JEE Main 2026 Jan 23 Shift 2 Question Paper with Solutions

Time Allowed :3 Hour	**Maximum Marks :**300	**Total Questions :**75

General Instructions

Read the following instructions very carefully and strictly follow them:

1. . The test is of 3 hours duration.
2. The question paper consists of 75 questions. The maximum marks are 300.
3. There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 25 questions in each part of equal weightage.
4. Each part (subject) has two sections.
   - Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and –1 mark for wrong answer.
   - Section-B: This section contains 5 questions. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and –1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer

Mathematics Section A

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1. The system of linear equations
$x + y + z = 6$
$2x + 5y + az = 36$
$x + 2y + 3z = b$
has
(A) infinitely many solutions for $a = 8$ and $b = 14$
(B) infinitely many solutions for $a = 8$ and $b = 16$
(C) unique solution for $a = 8$ and $b = 16$
(D) unique solution for $a = 8$ and $b = 14$

Correct Answer: (A) infinitely many solutions for $a = 8$ and $b = 14$

Solution:

Write the augmented matrix:

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 2 & 5 & a & 36 \\ 1 & 2 & 3 & b \end{array}\right]$$

Apply row operations:

$$R_2 \to R_2 - 2R_1, \quad R_3 \to R_3 - R_1$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 3 & a-2 & 24 \\ 0 & 1 & 2 & b-6 \end{array}\right]$$

Swap R_2 and R_3:

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & b-6 \\ 0 & 3 & a-2 & 24 \end{array}\right]$$

Apply:

$$R_3 \to R_3 - 3R_2$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & b-6 \\ 0 & 0 & a-8 & 42-3b \end{array}\right]$$

For infinitely many solutions, the last row must be zero:

$$a - 8 = 0, \quad 42 - 3b = 0$$

$$a = 8, \quad b = 14$$

$\therefore$ Option (A) is correct.

Quick Tip

For a system $AX = B$, infinite solutions exist if $|A| = 0$ and $(\text{adj } A)B = 0$. In matrix form, the last row of the row-reduced augmented matrix must be all zeros.

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2. If the mean and the variance of the data

Class	4–8	8–12	12–16	16–20
Frequency	3	λ	4	7

are μ and 19 respectively, then the value of $\lambda + \mu$ is

(A) 21

(B) 18

(C) 19

(D) 20

Correct Answer: (C) 19

Solution:

Class midpoints are:

$$x_i = 6, 10, 14, 18$$

Frequencies:

$$f_i = 3, \lambda, 4, 7$$

Total frequency:

$$N = 14 + \lambda$$

Assume mean $A = 14$, class width $h = 4$.

$$u_i = \frac{x_i - A}{h} = -2, -1, 0, 1$$

$$\sum f_i u_i = -6 - \lambda + 7 = 1 - \lambda$$

$$\bar{u} = \frac{1 - \lambda}{14 + \lambda}$$

$$\sum f_i u_i^2 = 12 + \lambda + 7 = 19 + \lambda$$

Variance formula:

$$19 = 16 \left[\frac{19 + \lambda}{14 + \lambda} - \left(\frac{1 - \lambda}{14 + \lambda} \right)^2 \right]$$

Solving gives:

$$\lambda = 6$$

$$\bar{u} = -\frac{1}{4} \Rightarrow \mu = 14 - 1 = 13$$

$$\lambda + \mu = 6 + 13 = \boxed{19}$$

Quick Tip

Variance is independent of change of origin but depends on the scale. $\text{Var}(ax + b) = a^2 \text{Var}(x)$.

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3. Let $I(x) = \int \frac{3dx}{(4x+6)\sqrt{4x^2+8x+3}}$ **and** $I(0) = \frac{\sqrt{3}}{4} + 20$**. If** $I\left(\frac{1}{2}\right) = \frac{a\sqrt{2}}{b} + c$**, where** $a, b, c \in \mathbb{N}, \gcd(a, b) = 1$**, then** $a + b + c$ **is equal to**

(A) 29
(B) 28
(C) 30
(D) 31

Correct Answer: (D) 31

Solution:
Given

$$I(x) = \int \frac{3dx}{(4x+6)\sqrt{4x^2+8x+3}}$$

Rewrite:

$$4x^2 + 8x + 3 = (2x+2)^2 - 1$$

Let:

$$2x + 2 = \sec\theta \Rightarrow dx = \frac{1}{2}\sec\theta\tan\theta d\theta$$

Then:

$$I = \frac{3}{4}\int \frac{\sec\theta}{\sec\theta + 1}d\theta$$

$$= \frac{3}{4}\int \frac{1}{1+\cos\theta}d\theta$$

$$= \frac{3}{8}\int \sec^2(\theta/2)d\theta = \frac{3}{4}\tan(\theta/2) + C$$

$$\tan(\theta/2) = \sqrt{\frac{2x+1}{2x+3}}$$

Using $I(0) = \frac{\sqrt{3}}{4} + 20$, we get $C = 20$.

$$I\left(\frac{1}{2}\right) = \frac{3\sqrt{2}}{8} + 20$$

$$a + b + c = 3 + 8 + 20 = \boxed{31}$$

Quick Tip

$\int \frac{dx}{(ax+b)\sqrt{px^2+qx+r}}$: Substitute $ax + b = 1/t$ or use trig substitution like $2x + 2 = \sec\theta$.

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4. An equilateral triangle OAB is inscribed in the parabola $y^2 = 4x$ with the vertex O at the vertex of the parabola. Then the minimum distance of the circle having AB as a diameter from the origin is
(A) $4(6 + \sqrt{3})$

(B) $4(3-\sqrt{3})$
(C) $2(8-3\sqrt{3})$
(D) $2(3+\sqrt{3})$

Correct Answer: (B) $4(3-\sqrt{3})$

Solution:
4.
Let:

$$A(t^2, 2t),\ B(t^2, -2t)$$

Slope of OA:

$$\frac{2t}{t^2} = \frac{1}{\sqrt{3}} \Rightarrow t = 2\sqrt{3}$$

Coordinates:

$$A(12, 4\sqrt{3}),\ B(12, -4\sqrt{3})$$

Center of circle:

$$C(12, 0)$$

Radius:

$$r = 4\sqrt{3}$$

Distance from origin:

$$12 - 4\sqrt{3} = 4(3-\sqrt{3})$$

$$\boxed{4(3-\sqrt{3})}$$

> Quick Tip
>
> For an equilateral triangle in $y^2 = 4ax$ with a vertex at $(0,0)$, the side length is $8a\sqrt{3}$.

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5. The sum of all the real solutions of the equation $\log_{(x+3)}(6x^2+28x+30) = 5 - 2\log_{(6x+10)}(x^2+6x+9)$ is equal to
(A) 1
(B) 0
(C) 2
(D) 4

Correct Answer: (B) 0

Solution:
Factor:

$$6x^2 + 28x + 30 = (x+3)(6x+10)$$

$$x^2 + 6x + 9 = (x+3)^2$$

Substitute:

$$1 + \log_{x+3}(6x+10) = 5 - 4\log_{6x+10}(x+3)$$

Let:

$$y = \log_{x+3}(6x+10) \Rightarrow \log_{6x+10}(x+3) = \frac{1}{y}$$

$$1 + y = 5 - \frac{4}{y} \Rightarrow (y-2)^2 = 0 \Rightarrow y = 2$$

$$6x + 10 = (x+3)^2 \Rightarrow x = \pm 1$$

Both valid.

$$\text{Sum of real solutions} = 1 + (-1) = \boxed{0}$$

> **Quick Tip**
>
> Always simplify log arguments by factoring. Remember $\log_a b = 1/\log_b a$.

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6. The least value of $(\cos^2\theta - 6\sin\theta\cos\theta + 3\sin^2\theta + 2)$ **is**
(A) -1
(B) $4 + \sqrt{10}$
(C) $4 - \sqrt{10}$
(D) 1

Correct Answer: (C) $4 - \sqrt{10}$

Solution:

Start by converting the trigonometric terms using identities:

$$\cos^2\theta = \frac{1+\cos 2\theta}{2}, \quad \sin^2\theta = \frac{1-\cos 2\theta}{2}, \quad 2\sin\theta\cos\theta = \sin 2\theta$$

Substitute into the given expression:

$$f(\theta) = \frac{1+\cos 2\theta}{2} - 3\sin 2\theta + 3\left(\frac{1-\cos 2\theta}{2}\right) + 2$$

Simplify constants:

$$f(\theta) = \left(\frac{1}{2} + \frac{3}{2} + 2\right) + \left(\frac{1}{2} - \frac{3}{2}\right)\cos 2\theta - 3\sin 2\theta$$

$$f(\theta) = 4 - \cos 2\theta - 3\sin 2\theta$$

The expression $a\cos x + b\sin x$ has range:

$$[-\sqrt{a^2+b^2},\ \sqrt{a^2+b^2}]$$

Here:

$$\sqrt{(-1)^2+(-3)^2} = \sqrt{10}$$

Hence:

$$\text{Minimum value} = 4 - \sqrt{10}$$

> **Quick Tip**
>
> Minimum value of $a\cos x + b\sin x + c$ is $c - \sqrt{a^2+b^2}$.

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7. Let $A = \{0, 1, 2, ..., 9\}$. Let R be a relation on A defined by $(x, y) \in R$ if and only if $|x - y|$ is a multiple of 3.
Statement I: $n(R) = 36$.
Statement II: R is an equivalence relation.
(A) Both Statement I and Statement II are correct
(B) Both Statement I and Statement II are incorrect
(C) Statement I is incorrect but Statement II is correct
(D) Statement I is correct but Statement II is incorrect

Correct Answer: (C) Statement I is incorrect but Statement II is correct

Solution:
Let $A = \{0, 1, 2, \ldots, 9\}$ and define a relation R by

$$(x, y) \in R \iff |x - y| \text{ is a multiple of 3.}$$

Statement I: $n(R) = 36$
Statement II: R is an equivalence relation

Correct Answer: (C)

Solution:

The relation represents:

$$x \equiv y \pmod 3$$

Such a relation is:

- Reflexive
- Symmetric

- Transitive

Hence, **Statement II is correct**.
Now, partition the set A into equivalence classes:

$$C_0 = \{0, 3, 6, 9\}, \quad C_1 = \{1, 4, 7\}, \quad C_2 = \{2, 5, 8\}$$

Number of ordered pairs in R:

$$n(R) = |C_0|^2 + |C_1|^2 + |C_2|^2$$

$$= 4^2 + 3^2 + 3^2 = 16 + 9 + 9 = 34$$

Since Statement I claims 36, it is incorrect.

$$\therefore \text{Option (C) is correct.}$$

> Quick Tip
>
> For an equivalence relation, the graph is a union of disjoint squares corresponding to the equivalence classes.

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8. The area of the region enclosed between the circles $x^2+y^2 = 4$ and $x^2+(y-2)^2 = 4$ is:
(A) $\frac{4}{3}(2\pi - 3\sqrt{3})$
(B) $\frac{2}{3}(4\pi - 3\sqrt{3})$
(C) $\frac{4}{3}(2\pi - \sqrt{3})$
(D) $\frac{2}{3}(2\pi - 3\sqrt{3})$

Correct Answer: (B) $\frac{2}{3}(4\pi - 3\sqrt{3})$

Solution:

The area enclosed between the circles

$$x^2 + y^2 = 4 \quad \text{and} \quad x^2 + (y-2)^2 = 4$$

is:

Correct Answer: (B) $\frac{2}{3}(4\pi - 3\sqrt{3})$

Solution:

Find points of intersection:

$$x^2 + y^2 = x^2 + (y-2)^2 \Rightarrow y = 1$$

Substitute into $x^2 + y^2 = 4$:

$$x^2 + 1 = 4 \Rightarrow x = \pm\sqrt{3}$$

Each circle has radius $r = 2$. Distance of chord from center:

$$d = 1$$

$$\cos\alpha = \frac{d}{r} = \frac{1}{2} \Rightarrow \alpha = \frac{\pi}{3}$$

Central angle $= 2\alpha = \frac{2\pi}{3}$.
Area of one segment:

$$\frac{1}{2}r^2(2\alpha) - \frac{1}{2}r^2\sin(2\alpha) = \frac{4\pi}{3} - \sqrt{3}$$

Total area (two identical segments):

$$2\left(\frac{4\pi}{3} - \sqrt{3}\right) = \frac{2}{3}(4\pi - 3\sqrt{3})$$

> **Quick Tip**
>
> Symmetry plays a huge role in area between curves. Always check if the area is composed of identical parts.

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9. Bag A contains 9 white and 8 black balls, while bag B contains 6 white and 4 black balls. One ball is picked from B and put in A. Then a ball is drawn from A. Probability it is white is p/q. Find $p + q$.
(A) 23
(B) 22
(C) 21
(D) 24

Correct Answer: (A) 23

Solution:

Bag A: $9W, 8B$
Bag B: $6W, 4B$

Correct Answer: (A) 23

Solution:

Probability of transferring a white ball from B:

$$P(T_W) = \frac{6}{10} = \frac{3}{5}$$

Probability of transferring a black ball:

$$P(T_B) = \frac{4}{10} = \frac{2}{5}$$

Case 1: White transferred Bag A becomes $10W, 8B$:

$$P(W|T_W) = \frac{10}{18} = \frac{5}{9}$$

Case 2: Black transferred Bag A becomes $9W, 9B$:

$$P(W|T_B) = \frac{9}{18} = \frac{1}{2}$$

Using total probability theorem:

$$P(W) = \frac{5}{9} \cdot \frac{3}{5} + \frac{1}{2} \cdot \frac{2}{5} = \frac{1}{3} + \frac{1}{5} = \frac{8}{15}$$

$$p + q = 8 + 15 = 23$$

> Quick Tip
>
> Use the Total Probability Theorem for multi-stage random experiments.

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10. Points of intersection of ellipses $x^2 + 2y^2 - 6x - 12y + 23 = 0$ and $4x^2 + 2y^2 - 20x - 12y + 35 = 0$ lie on a circle. Value of $ab + 18r^2$ is
(A) 53
(B) 51
(C) 55
(D) 52

Correct Answer: (C) 55

Solution:

Equation of family through intersection:

$$S_2 + \lambda S_1 = 0$$

For a circle:

$$\text{Coeff of } x^2 = \text{Coeff of } y^2$$

$$4 + \lambda = 2 + 2\lambda \Rightarrow \lambda = 2$$

Substitute:

$$6x^2 + 6y^2 - 32x - 36y + 81 = 0$$

Divide by 6:

$$x^2 + y^2 - \frac{16}{3}x - 6y + \frac{27}{2} = 0$$

Center:

$$(a, b) = \left(\frac{8}{3}, 3\right)$$

Radius:

$$r^2 = \frac{64}{9} + 9 - \frac{27}{2}$$

Required value:

$$ab + 18r^2 = 55$$

> **Quick Tip**
>
> Family of curves $S_1 + \lambda S_2 = 0$ is powerful for finding curves through intersections without finding the points explicitly.

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11. If $f(x) = \begin{cases} \frac{a|x| + x^2 - 2(\sin|x|)(\cos|x|)}{x} & , x \neq 0 \\ b & , x = 0 \end{cases}$ **is continuous at** $x = 0$**, then** $a + b$ **is equal to**

(A) 0
(B) 1
(C) 4
(D) 2

Correct Answer: (D) 2

Solution:

First simplify the numerator using the identity:

$$2\sin|x|\cos|x| = \sin(2|x|)$$

So for $x \neq 0$,

$$f(x) = \frac{a|x| + x^2 - \sin(2|x|)}{x}$$

Right Hand Limit ($x \to 0^+$)**:** Here $|x| = x$.

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \frac{ax + x^2 - \sin(2x)}{x}$$

Divide each term by x:

$$= a + 0 - \frac{\sin(2x)}{x}$$

Using $\lim_{x\to 0} \frac{\sin(2x)}{x} = 2$,

$$\text{RHL} = a - 2$$

Left Hand Limit ($x \to 0^-$): Here $|x| = -x$.

