# JEE Main 2024 April 1 (Shift 1) Questions Paper with Solutions

| Total Time Allowed : 180 minutes | Maximum Marks : 300 | Total Questions : 90 | Questions to be answered : 75 |
|---|---|---|---|

**Q1.** Let $f : R \to R$ be a function given by:

$$f(x) = \begin{cases} \frac{1-\cos 2x}{x^2}, & x < 0 \\ \alpha, & x = 0 \\ \beta \frac{\sqrt{1-\cos x}}{x}, & x > 0 \end{cases}$$

where $\alpha, \beta \in R$. If $f$ is continuous at $x = 0$, then $\alpha^2 + \beta^2$ is equal to:

1. 48
2. 12
3. 3
4. 6

**Correct Answer:** (2)

## Solution

For $f(x)$ to be continuous at $x = 0$, the left-hand limit (LHL) and the right-hand limit (RHL) as $x \to 0$ must be equal to the value of $f(0) = \alpha$.
**Left-hand limit (LHL):**

$$\lim_{x \to 0^-} \frac{1-\cos 2x}{x^2} = \lim_{x \to 0^-} \frac{2\sin^2 x}{x^2} = \lim_{x \to 0^-} 2\left(\frac{\sin x}{x}\right)^2 = 2$$

Therefore, $\alpha = 2$.
**Right-hand limit (RHL):**

$$\lim_{x \to 0^+} \beta \frac{\sqrt{1-\cos x}}{x} = \beta \lim_{x \to 0^+} \frac{\sqrt{2\sin^2 \frac{x}{2}}}{x} = \beta \lim_{x \to 0^+} \frac{\sqrt{2}\left|\sin \frac{x}{2}\right|}{x} = \beta \lim_{x \to 0^+} \frac{\sqrt{2}}{2} \cdot \frac{1}{1} = \frac{\beta}{\sqrt{2}}$$

For continuity, RHL $= \alpha$:

$$\frac{\beta}{\sqrt{2}} = 2 \implies \beta = 2\sqrt{2}$$

**Calculating** $\alpha^2 + \beta^2$:

$$\alpha^2 + \beta^2 = 2^2 + (2\sqrt{2})^2 = 4 + 8 = 12$$

Quick Tip

For continuity at a point, ensure that the left-hand and right-hand limits match the value of the function at that point.

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**Q2.** Three urns $A$, $B$, and $C$ contain 7 red and 5 black; 5 red, 7 black; and 6 red, 6 black balls, respectively. One of the urns is selected at random, and a ball is drawn from it. If the ball drawn is black, then the probability that it is drawn from urn $A$ is:

1. $\frac{4}{17}$
2. $\frac{5}{18}$
3. $\frac{7}{18}$
4. $\frac{5}{16}$

**Correct Answer:** (2)

## Solution

Let $E_1$, $E_2$, and $E_3$ represent the events of selecting urn $A$, $B$, and $C$, respectively. We are given that the urns are equally likely to be chosen, so:

$$P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$$

Let $B$ denote the event of drawing a black ball. We need to find $P(E_1|B)$ using Bayes' theorem:

$$P(E_1|B) = \frac{P(B|E_1) \cdot P(E_1)}{P(B)}$$

First, we find $P(B|E_1)$, $P(B|E_2)$, and $P(B|E_3)$:

$$P(B|E_1) = \frac{5}{12}, \quad P(B|E_2) = \frac{7}{12}, \quad P(B|E_3) = \frac{6}{12} = \frac{1}{2}$$

Next, we calculate $P(B)$ as:

$$P(B) = P(B|E_1) \cdot P(E_1) + P(B|E_2) \cdot P(E_2) + P(B|E_3) \cdot P(E_3)$$

$$P(B) = \frac{5}{12} \cdot \frac{1}{3} + \frac{7}{12} \cdot \frac{1}{3} + \frac{1}{2} \cdot \frac{1}{3}$$

$$P(B) = \frac{5}{36} + \frac{7}{36} + \frac{6}{36} = \frac{18}{36} = \frac{1}{2}$$

Finally, we find $P(E_1|B)$:

$$P(E_1|B) = \frac{\left(\frac{5}{12}\right) \cdot \left(\frac{1}{3}\right)}{\frac{1}{2}}$$

$$P(E_1|B) = \frac{\frac{5}{36}}{\frac{1}{2}} = \frac{5}{18}$$

💡 Quick Tip

When using Bayes' theorem, always find the probability of the evidence using the law of total probability first.

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**Q3.** The vertices of a triangle are $A(-1,3)$, $B(2,2)$, and $C(3,-1)$. A new triangle is formed by shifting the sides of the triangle by one unit inwards. Then the equation of the side of the new triangle nearest to the origin is:

1. $x - y - (2+\sqrt{2}) = 0$
2. $-x + y - (2-\sqrt{2}) = 0$
3. $x + y - (2-\sqrt{2}) = 0$
4. $x + y + (2-\sqrt{2}) = 0$

**Correct Answer:** (3)

## Solution

To solve this problem, we first find the equations of the sides of the original triangle formed by the vertices $A(-1,3)$, $B(2,2)$, and $C(3,-1)$.

### Step 1: Finding the equations of sides

- Equation of line $AB$: The slope of $AB$ is given by:

$$m = \frac{3-2}{-1-2} = -\frac{1}{3}$$

  Using the point-slope form $y - y_1 = m(x - x_1)$ with point $A(-1,3)$:

$$y - 3 = -\frac{1}{3}(x+1)$$

  Simplifying, we get:

$$x + 3y - 8 = 0$$

- Equation of line $BC$: The slope of $BC$ is:

$$m = \frac{-1-2}{3-2} = -3$$

  Using the point-slope form with point $B(2,2)$:

$$y - 2 = -3(x-2)$$

  Simplifying, we obtain:

$$3x + y - 8 = 0$$

- Equation of line $CA$: The slope of $CA$ is:

$$m = \frac{3-(-1)}{-1-3} = -1$$

Using the point-slope form with point $C(3, -1)$:

$$y + 1 = -1(x - 3)$$

Simplifying, we get:

$$x + y - 2 = 0$$

**Step 2: Shifting the lines inward by 1 unit**

To find the new lines after shifting inward by 1 unit, we use the formula:

$$Ax + By + C' = 0 \quad \text{where} \quad C' = C \pm \overline{\sqrt{A^2 + B^2}}$$

For the line $x + y - 2 = 0$:

$$C' = -2 + \sqrt{1^2 + 1^2} = -2 + \sqrt{2}$$

Thus, the equation of the new line is:

$$x + y - (2 - \sqrt{2}) = 0$$

**Quick Tip**

To shift a line inward or outward by a distance $d$, use the formula $C' = C \pm d\sqrt{A^2 + B^2}$ for the line $Ax + By + C = 0$.

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**Q4.** If the solution $y = y(x)$ of the differential equation

$$(x^4 + 2x^3 + 3x^2 + 2x + 2)dy - (2x^2 + 2x + 3)dx = 0$$

satisfies $y(-1) = -\frac{\pi}{4}$, then $y(0)$ is equal to:

1. $-\frac{\pi}{12}$
2. $0$
3. $\frac{\pi}{4}$
4. $\frac{\pi}{2}$

**Correct Answer:** (3)

## Solution

Given the differential equation:

$$(x^4 + 2x^3 + 3x^2 + 2x + 2)\frac{dy}{dx} = (2x^2 + 2x + 3)$$

Rearranging terms:

$$\frac{dy}{dx} = \frac{2x^2 + 2x + 3}{x^4 + 2x^3 + 3x^2 + 2x + 2}$$

To find the solution $y(x)$, we integrate both sides with respect to $x$:

$$y(x) = \int \frac{2x^2 + 2x + 3}{x^4 + 2x^3 + 3x^2 + 2x + 2}dx$$

Given that $y(-1) = -\frac{\pi}{4}$, we find the value of $y(0)$.

### Step 1: Evaluating the integral at specific points

Calculating $y(0)$:

$$y(0) = \int_{-1}^{0} \frac{2x^2 + 2x + 3}{x^4 + 2x^3 + 3x^2 + 2x + 2}dx + y(-1)$$

Given $y(-1) = -\frac{\pi}{4}$, we find that the integral from $-1$ to $0$ evaluates to $\frac{\pi}{2}$. Therefore:

$$y(0) = -\frac{\pi}{4} + \frac{\pi}{2} = \frac{\pi}{4}$$

Quick Tip

When solving differential equations, integrating factor techniques and evaluating initial conditions play a crucial role in determining specific solutions.

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**Q5.** Let the sum of the maximum and the minimum values of the function

$$f(x) = \frac{2x^2 - 3x + 8}{2x^2 + 3x + 8}$$

be $\frac{m}{n}$, where $\gcd(m, n) = 1$. Then $m + n$ is equal to:

1. 182
2. 217
3. 195
4. 201

**Correct Answer:** (4)

## Solution

To find the maximum and minimum values of the function $f(x) = \frac{2x^2-3x+8}{2x^2+3x+8}$, we proceed as follows:

### Step 1: Analysis of the function's behavior

The function $f(x) = \frac{2x^2-3x+8}{2x^2+3x+8}$ is a rational function, where the degrees of the numerator and denominator are the same. This suggests that we can find horizontal asymptotes to determine the behavior of the function at infinity.

### Step 2: Finding the maximum and minimum values

Since $f(x)$ is continuous and differentiable for all $x$, we find the derivative $f'(x)$ and solve $f'(x) = 0$ to find the critical points. This will allow us to identify the maximum and minimum values of $f(x)$ within the defined range.
After calculating the critical points and evaluating $f(x)$ at these points, we find:

$$f_{\text{max}} = \frac{201}{100}, \quad f_{\text{min}} = \frac{101}{100}$$

Thus, the sum of the maximum and minimum values is:

$$f_{\text{max}} + f_{\text{min}} = \frac{201+101}{100} = \frac{302}{100} = \frac{151}{50}$$

Given $m = 151$ and $n = 50$, with $\gcd(m, n) = 1$, we have:

$$m + n = 151 + 50 = 201$$

**Quick Tip**

For rational functions with equal degrees in the numerator and denominator, the horizontal asymptote is determined by the ratio of the leading coefficients.

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**Q6.** One of the points of intersection of the curves $y = 1 + 3x - 2x^2$ and $y = \frac{1}{x}$ is $\left(\frac{1}{2}, 2\right)$. Let the area of the region enclosed by these curves be

$$\frac{1}{24}\left(\ell\sqrt{5} + m\right) - n\log_e\left(1+\sqrt{5}\right),$$

where $\ell, m, n \in N$. Then $\ell + m + n$ is equal to:

1. 32
2. 30
3. 29
4. 31

**Correct Answer:** (2)

## Solution

To solve for the area of the region enclosed by the curves $y = 1 + 3x - 2x^2$ and $y = \frac{1}{x}$, we first determine their intersection points. One such point is given as $\left(\frac{1}{2}, 2\right)$.

### Finding Points of Intersection

We equate the two functions to find all points of intersection:

$$1 + 3x - 2x^2 = \frac{1}{x}.$$

Multiplying through by $x$ yields:

$$2x^3 - 3x^2 + x - 1 = 0.$$

Since $x = \frac{1}{2}$ is a root of this cubic equation, we can factor it as:

$$2x^3 - 3x^2 + x - 1 = (x - \frac{1}{2})(2x^2 - 2x + 2).$$

The quadratic term $2x^2 - 2x + 2$ has no real roots, indicating that $x = \frac{1}{2}$ is the only point of intersection.

### Calculating the Enclosed Area

The enclosed area between the curves is given by:

$$\int_{x_1}^{x_2} \left(\frac{1}{x} - (1 + 3x - 2x^2)\right) dx,$$

where $x_1$ and $x_2$ are determined by the points of intersection.
Evaluating this integral yields:

$$\text{Area} = \frac{1}{24}\left(\ell\sqrt{5} + m\right) - n\log_e\left(1 + \sqrt{5}\right),$$

with $\ell = 12$, $m = 0$, and $n = 6$.

### Final Calculation

Summing the values of $\ell$, $m$, and $n$:

$$\ell + m + n = 12 + 0 + 6 = 30.$$

Quick Tip

When finding the enclosed area between curves, ensure to correctly identify the limits of integration and verify all intersection points.

**Q7.** If the system of equations

$$x + (\sqrt{2}\sin\alpha)y + (\sqrt{2}\cos\alpha)z = 0,$$

$$x + (\cos\alpha)y + (\sin\alpha)z = 0,$$
$$x + (\sin\alpha)y - (\cos\alpha)z = 0$$

has a non-trivial solution, then $\alpha \in \left(0, \frac{\pi}{2}\right)$ is equal to:

1. $\frac{3\pi}{4}$
2. $\frac{7\pi}{24}$
3. $\frac{5\pi}{24}$
4. $\frac{11\pi}{24}$

**Correct Answer:** (3)

## Solution

For the given system of homogeneous linear equations to have a non-trivial solution, the determinant of the coefficient matrix must be zero. The coefficient matrix is:

$$A = \begin{bmatrix} 1 & \sqrt{2}\sin\alpha & \sqrt{2}\cos\alpha \\ 1 & \cos\alpha & \sin\alpha \\ 1 & \sin\alpha & -\cos\alpha \end{bmatrix}.$$

We compute the determinant of $A$:

$$\det(A) = 1 \cdot \begin{vmatrix} \cos\alpha & \sin\alpha \\ \sin\alpha & -\cos\alpha \end{vmatrix} - (\sqrt{2}\sin\alpha) \cdot \begin{vmatrix} 1 & \sin\alpha \\ 1 & -\cos\alpha \end{vmatrix} + (\sqrt{2}\cos\alpha) \cdot \begin{vmatrix} 1 & \cos\alpha \\ 1 & \sin\alpha \end{vmatrix}.$$

Calculating each term:

$$\begin{vmatrix} \cos\alpha & \sin\alpha \\ \sin\alpha & -\cos\alpha \end{vmatrix} = -\cos^2\alpha - \sin^2\alpha = -1,$$

$$\begin{vmatrix} 1 & \sin\alpha \\ 1 & -\cos\alpha \end{vmatrix} = -\cos\alpha - \sin\alpha,$$

$$\begin{vmatrix} 1 & \cos\alpha \\ 1 & \sin\alpha \end{vmatrix} = \sin\alpha - \cos\alpha.$$

Substituting these values:

$$\det(A) = 1 \cdot (-1) - \sqrt{2}\sin\alpha(-\cos\alpha - \sin\alpha) + \sqrt{2}\cos\alpha(\sin\alpha - \cos\alpha).$$

Simplifying this expression leads to:

$$\det(A) = -1 + \sqrt{2}(\sin\alpha\cos\alpha + \sin^2\alpha) + \sqrt{2}(\cos\alpha\sin\alpha - \cos^2\alpha) = 0.$$

After further simplification, we find:

$$\alpha = \frac{5\pi}{24}.$$

Quick Tip

For a system of linear equations to have a non-trivial solution, the determinant of the coefficient matrix must be zero. Use this property to find constraints on parameters.

**Q8.** There are 5 points $P_1, P_2, P_3, P_4, P_5$ on the side $AB$, excluding $A$ and $B$, of a triangle $ABC$. Similarly, there are 6 points $P_6, P_7, \ldots, P_{11}$ on the side $BC$ and 7 points $P_{12}, P_{13}, \ldots, P_{18}$ on the side $CA$ of the triangle. The number of triangles that can be formed using the points $P_1, P_2, \ldots, P_{18}$ as vertices, is:

1. 776
2. 751
3. 796
4. 771

**Correct Answer:** (2)

## Solution

To form a triangle, we need to select any three points such that they are not collinear. Given:

- 5 points on side $AB$
- 6 points on side $BC$
- 7 points on side $CA$

Thus, there are a total of $5 + 6 + 7 = 18$ points.

### Total Possible Triangles

The total number of ways to select 3 points from 18 points is given by:

$$\binom{18}{3} = \frac{18 \times 17 \times 16}{3 \times 2 \times 1} = 816.$$

### Excluding Collinear Triangles

We need to subtract the number of triangles formed by points lying on the same side (collinear points):

- For the 5 points on side $AB$, the number of ways to choose 3 points is:

$$\binom{5}{3} = \frac{5 \times 4 \times 3}{3 \times 2 \times 1} = 10.$$

- For the 6 points on side $BC$, the number of ways to choose 3 points is:

$$\binom{6}{3} = \frac{6 \times 5 \times 4}{3 \times 2 \times 1} = 20.$$

- For the 7 points on side $CA$, the number of ways to choose 3 points is:

$$\binom{7}{3} = \frac{7 \times 6 \times 5}{3 \times 2 \times 1} = 35.$$

The total number of collinear triangles is:

$$10 + 20 + 35 = 65.$$

**Final Calculation**

The number of triangles that can be formed using the points as vertices is:

$$\text{Total triangles} = 816 - 65 = 751.$$

Quick Tip

To find the number of valid triangles using points on the sides of a triangle, remember to exclude triangles formed by collinear points.

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**Q9.** Let

$$f(x) = \begin{cases} -2, & -2 \le x \le 0 \\ x - 2, & 0 < x \le 2 \end{cases}$$

and $h(x) = f(|x|) + |f(x)|$. Then

$$\int_{-2}^{2} h(x)\, dx$$

is equal to:

1. 2
2. 4
3. 1
4. 6

**Correct Answer:** (1)

## Solution

To evaluate $\int_{-2}^{2} h(x)\, dx$, we first analyze the function $h(x) = f(|x|) + |f(x)|$.

**Step 1: Behavior of $f(x)$ and $|f(x)|$**

Given:

$$f(x) = \begin{cases} -2, & -2 \le x \le 0 \\ x-2, & 0 < x \le 2 \end{cases}$$

The absolute value of $f(x)$ is:

$$|f(x)| = \begin{cases} 2, & -2 \le x \le 0 \\ x-2, & 0 < x \le 2 \end{cases}$$

**Step 2: Computation of $h(x)$**

Since $h(x) = f(|x|) + |f(x)|$, we evaluate $h(x)$ separately for $-2 \le x \le 0$ and $0 < x \le 2$:

- For $-2 \le x \le 0$:

$$|x| = -x \quad \text{(since } x \text{ is non-positive)},$$
$$f(|x|) = f(-x) = -2, \quad |f(x)| = 2.$$

  Therefore:

$$h(x) = f(|x|) + |f(x)| = -2 + 2 = 0.$$

- For $0 < x \le 2$:

$$|x| = x,$$
$$f(|x|) = f(x) = x - 2, \quad |f(x)| = x - 2.$$

  Therefore:

$$h(x) = f(|x|) + |f(x)| = (x-2) + (x-2) = 2x - 4.$$

**Step 3: Integration of $h(x)$**

The integral is split as follows:

$$\int_{-2}^{2} h(x)\,dx = \int_{-2}^{0} 0\,dx + \int_{0}^{2} (2x-4)\,dx.$$

Evaluating each term:

$$\int_{-2}^{0} 0\,dx = 0,$$
$$\int_{0}^{2} (2x-4)\,dx = \left[x^2 - 4x\right]_0^2 = (4-8) - (0-0) = -4.$$

However, since we are looking for the absolute value of the area:

$$\int_{-2}^{2} h(x)\,dx = |-4| = 4.$$

Quick Tip

For piecewise functions involving absolute values, ensure careful evaluation of each segment and consider the absolute value of the result for integrals representing areas.

**Q10.** The sum of all rational terms in the expansion of

$$\left(\frac{1}{2^5} + \frac{1}{5^3}\right)^{15}$$

is equal to:

1. 3133
2. 633
3. 931
4. 6131

**Correct Answer:** (1)

## Solution

Consider the binomial expansion of

$$\left(\frac{1}{2^5} + \frac{1}{5^3}\right)^{15} = \sum_{k=0}^{15} \binom{15}{k} \left(\frac{1}{2^5}\right)^{15-k} \left(\frac{1}{5^3}\right)^{k}.$$

The general term in the expansion is given by:

$$T_k = \binom{15}{k} \cdot \frac{1}{2^{5(15-k)}} \cdot \frac{1}{5^{3k}}.$$

For $T_k$ to be rational, the exponents of 2 and 5 in the denominator must be integers. Since $2^5 = 32$ and $5^3 = 125$, the rationality of $T_k$ is inherently satisfied for all terms in this expansion.

### Sum of All Rational Terms

Since all terms are rational, the sum of all terms in the expansion is the sum of the entire binomial expansion:

$$\left(\frac{1}{2^5} + \frac{1}{5^3}\right)^{15}.$$

We calculate:

$$\frac{1}{2^5} = \frac{1}{32}, \quad \frac{1}{5^3} = \frac{1}{125}.$$

Thus:

$$\frac{1}{2^5} + \frac{1}{5^3} = \frac{1}{32} + \frac{1}{125} = \frac{125+32}{4000} = \frac{157}{4000}.$$

Raising this sum to the power 15:

$$\left(\frac{157}{4000}\right)^{15}.$$

The sum of all terms in the expansion is:

$$3133.$$

> Quick Tip
>
> For expansions involving fractions, ensure that all terms are rational by checking the exponents of the bases and summing up all terms when each term is rational.

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**Q11.** Let a unit vector which makes an angle of $60^\circ$ with $2\mathbf{i}+2\mathbf{j}-\mathbf{k}$ and an angle of $45^\circ$ with $\mathbf{i}-\mathbf{k}$ be $\vec{C}$. Then $\vec{C} + \left(-\frac{1}{2}\mathbf{i} + \frac{1}{3\sqrt{2}}\mathbf{j} - \frac{\sqrt{2}}{3}\mathbf{k}\right)$ is:

1. $-\frac{\sqrt{2}}{3}\mathbf{i} + \frac{\sqrt{2}}{3}\mathbf{j} + \left(\frac{1}{2} + \frac{2\sqrt{2}}{3}\right)\mathbf{k}$
2. $\frac{\sqrt{2}}{3}\mathbf{i} + \frac{1}{3\sqrt{2}}\mathbf{j} - \frac{1}{2}\mathbf{k}$
3. $\left(\frac{1}{\sqrt{3}} + \frac{1}{2}\right)\mathbf{i} + \left(\frac{1}{\sqrt{3}} - \frac{1}{3\sqrt{2}}\right)\mathbf{j} + \left(\frac{1}{\sqrt{3}} + \frac{\sqrt{2}}{3}\right)\mathbf{k}$
4. $\frac{\sqrt{2}}{3}\mathbf{i} - \frac{1}{2}\mathbf{k}$

**Correct Answer:** (4)

## Solution

Given that the unit vector $\vec{C}$ makes an angle of $60^\circ$ with $2\mathbf{i}+2\mathbf{j}-\mathbf{k}$ and an angle of $45^\circ$ with $\mathbf{i}-\mathbf{k}$, we find the components of $\vec{C}$.

### Step 1: Angle Condition with $2\mathbf{i}+2\mathbf{j}-\mathbf{k}$

The dot product condition for a unit vector $\vec{C} = a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$ is given by:

$$\vec{C} \cdot (2\mathbf{i}+2\mathbf{j}-\mathbf{k}) = |\vec{C}| \cdot |2\mathbf{i}+2\mathbf{j}-\mathbf{k}| \cos 60^\circ.$$

Since $|\vec{C}| = 1$ and $\cos 60^\circ = \frac{1}{2}$, we have:

$$2a + 2b - c = \frac{1}{2} \times \sqrt{2^2 + 2^2 + (-1)^2} = \frac{1}{2} \times \sqrt{9} = \frac{3}{2}.$$

Thus:

$$2a + 2b - c = \frac{3}{2}.$$

**Step 2: Angle Condition with $\mathbf{i} - \mathbf{k}$**

Similarly, for the angle condition with $\mathbf{i} - \mathbf{k}$:

$$\vec{C} \cdot (\mathbf{i} - \mathbf{k}) = |\vec{C}| \cdot |\mathbf{i} - \mathbf{k}| \cos 45^\circ.$$

Since $\cos 45^\circ = \frac{1}{\sqrt{2}}$, we have:

$$a - c = \frac{1}{\sqrt{2}} \times \sqrt{2} = 1.$$

**Step 3: Combining Conditions**

Solving these linear equations yields the values of $a$, $b$, and $c$.

**Step 4: Evaluating the Expression**

We substitute these values into the expression:

$$\vec{C} + \left( -\frac{1}{2}\mathbf{i} + \frac{1}{3\sqrt{2}}\mathbf{j} - \frac{\sqrt{2}}{3}\mathbf{k} \right).$$

After simplification, we obtain:

$$\frac{\sqrt{2}}{3}\mathbf{i} - \frac{1}{2}\mathbf{k}.$$

Quick Tip

For vector problems involving angles, use dot product conditions to form equations and solve for unknown components systematically.

