JEE Main 2026 Jan 21 Shift 2 Question Paper with Solutions

Time Allowed : 3 Hour	**Maximum Marks :** 300	**Total Questions :** 75

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The test is of 3 hours duration.
2. This test paper consists of 75 questions. Each subject (PCM) has 25 questions. The maximum marks are 300.
3. This question paper contains Three Parts. Part-A is Physics, Part-B is Chemistry and Part-C is Mathematics. Each part has only two sections: Section-A and Section-B.
4. Section - A : Attempt all questions.
5. Section - B : Attempt all questions.
6. Section - A (01 – 20) contains 20 multiple choice questions which have only one correct answer. Each question carries +4 marks for correct answer and –1 mark for wrong answer.
7. Section - B (21 – 25) contains 5 Numerical value based questions. The answer to each question should be rounded off to the nearest integer. Each question carries +4 marks for correct answer and –1 mark for wrong answer.

Mathematics Section A

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1. Let $A = \{2, 3, 5, 7, 9\}$. Let R be the relation on A defined by xRy if and only if $2x \leq 3y$. Let l be the number of elements in R, and m be the minimum number of elements required to be added in R to make it a symmetric relation. Then $l + m$ is equal to :

(A) 23
(B) 21
(C) 25
(D) 27

Correct Answer: (C) 25

Solution:

Step 1: Understanding the Concept:
A relation R on a set A is a subset of $A \times A$.

The number of elements l is the count of ordered pairs (x, y) that satisfy the inequality $2x \leq 3y$.
A relation is symmetric if for every $(x, y) \in R$, the pair (y, x) must also be in R.
To make R symmetric, we must add the "missing" symmetric pairs.

Step 2: Detailed Explanation:
The set is $A = \{2, 3, 5, 7, 9\}$. We check each $x \in A$:
- If $x = 2$, $2(2) = 4 \leq 3y \implies y \in \{2, 3, 5, 7, 9\}$. (5 pairs: (2,2), (2,3), (2,5), (2,7), (2,9))
- If $x = 3$, $2(3) = 6 \leq 3y \implies y \in \{2, 3, 5, 7, 9\}$. (5 pairs: (3,2), (3,3), (3,5), (3,7), (3,9))
- If $x = 5$, $2(5) = 10 \leq 3y \implies y \in \{5, 7, 9\}$. (3 pairs: (5,5), (5,7), (5,9))
- If $x = 7$, $2(7) = 14 \leq 3y \implies y \in \{5, 7, 9\}$. (3 pairs: (7,5), (7,7), (7,9))
- If $x = 9$, $2(9) = 18 \leq 3y \implies y \in \{7, 9\}$. (2 pairs: (9,7), (9,9))
Total elements $l = 5 + 5 + 3 + 3 + 2 = 18$.

Now, we identify pairs $(x, y) \in R$ where $(y, x) \notin R$:
- Symmetric pairs already in R: (2,2), (3,3), (5,5), (7,7), (9,9), (2,3) & (3,2), (5,7) & (7,5), (7,9) & (9,7).
- Pairs needing their symmetric counterparts:
1. $(2, 5) \in R$ but $(5, 2) \notin R$ (since $10 > 6$).
2. $(2, 7) \in R$ but $(7, 2) \notin R$ (since $14 > 6$).
3. $(2, 9) \in R$ but $(9, 2) \notin R$ (since $18 > 6$).
4. $(3, 5) \in R$ but $(5, 3) \notin R$ (since $10 > 9$).
5. $(3, 7) \in R$ but $(7, 3) \notin R$ (since $14 > 9$).
6. $(3, 9) \in R$ but $(9, 3) \notin R$ (since $18 > 9$).
7. $(5, 9) \in R$ but $(9, 5) \notin R$ (since $18 > 15$).
Thus, $m = 7$.

Step 3: Final Answer:

$$l + m = 18 + 7 = 25$$

> Quick Tip
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> To find m quickly, count the number of pairs (x, y) in R where $x \neq y$ and (y, x) is NOT in R. Symmetry requires the relation to be balanced across the diagonal $y = x$.

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2. Let z be the complex number satisfying $|z - 5| \leq 3$ and having maximum positive principal argument. Then $34 \left| \frac{5z-12}{5iz+16} \right|^2$ is equal to :

(A) 12
(B) 16
(C) 26
(D) 20

Correct Answer: (D) 20

Solution:

Step 1: Understanding the Concept:
The inequality $|z-5| \le 3$ represents a circle centered at $(5,0)$ with radius 3.
The maximum principal argument θ occurs when the line from the origin to z is tangent to the circle in the first quadrant.

Step 2: Key Formula or Approach:
In the right triangle formed by the origin $O(0,0)$, the center $C(5,0)$, and the point of tangency T:
- $OC = 5$ (hypotenuse)
- $CT = 3$ (radius/perpendicular)
- $OT = \sqrt{5^2 - 3^2} = 4$ (base)
Thus, $\cos\theta = \frac{4}{5}$ and $\sin\theta = \frac{3}{5}$.
The point z is $4(\cos\theta + i\sin\theta) = 4(\frac{4}{5} + i\frac{3}{5}) = \frac{16+12i}{5}$.

Step 3: Detailed Explanation:
From the above, $5z = 16 + 12i$.
The expression to evaluate is $34\left|\frac{5z-12}{5iz+16}\right|^2$.
Numerator: $5z - 12 = (16+12i) - 12 = 4 + 12i$.
Denominator: $5iz + 16 = i(5z) + 16 = i(16+12i) + 16 = 16i - 12 + 16 = 4 + 16i$.
Modulus squared:

$$\left|\frac{4+12i}{4+16i}\right|^2 = \frac{|4+12i|^2}{|4+16i|^2} = \frac{4^2+12^2}{4^2+16^2} = \frac{16+144}{16+256} = \frac{160}{272}$$

Multiply by 34:

$$E = 34 \times \frac{160}{272} = \frac{34 \times 160}{34 \times 8} = \frac{160}{8} = 20$$

Step 4: Final Answer:
The required value is 20.

> Quick Tip
>
> For circles $|z - z_0| = r$, the tangent from the origin helps find the complex number with extreme arguments. Geometry is usually faster than algebraic substitution of $z = x + iy$.

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3. Let α and β be the roots of the equation $x^2+2ax+(3a+10) = 0$ such that $\alpha < 1 < \beta$. Then the set of all possible values of a is :

(A) $(-\infty, -\frac{11}{5}) \cup (5, \infty)$
(B) $(-\infty, -3)$
(C) $(-\infty, -2) \cup (5, \infty)$
(D) $(-\infty, -\frac{11}{5})$

Correct Answer: (D) $(-\infty, -\frac{11}{5})$

Solution:

Step 1: Understanding the Concept:
For a quadratic $f(x) = x^2 + 2ax + 3a + 10$ to have roots α and β such that 1 lies between them ($\alpha < 1 < \beta$), the value of the function at $x = 1$ must be negative, provided the coefficient of x^2 is positive.

Step 2: Key Formula or Approach:
Since the coefficient of x^2 is $1 > 0$, the condition for a value k to lie between roots is $f(k) < 0$. Set $k = 1$.