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \frac{-ax + x^2 + \sin(2x)}{x}$$

$$= -a + 0 + \frac{\sin(2x)}{x} = -a + 2$$

For continuity at $x = 0$:

$$\text{LHL} = \text{RHL} \Rightarrow a - 2 = -a + 2 \Rightarrow a = 2$$

The common limit value is:

$$\lim_{x\to 0} f(x) = a - 2 = 0$$

For continuity:

$$b = 0$$

$$\therefore a + b = 2 + 0 = \boxed{2}$$

> Quick Tip
>
> Use Maclaurin series expansion or L'Hospital's rule for limits of the form $0/0$.

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12. Let $\vec{a}, \vec{b}, \vec{c}$ be vectors such that $\vec{a} \times \vec{b} = 2(\vec{a} \times \vec{c})$. $|\vec{a}| = 1, |\vec{b}| = 4, |\vec{c}| = 2$, angle between $\vec{b}, \vec{c}$ is $60°$. Find $|\vec{b} - 2\vec{c}|$.

(A) 4
(B) 2
(C) 0
(D) 1

Correct Answer: (A) 4

Solution:

Given:

$$\vec{a} \times \vec{b} - 2\vec{a} \times \vec{c} = 0$$

Factor out $\vec{a}$:

$$\vec{a} \times (\vec{b} - 2\vec{c}) = 0$$

This implies $\vec{b} - 2\vec{c}$ is parallel to $\vec{a}$. So,

$$\vec{b} - 2\vec{c} = \lambda\vec{a}$$

Taking magnitudes:

$$|\vec{b} - 2\vec{c}|^2 = \lambda^2|\vec{a}|^2 = \lambda^2$$

Now expand:

$$|\vec{b} - 2\vec{c}|^2 = |\vec{b}|^2 + 4|\vec{c}|^2 - 4(\vec{b} \cdot \vec{c})$$

Compute dot product:

$$\vec{b} \cdot \vec{c} = |\vec{b}||\vec{c}| \cos 60^\circ = 4 \times 2 \times \frac{1}{2} = 4$$

So,

$$|\vec{b} - 2\vec{c}|^2 = 16 + 16 - 16 = 16$$

$$|\vec{b} - 2\vec{c}| = \boxed{4}$$

> **Quick Tip**
>
> $|\vec{u} - \vec{v}|^2 = u^2 + v^2 - 2\vec{u} \cdot \vec{v}$ is a fundamental vector identity.

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13. Let $\vec{a}, \vec{b}, \vec{c}$ be defined. $\vec{v} = \vec{a} \times \vec{b}$. $\vec{v} \cdot \vec{c} = 11$. Projection of $\vec{b}$ on $\vec{c}$ is p. Find $9p^2$.
(A) 12
(B) 4
(C) 9
(D) 6

Correct Answer: (A) 12

Solution:
First compute the cross product:

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 3 \\ 2 & 1 & -1 \end{vmatrix} = (-1, 7, 5)$$

Now use scalar triple product:

$$(\vec{a} \times \vec{b}) \cdot \vec{c} = (-1, 7, 5) \cdot (\lambda, 1, 1) = -\lambda + 12$$

Given:

$$-\lambda + 12 = 11 \Rightarrow \lambda = 1$$

So,

$$\vec{c} = (1, 1, 1), \quad |\vec{c}| = \sqrt{3}$$

Projection of $\vec{b}$ on $\vec{c}$:

$$p = \frac{\vec{b}\cdot\vec{c}}{|\vec{c}|} = \frac{2+1-1}{\sqrt{3}} = \frac{2}{\sqrt{3}}$$

$$9p^2 = 9\left(\frac{4}{3}\right) = \boxed{12}$$

> Quick Tip
>
> Scalar Triple Product $[\vec{a}\vec{b}\vec{c}] = (\vec{a}\times\vec{b})\cdot\vec{c}$.

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14. PQ is chord of hyperbola $\frac{x^2}{4} - \frac{y^2}{b^2} = 1$ perpendicular to x-axis. $\triangle OPQ$ is equilateral ($e = \sqrt{3}$). Area OPQ is
(A) $\frac{11}{5}$
(B) $\frac{9}{5}$
(C) $\frac{8\sqrt{3}}{5}$
(D) $2\sqrt{3}$

Correct Answer: (C) $\frac{8\sqrt{3}}{5}$

Solution:

For a hyperbola:

$$b^2 = a^2(e^2 - 1)$$

Here $a^2 = 4,\ e^2 = 3$:

$$b^2 = 4(2) = 8$$

Equation becomes:

$$\frac{x^2}{4} - \frac{y^2}{8} = 1$$

Let the vertical chord be $x = k$. Points:

$$P(k, y), \quad Q(k, -y)$$

Since $\triangle OPQ$ is equilateral and symmetric about x-axis,

$$\frac{y}{k} = \tan 30^\circ = \frac{1}{\sqrt{3}} \Rightarrow k^2 = 3y^2$$

Substitute in hyperbola:

$$\frac{3y^2}{4} - \frac{y^2}{8} = 1 \Rightarrow \frac{5y^2}{8} = 1 \Rightarrow y^2 = \frac{8}{5}$$

Side length $PQ = 2y$.

Area of equilateral triangle:

$$\text{Area} = \frac{\sqrt{3}}{4}(2y)^2 = \sqrt{3}\,y^2 = \sqrt{3}\left(\frac{8}{5}\right) = \boxed{\frac{8\sqrt{3}}{5}}$$

> **Quick Tip**
>
> Equilateral triangle with vertex at origin and symmetry axis X: Side vertices have $y/x = \pm\tan 30°$.

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15. Let $\frac{\pi}{2} < \theta < \pi$ and $\cot\theta = -\frac{1}{2\sqrt{2}}$. Value of expression involving $\frac{15\theta}{2}$ and 8θ.

(A) $\frac{\sqrt{2}-1}{\sqrt{3}}$

(B) $\frac{\sqrt{2}}{\sqrt{3}}$

(C) $\frac{1-\sqrt{2}}{\sqrt{3}}$

(D) $\frac{\sqrt{2}}{\sqrt{3}}$

Correct Answer: (C) $\frac{1-\sqrt{2}}{\sqrt{3}}$

Solution:

The expression can be grouped as:

$$(\sin A\cos B - \cos A\sin B) + (\cos A\cos B + \sin A\sin B)$$

$$= \sin(A-B) + \cos(A-B)$$

Here:

$$A = \frac{15\theta}{2}, \quad B = 8\theta \Rightarrow A - B = -\frac{\theta}{2}$$

So the expression becomes:

$$\sin\left(-\frac{\theta}{2}\right) + \cos\left(-\frac{\theta}{2}\right) = \cos\frac{\theta}{2} - \sin\frac{\theta}{2}$$

Given $\cot\theta = -\frac{1}{2\sqrt{2}}$,

$$\cos\theta = -\frac{1}{3} \quad (\theta \text{ in second quadrant})$$

Using half-angle formulas:

$$\sin\frac{\theta}{2} = \sqrt{\frac{1-\cos\theta}{2}} = \sqrt{\frac{2}{3}}$$

$$\cos\frac{\theta}{2} = \sqrt{\frac{1+\cos\theta}{2}} = \sqrt{\frac{1}{3}}$$

Thus,

$$\cos\frac{\theta}{2} - \sin\frac{\theta}{2} = \frac{1-\sqrt{2}}{\sqrt{3}}$$

$$\therefore \text{Answer} = \boxed{\frac{1-\sqrt{2}}{\sqrt{3}}}$$

> **Quick Tip**
>
> $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$.

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16. If $z = \frac{\sqrt{3}}{2} + \frac{i}{2}$, then $(z^{201} - i)^8$ is equal to
(A) 0
(B) 256
(C) -1
(D) 1

Correct Answer: (B) 256

Solution:

First write z in trigonometric (polar) form.

$$z = \cos 30^\circ + i \sin 30^\circ = e^{i\pi/6}$$

Now raise z to the power 201:

$$z^{201} = e^{i\frac{201\pi}{6}}$$

Reduce the angle modulo 2π:

$$\frac{201\pi}{6} = \frac{67\pi}{2} = 32\pi + \frac{3\pi}{2}$$

Thus,

$$z^{201} = \cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2} = -i$$

Now evaluate the given expression:

$$(z^{201} - i)^8 = (-i - i)^8 = (-2i)^8$$

$$= (-2)^8 i^8 = 256 \times 1 = 256$$

$$\therefore \text{the required value is } \boxed{256}.$$

Quick Tip

Powers of i: $i^1 = i, i^2 = -1, i^3 = -i, i^4 = 1$.

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17. Sets $A = \{x \in Z : ||x-3|-3| \le 1\}$ and $B = \{x : \text{roots of eq}\}$. Number of onto functions $A \to B$.
(A) 32
(B) 62
(C) 81
(D) 79

Correct Answer: (B) 62

Solution:

Step 1: Find set A

$$||x-3|-3| \le 1 \Rightarrow 2 \le |x-3| \le 4$$

Case 1:

$$2 \le x-3 \le 4 \Rightarrow x \in \{5,6,7\}$$

Case 2:

$$-4 \le x-3 \le -2 \Rightarrow x \in \{-1,0,1\}$$

Hence,

$$A = \{-1,0,1,5,6,7\}, \quad |A| = 6$$

Step 2: Find set B
Equation:

$$\frac{(x-2)(x-4)}{x-1} \ln|x-2| = 0$$

Domain restriction:

$$x \neq 1,\ x \neq 2$$

Roots:

$$x-4 = 0 \Rightarrow x = 4$$

$$\ln|x-2| = 0 \Rightarrow |x-2| = 1 \Rightarrow x = 3$$

Thus,

$$B = \{3,4\}, \quad |B| = 2$$

Step 3: Count onto functions
Total functions:

$$2^6 = 64$$

Non-onto functions (all elements map to only one element of B):

$$2$$

Onto functions:

$$64 - 2 = 62$$

$$\therefore \text{Number of onto functions} = \boxed{62}.$$

Quick Tip

Number of surjections from size m to size n: $\sum_{k=0}^{n}(-1)^k\binom{n}{k}(n-k)^m$.

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18. Rhombus vertices A(1,2), C(-3,-6). Line AD parallel to $7x - y = 14$. Find $|\alpha + \beta + \gamma + \delta|$.
(A) 6
(B) 1
(C) 9
(D) 3

Correct Answer: (A) 6

Solution:

In a rhombus (or any parallelogram), the diagonals bisect each other.
Midpoint of diagonal AC:

$$M = \left(\frac{1 + (-3)}{2}, \frac{2 + (-6)}{2}\right) = (-1, -2)$$

Let the coordinates of B and D be (α, β) and (γ, δ).
Using midpoint property:

$$\alpha + \gamma = 1 + (-3) = -2$$
$$\beta + \delta = 2 + (-6) = -4$$

Hence,

$$\alpha + \beta + \gamma + \delta = -2 - 4 = -6$$

$$|\alpha + \beta + \gamma + \delta| = \boxed{6}$$

Quick Tip

Midpoint of diagonals coincides in parallelograms.

---

19. $\sum_{k=1}^{n} a_k = \alpha n^2 + \beta n$. $a_{10} = 59, a_6 = 7a_1$. **Find** $\alpha + \beta$.
(A) 3
(B) 5
(C) 7
(D) 12

Correct Answer: (B) 5

Solution:

Since S_n is quadratic in n, the sequence $\{a_n\}$ is an A.P.

$$a_n = S_n - S_{n-1}$$

$$a_n = \alpha(2n-1) + \beta$$

Given:

$$a_{10} = 19\alpha + \beta = 59 \quad (1)$$

Also,

$$a_6 = 11\alpha + \beta, \quad a_1 = \alpha + \beta$$

Condition:

$$11\alpha + \beta = 7(\alpha + \beta)$$

$$4\alpha = 6\beta \Rightarrow \beta = \frac{2}{3}\alpha$$

Substitute into (1):

$$19\alpha + \frac{2}{3}\alpha = 59 \Rightarrow \alpha = 3$$

$$\beta = 2$$

$$\alpha + \beta = \boxed{5}$$

Quick Tip

For $S_n = An^2 + Bn$, common difference $d = 2A$, first term $a = A + B$.

---

20. The number of ways 16 oranges distributed to 4 children, each gets at least one.
(A) 403
(B) 384
(C) 429

(D) 455

Correct Answer: (D) 455

Solution:

Let x_1, x_2, x_3, x_4 denote the number of oranges received by the children.

$$x_1 + x_2 + x_3 + x_4 = 16, \quad x_i \geq 1$$

Using the **stars and bars** method, the number of solutions is:

$$\binom{16-1}{4-1} = \binom{15}{3}$$

$$\binom{15}{3} = \frac{15 \times 14 \times 13}{3 \times 2 \times 1} = 455$$

$\therefore$ the required number of ways is $\boxed{455}$.

> Quick Tip
>
> Stars and bars method: For $x_1 + ... + x_r = n, x_i \geq 1$, ways $= \binom{n-1}{r-1}$.

---

1 Mathematics Section B

21. Let S denote the set of 4-digit numbers abcd such that $a > b > c > d$ and P denote the set of 5-digit numbers having product of its digits equal to 20. Then $n(S) + n(P)$ is equal to ___
Correct Answer: 260

Solution:
Step 1: Find $n(S)$
Digits are chosen from $\{0, 1, 2, \ldots, 9\}$.
Since $a > b > c > d$, the order of digits is **strictly fixed**. Hence, each choice of 4 distinct digits gives exactly one valid number.
Therefore,

$$n(S) = \binom{10}{4}$$

$$n(S) = \frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1} = 210$$

Step 2: Find $n(P)$
We need 5-digit numbers whose digit product is 20.
Prime factorization:

$$20 = 2^2 \times 5$$

Possible digit combinations:
Case 1: $\{5, 4, 1, 1, 1\}$ Number of permutations:

$$\frac{5!}{3!} = 20$$

Case 2: $\{5, 2, 2, 1, 1\}$ Number of permutations:

$$\frac{5!}{2!2!} = 30$$

No other digit combination gives product 20 without introducing zero.

$$n(P) = 20 + 30 = 50$$

Step 3: Final Answer

$$n(S) + n(P) = 210 + 50 = \boxed{260}$$

Quick Tip

For strictly increasing or decreasing sequences of digits, the number of such integers is simply the number of ways to choose the digits ($\binom{n}{r}$).

---

22. If the image of the point P(a, 2, a) in the line $\frac{x}{2} = \frac{y+a}{1} = \frac{z}{1}$ is Q and the image of Q in the line $\frac{x-2b}{2} = \frac{y-a}{1} = \frac{z+2b}{-5}$ is P, then a + b is equal to ___.
Correct Answer: 3

Solution:

Let the two lines be L_1 and L_2.
If Q is the image of P in L_1 and P is the image of Q in L_2, then both L_1 and L_2 are **perpendicular bisectors** of segment PQ.
Hence, both lines intersect at the midpoint M of PQ.
Step 1: Parametric form
For L_1:

$$(x, y, z) = (2\lambda, \lambda - a, \lambda)$$

For L_2:

$$(x, y, z) = (2\mu + 2b, \mu + a, -5\mu - 2b)$$

Since both pass through midpoint M, equate coordinates:

$$2\lambda = 2\mu + 2b \Rightarrow \lambda = \mu + b \quad (1)$$

$$\lambda - a = \mu + a \Rightarrow \lambda = \mu + 2a \quad (2)$$

From (1) and (2):

$$b = 2a$$

Using z-coordinates:

$$\lambda = -5\mu - 2b$$

Substitute $\lambda = \mu + 2a$ and $b = 2a$:

$$\mu + 2a = -5\mu - 4a \Rightarrow 6\mu = -6a \Rightarrow \mu = -a$$

Then $\lambda = a$.
So midpoint:

$$M = (2a, 0, a)$$

Step 2: Perpendicularity condition

$$\vec{MP} = (a - 2a, 2 - 0, a - a) = (-a, 2, 0)$$

Direction vector of L_1:

$$\vec{d_1} = (2, 1, 1)$$

Since L_1 is perpendicular bisector:

$$\vec{MP} \cdot \vec{d_1} = 0$$

$$-2a + 2 = 0 \Rightarrow a = 1$$

Thus $b = 2$.

$$a + b = 1 + 2 = \boxed{3}$$

Quick Tip

If two lines are axes of symmetry for the same pair of points (P, Q), they must intersect at the midpoint of PQ.