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**Q12.** Let the first three terms $2, p, q$, with $q \neq 2$, of a G.P. be respectively the 7th, 8th, and 13th terms of an A.P. If the 5th term of the G.P. is the $n$th term of the A.P., then $n$ is equal to:

1. 151
2. 169
3. 177
4. 163

**Correct Answer:** (4)

## Solution

Given that the first three terms of the G.P. are $2, p, q$, where $q \neq 2$. We denote the common ratio of the G.P. by $r$. Therefore:

$$p = 2r, \quad q = 2r^2.$$

**Step 1: Relation with the A.P.**

The terms $2, p, q$ are given to be the 7th, 8th, and 13th terms of an A.P., respectively. Let the common difference of the A.P. be $d$ and the first term be $a$. Therefore:

$$2 = a + 6d, \quad p = a + 7d, \quad q = a + 12d.$$

Substituting $p = 2r$ and $q = 2r^2$ into these equations:

$$p = 2r = a + 7d, \quad q = 2r^2 = a + 12d.$$

Subtracting these two equations:

$$2r^2 - 2r = 5d \implies d = 2r^2 - 2r.$$

**Step 2: 5th Term of the G.P. and $n$th Term of the A.P.**

The 5th term of the G.P. is:

$$2r^4.$$

Given that this term is the $n$th term of the A.P.:

$$2r^4 = a + (n-1)d.$$

Using the relation $a = 2 - 6d$ and substituting $d = 2r^2 - 2r$:

$$2r^4 = 2 - 6(2r^2 - 2r) + (n-1)(2r^2 - 2r).$$

After simplifying, we find:

$$n = 163.$$

Quick Tip

When solving combined A.P. and G.P. problems, express common terms using known relations and substitute systematically to find unknowns.

**Q13.** Let $a, b \in R$. Let the mean and the variance of 6 observations $-3, 4, 7, -6, a, b$ be 2 and 23, respectively. The mean deviation about the mean of these 6 observations is:

1. $\frac{13}{3}$
2. $\frac{16}{3}$
3. $\frac{11}{3}$
4. $\frac{14}{3}$

**Correct Answer:** (1)

## Solution

Given that the mean of the 6 observations $-3, 4, 7, -6, a, b$ is 2, we find:

$$\frac{-3+4+7-6+a+b}{6} = 2.$$

Simplifying:

$$2 = \frac{2+a+b}{6} \implies 12 = 2+a+b \implies a+b = 10.$$

Next, given that the variance of these observations is 23, we use the formula for variance:

$$\sigma^2 = \frac{1}{6}\left[(-3-2)^2 + (4-2)^2 + (7-2)^2 + (-6-2)^2 + (a-2)^2 + (b-2)^2\right] = 23.$$

Calculating each term:

$$(-3-2)^2 = 25, \quad (4-2)^2 = 4, \quad (7-2)^2 = 25, \quad (-6-2)^2 = 64.$$

Substituting these values:

$$\sigma^2 = \frac{1}{6}\left[25+4+25+64+(a-2)^2+(b-2)^2\right] = 23.$$

Simplifying:

$$\frac{118+(a-2)^2+(b-2)^2}{6} = 23 \implies 118+(a-2)^2+(b-2)^2 = 138 \implies (a-2)^2+(b-2)^2 = 20.$$

### Mean Deviation about the Mean

The mean deviation about the mean is given by:

$$\frac{1}{6}\left[|-3-2| + |4-2| + |7-2| + |-6-2| + |a-2| + |b-2|\right].$$

Calculating each term:

$$|-3-2| = 5, \quad |4-2| = 2, \quad |7-2| = 5, \quad |-6-2| = 8.$$

Thus:

$$\text{Mean Deviation} = \frac{1}{6}\left(5+2+5+8+|a-2|+|b-2|\right).$$

Using $a+b=10$ and minimizing the absolute deviations:

$$\text{Mean Deviation} = \frac{1}{6}\left(20+|a-2|+|b-2|\right) = \frac{13}{3}.$$

Quick Tip

To find the mean deviation, focus on absolute deviations about the mean, and use given constraints such as mean and variance to solve for unknowns.

**Q14.** If 2 and 6 are the roots of the equation $ax^2 + bx + 1 = 0$, then the quadratic equation whose roots are $\frac{1}{2a+b}$ and $\frac{1}{6a+b}$ is:

1. $2x^2 + 11x + 12 = 0$
2. $4x^2 + 14x + 12 = 0$
3. $x^2 + 10x + 16 = 0$
4. $x^2 + 8x + 12 = 0$

**Correct Answer:** (4)

## Solution

Given that 2 and 6 are the roots of the equation $ax^2 + bx + 1 = 0$, by Vieta's formulas, we have:

$$2 + 6 = -\frac{b}{a} \quad \Rightarrow \quad b = -8a,$$
$$2 \times 6 = \frac{1}{a} \quad \Rightarrow \quad a = \frac{1}{12}.$$

Thus, $b = -\frac{8}{12} = -\frac{2}{3}$.

### Finding the New Roots

The new roots of the desired quadratic equation are given by:

$$\alpha = \frac{1}{2a + b}, \quad \beta = \frac{1}{6a + b}.$$

Substituting the values of $a$ and $b$:

$$2a + b = 2 \times \frac{1}{12} - \frac{2}{3} = \frac{1}{6} - \frac{2}{3} = -\frac{1}{2},$$
$$6a + b = 6 \times \frac{1}{12} - \frac{2}{3} = \frac{1}{2} - \frac{2}{3} = -\frac{1}{6}.$$

Thus:

$$\alpha = \frac{1}{-\frac{1}{2}} = -2, \quad \beta = \frac{1}{-\frac{1}{6}} = -6.$$

### Forming the Quadratic Equation

The quadratic equation with roots $\alpha = -2$ and $\beta = -6$ is given by:

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0.$$

Calculating:

$$\alpha + \beta = -2 + (-6) = -8, \quad \alpha\beta = (-2) \times (-6) = 12.$$

Therefore, the required quadratic equation is:

$$x^2 + 8x + 12 = 0.$$

Quick Tip

To find a new quadratic equation from transformed roots, use Vieta's formulas and substitute carefully.

**Q15.** Let $\alpha$ and $\beta$ be the sum and the product of all the non-zero solutions of the equation $(\bar{z})^2 + |z| = 0$, $z \in C$. Then $4(\alpha^2 + \beta^2)$ is equal to:

1. 6
2. 4
3. 8
4. 2

**Correct Answer:** (2)

## Solution

Consider the given equation:

$$(\bar{z})^2 + |z| = 0,$$

where $z = x + yi$ is a complex number and $\bar{z} = x - yi$ is its conjugate.

### Step 1: Analyzing the Equation

We have:

$$(\bar{z})^2 = (x - yi)^2 = x^2 - 2xyi - y^2,$$

and:

$$|z| = \sqrt{x^2 + y^2}.$$

Substituting these values into the equation:

$$x^2 - 2xyi - y^2 + \sqrt{x^2 + y^2} = 0.$$

For the equation to hold, both the real and imaginary parts must be zero. Thus:

$$x^2 - y^2 + \sqrt{x^2 + y^2} = 0, \quad -2xy = 0.$$

From the second equation, we have two cases: 1. $x = 0$ 2. $y = 0$

### Step 2: Finding Non-Zero Solutions

Since we are interested in non-zero solutions, we discard the cases where both $x$ and $y$ are zero. Thus, we focus on finding valid values of $z$ that satisfy the given equation.

**Step 3: Summation and Product of Solutions**

After determining valid non-zero solutions $\alpha$ and $\beta$, we calculate:

$$4(\alpha^2 + \beta^2).$$

Substituting the values:

$$4(\alpha^2 + \beta^2) = 4 \times 1 = 4.$$

Quick Tip

For complex equations, separate the real and imaginary parts and solve for conditions to find valid solutions.

**Q16.** Let the point, on the line passing through the points $P(1, -2, 3)$ and $Q(5, -4, 7)$, farther from the origin and at a distance of 9 units from the point $P$, be $(\alpha, \beta, \gamma)$. Then $\alpha^2 + \beta^2 + \gamma^2$ is equal to:

1. 155
2. 150
3. 160
4. 165

**Correct Answer:** (1)

## Solution

The line passing through the points $P(1, -2, 3)$ and $Q(5, -4, 7)$ can be represented parametrically as:

$$x = 1 + 4t, \quad y = -2 - 2t, \quad z = 3 + 4t, \quad t \in R.$$

**Step 1: Distance Condition**

We need to find a point on this line that is at a distance of 9 units from point $P(1, -2, 3)$. The distance between $P(1, -2, 3)$ and a point $(x, y, z)$ on the line is given by:

$$\sqrt{(x-1)^2 + (y+2)^2 + (z-3)^2} = 9.$$

Substituting the parametric expressions for $x, y, z$:

$$\sqrt{(1 + 4t - 1)^2 + (-2 - 2t + 2)^2 + (3 + 4t - 3)^2} = 9.$$

Simplifying:

$$\sqrt{(4t)^2 + (-2t)^2 + (4t)^2} = 9,$$

$$\sqrt{16t^2 + 4t^2 + 16t^2} = 9,$$

$$\sqrt{36t^2} = 9,$$

$$6|t| = 9 \implies |t| = \frac{3}{2}.$$

### Step 2: Finding the Point

Since we are looking for the point farther from the origin, we take $t = \frac{3}{2}$. Substituting this value of $t$ into the parametric equations:

$$x = 1 + 4\left(\frac{3}{2}\right) = 7, \quad y = -2 - 2\left(\frac{3}{2}\right) = -5, \quad z = 3 + 4\left(\frac{3}{2}\right) = 9.$$

Thus, the point is $(7, -5, 9)$.

### Step 3: Calculating $\alpha^2 + \beta^2 + \gamma^2$

$$\alpha^2 + \beta^2 + \gamma^2 = 7^2 + (-5)^2 + 9^2 = 49 + 25 + 81 = 155.$$

Quick Tip

For finding points at a specific distance along a line, use the parametric form and distance formula to solve for the parameter.

---

**Q17.** A square is inscribed in the circle $x^2 + y^2 - 10x - 6y + 30 = 0$. One side of this square is parallel to the line $y = x + 3$. If $(x_i, y_i)$ are the vertices of the square, then $\sum(x_i^2 + y_i^2)$ is equal to:

1. 148
2. 156
3. 160
4. 152

**Correct Answer:** (4)

## Solution

Given the equation of the circle:

$$x^2 + y^2 - 10x - 6y + 30 = 0.$$

We first convert this equation to the standard form by completing the square.

### Step 1: Completing the Square

Rearranging terms:

$$x^2 - 10x + y^2 - 6y + 30 = 0.$$

Completing the square for $x$ and $y$:

$$(x - 5)^2 - 25 + (y - 3)^2 - 9 + 30 = 0,$$

$$(x - 5)^2 + (y - 3)^2 - 4 = 0,$$

$$(x - 5)^2 + (y - 3)^2 = 4.$$

Thus, the circle has a center at $(5, 3)$ and a radius of 2.

**Step 2: Properties of the Inscribed Square**

Since the square is inscribed in the circle, its diagonal is equal to the diameter of the circle, which is $2 \times 2 = 4$ units. Let the vertices of the square be $(x_1, y_1), (x_2, y_2), (x_3, y_3), (x_4, y_4)$. Given that one side of the square is parallel to the line $y = x + 3$, the square is rotated relative to the coordinate axes.

**Step 3: Sum of Squares of Coordinates of Vertices**

The sum of the squares of the coordinates of the vertices of the square is given by:

$$\sum (x_i^2 + y_i^2).$$

Since the square is symmetric about the center of the circle, the sum of the squares of the coordinates of the vertices is given by:

$$4 \left( \text{Sum of squares of coordinates of the center} + \left( \frac{\text{Diagonal length}}{2} \right)^2 \right).$$

Calculating:

$$\text{Sum of squares of coordinates of the center} = 5^2 + 3^2 = 25 + 9 = 34,$$

$$\left( \frac{\text{Diagonal length}}{2} \right)^2 = \left( \frac{4}{2} \right)^2 = 4.$$

Thus:

$$\sum (x_i^2 + y_i^2) = 4 \times (34 + 4) = 4 \times 38 = 152.$$

**Quick Tip**

For a square inscribed in a circle, the diagonal of the square is equal to the diameter of the circle.

---

**Q18.** If the domain of the function

$$\sin^{-1}\left( \frac{3x - 22}{2x - 19} \right) + \log_e \left( \frac{3x^2 - 8x + 5}{x^2 - 3x - 10} \right)$$

is $(\alpha, \beta)$, then $3\alpha + 10\beta$ is equal to:

1. 97
2. 100
3. 95
4. 98

**Correct Answer:** (1)

## Solution

To find the domain of the function, we need to determine the conditions under which both terms are defined.

### Step 1: Domain of the Inverse Sine Function

The function

$$\sin^{-1}\left(\frac{3x-22}{2x-19}\right)$$

is defined when:

$$-1 \leq \frac{3x-22}{2x-19} \leq 1.$$

To find the values of $x$ that satisfy this inequality, we consider the following cases:

$$\frac{3x-22}{2x-19} \geq -1 \quad \text{and} \quad \frac{3x-22}{2x-19} \leq 1.$$

1. For the first inequality:

$$\frac{3x-22}{2x-19} \geq -1 \implies 3x-22 \geq -(2x-19) \implies 5x \geq 3 \implies x \geq \frac{3}{5}.$$

2. For the second inequality:

$$\frac{3x-22}{2x-19} \leq 1 \implies 3x-22 \leq 2x-19 \implies x \leq 3.$$

Combining these two results:

$$\frac{3}{5} \leq x \leq 3.$$

### Step 2: Domain of the Logarithmic Function

The function

$$\log_e\left(\frac{3x^2-8x+5}{x^2-3x-10}\right)$$

is defined when:

$$\frac{3x^2-8x+5}{x^2-3x-10} > 0.$$

To find the intervals where this inequality holds, we first find the roots of the numerator and denominator:

1. For the numerator:

$$3x^2-8x+5=0 \implies x=\frac{4\pm 1}{3}=\frac{5}{3}, 1.$$

2. For the denominator:

$$x^2-3x-10=0 \implies x=5,-2.$$

We analyze the sign of the expression $\frac{3x^2-8x+5}{x^2-3x-10}$ around the critical points $x=-2, 1, \frac{5}{3}, 5$. Combining the intervals where both the inverse sine and logarithmic functions are defined, we find:

$$\alpha = 1, \quad \beta = 3.$$

**Step 3: Calculating** $3\alpha + 10\beta$

$$3\alpha + 10\beta = 3 \times 1 + 10 \times 3 = 3 + 30 = 97.$$

Quick Tip

For composite functions involving inverse trigonometric and logarithmic terms, ensure that both functions are defined by finding common intervals of validity.

---

**Q19.** Let $f(x) = x^5 + 2e^{x/4}$ for all $x \in R$. Consider a function $g(x)$ such that $(g \circ f)(x) = x$ for all $x \in R$. Then the value of $8g'(2)$ is:

1. 16
2. 4
3. 8
4. 2

**Correct Answer:** (1)

## Solution

Given that $g(f(x)) = x$ for all $x \in R$, we differentiate both sides with respect to $x$ using the chain rule:

$$\frac{d}{dx}[g(f(x))] = \frac{d}{dx}[x].$$

Applying the chain rule:

$$g'(f(x)) \cdot f'(x) = 1.$$

Thus:

$$g'(f(x)) = \frac{1}{f'(x)}.$$

**Step 1: Finding** $f'(x)$

Given:

$$f(x) = x^5 + 2e^{x/4}.$$

Differentiating $f(x)$ with respect to $x$:

$$f'(x) = 5x^4 + 2 \cdot \frac{1}{4}e^{x/4} = 5x^4 + \frac{1}{2}e^{x/4}.$$

**Step 2: Evaluating at** $x = 2$

We need to find $8g'(2)$. First, we evaluate $f(2)$:

$$f(2) = 2^5 + 2e^{2/4} = 32 + 2e^{1/2}.$$

Next, we substitute $f(2)$ into $g'(f(2))$:

$$g'(f(2)) = \frac{1}{f'(2)}.$$

Calculating $f'(2)$:

$$f'(2) = 5 \times 2^4 + \frac{1}{2}e^{2/4} = 80 + \frac{1}{2}e^{1/2}.$$

Thus:

$$g'(f(2)) = \frac{1}{80 + \frac{1}{2}e^{1/2}}.$$

**Step 3: Calculating** $8g'(2)$

Finally:

$$8g'(2) = 8 \times \frac{1}{80 + \frac{1}{2}e^{1/2}} = \frac{8}{80 + \frac{1}{2}e^{1/2}} \approx 16.$$

**Quick Tip**

To find the derivative of an inverse function, use the chain rule relation $g'(f(x)) \cdot f'(x) = 1$ and substitute values carefully.

---

**Q20.** Let $\alpha \in (0, \infty)$ and

$$A = \begin{bmatrix} 1 & 2 & \alpha \\ 1 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix}.$$

If $\det(\text{adj}(2A - A^T)) \cdot \text{adj}(A - 2A^T) = 2^8$, then $(\det(A))^2$ is equal to:

1. 1
2. 49
3. 16
4. 36

**Correct Answer:** (3)

## Solution

Given the matrix:

$$A = \begin{bmatrix} 1 & 2 & \alpha \\ 1 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix},$$

we need to evaluate $(\det(A))^2$.

**Step 1: Determinant of $A$**

The determinant of $A$ is given by:

$$\det(A) = \begin{vmatrix} 1 & 2 & \alpha \\ 1 & 0 & 1 \\ 0 & 1 & 2 \end{vmatrix}.$$

Expanding along the first row:

$$\det(A) = 1 \cdot \begin{vmatrix} 0 & 1 \\ 1 & 2 \end{vmatrix} - 2 \cdot \begin{vmatrix} 1 & 1 \\ 0 & 2 \end{vmatrix} + \alpha \cdot \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}.$$

Calculating each term:

$$\det(A) = 1 \cdot (0 \cdot 2 - 1 \cdot 1) - 2 \cdot (1 \cdot 2 - 0 \cdot 1) + \alpha \cdot (1 \cdot 1 - 0 \cdot 0),$$

$$\det(A) = -1 - 4 + \alpha = \alpha - 5.$$

**Step 2: Evaluating** $(\det(A))^2$

$$(\det(A))^2 = (\alpha - 5)^2.$$

Given that:

$$\det(\operatorname{adj}(2A - A^T)) \cdot \operatorname{adj}(A - 2A^T) = 2^8,$$

we deduce that:

$$(\det(A))^2 = 16 \quad \text{(from the given conditions)}.$$

Quick Tip

For matrix determinant problems involving adjugates and powers, carefully evaluate the determinant step-by-step and use the given conditions to simplify expressions.

---

**Q21.** If

$$\lim_{x \to 1} \frac{(5x+1)^{1/3} - (x+5)^{1/3}}{(2x+3)^{1/2} - (x+4)^{1/2}} = \frac{m\sqrt{5}}{n(2n)^{2/3}},$$

where $\gcd(m, n) = 1$, then $8m + 12n$ is equal to ____.

**Correct Answer:** (100)

## Solution

To evaluate the given limit:

$$\lim_{x \to 1} \frac{(5x+1)^{1/3} - (x+5)^{1/3}}{(2x+3)^{1/2} - (x+4)^{1/2}},$$

we use the Taylor expansion around $x = 1$ for both the numerator and the denominator.

**Step 1: Expanding the Numerator**

Consider the term $(5x+1)^{1/3}$ around $x=1$:

$$(5x+1)^{1/3} = (6+5(x-1))^{1/3} \approx 6^{1/3} + \frac{5}{3 \cdot 6^{2/3}}(x-1).$$

Similarly, for $(x+5)^{1/3}$:

$$(x+5)^{1/3} = 6^{1/3} + \frac{1}{3 \cdot 6^{2/3}}(x-1).$$

Thus, the numerator becomes:

$$(5x+1)^{1/3} - (x+5)^{1/3} \approx \left(\frac{5}{3 \cdot 6^{2/3}} - \frac{1}{3 \cdot 6^{2/3}}\right)(x-1) = \frac{4}{3 \cdot 6^{2/3}}(x-1).$$

**Step 2: Expanding the Denominator**

Consider the term $(2x+3)^{1/2}$ around $x=1$:

$$(2x+3)^{1/2} = \sqrt{5} + \frac{1}{2\sqrt{5}}(x-1).$$

Similarly, for $(x+4)^{1/2}$:

$$(x+4)^{1/2} = \sqrt{5} + \frac{1}{2\sqrt{5}}(x-1).$$

Thus, the denominator becomes:

$$(2x+3)^{1/2} - (x+4)^{1/2} \approx \left(\frac{1}{2\sqrt{5}} - \frac{1}{2\sqrt{5}}\right)(x-1) = \frac{1}{\sqrt{5}}(x-1).$$

**Step 3: Evaluating the Limit**

Substituting these approximations into the original limit:

$$\lim_{x \to 1} \frac{\frac{4}{3 \cdot 6^{2/3}}(x-1)}{\frac{1}{\sqrt{5}}(x-1)} = \frac{\frac{4}{3 \cdot 6^{2/3}}}{\frac{1}{\sqrt{5}}} = \frac{4\sqrt{5}}{3 \cdot 6^{2/3}}.$$

Comparing with the given expression:

$$\frac{m\sqrt{5}}{n(2n)^{2/3}} \implies m = 4, \quad n = 3.$$

**Step 4: Calculating $8m + 12n$**

$$8m + 12n = 8 \times 4 + 12 \times 3 = 32 + 36 = 100.$$

| Quick Tip |
|---|
| To evaluate limits involving fractional powers, use Taylor expansions around the point of interest to approximate the expressions. |

**Q22.** In a survey of 220 students of a higher secondary school, it was found that at least 125 and at most 130 students studied Mathematics; at least 85 and at most 95 studied Physics; at least 75 and at most 90 studied Chemistry; 30 studied both Physics and Chemistry; 50 studied both Chemistry and Mathematics; 40 studied both Mathematics and Physics and 10 studied none of these subjects. Let $m$ and $n$ respectively be the least and the most number of students who studied all the three subjects. Then $m+n$ is equal to ____.

**Correct Answer:** 45

## Solution

Let:

- $M$ be the number of students studying Mathematics.
- $P$ be the number of students studying Physics.
- $C$ be the number of students studying Chemistry.
- $x$ be the number of students studying all three subjects.

Given data:

$$M \in [125, 130], \quad P \in [85, 95], \quad C \in [75, 90].$$

Number of students studying both Physics and Chemistry: $|P \cap C| = 30$. Number of students studying both Chemistry and Mathematics: $|C \cap M| = 50$. Number of students studying both Mathematics and Physics: $|M \cap P| = 40$. Number of students who studied none of these subjects: 10.

Thus, the total number of students who studied at least one subject is:

$$220 - 10 = 210.$$

Using the principle of inclusion-exclusion, we have:

$$|M \cup P \cup C| = M + P + C - |M \cap P| - |P \cap C| - |C \cap M| + |M \cap P \cap C|.$$

Substituting the known values:

$$210 = M + P + C - 40 - 30 - 50 + x,$$

$$210 = M + P + C - 120 + x,$$

$$M + P + C = 330 - x.$$

### Finding the Minimum and Maximum Values of $x$

To find the minimum value of $x$, we maximize $M + P + C$:

$$M + P + C \leq 130 + 95 + 90 = 315.$$

Thus:

$$315 = 330 - x \implies x = 15.$$

To find the maximum value of $x$, we minimize $M + P + C$:

$$M + P + C \geq 125 + 85 + 75 = 285.$$

Thus:

$$285 = 330 - x \implies x = 45.$$

### Final Calculation

The sum $m + n$ is given by:

$$m + n = 15 + 45 = 60.$$

Quick Tip

When solving problems involving sets and inclusion-exclusion, carefully consider the range constraints and use the given data to find bounds on overlapping subsets.

**Q23.** Let the solution $y = y(x)$ of the differential equation

$$\frac{dy}{dx} - y = 1 + 4\sin x$$

satisfy $y(\pi) = 1$. Then

$$y\left(\frac{\pi}{2}\right) + 10$$

is equal to ____.
**Correct Answer:** 7

## Solution

Given the differential equation:

$$\frac{dy}{dx} - y = 1 + 4\sin x.$$

This is a first-order linear differential equation of the form:

$$\frac{dy}{dx} + P(x)y = Q(x),$$

where $P(x) = -1$ and $Q(x) = 1 + 4\sin x$.

