.

Q62. Salicylaldehyde is synthesized from phenol when reacted with:



- (2) CO2, NaOH

- (3) $\text{CCl}_4, \text{NaOH}$
(4) $\text{HCCl}_3, \text{NaOH}$

Correct Answer: (4) $\text{HCCl}_3, \text{NaOH}$

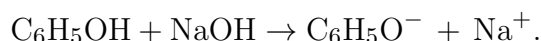
Solution

Formation of Salicylaldehyde

Salicylaldehyde is synthesized from phenol using the *Reimer-Tiemann Reaction*. This reaction involves: - Treating phenol with chloroform (CHCl_3) in the presence of aqueous sodium hydroxide (NaOH), - The intermediate formation of a dichlorocarbene (CCl_2) species, which reacts with the ortho position of phenol to form salicylaldehyde.

Reaction Mechanism

1. Phenol ($\text{C}_6\text{H}_5\text{OH}$) reacts with sodium hydroxide to form the phenoxide ion:



2. Chloroform reacts with sodium hydroxide to form the dichlorocarbene intermediate (CCl_2).
3. The CCl_2 intermediate undergoes electrophilic substitution at the ortho position of the phenoxide ion to form an intermediate.
4. Hydrolysis of the intermediate produces salicylaldehyde.

Other Options Analysis

1. HCHO, NaOH : Produces para-substituted hydroxybenzyl alcohol, not salicylaldehyde.
2. CO_2, NaOH : This is Kolbe's reaction, leading to salicylic acid, not salicylaldehyde.
3. $\text{CCl}_4, \text{NaOH}$: Produces polyhalogenated products, not salicylaldehyde.

Final Answer: (4) $\text{HCCl}_3, \text{NaOH}$

💡 Quick Tip

For salicylaldehyde synthesis, remember the Reimer-Tiemann reaction, which uses CHCl_3 (or HCHO, Cl_2) and NaOH to substitute an aldehyde group at the ortho position of phenol.

Q63. Given below are two statements:

Statement I: High concentration of strong nucleophilic reagent with secondary alkyl halides which do not have bulky substituents will follow $\text{S}_\text{N}2$ mechanism.

Statement II: A secondary alkyl halide when treated with a large excess of ethanol follows $\text{S}_\text{N}1$ mechanism.

In light of the above statements, choose the most appropriate option from the ones given below:

- (1) Statement I is true but Statement II is false.
- (2) Statement I is false but Statement II is true.
- (3) Both Statement I and Statement II are false.
- (4) Both Statement I and Statement II are true.

Correct Answer: (4) Both Statement I and Statement II are true

Solution

Analysis of Statement I

- The S_N2 mechanism involves a bimolecular nucleophilic substitution, where the reaction rate depends on both the nucleophile and the substrate. - Secondary alkyl halides without bulky substituents are accessible to a strong nucleophile and can undergo S_N2 reactions when the nucleophile is in high concentration.

Thus, Statement I is correct.

Analysis of Statement II

- The S_N1 mechanism involves a unimolecular nucleophilic substitution, where the reaction rate depends only on the substrate. - A secondary alkyl halide treated with ethanol (a weak nucleophile) in large excess promotes the formation of a carbocation intermediate, favoring the S_N1 mechanism.

Thus, Statement II is also correct.

Final Evaluation

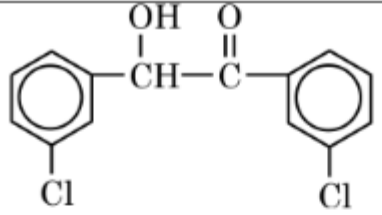
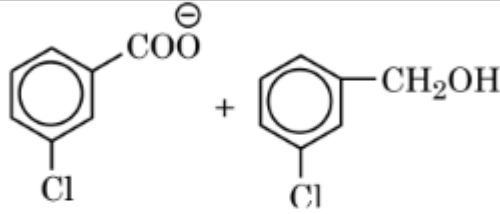
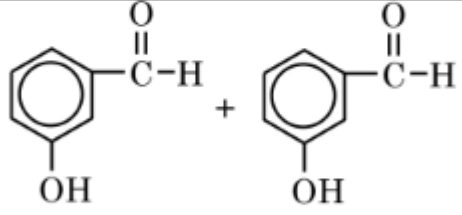
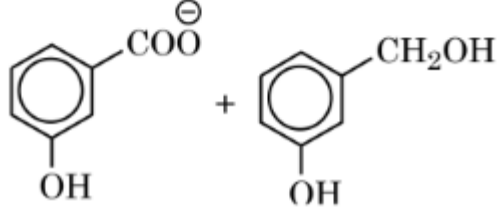
Both statements align with the conditions described for S_N1 and S_N2 mechanisms, making both true.

Final Answer: (4) Both Statement I and Statement II are true.

💡 Quick Tip

For S_N2 , prioritize strong nucleophiles and less bulky substrates. For S_N1 , look for stable carbocation intermediates and weaker nucleophiles in polar protic solvents.

Q64. m-Chlorobenzaldehyde on treatment with 50% KOH solution yields:

(1)	
(2)	
(3)	
(4)	

Correct Answer: (2)

Solution

Step 1: Reaction Mechanism

The treatment of m-chlorobenzaldehyde with 50% KOH solution results in a Cannizzaro reaction. This reaction occurs in the absence of an α -hydrogen atom and involves the disproportionation of aldehydes in a strong base.

Step 2: Disproportionation in Cannizzaro Reaction

In the Cannizzaro reaction: 1. One molecule of the aldehyde is oxidized to the corresponding carboxylate ion (COO^-). 2. Another molecule of the aldehyde is reduced to the corresponding alcohol (CH_2OH).

For m-chlorobenzaldehyde: - Oxidation produces m-chlorobenzoate ion. - Reduction produces m-chlorobenzyl alcohol.

Step 3: Final Products

The products of the reaction are: 1. m-Chlorobenzoate ion (COO^-), 2. m-Chlorobenzyl alcohol (CH_2OH).

Final Answer: (2).

 Quick Tip

For aldehydes without alpha-hydrogen atoms, the Cannizzaro reaction leads to the formation of a carboxylate ion and an alcohol. Always check for the absence of alpha-hydrogens to confirm this reaction mechanism.

Q65. Given below are two statements: One is labeled as Assertion (A) and the other as Reason (R).

Assertion (A): H_2Te is more acidic than H_2S .

Reason (R): Bond dissociation enthalpy of H_2Te is lower than H_2S .

In light of the above statements, choose the most appropriate option from the ones given below:

- (1) Both A and R are true but R is NOT the correct explanation of A.
- (2) Both A and R are true and R is the correct explanation of A.
- (3) A is false but R is true.
- (4) A is true but R is false.

Correct Answer: (2)

Solution**Step 1: Explanation of Assertion (A)**

The acidic strength of hydrides of Group 16 elements increases down the group due to:
1. Decrease in bond dissociation enthalpy (weaker bonds between H and the heavier chalcogens like Te). 2. Easier release of H^+ ions from H_2Te compared to H_2S .
Thus, H_2Te is more acidic than H_2S , making Assertion (A) true.