Step 3: Detailed Explanation:
Calculate $f(1)$:

$$f(1) = 1^2 + 2a(1) + 3a + 10 < 0$$

$$1 + 2a + 3a + 10 < 0$$

$$5a + 11 < 0 \implies 5a < -11 \implies a < -\frac{11}{5}$$

Check the discriminant $D > 0$ for real roots:

$$D = (2a)^2 - 4(3a + 10) = 4a^2 - 12a - 40 > 0$$

$$a^2 - 3a - 10 > 0 \implies (a - 5)(a + 2) > 0$$

$$a \in (-\infty, -2) \cup (5, \infty)$$

Since $-\frac{11}{5} = -2.2$, any $a < -2.2$ automatically satisfies $a < -2$.
Thus, the condition is $a < -\frac{11}{5}$.

Step 4: Final Answer:
The set of all possible values of a is $(-\infty, -\frac{11}{5})$.

> 💡 Quick Tip
>
> Location of roots: If a number k is between the roots of $f(x) = ax^2 + bx + c$, then $a \cdot f(k) < 0$. This single condition implies that the roots are real and distinct.

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4. If the line $\alpha x + 4y = \sqrt{7}$, where $\alpha \in \mathbf{R}$, touches the ellipse $3x^2 + 4y^2 = 1$ at the point P in the first quadrant, then one of the focal distances of P is :

(A) $\frac{1}{\sqrt{3}} + \frac{1}{2\sqrt{5}}$
(B) $\frac{1}{\sqrt{3}} + \frac{1}{2\sqrt{7}}$
(C) $\frac{1}{\sqrt{3}} - \frac{1}{2\sqrt{5}}$
(D) $\frac{1}{\sqrt{3}} - \frac{1}{2\sqrt{11}}$

Correct Answer: (B) $\frac{1}{\sqrt{3}} + \frac{1}{2\sqrt{7}}$

Solution:

Step 1: Understanding the Concept:
The ellipse equation is $3x^2 + 4y^2 = 1 \implies \frac{x^2}{1/3} + \frac{y^2}{1/4} = 1$.
Here $a^2 = 1/3$ and $b^2 = 1/4$.
The focal distances of a point $P(x, y)$ are given by $a \pm ex$.

Step 2: Key Formula or Approach:
Tangency condition for $y = mx + c$: $c^2 = a^2m^2 + b^2$.
Line: $4y = -\alpha x + \sqrt{7} \implies y = -\frac{\alpha}{4}x + \frac{\sqrt{7}}{4}$.
Set $m = -\frac{\alpha}{4}$ and $c = \frac{\sqrt{7}}{4}$.

Step 3: Detailed Explanation:
Using the tangency condition:

$$(\frac{\sqrt{7}}{4})^2 = (\frac{1}{3})(-\frac{\alpha}{4})^2 + \frac{1}{4}$$

$$\frac{7}{16} = \frac{\alpha^2}{48} + \frac{4}{16} \implies \frac{3}{16} = \frac{\alpha^2}{48} \implies \alpha^2 = 9 \implies \alpha = 3$$

Point of contact $x_P = -\frac{a^2m}{c} = -\frac{(1/3)(-3/4)}{\sqrt{7}/4} = \frac{1/4}{\sqrt{7}/4} = \frac{1}{\sqrt{7}}$.
Eccentricity e: $e^2 = 1 - \frac{b^2}{a^2} = 1 - \frac{1/4}{1/3} = 1 - \frac{3}{4} = \frac{1}{4} \implies e = \frac{1}{2}$.

Focal distances: $r = a \pm ex = \frac{1}{\sqrt{3}} \pm \frac{1}{2}(\frac{1}{\sqrt{7}}) = \frac{1}{\sqrt{3}} \pm \frac{1}{2\sqrt{7}}$.

Step 4: Final Answer:
One of the focal distances is $\frac{1}{\sqrt{3}} + \frac{1}{2\sqrt{7}}$.

> Quick Tip
>
> The sum of focal distances of any point on an ellipse equals the major axis ($2a$). Use the formula $r = a \pm ex$ to find individual distances once the point coordinates are known.

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5. Let $A = \{x : |x^2 - 10| \leq 6\}$ and $B = \{x : |x - 2| > 1\}$. Then

(A) $A - B = [2, 3)$
(B) $A \cap B = [-4, -2] \cup (3, 4]$
(C) $B - A = (-\infty, -4) \cup (-2, 1) \cup (4, \infty)$
(D) $A \cup B = (-\infty, 1] \cup (2, \infty)$

Correct Answer: (B) $A \cap B = [-4, -2] \cup (3, 4]$

Solution:

Step 1: Understanding the Concept:
We need to determine the solution sets for inequalities A and B and then find their intersection.

Step 2: Detailed Explanation:
Set A: $|x^2 - 10| \leq 6 \implies -6 \leq x^2 - 10 \leq 6$.
Add 10: $4 \leq x^2 \leq 16$.
Taking square root: $x \in [-4, -2] \cup [2, 4]$.
Set B: $|x - 2| > 1 \implies x - 2 > 1$ or $x - 2 < -1$.
Solving gives: $x > 3$ or $x < 1$.
So, $B = (-\infty, 1) \cup (3, \infty)$.

Now, $A \cap B$:
- Portion of A in $(-\infty, 1)$: $[-4, -2]$.
- Portion of A in $(3, \infty)$: $(3, 4]$.
Combining these, $A \cap B = [-4, -2] \cup (3, 4]$.

Step 3: Final Answer:
The intersection is $A \cap B = [-4, -2] \cup (3, 4]$.

Quick Tip

When working with multiple set operations, visualize the sets on a real number line to clearly see where intervals overlap (Intersection) or join (Union).

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6. Let $f(x) = x^3 + x^2 f'(1) + 2xf''(2) + f'''(3), x \in \mathbf{R}$. Then the value of $f'(5)$ is :

(A) $\frac{62}{5}$
(B) $\frac{657}{5}$
(C) $\frac{2}{5}$
(D) $\frac{117}{5}$

Correct Answer: (D) $\frac{117}{5}$

Solution:

Step 1: Understanding the Concept:
The polynomial $f(x)$ is defined by constants that are its own derivatives. We define these constants as variables and solve a system of linear equations.

Step 2: Detailed Explanation:
Let $f'(1) = a$, $f''(2) = b$, and $f'''(3) = c$.
Then $f(x) = x^3 + ax^2 + 2bx + c$.
Derivatives:
$f'(x) = 3x^2 + 2ax + 2b$
$f''(x) = 6x + 2a$
$f'''(x) = 6$
Comparing definitions:
1. $f'''(3) = c \implies c = 6$.
2. $f''(2) = b \implies 6(2) + 2a = b \implies 12 + 2a = b$.
3. $f'(1) = a \implies 3(1)^2 + 2a(1) + 2b = a \implies 3 + 2a + 2b = a \implies a + 2b = -3$.
Substitute $b = 12 + 2a$ into the third equation:

$$a + 2(12 + 2a) = -3 \implies 5a + 24 = -3 \implies 5a = -27 \implies a = -27/5$$

Calculate b: $b = 12 + 2(-27/5) = (60 - 54)/5 = 6/5$.
Thus, $f'(x) = 3x^2 + 2(-27/5)x + 2(6/5) = 3x^2 - \frac{54}{5}x + \frac{12}{5}$.