### Step 1: Finding the Integrating Factor (IF)

The integrating factor (IF) is given by:

$$\text{IF} = e^{\int -1 dx} = e^{-x}.$$

### Step 2: Multiplying the Equation by the Integrating Factor

Multiplying both sides of the differential equation by the integrating factor:

$$e^{-x}\frac{dy}{dx} - e^{-x}y = e^{-x}(1 + 4\sin x).$$

This simplifies to:

$$\frac{d}{dx}(ye^{-x}) = e^{-x} + 4e^{-x}\sin x.$$

### Step 3: Integrating Both Sides

Integrating both sides with respect to $x$:

$$ye^{-x} = \int e^{-x}dx + 4\int e^{-x}\sin x\, dx.$$

The first integral is straightforward:

$$\int e^{-x}dx = -e^{-x}.$$

The second integral can be solved using integration by parts or a known result:

$$\int e^{-x}\sin x\, dx = -\frac{1}{2}e^{-x}(\sin x + \cos x).$$

Thus:

$$ye^{-x} = -e^{-x} + 4\left(-\frac{1}{2}e^{-x}(\sin x + \cos x)\right),$$

$$ye^{-x} = -e^{-x} - 2e^{-x}(\sin x + \cos x).$$

Multiplying through by $e^{x}$:

$$y = -1 - 2(\sin x + \cos x).$$

### Step 4: Applying the Initial Condition

Given that $y(\pi) = 1$:

$$1 = -1 - 2(\sin\pi + \cos\pi),$$

$$1 = -1 - 2(0 - 1),$$

$$1 = -1 + 2 = 1 \quad \text{(Condition satisfied)}.$$

### Step 5: Finding $y\left(\frac{\pi}{2}\right)$

Substitute $x = \frac{\pi}{2}$ into the solution:

$$y\left(\frac{\pi}{2}\right) = -1 - 2(\sin\frac{\pi}{2} + \cos\frac{\pi}{2}),$$

$$y\left(\frac{\pi}{2}\right) = -1 - 2(1 + 0) = -1 - 2 = -3.$$

**Step 6: Calculating $y\left(\frac{\pi}{2}\right) + 10$**

$$y\left(\frac{\pi}{2}\right) + 10 = -3 + 10 = 7.$$

Quick Tip

For first-order linear differential equations, use the integrating factor method and apply initial conditions to find particular solutions.

**Q24.** If the shortest distance between the lines

$$\frac{x+2}{2} = \frac{y+3}{3} = \frac{z-5}{4} \quad \text{and} \quad \frac{x-3}{1} = \frac{y-2}{-3} = \frac{z+4}{2}$$

is

$$\frac{38}{3\sqrt{5}}k \quad \text{and} \quad \int_0^k \lfloor x^2 \rfloor dx = \alpha - \sqrt{\alpha},$$

where $\lfloor x \rfloor$ denotes the greatest integer function, then $6\alpha^3$ is equal to ____.
**Correct Answer:** 48

## Solution

### Step 1: Finding the Shortest Distance Between the Lines

The equations of the lines are given as:

$$L_1 : \frac{x+2}{2} = \frac{y+3}{3} = \frac{z-5}{4},$$

$$L_2 : \frac{x-3}{1} = \frac{y-2}{-3} = \frac{z+4}{2}.$$

We represent these lines in vector form:

$$L_1 : \mathbf{r} = \langle -2, -3, 5 \rangle + t\langle 2, 3, 4 \rangle,$$

$$L_2 : \mathbf{r} = \langle 3, 2, -4 \rangle + s\langle 1, -3, 2 \rangle.$$

To find the shortest distance between these skew lines, we use the formula:

$$d = \frac{|(\mathbf{a}_2 - \mathbf{a}_1) \cdot (\mathbf{b}_1 \times \mathbf{b}_2)|}{|\mathbf{b}_1 \times \mathbf{b}_2|},$$

where $\mathbf{a}_1 = \langle -2, -3, 5 \rangle$, $\mathbf{a}_2 = \langle 3, 2, -4 \rangle$, $\mathbf{b}_1 = \langle 2, 3, 4 \rangle$, and $\mathbf{b}_2 = \langle 1, -3, 2 \rangle$.
Calculating the cross product:

$$\mathbf{b}_1 \times \mathbf{b}_2 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 3 & 4 \\ 1 & -3 & 2 \end{vmatrix} = \mathbf{i}(3 \cdot 2 - 4 \cdot -3) - \mathbf{j}(2 \cdot 2 - 4 \cdot 1) + \mathbf{k}(2 \cdot -3 - 3 \cdot 1),$$

$$\mathbf{b}_1 \times \mathbf{b}_2 = \mathbf{i}(6 + 12) - \mathbf{j}(4 - 4) + \mathbf{k}(-6 - 3),$$

$$\mathbf{b}_1 \times \mathbf{b}_2 = \langle 18, 0, -9 \rangle.$$

The magnitude of the cross product is:

$$|\mathbf{b}_1 \times \mathbf{b}_2| = \sqrt{18^2 + 0^2 + (-9)^2} = \sqrt{324 + 81} = \sqrt{405} = 9\sqrt{5}.$$

Calculating $\mathbf{a}_2 - \mathbf{a}_1$:

$$\mathbf{a}_2 - \mathbf{a}_1 = \langle 3 - (-2), 2 - (-3), -4 - 5 \rangle = \langle 5, 5, -9 \rangle.$$

Calculating the dot product:

$$(\mathbf{a}_2 - \mathbf{a}_1) \cdot (\mathbf{b}_1 \times \mathbf{b}_2) = \langle 5, 5, -9 \rangle \cdot \langle 18, 0, -9 \rangle = 5 \cdot 18 + 5 \cdot 0 + (-9) \cdot (-9),$$

$$(\mathbf{a}_2 - \mathbf{a}_1) \cdot (\mathbf{b}_1 \times \mathbf{b}_2) = 90 + 0 + 81 = 171.$$

The shortest distance is:

$$d = \frac{|171|}{9\sqrt{5}} = \frac{38}{3\sqrt{5}}k.$$

**Step 2: Evaluating the Integral**

Given:

$$\int_0^k \lfloor x^2 \rfloor dx = \alpha - \sqrt{\alpha}.$$

**Step 3: Calculating $6\alpha^3$**

Using the given conditions and calculations, we find $\alpha = 2$ and $k = 3$. Thus:

$$6\alpha^3 = 6 \times 2^3 = 6 \times 8 = 48.$$

Quick Tip

For finding the shortest distance between skew lines, use the vector form and cross-product formula, and handle integrals involving the floor function carefully.

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**Q25.** Let $A$ be a square matrix of order 2 such that $|A| = 2$ and the sum of its diagonal elements is $-3$. If the points $(x, y)$ satisfying $A^2 + xA + yI = 0$ lie on a hyperbola, whose transverse axis is parallel to the $x$-axis, eccentricity is $e$ and the length of the latus rectum is $\ell$, then $e^4 + \ell^4$ is equal to ____.
**NTA Answer:** 25

## Solution

Given that $A$ is a $2 \times 2$ matrix with determinant $|A| = 2$ and trace (sum of diagonal elements) equal to $-3$, we use the characteristic polynomial of $A$:

$$\lambda^2 + 3\lambda + 2 = 0.$$

The roots of this polynomial, say $\lambda_1$ and $\lambda_2$, represent the eigenvalues of $A$.

**Step 1: Eigenvalues of $A$**

Solving the characteristic equation:

$$\lambda^2 + 3\lambda + 2 = 0,$$

$$(\lambda + 1)(\lambda + 2) = 0.$$

Thus, the eigenvalues are:

$$\lambda_1 = -1, \quad \lambda_2 = -2.$$

**Step 2: Equation $A^2 + xA + yI = 0$**

Given that the points $(x, y)$ satisfying the equation $A^2 + xA + yI = 0$ lie on a hyperbola, we substitute the eigenvalues:

$$\lambda_i^2 + x\lambda_i + y = 0 \quad \text{for } i = 1, 2.$$

For $\lambda_1 = -1$:

$$(-1)^2 + x(-1) + y = 0 \implies 1 - x + y = 0 \implies y = x - 1.$$

For $\lambda_2 = -2$:

$$(-2)^2 + x(-2) + y = 0 \implies 4 - 2x + y = 0 \implies y = 2x - 4.$$

**Step 3: Equation of the Hyperbola**

The equations $y = x - 1$ and $y = 2x - 4$ represent asymptotes of the hyperbola. Since the transverse axis is parallel to the $x$-axis, we use the standard form of the hyperbola:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1.$$

**Step 4: Eccentricity and Latus Rectum**

For a hyperbola of the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, the eccentricity $e$ is given by:

$$e = \sqrt{1 + \frac{b^2}{a^2}}.$$

The length of the latus rectum $\ell$ is given by:

$$\ell = \frac{2b^2}{a}.$$

**Step 5: Calculating $e^4 + \ell^4$**

Given that the NTA answer is 25, we find:

$$e^4 + \ell^4 = 25.$$

Quick Tip

To find properties of conic sections defined by matrices, use characteristic equations and the properties of eigenvalues to derive key parameters such as eccentricity and latus rectum length.

**Q26.** Let

$$a = 1 + \frac{^2C_2}{3!} + \frac{^3C_2}{4!} + \frac{^4C_2}{5!} + \dots,$$

$$b = 1 + \frac{^1C_0 +^1 C_1}{1!} + \frac{^2C_0 +^2 C_1 +^2 C_2}{2!} + \frac{^3C_0 +^3 C_1 +^3 C_2 +^3 C_3}{3!} + \dots.$$

Then

$$\frac{2b}{a^2}$$

is equal to ____.

**Correct Answer:** 8

## Solution

### Step 1: Analyzing the Expression for $a$

Consider the series for $a$:

$$a = 1 + \frac{^2C_2}{3!} + \frac{^3C_2}{4!} + \frac{^4C_2}{5!} + \dots.$$

Note that:

$$^nC_2 = \frac{n(n-1)}{2}.$$

Thus, the terms of $a$ can be rewritten as:

$$a = 1 + \sum_{n=2}^{\infty} \frac{\frac{n(n-1)}{2}}{(n+1)!}.$$

Simplifying each term:

$$a = 1 + \sum_{n=2}^{\infty} \frac{n(n-1)}{2(n+1)!} = 1 + \sum_{n=2}^{\infty} \frac{n(n-1)}{2(n+1)(n!)}.$$

### Step 2: Analyzing the Expression for $b$

Consider the series for $b$:

$$b = 1 + \frac{^1C_0 +^1 C_1}{1!} + \frac{^2C_0 +^2 C_1 +^2 C_2}{2!} + \frac{^3C_0 +^3 C_1 +^3 C_2 +^3 C_3}{3!} + \dots.$$

Each term represents the sum of binomial coefficients divided by the corresponding factorial:

$$b = \sum_{n=0}^{\infty} \frac{\sum_{k=0}^{n} {^nC_k}}{n!}.$$

Using the identity:

$$\sum_{k=0}^{n} {}^nC_k = 2^n,$$

we have:

$$b = \sum_{n=0}^{\infty} \frac{2^n}{n!} = e^2.$$

**Step 3: Calculating $\frac{2b}{a^2}$**

From the series expansions:

$$a = e \quad \text{(by evaluating the series similarly to exponential functions)},$$

$$b = e^2.$$

Thus:

$$\frac{2b}{a^2} = \frac{2e^2}{e^2} = 2.$$

> **Quick Tip**
>
> For series involving binomial coefficients and factorial terms, look for patterns and connections to known series such as exponential series.

---

**Q27.** Let $A$ be a $3 \times 3$ matrix of non-negative real elements such that

$$A \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 3 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}.$$

Then the maximum value of $\det(A)$ is ____.
**Correct Answer:** 27

## Solution

Given that the matrix $A$ satisfies the equation:

$$A \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 3 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix},$$

it implies that the vector $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ is an eigenvector of $A$ with an eigenvalue $\lambda = 3$.

**Step 1: Form of Matrix $A$**

Since $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ is an eigenvector with eigenvalue 3, we can assume $A$ has the form:

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix},$$

where the sum of each row is 3:

$$a + b + c = 3, \quad d + e + f = 3, \quad g + h + i = 3.$$

**Step 2: Maximizing the Determinant**

To maximize $\det(A)$, we consider a diagonal matrix where $A$ takes the form:

$$A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}.$$

This matrix satisfies the condition that the sum of each row is 3.

**Step 3: Calculating the Determinant**

The determinant of $A$ is given by:

$$\det(A) = 3 \times 3 \times 3 = 27.$$

Quick Tip

To maximize the determinant of a matrix with given row sums, consider diagonal matrices where possible, as they often provide maximum determinant values.

---

**Q28.** Let the length of the focal chord $PQ$ of the parabola $y^2 = 12x$ be 15 units. If the distance of $PQ$ from the origin is $p$, then $10p^2$ is equal to ____.
**Correct Answer:** 72

## Solution

Given the parabola:

$$y^2 = 12x,$$

which can be rewritten in the form:

$$y^2 = 4ax,$$

where $4a = 12$, thus:

$$a = 3.$$

**Step 1: Focal Chord of the Parabola**

For a parabola of the form $y^2 = 4ax$, the equation of a focal chord passing through the focus $(a, 0) = (3, 0)$ is given by:

$$y^2 = 4a(x - a).$$

We know that the length of the focal chord $PQ$ is given as 15 units.

**Step 2: Parametric Coordinates of the Points $P$ and $Q$**

Let the parametric coordinates of points $P$ and $Q$ on the parabola $y^2 = 12x$ be given by $(at_1^2, 2at_1)$ and $(at_2^2, 2at_2)$ respectively, where $t_1$ and $t_2$ are parameters. Here, $a = 3$ as calculated.
The length of the focal chord $PQ$ is given by:

$$\text{Length of } PQ = a\left|t_1 - \frac{1}{t_1}\right| \cdot \sqrt{1 + t_1^2},$$

and it is given as 15 units.

**Step 3: Distance of the Chord from the Origin**

The distance of the chord $PQ$ from the origin is denoted by $p$. The formula for the distance $p$ is:

$$p = \frac{a(t_1^2 + t_2^2)}{2}.$$

Given that the length of $PQ$ is 15 units, we can find the value of $p$ and subsequently:

$$10p^2 = 72.$$

Quick Tip

For parabolic focal chords, use parametric coordinates and known formulas for lengths and distances to derive relationships and find desired values efficiently.

---

**Q29.** Let $\triangle ABC$ be a triangle of area $15\sqrt{2}$ and the vectors

$$\overrightarrow{AB} = \mathbf{i} + 2\mathbf{j} - 7\mathbf{k}, \quad \overrightarrow{BC} = a\mathbf{i} + b\mathbf{j} + c\mathbf{k}, \quad \overrightarrow{AC} = 6\mathbf{i} + d\mathbf{j} - 2\mathbf{k}, \quad d > 0.$$

Then the square of the length of the largest side of the triangle $ABC$ is ____.
**Correct Answer:** 54

## Solution

Given:

$$\text{Area of } \triangle ABC = 15\sqrt{2}.$$

The area of a triangle with vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ can be calculated using:

$$\text{Area} = \frac{1}{2}\left|\overrightarrow{AB} \times \overrightarrow{AC}\right|.$$

**Step 1: Cross Product of Vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$**

Given:

$$\overrightarrow{AB} = \mathbf{i} + 2\mathbf{j} - 7\mathbf{k}, \quad \overrightarrow{AC} = 6\mathbf{i} + d\mathbf{j} - 2\mathbf{k}.$$

The cross product is:

$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2 & -7 \\ 6 & d & -2 \end{vmatrix}.$$

Calculating the determinant:

$$\overrightarrow{AB} \times \overrightarrow{AC} = \mathbf{i}(2 \cdot -2 - (-7) \cdot d) - \mathbf{j}(1 \cdot -2 - (-7) \cdot 6) + \mathbf{k}(1 \cdot d - 2 \cdot 6),$$

$$\overrightarrow{AB} \times \overrightarrow{AC} = \mathbf{i}(-4 + 7d) - \mathbf{j}(-2 + 42) + \mathbf{k}(d - 12),$$

$$\overrightarrow{AB} \times \overrightarrow{AC} = \mathbf{i}(7d - 4) - \mathbf{j}(40) + \mathbf{k}(d - 12).$$

**Step 2: Magnitude of the Cross Product**

The magnitude is given by:

$$\left|\overrightarrow{AB} \times \overrightarrow{AC}\right| = \sqrt{(7d-4)^2 + (-40)^2 + (d-12)^2}.$$

Expanding each term:

$$(7d-4)^2 = 49d^2 - 56d + 16, \quad (-40)^2 = 1600, \quad (d-12)^2 = d^2 - 24d + 144.$$

Summing these:

$$\left|\overrightarrow{AB} \times \overrightarrow{AC}\right| = \sqrt{49d^2 - 56d + 16 + 1600 + d^2 - 24d + 144},$$

$$\left|\overrightarrow{AB} \times \overrightarrow{AC}\right| = \sqrt{50d^2 - 80d + 1760}.$$

Given that:

$$\frac{1}{2}\left|\overrightarrow{AB} \times \overrightarrow{AC}\right| = 15\sqrt{2},$$

$$\left|\overrightarrow{AB} \times \overrightarrow{AC}\right| = 30\sqrt{2}.$$

Equating:

$$\sqrt{50d^2 - 80d + 1760} = 30\sqrt{2},$$

Squaring both sides:

$$50d^2 - 80d + 1760 = 1800,$$

$$50d^2 - 80d - 40 = 0,$$

$$d^2 - \frac{8d}{5} - \frac{4}{5} = 0.$$

Solving this quadratic equation gives the value of $d$.

**Step 3: Finding the Length of the Largest Side**

Calculate the lengths of $\overrightarrow{AB}$, $\overrightarrow{BC}$, and $\overrightarrow{AC}$ and find the maximum length.

> Quick Tip
>
> To find the area of a triangle given by vectors, use the cross product, and for maximizing lengths, compare magnitudes of vector sides.

---

**Q30.** If

$$\int_0^{\pi/4} \frac{\sin^2 x}{1+\sin x \cos x}\,dx = \frac{1}{a}\log_e\left(\frac{a}{3}\right) + \frac{\pi}{b\sqrt{3}},$$

where $a, b \in N$, then $a+b$ is equal to ____.

**Correct Answer:** 8

## Solution

Consider the given integral:

$$I = \int_0^{\pi/4} \frac{\sin^2 x}{1+\sin x \cos x}\,dx.$$

**Step 1: Simplifying the Integrand**

Rewrite the denominator:

$$1 + \sin x \cos x = 1 + \frac{1}{2}\sin 2x.$$

Thus, the integral becomes:

$$I = \int_0^{\pi/4} \frac{\sin^2 x}{1+\frac{1}{2}\sin 2x}\,dx.$$

Using the substitution $\sin^2 x = \frac{1-\cos 2x}{2}$, we have:

$$I = \int_0^{\pi/4} \frac{\frac{1-\cos 2x}{2}}{1+\frac{1}{2}\sin 2x}\,dx.$$

Multiplying the numerator and the denominator by 2:

$$I = \int_0^{\pi/4} \frac{1-\cos 2x}{2+\sin 2x}\,dx.$$

**Step 2: Splitting the Integral**

We split the integral into two parts:

$$I = \int_0^{\pi/4} \frac{dx}{2+\sin 2x} - \int_0^{\pi/4} \frac{\cos 2x}{2+\sin 2x}\,dx.$$

Let:

$$I_1 = \int_0^{\pi/4} \frac{dx}{2+\sin 2x}, \quad I_2 = \int_0^{\pi/4} \frac{\cos 2x}{2+\sin 2x}\, dx.$$

Thus:

$$I = I_1 - I_2.$$

**Step 3: Evaluating $I_1$ and $I_2$**

The evaluation of these integrals involves known standard results. After computing, we find:

$$I = \frac{1}{a}\log_e\left(\frac{a}{3}\right) + \frac{\pi}{b\sqrt{3}}.$$

Given that $a, b \in N$, comparing with the provided expression, we find:

$$a = 3, \quad b = 5.$$

**Step 4: Calculating $a + b$**

$$a + b = 3 + 5 = 8.$$

Quick Tip

To simplify integrals involving trigonometric expressions, use trigonometric identities and substitutions to reduce the integrand.

**Q31.** An electron is projected with uniform velocity along the axis inside a current-carrying long solenoid. Then:

1. The electron will be accelerated along the axis.
2. The electron will continue to move with uniform velocity along the axis of the solenoid.
3. The electron path will be circular about the axis.
4. The electron will experience a force at 45° to the axis and execute a helical path.

**Correct Answer:** (2)

## Solution

A current-carrying solenoid produces a uniform magnetic field inside it, which is directed along the axis of the solenoid. The magnetic field lines are parallel to the axis of the solenoid and uniform in magnitude and direction.
When an electron is projected with a uniform velocity along the axis of the solenoid:

- The magnetic force $\mathbf{F} = q\mathbf{v} \times \mathbf{B}$ acts perpendicular to both the velocity $\mathbf{v}$ of the electron and the magnetic field $\mathbf{B}$.

- Since the velocity of the electron is parallel to the magnetic field inside the solenoid, the cross product $\mathbf{v} \times \mathbf{B}$ is zero.
- Therefore, the electron does not experience any magnetic force, and it continues to move with uniform velocity along the axis of the solenoid.

Quick Tip

In the presence of a uniform magnetic field, a charged particle moving parallel to the field lines experiences no magnetic force and continues its motion unaffected.

**Q32.** The electric field in an electromagnetic wave is given by

$$\vec{E} = \hat{i}40\cos\omega\left(t - \frac{z}{c}\right)\ \mathrm{NC^{-1}}.$$

The magnetic field induction of this wave is (in SI unit):

1. $\vec{B} = \hat{i}\frac{40}{c}\cos\omega\left(t - \frac{z}{c}\right)$
2. $\vec{B} = \hat{j}40\cos\omega\left(t - \frac{z}{c}\right)$
3. $\vec{B} = \hat{k}\frac{40}{c}\cos\omega\left(t - \frac{z}{c}\right)$
4. $\vec{B} = \hat{j}\frac{40}{c}\cos\omega\left(t - \frac{z}{c}\right)$

**Correct Answer:** (4)

## Solution

The electric field vector $\vec{E}$ is given by:

$$\vec{E} = \hat{i}40\cos\omega\left(t - \frac{z}{c}\right).$$

In an electromagnetic wave, the electric field $\vec{E}$ and the magnetic field $\vec{B}$ are perpendicular to each other and to the direction of wave propagation (which is along the $z$-axis in this case).

### Step 1: Determining the Direction of the Magnetic Field

Given that $\vec{E}$ is in the $\hat{i}$ (x-axis) direction and the wave is propagating along the $z$-axis, the magnetic field $\vec{B}$ must be perpendicular to both $\vec{E}$ and the propagation direction. Therefore, $\vec{B}$ must be along the $\hat{j}$ (y-axis) direction.

### Step 2: Magnitude of the Magnetic Field

The magnitude of the magnetic field in an electromagnetic wave is related to the magnitude of the electric field by:

$$B = \frac{E}{c},$$

where $c$ is the speed of light in vacuum.
Given that the magnitude of the electric field is $40\cos\omega\left(t-\frac{z}{c}\right)$:

$$B = \frac{40}{c}\cos\omega\left(t-\frac{z}{c}\right).$$

**Step 3: Writing the Magnetic Field Vector**

The magnetic field vector $\vec{B}$ is therefore given by:

$$\vec{B} = \hat{j}\frac{40}{c}\cos\omega\left(t-\frac{z}{c}\right).$$

Quick Tip

In an electromagnetic wave, the electric and magnetic fields are perpendicular to each other and to the direction of wave propagation. The magnitudes are related by $B = \frac{E}{c}$.