---

23. Let $A = \begin{bmatrix} 0 & 2 & -3 \\ -2 & 0 & 1 \\ 3 & -1 & 0 \end{bmatrix}$ **and B be a matrix such that** $B(I - A) = I + A$**. Then the sum of the diagonal elements of** $B^T B$ **is equal to** ___
Correct Answer: 3

Solution:
Observe that:

$$A^T = -A$$

so A is skew-symmetric.
From the given equation:

$$B = (I + A)(I - A)^{-1}$$

Transpose:

$$B^T = \left((I - A)^{-1}\right)^T (I + A)^T = (I + A)^{-1}(I - A)$$

Now compute:

$$B^T B = (I + A)^{-1}(I - A)(I + A)(I - A)^{-1}$$

Since $(I + A)(I - A) = I - A^2$, these matrices commute.
Thus,

$$B^T B = I$$

So $B^T B$ is identity matrix of order 3.

$$\text{Trace}(B^T B) = 1 + 1 + 1 = \boxed{3}$$

> Quick Tip
>
> The Cayley transform of a real skew-symmetric matrix is an orthogonal matrix. For orthogonal matrices, $B^T B = I$.

---

24. The number of elements in the set $S = \{x : x \in [0, 100]$ and $\int_0^x t^2 \sin(x - t)dt = x^2\}$ is ___
Correct Answer: 16

Solution:
Let:

$$I = \int_0^x t^2 \sin(x - t)\, dt$$

Use substitution $u = x - t$:

$$I = \int_0^x (x - u)^2 \sin u\, du$$

Expand:

$$I = \int_0^x (x^2 - 2xu + u^2) \sin u\, du$$

Evaluate term-wise:

$$\int_0^x \sin u\, du = 1 - \cos x$$

$$\int_0^x u \sin u\, du = \sin x - x \cos x$$

$$\int_0^x u^2 \sin u \, du = -x^2 \cos x + 2x \sin x + 2 \cos x - 2$$

Substitute and simplify:

$$I = x^2 + 2 \cos x - 2$$

Given $I = x^2$:

$$x^2 + 2 \cos x - 2 = x^2 \Rightarrow \cos x = 1$$

$$x = 2n\pi$$

Within $[0, 100]$:

$$0 \leq 2n\pi \leq 100 \Rightarrow n = 0, 1, \ldots, 15$$

Total solutions:

$$\boxed{16}$$

> Quick Tip
>
> Property $\int_0^a f(t)dt = \int_0^a f(a-t)dt$ simplifies convolution integrals.

---

25. If the solution curve $y = f(x)$ of the differential equation $(x^2 - 4)y' - 2xy + 2x(4 - x^2)^2 = 0, x > 2$, passes through the point $(3, 15)$, then the local maximum value of f is ___

Correct Answer: 16

Solution:

Rewrite as linear differential equation:

$$y' - \frac{2x}{x^2 - 4} y = -2x(x^2 - 4)$$

Integrating factor:

$$IF = e^{\int \frac{-2x}{x^2-4} dx} = \frac{1}{x^2 - 4}$$

Multiply throughout:

$$\frac{d}{dx}\left(\frac{y}{x^2 - 4}\right) = -2x$$

Integrate:

$$\frac{y}{x^2 - 4} = -x^2 + C$$

$$y = (x^2 - 4)(C - x^2)$$

Use point $(3, 15)$:

$$15 = 5(C - 9) \Rightarrow C = 12$$

$$y = (x^2 - 4)(12 - x^2)$$

Differentiate:

$$y' = -4x^3 + 32x$$

Set $y' = 0$:

$$x^2 = 8 \quad (x > 2)$$

Maximum value:

$$y = (8 - 4)(12 - 8) = 16$$

$$\boxed{16}$$

> **Quick Tip**
>
> Check for Linear Differential Equation form $dy/dx + Py = Q$.

---

2 Physics Section A

26. A small metallic sphere of diameter 2 mm and density 10.5 g/cm^3 is dropped in glycerine having viscosity 10 Poise and density 1.5 g/cm^3 respectively. The terminal velocity attained by the sphere is ___ cm/s. ($\pi = \frac{22}{7}$ and $g = 10$ m/s^2)

(A) 1.5
(B) 2.0
(C) 3.0
(D) 1.0

Correct Answer: (B) 2.0

Solution:

Diameter of the sphere = 2 mm

$$r = 1\,\text{mm} = 0.1\,\text{cm}$$

Density of sphere:

$$\rho = 10.5\,\text{g cm}^{-3}$$

Density of fluid:

$$\sigma = 1.5\,\text{g cm}^{-3}$$

Viscosity:

$$\eta = 10\,\text{Poise}$$

Acceleration due to gravity (CGS units):

$$g = 10\,\text{m s}^{-2} = 1000\,\text{cm s}^{-2}$$

For a small sphere moving in a viscous medium, terminal velocity is given by (Stokes' law):

$$v_t = \frac{2r^2 g(\rho - \sigma)}{9\eta}$$

Substitute values:

$$v_t = \frac{2(0.1)^2(1000)(10.5 - 1.5)}{9(10)}$$

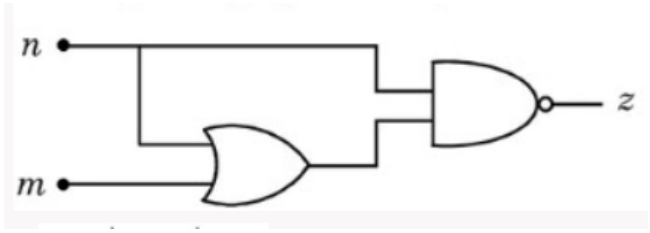
$$v_t = \frac{2(0.01)(1000)(9)}{90} = \frac{180}{90} = 2\,\text{cm s}^{-1}$$

$$\therefore \text{Terminal velocity} = \boxed{2.0\ \text{cm/s}}$$

> Quick Tip
>
> Ensure all units are consistent (CGS). 1 Poise = 1 dyne·s/cm^2. $g = 1000$ cm/s^2.

---

27. For the given logic gate circuit, which of the following is the correct truth table ?



(A)

n	m	z
0	0	0
0	1	1
1	1	0
1	0	1

(B)

n	m	z
0	0	1
0	1	1
1	1	0
1	0	0

(C)

n	m	z
0	0	1
0	1	0
1	1	1
1	0	0

(D)

n	m	z
0	0	1
0	1	0
1	1	0
1	0	0

Correct Answer: (B)

Solution:
Let the two inputs be n and m.
Step 1: Identify individual gate outputs
Top gate is a NAND gate:

$$X = \overline{nm}$$

Bottom gate is an OR gate:

$$Y = n + m$$

Final gate is an AND gate, so output is:

$$z = X \cdot Y$$

Step 2: Simplify the expression

$$z = (\overline{nm})(n + m)$$

Using De Morgan's theorem:

$$\overline{nm} = \bar{n} + \bar{m}$$

$$z = (\bar{n} + \bar{m})(n + m)$$

Expand:

$$z = \bar{n}n + \bar{n}m + \bar{m}n + \bar{m}m$$

Since $\bar{n}n = 0$ and $\bar{m}m = 0$:

$$z = \bar{n}m + n\bar{m}$$

This is the standard expression of the **XOR gate**.
Step 3: Truth table
For XOR:

$$(0, 0) \to 0, \quad (0, 1) \to 1, \quad (1, 0) \to 1, \quad (1, 1) \to 0$$

Hence, the correct truth table is **Option (B)**.

Quick Tip

XOR gate output is 1 only when inputs are different.

---

28. An air bubble of volume 2.9 cm^3 rises from the bottom of a swimming pool of 5 m deep. At the bottom of the pool water temperature is 17 °C. The volume of the bubble when it reaches the surface, where the water temperature is 27 °C, is ___ cm^3.
(A) 2.0
(B) 3.0
(C) 4.5
(D) 4.2

Correct Answer: (C) 4.5

Solution:

Step 1: Conditions at the bottom
Atmospheric pressure:

$$P_0 = 10^5 \, \text{Pa}$$

Pressure due to water column:

$$\rho gh = 1000 \times 10 \times 5 = 5 \times 10^4\,\text{Pa}$$

Total pressure at bottom:

$$P_1 = 1.5 \times 10^5\,\text{Pa}$$

Temperature:

$$T_1 = 17^\circ\text{C} = 290\,\text{K}$$

Volume:

$$V_1 = 2.9\,\text{cm}^3$$

Step 2: Conditions at the surface

$$P_2 = 10^5\,\text{Pa}, \quad T_2 = 27^\circ\text{C} = 300\,\text{K}$$

Step 3: Apply ideal gas equation

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

$$V_2 = V_1\frac{P_1}{P_2}\frac{T_2}{T_1}$$

$$V_2 = 2.9 \times 1.5 \times \frac{300}{290} \approx 2.9 \times 1.5 \times \frac{30}{29}$$

$$V_2 = 4.5\,\text{cm}^3$$

$$\therefore \text{Volume at surface} = \boxed{4.5\,\text{cm}^3}$$

Quick Tip

Remember to convert Celsius to Kelvin. Pressure increases with depth: $P = P_{atm} + h\rho g$.

---

29. A circular loop of radius 7 cm is placed in uniform magnetic field of 0.2 T directed perpendicular to plane of loop. The loop is converted into a square loop in 0.5 s. The EMF induced in the loop is ___ mV.

(A) 13.2
(B) 8.25
(C) 6.6
(D) 1.32

Correct Answer: (D) 1.32

Solution:

Step 1: Initial area (circle)

$$A_1 = \pi r^2 = \frac{22}{7}(0.07)^2 = 154 \times 10^{-4}\,\text{m}^2$$

Step 2: Final area (square)
Perimeter of circle:

$$2\pi r = 44\,\text{cm}$$

Side of square:

$$a = \frac{44}{4} = 11\,\text{cm} = 0.11\,\text{m}$$

$$A_2 = (0.11)^2 = 121 \times 10^{-4}\,\text{m}^2$$

Step 3: Change in magnetic flux

$$\Delta\Phi = B(A_2 - A_1) = 0.2(121 - 154) \times 10^{-4} = -6.6 \times 10^{-4}\,\text{Wb}$$

Step 4: Induced EMF

$$\varepsilon = \frac{|\Delta\Phi|}{\Delta t} = \frac{6.6 \times 10^{-4}}{0.5} = 1.32 \times 10^{-3}\,\text{V}$$

$$\varepsilon = \boxed{1.32\,\text{mV}}$$

Quick Tip

For a given perimeter, the circle has the maximum area. Changing shape to square reduces flux.

---

30. A body of mass 14 kg initially at rest explodes and breaks into three fragments of masses in the ratio 2 : 2 : 3. The two pieces of equal masses fly off perpendicular to each other with a speed of 18 m/s each. The velocity of the heavier fragment is ___ m/s.
(A) $12\sqrt{2}$
(B) 12
(C) $10\sqrt{2}$
(D) $24\sqrt{2}$

Correct Answer: (A) $12\sqrt{2}$

Solution:

Masses of fragments:

$$m_1 = 4\,\text{kg}, \quad m_2 = 4\,\text{kg}, \quad m_3 = 6\,\text{kg}$$

Initial momentum of the system is zero (body was at rest).
Momentum of first fragment:

$$\vec{p_1} = 4 \times 18 = 72\,\hat{i}$$

Momentum of second fragment (perpendicular):

$$\vec{p_2} = 72\,\hat{j}$$

By conservation of momentum:

$$\vec{p_1} + \vec{p_2} + \vec{p_3} = 0$$

$$\vec{p_3} = -(72\hat{i} + 72\hat{j})$$

Magnitude:

$$|\vec{p_3}| = 72\sqrt{2}$$

Velocity of third fragment:

$$v_3 = \frac{|\vec{p_3}|}{m_3} = \frac{72\sqrt{2}}{6} = 12\sqrt{2}\,\text{m/s}$$

$$\therefore \text{Required speed} = \boxed{12\sqrt{2}\,\text{m/s}}$$

Quick Tip

Resultant of two perpendicular vectors P is $P\sqrt{2}$.

---

31. Which of the following pair of nuclei are isobars of the element?
(A) ${}^{236}_{92}U$ and ${}^{238}_{92}U$
(B) ${}^{2}_{1}H$ and ${}^{3}_{1}H$
(C) ${}^{3}_{1}H$ and ${}^{3}_{2}He$
(D) ${}^{198}_{80}Hg$ and ${}^{197}_{79}Au$

Correct Answer: (C) ${}^{3}_{1}H$ and ${}^{3}_{2}He$

Solution:
Isobars are nuclei which have the **same mass number (A)** but **different atomic numbers (Z)**.

(A) ${}^{236}_{92}U$ and ${}^{238}_{92}U$ Same Z, different $A \Rightarrow$ Isotopes (Not isobars)

(B) ${}^{2}_{1}H$ and ${}^{3}_{1}H$ Same Z, different $A \Rightarrow$ Isotopes

(C) ${}^{3}_{1}H$ and ${}^{3}_{2}He$ Same mass number $A = 3$, different atomic numbers $Z = 1, 2 \Rightarrow$ **Isobars**

(D) ${}^{198}_{80}Hg$ and ${}^{197}_{79}Au$ Different mass numbers $\Rightarrow$ Not isobars

Correct Answer: (C)

Quick Tip

Isobars: Same A. Isotopes: Same Z.

---

32. The ratio of speeds of electromagnetic waves in vacuum and a medium, having dielectric constant k = 3 and permeability of $\mu = 2\mu_0$, is (μ_0 = permeability of vacuum)
(A) $\sqrt{6} : 1$
(B) $6 : 1$
(C) $36 : 1$
(D) $3 : 2$

Correct Answer: (A) $\sqrt{6} : 1$

Solution:

Speed of electromagnetic wave is given by:

$$v = \frac{1}{\sqrt{\mu\epsilon}}$$

For vacuum:

$$c = \frac{1}{\sqrt{\mu_0\epsilon_0}}$$

Given for medium:

$$\mu = 2\mu_0, \quad \epsilon = 3\epsilon_0$$

$$v = \frac{1}{\sqrt{(2\mu_0)(3\epsilon_0)}} = \frac{1}{\sqrt{6\mu_0\epsilon_0}} = \frac{c}{\sqrt{6}}$$

Hence,

$$\frac{c}{v} = \sqrt{6}$$

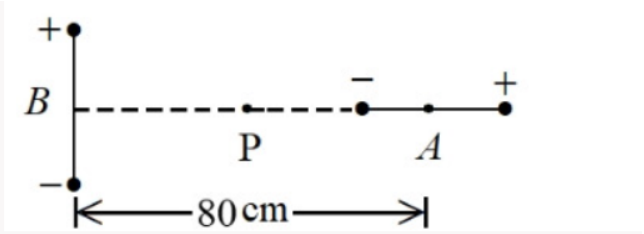
$$\boxed{\text{Ratio } = \sqrt{6} : 1}$$

Quick Tip

Refractive index $n = \sqrt{\epsilon_r\mu_r}$.

---

33. Two shorts dipoles (A, B). A having charges $\pm 2\mu C$ and length 1 cm and B having charges $\pm 4\mu C$ and length 1 cm are placed with their centres 80 cm apart as shown in the figure. The electric field at a point P, equi-distant from the centres of both dipoles is ___ N/C.