Step 3: Final Answer:
Calculate $f'(5)$:

$$f'(5) = 3(25) - \frac{54}{5}(5) + \frac{12}{5} = 75 - 54 + \frac{12}{5} = 21 + 2.4 = 23.4 = \frac{117}{5}$$

💡 Quick Tip

Treat fixed derivative values like $f'(1)$ as specific numerical constants (a, b, c). Once you differentiate the general form, equate the calculated derivatives back to these constants to solve.

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7. Let the line L_1 be parallel to the vector $-3\hat{i} + 2\hat{j} + 4\hat{k}$ and pass through the point $(2, 6, 7)$, and the line L_2 be parallel to the vector $2\hat{i} + \hat{j} + 3\hat{k}$ and pass through the point $(4, 3, 5)$. If the line L_3 is parallel to the vector $-3\hat{i} + 5\hat{j} + 16\hat{k}$ and intersects the lines L_1 and L_2 at the points C and D, respectively, then $|\vec{CD}|^2$ is equal to :

(A) 290
(B) 89
(C) 312
(D) 171

Correct Answer: (A) 290

Solution:

Step 1: Understanding the Concept:
Points C and D lie on L_1 and L_2. Vector $\vec{CD}$ must be parallel to the direction of L_3.

Step 2: Detailed Explanation:
Point C on $L_1 = (2 - 3\lambda, 6 + 2\lambda, 7 + 4\lambda)$.
Point D on $L_2 = (4 + 2\mu, 3 + \mu, 5 + 3\mu)$.
Vector $\vec{CD} = (2 + 2\mu + 3\lambda)\hat{i} + (-3 + \mu - 2\lambda)\hat{j} + (-2 + 3\mu - 4\lambda)\hat{k}$.
Since $\vec{CD} \parallel (-3, 5, 16)$:

$$\frac{2 + 2\mu + 3\lambda}{-3} = \frac{-3 + \mu - 2\lambda}{5} = \frac{-2 + 3\mu - 4\lambda}{16} = k$$

From the first two ratios: $10 + 10\mu + 15\lambda = 9 - 3\mu + 6\lambda \implies 13\mu + 9\lambda = -1$.
From the first and third ratios: $32 + 32\mu + 48\lambda = 6 - 9\mu + 12\lambda \implies 41\mu + 36\lambda = -26$.
Solving the equations: Multiply first by 4: $52\mu + 36\lambda = -4$.
Subtract second from this: $11\mu = 22 \implies \mu = 2$.
Substitute $\mu = 2$: $13(2) + 9\lambda = -1 \implies 9\lambda = -27 \implies \lambda = -3$.
Ratio $k = \frac{2+2(2)+3(-3)}{-3} = \frac{-3}{-3} = 1$.
Thus $\vec{CD} = -3\hat{i} + 5\hat{j} + 16\hat{k}$.

Step 3: Final Answer:

$$|\vec{CD}|^2 = (-3)^2 + 5^2 + 16^2 = 9 + 25 + 256 = 290$$

Quick Tip

When a vector is parallel to another, their corresponding components are in a fixed ratio k. Use this property to set up linear equations for the unknown parameters λ and μ.

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8. Let $y = y(x)$ be the solution of the differential equation $\sec x \frac{dy}{dx} - 2y = 2 + 3\sin x$, $x \in (-\frac{\pi}{2}, \frac{\pi}{2})$, $y(0) = -\frac{7}{4}$. Then $y(\frac{\pi}{6})$ is equal to :

(A) $-\frac{5}{2}$
(B) $-\frac{5}{4}$
(C) $-3\sqrt{2} - 7$
(D) $-3\sqrt{3} - 7$

Correct Answer: (A) $-\frac{5}{2}$

Solution:

Step 1: Understanding the Concept:
Rewrite in standard linear form $\frac{dy}{dx} + P(x)y = Q(x)$.
Divide by $\sec x$ (multiply by $\cos x$):

$$\frac{dy}{dx} - (2\cos x)y = (2 + 3\sin x)\cos x$$

Step 2: Detailed Explanation:
Integrating factor (I.F.): $e^{\int -2\cos x dx} = e^{-2\sin x}$.
General solution:

$$y \cdot e^{-2\sin x} = \int (2 + 3\sin x)\cos x \cdot e^{-2\sin x} dx$$

Let $t = -2\sin x \implies dt = -2\cos x dx$.
Integral becomes: $\int (2 - \frac{3t}{2})e^t(-\frac{1}{2})dt = -\frac{1}{2}\int (2e^t - \frac{3}{2}te^t)dt$.
Using $\int te^t dt = (t-1)e^t$:
$ye^{-2\sin x} = -e^t + \frac{3}{4}(t-1)e^t + C = e^t(\frac{3}{4}t - \frac{7}{4}) + C$.
Substitute $t = -2\sin x$: $y = -\frac{3}{2}\sin x - \frac{7}{4} + Ce^{2\sin x}$.
Given $y(0) = -7/4 \implies -7/4 = 0 - 7/4 + C(1) \implies C = 0$.

Step 3: Final Answer:
Function is $y(x) = -\frac{3}{2}\sin x - \frac{7}{4}$.
At $x = \pi/6$, $\sin x = 1/2$:

$$y(\pi/6) = -\frac{3}{2}(1/2) - 7/4 = -3/4 - 7/4 = -10/4 = -5/2$$

Quick Tip

For linear ODEs with $\sin x$ and $\cos x$, the integrating factor often simplifies the RHS integral through a u-substitution. Always check the initial condition $y(0)$ early to eliminate the constant C.

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9. If the area of the region $\{(x, y) : 1 - 2x \leq y \leq 4 - x^2, x \geq 0, y \geq 0\}$ is $\frac{\alpha}{\beta}$, $\alpha, \beta \in \mathbf{N}$, $\gcd(\alpha, \beta) = 1$, then the value of $(\alpha + \beta)$ is :

(A) 67
(B) 85
(C) 91
(D) 73

Correct Answer: (D) 73

Solution:

Step 1: Understanding the Concept:
The region is in the first quadrant, bounded above by the parabola $y = 4 - x^2$ and below by the line $y = 1 - 2x$ when $y \geq 0$.

Step 2: Detailed Explanation:
Area $= \int(\text{Upper Curve} - \text{Lower Curve})dx$.
Upper limit: $4 - x^2 = 0 \implies x = 2$.
Lower boundary is composed of parts:
From $x = 0$ to $x = 0.5$, the line $y = 1 - 2x$ is above $y = 0$.
From $x = 0.5$ to $x = 2$, the lower boundary is simply $y = 0$.
Total Area $= \int_0^2 (4 - x^2)dx -$ (Area under line in first quadrant).
Area under parabola: $[4x - \frac{x^3}{3}]_0^2 = 8 - \frac{8}{3} = \frac{16}{3}$.
Area under line (triangle with vertices (0,0), (0.5,0), (0,1)): $\frac{1}{2} \times 0.5 \times 1 = \frac{1}{4}$.
Net Area $= \frac{16}{3} - \frac{1}{4} = \frac{64-3}{12} = \frac{61}{12}$.
So $\alpha = 61, \beta = 12$.

Step 3: Final Answer:

$$\alpha + \beta = 61 + 12 = 73$$

Quick Tip

Geometric shortcuts can save time! If part of the boundary is a straight line forming a triangle with axes, subtract the triangle area from the total integral instead of doing piecewise integration.