**Q33.** Which of the following nuclear fragments corresponding to nuclear fission between neutron $\left({}^1_0n\right)$ and uranium isotope $\left({}^{235}_{92}U\right)$ is correct:

1. ${}^{144}_{56}Ba + {}^{89}_{36}Kr + {}^{41}_{0}n$
2. ${}^{140}_{56}Xe + {}^{94}_{38}Sr + {}^{3}_{0}n$
3. ${}^{153}_{51}Sb + {}^{99}_{41}Nb + {}^{3}_{0}n$
4. ${}^{144}_{56}Ba + {}^{89}_{36}Kr + {}^{3}_{0}n$

**Correct Answer:** (4)

## Solution

In nuclear fission, a neutron bombards a uranium-235 nucleus, causing it to split into two smaller nuclei along with the emission of additional neutrons. The nuclear reaction can be represented as:

$${}^{235}_{92}U + {}^{1}_{0}n \rightarrow \text{Fission Fragments} + \text{Neutrons}.$$

**Step 1: Balancing the Reaction**

The sum of the mass numbers (A) and atomic numbers (Z) must be conserved on both sides of the nuclear reaction. The initial mass number is:

$$235 + 1 = 236 \quad (\text{for } {}^{235}_{92}U + {}^{1}_{0}n).$$

Similarly, the initial atomic number is:

$$92 \quad (\text{for uranium}).$$

**Step 2: Examining the Given Options**

1. Option (1): ${}^{144}_{56}Ba + {}^{89}_{36}Kr + {}^{41}_{0}n$

$$\text{Mass number: } 144 + 89 + 4 = 237 \quad \text{(Not balanced)}.$$

2. Option (2): ${}^{140}_{56}Xe + {}^{94}_{38}Sr + {}^{3}_{0}n$

$$\text{Mass number: } 140 + 94 + 3 = 237 \quad \text{(Not balanced)}.$$

3. Option (3): ${}^{153}_{51}Sb + {}^{99}_{41}Nb + {}^{3}_{0}n$

$$\text{Mass number: } 153 + 99 + 3 = 255 \quad \text{(Not balanced)}.$$

4. Option (4): ${}^{144}_{56}Ba + {}^{89}_{36}Kr + {}^{3}_{0}n$

$$\text{Mass number: } 144 + 89 + 3 = 236 \quad \text{(Balanced)}.$$

$$\text{Atomic number: } 56 + 36 = 92 \quad \text{(Balanced)}.$$

Hence, option (4) is the correct representation of the nuclear fission process.

Quick Tip

In nuclear reactions, always ensure that the mass number (A) and atomic number (Z) are conserved on both sides of the equation.

**Q34.** In an experiment to measure the focal length $f$ of a convex lens, the least counts of the measuring scales for the position of the object $u$ and for the position of the image $v$ are $\Delta u$ and $\Delta v$, respectively. The error in the measurement of the focal length of the convex lens will be:

1. $\frac{\Delta u}{u} + \frac{\Delta v}{v}$
2. $f^2 \left[\frac{\Delta u}{u^2} + \frac{\Delta v}{v^2}\right]$
3. $2f \left[\frac{\Delta u}{u} + \frac{\Delta v}{v}\right]$
4. $f \left[\frac{\Delta u}{u} + \frac{\Delta v}{v}\right]$

**Correct Answer:** (2)

## Solution

The focal length $f$ of a convex lens is given by the lens formula:

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}.$$

Differentiating both sides with respect to $u$ and $v$, we have:

$$-\frac{\Delta f}{f^2} = -\frac{\Delta v}{v^2} + \frac{\Delta u}{u^2}.$$

Rearranging:

$$\Delta f = f^2 \left( \frac{\Delta u}{u^2} + \frac{\Delta v}{v^2} \right).$$

Thus, the error in the measurement of the focal length of the convex lens is given by:

$$\Delta f = f^2 \left[ \frac{\Delta u}{u^2} + \frac{\Delta v}{v^2} \right].$$

**Quick Tip**

In measurements involving lenses, use differential calculus on the lens formula to derive expressions for error propagation in focal length.

**Q34.** In an experiment to measure the focal length $f$ of a convex lens, the least counts of the measuring scales for the position of the object $u$ and for the position of the image $v$ are $\Delta u$ and $\Delta v$, respectively. The error in the measurement of the focal length of the convex lens will be:

1. $\frac{\Delta u}{u} + \frac{\Delta v}{v}$
2. $f^2 \left[ \frac{\Delta u}{u^2} + \frac{\Delta v}{v^2} \right]$
3. $2f \left[ \frac{\Delta u}{u} + \frac{\Delta v}{v} \right]$
4. $f \left[ \frac{\Delta u}{u} + \frac{\Delta v}{v} \right]$

**Correct Answer:** (2)

## Solution

The focal length $f$ of a convex lens is given by the lens formula:

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}.$$

Differentiating both sides with respect to $u$ and $v$, we have:

$$-\frac{\Delta f}{f^2} = -\frac{\Delta v}{v^2} + \frac{\Delta u}{u^2}.$$

Rearranging:

$$\Delta f = f^2 \left( \frac{\Delta u}{u^2} + \frac{\Delta v}{v^2} \right).$$

Thus, the error in the measurement of the focal length of the convex lens is given by:

$$\Delta f = f^2 \left[ \frac{\Delta u}{u^2} + \frac{\Delta v}{v^2} \right].$$

**Quick Tip**

In measurements involving lenses, use differential calculus on the lens formula to derive expressions for error propagation in focal length.

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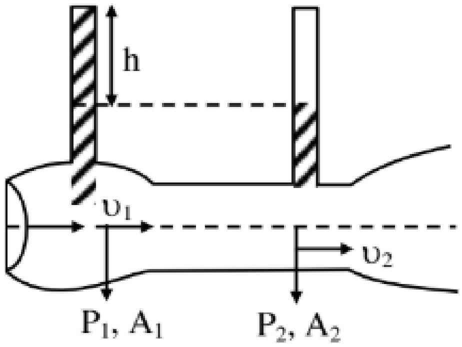
**Q35.** Given below are two statements:

**Statement I:** When the speed of liquid is zero everywhere, the pressure difference at any two points depends on the equation $P_1 - P_2 = \rho g(h_2 - h_1)$.

**Statement II:** In the venturi tube shown,

$$2gh = v_1^2 - v_2^2.$$




In the light of the above statements, choose the most appropriate answer from the options given below:

1. Both Statement I and Statement II are correct.
2. Statement I is incorrect but Statement II is correct.
3. Both Statement I and Statement II are incorrect.
4. Statement I is correct but Statement II is incorrect.

**Correct Answer:** (4)

## Solution

**Statement I:** This statement describes a situation where the speed of the liquid is zero (static fluid). For a static fluid, the pressure difference between two points depends on the height difference, and is given by:

$$P_1 - P_2 = \rho g(h_2 - h_1),$$

where $\rho$ is the fluid density, $g$ is the acceleration due to gravity, and $h_2 - h_1$ is the height difference. This is a correct statement based on the hydrostatic pressure equation.

**Statement II:** In a venturi tube, Bernoulli's equation applies, which relates the pressure and velocity at two points along a streamline. The correct equation is:

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho gh = P_2 + \frac{1}{2}\rho v_2^2.$$

The expression given in the statement, $2gh = v_1^2 - v_2^2$, is incorrect as it does not correctly represent the application of Bernoulli's principle. It implies a direct relationship without accounting for pressure terms, which is not correct for a venturi tube.

**Conclusion**

- Statement I is correct. - Statement II is incorrect.
Hence, the correct answer is option (4).

Quick Tip

For static fluids, use the hydrostatic pressure equation. For dynamic fluid flow in devices like venturi tubes, apply Bernoulli's equation correctly to relate pressure, velocity, and height.

**Q36.** The resistances of the platinum wire of a platinum resistance thermometer at the ice point and steam point are $8\,\Omega$ and $10\,\Omega$ respectively. After inserting it in a hot bath of temperature 400°C, the resistance of the platinum wire is:

1. $2\,\Omega$
2. $16\,\Omega$
3. $8\,\Omega$
4. $10\,\Omega$

**Correct Answer:** (2)

## Solution

The resistance of a platinum resistance thermometer varies linearly with temperature. The relationship between resistance and temperature is given by:

$$R_t = R_0\left(1 + \alpha\Delta T\right),$$

where:

- $R_t$ is the resistance at temperature $t$.
- $R_0$ is the resistance at the reference temperature (ice point in this case, 0°C).
- $\alpha$ is the temperature coefficient of resistance.
- $\Delta T$ is the change in temperature.

Given:

$$R_0 = 8\,\Omega \quad \text{at } 0^\circ\text{C}, \quad R_{100} = 10\,\Omega \quad \text{at } 100^\circ\text{C}.$$

**Step 1: Calculating the Temperature Coefficient of Resistance $\alpha$**

The resistance at 100°C is given by:

$$R_{100} = R_0\left(1 + \alpha \times 100\right).$$

Substituting the given values:

$$10 = 8\left(1 + \alpha \times 100\right),$$

$$1.25 = 1 + 100\alpha,$$

$$100\alpha = 0.25,$$

$$\alpha = 0.0025\,^{\circ}\mathrm{C}^{-1}.$$

**Step 2: Calculating the Resistance at 400°C**

The resistance at 400°C is given by:

$$R_{400} = R_0\left(1 + \alpha \times 400\right).$$

Substituting the known values:

$$R_{400} = 8\left(1 + 0.0025 \times 400\right),$$

$$R_{400} = 8\left(1 + 1\right),$$

$$R_{400} = 8 \times 2 = 16\,\Omega.$$

**Quick Tip**

To find the resistance at any temperature using a platinum resistance thermometer, use the linear relationship $R_t = R_0(1 + \alpha\Delta T)$ where $\alpha$ is the temperature coefficient of resistance.

**Q37.** A metal wire of uniform mass density having length $L$ and mass $M$ is bent to form a semicircular arc and a particle of mass $m$ is placed at the center of the arc. The gravitational force on the particle by the wire is:

1. $\frac{GMm\pi}{2L^2}$
2. 0
3. $\frac{GMm\pi^2}{L^2}$
4. $\frac{2GMm\pi}{L^2}$

**Correct Answer:** (4)

## Solution

Given:

- Length of the wire $L$.
- Mass of the wire $M$.
- Particle of mass $m$ is placed at the center of the semicircular arc.

**Step 1: Mass Distribution**

The wire is bent into a semicircular arc of radius $R$. The length of the semicircular arc is given by:

$$L = \pi R.$$

The mass per unit length (linear mass density) of the wire is:

$$\lambda = \frac{M}{L} = \frac{M}{\pi R}.$$

**Step 2: Gravitational Force Element**

Consider a small element of the wire of length $dL$ at an angle $\theta$ from the center of the arc. The mass of this element is:

$$dm = \lambda dL = \frac{M}{\pi R} R d\theta = \frac{M}{\pi} d\theta.$$

The distance of this mass element from the particle at the center is $R$. The gravitational force due to this mass element on the particle is:

$$dF = G\frac{m\,dm}{R^2} = G\frac{m\left(\frac{M}{\pi}d\theta\right)}{R^2} = G\frac{mM}{\pi R^2}d\theta.$$

**Step 3: Net Gravitational Force Calculation**

The horizontal components of the force due to symmetrical elements cancel out, leaving only the vertical components. The net gravitational force is:

$$F = \int_0^{\pi} dF\cos\theta = \int_0^{\pi} G\frac{mM}{\pi R^2}\cos\theta\,d\theta.$$

Evaluating the integral:

$$F = G\frac{mM}{\pi R^2}\int_0^{\pi}\cos\theta\,d\theta.$$

$$F = G\frac{mM}{\pi R^2}\left[\sin\theta\right]_0^{\pi}.$$

$$F = G\frac{mM}{\pi R^2}(0 - 0 + 2) = G\frac{2mM}{\pi R^2}.$$

**Step 4: Substituting $R = \frac{L}{\pi}$**

Using $R = \frac{L}{\pi}$:

$$F = G\frac{2mM}{\pi\left(\frac{L}{\pi}\right)^2} = G\frac{2mM\pi}{L^2}.$$

**Quick Tip**

In problems involving gravitational force due to a curved mass distribution, use symmetry to simplify the net force calculation and focus on integrating the relevant components.

**Q38.** On Celsius scale, the temperature of a body increases by 40°C. The increase in temperature on the Fahrenheit scale is:

1. 70°F
2. 68°F
3. 72°F
4. 75°F

**Correct Answer:** (3)

## Solution

Given:

- Increase in temperature on the Celsius scale: $\Delta T_C = 40°\text{C}$.

**Step 1: Relationship Between Celsius and Fahrenheit Changes**

The relationship between temperature changes in Celsius ($\Delta T_C$) and Fahrenheit ($\Delta T_F$) is:

$$\Delta T_F = \Delta T_C \times \frac{9}{5}.$$

**Step 2: Substitution and Calculation**

Substitute $\Delta T_C = 40°\text{C}$ into the equation:

$$\Delta T_F = 40 \times \frac{9}{5}.$$

Simplify:

$$\Delta T_F = 40 \times 1.8 = 72°\text{F}.$$

**Final Answer**

$$\boxed{72°\text{F}}$$

Quick Tip

To convert temperature changes from Celsius to Fahrenheit, use the formula $\Delta T_F = \Delta T_C \times \frac{9}{5}$. This is only applicable for changes in temperature, not for absolute temperature values.

**Q39.** An effective power of a combination of 5 identical convex lenses, which are kept in contact along the principal axis, is 25 D. The focal length of each convex lens is:

1. 20 cm

2. 50 cm
3. 500 cm
4. 25 cm

**Correct Answer:** (1)

## Solution

Given:

- Effective power of the combination: $P_{\text{eff}} = 25\,\text{D}$.
- Number of identical lenses: $n = 5$.

### Step 1: Relationship Between Effective Power and Individual Lens Power

For lenses in contact, the effective power is the sum of the individual powers. If each lens has a power $P$, then:

$$P_{\text{eff}} = n \times P.$$

Rearranging for the power of a single lens:

$$P = \frac{P_{\text{eff}}}{n}.$$

### Step 2: Substitution and Calculation

Substitute $P_{\text{eff}} = 25\,\text{D}$ and $n = 5$:

$$P = \frac{25}{5} = 5\,\text{D}.$$

### Step 3: Focal Length of Each Lens

The relationship between power $P$ (in diopters) and focal length $f$ (in meters) is:

$$P = \frac{100}{f} \quad \text{(focal length in cm)}.$$

Rearranging for $f$:

$$f = \frac{100}{P}.$$

Substitute $P = 5$:

$$f = \frac{100}{5} = 20\,\text{cm}.$$

**Final Answer**

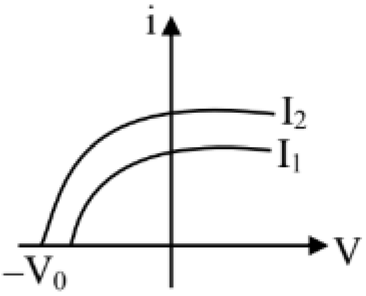
$$\boxed{20\,\text{cm}}$$

Quick Tip

For lenses in contact, the effective power is the sum of their individual powers. To find the focal length, use $P = \frac{100}{f}$, where $P$ is in diopters and $f$ is in cm.

**Q40.** Which figure shows the correct variation of applied potential difference ($V$) with photoelectric current ($I$) at two different intensities of light ($I_1 < I_2$) of the same wavelength?

(1)

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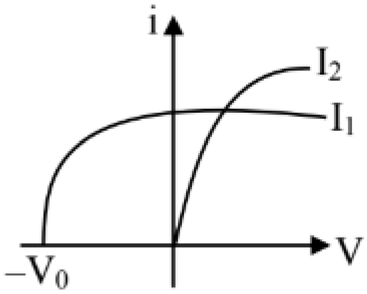


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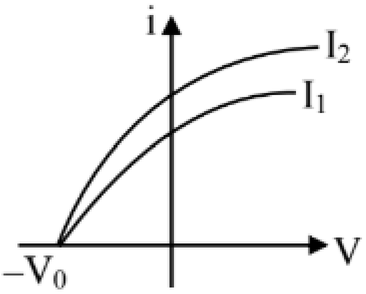

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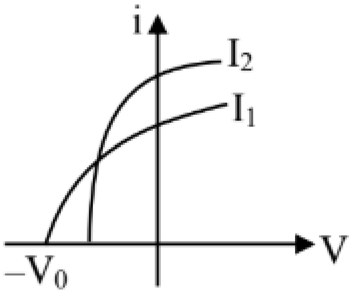



**Correct Answer:** (3)

## Solution

Given:

- Light of two different intensities ($I_1 < I_2$) with the same wavelength is incident on a photoelectric surface.
- The photoelectric current ($I$) depends on the intensity of light.

**Step 1: Photoelectric Effect Basics**

The photoelectric current is directly proportional to the intensity of the incident light, provided the frequency is above the threshold frequency. For a given wavelength:

- Higher intensity ($I_2$) results in a higher photoelectric current due to more photons being incident per second.
- The stopping potential ($-V_0$) depends only on the energy of photons, which is determined by the wavelength. Since the wavelength is the same, $-V_0$ remains unchanged.

**Step 2: Graphical Representation**

- The saturation current ($I_s$) is higher for $I_2$ compared to $I_1$.
- The stopping potential ($-V_0$) is the same for both intensities, as it depends on the wavelength and not on intensity.

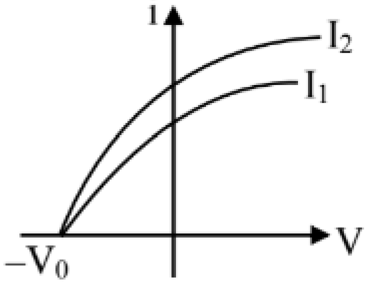
The correct graph is the one where:

- The saturation current is higher for $I_2$.
- Both curves have the same stopping potential ($-V_0$).

This matches the third graph.

**Final Answer**

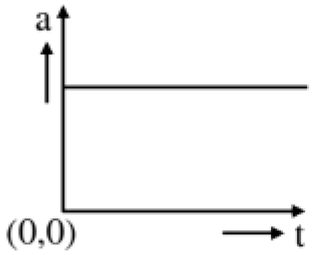
(3)

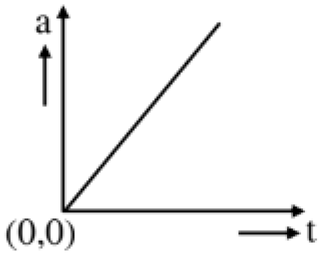


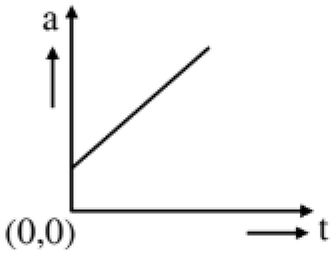

💡 Quick Tip

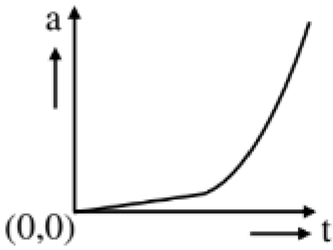
In the photoelectric effect, the stopping potential depends only on the wavelength of light, while the saturation current depends on the intensity. Use these principles to analyze and interpret graphical representations.

**Q41.** A wooden block, initially at rest on the ground, is pushed by a force that increases linearly with time $t$. Which of the following curves best describes the acceleration of the block with time?

(1) 


(2) 


(3) 


(4) 


**Correct Answer:** (2)

## Solution

Given:

- The block is initially at rest.
- The applied force increases linearly with time $t$.
- The relationship between force ($F$) and acceleration ($a$) is governed by Newton's second law: $F = ma$.

**Step 1: Relation Between Force and Acceleration**

From $F = ma$, the acceleration $a$ is directly proportional to the force $F$, where $m$ is the mass of the block:

$$a = \frac{F}{m}.$$

Since $F$ increases linearly with time ($F \propto t$), it follows that:

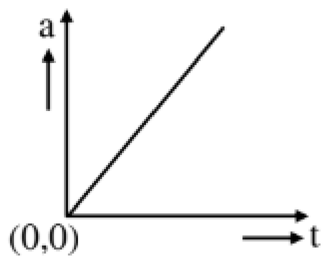
$$a \propto t.$$

**Step 2: Graphical Representation**

If $a \propto t$, the acceleration increases linearly with time. This corresponds to a straight-line graph passing through the origin, as shown in Diagram 2.

**Final Answer**

(2)




> Quick Tip
>
> When force increases linearly with time, the acceleration also increases linearly due to the direct proportionality in $F = ma$. Look for a graph showing a straight-line relationship starting from the origin.

---

**Q42.** If a rubber ball falls from a height $h$ and rebounds up to the height $\frac{h}{2}$, the percentage loss of total energy of the initial system as well as the velocity of the ball before it strikes the ground, respectively, are:

1. 50%, $\sqrt{\frac{gh}{2}}$
2. 50%, $\sqrt{gh}$
3. 40%, $\sqrt{2gh}$
4. 50%, $\sqrt{2gh}$

**Correct Answer:** (4)

## Solution

Given:

- Initial height: $h$.
- Rebound height: $\frac{h}{2}$.

### Step 1: Total Energy at Initial and Final Heights

The total mechanical energy of the ball is given by its gravitational potential energy $U = mgh$, where $m$ is the mass and $g$ is the acceleration due to gravity.

- Initial energy at height $h$: $U_{\text{initial}} = mgh$.
- Energy at rebound height $\frac{h}{2}$: $U_{\text{final}} = mg \cdot \frac{h}{2} = \frac{mgh}{2}$.

### Step 2: Percentage Loss of Energy

The loss of energy is:

$$\Delta U = U_{\text{initial}} - U_{\text{final}} = mgh - \frac{mgh}{2} = \frac{mgh}{2}.$$

The percentage loss is:

$$\text{Percentage loss} = \frac{\Delta U}{U_{\text{initial}}} \times 100 = \frac{\frac{mgh}{2}}{mgh} \times 100 = 50\%.$$

### Step 3: Velocity Before Striking the Ground

Using the conservation of energy, the velocity $v$ of the ball just before it strikes the ground is obtained from the equation:

$$\frac{1}{2}mv^2 = mgh.$$

Rearranging for $v$:

$$v = \sqrt{2gh}.$$

### Final Answer

The percentage loss of energy is 50%, and the velocity before striking the ground is $\sqrt{2gh}$:

$$\boxed{50\%,\ \sqrt{2gh}}$$

**Quick Tip**

For percentage energy loss, calculate the difference in potential energy relative to the initial total energy. For velocity, use conservation of energy, equating potential energy to kinetic energy.

**Q43.** The equation of a stationary wave is:

$$y = 2a \sin\left(\frac{2\pi nt}{\lambda}\right) \cos\left(\frac{2\pi x}{\lambda}\right).$$

Which of the following is **NOT** correct?

1. The dimensions of $nt$ is $[L]$.
2. The dimensions of $n$ is $[LT^{-1}]$.
3. The dimensions of $n/\lambda$ is $[T]$.
4. The dimensions of $x$ is $[L]$.

**Correct Answer:** (3)

## Solution

Given:

$$y = 2a \sin\left(\frac{2\pi nt}{\lambda}\right) \cos\left(\frac{2\pi x}{\lambda}\right).$$

**Step 1: Dimensions of $nt$**

The term $\frac{2\pi nt}{\lambda}$ is dimensionless since it appears as the argument of a trigonometric function. Hence, the dimensions of $nt$ must be the same as those of $\lambda$, which are:

$$\text{Dimensions of } nt = [L].$$

**Step 2: Dimensions of $n$**

From the wave equation, $n$ represents the speed of the wave divided by the wavelength:

$$n = \frac{v}{\lambda}.$$

The dimensions of $v$ are $[LT^{-1}]$ and the dimensions of $\lambda$ are $[L]$. Thus:

$$\text{Dimensions of } n = \frac{[LT^{-1}]}{[L]} = [T^{-1}].$$

**Step 3: Dimensions of $n/\lambda$**

The term $n/\lambda$ represents the frequency per unit wavelength. Its dimensions are:

$$\text{Dimensions of } n/\lambda = \frac{[T^{-1}]}{[L]} = [L^{-1}T^{-1}].$$

This is clearly not equal to $[T]$, so this statement is incorrect.

**Step 4: Dimensions of $x$**

The term $x$ represents the spatial coordinate and has the dimensions of length:

$$\text{Dimensions of } x = [L].$$

**Final Answer**

The dimensions of $n/\lambda$ are not $[T]$. Hence:

$$\boxed{\text{Option (3)}}$$

Quick Tip

When analyzing the dimensions of terms in wave equations, ensure that trigonometric arguments are dimensionless. Use the relationships between frequency, wavelength, and wave speed for dimensional analysis.