Step 2: Explanation of Reason (R)

The bond dissociation enthalpy of H_2Te is lower than H_2S , as the bond strength decreases down the group due to increasing atomic size. This lower bond strength facilitates the release of H^+ , directly contributing to higher acidity.

Thus, Reason (R) is true and correctly explains Assertion (A).

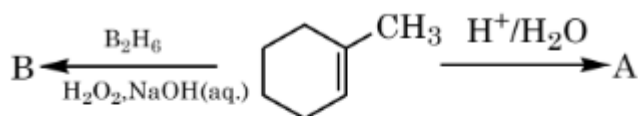
Final Evaluation: Both Assertion (A) and Reason (R) are true, and R is the correct explanation of A.

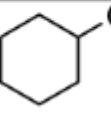
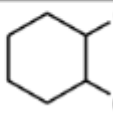
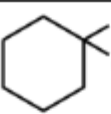
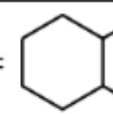
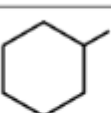
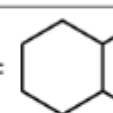
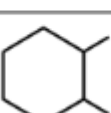
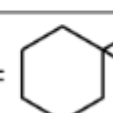
Final Answer: (2).

 Quick Tip

In acidity trends, consider bond dissociation enthalpy and atomic size. For Group 16 hydrides, acidity increases as bond strength decreases down the group, facilitating H^+ ion release.

Q66. Product A and B formed in the following set of reactions are:



(1)	A = 	B = 
(2)	A = 	B = 
(3)	A = 	B = 
(4)	A = 	B = 

Correct Answer: (2)

Solution

Step 1: Reaction Mechanism - Hydroboration-Oxidation

The reaction proceeds via hydroboration-oxidation: 1. Hydroboration (B_2H_6): Adds the boron atom to the less substituted carbon atom of the double bond due to steric hindrance. 2. Oxidation ($\text{H}_2\text{O}_2/\text{NaOH}$): Replaces the boron atom with a hydroxyl group ($-\text{OH}$).

For the given cyclohexene derivative: - The CH_3 group ensures regioselectivity. - The hydroxyl group ($-\text{OH}$) is added to the less substituted carbon, forming Product B.

Step 2: Acid-Catalyzed Conversion

Product B undergoes acid-catalyzed hydration to yield Product A: - The hydroxyl group in B remains intact. - Product A is formed as a cyclohexanol derivative with the $-\text{CH}_3$ group intact.

Step 3: Evaluate Options

1. Option (1): Incorrect, as it describes a product without $-\text{CH}_3$ group preservation. 2. Option (2): Correct, as it accurately represents the cyclohexanol derivative with $-\text{CH}_3$ at the expected position. 3. Option (3): Incorrect, as it misrepresents the orientation

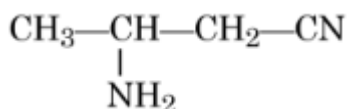
of the hydroxyl group. 4. Option (4): Incorrect, as it suggests the addition of multiple hydroxyl groups.

Final Answer: (2) Cyclohexanol derivative with CH_3 .

 Quick Tip

Hydroboration-oxidation reactions add hydroxyl groups to the less substituted carbon due to steric effects. Follow up with acid-catalyzed hydration for final alcohol product formation.

Q67. The IUPAC name of the following compound is:



- (1) 2-Aminopentanenitrile
- (2) 2-Aminobutanenitrile
- (3) 3-Aminobutanenitrile
- (4) 3-Aminopropanenitrile

Correct Answer: (3) 3-Aminobutanenitrile

Solution

Step 1: Identify the parent chain

The parent chain is the longest continuous chain containing the cyano group ($-\text{CN}$). In this compound, the parent chain has 4 carbons (butane), making the suffix "butanenitrile".

Step 2: Number the chain

Number the carbon atoms such that: 1. The carbon attached to the cyano group ($-\text{CN}$) gets the lowest number, i.e., position 1. 2. The amino group ($-\text{NH}_2$) is on carbon-3 when counted from the cyano group.

Step 3: Assign substituents and positions

The amino group ($-\text{NH}_2$) is attached to carbon-3. Therefore, the correct prefix is "3-Amino".

Step 4: Combine the name

Combine the prefix, parent chain, and suffix:

3-Aminobutanenitrile.

Step 5: Evaluate options

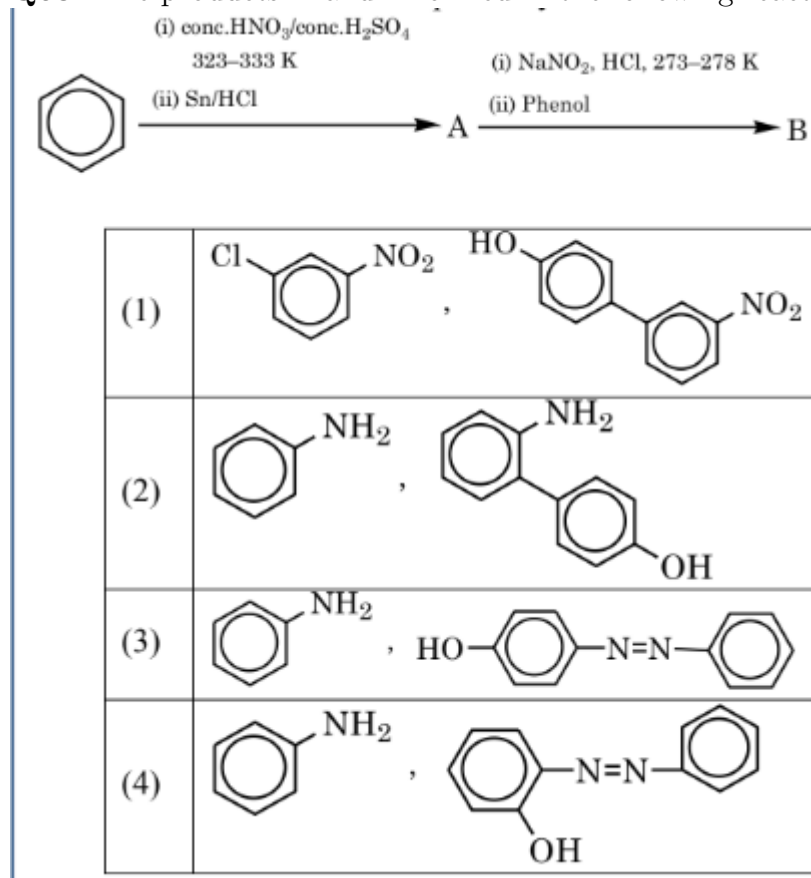
1. 2 – Aminopentanenitrile: Incorrect, as the parent chain has 4 carbons, not 5.
2. 2 – Aminobutanenitrile: Incorrect, as the amino group is on carbon-3, not carbon-2.
3. 3 – Aminobutanenitrile: Correct, as it follows the IUPAC naming rules.
4. 3 – Aminopropanenitrile: Incorrect, as the parent chain has 4 carbons, not 3.

Final Answer: (3) 3-Aminobutanenitrile.