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10. Let $a_1, \frac{a_2}{2}, \frac{a_3}{2^2}, \ldots, \frac{a_{10}}{2^9}$ be a G.P. of common ratio $\frac{1}{\sqrt{2}}$. If $a_1+a_2+\cdots+a_{10}=62$, then a_1 is equal to :

(A) $2-\sqrt{2}$
(B) $2(2-\sqrt{2})$
(C) $\sqrt{2}-1$
(D) $2(\sqrt{2}-1)$

Correct Answer: (D) $2(\sqrt{2}-1)$

Solution:

Step 1: Understanding the Concept:
Let the general term of the G.P. be $b_n = \frac{a_n}{2^{n-1}}$.
Since $b_n = b_1 r^{n-1}$ with $r = 1/\sqrt{2}$, we find the form of a_n.

Step 2: Detailed Explanation:
$a_n = 2^{n-1} \cdot b_n = 2^{n-1} \cdot a_1 \cdot (\frac{1}{\sqrt{2}})^{n-1} = a_1 \cdot (\frac{2}{\sqrt{2}})^{n-1} = a_1 \cdot (\sqrt{2})^{n-1}$.
The sequence $a_1, a_2, \ldots$ is itself a G.P. with common ratio $R=\sqrt{2}$.
Sum $S_{10} = a_1 \frac{R^{10}-1}{R-1} = a_1 \frac{(\sqrt{2})^{10}-1}{\sqrt{2}-1} = a_1 \frac{2^5-1}{\sqrt{2}-1} = a_1 \frac{31}{\sqrt{2}-1}$.
Set $S_{10} = 62$:

$$62 = \frac{31a_1}{\sqrt{2}-1} \implies 2 = \frac{a_1}{\sqrt{2}-1} \implies a_1 = 2(\sqrt{2}-1)$$

Step 3: Final Answer:
The value of a_1 is $2(\sqrt{2}-1)$.

Quick Tip

If $\frac{a_n}{k^n}$ is a G.P. with ratio r, then a_n is a G.P. with ratio kr. This transformation allows for standard sum formulas to be applied easily.

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11. Let $f : \mathbf{R} \to \mathbf{R}$ be a twice differentiable function such that $f''(x) > 0$ for all $x \in \mathbf{R}$ and $f'(a-1) = 0$, where a is a real number. Let $g(x) = f(\tan^2 x - 2\tan x + a), 0 < x < \frac{\pi}{2}$.

Consider the following two statements :
(I) g **is increasing in** $(0, \frac{\pi}{4})$
(II) g **is decreasing in** $(\frac{\pi}{4}, \frac{\pi}{2})$
Then,

(A) Only (II) is True
(B) Only (I) is True
(C) Both (I) and (II) are True
(D) Neither (I) nor (II) is True

Correct Answer: (D) Neither (I) nor (II) is True

Solution:

Step 1: Understanding the Concept:
$f''(x) > 0$ means $f'(x)$ is strictly increasing.
Since $f'(a-1) = 0$, it follows that $f'(u) > 0$ for $u > a-1$ and $f'(u) < 0$ for $u < a-1$.

Step 2: Detailed Explanation:
Let $u(x) = \tan^2 x - 2\tan x + a = (\tan x - 1)^2 + a - 1$.
Then $g(x) = f(u(x))$. Derivative $g'(x) = f'(u) \cdot u'(x)$.
$u'(x) = 2\tan x \sec^2 x - 2\sec^2 x = 2\sec^2 x(\tan x - 1)$.
Notice $(\tan x - 1)^2 \geq 0$, so $u(x) \geq a - 1$. Thus $f'(u(x)) \geq 0$ for all x.

Case (I): $x \in (0, \pi/4)$
$\tan x < 1 \implies \tan x - 1 < 0 \implies u'(x) < 0$.
$g'(x) = (\text{Positive}) \cdot (\text{Negative}) = \text{Negative}$.
Thus g is **decreasing** in $(0, \pi/4)$. Statement (I) is False.

Case (II): $x \in (\pi/4, \pi/2)$
$\tan x > 1 \implies \tan x - 1 > 0 \implies u'(x) > 0$.
$g'(x) = (\text{Positive}) \cdot (\text{Positive}) = \text{Positive}$.
Thus g is **increasing** in $(\pi/4, \pi/2)$. Statement (II) is False.

Step 3: Final Answer:
Neither statement is true.

> Quick Tip
>
> For composite functions $f(h(x))$, monotonicity depends on the sign of $f'(h(x)) \cdot h'(x)$. Analyze the range of the inner function $h(x)$ to find the sign of the outer derivative f'.

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12. For the matrices $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ **and** $B = \begin{bmatrix} -29 & 49 \\ -13 & 18 \end{bmatrix}$**, if** $(A^{15} + B)\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$**, then**

among the following which one is true ?

(A) $x = 16, y = 3$
(B) $x = 18, y = 11$
(C) $x = 5, y = 7$
(D) $x = 11, y = 2$

Correct Answer: (D) $x = 11, y = 2$

Solution:

Step 1: Understanding the Concept:
We need to find a pattern or expression for A^n.
Decompose $A = I + N$, where $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $N = \begin{bmatrix} 2 & -4 \\ 1 & -2 \end{bmatrix}$.

Step 2: Detailed Explanation:
Check N^2: $N^2 = \begin{bmatrix} 2 & -4 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} 2 & -4 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.
N is nilpotent. Using Binomial expansion:
$A^{15} = (I + N)^{15} = I^{15} + 15I^{14}N + \dots$ (all further terms are 0).
$A^{15} = I + 15N = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 30 & -60 \\ 15 & -30 \end{bmatrix} = \begin{bmatrix} 31 & -60 \\ 15 & -29 \end{bmatrix}$.
Now find $A^{15} + B$:
$A^{15} + B = \begin{bmatrix} 31 & -60 \\ 15 & -29 \end{bmatrix} + \begin{bmatrix} -29 & 49 \\ -13 & 18 \end{bmatrix} = \begin{bmatrix} 2 & -11 \\ 2 & -11 \end{bmatrix}$.
The equation is $\begin{bmatrix} 2 & -11 \\ 2 & -11 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.
This gives the linear equation: $2x - 11y = 0 \implies 2x = 11y$.

Step 3: Final Answer:
From the options, if $x = 11$ and $y = 2$, then $2(11) = 22$ and $11(2) = 22$.
Thus, $(x, y) = (11, 2)$ is the correct choice.

> Quick Tip
>
> Matrix decomposition $A = \lambda I + N$ is very powerful for finding high powers A^n. If N is nilpotent ($N^k = 0$), the binomial expansion terminates early, making calculations simple.

---

13. Let one end of a focal chord of the parabola $y^2 = 16x$ be $(16, 16)$. If $P(\alpha, \beta)$ divides this focal chord internally in the ratio $5 : 2$, then the minimum value of $\alpha + \beta$ is equal to :

(A) 5
(B) 7
(C) 16
(D) 22

Correct Answer: (B) 7

Solution:

Step 1: Understanding the Concept:
The focal chord of a parabola passes through the focus. If one end is given as $(at^2, 2at)$, the other end is $(a/t^2, -2a/t)$. The point P divides the segment joining these two points in a given ratio. We need to find the coordinates of P for both possible arrangements of the ratio and identify the minimum sum.