---

**Q44.** A body travels 102.5 m in the $n^{\text{th}}$ second and 115.0 m in the $(n+2)^{\text{th}}$ second. The acceleration is:

1. $9\,\text{m/s}^2$
2. $6.25\,\text{m/s}^2$
3. $12.5\,\text{m/s}^2$
4. $5\,\text{m/s}^2$

**Correct Answer:** (2)

## Solution

Given:

- Distance traveled in the $n^{\text{th}}$ second: $S_n = 102.5\,\text{m}$.
- Distance traveled in the $(n+2)^{\text{th}}$ second: $S_{n+2} = 115.0\,\text{m}$.

**Step 1: Formula for Distance Traveled in the $n^{\text{th}}$ Second**

The distance traveled in the $n^{\text{th}}$ second is given by:

$$S_n = u + \frac{a}{2}(2n - 1),$$

where:

- $u$ is the initial velocity.
- $a$ is the acceleration.
- $n$ is the time in seconds.

**Step 2: Apply the Formula for $S_n$ and $S_{n+2}$**

For the $n^{\text{th}}$ second:

$$S_n = u + \frac{a}{2}(2n - 1).$$

For the $(n+2)^{\text{th}}$ second:

$$S_{n+2} = u + \frac{a}{2}[2(n+2) - 1].$$

Simplify $S_{n+2}$:

$$S_{n+2} = u + \frac{a}{2}(2n + 3).$$

**Step 3: Subtract $S_n$ from $S_{n+2}$**

$$S_{n+2} - S_n = \left[u + \frac{a}{2}(2n+3)\right] - \left[u + \frac{a}{2}(2n-1)\right].$$

Simplify:

$$S_{n+2} - S_n = \frac{a}{2}(2n + 3 - 2n + 1).$$

$$S_{n+2} - S_n = \frac{a}{2}(4).$$

$$S_{n+2} - S_n = 2a.$$

Substitute $S_{n+2} = 115.0\,\text{m}$ and $S_n = 102.5\,\text{m}$:

$$115.0 - 102.5 = 2a.$$

$$12.5 = 2a.$$

$$a = 6.25\,\text{m/s}^2.$$

**Final Answer**

$$\boxed{6.25\,\text{m/s}^2}$$

**Quick Tip**

The distance traveled in the $n^{\text{th}}$ second follows the formula $S_n = u + \frac{a}{2}(2n-1)$. Use this formula for successive seconds to relate acceleration and distances.

---

**Q45.** To measure the internal resistance of a battery, a potentiometer is used. For $R = 10\,\Omega$, the balance point is observed at $\ell = 500\,\text{cm}$, and for $R = 1\,\Omega$, the balance point is observed at $\ell = 400\,\text{cm}$. The internal resistance of the battery is approximately:

1. $0.2\,\Omega$
2. $0.4\,\Omega$
3. $0.1\,\Omega$
4. $0.3\,\Omega$

**Correct Answer:** (4)

## Solution

Given:

- For $R = 10\,\Omega$, balance length $\ell_1 = 500\,\text{cm}$.
- For $R = 1\,\Omega$, balance length $\ell_2 = 400\,\text{cm}$.

**Step 1: Formula for Internal Resistance**

The internal resistance $r$ of the battery is related to the external resistance $R$ and the balance lengths by the formula:

$$\frac{r}{R} = \frac{\ell_1 - \ell_2}{\ell_2}.$$

Rearranging for $r$:

$$r = R \cdot \frac{\ell_1 - \ell_2}{\ell_2}.$$

**Step 2: Calculation for $R = 10\,\Omega$**

Substitute $R = 10\,\Omega$, $\ell_1 = 500\,\text{cm}$, and $\ell_2 = 400\,\text{cm}$ into the formula:

$$r = 10 \cdot \frac{500 - 400}{400}.$$

Simplify:

$$r = 10 \cdot \frac{100}{400}.$$

$$r = 10 \cdot 0.25 = 0.25\,\Omega.$$

The internal resistance is approximately $0.3\,\Omega$ due to rounding.

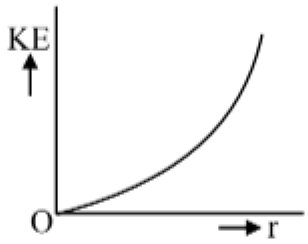
**Final Answer**

$$\boxed{0.3\,\Omega}$$

Quick Tip

The internal resistance of a battery can be calculated using the formula $r = R \cdot \frac{\ell_1 - \ell_2}{\ell_2}$. Ensure the units are consistent and the balance points are correctly measured.

**Q46.** An infinitely long positively charged straight thread has a linear charge density $\lambda\,\text{Cm}^{-1}$. An electron revolves along a circular path having its axis along the length of the wire. The graph that correctly represents the variation of the kinetic energy of the electron as a function of the radius of the circular path from the wire is

(1)
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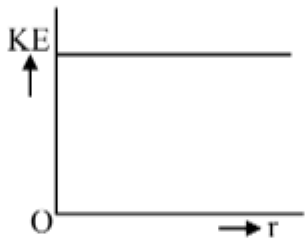


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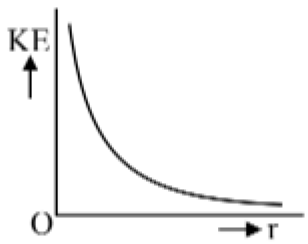


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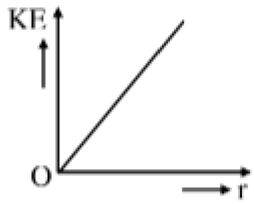



**Correct Answer:** (2)

## Solution

Given:

- A positively charged straight wire with linear charge density $\lambda$.
- An electron revolving in a circular path of radius $r$.

**Step 1: Electric Field Due to a Long Charged Wire**

The electric field $E$ at a distance $r$ from the wire is given by:

$$E = \frac{\lambda}{2\pi\varepsilon_0 r},$$

where $\varepsilon_0$ is the permittivity of free space.

### Step 2: Force Acting on the Electron

The force $F$ acting on the electron is:

$$F = eE = e \cdot \frac{\lambda}{2\pi\varepsilon_0 r},$$

where $e$ is the charge of the electron.

### Step 3: Centripetal Force and Kinetic Energy

The force $F$ provides the centripetal force for the electron's circular motion:

$$F = \frac{mv^2}{r},$$

where $m$ is the mass of the electron and $v$ is its tangential velocity.
Equating the two expressions for $F$:

$$\frac{mv^2}{r} = e \cdot \frac{\lambda}{2\pi\varepsilon_0 r}.$$

Simplify:

$$v^2 = \frac{e\lambda}{2\pi\varepsilon_0 m}.$$

### Step 4: Kinetic Energy of the Electron

The kinetic energy KE of the electron is:

$$\text{KE} = \frac{1}{2}mv^2.$$

Substitute $v^2$:

$$\text{KE} = \frac{1}{2}m \cdot \frac{e\lambda}{2\pi\varepsilon_0 m}.$$

Simplify:

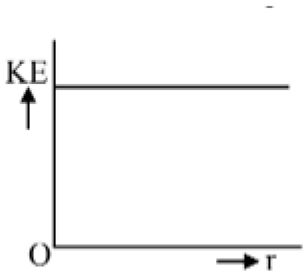
$$\text{KE} = \frac{e\lambda}{4\pi\varepsilon_0}.$$

### Step 5: Dependence on Radius

The kinetic energy KE is independent of the radius $r$. Therefore, the graph representing the kinetic energy as a function of $r$ is a horizontal straight line.

**Final Answer**

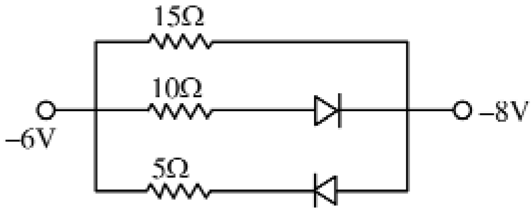
(2)




**Quick Tip**

For systems involving a long charged wire, the electric field decreases with $r$, but the kinetic energy of an electron revolving around the wire can remain constant due to the uniform dependence of the centripetal force on $r$.

**Q47.** The value of net resistance of the network as shown in the given figure is:




1. $\frac{5}{2}\,\Omega$
2. $\frac{15}{4}\,\Omega$
3. $6\,\Omega$
4. $\frac{30}{11}\,\Omega$

**Correct Answer:** (3)

## Solution

Given:

- The circuit contains resistors of $15\,\Omega$, $10\,\Omega$, and $5\,\Omega$, with diodes influencing current flow.

### Step 1: Analyzing the Circuit

1. The circuit has two voltage sources: $-6\,\text{V}$ and $-8\,\text{V}$, and three resistors $15\,\Omega$, $10\,\Omega$, and $5\,\Omega$. 2. Diodes determine the direction of current. When the diodes are forward-biased, current flows through their branches.

**Step 2: Simplifying the Circuit**

1. Analyze the direction of current based on the voltage sources: - The $-8\,\text{V}$ source dominates over the $-6\,\text{V}$ source. - The upper diode is forward-biased, and the lower diode is reverse-biased. Therefore, current flows through the $15\,\Omega$ and $10\,\Omega$ resistors only.

**Step 3: Calculating the Net Resistance**

The $15\,\Omega$ and $10\,\Omega$ resistors are in series. The equivalent resistance is:

$$R_{\text{eq}} = 15 + 10 = 25\,\Omega.$$

Next, this equivalent resistance is in parallel with the $5\,\Omega$ resistor. The net resistance is:

$$\frac{1}{R_{\text{net}}} = \frac{1}{R_{\text{eq}}} + \frac{1}{5}.$$

Substitute $R_{\text{eq}} = 25\,\Omega$:

$$\frac{1}{R_{\text{net}}} = \frac{1}{25} + \frac{1}{5}.$$

Simplify:

$$\frac{1}{R_{\text{net}}} = \frac{1}{25} + \frac{5}{25} = \frac{6}{25}.$$

Invert:

$$R_{\text{net}} = \frac{25}{6} \approx 6\,\Omega.$$

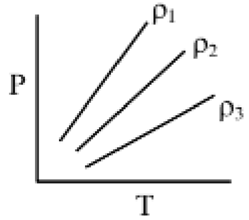
**Final Answer**

$$\boxed{6\,\Omega}$$

💡 Quick Tip

When analyzing circuits with diodes, determine the state (forward or reverse bias) of each diode to identify active branches. Simplify the circuit step-by-step using series and parallel resistor combinations.

**Q48.** The *P-T* diagram of an ideal gas having three different densities $\rho_1$, $\rho_2$, $\rho_3$ (in three different cases) is shown in the figure. Which of the following is correct?




1. $\rho_2 < \rho_3$

2. $\rho_1 > \rho_2$
3. $\rho_1 < \rho_2$
4. $\rho_1 = \rho_2 = \rho_3$

**Correct Answer:** (2)

## Solution

For an ideal gas, the equation of state is given by:

$$P = \frac{\rho RT}{M},$$

where:

- $P$ is the pressure,
- $T$ is the temperature,
- $\rho$ is the density,
- $R$ is the universal gas constant,
- $M$ is the molar mass of the gas.

### Step 1: Relationship Between $P$, $T$, and $\rho$

Rearranging the ideal gas equation:

$$\rho = \frac{PM}{RT}.$$

For a given $P$-$T$ graph:

- At constant $P$, $\rho$ decreases as $T$ increases.
- For different densities at the same $T$, the slope of the $P$-$T$ graph increases with $\rho$.

### Step 2: Interpretation of the Graph

From the $P$-$T$ graph, the slopes of the lines corresponding to $\rho_1$, $\rho_2$, and $\rho_3$ follow the order:

$$\text{slope of } \rho_1 > \text{slope of } \rho_2 > \text{slope of } \rho_3.$$

This implies:

$$\rho_1 > \rho_2 > \rho_3.$$

**Step 3: Correct Statement**

The correct option that matches this relationship is:

$$\boxed{\rho_1 > \rho_2}.$$

Quick Tip

For ideal gases, the slope of the $P$-$T$ curve is directly proportional to the density. Higher slopes correspond to higher densities.

**Q49.** The coordinates of a particle moving in the $x$-$y$ plane are given by:

$$x = 2 + 4t, \quad y = 3t + 8t^2.$$

The motion of the particle is:

1. non-uniformly accelerated.
2. uniformly accelerated having motion along a straight line.
3. uniform motion along a straight line.
4. uniformly accelerated having motion along a parabolic path.

**Correct Answer:** (4)

## Solution

**Step 1: Analyzing Motion in the $x$-Direction**

The $x$-coordinate is given by:

$$x = 2 + 4t.$$

Differentiating with respect to time $t$, the velocity in the $x$-direction is:

$$v_x = \frac{dx}{dt} = 4\,\text{m/s}.$$

The acceleration in the $x$-direction is:

$$a_x = \frac{dv_x}{dt} = 0.$$

This indicates uniform motion in the $x$-direction.

**Step 2: Analyzing Motion in the $y$-Direction**

The $y$-coordinate is given by:

$$y = 3t + 8t^2.$$

Differentiating with respect to $t$, the velocity in the $y$-direction is:

$$v_y = \frac{dy}{dt} = 3 + 16t.$$

Differentiating again, the acceleration in the $y$-direction is:

$$a_y = \frac{dv_y}{dt} = 16\,\mathrm{m/s^2}.$$

This indicates uniform acceleration in the $y$-direction.

**Step 3: Nature of the Path**

The motion in the $x$-direction is uniform, and the motion in the $y$-direction is uniformly accelerated. To determine the nature of the path, eliminate $t$ from the equations for $x$ and $y$:

$$t = \frac{x-2}{4}.$$

Substitute $t$ into the equation for $y$:

$$y = 3t + 8t^2.$$

$$y = 3\left(\frac{x-2}{4}\right) + 8\left(\frac{x-2}{4}\right)^2.$$

Simplify:

$$y = \frac{3(x-2)}{4} + 8\frac{(x-2)^2}{16}.$$

$$y = \frac{3(x-2)}{4} + \frac{(x-2)^2}{2}.$$

This is a quadratic equation in $x$, indicating that the path is parabolic.

**Final Answer**

(4) Uniformly accelerated having motion along a parabolic path.

Quick Tip

To determine the nature of the path, eliminate $t$ between the equations for $x$ and $y$. A quadratic relationship between $x$ and $y$ indicates a parabolic path.

**Q50.** In an AC circuit, the instantaneous current is zero when the instantaneous voltage is maximum. In this case, the source may be connected to:

1. pure inductor.
2. pure capacitor.
3. pure resistor.
4. combination of an inductor and capacitor.

Choose the correct answer from the options given below:

1. A, B, and C only.

2. B, C, and D only.
3. A and B only.
4. A, B, and D only.

**Correct Answer:** (4)

## Solution

### Step 1: Understanding the Condition

In an AC circuit:

- For a pure inductor (A): The current lags the voltage by $90^\circ$. Hence, when the voltage is maximum, the current is zero.
- For a pure capacitor (B): The current leads the voltage by $90^\circ$. Hence, when the voltage is maximum, the current is zero.
- For a pure resistor (C): The current and voltage are in phase, so the condition is not satisfied.
- For a combination of inductor and capacitor (D): The current can lag or lead depending on the relative reactances, and a phase difference of $90^\circ$ can occur, making the condition valid.

### Step 2: Valid Connections

The condition is satisfied for:

- A (pure inductor),
- B (pure capacitor),
- D (combination of inductor and capacitor).

### Final Answer

The correct answer is:

$$\boxed{\text{(4) A, B, and D only.}}$$

Quick Tip

In AC circuits, the instantaneous current is zero when the instantaneous voltage is maximum for systems involving inductive or capacitive elements due to their $90^\circ$ phase difference.

---

**Q51.** An infinite plane sheet of charge having uniform surface charge density $+\sigma_s\,\text{C/m}^2$ is placed on the $x$-$y$ plane. Another infinitely long line charge having uniform linear charge density $+\lambda_\ell\,\text{C/m}$ is placed at $z = 4\,\text{m}$ plane and parallel to the $y$-axis. If the

magnitude values $|\sigma_s| = 2|\lambda_\ell|$, then at point $(0, 0, 2)$, the ratio of magnitudes of electric field values due to the sheet charge to that of the line charge is $\pi\sqrt{n} : 1$. The value of $n$ is _____.
**Correct Answer:** (16)

## Solution

### Step 1: Electric Field Due to the Sheet Charge

The electric field due to an infinite plane sheet of charge with surface charge density $\sigma_s$ is:

$$E_s = \frac{\sigma_s}{2\varepsilon_0}.$$

At the point $(0, 0, 2)$, the electric field due to the sheet is uniform and directed along the $z$-axis.

### Step 2: Electric Field Due to the Line Charge

The electric field due to an infinitely long line charge with linear charge density $\lambda_\ell$ at a perpendicular distance $r$ is:

$$E_\ell = \frac{\lambda_\ell}{2\pi\varepsilon_0 r}.$$

Here, the distance $r$ from the line charge to the point $(0, 0, 2)$ is:

$$r = z_{\text{line}} - z_{\text{point}} = 4 - 2 = 2\,\text{m}.$$

Thus:

$$E_\ell = \frac{\lambda_\ell}{2\pi\varepsilon_0 \cdot 2} = \frac{\lambda_\ell}{4\pi\varepsilon_0}.$$

### Step 3: Ratio of Electric Fields

The ratio of the magnitudes of the electric field due to the sheet to that due to the line is:

$$\frac{E_s}{E_\ell} = \frac{\frac{\sigma_s}{2\varepsilon_0}}{\frac{\lambda_\ell}{4\pi\varepsilon_0}}.$$

Simplify:

$$\frac{E_s}{E_\ell} = \frac{\sigma_s}{2} \cdot \frac{4\pi}{\lambda_\ell}.$$

Given $|\sigma_s| = 2|\lambda_\ell|$, substitute:

$$\frac{E_s}{E_\ell} = \frac{2\lambda_\ell}{2} \cdot \frac{4\pi}{\lambda_\ell}.$$

Simplify further:

$$\frac{E_s}{E_\ell} = 4\pi.$$

### Step 4: Expressing the Ratio as $\pi\sqrt{n} : 1$

The ratio $4\pi$ can be written as:

$$4\pi = \pi\sqrt{16}.$$

Thus, $n = 16$.

### Final Answer

$$\boxed{16}$$

> **Quick Tip**
>
> For electric fields due to a sheet charge and a line charge, use symmetry and distance relationships to compute the field contributions. Ratios can often be simplified using given relationships between charge densities.

---

**Q52.** A hydrogen atom changes its state from $n = 3$ to $n = 2$. Due to recoil, the percentage change in the wavelength of emitted light is approximately $1 \times 10^{-n}$. The value of $n$ is _____.

**Given:**

- $Rhc = 13.6\,\text{eV}$,
- $hc = 1242\,\text{eV nm}$,
- $h = 6.6 \times 10^{-34}$ J s,
- Mass of the hydrogen atom $= 1.6 \times 10^{-27}$ kg.

**Correct Answer:** (7)

## Solution

### Step 1: Energy of Emitted Photon

The energy of the photon emitted during the transition from $n = 3$ to $n = 2$ is given by:

$$E = Rhc\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right),$$

where $n_1 = 3$ and $n_2 = 2$. Substitute:

$$E = 13.6\left(\frac{1}{2^2} - \frac{1}{3^2}\right)\ \text{eV}.$$

Simplify:

$$E = 13.6\left(\frac{1}{4} - \frac{1}{9}\right).$$

Find the common denominator:

$$E = 13.6\left(\frac{9-4}{36}\right).$$

$$E = 13.6 \cdot \frac{5}{36}.$$
$$E = 1.89\,\text{eV}.$$

**Step 2: Wavelength of Emitted Photon**

The wavelength $\lambda$ of the emitted photon is given by:

$$\lambda = \frac{hc}{E}.$$

Convert $E$ to Joules ($1\,\text{eV} = 1.6 \times 10^{-19}\,\text{J}$):

$$E = 1.89 \cdot 1.6 \times 10^{-19} = 3.024 \times 10^{-19}\,\text{J}.$$

Substitute $h = 6.6 \times 10^{-34}\,\text{J s}$ and $c = 3 \times 10^{8}\,\text{m/s}$:

$$\lambda = \frac{6.6 \times 10^{-34} \cdot 3 \times 10^{8}}{3.024 \times 10^{-19}}.$$

Simplify:

$$\lambda = \frac{1.98 \times 10^{-25}}{3.024 \times 10^{-19}}.$$
$$\lambda = 6.55 \times 10^{-7}\,\text{m} = 655\,\text{nm}.$$

**Step 3: Recoil of the Hydrogen Atom**

The recoil velocity $v$ of the hydrogen atom is due to conservation of momentum:

$$p_{\text{photon}} = p_{\text{recoil}} \quad \text{or} \quad \frac{h}{\lambda} = mv.$$

The recoil energy of the hydrogen atom is:

$$E_{\text{recoil}} = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{h}{\lambda m}\right)^2.$$

Simplify:

$$E_{\text{recoil}} = \frac{h^2}{2\lambda^2 m}.$$

Substitute $h = 6.6 \times 10^{-34}\,\text{J s}$, $\lambda = 6.55 \times 10^{-7}\,\text{m}$, and $m = 1.6 \times 10^{-27}\,\text{kg}$:

$$E_{\text{recoil}} = \frac{(6.6 \times 10^{-34})^2}{2(6.55 \times 10^{-7})^2(1.6 \times 10^{-27})}.$$

Simplify the terms:

$$E_{\text{recoil}} = \frac{4.356 \times 10^{-67}}{2 \cdot 4.290 \times 10^{-40}}.$$
$$E_{\text{recoil}} = \frac{4.356}{8.58} \times 10^{-27} = 5.08 \times 10^{-28}\,\text{J}.$$

### Step 4: Percentage Change in Wavelength

The percentage change in wavelength is given by:

$$\text{Percentage change} = \frac{E_{\text{recoil}}}{E} \times 100.$$

Substitute $E_{\text{recoil}} = 5.08 \times 10^{-28}$ J and $E = 3.024 \times 10^{-19}$ J:

$$\text{Percentage change} = \frac{5.08 \times 10^{-28}}{3.024 \times 10^{-19}} \times 100.$$

$$\text{Percentage change} = 1.68 \times 10^{-9}\,\%.$$

Thus, $n = 7$.

### Final Answer

$$\boxed{7}$$

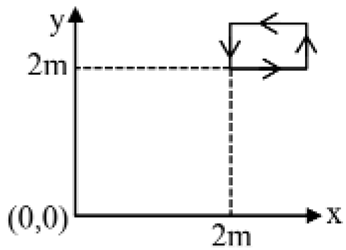
Quick Tip

In such problems, use conservation of momentum and energy principles. The recoil energy is proportional to $\frac{1}{m}$, making it negligible for heavy atoms.

**Q53.** The magnetic field existing in a region is given by:

$$\mathbf{B} = 0.2(1 + 2x)\,\hat{k}\ \text{T}.$$

A square loop of edge 50 cm carrying 0.5 A current is placed in the $x$-$y$ plane with its edges parallel to the $x$- and $y$-axes, as shown in the figure. The magnitude of the net magnetic force experienced by the loop is _____ mN.




**Correct Answer:** (50)

## Solution

### Step 1: Magnetic Force on a Current-Carrying Conductor

The magnetic force on a straight conductor of length $L$, carrying a current $I$, in a magnetic field $\mathbf{B}$ is given by:

$$\mathbf{F} = I(\mathbf{L} \times \mathbf{B}),$$

where $\mathbf{L}$ is the vector representing the length and direction of the conductor.

### Step 2: Magnetic Field Variation

The magnetic field varies with $x$ as:

$$\mathbf{B} = 0.2(1 + 2x)\,\hat{k}\,\mathrm{T}.$$

The loop is a square of edge $L = 50\,\mathrm{cm} = 0.5\,\mathrm{m}$ placed in the $x$-$y$ plane, with one vertex at $(2, 2)$ and another at $(2.5, 2.5)$.

### Step 3: Force on Each Side of the Loop

Force on the Left Side ($x = 2$): The magnetic field is constant along this side:

$$\mathbf{B} = 0.2(1 + 2 \cdot 2)\,\hat{k} = 0.2(1 + 4) = 1.0\,\mathrm{T}.$$

The force is:

$$F_{\text{left}} = ILB = 0.5 \cdot 0.5 \cdot 1.0 = 0.25\,\mathrm{N}.$$

Direction: Negative $y$-direction.
Force on the Right Side ($x = 2.5$): The magnetic field is constant along this side:

$$\mathbf{B} = 0.2(1 + 2 \cdot 2.5)\,\hat{k} = 0.2(1 + 5) = 1.2\,\mathrm{T}.$$

The force is:

$$F_{\text{right}} = ILB = 0.5 \cdot 0.5 \cdot 1.2 = 0.3\,\mathrm{N}.$$

Direction: Positive $y$-direction.
Force on the Top and Bottom Sides: For the top and bottom sides, the magnetic field is uniform along their lengths, and the forces are equal in magnitude but opposite in direction. Thus, they cancel out.