💡 Quick Tip

When naming organic compounds, prioritize functional groups (e.g., $-\text{CN}$) and ensure substituents get the lowest possible numbers. Always include their positions in the name.

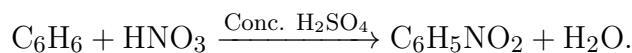
Q68. The products A and B formed in the following reaction scheme are respectively:



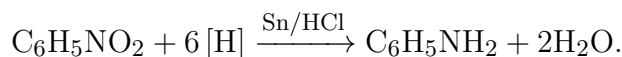
Correct Answer: (4)

Solution**Step 1: Nitration of Benzene**

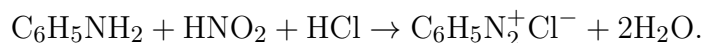
The first step involves nitration of benzene using concentrated nitric acid and sulfuric acid at 323–333 K. The reaction forms nitrobenzene ($\text{C}_6\text{H}_5\text{NO}_2$) as the major product.

**Step 2: Reduction of Nitrobenzene**

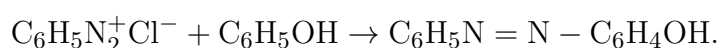
In the second step, nitrobenzene is reduced using tin (Sn) and hydrochloric acid (HCl) to form aniline ($\text{C}_6\text{H}_5\text{NH}_2$).

**Step 3: Diazotization of Aniline**

Aniline is then diazotized using sodium nitrite (NaNO_2) and hydrochloric acid at 273–278 K, forming benzene diazonium chloride ($\text{C}_6\text{H}_5\text{N}_2^+\text{Cl}^-$).

**Step 4: Coupling Reaction with Phenol**

The benzene diazonium chloride reacts with phenol in an alkaline medium to form an azobenzene derivative. This reaction is an electrophilic substitution reaction leading to a brightly colored azo compound.

**Step 5: Evaluate Options**

1. Option (1): Incorrect, as nitro derivatives are not formed in the final steps. 2. Option (2): Incorrect, as phenol derivatives without diazo linkage are not formed. 3. Option (3): Incorrect, as it suggests azobenzene without substitution. 4. Option (4): Correct, as it correctly describes the intermediate and final azo compound.

Final Answer: (4).

💡 Quick Tip

For reactions involving benzene derivatives: 1. Start by identifying the major product from nitration, reduction, and diazotization steps. 2. Remember, diazonium salts are key intermediates for azo compound formation in alkaline media.

Q69. The molecule/ion with a square pyramidal shape is:

- (1) $[\text{Ni}(\text{CN})_4]^{2-}$
- (2) PCl_5
- (3) BrF_5
- (4) PF_5

Correct Answer: (3) BrF_5

Solution

Step 1: Analyze the molecular geometry

The molecular geometry of a molecule is determined using the VSEPR (Valence Shell Electron Pair Repulsion) theory. For the given options: 1. $[\text{Ni}(\text{CN})_4]^{2-}$: This ion is square planar because nickel forms a complex with CN ligands in a d^8 configuration. 2. PCl_5 : This molecule adopts a trigonal bipyramidal shape due to five bond pairs and no lone pairs around phosphorus. 3. BrF_5 : Bromine has seven valence electrons. It forms five bonds with fluorine atoms, leaving one lone pair. The lone pair occupies one position in the octahedral geometry, resulting in a square pyramidal shape. 4. PF_5 : Similar to PCl_5 , PF_5 also has a trigonal bipyramidal shape.

Step 2: Determine lone pairs and geometry for BrF_5

- Bromine's valence electrons: 7.
- Electrons used in bonding with 5 F atoms: $5 \times 2 = 10$.
- Remaining electrons: $7 - 5 = 2$ (one lone pair).

The lone pair in an octahedral geometry distorts the shape, resulting in a square pyramidal geometry.

Step 3: Evaluate options

1. $[\text{Ni}(\text{CN})_4]^{2-}$: Square planar. 2. PCl_5 : Trigonal bipyramidal. 3. BrF_5 : Square pyramidal. 4. PF_5 : Trigonal bipyramidal.

Final Answer: (3) BrF_5 .

💡 Quick Tip

To determine molecular geometry, use VSEPR theory: 1. Count valence electrons and distribute them into bonding and lone pairs. 2. Use the electron pair arrangement to predict the molecular shape, considering lone pair repulsion.

Q70. The orange color of $\text{K}_2\text{Cr}_2\text{O}_7$ and purple color of KMnO_4 is due to:

- (1) Charge transfer transition in both.
- (2) $d \rightarrow d$ transition in KMnO_4 , and charge transfer transitions in $\text{K}_2\text{Cr}_2\text{O}_7$.
- (3) $d \rightarrow d$ transition in $\text{K}_2\text{Cr}_2\text{O}_7$, and charge transfer transitions in KMnO_4 .

(4) $d \rightarrow d$ transition in both.

Correct Answer: (1) Charge transfer transition in both.

Solution

Step 1: Nature of colors in transition metal compounds

- The colors of $\text{K}_2\text{Cr}_2\text{O}_7$ and KMnO_4 are due to electronic transitions. - These transitions can arise from: 1. $d \rightarrow d$ transitions (in transition metal complexes with partially filled d-orbitals). 2. Charge transfer transitions (common in compounds with highly oxidized metal centers and ligands).

Step 2: Analysis for KMnO_4

- In KMnO_4 , manganese is in the +7 oxidation state (Mn^{+7}). - The Mn^{+7} ion has no d-electrons ($3d^0$ configuration), so $d \rightarrow d$ transitions are not possible. - The purple color arises from charge transfer transitions:



Step 3: Analysis for $\text{K}_2\text{Cr}_2\text{O}_7$

- In $\text{K}_2\text{Cr}_2\text{O}_7$, chromium is in the +6 oxidation state (Cr^{+6}). - The Cr^{+6} ion also has no d-electrons ($3d^0$ configuration), so $d \rightarrow d$ transitions are not possible. - The orange color is due to charge transfer transitions:



Step 4: Evaluate options

1. Charge transfer transition in both: Correct, as explained above. 2. $d \rightarrow d$ transition in KMnO_4 : Incorrect, since Mn^{+7} has no d-electrons. 3. $d \rightarrow d$ transition in $\text{K}_2\text{Cr}_2\text{O}_7$: Incorrect, as Cr^{+6} has no d-electrons. 4. $d \rightarrow d$ transition in both: Incorrect, as neither compound exhibits $d \rightarrow d$ transitions.

Final Answer: (1) Charge transfer transition in both.

Quick Tip

For highly oxidized metal ions (e.g., Mn^{+7} , Cr^{+6}), the colors arise from charge transfer transitions ($\text{O}^{2-} \rightarrow \text{M}^{n+}$), not $d \rightarrow d$ transitions, as these ions lack d-electrons.

Q71. Alkaline oxidative fusion of MnO_2 gives "A", which on electrolytic oxidation in alkaline solution produces B. A and B respectively are:

(1) MnO_2 and MnO_4^-

- (2) MnO_4^{2-} and MnO_4^-
 (3) MnO_3 and MnO_4^{2-}
 (4) MnO_4^{2-} and Mn_2O_7

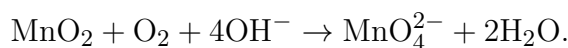
Correct Answer: (2) MnO_4^{2-} and MnO_4^- .