Step 2: Key Formula or Approach:
For $y^2 = 16x$, $4a = 16 \implies a = 4$.
The focus is $S(4, 0)$.
Let the ends of the focal chord be $A(x_1, y_1)$ and $B(x_2, y_2)$.
If $A = (16, 16)$, then $2at_1 = 16 \implies 2(4)t_1 = 16 \implies t_1 = 2$.
The parameter for the other end B is $t_2 = -1/t_1 = -1/2$.
Coordinates of $B = (at_2^2, 2at_2) = (4(-1/2)^2, 2(4)(-1/2)) = (1, -4)$.

Step 3: Detailed Explanation:
The ends of the focal chord are $A(16, 16)$ and $B(1, -4)$.
Point $P(\alpha, \beta)$ divides AB internally in the ratio $5 : 2$.
Case 1: Ratio 5:2 from A to B

$$\alpha = \frac{5(1) + 2(16)}{5 + 2} = \frac{5 + 32}{7} = \frac{37}{7}$$

$$\beta = \frac{5(-4) + 2(16)}{5 + 2} = \frac{-20 + 32}{7} = \frac{12}{7}$$

Sum $\alpha + \beta = \frac{37+12}{7} = \frac{49}{7} = 7$.
Case 2: Ratio 5:2 from B to A

$$\alpha = \frac{5(16) + 2(1)}{5 + 2} = \frac{80 + 2}{7} = \frac{82}{7}$$

$$\beta = \frac{5(16) + 2(-4)}{5 + 2} = \frac{80 - 8}{7} = \frac{72}{7}$$

Sum $\alpha + \beta = \frac{82+72}{7} = \frac{154}{7} = 22$.

Step 4: Final Answer:
Comparing the sums from both cases, the minimum value of $\alpha + \beta$ is 7.

💡 Quick Tip

For a parabola $y^2 = 4ax$, if one end of a focal chord is t_1, the other is $t_2 = -1/t_1$. Always consider both directions of the internal division ratio unless specified, as the "minimum" value often comes from the point closer to the smaller coordinates.

---

14. Let $y^2 = 12x$ be the parabola with its vertex at O. Let P be a point on the parabola and A be a point on the x-axis such that $\angle OPA = 90°$. Then the locus of the centroid of such triangles OPA is :

(A) $y^2 - 4x + 8 = 0$
(B) $y^2 - 6x + 4 = 0$
(C) $y^2 - 9x + 6 = 0$
(D) $y^2 - 2x + 8 = 0$

Correct Answer: (D) $y^2 - 2x + 8 = 0$

Solution:

Step 1: Understanding the Concept:
Let $O(0,0)$ be the vertex. Any point P on the parabola $y^2 = 12x$ can be represented as $(3t^2, 6t)$. Since A is on the x-axis, let its coordinates be $(x_A, 0)$. We use the perpendicularity condition $\angle OPA = 90°$ to relate x_A and t, then find the centroid of $\triangle OPA$.

Step 2: Key Formula or Approach:
Coordinates: $O(0,0)$, $P(3t^2, 6t)$, $A(x_A, 0)$.
Slope of OP: $m_{OP} = \frac{6t-0}{3t^2-0} = \frac{2}{t}$.
Slope of PA: $m_{PA} = \frac{6t-0}{3t^2-x_A} = \frac{6t}{3t^2-x_A}$.
Condition for perpendicularity: $m_{OP} \cdot m_{PA} = -1$.

Step 3: Detailed Explanation:

$$\frac{2}{t} \cdot \frac{6t}{3t^2 - x_A} = -1 \implies \frac{12}{3t^2 - x_A} = -1$$

$$12 = -(3t^2 - x_A) = x_A - 3t^2 \implies x_A = 3t^2 + 12$$

So, $A = (3t^2 + 12, 0)$.
Let the centroid of $\triangle OPA$ be $G(h, k)$:

$$h = \frac{0 + 3t^2 + (3t^2 + 12)}{3} = \frac{6t^2 + 12}{3} = 2t^2 + 4$$

$$k = \frac{0 + 6t + 0}{3} = 2t \implies t = \frac{k}{2}$$

Substitute t in the expression for h:

$$h = 2\left(\frac{k}{2}\right)^2 + 4 = 2\left(\frac{k^2}{4}\right) + 4 = \frac{k^2}{2} + 4$$

$$2h = k^2 + 8 \implies k^2 - 2h + 8 = 0$$

Step 4: Final Answer:
The locus is $y^2 - 2x + 8 = 0$.

> Quick Tip
>
> For locus problems involving a triangle's centroid, express the coordinates of all vertices in terms of a single parameter. The centroid (h, k) will then give you parametric equations that you can combine by eliminating the parameter.

---

15. The positive integer n, for which the solutions of the equation $x(x+2) + (x+2)(x+4) + \cdots + (x+2n-2)(x+2n) = \frac{8n}{3}$ are two consecutive even integers, is :

(A) 9
(B) 3
(C) 12
(D) 6

Correct Answer: (B) 3

Solution:

Step 1: Understanding the Concept:
The given equation is a sum of n terms of the form $T_k = (x+2k-2)(x+2k)$ for $k = 1, 2, \ldots, n$. We simplify this summation into a quadratic equation in x and use the properties of its roots.

Step 2: Key Formula or Approach: General term $T_k = x^2 + (4k-2)x + 4k(k-1)$.
Summation: $\sum_{k=1}^{n} T_k = nx^2 + x\sum_{k=1}^{n}(4k-2) + \sum_{k=1}^{n}(4k^2 - 4k) = \frac{8n}{3}$.

Step 3: Detailed Explanation:
Linear term coefficient: $\sum_{k=1}^{n}(4k-2) = 4\frac{n(n+1)}{2} - 2n = 2n^2 + 2n - 2n = 2n^2$.
Constant term sum: $4\sum_{k=1}^{n} k^2 - 4\sum_{k=1}^{n} k = 4\frac{n(n+1)(2n+1)}{6} - 4\frac{n(n+1)}{2}$.

$$= \frac{2n(n+1)}{3}\left[(2n+1) - 3\right] = \frac{2n(n+1)(2n-2)}{3} = \frac{4n(n+1)(n-1)}{3} = \frac{4n(n^2-1)}{3}$$

The equation is: $nx^2 + 2n^2x + \frac{4n(n^2-1)}{3} = \frac{8n}{3}$.
Dividing by n: $x^2 + 2nx + \frac{4(n^2-1)}{3} - \frac{8}{3} = 0 \implies x^2 + 2nx + \frac{4n^2-12}{3} = 0$.
Let the roots be α and $\alpha + 2$. Their difference is 2.

$$(\text{Difference of roots})^2 = 4 \implies (\alpha_1 + \alpha_2)^2 - 4\alpha_1\alpha_2 = 4$$

$$(-2n)^2 - 4\frac{4n^2-12}{3} = 4 \implies 4n^2 - \frac{16n^2-48}{3} = 4$$

$$12n^2 - 16n^2 + 48 = 12 \implies -4n^2 = -36 \implies n^2 = 9 \implies n = 3$$

Step 4: Final Answer:
The positive integer n is 3.

Quick Tip

For quadratic equations where roots are "consecutive even integers", the difference between roots is always 2. Use the relation $|\alpha - \beta| = \frac{\sqrt{D}}{|a|}$ to solve for the unknown parameter quickly.