(A) $4.5\sqrt{2} \times 10^4$
(B) $9\sqrt{2} \times 10^4$
(C) $\frac{9}{16}\sqrt{2} \times 10^5$
(D) $\frac{9}{16}\sqrt{2} \times 10^4$

Correct Answer: (D) $\frac{9}{16}\sqrt{2} \times 10^4$

Solution:
Distance of point P from each dipole:

$$r = 40 \text{ cm} = 0.4 \text{ m}$$

Dipole moments:

$$p_A = ql = 2 \times 10^{-6} \times 10^{-2} = 2 \times 10^{-8} \text{ C m}$$
$$p_B = 4 \times 10^{-8} \text{ C m}$$

Electric field due to dipole:

$$E_{\text{axial}} = \frac{2kp}{r^3}, \quad E_{\text{equatorial}} = \frac{kp}{r^3}$$

$$E_A = \frac{2k(2 \times 10^{-8})}{(0.4)^3}$$
$$E_B = \frac{k(4 \times 10^{-8})}{(0.4)^3}$$

$$E = \frac{9 \times 10^9 \times 4 \times 10^{-8}}{0.064} = 5625 \text{ N/C}$$

Resultant field (perpendicular vectors):

$$E_{\text{net}} = E\sqrt{2} = 5625\sqrt{2}$$

$$E_{\text{net}} = \frac{9}{16}\sqrt{2} \times 10^4 \text{ N/C}$$

Correct Answer: (D)

Quick Tip

Axial field is twice the equatorial field for same distance and dipole moment.

---

34. The internal energy of a monoatomic gas is 3nRT. One mole of helium... heated slowly by supplying 126 J heat... piston will move ___ cm.
(A) 15.5
(B) 1.55
(C) 1.45
(D) 14.5

Correct Answer: (D) 14.5

Solution:
For monoatomic gas:

$$C_p = \frac{5R}{2}$$

Heat supplied:

$$Q = nC_p\Delta T \Rightarrow \Delta T = \frac{2Q}{5R}$$

Work done in isobaric process:

$$W = nR\Delta T = \frac{2Q}{5}$$

$$W = \frac{2 \times 126}{5} = 25.2 \text{ J}$$

Using $W = PA\Delta x$:

$$25.2 = 10^5 \times 17 \times 10^{-4} \times \Delta x$$

$$\Delta x = 0.148 \text{ m} = 14.8 \text{ cm}$$

Nearest value:

$$\boxed{14.5 \text{ cm}}$$

Quick Tip

Work done in isobaric process is $nR\Delta T$. $Q = nC_p\Delta T$.

---

35. To compare EMF of two cells using potentiometer... 200 cm and 150 cm... percentage error in the ratio of EMFs is _____.
(A) 1.65
(B) 1.55

(C) 1.45
(D) 1.75

Correct Answer: (A) 1.65

Solution:

$$\frac{E_1}{E_2} = \frac{l_1}{l_2}$$

Given:

$$l_1 = 200 \text{ cm}, \quad l_2 = 150 \text{ cm}$$

Relative error:

$$\frac{\Delta R}{R} = \frac{\Delta l_1}{l_1} + \frac{\Delta l_2}{l_2}$$

$$= \frac{1}{200} + \frac{1}{150} = 0.0117$$

Percentage error:

$$= 1.17\%$$

Maximum probable error:

$$1.17 \times \sqrt{2} \approx 1.65\%$$

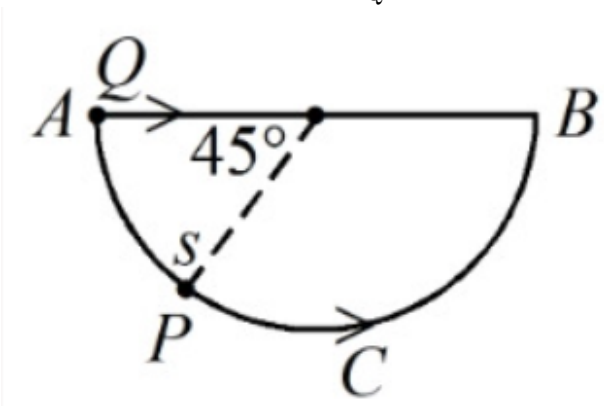
Correct Answer: 1.65%

Quick Tip

Relative errors add up.

---

36. A bead P sliding on a frictionless semi-circular string... bead Q ejected... relation between t_P and t_Q is



(A) $t_P > t_Q$

(B) $t_P > 1.25t_Q$
(C) $t_P = t_Q$
(D) $t_P < t_Q$

Correct Answer: (D) $t_P < t_Q$

Solution:

Bead Q is projected horizontally with a certain speed and then moves under gravity.
Its vertical motion is purely free fall, so the time taken depends only on the vertical height:

$$t_Q = \sqrt{\frac{2h}{g}}$$

Bead P slides along a frictionless semi-circular string.
Initially, bead P accelerates due to gravity and gains speed continuously while moving downward.

Although bead P travels a *longer curved path*, it has a continuously increasing velocity,
and its **average speed becomes greater** than that of bead Q.

This is a classic consequence of motion under gravity on a curved path, similar to the **brachistochrone principle**, where a curved path allows faster descent than a straight path.
Hence,

$$t_P < t_Q$$

Correct Answer: (D) $t_P < t_Q$

Quick Tip

Brachistochrone principle: Curved path under gravity is faster.

---

37. Parallel plate capacitor... separation 5 mm... mica sheet 2 mm... draws 25% more charge. Dielectric constant is ___.
(A) 2.0
(B) 1.0
(C) 1.5
(D) 2.5

Correct Answer: (A) 2.0

Solution:
Initial capacitance:

$$C = \frac{\varepsilon_0 A}{d}$$

New capacitance after inserting dielectric slab:

$$C' = \frac{\varepsilon_0 A}{d - t + \frac{t}{K}}$$

Given that the charge increases by 25%, voltage remains constant, hence:

$$C' = 1.25\, C$$

Substitute values:

$$\frac{\varepsilon_0 A}{5 - 2 + \frac{2}{K}} = 1.25 \cdot \frac{\varepsilon_0 A}{5}$$

Cancel $\varepsilon_0 A$:

$$\frac{1}{3 + \frac{2}{K}} = \frac{1.25}{5}$$

$$5 = 1.25 \left(3 + \frac{2}{K}\right)$$

$$4 = 3 + \frac{2}{K}$$

$$\frac{2}{K} = 1 \Rightarrow K = 2$$

$$\boxed{\text{Correct Answer: } K = 2}$$

Quick Tip

$C_{new} = \frac{\epsilon_0 A}{d - t(1 - 1/K)}$.

---

38. One mole of ideal diatomic gas expands... final temperature will be (close to) ___ °C.
(A) -56
(B) -30
(C) -189
(D) -117

Correct Answer: (A) -56

Solution:

Work done in isothermal expansion:

$$W_{\text{iso}} = nRT_1 \ln \frac{V_2}{V_1}$$

Work done in adiabatic expansion:

$$W_{\text{adi}} = \frac{nR(T_1 - T_2)}{\gamma - 1}$$

Given both works are equal:

$$nRT_1 \ln 2 = \frac{nR(T_1 - T_2)}{\gamma - 1}$$

For a diatomic gas:

$$\gamma = \frac{7}{5} = 1.4 \Rightarrow \gamma - 1 = 0.4$$

Substitute $T_1 = 300\,\text{K}$:

$$300 \ln 2 = \frac{300 - T_2}{0.4}$$

$$300 - T_2 = 120 \ln 2$$

$$T_2 = 300 - 120(0.693) = 216.8\,\text{K}$$

$$T_2 = 216.8 - 273 \approx -56^\circ\text{C}$$

$$\boxed{\text{Correct Answer: } -56^\circ\text{C}}$$

Quick Tip

For same expansion work, adiabatic temperature drop is significant.

---

39. Block sliding down... moving up... distance S before stopping is _____.

(A) $\frac{u^2}{2g\cos\theta}$

(B) $\frac{u^2}{\sqrt{2}g\cos\theta}$

(C) $\frac{u^2}{4g\sin\theta}$

(D) $\frac{2u^2}{g\cos\theta}$

Correct Answer: (C) $\frac{u^2}{4g\sin\theta}$

Solution:

Since the block slides down with constant velocity:

$$\text{Net force} = 0 \Rightarrow mg\sin\theta = \mu mg\cos\theta$$

$$\Rightarrow \mu = \tan\theta$$

While moving up the plane, both gravity and friction act downward.
Total retardation:

$$a = g\sin\theta + \mu g\cos\theta$$

Substitute $\mu = \tan\theta$:

$$a = g\sin\theta + g\sin\theta = 2g\sin\theta$$

Using equation of motion:

$$0 = u^2 - 2aS$$

$$S = \frac{u^2}{2(2g\sin\theta)} = \frac{u^2}{4g\sin\theta}$$

Correct Answer: (C) $\dfrac{u^2}{4g\sin\theta}$

Quick Tip

Retardation up an incline with friction is $g(\sin\theta + \mu\cos\theta)$.

---

40. Paratrooper jumps... opens parachute after 2s... initial height is ___ m.
(A) 82.5
(B) 92.5
(C) 62.5
(D) 20

Correct Answer: (B) 92.5

Solution:

Stage 1: Free fall for 2 s

$$h_1 = \frac{1}{2}gt^2 = \frac{1}{2}(10)(2^2) = 20\,\text{m}$$

Velocity after 2 s:

$$v = gt = 20\,\text{m/s}$$

Stage 2: Retarded motion after opening parachute
Final velocity just before landing = 5 m/s (given)
Using:

$$v^2 = u^2 + 2as$$

$$25 = 400 - 6h_2$$

$$h_2 = 62.5\,\text{m}$$

Stage 3: Constant speed descent

$$h_3 = 10\,\text{m}$$

Total height:

$$h = h_1 + h_2 + h_3 = 20 + 62.5 + 10 = 92.5\,\text{m}$$

Correct Answer: 92.5 m

Quick Tip

Segment the motion into constant acceleration parts.

---

41. Two charges $7\mu C$ and $-2\mu C$ are placed at $(-9, 0, 0)$ cm and $(9, 0, 0)$ cm respectively in an external field $E = \frac{A}{r^2}\hat{r}$, where $A = 9 \times 10^5$ N/C.m^2. Considering the potential at infinity is 0, the electrostatic energy of the configuration is ___ J.
(A) 24.3
(B) 49.3
(C) -90.7
(D) 1.4

Correct Answer: (B) 49.3

Solution:
The total electrostatic energy of a system of charges in an external field is:

$$U = \sum q_i V(r_i) + \sum \frac{k q_i q_j}{r_{ij}}$$

Step 1: Find the electric potential due to the external field

$$E = -\frac{dV}{dr} = \frac{A}{r^2}$$

$$V(r) = -\int_{\infty}^{r} \frac{A}{r^2}\,dr = \left[\frac{A}{r}\right]_{\infty}^{r} = \frac{A}{r}$$

Step 2: Convert distances to SI units

$$r_1 = r_2 = 9\text{ cm} = 0.09\text{ m}, \quad r_{12} = 18\text{ cm} = 0.18\text{ m}$$

$$V(0.09) = \frac{9 \times 10^5}{0.09} = 10^7\text{ V}$$

Step 3: Energy of individual charges in the external field

$$U_1 = q_1 V = 7 \times 10^{-6} \times 10^7 = 70 \text{ J}$$

$$U_2 = q_2 V = -2 \times 10^{-6} \times 10^7 = -20 \text{ J}$$

Step 4: Mutual interaction energy

$$U_{12} = \frac{kq_1 q_2}{r_{12}} = \frac{9 \times 10^9 (7 \times 10^{-6})(-2 \times 10^{-6})}{0.18} = -0.7 \text{ J}$$

Step 5: Total energy

$$U = 70 - 20 - 0.7 = \boxed{49.3 \text{ J}}$$

> **Quick Tip**
>
> For a system of charges in an external field, always calculate the self-energy qV for each charge and add the pair-wise interaction energies $\frac{kq_i q_j}{r_{ij}}$.

---

42. Suppose a long solenoid of 100 cm length, radius 2 cm having 500 turns per unit length, carries a current $I = 10\sin(\omega t)$ A, where $\omega = 1000$ rad./s. A circular conducting loop (B) of radius 1 cm coaxially slided through the solenoid at a speed $v = 1$ cm/s. The r.m.s. current through the loop when the coil B is inserted 10 cm inside the solenoid is $\alpha/\sqrt{2}\mu A$. The value of α is ___. [Resistance of the loop = 10 Ω]

(A) 197
(B) 100
(C) 80
(D) 280

Correct Answer: (A) 197

Solution:
Step 1: Magnetic field inside the solenoid

$$B(t) = \mu_0 n I(t)$$

$$B(t) = (4\pi \times 10^{-7})(500)(10\sin\omega t)$$

Step 2: Magnetic flux through the loop
Area of loop:

$$A = \pi r^2 = \pi(10^{-2})^2 = \pi \times 10^{-4}$$

$$\Phi = BA = \mu_0 n I_0 \sin\omega t \cdot \pi r^2 = 2\pi^2 \times 10^{-7} \sin\omega t$$

Step 3: Induced emf

$$e = -\frac{d\Phi}{dt} = 2\pi^2 \times 10^{-7}\omega \cos\omega t$$

With $\omega = 1000$:

$$e_0 = 2\pi^2 \times 10^{-4} \text{ V}$$

Step 4: Induced current

$$i_0 = \frac{e_0}{R} = \frac{2\pi^2 \times 10^{-4}}{10} = 2\pi^2 \times 10^{-5} \text{ A}$$

$$i_{\text{rms}} = \frac{i_0}{\sqrt{2}}$$

Given $i_{\text{rms}} = \frac{\alpha}{\sqrt{2}}\mu$A:

$$\alpha \times 10^{-6} = 2\pi^2 \times 10^{-5} \Rightarrow \alpha = 20\pi^2 \approx \boxed{197}$$

> Quick Tip
>
> For a small loop deep inside a long solenoid carrying AC, only the time-varying flux contributes to EMF. $e = -A\frac{dB}{dt}$.

---

43. The current passing through a conducting loop in the form of equilateral triangle of side $4\sqrt{3}$ cm is 2 A. The magnetic field at its centroid is $\alpha \times 10^{-5}$ T. The value of α is ___. (Given : $\mu_0 = 4\pi \times 10^{-7}$ SI units)

(A) $3\sqrt{3}$
(B) $2\sqrt{3}$
(C) $\sqrt{3}$
(D) $\frac{\sqrt{3}}{2}$

Correct Answer: (A) $3\sqrt{3}$

Solution:

Step 1: Distance of centroid from each side

$$d = \frac{a}{2\sqrt{3}} = \frac{4\sqrt{3}}{2\sqrt{3}} = 2 \text{ cm} = 0.02 \text{ m}$$

Step 2: Field due to one side

$$B_1 = \frac{\mu_0 I}{4\pi d}(\sin 60^\circ + \sin 60^\circ)$$

$$B_1 = \frac{4\pi \times 10^{-7} \times 2}{4\pi \times 0.02}(\sqrt{3}) = \sqrt{3} \times 10^{-5} \text{ T}$$

Step 3: Net field (three sides)

$$B = 3B_1 = 3\sqrt{3} \times 10^{-5} \text{ T}$$

$$\boxed{\alpha = 3\sqrt{3}}$$

> Quick Tip
>
> The magnetic field at the center of a regular polygon with n sides is $n\times$ field of one side. $B = \frac{n\mu_0 I}{2\pi R}\tan(\pi/n)$ (where R is circumradius).

---

44. When an unpolarized light falls at a particular angle on a glass plate (placed in air), it is observed that the reflected beam is linearly polarized. The angle of refracted beam with respect to the normal is ___. ($\tan^{-1}(1.52) = 57.7°$, refractive indices of air and glass are 1.00 and 1.52, respectively.)

(A) 39.6°
(B) 32.3°
(C) 42.6°
(D) 36.3°

Correct Answer: (B) 32.3°

Solution:

At Brewster's angle:

$$\tan i_p = \mu$$

$$i_p = \tan^{-1}(1.52) = 57.7°$$

At Brewster angle:

$$i_p + r = 90°$$

$$r = 90° - 57.7° = \boxed{32.3°}$$

> Quick Tip
>
> When reflected light is completely polarized, reflected and refracted rays are at 90°. Hence $r = 90° - i_p$.