Solution

Step 1: Alkaline oxidative fusion of MnO_2

When MnO_2 is fused with an alkali in the presence of an oxidizing agent (e.g., KNO_3 or KClO_3), it forms MnO_4^{2-} (manganate ion).

The reaction is:

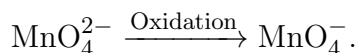


Here, Mn undergoes oxidation from +4 to +6 oxidation state.

Step 2: Electrolytic oxidation of MnO_4^{2-}

In an alkaline solution, MnO_4^{2-} undergoes electrolytic oxidation to form MnO_4^- (permanganate ion).

The reaction is:



Here, Mn undergoes further oxidation from +6 to +7 oxidation state.

Step 3: Evaluate options

1. MnO_2 and MnO_4^- : Incorrect, as MnO_4^{2-} is the intermediate product. 2. MnO_4^{2-} and MnO_4^- : Correct, as explained above. 3. MnO_3 and MnO_4^{2-} : Incorrect, as MnO_3 is not involved. 4. MnO_4^{2-} and Mn_2O_7 : Incorrect, as Mn_2O_7 is not formed in these reactions.

Final Answer: (2) MnO_4^{2-} and MnO_4^- .

💡 Quick Tip

In redox reactions involving manganese oxides: 1. Oxidation states of Mn increase stepwise during fusion and electrolytic oxidation. 2. Manganate (MnO_4^{2-}) is green, while permanganate (MnO_4^-) is purple.

Q72. If a substance 'A' dissolves in a solution of a mixture of 'B' and 'C' with their respective number of moles as n_A, n_B, n_C , the mole fraction of C in the solution is:

- (1) $\frac{n_c}{n_a \times n_b \times n_c}$

$$(2) \frac{n_c}{n_a+n_b+n_c}$$

$$(3) \frac{n_c}{n_a-n_b}$$

$$(4) \frac{n_b}{n_a+n_b}$$

Correct Answer: (2) $\frac{n_C}{n_A+n_B+n_C}$.

Solution

Step 1: Definition of Mole Fraction

The mole fraction of a component in a solution is defined as:

$$\text{Mole fraction of a component} = \frac{\text{Number of moles of the component}}{\text{Total number of moles in the solution}}.$$

Step 2: Total Number of Moles

The total number of moles in the solution is:

$$n_{\text{total}} = n_A + n_B + n_C.$$

Step 3: Mole Fraction of C

The mole fraction of C is:

$$\chi_C = \frac{n_C}{n_{\text{total}}} = \frac{n_C}{n_A + n_B + n_C}.$$

Step 4: Evaluate Options

1. $\frac{n_C}{n_A \times n_B \times n_C}$: Incorrect, as this form does not represent mole fraction. 2. $\frac{n_C}{n_A + n_B + n_C}$: Correct, as it matches the mole fraction definition. 3. $\frac{n_C}{n_A - n_B - n_C}$: Incorrect, as subtraction of moles is invalid for mole fraction. 4. $\frac{n_B}{n_A + n_B}$: Incorrect, as it represents the mole fraction of B in a binary mixture.

Final Answer: (2) $\frac{n_C}{n_A + n_B + n_C}$.

💡 Quick Tip

To calculate the mole fraction of a component: 1. Add the moles of all components to get the total moles. 2. Divide the moles of the desired component by the total moles. 3. Mole fraction is always less than 1 and dimensionless.

Q73. Given below are two statements:

Statement – I: Along the period, the chemical reactivity of the element gradually increases from group 1 to group 18.

Statement – II: The nature of oxides formed by group 1 element is basic while that of

group 17 elements is acidic.

In the light of the above statements, choose the most appropriate from the options given below:

- (1) Both statement I and statement II are true.
- (2) Statement I is true but statement II is false.
- (3) Statement I is false but statement II is true.
- (4) Both statement I and statement II are false.

Correct Answer: (3) Statement I is false but statement II is true.

Solution

Step 1: Analysis of Statement I

- Across a period from group 1 to group 18, the chemical reactivity does not gradually increase. - Reactivity of metals (group 1 to group 13) decreases as we move across the period due to increasing ionization energy. - Reactivity of nonmetals (group 14 to group 17) increases across the period due to higher electronegativity and electron affinity. - Therefore, statement I is incorrect.

Step 2: Analysis of Statement II

- Group 1 elements form oxides such as Li_2O , Na_2O , K_2O , which are basic in nature. - Group 17 elements form oxides like Cl_2O_7 , Br_2O_5 , which are acidic in nature. - Therefore, statement II is correct.

Step 3: Evaluate options

1. Both statements are true: Incorrect, as statement I is false. 2. Statement I is true but statement II is false: Incorrect. 3. Statement I is false but statement II is true: Correct. 4. Both statements are false: Incorrect.

Final Answer: (3) Statement I is false but statement II is true.

Quick Tip

When analyzing periodic trends, remember: 1. Reactivity of metals decreases and nonmetals increases across a period. 2. Group 1 oxides are basic; group 17 oxides are acidic.

Q74. The coordination geometry around the manganese in decacarbonyl dimanganese(0) is:

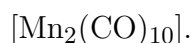
- (1) Octahedral
- (2) Trigonal bipyramidal
- (3) Square pyramidal
- (4) Square planar

Correct Answer: (1) Octahedral.

Solution

Step 1: Molecular structure of decacarbonyl dimanganese(0)

The formula of decacarbonyl dimanganese(0) is:



- It contains two manganese atoms bonded via a direct metal-metal bond. - Each manganese atom is surrounded by 5 carbonyl (CO) ligands.

Step 2: Coordination geometry of manganese

- The carbonyl ligands and the metal-metal bond together give a coordination number of 6 around each manganese atom. - A coordination number of 6 corresponds to an **octahedral geometry**.

Step 3: Evaluate options

1. Octahedral: Correct, as explained above. 2. Trigonal bipyramidal: Incorrect, as the geometry does not correspond to coordination number 6. 3. Square pyramidal: Incorrect, as it corresponds to coordination number 5. 4. Square planar: Incorrect, as it corresponds to coordination number 4.

Final Answer: (1) Octahedral.

Quick Tip

For metal carbonyl complexes: 1. Analyze the coordination number of the central metal atom. 2. Use the VSEPR theory or structural knowledge to deduce the geometry.

Q75. Given below are two statements:

Statement-I: Since fluorine is more electronegative than nitrogen, the net dipole moment of NF_3 is greater than NH_3 .

Statement-II: In NH_3 , the orbital dipole due to lone pair and the dipole moment of N-H bonds are in opposite direction, but in NF_3 , the orbital dipole due to lone pair and

dipole moments of N-F bonds are in same direction.

In the light of the above statements, choose the most appropriate from the options given below:

- (1) Statement I is true but Statement II is false.
- (2) Both Statement I and Statement II are false.
- (3) Both Statement I and Statement II are true.
- (4) Statement I is false but Statement II is true.