---

16. A random variable X takes values 0, 1, 2, 3 with probabilities $\frac{2a+1}{30}, \frac{8a-1}{30}, \frac{4a+1}{30}, b$ respectively, where $a, b \in \mathbf{R}$. Let μ and σ respectively be the mean and standard deviation of X such that $\sigma^2 + \mu^2 = 2$. Then $\frac{a}{b}$ is equal to :

(A) 12
(B) 3
(C) 60
(D) 30

Correct Answer: (C) 60

Solution:

Step 1: Understanding the Concept:
The sum of all probabilities in a distribution must equal 1. This allows us to express b in terms of a. The variance formula is $\sigma^2 = E[X^2] - \mu^2$, which implies $\sigma^2 + \mu^2 = E[X^2]$. We equate $E[X^2]$ to 2 to solve for a and b.

Step 2: Key Formula or Approach:
1. $\sum P(X) = 1$.

2. $E[X^2] = \sum x^2 P(x) = \sigma^2 + \mu^2$.

Step 3: Detailed Explanation:
Sum of probabilities:

$$\frac{2a+1}{30} + \frac{8a-1}{30} + \frac{4a+1}{30} + b = 1 \implies \frac{14a+1}{30} + b = 1 \implies b = 1 - \frac{14a+1}{30} = \frac{29-14a}{30}$$

Given $\sigma^2 + \mu^2 = 2 \implies E[X^2] = 2$.

$$E[X^2] = 0^2 \cdot P(0) + 1^2 \cdot P(1) + 2^2 \cdot P(2) + 3^2 \cdot P(3) = 2$$

$$1 \cdot \frac{8a-1}{30} + 4 \cdot \frac{4a+1}{30} + 9 \cdot b = 2$$

$$\frac{8a-1+16a+4}{30} + 9\left(\frac{29-14a}{30}\right) = 2$$

$$\frac{24a+3+261-126a}{30} = 2 \implies \frac{264-102a}{30} = 2$$

$$264 - 102a = 60 \implies 102a = 204 \implies a = 2$$

Now, calculate b:

$$b = \frac{29-14(2)}{30} = \frac{29-28}{30} = \frac{1}{30}$$

Step 4: Final Answer:

$$\frac{a}{b} = \frac{2}{1/30} = 60$$

Quick Tip

The identity $E[X^2] = \sigma^2 + \mu^2$ is a common shortcut in probability questions involving mean and variance. Always start by ensuring the sum of probabilities equals 1 to link unknown variables.

---

17. Let the line L pass through the point $(-3, 5, 2)$ and make equal angles with the positive coordinate axes. If the distance of L from the point $(-2, r, 1)$ is $\sqrt{\frac{14}{3}}$, then the sum of all possible values of r is :

(A) 16
(B) 12
(C) 10
(D) 6

Correct Answer: (C) 10

Solution:

Step 1: Understanding the Concept:
A line making equal angles with positive coordinate axes has direction cosines $l = m = n = \frac{1}{\sqrt{3}}$.
The distance from a point A to a line passing through P with direction unit vector $\hat{u}$ is given by $d^2 = |\vec{PA}|^2 - (\vec{PA} \cdot \hat{u})^2$.

Step 2: Key Formula or Approach:
Line passes through $P(-3, 5, 2)$ with $\hat{u} = \frac{1}{\sqrt{3}}(1, 1, 1)$.
Target point $A(-2, r, 1)$.
Vector $\vec{PA} = (-2 - (-3), r - 5, 1 - 2) = (1, r - 5, -1)$.

Step 3: Detailed Explanation:

$$|\vec{PA}|^2 = 1^2 + (r - 5)^2 + (-1)^2 = 2 + (r - 5)^2$$

$$\vec{PA} \cdot \hat{u} = \frac{1(1) + 1(r - 5) + 1(-1)}{\sqrt{3}} = \frac{r - 5}{\sqrt{3}}$$

Distance squared $d^2 = \frac{14}{3}$:

$$\frac{14}{3} = [2 + (r - 5)^2] - \frac{(r - 5)^2}{3}$$

$$\frac{14}{3} - 2 = \frac{2}{3}(r - 5)^2 \implies \frac{8}{3} = \frac{2}{3}(r - 5)^2$$

$$(r - 5)^2 = 4 \implies r - 5 = \pm 2$$

$$r_1 = 7, r_2 = 3$$

Step 4: Final Answer:
The sum of values is $7 + 3 = 10$.

💡 Quick Tip

For a line with equal inclination to axes, the direction vector is $(1, 1, 1)$. The distance formula $d = \frac{|\vec{PA} \times \vec{b}|}{|\vec{b}|}$ can also be used, where $\vec{b}$ is the direction vector.

---

18. The largest $n \in \mathbf{N}$, for which 7^n divides $101!$, is :

(A) 15
(B) 19
(C) 16
(D) 18

Correct Answer: (C) 16

Solution:

Step 1: Understanding the Concept:
To find the highest power of a prime p that divides $m!$, we use Legendre's Formula (or de Polignac's formula), which involves summing the floor values of the quotients of m by powers of p.

Step 2: Key Formula or Approach:

$$n = \sum_{i=1}^{\infty} \left\lfloor \frac{m}{p^i} \right\rfloor$$

Here $m = 101$ and $p = 7$.

Step 3: Detailed Explanation:
We calculate the terms of the sum until the divisor exceeds 101:
1. $\left\lfloor \frac{101}{7} \right\rfloor = \lfloor 14.42 \dots \rfloor = 14$
2. $\left\lfloor \frac{101}{7^2} \right\rfloor = \lfloor \frac{101}{49} \rfloor = \lfloor 2.06 \dots \rfloor = 2$
3. $\left\lfloor \frac{101}{7^3} \right\rfloor = \lfloor \frac{101}{343} \rfloor = 0$
Summing these up: $n = 14 + 2 = 16$.

Step 4: Final Answer:
The largest n is 16.

Quick Tip

Always remember to continue dividing by increasing powers $(p, p^2, p^3 \ldots)$ until the denominator is larger than the number itself. Each term represents how many multiples of that power exist in the product.

---

19. For a triangle ABC, let $\vec{p} = \vec{BC}$, $\vec{q} = \vec{CA}$ and $\vec{r} = \vec{AB}$. If $|\vec{p}| = 2\sqrt{3}$, $|\vec{q}| = 2$ and $\cos\theta = \frac{1}{\sqrt{3}}$, where θ is the angle between $\vec{p}$ and $\vec{q}$, then $|\vec{p} \times (\vec{q} - \vec{r})|^2 + 3|\vec{r}|^2$ is equal to :

(A) 340
(B) 220
(C) 200
(D) 410

Correct Answer: (C) 200

Solution:

Step 1: Understanding the Concept:
In any triangle, the sum of the side vectors taken in order is zero: $\vec{p}+\vec{q}+\vec{r} = 0 \implies \vec{r} = -(\vec{p}+\vec{q})$. We use this to simplify the cross product and determine the magnitude of $\vec{r}$.

Step 2: Key Formula or Approach:
1. $|\vec{r}|^2 = |\vec{p} + \vec{q}|^2 = |\vec{p}|^2 + |\vec{q}|^2 + 2\vec{p} \cdot \vec{q}$.
2. $\vec{p} \cdot \vec{q} = |\vec{p}||\vec{q}| \cos\theta$.
3. $|\vec{p} \times \vec{q}|^2 = |\vec{p}|^2|\vec{q}|^2 \sin^2\theta$.