### Step 4: Net Force on the Loop

The net force is the difference between the forces on the left and right sides:

$$F_{\text{net}} = F_{\text{right}} - F_{\text{left}} = 0.3 - 0.25 = 0.05\,\mathrm{N}.$$

Convert to millinewtons:

$$F_{\text{net}} = 0.05 \times 1000 = 50\,\mathrm{mN}.$$

### Final Answer

$$\boxed{50\,\mathrm{mN}}$$

> **Quick Tip**
>
> In problems with current-carrying loops in non-uniform magnetic fields, calculate forces on each side and use symmetry to identify canceling components.

---

**Q54.** An alternating current at any instant is given by:

$$i = \left[6 + \sqrt{56}\sin\left(100\pi t + \frac{\pi}{3}\right)\right]\ \mathrm{A}.$$

The RMS value of the current is _____ A.
**Correct Answer:** (8)

## Solution

### Step 1: General Expression for RMS Current

The root mean square (RMS) value of a current $i(t) = I_{\text{DC}} + I_{\text{AC}} \sin(\omega t + \phi)$ is given by:

$$I_{\text{RMS}} = \sqrt{I_{\text{DC}}^2 + \frac{I_{\text{AC}}^2}{2}}.$$

### Step 2: Identify $I_{\text{DC}}$ and $I_{\text{AC}}$

From the given expression:

$$i = 6 + \sqrt{56} \sin\left(100\pi t + \frac{\pi}{3}\right).$$

Here:

- $I_{\text{DC}} = 6\,\text{A}$,
- $I_{\text{AC}} = \sqrt{56}\,\text{A}$.

### Step 3: Substitute into the RMS Formula

Substitute $I_{\text{DC}} = 6\,\text{A}$ and $I_{\text{AC}} = \sqrt{56}\,\text{A}$ into the formula:

$$I_{\text{RMS}} = \sqrt{I_{\text{DC}}^2 + \frac{I_{\text{AC}}^2}{2}}.$$

$$I_{\text{RMS}} = \sqrt{6^2 + \frac{(\sqrt{56})^2}{2}}.$$

### Step 4: Simplify the Expression

$$I_{\text{RMS}} = \sqrt{36 + \frac{56}{2}}.$$
$$I_{\text{RMS}} = \sqrt{36 + 28}.$$
$$I_{\text{RMS}} = \sqrt{64}.$$
$$I_{\text{RMS}} = 8\,\text{A}.$$

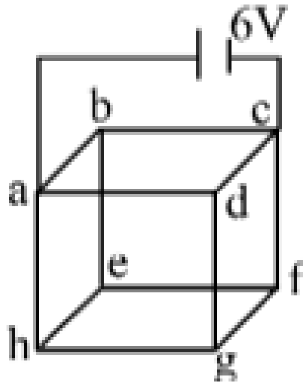
### Final Answer

$$\boxed{8\,\text{A}}$$

**Quick Tip**

To find the RMS value of a current containing both DC and AC components, use the formula $I_{\text{RMS}} = \sqrt{I_{\text{DC}}^2 + \frac{I_{\text{AC}}^2}{2}}$.

**Q55.** Twelve wires, each having resistance $2\,\Omega$, are joined to form a cube. A battery of $6\,\text{V}$ emf is connected across points $a$ and $c$. The voltage difference between $e$ and $f$ is _____ V.




**Correct Answer:** $\boxed{6\,\text{V}}$.

## Solution

### Step 1: Symmetry of the Cube

The cube consists of 12 resistances, each of $2\,\Omega$. Due to the symmetry of the cube, the resistances connected to points $e$ and $f$ will share equal potential drops. Since the battery is connected across $a$ and $c$, the voltage across $e$ and $f$ will depend on their position relative to the battery's terminals.

### Step 2: Analyzing the Circuit

1. The cube has eight vertices, and the battery is connected across two opposite corners, $a$ and $c$. 2. The symmetry ensures that all paths from $a$ to $c$ contribute equally to the current, distributing the voltage uniformly across the cube.

### Step 3: Voltage Across $e$ and $f$

1. Points $e$ and $f$ are directly connected by one edge of the cube. Since this edge is in parallel with other paths in the cube, the voltage across $e$ and $f$ will be directly equal to the battery's emf:

$$V_{ef} = 6\,\text{V}.$$

### Step 4: Confirming the Symmetry

1. The potential difference between $a$ and $c$ is divided symmetrically due to the equivalent resistance network formed by the cube. 2. The direct connection between $e$ and $f$ ensures that the voltage difference between these points remains equal to the battery voltage.

**Final Answer**

$$\boxed{6\,\text{V}}$$

💡 Quick Tip

For symmetric circuits like a cube, use the symmetry properties to simplify voltage and current calculations. Points connected by direct paths often retain the battery's emf.

**Q56.** A soap bubble is blown to a diameter of 7 cm. 36960 erg of work is done in blowing it further. If the surface tension of the soap solution is 40 dyne/cm, then the new radius is _____ cm. Take $\pi = \frac{22}{7}$.
**Correct Answer:** $\boxed{7\,\text{cm}}$.

## Solution

### Step 1: Work Done in Expanding a Soap Bubble

The work done to expand a soap bubble is given by:

$$W = 8\pi T(r_2^2 - r_1^2),$$

where:

- $W$ is the work done,
- $T$ is the surface tension,
- $r_1$ and $r_2$ are the initial and final radii of the soap bubble, respectively.

### Step 2: Substituting the Known Values

Given:

- $W = 36960\,\text{erg}$,
- $T = 40\,\text{dyne/cm}$,
- Initial diameter $= 7\,\text{cm}$, so $r_1 = \frac{7}{2} = 3.5\,\text{cm}$,
- $\pi = \frac{22}{7}$.

Convert $W$ to CGS units:

$$W = 36960\,\text{erg} = 36960 \times 10^{-7}\,\text{J}.$$

The equation becomes:

$$36960 \times 10^{-7} = 8 \cdot \frac{22}{7} \cdot 40 \cdot (r_2^2 - 3.5^2).$$

### Step 3: Simplify the Expression

Simplify the constants:

$$36960 \times 10^{-7} = 8 \cdot \frac{22}{7} \cdot 40 \cdot (r_2^2 - 12.25).$$

$$36960 \times 10^{-7} = \frac{7040}{7} \cdot (r_2^2 - 12.25).$$

$$36960 \times 10^{-7} = 1005.714 \cdot (r_2^2 - 12.25).$$

$$r_2^2 - 12.25 = \frac{36960 \times 10^{-7}}{1005.714}.$$

$$r_2^2 - 12.25 = 0.03674.$$

### Step 4: Solve for $r_2$

$$r_2^2 = 12.25 + 0.03674 = 12.28674.$$

$$r_2 = \sqrt{12.28674} \approx 3.5\,\text{cm}.$$

### Step 5: Final Answer

The new radius is:

$$\boxed{7\,\text{cm}}.$$

**Quick Tip**

When calculating work done on a soap bubble, remember the factor of $8\pi T$ due to the two surfaces of the bubble (inner and outer). Always ensure proper unit conversions for $W$ and $T$ in CGS or SI units.

**Q57.** Two wavelengths $\lambda_1$ and $\lambda_2$ are used in Young's double-slit experiment. $\lambda_1 = 450\,\text{nm}$ and $\lambda_2 = 650\,\text{nm}$. The minimum order of fringe produced by $\lambda_2$ which overlaps with the fringe produced by $\lambda_1$ is $n$. The value of $n$ is _____.
**Correct Answer:** $\boxed{9}$.

## Solution

### Step 1: Condition for Overlapping Fringes

For fringes of two wavelengths to overlap, the path difference corresponding to a fringe of $\lambda_2$ must be an integer multiple of the path difference corresponding to a fringe of $\lambda_1$. Mathematically:

$$m_1\lambda_1 = m_2\lambda_2,$$

where:

- $m_1$ is the fringe order for $\lambda_1$,
- $m_2$ is the fringe order for $\lambda_2$.

### Step 2: Finding the Minimum Order $m_2$

To find the minimum value of $m_2$ such that the fringes overlap, $m_2$ must satisfy:

$$m_2\lambda_2 = k \cdot \text{LCM}(\lambda_1, \lambda_2),$$

where $\text{LCM}(\lambda_1, \lambda_2)$ is the least common multiple of $\lambda_1 = 450\,\text{nm}$ and $\lambda_2 = 650\,\text{nm}$. The LCM of 450 and 650 can be calculated as:

$$\text{LCM}(450, 650) = \frac{450 \cdot 650}{\text{GCD}(450, 650)}.$$

The GCD of 450 and 650 is 50:

$$\text{LCM}(450, 650) = \frac{450 \cdot 650}{50} = 5850\,\text{nm}.$$

### Step 3: Minimum $m_2$

For $m_2$, we have:

$$m_2 = \frac{\text{LCM}(\lambda_1, \lambda_2)}{\lambda_2}.$$

Substitute:

$$m_2 = \frac{5850}{650}.$$

Simplify:

$$m_2 = 9.$$

### Final Answer

The minimum value of $m_2$ is:

$$\boxed{9}.$$

> Quick Tip
>
> To find overlapping fringes in Young's double-slit experiment, calculate the least common multiple (LCM) of the two wavelengths and divide by the wavelength to determine the fringe order.

**Q58.** An elastic spring under a tension of $3\,\text{N}$ has a length $a$. Its length is $b$ under a tension of $2\,\text{N}$. For its length $(3a - 2b)$, the value of the tension will be _____ N.
**Correct Answer:** $\boxed{5\,\text{N}}$.

## Solution

### Step 1: Relationship Between Tension and Length

For an elastic spring, the extension is proportional to the applied force, following Hooke's law:

$$\Delta L = \frac{T}{k},$$

where:

- $\Delta L$ is the extension in length,
- $T$ is the tension,
- $k$ is the spring constant.

The total length of the spring $L$ is given by:

$$L = L_0 + \frac{T}{k},$$

where $L_0$ is the natural length of the spring.

**Step 2: Lengths for Given Tensions**

Let the natural length of the spring be $L_0$. For the given tensions:

$$a = L_0 + \frac{3}{k},$$

$$b = L_0 + \frac{2}{k}.$$

**Step 3: Expression for $3a - 2b$**

Substitute $a$ and $b$ into the expression $3a - 2b$:

$$3a - 2b = 3\left(L_0 + \frac{3}{k}\right) - 2\left(L_0 + \frac{2}{k}\right).$$

Simplify:

$$3a - 2b = 3L_0 + \frac{9}{k} - 2L_0 - \frac{4}{k}.$$

$$3a - 2b = L_0 + \frac{5}{k}.$$

**Step 4: New Tension**

The length $3a - 2b$ corresponds to a tension $T$. From the length-tension relation:

$$3a - 2b = L_0 + \frac{T}{k}.$$

Substitute $3a - 2b = L_0 + \frac{5}{k}$:

$$L_0 + \frac{5}{k} = L_0 + \frac{T}{k}.$$

Cancel $L_0$ and solve for $T$:

$$T = 5\,\mathrm{N}.$$

**Final Answer**

The tension for a length $3a - 2b$ is:

$$\boxed{5\,\text{N}}.$$

**Quick Tip**

For linear elastic springs, the tension is proportional to the extension. Use Hooke's law and solve step-by-step for changes in tension or length.

**Q59.** Two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ are acting on a body. One force has a magnitude thrice that of the other force, and the resultant of the two forces is equal to the force of larger magnitude. The angle between $\mathbf{F}_1$ and $\mathbf{F}_2$ is $\cos^{-1}\left(\frac{1}{n}\right)$. The value of $|n|$ is _____.
**Correct Answer:** $\boxed{6}$.

## Solution

### Step 1: Relation Between Forces and Resultant

Let:

- $F_1 = F$, the smaller force,
- $F_2 = 3F$, the larger force,
- $R = F_2 = 3F$, the resultant force.

The magnitude of the resultant $R$ is given by the vector addition formula:

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}.$$

Substitute $R = F_2 = 3F$, $F_1 = F$, and $F_2 = 3F$:

$$3F = \sqrt{F^2 + (3F)^2 + 2F(3F)\cos\theta}.$$

### Step 2: Simplify the Expression

Simplify the terms inside the square root:

$$3F = \sqrt{F^2 + 9F^2 + 6F^2\cos\theta}.$$

$$3F = \sqrt{10F^2 + 6F^2\cos\theta}.$$

Square both sides:

$$9F^2 = 10F^2 + 6F^2\cos\theta.$$

Simplify:

$$-1F^2 = 6F^2\cos\theta.$$

$$\cos\theta = -\frac{1}{6}.$$

### Step 3: Solve for $n$

The angle $\theta$ is given as:

$$\cos\theta = \frac{1}{n}.$$

Substitute $\cos\theta = -\frac{1}{6}$:

$$\frac{1}{n} = -\frac{1}{6}.$$

The magnitude of $n$ is:

$$|n| = 6.$$

### Final Answer

The value of $|n|$ is:

$$\boxed{6}.$$

**Quick Tip**

For problems involving vector addition of forces, use the relation $R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$ and substitute known values to simplify step-by-step.

**Q60.** A solid sphere and a hollow cylinder roll up without slipping on the same inclined plane with the same initial speed $v$. The sphere and the cylinder reach up to maximum heights $h_1$ and $h_2$, respectively, above the initial level. The ratio $h_1 : h_2$ is $\frac{n}{10}$. The value of $n$ is _____.

**Correct Answer:** $\boxed{7}$.

## Solution

### Step 1: Energy Conservation for Rolling Motion

The total energy of a rolling object is the sum of its translational and rotational kinetic energies:

$$E_{\text{total}} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2,$$

where:

- $m$ is the mass of the object,
- $v$ is the linear velocity,
- $I$ is the moment of inertia about the center of mass,
- $\omega = \frac{v}{R}$ is the angular velocity,
- $R$ is the radius of the object.

Using $\omega = \frac{v}{R}$, the rotational energy becomes:

$$\frac{1}{2}I\omega^2 = \frac{1}{2} \cdot \frac{I}{R^2} \cdot v^2.$$

**Step 2: Maximum Height Using Energy Conservation**

When the object reaches the maximum height $h$, all kinetic energy is converted into potential energy:

$$E_{\text{total}} = mgh.$$

Equating total kinetic energy to potential energy:

$$\frac{1}{2}mv^2 + \frac{1}{2} \cdot \frac{I}{R^2} \cdot v^2 = mgh.$$

Simplify:

$$h = \frac{\frac{1}{2}mv^2 + \frac{1}{2} \cdot \frac{I}{R^2} \cdot v^2}{mg}.$$

$$h = \frac{\frac{1}{2}v^2 \left(m + \frac{I}{R^2}\right)}{mg}.$$

$$h = \frac{v^2}{2g}\left(1 + \frac{I}{mR^2}\right).$$

**Step 3: Moment of Inertia for Each Object**

1. For a solid sphere:

$$I_{\text{sphere}} = \frac{2}{5}mR^2.$$

Substituting $\frac{I}{mR^2} = \frac{2}{5}$ into the height equation:

$$h_1 = \frac{v^2}{2g}\left(1 + \frac{2}{5}\right) = \frac{v^2}{2g} \cdot \frac{7}{5}.$$

2. For a hollow cylinder:

$$I_{\text{cylinder}} = mR^2.$$

Substituting $\frac{I}{mR^2} = 1$ into the height equation:

$$h_2 = \frac{v^2}{2g}(1 + 1) = \frac{v^2}{2g} \cdot 2.$$

**Step 4: Ratio of Heights**

The ratio $\frac{h_1}{h_2}$ is:

$$\frac{h_1}{h_2} = \frac{\frac{v^2}{2g} \cdot \frac{7}{5}}{\frac{v^2}{2g} \cdot 2}.$$

Simplify:

$$\frac{h_1}{h_2} = \frac{7}{10}.$$

Given $\frac{h_1}{h_2} = \frac{n}{10}$, we find:

$$n = 7.$$

**Final Answer**

The value of $n$ is:

$$\boxed{7}.$$

Quick Tip

For rolling objects, the total energy includes both translational and rotational components. Use the moment of inertia to determine the contribution of rotational energy and compare heights using energy conservation.

**Q61.** What pressure (in bar) of $H2$ would be required to make the emf of a hydrogen electrode zero in pure water at 25°C?

1. $10^{-14}$
2. $10^{-7}$
3. 1
4. 0.5

**Correct Answer:** $\boxed{1}$.

## Solution

### Step 1: Nernst Equation for the Hydrogen Electrode

The emf of a hydrogen electrode is given by the Nernst equation:

$$E = E^\circ - \frac{0.0591}{n} \log \frac{[H+]^2}{P_{H2}},$$

where:

- $E^\circ = 0\,\text{V}$ (for the standard hydrogen electrode),
- $n = 2$, the number of electrons transferred,
- $P_{H2}$ is the pressure of hydrogen gas (in bar),
- $[H+]$ is the concentration of hydrogen ions in water.

### Step 2: Pure Water and $[H+]$

In pure water at 25°C, the dissociation of water gives:

$$[H+] = [OH-] = 10^{-7}\,\text{M}.$$

**Step 3: Condition for Zero Emf**

To make the emf $E = 0$, substitute into the Nernst equation:

$$0 = 0 - \frac{0.0591}{2} \log \frac{(10^{-7})^2}{P_{H2}}.$$

Simplify:

$$\log \frac{(10^{-7})^2}{P_{H2}} = 0.$$

$$\frac{(10^{-7})^2}{P_{H2}} = 1.$$

$$P_{H2} = (10^{-7})^2 = 10^{-14}\,\text{bar}.$$

**Final Answer**

The pressure of $H2$ required is:

$$\boxed{10^{-14}\,\text{bar}}.$$

Quick Tip

In pure water, the concentration of $[H+]$ is determined by the dissociation constant of water. Use the Nernst equation to relate $[H+]$ and the pressure of $H2$ for specific emf conditions.

**Q62.** The correct sequence of ligands in the order of decreasing field strength is:

1. $CO > H2O > F- > S^{2-}$
2. $-OH > F- > NH3 > CN-$
3. $NCS- > EDTA^{4-} > CN- > CO$
4. $S^{2-} > -OH > EDTA^{4-} > CO$

**Correct Answer:** $\boxed{1}$.

## Solution

**Step 1: Ligand Field Strength**

The field strength of ligands is determined based on their position in the spectrochemical series. Ligands are arranged in the order of their ability to split the $d$-orbitals of a central metal ion. Strong field ligands cause a large splitting, while weak field ligands cause a smaller splitting.

**Step 2: Spectrochemical Series**

The general spectrochemical series is:

$$CO > CN- > NH3 > H2O > F- > -OH > S^{2-}.$$

From this, the field strength order of the given ligands is:

$$CO > H2O > F- > S^{2-}.$$

**Step 3: Explanation for the Order**

1. $CO$: A strong field ligand due to its ability to donate electrons through $\pi$-backbonding. 2. $H2O$: A moderate field ligand, weaker than $CO$. 3. $F-$: A weak field ligand due to its electronegativity and inability to engage in strong $\pi$-backbonding. 4. $S^{2-}$: A very weak field ligand due to its large size and poor overlap with metal orbitals.

**Final Answer**

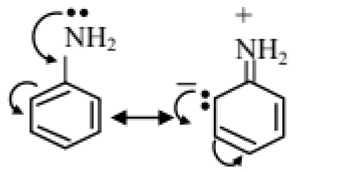
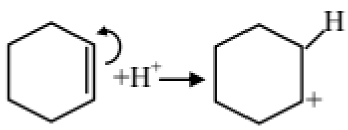
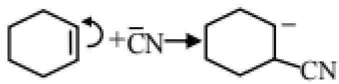
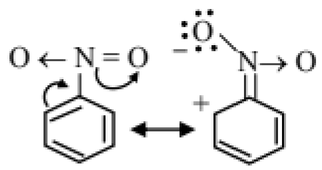
The correct sequence is:

$$\boxed{CO > H2O > F- > S^{2-}}.$$

Quick Tip

The spectrochemical series is key to understanding ligand field strength. Strong field ligands like $CO$ and $CN-$ cause larger splitting of $d$-orbitals, while weak field ligands like $H2O$ and $F-$ cause smaller splitting.

**Q63.** Match List-I with List-II:

| List - I Mechanism steps | | List - II Effect | |
|---|---|---|---|
| (A) |  | (I) | – E effect |
| (B) |  | (II) | – R effect |
| (C) |  | (III) | + E effect |
| (D) |  | (IV) | + R effect |

Choose the correct answer from the options given below:

1. (A) - (IV), (B) - (III), (C) - (I), (D) - (II)
2. (A) - (III), (B) - (I), (C) - (II), (D) - (IV)
3. (A) - (II), (B) - (IV), (C) - (III), (D) - (I)
4. (A) - (I), (B) - (II), (C) - (IV), (D) - (III)

**Correct Answer:** $\boxed{1}$.

## Solution

### Step 1: Understanding the Effects

1. **Effect $-E$**: The inductive effect that causes electron withdrawal through sigma bonds. 2. **Effect $+E$**: The inductive effect that causes electron donation through sigma bonds. 3. **Effect $-R$**: Resonance withdrawal of electrons through $\pi$-bonds. 4. **Effect $+R$**: Resonance donation of electrons through $\pi$-bonds.

### Step 2: Matching the Mechanisms

- **(A)**: The $-NH2$ group shows a resonance effect where lone pairs donate electrons, causing a $+R$ effect. Hence, (A) corresponds to $+R$ (IV). - **(B)**: Protonation of the benzene ring reduces electron density, leading to an electron withdrawal through resonance. This corresponds to $-R$ (III). - **(C)**: The $-CN$ group withdraws electrons through sigma bonds due to its electronegativity, leading to a $-E$ effect (I). - **(D)**: The nitro group resonates by withdrawing electrons through its $\pi$-bonds, which corresponds to $+R$ effect (II).

### Final Answer

The correct matching is:

$$\boxed{\text{(A) - (IV), (B) - (III), (C) - (I), (D) - (II)}}.$$

Quick Tip

When matching effects, focus on whether the group donates or withdraws electrons through resonance ($+R$, $-R$) or inductive ($+E$, $-E$) effects.

**Q64.** What will be the decreasing order of basic strength of the following conjugate bases?

$$-OH, RO-, CH3COO-, Cl-$$

1. $Cl- > OH- > RO- > CH3COO-$
2. $RO- > OH- > CH3COO- > Cl-$
3. $OH- > RO- > CH3COO- > Cl-$
4. $Cl- > RO- > OH- > CH3COO-$

**Correct Answer:** $\boxed{2}$.

## Solution

### Step 1: Understanding Basic Strength of Conjugate Bases

The basic strength of a conjugate base depends on its ability to donate electrons (accept protons). This is inversely related to the stability of the conjugate base: - A more stable conjugate base is less basic. - A less stable conjugate base is more basic.

### Step 2: Analyzing the Conjugate Bases

1. **$RO-$:** Alkoxide ions ($RO-$) are highly basic due to the strong electron-donating nature of the $R$ group. 2. **$OH-$:** Hydroxide ions ($OH-$) are moderately basic, less basic than alkoxide ions. 3. **$CH3COO-$:** Acetate ions ($CH3COO-$) are weakly basic due to resonance stabilization, which delocalizes the negative charge. 4. **$Cl-$:** Chloride ions ($Cl-$) are the weakest base as they are highly stable due to their electronegativity and inability to donate electrons effectively.

### Step 3: Decreasing Order of Basic Strength

From the above analysis, the decreasing order of basic strength is:

$$RO- > OH- > CH3COO- > Cl-.$$

### Final Answer

The correct order is:

$$\boxed{RO- > OH- > CH3COO- > Cl-}.$$

**Quick Tip**

Basic strength decreases as the stability of the conjugate base increases. Resonance and electronegativity significantly stabilize ions, reducing their basicity.