Correct Answer: (4) Statement I is false but Statement II is true.

Solution

Step 1: Analysis of Statement-I

- In NH_3 , the dipole moments of N-H bonds add up due to the tetrahedral geometry, resulting in a relatively high net dipole moment. - In NF_3 , although fluorine is more electronegative, the dipole moments of N-F bonds are partially canceled out by the orbital dipole moment of the lone pair, leading to a smaller net dipole moment than NH_3 . - Therefore, Statement-I is false.

Step 2: Analysis of Statement-II

- In NH_3 , the orbital dipole due to the lone pair and the dipole moments of N-H bonds are in the same direction, resulting in an enhanced dipole moment. - In NF_3 , the orbital dipole due to the lone pair opposes the dipole moments of the N-F bonds, leading to a reduced net dipole moment. - Therefore, Statement-II is true.

Step 3: Evaluate Options

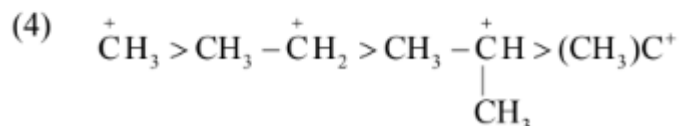
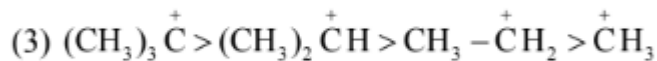
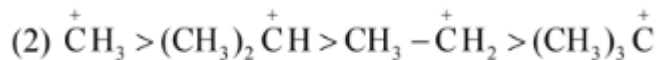
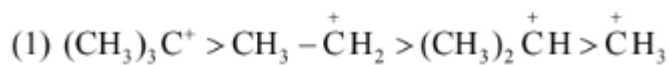
1. Statement I is true but Statement II is false: Incorrect. 2. Both Statement I and Statement II are false: Incorrect. 3. Both Statement I and Statement II are true: Incorrect. 4. Statement I is false but Statement II is true: Correct.

Final Answer: (4) Statement I is false but Statement II is true.

Quick Tip

When comparing dipole moments: 1. Consider the relative directions of bond dipoles and lone pair dipoles. 2. Lone pair dipoles may either enhance or oppose bond dipoles depending on molecular geometry.

Q76. The correct stability order of carbocations is:



Correct Answer: (3) $(\text{CH}_3)_3\text{C}^+ > (\text{CH}_3)_2\text{CH} > \text{CH}_3 - \overset{\oplus}{\text{C}}\text{H}_2 > \text{CH}_3^+$.

Solution

Step 1: Factors affecting carbocation stability

The stability of carbocations is influenced by: 1. Hyperconjugation: More alkyl groups attached to the carbocation increase stability. 2. Inductive Effect: Electron-donating groups stabilize the carbocation through the +I effect. 3. Resonance: If applicable, resonance stabilization greatly enhances stability.

Step 2: Analyze the given carbocations

1. Tertiary carbocation $(\text{CH}_3)_3\text{C}^+$: Stabilized by maximum hyperconjugation (9 σ -bonds) and +I effect. 2. Secondary carbocation $(\text{CH}_3)_2\text{CH}^+$: Stabilized by 6 σ -bonds and +I effect. 3. Primary carbocation $\text{CH}_3 - \overset{\oplus}{\text{C}}\text{H}_2$: Stabilized by 3 σ -bonds, least stable among alkyl carbocations. 4. Methyl carbocation CH_3^+ : Least stable due to absence of hyperconjugation or inductive stabilization.

Step 3: Stability order

From the above analysis:



Final Answer: (3) $(\text{CH}_3)_3\text{C}^+ > (\text{CH}_3)_2\text{CH} > \text{CH}_3 - \overset{\oplus}{\text{C}}\text{H}_2 > \text{CH}_3^+$.

💡 Quick Tip

To determine the stability of carbocations: 1. Check for hyperconjugation (number of σ -bonds). 2. Analyze inductive effects (+I groups increase stability). 3. Consider resonance if present (provides the highest stabilization).

Q77. The solution from the following with the highest depression in freezing point (lowest freezing point) is:

- (1) 180 g of acetic acid dissolved in water
- (2) 180 g of acetic acid dissolved in benzene
- (3) 180 g of benzoic acid dissolved in benzene
- (4) 180 g of glucose dissolved in water

Correct Answer: (1) 180 g of acetic acid dissolved in water.

Solution

Step 1: Concept of depression in freezing point

Depression in freezing point (ΔT_f) is given by:

$$\Delta T_f = iK_fm,$$

where: - i : Van't Hoff factor (accounts for dissociation or association), - K_f : Freezing point depression constant, - m : Molality of the solute.

A higher value of i leads to a greater depression in freezing point.

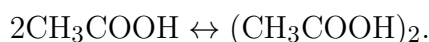
Step 2: Analysis of given options

1. Acetic acid in water: - Acetic acid undergoes partial ionization in water:



- The value of i is slightly greater than 1, increasing ΔT_f .

2. Acetic acid in benzene: - Acetic acid forms dimers in benzene due to hydrogen bonding:



- The value of i is less than 1, reducing ΔT_f .

3. Benzoic acid in benzene: - Benzoic acid also forms dimers in benzene, leading to $i < 1$.
4. Glucose in water: - Glucose does not dissociate or associate in water. Thus, $i = 1$.

Step 3: Comparison of ΔT_f

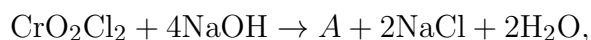
- Acetic acid in water has the highest i , as it partially ionizes. - Acetic acid and benzoic acid in benzene have $i < 1$ due to dimerization. - Glucose in water has $i = 1$, lower than ionized acetic acid.

Final Answer: (1) 180 g of acetic acid dissolved in water.

 Quick Tip

For freezing point depression: 1. Consider whether the solute ionizes ($i > 1$) or associates ($i < 1$). 2. Higher i results in a greater depression in freezing point.

Q78. A and B formed in the following reactions are:



- (1) $A = \text{Na}_2\text{CrO}_4, B = \text{CrO}_5$
- (2) $A = \text{Na}_2\text{Cr}_2\text{O}_4, B = \text{CrO}_4$
- (3) $A = \text{Na}_2\text{Cr}_2\text{O}_7, B = \text{CrO}_3$
- (4) $A = \text{Na}_2\text{CrO}_7, B = \text{CrO}_5$

Correct Answer: (1) $A = \text{Na}_2\text{CrO}_4, B = \text{CrO}_5$.

—
Solution

Step 1: Reaction 1 — Formation of A

The first reaction is:

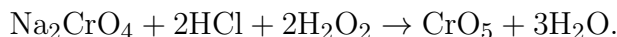


In this reaction: - Chromyl chloride (CrO_2Cl_2) reacts with sodium hydroxide (NaOH) to form sodium chromate (Na_2CrO_4). Thus:

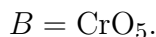


Step 2: Reaction 2 — Formation of B

The second reaction is:

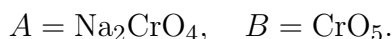


In this reaction: - Sodium chromate (Na_2CrO_4) reacts with hydrogen peroxide (H_2O_2) and hydrochloric acid (HCl) to form chromium pentoxide (CrO_5), which is a blue-colored compound. Thus:



Step 3: Verify the correct answer

From the above steps:



Final Answer: (1) $A = \text{Na}_2\text{CrO}_4$, $B = \text{CrO}_5$.