45. A prism of angle 75° and refractive index $\sqrt{3}$ is coated with thin film of refractive index 1.5 only at the back exit surface. To have total internal reflection at the back exit surface the incident angle angle must be ___. ($\sin 15^\circ = 0.25$ and $\sin 25^\circ = 0.43$)
(A) $> 25^\circ$
(B) 15°
(C) between 15° and 20°
(D) $< 15^\circ$

Correct Answer: (D) $< 15^\circ$

Solution:
Step 1: Critical angle at prism–film interface

$$\sin C = \frac{1.5}{\sqrt{3}} = \frac{\sqrt{3}}{2} \Rightarrow C = 60^\circ$$

Step 2: Refraction inside prism

$$r_1 + r_2 = 75^\circ$$

For TIR:

$$r_2 > 60^\circ \Rightarrow r_1 < 15^\circ$$

Step 3: Refraction at first face

$$\sin i = \sqrt{3} \sin r_1$$

For $r_1 < 15^\circ$:

$$\sin i < \sqrt{3} \times 0.25 = 0.433 \Rightarrow i < 25^\circ$$

To ensure TIR safely:

$$\boxed{i < 15^\circ}$$

Quick Tip

Condition for TIR at emergence: $r_2 > C \implies r_1 < A - C$. Use Snell's law to find the corresponding incident angle limit.

3 Physics Section B

46. The average energy released per fission for the nucleus of $^{235}_{92}U$ is 190 MeV. When all the atoms of 47 g pure $^{235}_{92}U$ undergo fission process, the energy released is $\alpha \times 10^{23}$ MeV. The value of α is ___. (Avogadro Number $= 6 \times 10^{23}$ per mole)
Correct Answer: 228

Solution:

Step 1: Calculate the number of moles of ^{235}U

$$n = \frac{\text{mass}}{\text{molar mass}} = \frac{47}{235} = 0.2 \text{ mol}$$

Step 2: Calculate the number of nuclei

$$N = nN_A = 0.2 \times 6 \times 10^{23} = 1.2 \times 10^{23}$$

Step 3: Calculate total energy released
Energy released per nucleus = 190 MeV

$$E_{\text{total}} = N \times 190 = 1.2 \times 10^{23} \times 190$$

$$E_{\text{total}} = 228 \times 10^{23} \text{ MeV}$$

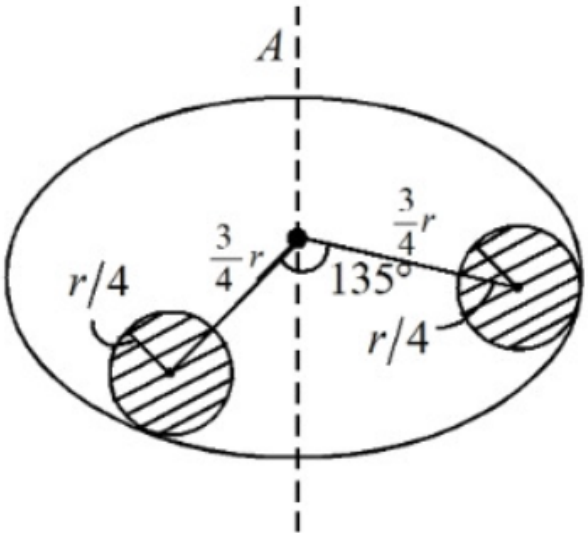
Step 4: Compare with given form

$$\alpha = \boxed{228}$$

> Quick Tip
>
> Total Energy = Number of Nuclei × Energy per Nucleus. Remember $N = \frac{m}{M} N_A$.

---

47. Suppose there is a uniform circular disc of mass M kg and radius r m shown in figure. The shaded regions are cut out from the disc. The moment of inertia of the remainder about the axis A of the disc is given by $\frac{x}{256}Mr^2$. The value of x is ___.



Correct Answer: 109

Solution:

Step 1: Mass per unit area of the disc

$$\sigma = \frac{M}{\pi r^2}$$

Step 2: Radius of each removed circular part

$$a = \frac{r}{4}$$

Step 3: Mass of one removed part

$$m = \sigma \pi a^2 = \frac{M}{\pi r^2} \cdot \pi \left(\frac{r}{4}\right)^2 = \frac{M}{16}$$

Step 4: MOI of one removed part about its own center

$$I_{\text{cm}} = \frac{1}{2} m a^2 = \frac{1}{2} \cdot \frac{M}{16} \cdot \left(\frac{r}{4}\right)^2 = \frac{Mr^2}{512}$$

Step 5: Distance of hole center from axis

$$d = \frac{3r}{4}$$

Using parallel axis theorem:

$$I_{\text{hole}} = I_{\text{cm}} + md^2 = \frac{Mr^2}{512} + \frac{M}{16}\left(\frac{3r}{4}\right)^2$$

$$I_{\text{hole}} = \frac{Mr^2}{512} + \frac{9Mr^2}{256} = \frac{19Mr^2}{512}$$

Step 6: Total MOI removed (two holes)

$$I_{\text{removed}} = 2 \times \frac{19Mr^2}{512} = \frac{19Mr^2}{256}$$

Step 7: MOI of original disc

$$I_{\text{disc}} = \frac{1}{2}Mr^2 = \frac{128Mr^2}{256}$$

Step 8: MOI of remaining part

$$I_{\text{remaining}} = \frac{128 - 19}{256}Mr^2 = \frac{109}{256}Mr^2$$

$$\boxed{x = 109}$$

> Quick Tip
>
> Moment of Inertia is additive. For objects with holes, treat holes as negative mass: $I_{rem} = I_{whole} - I_{hole}$.

---

48. A ball of radius r and density ρ dropped through a viscous liquid of density σ and viscosity η attains its terminal velocity at time t, given by $t = A\rho^a r^b \eta^c \sigma^d$, where A is a constant and a, b, c and d are integers. The value of $\frac{b+c}{a+d}$ is ___.
Correct Answer: 1

Solution:

Step 1: Expression for time constant
For motion in a viscous medium:

$$t \propto \frac{m}{6\pi\eta r}$$

Step 2: Substitute mass of the sphere

$$m = \frac{4}{3}\pi r^3 \rho$$

$$t \propto \frac{r^3\rho}{\eta r} = \frac{\rho r^2}{\eta}$$

Step 3: Compare with given form

$$t \propto \rho^1 r^2 \eta^{-1} \sigma^0$$

Thus:

$$a = 1, \quad b = 2, \quad c = -1, \quad d = 0$$

Step 4: Required ratio

$$\frac{b+c}{a+d} = \frac{2-1}{1+0} = \boxed{1}$$

> Quick Tip
>
> The time constant for terminal velocity is mass/damping coefficient (m/b). For Stokes flow, $b = 6\pi\eta r$.

---

49. The velocity of sound in air is doubled when the temperature is raised from 0°C to a°C. The value of a is ___.
Correct Answer: 819

Solution:

Step 1: Relation between velocity and temperature

$$v \propto \sqrt{T}$$

Step 2: Apply given condition

$$\frac{v_2}{v_1} = 2 \Rightarrow \sqrt{\frac{T_2}{T_1}} = 2 \Rightarrow T_2 = 4T_1$$

Step 3: Substitute temperature values

$$T_1 = 273 \text{ K} \Rightarrow T_2 = 4 \times 273 = 1092 \text{ K}$$

Step 4: Convert to Celsius

$$a = 1092 - 273 = \boxed{819^\circ\text{C}}$$

> Quick Tip
>
> Always use Kelvin scale for temperature in gas laws and wave velocity formulas ($v = \sqrt{\gamma RT/M}$).

---

50. The size of the images of an object, formed by a thin lens are equal when the object is placed at two different positions 8 cm and 24 cm from the lens. The focal length of the lens is ___ cm.
Correct Answer: 16

Solution:

Step 1: Key principle
Equal image sizes imply equal magnification magnitudes. One image is real and the other is virtual.
Step 2: Use lens formula symmetry
For two object positions u_1 and u_2 giving equal image size:

$$f = \frac{|u_1| + |u_2|}{2}$$

Step 3: Substitute values

$$f = \frac{8 + 24}{2} = \boxed{16 \text{ cm}}$$

Quick Tip

If magnification magnitudes are equal for object distances d_1 and d_2, then $f = \frac{d_1+d_2}{2}$.

Chemistry Section A

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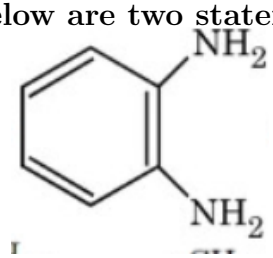
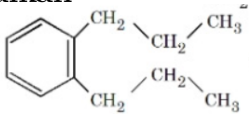
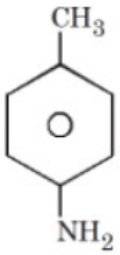
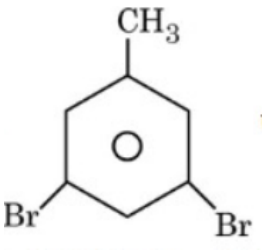
51. Given below are two statements:



Statement I: **Aniline can be synthesized from propylbenzene using simpler reagents in the order i) Acidic KMnO4, ii) Ammonia, iii) Bromine and alkali**







Statement II: Aniline can be converted into 1,3,5-tribromobenzene using reagents in the order i) Bromine-H2O ii) NaNO2/HCl (0 - 5 C) (iii) H3PO2.
In the light of the above statements, choose the correct answer from the options given below
(A) Statement I is false but Statement II is true
(B) Both Statement I and Statement II are false
(C) Statement I is true but Statement II is false
(D) Both Statement I and Statement II are true

Correct Answer: (D) Both Statement I and Statement II are true

Solution:
Statement I:
Propylbenzene on oxidation with acidic $KMnO_4$ undergoes **side-chain oxidation** to give benzoic acid.

$$\text{Propylbenzene} \xrightarrow{KMnO_4/H^+} \text{Benzoic acid}$$

Benzoic acid reacts with ammonia to form benzamide.

$$\text{Benzoic acid} + NH_3 \rightarrow \text{Benzamide}$$

Benzamide on Hoffmann bromamide degradation ($Br_2/NaOH$) loses one carbon atom and forms aniline.

$$\text{Benzamide} \xrightarrow{Br_2/NaOH} \text{Aniline}$$

Hence, the sequence mentioned in Statement I is chemically correct.

Statement II:
Aniline reacts with bromine water to give 2,4,6-tribromoaniline due to strong activating effect of $-NH_2$ group.

$$\text{Aniline} \xrightarrow{Br_2/H_2O} \text{2,4,6-tribromoaniline}$$

On diazotization using $NaNO_2/HCl$ at $0-5^\circ$C, the amino group is converted into diazonium salt.

$$\text{Ar-NH}_2 \rightarrow \text{Ar-N}_2^+Cl^-$$

Reduction with H_3PO_2 replaces the diazonium group by hydrogen.

$$\text{Ar-N}_2^+ \xrightarrow{H_3PO_2} \text{Ar-H}$$

Thus, 1,3,5-tribromobenzene is obtained.
Both statements are correct.

Correct answer: (D)

Quick Tip

Side chain oxidation of alkyl benzenes yields Benzoic acid. H_3PO_2 is a strong reducing agent for diazonium salts ($Ar - N_2^+ \rightarrow Ar - H$).

---

52. In Carius method 0.2425 g of an organic compound gave 0.5253 g silver chloride. The percentage of chlorine in the organic compound is
(A) 37.57%
(B) 34.79%
(C) 53.58%
(D) 87.65%

Correct Answer: (C) 53.58%

Solution:
Step 1: Write required formula

$$\%Cl = \frac{\text{Atomic mass of Cl}}{\text{Molar mass of AgCl}} \times \frac{\text{Mass of AgCl}}{\text{Mass of compound}} \times 100$$

Step 2: Substitute values
Atomic mass of Cl = 35.5 Molar mass of AgCl = 108 + 35.5 = 143.5

$$\%Cl = \frac{35.5}{143.5} \times \frac{0.5253}{0.2425} \times 100$$

Step 3: Calculate

$$\%Cl = 0.24738 \times 2.1662 \times 100 \approx 53.58\%$$

53.58%

Quick Tip

Formula for Carius estimation: $\%X = \frac{At.Mass}{Mol.Mass(AgX)} \times \frac{W_{AgX}}{W_{org}} \times 100$.

53. It is noticed that Pb^{2+} is more stable than Pb^{4+} but Sn^{2+} is less stable than Sn^{4+}. Observe the following reactions.
$PbO_2 + Pb \rightarrow 2PbO; \Delta_r G^\circ(1)$
$SnO_2 + Sn \rightarrow 2SnO; \Delta_r G^\circ(2)$
Identify the correct set from the following
(A) $\Delta_r G^\circ(1) > 0; \Delta_r G^\circ(2) < 0$
(B) $\Delta_r G^\circ(1) < 0; \Delta_r G^\circ(2) > 0$
(C) $\Delta_r G^\circ(1) > 0; \Delta_r G^\circ(2) > 0$
(D) $\Delta_r G^\circ(1) < 0; \Delta_r G^\circ(2) < 0$

Correct Answer: (B) $\Delta_r G^\circ(1) < 0; \Delta_r G^\circ(2) > 0$

Solution:
Key concept: Inert pair effect
Down the group (Group 14), the +2 oxidation state becomes more stable.

Reaction (1): Lead
Pb^{2+} is more stable than Pb^{4+}.
Hence, the conversion of Pb^{4+} to Pb^{2+} is spontaneous.

$$\Delta_r G^\circ(1) < 0$$

Reaction (2): Tin
Sn^{4+} is more stable than Sn^{2+}.
Thus, conversion of Sn^{4+} to Sn^{2+} is non-spontaneous.

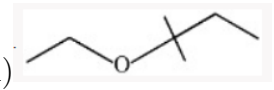
$$\Delta_r G^\circ(2) > 0$$

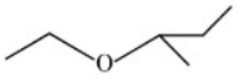
Correct option: (B)

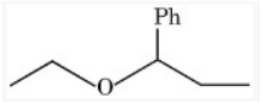
Quick Tip

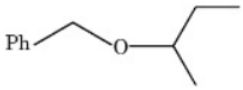
The stability of the lower oxidation state (+2) increases down group 14 due to the Inert Pair Effect.

54. A mixed ether (P), when heated with excess of hot concentrated hydrogen iodide produces two different alkyl iodides which when treated with aq. NaOH give compounds (Q) and (R) give yellow precipitate with NaOI. Identify the mixed ether (P):

(A) 

(B) 

(C) 

(D) 

Correct Answer: (B) [Isopropyl sec-butyl ether]

Solution:

Step 1: Ether cleavage

Ethers on heating with excess HI break into two alkyl iodides.

$$R - O - R' + HI \rightarrow RI + R'I$$

Step 2: Hydrolysis

Alkyl iodides react with aqueous NaOH to form alcohols.

Step 3: Iodoform test

Yellow precipitate with NaOI is given by compounds containing:

$$CH_3 - CH(OH) -$$

Check option (B): Isopropyl sec-butyl ether

Cleavage gives:

$$\text{Isopropyl iodide} \rightarrow \text{2-Propanol}$$

$$\text{sec-Butyl iodide} \rightarrow \text{2-Butanol}$$

Both 2-propanol and 2-butanol contain $CH_3 - CH(OH) -$ group and give positive iodoform test.

Hence, option (B) is correct.

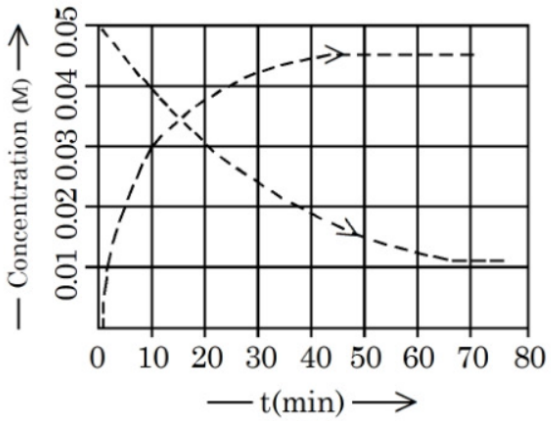
Correct answer: (B)

Quick Tip

Ethers cleave with HI to form alkyl iodides. Check the resulting alcohols for the methyl carbinol group for iodoform test.

---

55. Given above is the concentration vs time plot for a dissociation reaction : $A \rightarrow nB$. Based on the data of the initial phase of the reaction (initial 10 min), the value of n is ____.