Step 3: Detailed Explanation:
First, calculate products:
$\vec{p} \cdot \vec{q} = (2\sqrt{3})(2)\left(\frac{1}{\sqrt{3}}\right) = 4$.
$|\vec{r}|^2 = (2\sqrt{3})^2 + 2^2 + 2(4) = 12 + 4 + 8 = 24$.
Simplify the cross product term:
$\vec{q} - \vec{r} = \vec{q} - (-\vec{p} - \vec{q}) = \vec{p} + 2\vec{q}$.
$\vec{p} \times (\vec{q} - \vec{r}) = \vec{p} \times (\vec{p} + 2\vec{q}) = \vec{p} \times \vec{p} + 2(\vec{p} \times \vec{q}) = 2(\vec{p} \times \vec{q})$.
Square of the magnitude:
$|2(\vec{p} \times \vec{q})|^2 = 4|\vec{p} \times \vec{q}|^2 = 4[|\vec{p}|^2|\vec{q}|^2 - (\vec{p} \cdot \vec{q})^2] = 4[(12)(4) - 4^2] = 4[48 - 16] = 128$.
Total expression:

$$128 + 3|\vec{r}|^2 = 128 + 3(24) = 128 + 72 = 200$$

Step 4: Final Answer:
The result is 200.

Quick Tip

The relation $\vec{a} + \vec{b} + \vec{c} = 0$ implies $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$. Use this property to switch between different cross products to simplify vector expressions in triangle geometry.

---

20. If the system of equations
$3x + y + 4z = 3$
$2x + \alpha y - z = -3$
$x + 2y + z = 4$
has no solution, then the value of α is equal to :

(A) 19
(B) 13
(C) 4
(D) 23

Correct Answer: (A) 19

Solution:

Step 1: Understanding the Concept:
A system of linear equations has no solution if the determinant of the coefficients (Δ) is zero, but at least one of the Cramer's determinants ($\Delta_x, \Delta_y, \Delta_z$) is non-zero.

Step 2: Key Formula or Approach:
Set $\Delta = 0$:

$$\Delta = \begin{vmatrix} 3 & 1 & 4 \\ 2 & \alpha & -1 \\ 1 & 2 & 1 \end{vmatrix} = 0$$

Step 3: Detailed Explanation:
Expanding along the first row:

$$3(\alpha(1) - (-1)(2)) - 1(2(1) - (-1)(1)) + 4(2(2) - \alpha(1)) = 0$$

$$3(\alpha + 2) - 1(2 + 1) + 4(4 - \alpha) = 0$$

$$3\alpha + 6 - 3 + 16 - 4\alpha = 0 \implies 19 - \alpha = 0 \implies \alpha = 19$$

To verify "no solution", we check for inconsistency.
For $\alpha = 19$:

From eq 1: $3x + y + 4z = 3$
From eq 3: $x + 2y + z = 4 \implies 3x + 6y + 3z = 12$
Subtracting gives $5y - z = 9 \implies z = 5y - 9$.
Substituting into eq 2: $2x + 19y - (5y - 9) = -3 \implies 2x + 14y = -12 \implies x + 7y = -6$.
Check in eq 3: $(-7y - 6) + 2y + (5y - 9) = 4 \implies -15 = 4$.
This is a contradiction, confirming no solution exists.

Step 4: Final Answer:
The value of α is 19.

> Quick Tip
>
> If $\Delta = 0$, the system either has infinitely many solutions or no solution. Always substitute the found value of α back into the equations to verify a contradiction, which confirms the "no solution" case.

Mathematics Section B

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21. Let the maximum value of $(\sin^{-1} x)^2 + (\cos^{-1} x)^2$ **for** $x \in \left[-\frac{\sqrt{3}}{2}, \frac{1}{\sqrt{2}}\right]$ **be** $\frac{m}{n}\pi^2$**, where** $\gcd(m, n) = 1$**. Then** $m + n$ **is equal to** ________**.**

Correct Answer: 65

Solution:

Step 1: Understanding the Concept:
The problem involves finding the maximum value of an expression consisting of squares of inverse trigonometric functions.
Using the identity $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$, we can transform the expression into a quadratic form in terms of a single variable.

Step 2: Key Formula or Approach:
Let $y = \sin^{-1} x$. The given expression is $E = y^2 + \left(\frac{\pi}{2} - y\right)^2$.
Simplifying, we get:

$$E = y^2 + \frac{\pi^2}{4} - \pi y + y^2 = 2y^2 - \pi y + \frac{\pi^2}{4}$$

By completing the square:

$$E = 2\left(y^2 - \frac{\pi}{2}y + \frac{\pi^2}{8}\right) = 2\left(y - \frac{\pi}{4}\right)^2 + \frac{\pi^2}{8}$$

Step 3: Detailed Explanation:
The domain for x is $\left[-\frac{\sqrt{3}}{2}, \frac{1}{\sqrt{2}}\right]$.
Consequently, the range of $y = \sin^{-1} x$ is $\left[\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right), \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)\right] = \left[-\frac{\pi}{3}, \frac{\pi}{4}\right]$.
We need to maximize $f(y) = 2\left(y - \frac{\pi}{4}\right)^2 + \frac{\pi^2}{8}$ for $y \in \left[-\frac{\pi}{3}, \frac{\pi}{4}\right]$.
Since the vertex of this upward-opening parabola is at $y = \frac{\pi}{4}$ (the right endpoint of the interval), the function is decreasing throughout the interval.
The maximum value occurs at the left endpoint, $y = -\frac{\pi}{3}$:

$$f\left(-\frac{\pi}{3}\right) = 2\left(-\frac{\pi}{3} - \frac{\pi}{4}\right)^2 + \frac{\pi^2}{8}$$

$$f\left(-\frac{\pi}{3}\right) = 2\left(-\frac{7\pi}{12}\right)^2 + \frac{\pi^2}{8} = 2\left(\frac{49\pi^2}{144}\right) + \frac{\pi^2}{8}$$

$$f\left(-\frac{\pi}{3}\right) = \frac{49\pi^2}{72} + \frac{9\pi^2}{72} = \frac{58\pi^2}{72} = \frac{29\pi^2}{36}$$

Comparing this with $\frac{m}{n}\pi^2$, we have $m = 29$ and $n = 36$.
Since $\gcd(29, 36) = 1$, the values are correct.

Step 4: Final Answer:
The value of $m + n = 29 + 36 = 65$.

> Quick Tip
>
> For quadratic expressions of the form $a(x-h)^2 + k$ on a closed interval, the maximum always occurs at one of the endpoints. Simply test both endpoints to find the extreme value.

---

22. If $\left(\frac{1}{^{15}C_0} + \frac{1}{^{15}C_1}\right)\left(\frac{1}{^{15}C_1} + \frac{1}{^{15}C_2}\right)\ldots\left(\frac{1}{^{15}C_{12}} + \frac{1}{^{15}C_{13}}\right) = \frac{\alpha^{13}}{^{14}C_0 \cdot {}^{14}C_1 \cdots\cdots {}^{14}C_{12}}$, then 30α is equal to ________.

Correct Answer: 32

Solution:

Step 1: Understanding the Concept:
This problem involves an algebraic simplification of a product of reciprocal binomial coefficients.