 Quick Tip

For chromyl chloride reactions: 1. In basic medium, chromate (CrO_4^{2-}) is formed.
2. In the presence of H_2O_2 , chromium pentoxide (CrO_5) is produced, which has a distinct blue color.

Q79. Choose the correct statements about the hydrides of group 15 elements:

- A. The stability of the hydrides decreases in the order $\text{NH}_3 > \text{PH}_3 > \text{AsH}_3 > \text{SbH}_3 > \text{BiH}_3$.
- B. The reducing ability of the hydrides increases in the order $\text{NH}_3 < \text{PH}_3 < \text{AsH}_3 < \text{SbH}_3 < \text{BiH}_3$.
- C. Among the hydrides, NH_3 is a strong reducing agent while BiH_3 is a mild reducing agent.
- D. The basicity of the hydrides increases in the order $\text{NH}_3 < \text{PH}_3 < \text{AsH}_3 < \text{SbH}_3 < \text{BiH}_3$.

Choose the most appropriate from the options given below:

- (1) B and C only
- (2) C and D only
- (3) A and B only
- (4) A and D only

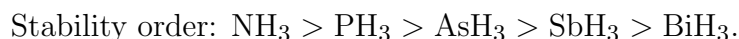
Correct Answer: (3) A and B only.

Solution

Step 1: Stability of hydrides

The stability of hydrides decreases down the group due to: - Increasing size of the central atom, leading to weaker E-H bonds. - Decreasing bond dissociation energy.

Thus:

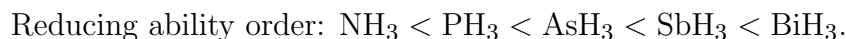


Statement A is correct.

Step 2: Reducing ability of hydrides

The reducing ability of hydrides increases down the group due to: - Decreasing bond dissociation energy. - Easier release of H^- ions from the heavier hydrides.

Thus:



Statement B is correct.

Step 3: Reducing strength of NH_3 and BiH_3

Among the hydrides: - NH_3 is not a strong reducing agent (it acts as a weak base instead).

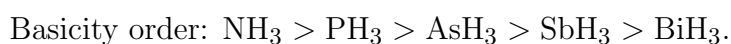
- BiH_3 is the strongest reducing agent among group 15 hydrides.

Thus, Statement C is incorrect.

Step 4: Basicity of hydrides

The basicity of hydrides decreases down the group due to: - Decreasing availability of the lone pair of electrons on the central atom. - Increasing atomic size, leading to lower electron density.

Thus:



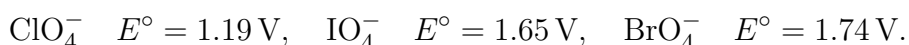
Statement D is incorrect.

Final Answer: (3) A and B only.

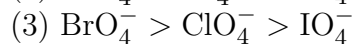
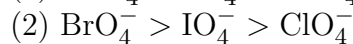
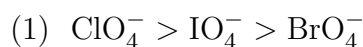
💡 Quick Tip

For group trends in hydrides: 1. Stability decreases down the group due to weaker bonds. 2. Reducing ability increases down the group due to easier release of H^- . 3. Basicity decreases down the group as lone-pair availability decreases.

Q80. Reduction potentials of ions are given below:



The correct order of their oxidising power is:



Correct Answer: (2) $\text{BrO}_4^- > \text{IO}_4^- > \text{ClO}_4^-$.

Solution

Step 1: Understanding oxidising power

The oxidising power of a species depends on its standard reduction potential E° : - A higher reduction potential indicates a stronger tendency to gain electrons (i.e., reduction).

- Thus, the species with a higher E° value is a stronger oxidising agent.

Step 2: Comparison of reduction potentials

The reduction potentials are:

$$E^\circ(\text{BrO}_4^-) = 1.74 \text{ V}, \quad E^\circ(\text{IO}_4^-) = 1.65 \text{ V}, \quad E^\circ(\text{ClO}_4^-) = 1.19 \text{ V}.$$

From these values:

$$\text{Oxidising power order: } \text{BrO}_4^- > \text{IO}_4^- > \text{ClO}_4^-.$$

Step 3: Verify the correct option

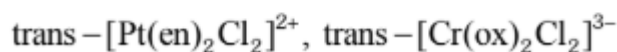
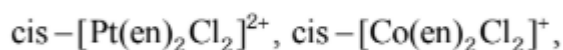
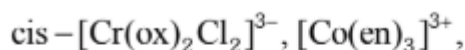
The given order matches Option (2).

Final Answer: (2) $\text{BrO}_4^- > \text{IO}_4^- > \text{ClO}_4^-$.

💡 Quick Tip

When comparing oxidising powers, remember: 1. A higher standard reduction potential (E°) implies a stronger oxidising agent. 2. Compare the values directly to determine the order.

Q81. Number of complexes which show optical isomerism among the following is :



Correct Answer: (4) 4.

Solution**0.1 Step 1: Optical isomerism in coordination complexes**

Optical isomerism occurs when a molecule is non-superimposable on its mirror image. This typically requires: - Absence of a plane of symmetry. - Specific geometries like cis-configurations or tetrahedral complexes with bidentate ligands.

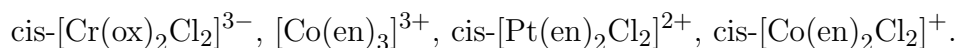
Step 2: Analyze the given complexes

1. $\text{cis} - [\text{Cr}(\text{ox})_2\text{Cl}_2]^{3-}$: - The oxalate (ox) ligand is bidentate. - The cis-configuration leads to a chiral structure. - Optically active.

2. $[\text{Co}(\text{en})_3]^{3+}$: - Contains three bidentate ethylenediamine (en) ligands. - Forms a chiral octahedral structure. - Optically active.
3. $\text{cis-}[\text{Pt}(\text{en})_2\text{Cl}_2]^{2+}$: - Two bidentate en ligands and cis-configuration ensure chirality. - Optically active.
4. $\text{cis-}[\text{Co}(\text{en})_2\text{Cl}_2]^+$: - Two bidentate en ligands and cis-configuration ensure chirality. - Optically active.
5. $\text{trans-}[\text{Pt}(\text{en})_2\text{Cl}_2]^{2+}$: - Trans-configuration provides a plane of symmetry. - Not optically active.
6. $\text{trans-}[\text{Cr}(\text{ox})_2\text{Cl}_2]^{3-}$: - Trans-configuration provides a plane of symmetry. - Not optically active.

Step 3: Count the optically active complexes

The optically active complexes are:



Total number of optically active complexes = 4.

Final Answer: (4) 4.

💡 Quick Tip

For optical isomerism in complexes: 1. Look for cis-configurations in octahedral complexes with bidentate ligands. 2. Trans-configurations and symmetrical structures generally lack chirality.