(A) 4
(B) 5
(C) 2
(D) 3

Correct Answer: (A) 4

Solution:
Step 1: Change in concentration of A
From graph:

$$[A] \text{ decreases from } 0.05 \rightarrow 0.04$$

$$-\Delta[A] = 0.01 \text{ M}$$

Step 2: Change in concentration of B

$$[B] \text{ increases from } 0 \rightarrow 0.04$$

$$\Delta[B] = 0.04 \text{ M}$$

Step 3: Stoichiometric relation

$$n = \frac{\Delta[B]}{-\Delta[A]}$$

$$n = \frac{0.04}{0.01} = 4$$

$$\boxed{n = 4}$$

Quick Tip

The ratio of changes in concentration $\Delta[Product]/|\Delta[Reactant]|$ gives the stoichiometric coefficient of the product (assuming reactant coefficient is 1).

56. The work functions of two metals (M_A and M_B) are in the $1:2$ ratio. When these metals are exposed to photons of energy 6 eV, the kinetic energy of liberated electrons of $M_A : M_B$ is in the ratio of $2.642:1$. The work functions (in eV) of M_A and M_B are respectively.

(A) $2.3, 4.6$
(B) $3.1, 6.2$
(C) $1.4, 2.8$
(D) $1.5, 3.0$

Correct Answer: (A) $2.3, 4.6$

Solution:

Step 1: Understanding the Concept:
This question is based on Einstein's Photoelectric Equation, which states that the energy of an incident photon is used in two parts: overcoming the work function (ϕ) of the metal and providing kinetic energy (K) to the emitted photoelectron.
The equation is given by:

$$E = \phi + K \text{ or } K = E - \phi$$

where E is the incident photon energy, ϕ is the work function, and K is the maximum kinetic energy.

Step 2: Key Formula or Approach:
Let the work function of metal M_A be ϕ_A and that of metal M_B be ϕ_B.
Given: 1. Ratio of work functions: $\frac{\phi_A}{\phi_B} = \frac{1}{2} \Rightarrow \phi_B = 2\phi_A$
2. Incident energy $E = 6$ eV for both.
3. Ratio of kinetic energies: $\frac{K_A}{K_B} = \frac{2.642}{1}$

Step 3: Detailed Explanation:
Using the photoelectric equation for both metals:
For M_A: $K_A = 6 - \phi_A$
For M_B: $K_B = 6 - \phi_B = 6 - 2\phi_A$
Now, take the ratio:

$$\frac{K_A}{K_B} = \frac{6 - \phi_A}{6 - 2\phi_A} = 2.642$$

Solving for ϕ_A:

$$6 - \phi_A = 2.642 \times (6 - 2\phi_A)$$

$$6 - \phi_A = 15.852 - 5.284\phi_A$$

$$5.284\phi_A - \phi_A = 15.852 - 6$$

$$4.284\phi_A = 9.852$$

$$\phi_A = \frac{9.852}{4.284} \approx 2.3 \text{ eV}$$

Since $\phi_B = 2\phi_A$:

$$\phi_B = 2 \times 2.3 = 4.6 \text{ eV}$$

Step 4: Final Answer:
The work functions are 2.3 eV for M_A and 4.6 eV for M_B.

Quick Tip

In photoelectric effect problems involving ratios, always write the fundamental equation $K = E - \phi$ for both cases and divide them. It helps eliminate constants and solves the variable directly.

---

57. Observe the following reactions at T(K).
I. A $\rightarrow$ products.
II. $5\mathbf{Br^-(aq)} + \mathbf{BrO_3^-(aq)} + 6\mathbf{H^+(aq)} \rightarrow 3\mathbf{Br_2(aq)} + 3\mathbf{H_2O}(l)$
Both the reactions are started at 10.00 am. The rates of these reactions at 10.10 am are same. The value of $-\frac{\Delta[\mathbf{Br^-}]}{\Delta t}$ at 10.10 am is 2×10^{-4} mol $\mathbf{L^{-1}}$ $\mathbf{min^{-1}}$. The concentration of A at 10.10 am is 10^{-2} mol $\mathbf{L^{-1}}$. What is the first order rate constant (in $\mathbf{min^{-1}}$) of reaction I?

(A) 2×10^{-3}
(B) 4×10^{-3}
(C) 10^{-2}
(D) 10^{-3}

Correct Answer: (B) 4×10^{-3}

Solution:

Step 1: Understanding the Concept:
In chemical kinetics, the overall rate of a reaction is related to the rate of disappearance of reactants or the rate of appearance of products, adjusted by their stoichiometric coefficients. For a reaction $a\text{A} + b\text{B} \rightarrow$ Products, the reaction rate r is given by:

$$r = -\frac{1}{a}\frac{\Delta[\text{A}]}{\Delta t} = -\frac{1}{b}\frac{\Delta[\text{B}]}{\Delta t}$$

Additionally, for a first-order reaction, the rate is proportional to the concentration of the reactant.

Step 2: Key Formula or Approach:
1. For Reaction II, the rate of reaction r_{II} is:

$$r_{\text{II}} = -\frac{1}{5}\frac{\Delta[\text{Br}^-]}{\Delta t}$$

2. For Reaction I, which is first order, the rate law is:

$$r_{\mathrm{I}} = k[\mathrm{A}]$$

Step 3: Detailed Explanation:
From the given data for Reaction II at 10.10 am:
The rate of disappearance of Br^- is $-\frac{\Delta[Br^-]}{\Delta t} = 2 \times 10^{-4}$ mol L^{-1} min^{-1}.
Thus, the overall rate of Reaction II (r_{II}) is:

$$r_{\mathrm{II}} = \frac{1}{5} \times \left(2 \times 10^{-4}\right)$$

$$r_{\mathrm{II}} = 0.4 \times 10^{-4} \text{ mol } \mathrm{L}^{-1} \text{ min}^{-1} = 4 \times 10^{-5} \text{ mol } \mathrm{L}^{-1} \text{ min}^{-1}$$

The problem states that at 10.10 am, the rates of both reactions are the same:

$$r_{\mathrm{I}} = r_{\mathrm{II}} = 4 \times 10^{-5} \text{ mol } \mathrm{L}^{-1} \text{ min}^{-1}$$

Now, using the first-order rate law for Reaction I:

$$r_{\mathrm{I}} = k[\mathrm{A}]$$

At 10.10 am, the concentration of A is $[\mathrm{A}] = 10^{-2}$ mol L^{-1}.
Substituting the values:

$$4 \times 10^{-5} = k \times 10^{-2}$$

Solving for the rate constant k:

$$k = \frac{4 \times 10^{-5}}{10^{-2}}$$

$$k = 4 \times 10^{-3} \text{ min}^{-1}$$

Step 4: Final Answer:
The first-order rate constant of reaction I is 4×10^{-3} min^{-1}.
This corresponds to option (B).

Quick Tip

Always differentiate between the 'rate of a specific reactant/product' and the 'overall rate of the reaction'. The overall rate must always be divided by the stoichiometric coefficient of the species being monitored. Skipping this step is a common error in competitive exams.

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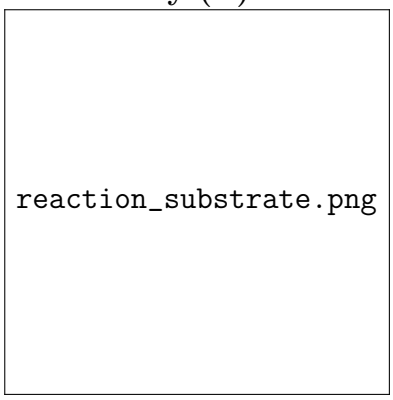
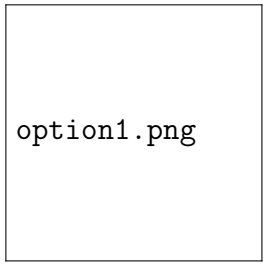
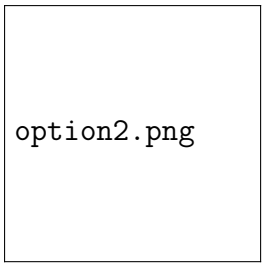
58. Identify (P) in the following reaction sequence:

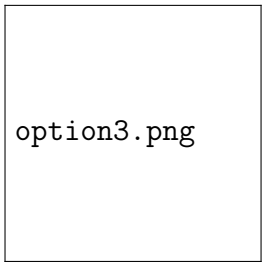


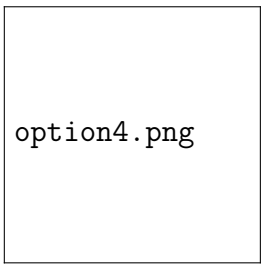
(Substrate: 1,2-dibromo-3,5-diethylcyclopentane)

(i) Zn, Δ $\rightarrow$ (ii) HBr $\rightarrow$ (P) Major Product

(A) 

(B) 

(C) 

(D) 

Correct Answer: (A) [Structure showing 1-bromo-1-ethyl-3-ethylcyclopentane]

Solution:

Step 1: Understanding the Concept:
Step (i) involves the dehalogenation of a vicinal dibromide using zinc and heat, which produces an alkene.
Step (ii) involves the addition of HBr to the alkene via a carbocation intermediate, followed by potential carbocation rearrangement to achieve greater stability.

Step 2: Key Formula or Approach:
1. Dehalogenation: Removal of two Br atoms from adjacent carbons to form a double bond.
2. Addition: H^+ adds to the double bond to form a carbocation.
3. Rearrangement: Check if a 1,2-shift (hydride or alkyl) can form a more stable (tertiary) carbocation.
4. Nucleophilic attack: Br^- attacks the most stable carbocation.

Step 3: Detailed Explanation:
1. **Reaction with Zn**: The 1,2-dibromo-3,5-diethylcyclopentane reacts with Zn to undergo β-elimination, forming 3,5-diethylcyclopent-1-ene.
2. **Reaction with HBr**: - H^+ adds to the double bond at C2 to form a secondary carbocation at C1. - The carbocation at C1 is adjacent to C5, which bears an ethyl group and a hydrogen. - A **1,2-hydride shift** occurs from C5 to C1. This converts the secondary carbocation at C1 into a **tertiary carbocation** at C5 (the carbon already attached to the ethyl group). - Br^- then attacks this tertiary carbocation. 3. **Result**: The product is 1-bromo-1-ethyl-3-ethylcyclopentane. This matches Option 1, where the bromine and one ethyl group are on the same carbon.

Step 4: Final Answer:
The major product (P) is 1-bromo-1-ethyl-3-ethylcyclopentane due to carbocation rearrangement.

Quick Tip

In addition reactions to alkenes involving carbocations, always look for the possibility of 1,2-hydride or 1,2-alkyl shifts. Tertiary carbocations are significantly more stable than secondary ones and will always form if a pathway exists.

59. Given below are two statements:
Statement I: $(CH_3)_3C^+$ is more stable than CH_3^+ as nine hyperconjugation interactions are possible in $(CH_3)_3C^+$.
Statement II: CH_3^+ is less stable than $(CH_3)_3C^+$ as only three hyperconjugation interactions are possible in CH_3^+.
In the light of the above statements, choose the correct answer from the options given below:

(A) Both Statement I and Statement II are true
(B) Both Statement I and Statement II are false
(C) Statement I is false but Statement II is true
(D) Statement I is true but Statement II is false

Correct Answer: (D) Statement I is true but Statement II is false

Solution:

Step 1: Understanding the Concept:
Carbocation stability is determined by inductive effects and hyperconjugation. Hyperconjugation involves the delocalization of electrons from C-H σ-bonds of alkyl groups attached to the carbocationic center. The number of hyperconjugative structures is equal to the number of α-hydrogens.

Step 2: Detailed Explanation:
Analysis of Statement I: In $(CH_3)_3C^+$ (tert-butyl carbocation), there are three methyl groups attached to the positive carbon. Each methyl group has 3 hydrogens. Total α-hydrogens $= 3 \times 3 = 9$. Thus, 9 hyperconjugation interactions are possible, providing significant stability. Statement I is **True**.

Analysis of Statement II: In CH_3^+ (methyl carbocation), the carbon with the positive charge is not attached to any other carbon atoms. Therefore, it has zero α-hydrogens. Zero hyperconjugation interactions are possible in CH_3^+. Statement II claims "three hyperconjugation interactions are possible", which is **False**.

Step 3: Final Answer:
Statement I is correct because tert-butyl has 9 alpha hydrogens, while Statement II is incorrect because methyl carbocation has no hyperconjugation.

Quick Tip

Stability order of carbocations: Tertiary > Secondary > Primary > Methyl. This is primarily due to the increasing number of α-hydrogens ($9 > 6 > 3 > 0$) allowing for more hyperconjugative resonance.

60. Which statements are NOT TRUE about XeO_2F_2?
A. It has a see-saw shape.
B. Xe has 5 electron pairs in its valence shell in XeO_2F_2.
C. The O-Xe-O bond angle is close to 180°.
D. The F-Xe-F bond angle is close to 180°.
E. Xe has 16 valence electrons in XeO_2F_2.
Choose the correct answer from the options given below:

(A) B and D Only
(B) B, C and E Only
(C) A and D Only
(D) B, D and E Only

Correct Answer: (B) B, C and E Only

Solution:

Step 1: Understanding the Concept:
We use the VSEPR theory to determine the geometry and properties of XeO_2F_2. Xenon (Xe) has 8 valence electrons. In XeO_2F_2: - 2 Oxygen atoms form 2 double bonds (using 4 electrons). - 2 Fluorine atoms form 2 single bonds (using 2 electrons). - Remaining electrons $= 8 - (4 + 2) = 2$ electrons (1 lone pair).
Step 2: Key Formula or Approach:
Steric Number (SN) = Number of lone pairs + Number of σ bonds $SN = 1(\text{LP}) + 4(\text{BP}) = 5$. Hybridization = sp^3d. Geometry = Trigonal Bipyramidal.

Step 3: Detailed Explanation:
1. **Shape (A)**: In sp^3d, the lone pair and more electropositive/double-bonded atoms (Oxygen) occupy equatorial positions to minimize repulsion. Fluorines occupy axial positions. The resulting shape is **See-saw**. (A is True)
2. **Electron Pairs (B)**: Xenon has 6 bonding pairs (2 double bonds, 2 single) and 1 lone pair, total 7 pairs. Steric number is 5 domains, but total pairs in valence shell is not 5. (B is Not True)
3. **O-Xe-O angle (C)**: Oxygen atoms are in equatorial positions. The equatorial-equatorial angle in TBP is 120°. Due to lone pair repulsion, it is $\approx 105^\circ$. It is NOT 180°. (C is Not True)
4. **F-Xe-F angle (D)**: Fluorine atoms are in axial positions. The axial-axial angle is $\approx 180^\circ$ (slightly bent to $\approx 174^\circ$ due to LP). It is close to 180°. (D is True)
5. **Valence Electrons (E)**: Xe has 8 original electrons + 4 from 2 O bonds + 2 from 2 F bonds = 14 electrons in its valence shell in this molecule, not 16. (E is Not True)

Step 4: Final Answer:
Statements B, C, and E are incorrect.

Quick Tip

For AX_4E systems like XeO_2F_2 or SF_4: 1. Lone pair always goes to the equatorial position. 2. More electronegative atoms (like F) prefer axial positions. 3. Double bonds (like O) prefer equatorial positions.

---

61. Identify the INCORRECT statements from the following:
A. Notation $^{24}_{12}Mg$ represents 24 protons and 12 neutrons.
B. Wavelength of a radiation of frequency 4.5×10^{15} s^{-1} is 6.7×10^{-8} m.
C. One radiation has wavelength $= \lambda_1$ (900 nm) and energy $= E_1$. Other radiation has wavelength $= \lambda_2$ (300 nm) and energy $= E_2$. $E_1 : E_2 = 3 : 1$.
D. Number of photons of light of wavelength 2000 pm that provides 1 J of energy is 1.006×10^{16}.
Choose the correct answer from the options given below:

(A) A and D Only
(B) A and C Only
(C) A and B Only
(D) B and C Only

Correct Answer: (B) A and C Only

Solution:

Step 1: Understanding the Concept:
This question tests fundamental concepts of atomic structure, electromagnetic radiation, and Planck's Quantum Theory. We need to evaluate each statement using basic formulas like $c = \nu\lambda$, $E = h\nu$, and atomic notation rules.