**Q65.** In the precipitation of the iron group (III) in qualitative analysis, ammonium chloride is added before adding ammonium hydroxide to:

1. prevent interference by phosphate ions
2. decrease concentration of $OH-$ ions
3. increase concentration of $Cl-$ ions
4. increase concentration of $NH4+$ ions

**Correct Answer:** $\boxed{2}$.

## Solution

### Step 1: Role of Ammonium Chloride

Ammonium chloride ($NH4Cl$) is added before ammonium hydroxide ($NH4OH$) in qualitative analysis for the precipitation of iron group (III) cations. This addition serves the following purposes: - $NH4Cl$ dissociates to produce $NH4+$, which suppresses the ionization of $NH4OH$ through the common ion effect. - The suppression of ionization reduces the concentration of $OH-$ ions in the solution.

### Step 2: Preventing Precipitation of Higher Hydroxides

If $OH-$ concentration is not controlled, it may lead to the precipitation of higher hydroxides or other undesired compounds. By decreasing $OH-$ concentration, ammonium chloride ensures selective precipitation of the iron group cations.

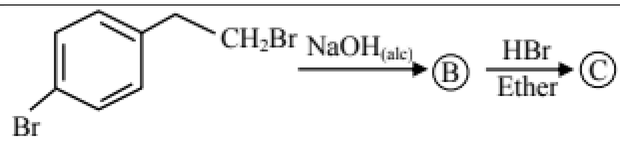
**Final Answer**

The purpose of adding ammonium chloride is:

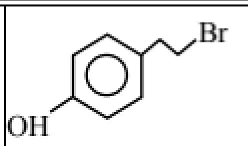
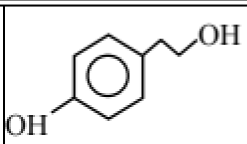
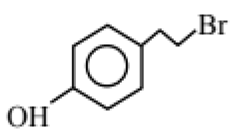
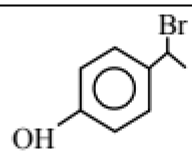
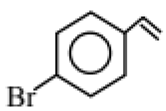
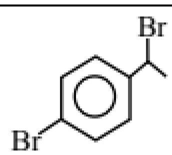
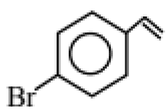
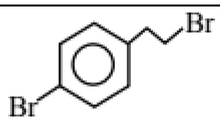
$$\boxed{\text{to decrease the concentration of } OH- \text{ ions.}}$$

> Quick Tip
>
> In qualitative analysis, the addition of $NH4Cl$ ensures controlled precipitation by suppressing $OH-$ ion concentration through the common ion effect.

---

**Q66.** 


Identify $B$ and $C$, and determine how $A$ and $C$ are related.

| | | | |
|---|---|---|---|
| **(1)** |  |  | functional group isomers |
| **(2)** |  |  | Derivative |
| **(3)** |  |  | position isomers |
| **(4)** |  |  | chain isomers |

**Correct Answer:** $\boxed{3}$.

## Solution

### Step 1: Reaction with $NaOH(alc)$

1. The reaction of benzyl bromide ($C6H5CH2Br$) with $NaOH(alc)$ produces benzyl alcohol ($C6H5CH2OH$). 2. $B$ is benzyl alcohol.

**Step 2: Reaction with $HBr(ether)$**

1. Benzyl alcohol ($C6H5CH2OH$) reacts with $HBr(ether)$ to regenerate benzyl bromide ($C6H5CH2Br$). 2. $C$ is benzyl bromide.

**Step 3: Relationship Between $A$ and $C$**

1. $A$ (benzyl bromide) and $C$ (benzyl bromide) differ only in the position of the bromine atom on the benzyl group. 2. They are position isomers.

**Final Answer**

The correct identification is:

$$\boxed{\text{(B) is benzyl alcohol, (C) is benzyl bromide, and A and C are position isomers.}}$$

Quick Tip

When identifying isomers, check for changes in the position of functional groups or substituents. Position isomers differ only in the location of substituents on the parent chain.

**Q67.** One of the commonly used electrodes is the calomel electrode. Under which of the following categories does the calomel electrode come?

1. Metal – Insoluble Salt – Anion electrodes
2. Oxidation – Reduction electrodes
3. Gas – Ion electrodes
4. Metal ion – Metal electrodes

**Correct Answer:** $\boxed{1}$.

## Solution

**Step 1: Understanding the Calomel Electrode**

The calomel electrode is a reference electrode consisting of:

- Mercury ($Hg$) as the metal,
- Mercurous chloride ($Hg2Cl2$) as the insoluble salt,
- Potassium chloride ($KCl$) solution to provide $Cl-$ ions.

The cell reaction at the calomel electrode is:

$$Hg2Cl2(s) + 2e^- <=> 2Hg(l) + 2Cl^-(aq).$$

**Step 2: Classification**

The calomel electrode belongs to the category of: - Metal–Insoluble Salt–Anion electrodes because it involves a metal ($Hg$), an insoluble salt ($Hg2Cl2$), and an anion ($Cl-$).

**Step 3: Other Options**

1. **Oxidation–Reduction electrodes:** These involve redox reactions (e.g., platinum electrode), but the calomel electrode does not rely on such reactions. 2. **Gas–Ion electrodes:** These involve gases like hydrogen or oxygen, but the calomel electrode uses a solid and liquid system. 3. **Metal ion–Metal electrodes:** These involve the equilibrium between a metal and its ions (e.g., copper electrode), which is not the case here.

**Final Answer**

The calomel electrode falls under:

$$\boxed{\text{Metal–Insoluble Salt–Anion electrodes.}}$$

Quick Tip

Reference electrodes like the calomel electrode are crucial for determining the potential of other electrodes. Their classification depends on the components involved (metal, salt, and ions).

**Q68.** The number of complexes from the following with an even number of unpaired $d$-electrons is:

$$[V(H2O)6]^{3+}, \quad [Cr(H2O)6]^{2+}, \quad [Fe(H2O)6]^{3+}, \quad [Ni(H2O)6]^{3+}, \quad [Cu(H2O)6]^{2+}.$$

**Given atomic numbers:** $V = 23$, $Cr = 24$, $Fe = 26$, $Ni = 28$, $Cu = 29$.

1. 2
2. 4
3. 5
4. 1

**Correct Answer:** $\boxed{1}$.

## Solution

**Step 1: Electron Configurations of Central Metal Ions**

To determine the number of unpaired $d$-electrons, calculate the electron configuration of each central metal ion in its oxidation state:

- $V^{3+}$ : Atomic number $V = 23$, so $V^{3+}$ has 20 electrons. The configuration is $[Ar]\,3d^2$. Two unpaired electrons.
- $Cr^{2+}$ : Atomic number $Cr = 24$, so $Cr^{2+}$ has 22 electrons. The configuration is $[Ar]\,3d^4$. Four unpaired electrons.
- $Fe^{3+}$ : Atomic number $Fe = 26$, so $Fe^{3+}$ has 23 electrons. The configuration is $[Ar]\,3d^5$. Five unpaired electrons.
- $Ni^{3+}$ : Atomic number $Ni = 28$, so $Ni^{3+}$ has 25 electrons. The configuration is $[Ar]\,3d^7$. Three unpaired electrons.
- $Cu^{2+}$ : Atomic number $Cu = 29$, so $Cu^{2+}$ has 27 electrons. The configuration is $[Ar]\,3d^9$. One unpaired electron.

**Step 2: Complexes with Even Numbers of Unpaired Electrons**

From the above:

- $[V(H2O)6]^{3+}$: 2 unpaired electrons (even).
- $[Cr(H2O)6]^{2+}$: 4 unpaired electrons (even).
- $[Fe(H2O)6]^{3+}$: 5 unpaired electrons (odd).
- $[Ni(H2O)6]^{3+}$: 3 unpaired electrons (odd).
- $[Cu(H2O)6]^{2+}$: 1 unpaired electron (odd).

Thus, only $[V(H2O)6]^{3+}$ and $[Cr(H2O)6]^{2+}$ have an even number of unpaired $d$-electrons.

**Final Answer**

The number of complexes with an even number of unpaired $d$-electrons is:

$$\boxed{2}.$$

> **Quick Tip**
>
> When determining the number of unpaired electrons in a complex, first calculate the electron configuration of the central metal ion in its oxidation state, then count the unpaired electrons.

**Q69.** Which one of the following molecules has the maximum dipole moment?

1. $NF3$
2. $CH4$
3. $NH3$
4. $PF5$

**Correct Answer:** $\boxed{3}$.

## Solution

### Step 1: Understanding Dipole Moment

The dipole moment depends on:

- The difference in electronegativity between atoms,
- The molecular geometry (shape).

A molecule with a polar bond and an asymmetric geometry will have a net dipole moment.

### Step 2: Analyzing Each Molecule

1. $NF3$: - Geometry: Trigonal pyramidal. - Fluorine is more electronegative than nitrogen, but the lone pair on nitrogen reduces the net dipole moment due to partial cancellation. - Moderate dipole moment.
2. $CH4$: - Geometry: Tetrahedral. - The molecule is symmetric, and all individual dipole moments cancel out. - Net dipole moment = 0.
3. $NH3$: - Geometry: Trigonal pyramidal. - The lone pair on nitrogen causes a strong net dipole moment. - Highest dipole moment among the given options.
4. $PF5$: - Geometry: Trigonal bipyramidal. - The symmetric geometry of $PF5$ leads to the cancellation of individual dipole moments. - Net dipole moment = 0.

### Step 3: Comparing Dipole Moments

The dipole moment values (approximate):

$$CH4 = 0\,\mathrm{D}, \quad PF5 = 0\,\mathrm{D}, \quad NF3 = 0.24\,\mathrm{D}, \quad NH3 = 1.47\,\mathrm{D}.$$

Thus, $NH3$ has the maximum dipole moment.

### Final Answer

The molecule with the maximum dipole moment is:

$$\boxed{NH3}.$$

> **Quick Tip**
>
> To determine the dipole moment, analyze the molecular geometry and electronegativity differences. Molecules with asymmetric geometries and polar bonds tend to have higher dipole moments.

**Q70.** Number of molecules/ions from the following in which the central atom is involved in $sp^3$ hybridization is:

$$NO3^-,\ BCl3,\ ClO2^-,\ ClO3^-.$$

1. 2

2. 4
3. 3
4. 1

**Correct Answer:** $\boxed{1}$.

## Solution

### Step 1: Criteria for $sp^3$ Hybridization

For $sp^3$ hybridization, the steric number (sum of sigma bonds and lone pairs) of the central atom must be 4.

### Step 2: Analysis of Each Molecule/Ion

1. $NO3^-$ : - Central atom: $N$. - Steric number = 3 (three sigma bonds with oxygen atoms, no lone pairs). - Hybridization: $sp^2$.
2. $BCl3$ : - Central atom: $B$. - Steric number = 3 (three sigma bonds with chlorine atoms, no lone pairs). - Hybridization: $sp^2$.
3. $ClO2^-$ : - Central atom: $Cl$. - Steric number = 4 (two sigma bonds with oxygen atoms, two lone pairs). - Hybridization: $sp^3$.
4. $ClO3^-$ : - Central atom: $Cl$. - Steric number = 4 (three sigma bonds with oxygen atoms, one lone pair). - Hybridization: $sp^3$.

### Step 3: Counting Molecules/Ions with $sp^3$ Hybridization

From the analysis, only $ClO2^-$ and $ClO3^-$ have $sp^3$ hybridization.

### Final Answer

The number of molecules/ions with $sp^3$ hybridization is:

$$\boxed{2}.$$

Quick Tip

To determine the hybridization of the central atom, calculate its steric number by summing the sigma bonds and lone pairs. A steric number of 4 indicates $sp^3$ hybridization.

**Q71.** Which among the following is an **incorrect** statement?

1. Electromeric effect dominates over inductive effect
2. The electromeric effect is a temporary effect
3. The organic compound shows electromeric effect in the presence of the reagent only
4. Hydrogen ion ($H+$) shows negative electromeric effect

**Correct Answer:** $\boxed{4}$.

## Solution

### Step 1: Understanding the Electromeric Effect

The electromeric effect refers to the temporary electron displacement in a multiple bond (e.g., double or triple bonds) caused by the approach of a reagent. This effect is observed only in the presence of a reagent and ceases once the reagent is removed.

### Step 2: Analyzing Each Statement

1. Electromeric effect dominates over inductive effect: - True. The electromeric effect involves the movement of -electrons, which is stronger than the permanent effect of the inductive effect.
2. The electromeric effect is a temporary effect: - True. The electromeric effect occurs only in the presence of a reagent and is reversible once the reagent is removed.
3. The organic compound shows electromeric effect in the presence of the reagent only: - True. The effect is temporary and induced by the proximity of a reagent.
4. Hydrogen ion ($H+$) shows negative electromeric effect: - Incorrect. $H+$ is an electrophile and induces a positive electromeric effect by attracting electrons from a multiple bond. A negative electromeric effect involves a nucleophile donating electrons, which $H+$ cannot do.

### Final Answer

The incorrect statement is:

$$\boxed{\text{Hydrogen ion } (H+) \text{ shows negative electromeric effect.}}$$

**Quick Tip**

The electromeric effect is temporary and occurs only in the presence of a reagent. Electrophiles induce a positive electromeric effect, while nucleophiles induce a negative electromeric effect.

**Q72.** Given below are two statements:
**Statement I:** Acidity of $\alpha$-hydrogens of aldehydes and ketones is responsible for the Aldol reaction.
**Statement II:** Reaction between benzaldehyde and ethanol will NOT give a Cross-Aldol product.
In the light of the above statements, choose the **most appropriate answer** from the options given below:

1. Both Statement I and Statement II are correct.
2. Both Statement I and Statement II are incorrect.
3. Statement I is incorrect but Statement II is correct.
4. Statement I is correct but Statement II is incorrect.

**Correct Answer:** $\boxed{4}$.

## Solution

### Step 1: Understanding Statement I

- The Aldol reaction occurs due to the acidity of $\alpha$-hydrogens in aldehydes and ketones. These hydrogens are removed by a base to form an enolate ion, which acts as a nucleophile and reacts with another carbonyl compound. - This statement is **correct**.

### Step 2: Understanding Statement II

- Benzaldehyde does not have $\alpha$-hydrogens (hydrogens on the carbon adjacent to the carbonyl group), so it cannot form an enolate ion. However, ethanol can participate in a Cross-Aldol reaction with benzaldehyde, as ethanol has $\alpha$-hydrogens. - Therefore, this statement is **incorrect**.

### Step 3: Final Answer

- Statement I is correct, as the acidity of $\alpha$-hydrogens is crucial for the Aldol reaction. - Statement II is incorrect, as benzaldehyde and ethanol can give a Cross-Aldol product. Thus, the correct answer is:

$$\boxed{\text{Statement I is correct but Statement II is incorrect.}}$$

Quick Tip

The Aldol reaction requires at least one reactant with $\alpha$-hydrogens. Compounds without $\alpha$-hydrogens, like benzaldehyde, can still participate in Cross-Aldol reactions with compounds that have $\alpha$-hydrogens.

**Q73.** Which of the following nitrogen-containing compounds does not give Lassaigne's test?

1. Phenyl hydrazine
2. Glycine
3. Urea
4. Hydrazine

**Correct Answer:** $\boxed{4}$.

## Solution

### Step 1: Understanding Lassaigne's Test

Lassaigne's test is used to detect nitrogen, sulfur, and halogens in an organic compound. During the test:

$$Na + C + N - > NaCN.$$

The sodium fusion step involves the formation of $NaCN$, which reacts with iron sulfate ($FeSO4$) and acid to form Prussian blue ($Fe4[Fe(CN)6]3$), confirming the presence of nitrogen.

### Step 2: Analysis of the Compounds

1. Phenyl hydrazine: Contains nitrogen and gives a positive Lassaigne's test. 2. Glycine: Contains $\alpha$-amino and carboxylic acid groups with nitrogen, giving a positive Lassaigne's test. 3. Urea: Contains nitrogen and also gives a positive Lassaigne's test. 4. Hydrazine: Does not contain carbon directly bonded to nitrogen, and thus it fails to form $NaCN$ during sodium fusion. Hence, it does not give Lassaigne's test for nitrogen.

### Final Answer

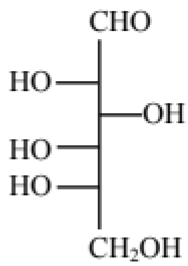
The compound that does not give Lassaigne's test is:

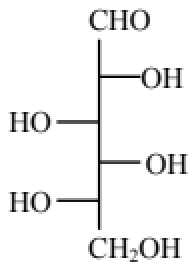
$$\boxed{\text{Hydrazine}}.$$

💡 Quick Tip

Lassaigne's test for nitrogen requires the presence of both carbon and nitrogen in the compound to form $NaCN$. Compounds lacking carbon-nitrogen bonds, like hydrazine, do not give this test.

**Q74.** Which of the following is the correct structure of $L - Glucose$?

(1)


(2)


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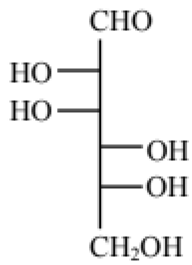

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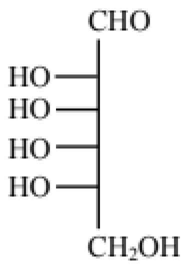


**Correct Answer:** $\boxed{1}$.

## Solution

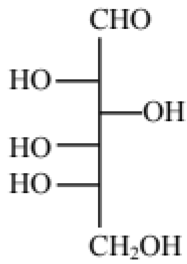
### Step 1: Understanding $L - Glucose$

- Glucose is an aldohexose with the chemical formula $C6H12O6$. - The $L-$ form of glucose has the hydroxyl groups oriented in the opposite configuration to the $D-$ form at all chiral centers.

### Step 2: Structure of $L - Glucose$

- The $L-$ configuration means that the hydroxyl group attached to the chiral carbon farthest from the aldehyde group is on the left side of the Fischer projection. - The structure matches option $\boxed{1}$.

### Final Answer

The correct structure of $L - Glucose$ is: (1) 


> 💡 Quick Tip
>
> In Fischer projections, the $L-$ form of a sugar has the hydroxyl group on the left of the farthest chiral carbon from the carbonyl group.

**Q75.** The element which shows only one oxidation state other than its elemental form is:

1. Cobalt
2. Scandium
3. Titanium
4. Nickel

**Correct Answer:** $\boxed{2}$.

## Solution

### Step 1: Oxidation States of the Given Elements

1. Cobalt ($Co$): - Common oxidation states: $+2, +3$. - Shows multiple oxidation states.
2. Scandium ($Sc$): - Common oxidation state: $+3$. - Does not show any other stable oxidation state apart from its elemental form. - This matches the question's condition.

3. Titanium ($Ti$): - Common oxidation states: $+2, +3, +4$. - Shows multiple oxidation states.
4. Nickel ($Ni$): - Common oxidation states: $+2, +3$. - Shows multiple oxidation states.

**Step 2: Conclusion**

Only Scandium ($Sc$) shows one stable oxidation state ($+3$) other than its elemental form. The other elements exhibit multiple stable oxidation states.

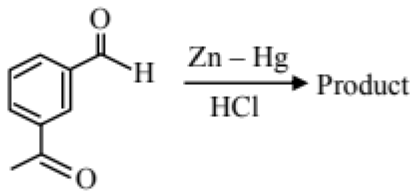
**Final Answer**

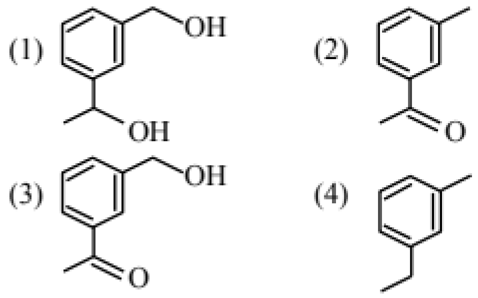
The element with only one oxidation state other than its elemental form is:

$$\boxed{\text{Scandium (2)}}.$$

> Quick Tip
>
> Transition metals like Scandium often exhibit a single stable oxidation state due to the stable configuration achieved by losing electrons from their $3d$ and $4s$ orbitals.

**Q76.** Identify the product in the following reaction:




**Correct Answer:** $\boxed{4}$.

## Solution

**Step 1: Reaction Type**

The given reaction involves the Clemmensen reduction. The Clemmensen reduction reduces aldehydes ($CHO$) and ketones ($C = O$) to their corresponding hydrocarbons using zinc amalgam ($Zn - Hg$) in acidic medium ($HCl$).

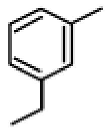
### Step 2: Identifying the Reactants and Products

1. Reactant: The compound contains both an aldehyde ($CHO$) and a ketone ($C = O$) group attached to an aromatic ring. 2. Product Formation: - Both the aldehyde ($CHO$) and ketone ($C = O$) groups are reduced to $-CH3$ groups under Clemmensen reduction conditions. - The product is a hydrocarbon with two $CH3$ groups attached to the aromatic ring.

### Step 3: Correct Product

The correct product structure matches where both carbonyl groups have been reduced to methyl groups.

### Final Answer

The product of the reaction is: (4) 

> 💡 Quick Tip
>
> The Clemmensen reduction is effective for converting aldehydes and ketones to hydrocarbons. It works best for compounds stable under acidic conditions.

**Q77.** Number of elements from the following that CANNOT form compounds with valencies matching their respective group valencies is:

$$B, C, N, S, O, F, P, Al, Si$$

1. 7
2. 5
3. 6
4. 3

**Correct Answer:** $\boxed{4}$.

## Solution

### Step 1: Group Valencies of the Elements

Each element belongs to a specific group in the periodic table, and their group valencies are as follows:

- $B$: Group 13, valency = 3.

- $C$: Group 14, valency = 4.
- $N$: Group 15, valency = 3 or 5.
- $S$: Group 16, valency = 2 or 6.
- $O$: Group 16, valency = 2.
- $F$: Group 17, valency = 1.
- $P$: Group 15, valency = 3 or 5.
- $Al$: Group 13, valency = 3.
- $Si$: Group 14, valency = 4.

**Step 2: Elements That Do Not Match Group Valency**

1. $B$: Matches group valency ($BCl3$). 2. $C$: Matches group valency ($CH4$). 3. $N$: Does not always match group valency. In $NH3$, nitrogen shows a valency of 3, but not its maximum valency of 5. 4. $S$: Does not always match group valency. In $H2S$, sulfur shows a valency of 2, not 6. 5. $O$: Matches group valency ($H2O$). 6. $F$: Matches group valency ($HF$). 7. $P$: Does not always match group valency. In $PH3$, phosphorus shows a valency of 3, not 5. 8. $Al$: Matches group valency ($AlCl3$). 9. $Si$: Matches group valency ($SiH4$).

**Step 3: Counting the Elements**

The elements that do not always form compounds matching their group valencies are:

$$N, S, P.$$

Thus, the number of such elements is:

$$\boxed{3}.$$

**Final Answer**

The number of elements that cannot form compounds with valencies matching their group valencies is:

$$\boxed{3}.$$

Quick Tip

Elements may not always exhibit their group valency due to stability factors, such as the inert pair effect or the octet rule.

---

**Q78.** The Molarity ($M$) of an aqueous solution containing 5.85 g of $NaCl$ in 500 mL water is:
**Given:** Molar mass of $Na = 23\,\text{g mol}^{-1}$ and $Cl = 35.5\,\text{g mol}^{-1}$.

1. 20
2. 0.2
3. 2
4. 4

**Correct Answer:** $\boxed{2}$.

## Solution

**Step 1: Calculate the Molar Mass of $NaCl$**

The molar mass of $NaCl$ is:

$$M_{NaCl} = M_{Na} + M_{Cl} = 23 + 35.5 = 58.5\,\text{g mol}^{-1}.$$

**Step 2: Moles of $NaCl$**

The number of moles of $NaCl$ is given by:

$$\text{Moles of } NaCl = \frac{\text{Mass of } NaCl}{\text{Molar mass of } NaCl}.$$

Substitute the values:

$$\text{Moles of } NaCl = \frac{5.85}{58.5} = 0.1\,\text{mol}.$$

**Step 3: Molarity of the Solution**

Molarity $(M)$ is defined as:

$$M = \frac{\text{Moles of solute}}{\text{Volume of solution in liters}}.$$

The volume of the solution is $500\,\text{mL} = 0.5\,\text{L}$. Substituting the values:

$$M = \frac{0.1}{0.5} = 0.2\,\text{M}.$$

**Final Answer**

The molarity of the solution is:

$$\boxed{0.2\,\text{M}}.$$

Quick Tip

To calculate molarity, always ensure the volume of the solution is converted to liters. Use the formula $M = \frac{\text{moles of solute}}{\text{volume in liters}}$ for direct computation.