Q82. NO_2 required for a reaction is produced by decomposition of N_2O_5 in CCl_4 as by equation:



The initial concentration of N_2O_5 is 3 mol L^{-1} and it is 2.75 mol L^{-1} after 30 minutes. The rate of formation of NO_2 is $x \times 10^{-3} \text{ mol L}^{-1}\text{min}^{-1}$, value of x is

Correct Answer: (17).

Solution

Step 1: Determine the change in N_2O_5 concentration

The decomposition of N_2O_5 causes a decrease in its concentration:

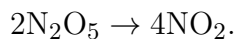
$$\Delta[\text{N}_2\text{O}_5] = [\text{N}_2\text{O}_5]_{\text{initial}} - [\text{N}_2\text{O}_5]_{\text{final}}.$$

Substitute:

$$\Delta[\text{N}_2\text{O}_5] = 3.00 - 2.75 = 0.25 \text{ mol L}^{-1}.$$

Step 2: Stoichiometric relationship

From the balanced equation:



The formation of NO_2 is twice the decrease in N_2O_5 :

$$\Delta[\text{NO}_2] = 2 \cdot \Delta[\text{N}_2\text{O}_5] = 2 \cdot 0.25 = 0.50 \text{ mol L}^{-1}.$$

Step 3: Rate of formation of NO_2

The rate of formation of NO_2 is given by:

$$\text{Rate} = \frac{\Delta[\text{NO}_2]}{\Delta t}.$$

Here, $\Delta t = 30$ minutes:

$$\text{Rate} = \frac{0.50}{30} = 0.0167 \text{ mol L}^{-1}\text{min}^{-1}.$$

Convert to scientific notation:

$$\text{Rate} = 16.7 \times 10^{-3} \text{ mol L}^{-1}\text{min}^{-1}.$$

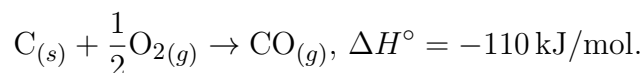
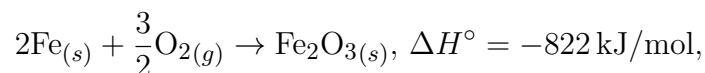
Thus, $x = 17$.

Final Answer: $x = 17$.

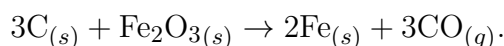
💡 Quick Tip

For rate calculations: 1. Use the stoichiometry of the reaction to relate the change in concentrations of reactants and products. 2. Divide the change in product concentration by the time interval to calculate the rate.

Q83. Two reactions are given below:



The enthalpy change for the following reaction is:

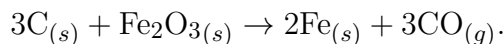


Correct Answer: (492).

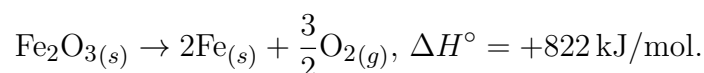
Solution

Step 1: Target reaction and reverse reactions

We need to calculate ΔH° for the reaction:



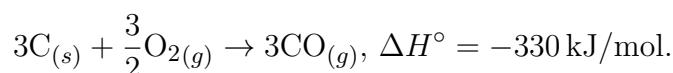
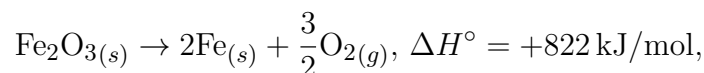
Reverse the first reaction, as we need $\text{Fe}_2\text{O}_{3(s)}$ as a reactant:



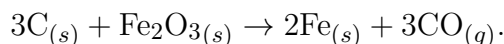
Keep the second reaction as it is and multiply it by 3, since we need 3 moles of CO:

**Step 2: Combine reactions**

Add the two modified reactions:



Resulting reaction:

**Step 3: Total enthalpy change**

The total enthalpy change is:

$$\Delta H^\circ = 822 + (-330) = 492 \text{ kJ/mol}.$$

Final Answer: $\Delta H^\circ = 492 \text{ kJ/mol}$.

💡 Quick Tip

For enthalpy calculations: 1. Reverse the reactions if required, and change the sign of ΔH° . 2. Multiply the reactions to match the stoichiometry, and scale ΔH° accordingly. 3. Add the modified reactions and their enthalpy changes to find the total ΔH° .

Q84. The total number of correct statements regarding nucleic acids is ____ .

Statements:

1. RNA is regarded as the reserve of genetic information.
2. DNA molecule self-duplicates during cell division.
3. DNA synthesizes proteins in the cell.

4. The message for the synthesis of particular proteins is present in DNA.
5. Identical DNA strands are transferred to daughter cells.

Correct Answer: (3)

Solution

Step 1: Analyze each statement

1. **False:** RNA is not regarded as the reserve of genetic information. DNA serves as the reserve of genetic information in most organisms.
2. **True:** DNA molecule undergoes self-replication (duplication) during cell division to ensure genetic material is passed to daughter cells.
3. **False:** DNA does not synthesize proteins directly. It contains the code for protein synthesis, which is carried out by RNA and ribosomes.
4. **True:** DNA contains the genetic instructions (genes) that provide the message for the synthesis of specific proteins.
5. **True:** Identical DNA strands are transferred to daughter cells after replication during cell division.

Step 2: Count correct statements

The correct statements are: B, D, and E.

Final Answer: 3.

 Quick Tip

For questions on nucleic acids: 1. Understand the roles of DNA and RNA in genetic information and protein synthesis. 2. DNA stores genetic information, self-replicates, and transfers identical strands to daughter cells. 3. RNA helps in protein synthesis but does not store genetic information in most organisms.

Q85. The pH of an aqueous solution containing 1 M benzoic acid ($pK_a = 4.20$) and 1 M sodium benzoate is 4.5. The volume of benzoic acid solution in 300 mL of this buffer solution is ____ mL.

Correct Answer: 100 mL

Solution

Step 1: Use the Henderson-Hasselbalch equation

The pH of a buffer solution is given by:

$$\text{pH} = \text{p}K_a + \log \left(\frac{[\text{Salt}]}{[\text{Acid}]} \right).$$

Here:

$$\text{pH} = 4.5, \quad \text{p}K_a = 4.20, \quad [\text{Salt}] = 1 \text{ M}, \quad [\text{Acid}] = 1 \text{ M}.$$

Substitute these values into the equation:

$$4.5 = 4.20 + \log \left(\frac{[\text{Salt}]}{[\text{Acid}]} \right).$$

Step 2: Solve for the ratio of salt to acid

Rearranging:

$$\log \left(\frac{[\text{Salt}]}{[\text{Acid}]} \right) = 4.5 - 4.20 = 0.30.$$

Take the antilog:

$$\frac{[\text{Salt}]}{[\text{Acid}]} = 10^{0.30} \approx 2.$$

Thus, the ratio of salt to acid is:

$$\frac{[\text{Salt}]}{[\text{Acid}]} = 2 : 1.$$

Step 3: Determine the volumes of acid and salt

The total volume of the buffer solution is 300 mL. Let the volume of benzoic acid solution be V_{acid} mL and the volume of sodium benzoate solution be V_{salt} mL.