Step 2: Key Formula or Approach:
1. Atomic notation: $^{A}_{Z}X$, where Z = number of protons and $A - Z$ = number of neutrons.
2. Wavelength-frequency relation: $\lambda = \frac{c}{\nu}$.
3. Energy of a photon: $E = \frac{hc}{\lambda}$.
4. Total energy: $E_{total} = n\frac{hc}{\lambda}$, where n is the number of photons.

Step 3: Detailed Explanation:
Statement A: For $^{24}_{12}Mg$, atomic number $Z = 12$ (protons) and mass number $A = 24$.
Number of neutrons $= A - Z = 24 - 12 = 12$.
The statement says 24 protons, which is incorrect. Thus, **Statement A is INCORRECT**.

Statement B: $\lambda = \frac{c}{\nu} = \frac{3\times10^{8} \text{ m/s}}{4.5\times10^{15} \text{ s}^{-1}} = 0.666 \times 10^{-7}$ m $\approx 6.7 \times 10^{-8}$ m.
Thus, **Statement B is CORRECT**.

Statement C: $E \propto \frac{1}{\lambda}$.

$$\frac{E_1}{E_2} = \frac{\lambda_2}{\lambda_1} = \frac{300 \text{ nm}}{900 \text{ nm}} = \frac{1}{3}$$

So, $E_1 : E_2 = 1 : 3$. The statement claims $3 : 1$.
Thus, **Statement C is INCORRECT**.

Statement D: $n = \frac{E \cdot \lambda}{h \cdot c}$.
Given: $E = 1$ J, $\lambda = 2000$ pm $= 2000 \times 10^{-12}$ m $= 2 \times 10^{-9}$ m.

$$n = \frac{1 \times 2 \times 10^{-9}}{6.626 \times 10^{-34} \times 3 \times 10^{8}} = \frac{2 \times 10^{-9}}{1.9878 \times 10^{-25}} \approx 1.006 \times 10^{16}$$

Thus, **Statement D is CORRECT**.

Step 4: Final Answer:
Statements A and C are incorrect.

Quick Tip

In atomic notation ${}^{A}_{Z}X$, the bottom number is ALWAYS the number of protons. For energy calculations, remember that Energy is inversely proportional to wavelength ($E \propto 1/\lambda$); a shorter wavelength means higher energy.

---

62. Both human DNA and RNA are chiral molecules. The chirality in DNA and RNA arises due to the presence of

(A) Base unit
(B) Chiral phosphate unit
(C) L-sugar component
(D) D-sugar component

Correct Answer: (D) D-sugar component

Solution:

Step 1: Understanding the Concept:
DNA (Deoxyribonucleic Acid) and RNA (Ribonucleic Acid) are biopolymers consisting of nucleotides. Each nucleotide has three parts: a nitrogenous base, a pentose sugar, and a phosphate group. Chirality in biological systems is often determined by the specific stereochemistry of the building blocks.

Step 2: Detailed Explanation:
The chirality of DNA and RNA molecules is primarily attributed to the sugar molecules present in their structure.

1. DNA contains **D-2-deoxyribose**.
2. RNA contains **D-ribose**.
These sugars are pentoses with several chiral centers. In nature, organisms specifically utilize the D-isomers of these sugars to build the backbone of nucleic acids. The asymmetric centers in the furanose ring of these sugars impart an overall chirality to the helical structure of the nucleic acids. The phosphate unit and the common nitrogenous bases (like Adenine, Guanine, etc.) are generally achiral or their contribution to the overall polymer's chirality is secondary to the sugar backbone.

Step 3: Final Answer:
The chirality arises due to the presence of the D-sugar component.

Quick Tip

Biological molecules are highly stereospecific. Remember: Sugars in DNA/RNA are always D-form, while Amino acids in proteins are mostly L-form. This is a common question in biomolecules.

---

63. The oxidation state of chromium in the final product formed in the reaction between KI and acidified $K_2Cr_2O_7$ solution is:

(A) +3
(B) +6
(C) +2
(D) +4

Correct Answer: (A) +3

Solution:

Step 1: Understanding the Concept:
Potassium dichromate ($K_2Cr_2O_7$) is a strong oxidizing agent in acidic medium. It oxidizes iodide ions (I^-) to molecular iodine (I_2) while being itself reduced.

Step 2: Key Formula or Approach:
Write the balanced redox reaction between dichromate and iodide in acidic medium to find the product containing Chromium.

Step 3: Detailed Explanation:
The ionic equation for the reaction is:

$$Cr_2O_7^{2-} + 6I^- + 14H^+ \rightarrow 2Cr^{3+} + 3I_2 + 7H_2O$$

The molecular equation can be written as:

$$K_2Cr_2O_7 + 6KI + 7H_2SO_4 \rightarrow Cr_2(SO_4)_3 + 4K_2SO_4 + 3I_2 + 7H_2O$$

In the reactant $K_2Cr_2O_7$, the oxidation state of Cr is +6.
In the final product $Cr_2(SO_4)_3$ (or the Cr^{3+} ion), the oxidation state of chromium is +3.

Step 4: Final Answer:
The oxidation state of chromium in the final product is +3.

Quick Tip

In acidic medium, Dichromate ($Cr_2O_7^{2-}$) always gets reduced to Cr^{3+} (green), regardless of the reducing agent used. This is a very useful general rule for inorganic chemistry.

---

64. Consider the above electrochemical cell where a metal electrode (M) is undergoing redox reaction by forming M^+ ($M \rightarrow M^+ + e^-$). The cation M^+ is present in two different concentrations c_1 and c_2 as shown above. Which of the following statement is correct for generating a positive cell potential?

(A) If c_1 is present at anode, then $c_1 > c_2$.
(B) If c_1 is present at cathode, then $c_1 > c_2$.
(C) If c_1 is present at anode, then $c_1 = c_2$.
(D) If c_1 is present at cathode, then $c_1 < c_2$.

Correct Answer: (B) If c_1 is present at cathode, then $c_1 > c_2$.

Solution:

Step 1: Understanding the Concept:
This is a concentration cell. A concentration cell consists of two electrodes of the same material in solutions of the same electrolyte but at different concentrations. For such cells, the standard cell potential E^o_{cell} is zero.

Step 2: Key Formula or Approach:
Nernst Equation for a concentration cell:

$$E_{cell} = E^o_{cell} - \frac{0.0591}{n} \log \frac{[M^+]_{anode}}{[M^+]_{cathode}}$$

Step 3: Detailed Explanation:
Since it's a concentration cell with the same metal, $E^o_{cell} = 0$.

For the cell to have a positive potential ($E_{cell} > 0$):

$$0 - \frac{0.0591}{1} \log \frac{c_{anode}}{c_{cathode}} > 0$$

$$- \log \frac{c_{anode}}{c_{cathode}} > 0 \implies \log \frac{c_{anode}}{c_{cathode}} < 0$$

This implies $\frac{c_{anode}}{c_{cathode}} < 1$, or $c_{cathode} > c_{anode}$.

Now, checking the options:
If c_1 is the concentration at the cathode and c_2 is at the anode, then for $E_{cell} > 0$, we must have $c_{cathode} > c_{anode}$, which means $c_1 > c_2$.
This matches Option (B).

Step 4: Final Answer:
For a positive potential, the cathode concentration must be greater than the anode concentration. Thus, if c_1 is at the cathode, $c_1 > c_2$.

Quick Tip

For any concentration cell to work spontaneously ($E_{cell} > 0$), "more concentrated" must be at the cathode and "less concentrated" must be at the anode. Think of it as the system trying to equalize concentrations through electron transfer.

---

65. A student has been given a compound "x" of molecular formula C_6H_7N. 'x' is sparingly soluble in water. However, on addition of dilute mineral acid, 'x' becomes soluble in water. 'x' when treated with $CHCl_3$ and KOH(alc), 'y' is produced. 'y' has a specific unpleasant smell. On treatment with benzenesulphonyl chloride, 'x' gives a compound 'z' which is soluble in alkali. The number of different "H" atoms present in 'z' is:

(A) 5
(B) 4
(C) 8
(D) 7

Correct Answer: (D) 7

Solution:

Step 1: Understanding the Concept:
Identify the compound "x" based on the qualitative tests: solubility in acid (basic nature), Carbylamine test (primary amine), and Hinsberg test (primary vs secondary amine).

Step 2: Detailed Explanation:
1. **Identification of "x":**
Molecular formula C_6H_7N and basic nature (soluble in dilute acid) suggests an amine.
Formation of "y" with an unpleasant smell upon treatment with $CHCl_3/KOH$ confirms "x" is a primary amine (Carbylamine reaction).
Specifically, $C_6H_5NH_2$ (Aniline) fits the formula and properties.

2. **Formation of "z":**
Reaction with benzenesulphonyl chloride ($C_6H_5SO_2Cl$):

$$C_6H_5NH_2 + C_6H_5SO_2Cl \rightarrow C_6H_5 - SO_2 - NH - C_6H_5 + HCl$$

The product "z" is N-phenylbenzenesulphonamide. This compound contains an acidic H on the nitrogen atom, making it soluble in alkali.

3. **Counting different "H" atoms in "z":**
The structure of "z" is: Ring A (from chloride) $- SO_2 - NH -$ Ring B (from aniline).
- In the benzenesulphonyl ring (Ring A), due to the $-SO_2-$ substituent, there are 3 types of protons: Ortho (2H), Meta (2H), and Para (1H). (3 sets)
- The $-NH-$ group has 1 proton. (1 set)
- In the aniline ring (Ring B), due to the $-NH - SO_2-$ substituent, there are 3 types of protons: Ortho (2H), Meta (2H), and Para (1H). (3 sets)
Total sets of chemically distinct hydrogen atoms = 3(Ring A) + 1(NH) + 3(Ring B) = 7.

Step 3: Final Answer:
The number of different types of hydrogen atoms in "z" is 7.

Quick Tip

The "solubility in alkali" part of the Hinsberg test is the key differentiator. Primary amines give sulfonamides with an acidic hydrogen ($-NH-$), hence they dissolve. Secondary amines give sulfonamides with no N-H bond, so they don't dissolve.

---

66. Which of the following statements are TRUE about Haloform reaction?:
A. Sodium hypochlorite reacts with KI to give KOI.
B. KOI is a reducing agent.
C. α,β-unsaturated methylketone ($CH_3 - CH = CH - C(= O) - CH_3$) will give iodoform reaction.
D. Isopropyl alcohol will not give iodoform test.
E. Methanoic acid will give positive iodoform test.
Choose the correct answer from the options given below:

(A) A, B & C Only
(B) B, D & E Only

(C) A & C Only
(D) A, C & E Only

Correct Answer: (C) A & C Only

Solution:

Step 1: Understanding the Concept:
The Haloform (specifically iodoform) reaction is given by compounds having the CH_3CO- group or the $CH_3CH(OH)-$ group.

Step 2: Detailed Explanation:
Statement A: Sodium hypochlorite ($NaOCl$) is an oxidizing agent. When reacted with KI, it can generate KOI (or IO^-) in situ.

$$NaOCl + KI \rightarrow NaCl + KOI$$

This is a standard way to prepare the reagent. **True**.

Statement B: KOI (Hypoiodite) acts as an **oxidizing and halogenating** agent in the haloform reaction, not a reducing agent. **False**.

Statement C: The molecule $CH_3-CH=CH-CO-CH_3$ contains the methyl ketone group ($-CO-CH_3$). Despite the unsaturation, it generally gives the haloform reaction to produce CHI_3. **True**.

Statement D: Isopropyl alcohol ($CH_3-CH(OH)-CH_3$) contains the $CH_3CH(OH)-$ group. It is oxidized to acetone, which then gives the iodoform test. The statement says it will **not** give the test. **False**.

Statement E: Methanoic acid ($HCOOH$) does not have the required methyl group. **False**.

Step 3: Final Answer:
Statements A and C are true.

Quick Tip

To quickly identify Iodoform-positive compounds, look for:
1. CH_3-CO-R (Methyl ketones)
2. $CH_3-CH(OH)-R$ (Secondary alcohols with a terminal methyl)
3. Ethanol and Acetaldehyde.

---

67. Elements X and Y belong to Group 15. The difference between the electronegativity values of 'X' and phosphorus is higher than that of the difference between

phosphorus and 'Y'. 'X' & 'Y' are respectively

(A) As & Bi
(B) Bi & N
(C) As & Sb
(D) N & As

Correct Answer: (D) N & As

Solution:

Step 1: Understanding the Concept:
Group 15 elements are Nitrogen (N), Phosphorus (P), Arsenic (As), Antimony (Sb), and Bismuth (Bi). Electronegativity decreases as we go down the group.

Step 2: Detailed Explanation:
The Pauling electronegativity values for Group 15 elements are approximately:
- N: 3.0
- P: 2.1
- As: 2.0
- Sb: 1.9
- Bi: 1.9

Condition given: $|EN_X - EN_P| > |EN_P - EN_Y|$.
Let's check Option (D): $X = N, Y = As$.
$|EN_N - EN_P| = |3.0 - 2.1| = 0.9$.
$|EN_P - EN_{As}| = |2.1 - 2.0| = 0.1$.
Since $0.9 > 0.1$, the condition is satisfied.

Checking other options for verification:
- (A) As & Bi: $|2.0 - 2.1| = 0.1$; $|2.1 - 1.9| = 0.2$. $0.1 < 0.2$ (Incorrect).
- (B) Bi & N: $|1.9 - 2.1| = 0.2$; $|2.1 - 3.0| = 0.9$. $0.2 < 0.9$ (Incorrect).
- (C) As & Sb: $|2.0 - 2.1| = 0.1$; $|2.1 - 1.9| = 0.2$. $0.1 < 0.2$ (Incorrect).

Step 3: Final Answer:
The elements X and Y are N and As respectively.

> Quick Tip
>
> Electronegativity drops sharply from N to P but the change becomes very small for heavier elements (As, Sb, Bi) due to the poor shielding of d and f electrons. Always expect the largest differences involving Nitrogen.

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68. Iodoform test can differentiate between
A. Methanol and Ethanol

B. CH_3COOH **and** CH_3CH_2COOH
C. Cyclohexene and cyclohexanone
D. Diethyl ether and Pentan-3-one
E. Anisole and acetone
Choose the correct answer from the options given below:

(A) B, C & E Only
(B) A, B & E Only
(C) A & E Only
(D) A & D Only

Correct Answer: (C) A & E Only

Solution:

Step 1: Understanding the Concept:
The iodoform test gives a positive result (yellow precipitate) only for specific structural units: $CH_3 - C(= O)-$ or $CH_3 - CH(OH)-$. To differentiate two compounds, one must give a positive result and the other must give a negative result.

Step 2: Detailed Explanation:
A. Methanol and Ethanol:
Methanol (CH_3OH): Negative.
Ethanol (CH_3CH_2OH): Positive (contains $CH_3CH(OH)-$).
Can differentiate.

B. CH_3COOH **and** CH_3CH_2COOH**:**
Both are carboxylic acids and do not give the iodoform test.
Cannot differentiate.

C. Cyclohexene and cyclohexanone:
Neither contains the required methyl group adjacent to the functional center (CH_3CO-).
Cannot differentiate.

D. Diethyl ether and Pentan-3-one:
Neither gives the test. Pentan-3-one ($CH_3CH_2COCH_2CH_3$) is a ketone but not a *methyl* ketone.
Cannot differentiate.

E. Anisole and acetone:
Anisole ($PhOCH_3$): Negative.
Acetone (CH_3COCH_3): Positive (contains CH_3CO-).
Can differentiate.

Step 3: Final Answer:
Pairs A and E can be differentiated by the iodoform test.

Quick Tip

For a ketone to give a positive iodoform test, it MUST be a "2-one" (like Propan-2-one, Butan-2-one). 3-ones or cyclic ketones like cyclohexanone usually give negative results.