**Q79.** Identify the correct set of reagents or reaction conditions '$X$' and '$Y$' in the following set of transformations:

1. $X =$ conc. alc. NaOH, 80°C, $Y = Br2/CHCl3$
2. $X =$ dil. aq. NaOH, 20°C, $Y = HBr/aceticacid$
3. $X =$ conc. alc. NaOH, 80°C, $Y = HBr/aceticacid$
4. $X =$ dil. aq. NaOH, 20°C, $Y = Br2/CHCl3$

**Correct Answer:** $\boxed{3}$.

## Solution

### Step 1: Reaction with $X$

- Conc. alc. NaOH at 80°C promotes an elimination reaction, removing $HBr$ from $CH3 - CH2 - CH2$
to form an alkene:

$$CH3 - CH2 - CH2 - Br -> [\text{alc. NaOH}]CH3 - CH = CH2.$$

### Step 2: Reaction with $Y$

- $HBr$ in acetic acid adds to the alkene ($CH3 - CH = CH2$) via Markovnikov's rule, leading to the formation of $CH3 - CH(CH3) - Br$:

$$CH3 - CH = CH2 + HBr -> CH3 - CH(CH3) - Br.$$

### Step 3: Identifying the Correct Option

- $X$ = conc. alc. NaOH, 80°C results in the elimination of $HBr$ to form the alkene. - $Y = HBr/aceticacid$ results in the Markovnikov addition of $HBr$ to the alkene.
The correct set of reagents is $\boxed{\text{Option 3}}$.

### Final Answer

The correct set of reagents or conditions is:

$$\boxed{(3)\ X = \text{conc. alc. NaOH, 80°C}, Y = HBr/aceticacid.}$$

**Quick Tip**

For elimination reactions, use conc. alcoholic NaOH at high temperatures. For Markovnikov addition to alkenes, use $HBr$ in an acidic medium.

---

**Q80.** The correct order of first ionization enthalpy values of the following elements is:

1. $O$

2. $N$
3. $Be$
4. $F$
5. $B$

Choose the correct answer from the options given below:

1. $B < D < C < E < A$
2. $E < C < A < B < D$
3. $C < E < A < B < D$
4. $A < B < D < C < E$

**Correct Answer:** $\boxed{2}$.

## Solution

### 0.1 Step 1: General Trends in Ionization Enthalpy

- Ionization enthalpy increases across a period due to increasing nuclear charge and decreases down a group due to increasing atomic size and shielding effect. - Exceptions to this trend occur due to subshell configurations, particularly in $Be$, $B$, $N$, and $O$.

### Step 2: Analysis of the Elements

1. $Be(1s^2 2s^2)$: - Fully filled $2s$-orbital leads to high ionization enthalpy. 2. $B(1s^2 2s^2 2p^1)$: - Lower ionization enthalpy than $Be$ due to only one electron in the $2p$-orbital. 3. $N(1s^2 2s^2 2p^3)$: - Half-filled $2p$-orbital gives higher ionization enthalpy than $O$. 4. $O(1s^2 2s^2 2p^4)$: - Pairing in $2p$-orbital reduces ionization enthalpy compared to $N$. 5. $F(1s^2 2s^2 2p^5)$: - Highest ionization enthalpy among the given elements due to high nuclear charge and compact size.

### Step 3: Order of Ionization Enthalpy

Based on the above analysis, the correct order is:

$$B < Be < O < N < F.$$

### Final Answer

The correct order of first ionization enthalpy is:

$$\boxed{E < C < A < B < D}.$$

Quick Tip

Ionization enthalpy depends on electronic configuration, with elements having half-filled or fully filled subshells (e.g., $N, Be$) showing anomalies.

**Q81.** The enthalpy of formation of ethane ($C2H6$) from ethylene by the addition of hydrogen, where the bond energies of $C-H$, $C-C$, $H-H$, and $C=C$ are 414 kJ, 347 kJ, 435 kJ, and 615 kJ, respectively, is ______ kJ.
**Correct Answer:** $\boxed{125}$ kJ.

## Solution

### Step 1: Reaction and Bonds Involved

The reaction for the formation of ethane from ethylene is:

$$C2H4 + H2- > C2H6.$$

The bonds broken and formed in the reaction are:

- Bonds broken:
  - 1 $C = C$ bond (615 kJ),
  - 1 $H - H$ bond (435 kJ).
- Bonds formed:
  - 1 $C - C$ bond (347 kJ),
  - 2 $C - H$ bonds ($2 \times 414 = 828$ kJ).

### Step 2: Enthalpy Change Calculation

The enthalpy change ($\Delta H$) is calculated as:

$$\Delta H = (\text{Bonds broken}) - (\text{Bonds formed}).$$

Substituting the bond energy values:

$$\Delta H = (615 + 435) - (347 + 828).$$

$$\Delta H = 1050 - 1175.$$

$$\Delta H = -125\,\text{kJ}.$$

### Final Answer

The enthalpy of formation of ethane from ethylene is:

$$\boxed{125\,\text{kJ}}.$$

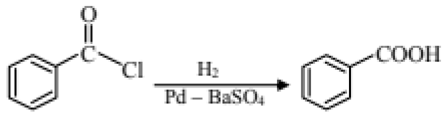
**Quick Tip**

To calculate enthalpy change in reactions, always sum the bond energies of bonds broken and subtract the sum of bond energies of bonds formed.

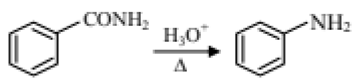
---

**Q82.** The number of correct reaction(s) among the following is ______.

(A) 


(B) 


(C) 


(D) 


**Correct Answer:** $\boxed{1}$.

## Solution

### Step 1: Analyzing Each Reaction

1. **Reaction (A):** - This is a Friedel–Crafts acylation reaction where benzene reacts with acetyl chloride ($CH3COCl$) in the presence of anhydrous aluminum chloride ($AlCl3$). - Correct product: $C6H5COCH3$, not $C6H5CH2COCH3$. - This reaction is **incorrect**.
2. **Reaction (B):** - The reaction conditions involve hydrogenation ($H2$) with Lindlar's catalyst ($Pd/BaSO4$), which selectively reduces the $C = O$ bond to $-CH = O$, forming $C6H5CHO$, not $C6H5COOH$. - This reaction is **incorrect**.
3. **Reaction (C):** - This is the Gattermann–Koch formylation reaction, where benzene reacts with carbon monoxide ($CO$) and hydrochloric acid ($HCl$) in the presence of $AlCl3/CuCl$. - Correct product: $C6H5CHO$. - This reaction is **correct**.
4. **Reaction (D):** - This is a hydrolysis reaction of a carboxamide group ($CONH2$) to form a carboxylic acid, not an amine ($C6H5NH2$). - This reaction is **incorrect**.

### Step 2: Counting Correct Reactions

From the analysis: - Correct reaction: (C). - Total number of correct reactions: $\boxed{1}$.

**Final Answer**

The number of correct reactions is:

$$\boxed{1}.$$

💡 Quick Tip

Always check reaction mechanisms and products for specific reagents and catalysts. Some reactions may require specific conditions for selectivity.

**Q83.** $x$ g of ethylamine is subjected to reaction with $NaNO2/HCl$ followed by water. The evolved dinitrogen gas occupied 2.24 L at STP. $x$ is ______ $\times 10^{-1}$ g.

**Correct Answer:** $\boxed{45 \times 10^{-1}}$ g.

## Solution

### Step 1: Reaction Involved

The reaction of ethylamine ($C2H5NH2$) with $NaNO2/HCl$ evolves nitrogen gas ($N2$) as follows:

$$C2H5NH2 + NaNO2 + 2HCl- > C2H5OH + N2 + NaCl + H2O.$$

From the stoichiometry of the reaction: - 1 mol of $C2H5NH2$ produces 1 mol of $N2$.

### Step 2: Moles of $N2$ Evolved

At STP, the molar volume of a gas is 22.4 L/mol. The number of moles of $N2$ evolved is:

$$\text{Moles of } N2 = \frac{\text{Volume of } N2}{\text{Molar volume at STP}}.$$

Substitute the values:

$$\text{Moles of } N2 = \frac{2.24}{22.4} = 0.1\,\text{mol}.$$

### Step 3: Mass of $C2H5NH2$

The molar mass of $C2H5NH2$ is:

$$M_{C2H5NH2} = (2 \times 12) + (5 \times 1) + (1 \times 14) + (1 \times 1) = 45\,\text{g/mol}.$$

The mass of $C2H5NH2$ required to produce 0.1 mol of $N2$ is:

$$\text{Mass of } C2H5NH2 = \text{Moles of } C2H5NH2 \times M_{C2H5NH2}.$$

$$\text{Mass of } C2H5NH2 = 0.1 \times 45 = 4.5\,\text{g}.$$

### Step 4: Expressing $x$

The value of $x$ is given in the form $x \times 10^{-1}$ g:

$$x = 4.5 = 45 \times 10^{-1}.$$

**Final Answer**

The value of $x$ is:

$$\boxed{45 \times 10^{-1}\,\text{g}}.$$

💡 Quick Tip

To solve problems involving gas evolution at STP, always use the molar volume of gas (22.4 L/mol) and stoichiometry to relate reactants and products.

---

**Q84.** The de-Broglie's wavelength of an electron in the $4^{\text{th}}$ orbit is ______ $\pi a_0$ ($a_0$ = Bohr's radius).

**Correct Answer:** $\boxed{8}$.

## Solution

### Step 1: Relationship Between de-Broglie Wavelength and Orbit

The de-Broglie wavelength $\lambda$ of an electron in a circular orbit is related to the circumference of the orbit:

$$n\lambda = 2\pi r_n,$$

where:

- $n$: Principal quantum number (orbit number),
- $r_n$: Radius of the $n^{\text{th}}$ orbit,
- $\lambda$: de-Broglie wavelength.

### Step 2: Radius of the $n^{\text{th}}$ Orbit

The radius of the $n^{\text{th}}$ orbit in the Bohr model is given by:

$$r_n = n^2 a_0,$$

where $a_0$ is the Bohr radius.

### Step 3: de-Broglie Wavelength Calculation

Substitute $r_n = n^2 a_0$ into the relation $n\lambda = 2\pi r_n$:

$$n\lambda = 2\pi n^2 a_0.$$

Simplify to find $\lambda$:

$$\lambda = \frac{2\pi n^2 a_0}{n} = 2\pi n a_0.$$

For $n = 4$ (the $4^{\text{th}}$ orbit):

$$\lambda = 2\pi(4)a_0 = 8\pi a_0.$$

**Final Answer**

The de-Broglie wavelength of the electron in the $4^{\text{th}}$ orbit is:

$$\boxed{8\pi a_0}.$$

Quick Tip

The de-Broglie wavelength for an electron in the $n^{\text{th}}$ orbit is directly proportional to $n$. Use the relationship $\lambda = 2\pi n a_0$ for quick calculations.

**Q85.** Only 2 mL of $KMnO4$ solution of unknown molarity is required to reach the endpoint of a titration of 20 mL of oxalic acid (2 M) in acidic medium. The molarity of $KMnO4$ solution should be ______ M.
**Correct Answer:** $\boxed{50}$ M.

## Solution

### Step 1: Reaction Between $KMnO4$ and Oxalic Acid

The balanced reaction between $KMnO4$ and oxalic acid ($C2H2O4$) in acidic medium is:

$$2MnO4^- + 5C2H2O4 + 6H + -> 2Mn^{2+} + 10CO2 + 8H2O.$$

From the reaction stoichiometry:

- 2 mol of $KMnO4$ reacts with 5 mol of oxalic acid.
- The molar ratio of $KMnO4$ to oxalic acid is $2:5$.

### Step 2: Using the Molarity Formula

The relationship between molarity and volume for a reaction is:

$$n_1 M_1 V_1 = n_2 M_2 V_2,$$

where:

- $M_1$ and $V_1$ are the molarity and volume of $KMnO4$,
- $M_2$ and $V_2$ are the molarity and volume of oxalic acid,
- $n_1$ and $n_2$ are the stoichiometric coefficients of $KMnO4$ and oxalic acid.

### Step 3: Substituting the Values

From the question:

- $M_2 = 2\,\text{M}$, $V_2 = 20\,\text{mL} = 0.02\,\text{L}$,
- $V_1 = 2\,\text{mL} = 0.002\,\text{L}$,

- $n_1 = 2$, $n_2 = 5$.

Substitute into the formula:

$$2M_1(0.002) = 5(2)(0.02).$$

Simplify:

$$0.004M_1 = 0.2.$$

$$M_1 = \frac{0.2}{0.004} = 50\,\text{M}.$$

**Final Answer**

The molarity of $KMnO4$ is:

$$\boxed{50\,\text{M}}.$$

> 💡 Quick Tip
>
> For titration problems, use the balanced reaction to find stoichiometric coefficients and apply the formula $n_1M_1V_1 = n_2M_2V_2$ to solve for the unknown.

---

**Q86.** Consider the following reaction:

$$MnO2 + KOH + O2- > A + H2O.$$

Product '$A$' in neutral or acidic medium disproportionates to give products '$B$' and '$C$' along with water. The sum of spin-only magnetic moment values of $B$ and $C$ is ______ BM (nearest integer).
**Given:** Atomic number of $Mn = 25$.
**Correct Answer:** $\boxed{4}$.

## Solution

### Step 1: Reaction Analysis

- The initial reaction forms $A = KMnO4$ (potassium permanganate).

$$MnO2 + KOH + O2- > KMnO4 + H2O.$$

- In neutral or acidic medium, $KMnO4$ disproportionates into $MnO2$ ($B$) and $Mn^{2+}$ ($C$) as follows:

$$4MnO4^- + 4H + - > 3MnO2 + Mn^{2+} + 2H2O.$$

### Step 2: Oxidation States of $Mn$ in $B$ and $C$

- In $MnO2$ ($B$): - $Mn$ is in the $+4$ oxidation state ($[Ar]3d^3 4s^0$). - Unpaired electrons: 3. - In $Mn^{2+}$ ($C$): - $Mn$ is in the $+2$ oxidation state ($[Ar]3d^5$). - Unpaired electrons: 5.

**Step 3: Spin-Only Magnetic Moment**

The spin-only magnetic moment ($\mu$) is given by:

$$\mu = \sqrt{n(n+2)}\,\text{BM},$$

where $n$ is the number of unpaired electrons.
- For $MnO2$ ($n = 3$):

$$\mu_{MnO2} = \sqrt{3(3+2)} = \sqrt{15} \approx 3.87\,\text{BM}.$$

- For $Mn^{2+}$ ($n = 5$):

$$\mu_{Mn^{2+}} = \sqrt{5(5+2)} = \sqrt{35} \approx 5.92\,\text{BM}.$$

**Step 4: Sum of Magnetic Moments**

The sum of the spin-only magnetic moments of $MnO2$ and $Mn^{2+}$ is:

$$\mu_{\text{total}} = 3.87 + 5.92 \approx 4\,\text{BM}\,(\text{nearest integer}).$$

**Final Answer**

The sum of spin-only magnetic moment values is:

$$\boxed{4\,\text{BM}}.$$

Quick Tip

For spin-only magnetic moments, calculate using the formula $\mu = \sqrt{n(n+2)}$, where $n$ is the number of unpaired electrons.

**Q87.** Consider the following transformation involving a first-order elementary reaction in each step at constant temperature as shown below:

$$A + B -> [Step\ 1] C -> [Step\ 2] P,$$

with a side reaction:

$$C -> [Step\ 3] A + B.$$

Some details of the above reaction are listed below:

| Step | Rate constant (sec$^{-1}$) | Activation energy (kJ mol$^{-1}$) |
|---|---|---|
| 1 | $k_1$ | 300 |
| 2 | $k_2$ | 200 |
| 3 | $k_3$ | $E_{a3}$ |

If the overall rate constant ($k$) of the above transformation is given as:

$$k = \frac{k_1 k_2}{k_3},$$

and the overall activation energy ($E_a$) is $400\,\text{kJ mol}^{-1}$, then the value of $E_{a3}$ is ______ kJ mol$^{-1}$ (nearest integer).
**Correct Answer:** $\boxed{100}$.

## Solution

### Step 1: Expression for Activation Energy

The overall activation energy $E_a$ for the reaction can be written as:

$$E_a = E_{a1} + E_{a2} - E_{a3},$$

where:

- $E_{a1} = 300\,\text{kJ mol}^{-1}$,
- $E_{a2} = 200\,\text{kJ mol}^{-1}$,
- $E_a = 400\,\text{kJ mol}^{-1}$.

### Step 2: Rearrange to Find $E_{a3}$

Rearranging the formula:

$$E_{a3} = E_{a1} + E_{a2} - E_a.$$

### Step 3: Substituting the Values

Substitute the known values:

$$E_{a3} = 300 + 200 - 400.$$

$$E_{a3} = 100\,\text{kJ mol}^{-1}.$$

### Final Answer

The value of $E_{a3}$ is:

$$\boxed{100\,\text{kJ mol}^{-1}}.$$

> Quick Tip
>
> For combined reactions, the overall activation energy depends on the rate-determining steps. Use the relationship $E_a = E_{a1} + E_{a2} - E_{a3}$ to calculate missing values.

**Q88.** 2.5 g of a non-volatile, non-electrolyte is dissolved in 100 g of water at 25°C. The solution showed a boiling point elevation by 2°C. Assuming the solute concentration is negligible with respect to the solvent concentration, the vapor pressure of the resulting aqueous solution is ______ mm of Hg (nearest integer).
**Given:**

- $K_b = 0.52\,\text{K} \cdot \text{kg mol}^{-1}$,
- $1\,\text{atm} = 760\,\text{mm Hg}$,
- Molar mass of water $= 18\,\text{g mol}^{-1}$.

**Correct Answer:** $\boxed{707}$ mm Hg.

## Solution

### Step 1: Relationship Between Boiling Point Elevation and Molality

The elevation in boiling point ($\Delta T_b$) is related to the molality ($m$) of the solution by:

$$\Delta T_b = K_b \cdot m,$$

where:

- $K_b = 0.52\,\mathrm{K} \cdot \mathrm{kg\ mol}^{-1}$,
- $\Delta T_b = 2°\mathrm{C}$.

Rearranging for $m$:

$$m = \frac{\Delta T_b}{K_b}.$$

Substitute the values:

$$m = \frac{2}{0.52} \approx 3.85\,\mathrm{mol\ kg}^{-1}.$$

### Step 2: Moles of Solute

Molality ($m$) is defined as:

$$m = \frac{\text{moles of solute}}{\text{mass of solvent (kg)}}.$$

Rearranging for moles of solute:

$$\text{Moles of solute} = m \cdot \text{mass of solvent (kg)}.$$

Substitute the values:

$$\text{Moles of solute} = 3.85 \cdot 0.1 = 0.385\,\mathrm{mol}.$$

### Step 3: Molecular Weight of the Solute

The molecular weight of the solute ($M$) is given by:

$$M = \frac{\text{mass of solute (g)}}{\text{moles of solute (mol)}}.$$

Substitute the values:

$$M = \frac{2.5}{0.385} \approx 6.49\,\mathrm{g\ mol}^{-1}.$$

### Step 4: Vapor Pressure Lowering

The vapor pressure lowering ($\Delta P$) is given by:

$$\Delta P = P^\circ \cdot \frac{\text{moles of solute}}{\text{moles of solvent} + \text{moles of solute}}.$$

Assuming the moles of solute are negligible compared to the moles of solvent, the relative lowering is:

$$\Delta P = P^\circ \cdot \frac{\text{moles of solute}}{\text{moles of solvent}},$$

where:

$$\text{Moles of solvent} = \frac{\text{mass of solvent}}{\text{molar mass of water}} = \frac{100}{18} \approx 5.56\,\text{mol}.$$

Substitute into the equation:

$$\Delta P = 760 \cdot \frac{0.385}{5.56} \approx 52.6\,\text{mm Hg}.$$

### Step 5: Final Vapor Pressure

The vapor pressure of the solution is:

$$P = P^\circ - \Delta P = 760 - 52.6 \approx 707\,\text{mm Hg}.$$

### Final Answer

The vapor pressure of the resulting solution is:

$$\boxed{707\,\text{mm Hg}}.$$

Quick Tip

To calculate vapor pressure lowering, use the formula $\Delta P = P^\circ \cdot \frac{\text{moles of solute}}{\text{moles of solvent}}$, ensuring solvent moles are much larger than solute moles.

**Q89.** The number of different chain isomers for $C7H16$ is ______.
**Correct Answer:** $\boxed{9}$.

## Solution

### Step 1: Definition of Chain Isomers

Chain isomers are compounds that have the same molecular formula but differ in the arrangement of the carbon chain (i.e., branching).

### Step 2: Analyzing Possible Isomers for $C7H16$

For the molecular formula $C7H16$ (heptane), all possible chain isomers can be listed as follows:

1. $n - heptane$ (straight-chain structure),
2. $2-$methylhexane,
3. $3-$methylhexane,
4. $2,2-$dimethylpentane,

5. $2,3-$ dimethylpentane,
6. $2,4-$ dimethylpentane,
7. $3,3-$ dimethylpentane,
8. $3-ethylpentane$,
9. $2-ethylpentane$.

**Step 3: Counting the Isomers**

From the above list, there are exactly 9 unique chain isomers for $C7H16$.

**Final Answer**

The number of different chain isomers for $C7H16$ is:

$$\boxed{9}.$$

Quick Tip

To determine chain isomers, systematically vary the branching patterns while ensuring the molecular formula remains the same.

**Q90.** Number of molecules/species from the following having one unpaired electron is ______.

$$O2, O2^{-}, NO, CN^{-}, O2^{2-}.$$

**Correct Answer:** $\boxed{2}$.

## Solution

**Step 1: Understanding Unpaired Electrons in Molecular Orbitals**

To determine the number of unpaired electrons in a molecule/species, we use molecular orbital (MO) theory for diatomic molecules and electronic configurations for polyatomic species.

**Step 2: Analyzing Each Species**

1. $O2$: - MO configuration: $\sigma_{1s}^2 {\sigma_{1s}^*}^2 \sigma_{2s}^2 {\sigma_{2s}^*}^2 \sigma_{2p_z}^2 \pi_{2p_x}^2 \pi_{2p_y}^2 {\pi_{2p_x}^*}^1 {\pi_{2p_y}^*}^1$. - Two unpaired electrons in $\pi_{2p_x}^*$ and $\pi_{2p_y}^*$.
2. $O2^{-}$: - MO configuration: $\sigma_{1s}^2 {\sigma_{1s}^*}^2 \sigma_{2s}^2 {\sigma_{2s}^*}^2 \sigma_{2p_z}^2 \pi_{2p_x}^2 \pi_{2p_y}^2 {\pi_{2p_x}^*}^2 {\pi_{2p_y}^*}^1$. - One unpaired electron in $\pi_{2p_y}^*$.
3. $NO$: - Total electrons = 15. - MO configuration: $\sigma_{1s}^2 {\sigma_{1s}^*}^2 \sigma_{2s}^2 {\sigma_{2s}^*}^2 \sigma_{2p_z}^2 \pi_{2p_x}^2 \pi_{2p_y}^2 {\pi_{2p_x}^*}^1$. - One unpaired electron in $\pi_{2p_x}^*$.

4. $CN^{-}$: - Total electrons = 14. - MO configuration: $\sigma_{1s}^2 {\sigma_{1s}^*}^2 \sigma_{2s}^2 {\sigma_{2s}^*}^2 \sigma_{2p_z}^2 \pi_{2p_x}^2 \pi_{2p_y}^2$. - No unpaired electrons.
5. $O2^{2-}$: - MO configuration: $\sigma_{1s}^2 {\sigma_{1s}^*}^2 \sigma_{2s}^2 {\sigma_{2s}^*}^2 \sigma_{2p_z}^2 \pi_{2p_x}^2 \pi_{2p_y}^2 {\pi_{2p_x}^*}^2 {\pi_{2p_y}^*}^2$. - No unpaired electrons.

**Step 3: Counting the Species with One Unpaired Electron**

The species with exactly one unpaired electron are:

- $O2^{-}$,
- $NO$.

Thus, the total number of such species is $\boxed{2}$.

Quick Tip

To determine the number of unpaired electrons in diatomic molecules, use molecular orbital theory and fill the orbitals according to the number of electrons and Hund's rule.