From the given ratio (2 : 1):

$$V_{\text{salt}} = 2V_{\text{acid}}.$$

Since the total volume is 300 mL:

$$V_{\text{acid}} + V_{\text{salt}} = 300.$$

Substitute $V_{\text{salt}} = 2V_{\text{acid}}$:

$$V_{\text{acid}} + 2V_{\text{acid}} = 300.$$

Solve for V_{acid} :

$$3V_{\text{acid}} = 300 \implies V_{\text{acid}} = 100 \text{ mL}.$$

Final Answer:

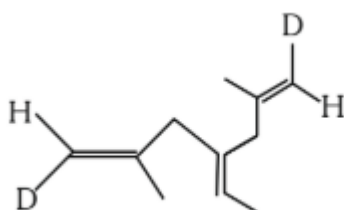
The volume of benzoic acid solution is:

100 mL.

💡 Quick Tip

For buffer problems, always use the Henderson-Hasselbalch equation. Determine the ratio of salt to acid, and use the total volume to calculate the respective volumes.

Q86. Number of geometrical isomers possible for the given structure is/are ____ :



Correct Answer: 4

Solution**Step 1: Identify double bonds with restricted rotation**

The given compound contains two double bonds. Both double bonds have distinct groups attached, allowing for cis-trans isomerism (geometrical isomerism).

Step 2: Count the possibilities for each double bond

- For the first double bond: Two possibilities exist (cis and trans). - For the second double bond: Two possibilities also exist (cis and trans).

Step 3: Combine the possibilities

Since these double bonds are independent of each other, the total number of geometrical isomers is:

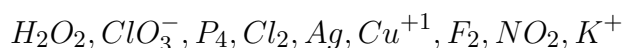
$$\text{Total isomers} = 2 \times 2 = 4.$$

Final Answer: 4.

💡 Quick Tip

For compounds with multiple double bonds, determine the number of cis-trans possibilities for each double bond. Multiply the possibilities for all double bonds to get the total number of geometrical isomers.

Q87. Total number of species from the following which can undergo disproportionation reaction ---- :



Correct Answer: 6

Solution**Step 1: Definition of disproportionation reaction**

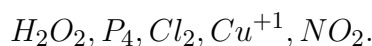
A disproportionation reaction is one in which a single species is both oxidized and reduced, resulting in two different products with varying oxidation states.

Step 2: Analyze the given species

- H_2O_2 : Undergoes disproportionation to form H_2O and O_2 . - ClO_3^- : Does not undergo disproportionation as the chlorine atom is in its highest oxidation state. - P_4 : Undergoes disproportionation to form PH_3 and H_3PO_4 . - Cl_2 : Undergoes disproportionation to form Cl^- and ClO^- . - Ag : Does not undergo disproportionation as it is in its elemental state. - Cu^{+1} : Undergoes disproportionation to form Cu^0 and Cu^{2+} . - F_2 : Does not undergo disproportionation as it is in its highest oxidation state. - NO_2 : Undergoes disproportionation to form NO_3^- and NO . - K^+ : Does not undergo disproportionation as it is in its lowest oxidation state.

Step 3: Count the species that undergo disproportionation

The species that undergo disproportionation are:

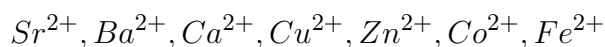


Total: 6.

💡 Quick Tip

To determine if a species undergoes disproportionation, check if it has an intermediate oxidation state that allows simultaneous oxidation and reduction.

Q88. Number of metal ions characterized by flame test among the following is ----:



Correct Answer: 4

Solution

Step 1: Metals that give characteristic flame color

Flame tests are typically used to identify alkali and alkaline earth metals, as well as a few transition metals. The following ions show characteristic flame colors: - Sr^{2+} : Gives a crimson flame. - Ba^{2+} : Gives a green flame. - Ca^{2+} : Gives a brick-red flame. - Cu^{2+} : Gives a blue-green flame.

Step 2: Metals that do not give characteristic flame color

- Zn^{2+} , Co^{2+} , and Fe^{2+} : These do not show characteristic colors in the flame test.

Step 3: Count the ions that give flame colors

The ions Sr^{2+} , Ba^{2+} , Ca^{2+} , and Cu^{2+} give characteristic flame colors.

Final Answer: 4.

 Quick Tip

Flame tests are particularly effective for alkali metals (e.g., Na, K) and alkaline earth metals (e.g., Ca, Sr, Ba). Transition metals rarely exhibit flame colors.

Q89. 2-chlorobutane + $Cl_2 \rightarrow C_4H_8Cl_2$ (isomers)

Total number of optically active isomers shown by $C_4H_8Cl_2$, obtained in the above reaction is

Correct Answer: 6

Solution

Step 1: Reaction details and possible products

2-chlorobutane undergoes chlorination with Cl_2 , producing $C_4H_8Cl_2$ as isomers. Chlorination introduces a second chlorine atom, forming different dichloro derivatives depending on the position of the chlorine atom.

Step 2: Structural isomers and optical activity

1. Positions of chlorination: The second chlorine atom can be added at different positions on the carbon chain, leading to multiple isomers. 2. **Optical activity**: Isomers with at least one chiral center are optically active.

- Case 1: Chlorine is added at the 1st and 3rd carbons of the chain, resulting in isomers with no chirality (optically inactive). - Case 2: Chlorine is added at the 2nd and 3rd carbons. This leads to two chiral centers, resulting in optically active isomers. - Case 3: Other combinations of positions may also lead to optically active forms.

Step 3: Counting optically active isomers

The total number of optically active isomers for $C_4H_8Cl_2$ is determined by the number of stereoisomers resulting from chiral centers. Based on the symmetry and positions of chlorination, 6 optically active isomers are possible.

Final Answer: 6.

💡 Quick Tip

For such reactions, determine the positions of substitution and check for chirality to find optically active isomers. Each chiral center doubles the number of possible optical isomers.

Q90. Number of spectral lines obtained in He^+ spectra, when an electron makes transition from fifth excited state to first excited state will be:

Correct Answer: 10

Solution**Step 1: Formula for spectral lines**

The number of spectral lines emitted when an electron transitions from a higher energy level n_2 to a lower energy level n_1 is given by:

$$N = \frac{(n_2 - n_1)(n_2 - n_1 + 1)}{2}.$$

Step 2: Values of n_2 and n_1

Here:

$n_2 = 6$ (fifth excited state corresponds to $n = 6$), $n_1 = 2$ (first excited state corresponds to $n = 2$).

Step 3: Calculate N

Substitute the values of n_2 and n_1 :

$$N = \frac{(6 - 2)(6 - 2 + 1)}{2} = \frac{4 \cdot 5}{2} = 10.$$

Final Answer: 10.

💡 Quick Tip

To calculate the number of spectral lines for transitions in an atom, use $N = \frac{(n_2 - n_1)(n_2 - n_1 + 1)}{2}$ and ensure that $n_2 > n_1$.