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69. Identify the CORRECT set of details from the following:
A. $[Co(NH_3)_6]^{3+}$**: Inner orbital complex;** d^2sp^3 **hybridized**
B. $[MnCl_6]^{3-}$**: Outer orbital complex;** sp^3d^2 **hybridized**
C. $[CoF_6]^{3-}$**: Outer orbital complex;** d^2sp^3 **hybridized**
D. $[FeF_6]^{3-}$**: Outer orbital complex;** sp^3d^2 **hybridized**
E. $[Ni(CN)_4]^{2-}$**: Inner orbital complex;** sp^3 **hybridized**
Choose the correct answer from the options given below:

(A) A, B, C, D & E
(B) C & D Only
(C) A, B & D Only
(D) A, C & E Only

Correct Answer: (C) A, B & D Only

Solution:

Step 1: Understanding the Concept:
Based on Valence Bond Theory (VBT), hybridization and complex type depend on the oxidation state of the metal and the strength of the ligand (Strong Field vs Weak Field).

Step 2: Detailed Explanation:
A. $[Co(NH_3)_6]^{3+}$**:** Co^{3+} is $3d^6$. NH_3 is a strong field ligand. It causes pairing, vacating two $3d$ orbitals. Hybridization: d^2sp^3. It is an **Inner orbital complex**. (Correct)

B. $[MnCl_6]^{3-}$**:** Mn^{3+} is $3d^4$. Cl^- is a weak field ligand. No pairing occurs. It uses $4d$ orbitals. Hybridization: sp^3d^2. It is an **Outer orbital complex**. (Correct)

C. $[CoF_6]^{3-}$**:** Co^{3+} is $3d^6$. F^- is a weak field ligand. No pairing. It uses $4d$ orbitals. Hybridization: sp^3d^2. The statement says d^2sp^3. (Incorrect)

D. $[FeF_6]^{3-}$**:** Fe^{3+} is $3d^5$. F^- is a weak field ligand. No pairing. Hybridization: sp^3d^2. It is an **Outer orbital complex**. (Correct)

E. $[Ni(CN)_4]^{2-}$**:** Ni^{2+} is $3d^8$. CN^- is a strong field ligand. It causes pairing, creating a square planar geometry with dsp^2 hybridization. The statement says sp^3. (Incorrect)

Step 3: Final Answer:
The correct statements are A, B, and D.

Quick Tip

Inner orbital (d^2sp^3) usually corresponds to strong field ligands (pairing), while Outer orbital (sp^3d^2) corresponds to weak field ligands (no pairing). Always check the ligand strength first!

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70. Given below are two statements:
Statement I: The second ionisation enthalpy of Na is larger than the corresponding ionisation enthalpy of Mg.
Statement II: The ionic radius of O^{2-} is larger than that of F^-.
In the light of the above statements, choose the correct answer from the options given below

(A) Statement I is true but Statement II is false
(B) Statement I is false but Statement II is true
(C) Both Statement I and Statement II are true
(D) Both Statement I and Statement II are false

Correct Answer: (C) Both Statement I and Statement II are true

Solution:

Step 1: Understanding the Concept:
Statement I concerns trends in Ionization Enthalpy (IE), especially for isoelectronic or stable configurations. Statement II concerns the comparison of ionic radii in isoelectronic species.

Step 2: Detailed Explanation:
Statement I:
For Sodium (Na): $[Ne]3s^1$.
First IE: $Na \rightarrow Na^+ + e^-$ (Na^+ is $[Ne]$ configuration).
Second IE: $Na^+ \rightarrow Na^{2+} + e^-$. This involves removing an electron from a stable noble gas core ($2s^2 2p^6$).
For Magnesium (Mg): $[Ne]3s^2$.
First IE: $Mg \rightarrow Mg^+ + e^-$.
Second IE: $Mg^+ \rightarrow Mg^{2+} + e^-$. This involves removing a $3s^1$ electron to reach noble gas stability.
Removing an electron from a stable octet (Na^+) requires significantly more energy than removing it from a valence shell (Mg^+). So, $IE_2(\text{Na}) > IE_2(\text{Mg})$.
Statement I is true.

Statement II:
O^{2-} (Atomic number 8, electrons 10) and F^- (Atomic number 9, electrons 10) are isoelectronic species.
For isoelectronic species, as the nuclear charge (Z) increases, the attractive force on the electrons increases, causing the radius to decrease.
$Z(O) = 8$, $Z(F) = 9$.

Since Fluorine has a higher nuclear charge than Oxygen, F^- will be smaller than O^{2-}.
Statement II is true.

Step 3: Final Answer:
Both Statement I and Statement II are true.

Quick Tip

For isoelectronic species:
Size $\propto \frac{1}{\text{Atomic Number (Z)}}$.
For ionization energy, always look for the change in the principal quantum number (n) or the removal of an electron from a stable half-filled or full-filled configuration.

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4 Chemistry Section B

71. Total number of unpaired electrons present in the central metal atoms/ions of $[Ni(CO)_4]$, $[NiCl_4]^{2-}$, $[PtCl_2(NH_3)_2]$, $[Ni(CN)_4]^{2-}$ **and** $[Pt(CN)_4]^{2-}$ **is** _____.

Correct Answer: 2

Solution:

Step 1: Understanding the Concept:
The number of unpaired electrons in a coordination complex depends on the oxidation state of the central metal, its electronic configuration, and the nature of the ligands (Strong Field vs. Weak Field) which determines the geometry and spin state.

Step 2: Key Formula or Approach:
1. Identify the oxidation state of the metal.
2. Determine the d-electron configuration.
3. Determine the geometry (Tetrahedral vs. Square Planar) based on the ligand strength and crystal field splitting energy ($CFSE$).

Step 3: Detailed Explanation:

1. $[Ni(CO)_4]$:
   Oxidation state of $Ni = 0$. Configuration: $[Ar]3d^8 4s^2$.
   CO is a strong field ligand. It causes pairing and shifts $4s$ electrons to $3d$, resulting in $3d^{10}$ configuration.
   Geometry: Tetrahedral (sp^3). Unpaired electrons = 0.

2. $[NiCl_4]^{2-}$:
   Oxidation state of $Ni = +2$. Configuration: $[Ar]3d^8$.
   Cl^- is a weak field ligand. No pairing occurs.
   Geometry: Tetrahedral (sp^3). Configuration: $(e_g)^4(t_2)^4$.
   Unpaired electrons = 2.

3. $[PtCl_2(NH_3)_2]$:
   Oxidation state of $Pt = +2$. Configuration: $[Xe]4f^{14}5d^8$.
   For $4d$ and $5d$ series metals, $CFSE$ is very high, making almost all ligands behave as strong field ligands.
   Geometry: Square planar (dsp^2). All electrons are paired.
   Unpaired electrons = 0.

4. $[Ni(CN)_4]^{2-}$:
   Oxidation state of $Ni = +2$. Configuration: $3d^8$.
   CN^- is a strong field ligand, causing pairing.
   Geometry: Square planar (dsp^2).
   Unpaired electrons = 0.

5. $[Pt(CN)_4]^{2-}$:
   Oxidation state of $Pt = +2$. Configuration: $5d^8$.
   Strong field ligand (CN^-) and $5d$ metal lead to square planar geometry.
   Unpaired electrons = 0.

Total unpaired electrons $= 0 + 2 + 0 + 0 + 0 = 2$.

Step 4: Final Answer:
The total number of unpaired electrons is 2.

Quick Tip

Remember that for $4d$ and $5d$ transition metals like Pd and Pt, the crystal field splitting is so large that d^8 complexes are almost always square planar and diamagnetic, regardless of whether the ligand is traditionally "weak" or "strong".

---

72. Consider the following reaction of benzene.

$Benzene + H_3C - C(=O) - CH_2 - CH_2 - C(=O) - Cl \xrightarrow{anhydrous AlCl_3} (P) \xrightarrow{aq. NaOH, \Delta} (Q)$

In compound (Q), the percentage of oxygen is _____% (Nearest integer)

Correct Answer: 10

Solution:

Step 1: Understanding the Concept:
The sequence involves a Friedel-Crafts acylation followed by an intramolecular Aldol condensation and dehydration to form a cyclic enone.

Step 2: Key Formula or Approach:
1. Perform Friedel-Crafts acylation at the acid chloride site.
2. Identify the intramolecular Aldol condensation product.
3. Calculate the percentage of oxygen using $\frac{\text{Mass of Oxygen}}{\text{Molar Mass}} \times 100$.

Step 3: Detailed Explanation:
1. **Formation of (P):**
Benzene reacts with 4-oxopentanoyl chloride ($H_3C-CO-CH_2-CH_2-COCl$) in the presence of $AlCl_3$. The acylation occurs at the chloride end.
$(P) = C_6H_5 - CO - CH_2 - CH_2 - CO - CH_3$ (1-phenylpentane-1,4-dione).

2. **Formation of (Q):**
Reaction with $aq.NaOH, \Delta$ triggers an intramolecular Aldol condensation.
Deprotonation occurs at the methyl group (CH_3) to form a carbanion, which attacks the carbonyl carbon adjacent to the phenyl ring to form a stable 5-membered ring.
After dehydration, the product (Q) is 3-phenylcyclopent-2-en-1-one.
Molecular formula of (Q): $C_{11}H_{10}O$.

3. **Mass Calculation:**
Molar mass of (Q) $= (11 \times 12) + (10 \times 1) + (1 \times 16)$
$M = 132 + 10 + 16 = 158$ g/mol.
Percentage of Oxygen $= \frac{16}{158} \times 100 \approx 10.126\%$.

Step 4: Final Answer:
The percentage of oxygen rounded to the nearest integer is 10.

> **Quick Tip**
>
> In intramolecular Aldol condensations of 1,4-diketones, 5-membered rings are kinetically and thermodynamically preferred over 3-membered rings. Always identify the most stable enone product.

---

73. 200 cc of $x \times 10^{-3}$ M potassium dichromate is required to oxidise 750 cc of 0.6 M Mohr's salt solution in acidic medium. Here $x =$ _____.

Correct Answer: 375

Solution:

Step 1: Understanding the Concept:
In a redox titration, the total equivalents of the oxidizing agent must equal the total equivalents of the reducing agent.

Step 2: Key Formula or Approach:
Equivalence law: $N_1V_1 = N_2V_2$ or $M_1V_1n_1 = M_2V_2n_2$.

Step 3: Detailed Explanation:
1. **For Potassium Dichromate ($K_2Cr_2O_7$):**
In acidic medium: $Cr_2O_7^{2-} + 14H^+ + 6e^- \rightarrow 2Cr^{3+} + 7H_2O$.
n-factor $(n_1) = 6$.
Molarity $(M_1) = x \times 10^{-3}$ M.
Volume $(V_1) = 200$ cc.

2. **For Mohr's Salt ($FeSO_4 \cdot (NH_4)_2SO_4 \cdot 6H_2O$):**
Oxidation: $Fe^{2+} \rightarrow Fe^{3+} + e^-$.
n-factor $(n_2) = 1$.
Molarity $(M_2) = 0.6$ M.
Volume $(V_2) = 750$ cc.

3. **Calculation:**

$$M_1V_1n_1 = M_2V_2n_2$$
$$(x \times 10^{-3}) \times 200 \times 6 = 0.6 \times 750 \times 1$$
$$1200x \times 10^{-3} = 450$$
$$1.2x = 450$$
$$x = \frac{450}{1.2} = 375$$

Step 4: Final Answer:
The value of x is 375.

Quick Tip

Mohr's salt is a preferred primary standard because it is resistant to air oxidation. Remember its n-factor is always 1 as only the Fe^{2+} ion is oxidized.

---

74. Two liquids A and B form an ideal solution. At 320 K, the vapour pressure of the solution, containing 3 mol of A and 1 mol of B is 500 mm Hg. At the same temperature, if 1 mol of A is further added to this solution, vapour pressure of the solution increases by 20 mm Hg. Vapour pressure (in mm Hg) of B in pure state

is _____. (Nearest integer)

Correct Answer: 200

Solution:

Step 1: Understanding the Concept:
For an ideal solution, the total vapor pressure is given by Raoult's Law: $P = P_A^o X_A + P_B^o X_B$.

Step 2: Key Formula or Approach:
Use the mole fraction formula $X_i = \frac{n_i}{n_{total}}$ and set up two linear equations for the two given conditions.

Step 3: Detailed Explanation:
Case 1:
$n_A = 3$, $n_B = 1$. Total moles = 4.
$X_A = \frac{3}{4}$, $X_B = \frac{1}{4}$.
$P_1 = 500 = P_A^o(\frac{3}{4}) + P_B^o(\frac{1}{4})$
Multiplying by 4: $3P_A^o + P_B^o = 2000$ — (Eq. 1)

Case 2:
1 mol of A is added. New $n_A = 3 + 1 = 4$, $n_B = 1$. Total moles = 5.
$X_A = \frac{4}{5}$, $X_B = \frac{1}{5}$.
Vapor pressure increases by 20 mm Hg, so $P_2 = 500 + 20 = 520$ mm Hg.
$520 = P_A^o(\frac{4}{5}) + P_B^o(\frac{1}{5})$
Multiplying by 5: $4P_A^o + P_B^o = 2600$ — (Eq. 2)

Solving the equations:
Subtract (Eq. 1) from (Eq. 2):
$(4P_A^o + P_B^o) - (3P_A^o + P_B^o) = 2600 - 2000$
$P_A^o = 600$ mm Hg.
Substitute P_A^o in (Eq. 1):
$3(600) + P_B^o = 2000$
$1800 + P_B^o = 2000$
$P_B^o = 200$ mm Hg.

Step 4: Final Answer:
The vapour pressure of B in pure state is 200 mm Hg.

Quick Tip

When dealing with "increases by" or "decreases to", be very careful with the signs. Adding a more volatile component always increases the total vapor pressure of an ideal solution.

75. $X_2(g) + Y_2(g) \rightleftharpoons 2Z(g)$
$X_2(g)$ and $Y_2(g)$ are added to a 1 L flask and it is found that the system attains the above equilibrium at T(K) with the number of moles of $X_2(g)$, $Y_2(g)$ and $Z(g)$ being 3, 3 and 9 mol respectively (equilibrium moles). Under this condition of equilibrium, 10 mol of Z(g) is added to the flask and the temperature is maintained at T(K). Then the number of moles of Z(g) in the flask when the new equilibrium is established is _____. (Nearest integer)

Correct Answer: 15

Solution:

Step 1: Understanding the Concept:
The equilibrium constant (K_c) depends only on temperature. Adding a product shifts the equilibrium to the left according to Le Chatelier's Principle.

Step 2: Key Formula or Approach:
1. Calculate initial K_c.
2. Set up the new equilibrium expression with the added amount.

Step 3: Detailed Explanation:
1. **Initial Equilibrium:**
$[X_2] = \frac{3}{1} = 3$ M, $[Y_2] = \frac{3}{1} = 3$ M, $[Z] = \frac{9}{1} = 9$ M.
$K_c = \frac{[Z]^2}{[X_2][Y_2]} = \frac{9^2}{3\times 3} = \frac{81}{9} = 9$.

2. **New Equilibrium Condition:**
10 moles of Z are added. Initial moles for new setup:
$n(X_2) = 3$, $n(Y_2) = 3$, $n(Z) = 9 + 10 = 19$.
Since product is added, reaction moves backward. Let $2x$ moles of Z react.
Moles at new equilibrium:
$n(X_2) = 3 + x$, $n(Y_2) = 3 + x$, $n(Z) = 19 - 2x$.

$$K_c = \frac{(19 - 2x)^2}{(3 + x)^2} = 9$$

Taking square root on both sides:

$$\frac{19 - 2x}{3 + x} = 3$$

(We take the positive root as x must result in positive concentrations)

$$19 - 2x = 3(3 + x)$$
$$19 - 2x = 9 + 3x$$
$$10 = 5x \Rightarrow x = 2$$

3. **Calculating New Z:**
Moles of Z at new equilibrium $= 19 - 2x = 19 - 2(2) = 15$ moles.

Step 4: Final Answer:
The number of moles of Z at the new equilibrium is 15.

Quick Tip

For reactions where $\Delta n_g = 0$, the volume term cancels out in the K_c expression. You can work directly with moles if the volume is constant or if you are only asked for molar ratios.