A key identity relating consecutive binomial coefficients is:

$$\frac{1}{^nC_r} + \frac{1}{^nC_{r+1}} = \frac{n+1}{n \cdot ^{n-1} C_r}$$

Step 2: Key Formula or Approach:
Verification of the identity:

$$\frac{1}{^nC_r} + \frac{1}{^nC_{r+1}} = \frac{r!(n-r)!}{n!} + \frac{(r+1)!(n-r-1)!}{n!}$$

$$= \frac{r!(n-r-1)!}{n!}[(n-r)+(r+1)] = \frac{r!(n-r-1)!(n+1)}{n!}$$

$$= \frac{n+1}{n} \cdot \frac{r!(n-r-1)!}{(n-1)!} = \frac{n+1}{n \cdot ^{n-1} C_r}$$

Step 3: Detailed Explanation:
In the given problem, $n = 15$. The general term in the product is:

$$\frac{1}{^{15}C_r} + \frac{1}{^{15}C_{r+1}} = \frac{15+1}{15 \cdot ^{15-1} C_r} = \frac{16}{15 \cdot ^{14} C_r}$$

The product is for $r = 0$ to $r = 12$:

$$P = \prod_{r=0}^{12} \left(\frac{16}{15 \cdot ^{14} C_r} \right) = \frac{\left(\frac{16}{15}\right)^{13}}{\prod_{r=0}^{12} {^{14}C_r}}$$

Given that this equals $\frac{\alpha^{13}}{^{14}C_0 \cdot ^{14}C_1 \cdots ^{14}C_{12}}$, we can compare the numerators:

$$\alpha^{13} = \left(\frac{16}{15}\right)^{13} \implies \alpha = \frac{16}{15}$$

Step 4: Final Answer:
We need to find 30α:

$$30\alpha = 30 \times \frac{16}{15} = 2 \times 16 = 32$$

Quick Tip

Remember the shortcut: $\frac{^nC_r + ^nC_{r+1}}{^nC_r \cdot ^nC_{r+1}} = \frac{^{n+1}C_{r+1}}{^nC_r \cdot ^nC_{r+1}}$. Converting terms into standard forms like $\frac{n+1}{n \cdot ^{n-1}C_r}$ often reveals a common factor that can be pulled out of a product.

23. Let $[\cdot]$ denote the greatest integer function and $f(x) = \lim_{n\to\infty} \frac{1}{n^3} \sum_{k=1}^{n} \left[\frac{k^2}{3^x}\right]$. Then $12\sum_{j=1}^{\infty} f(j)$ is equal to ________.

Correct Answer: 2

Solution:

Step 1: Understanding the Concept:
The limit of a sum can be evaluated using the property of the greatest integer function $[x] = x - \{x\}$, where $\{x\}$ is the fractional part. For large n, the contribution of the fractional parts becomes negligible.

Step 2: Key Formula or Approach:
Consider the expression inside the limit:

$$S_n = \frac{1}{n^3} \sum_{k=1}^{n} \left[\frac{k^2}{3^x}\right] = \frac{1}{n^3} \sum_{k=1}^{n} \left(\frac{k^2}{3^x} - \left\{\frac{k^2}{3^x}\right\}\right)$$

$$S_n = \frac{1}{3^x \cdot n^3} \sum_{k=1}^{n} k^2 - \frac{1}{n^3} \sum_{k=1}^{n} \left\{\frac{k^2}{3^x}\right\}$$

Step 3: Detailed Explanation:
Using the sum of squares formula $\sum k^2 = \frac{n(n+1)(2n+1)}{6} \approx \frac{n^3}{3}$ for large n:
The first part tends to:

$$\lim_{n\to\infty} \frac{n(n+1)(2n+1)}{6 \cdot 3^x \cdot n^3} = \frac{2}{6 \cdot 3^x} = \frac{1}{3 \cdot 3^x} = \frac{1}{3^{x+1}}$$

For the second part, since $0 \leq \{y\} < 1$:

$$0 \leq \frac{1}{n^3} \sum_{k=1}^{n} \left\{\frac{k^2}{3^x}\right\} < \frac{n}{n^3} = \frac{1}{n^2}$$

As $n \to \infty$, this term tends to 0.
Thus, $f(x) = \frac{1}{3^{x+1}}$.
Now evaluate the sum:

$$\sum_{j=1}^{\infty} f(j) = \sum_{j=1}^{\infty} \frac{1}{3^{j+1}} = \frac{1}{3^2} + \frac{1}{3^3} + \frac{1}{3^4} + \dots$$

This is an infinite geometric progression with $a = \frac{1}{9}$ and $r = \frac{1}{3}$:

$$\text{Sum} = \frac{1/9}{1-1/3} = \frac{1/9}{2/3} = \frac{1}{6}$$

Step 4: Final Answer:
The final value is $12 \times \frac{1}{6} = 2$.

> Quick Tip
>
> In limit problems involving $[f(k)]$, if the denominator is of a higher power than the sum of fractional parts (which is at most n), you can ignore the brackets and treat it as a continuous sum or integral.

---

24. If P is a point on the circle $x^2 + y^2 = 4$, Q is a point on the straight line $5x + y + 2 = 0$ and $x - y + 1 = 0$ is the perpendicular bisector of PQ, then 13 times the sum of abscissa of all such points P is ________.

Correct Answer: 2

Solution:

Step 1: Understanding the Concept:
If a line L is the perpendicular bisector of segment PQ, then point Q is the reflection of point P across line L.
We find the image of a general point $P(x_1, y_1)$ on the circle and impose the condition that its image Q lies on the given line.

Step 2: Key Formula or Approach:
The image of (x_1, y_1) across line $ax + by + c = 0$ is (α, β) where:

$$\frac{\alpha - x_1}{a} = \frac{\beta - y_1}{b} = \frac{-2(ax_1 + by_1 + c)}{a^2 + b^2}$$

For $L : x - y + 1 = 0$, we have $a = 1, b = -1, c = 1$:

$$\frac{\alpha - x_1}{1} = \frac{\beta - y_1}{-1} = \frac{-2(x_1 - y_1 + 1)}{2} = -(x_1 - y_1 + 1) = y_1 - x_1 - 1$$

$$\alpha = y_1 - 1, \quad \beta = x_1 + 1$$

Step 3: Detailed Explanation:
Point $Q(\alpha, \beta)$ lies on $5x + y + 2 = 0$:

$$5(y_1 - 1) + (x_1 + 1) + 2 = 0 \implies 5y_1 - 5 + x_1 + 3 = 0 \implies x_1 + 5y_1 - 2 = 0$$

So, $x_1 = 2 - 5y_1$.
Since $P(x_1, y_1)$ is on the circle $x^2 + y^2 = 4$:

$$(2 - 5y_1)^2 + y_1^2 = 4 \implies 4 - 20y_1 + 25y_1^2 + y_1^2 = 4$$

$$26y_1^2 - 20y_1 = 0 \implies 2y_1(13y_1 - 10) = 0$$

This gives two possible values for y_1: 0 and $\frac{10}{13}$.
Corresponding x-coordinates (abscissa):
1. For $y_1 = 0$, $x_1 = 2 - 0 = 2$.
2. For $y_1 = \frac{10}{13}$, $x_1 = 2 - 5\left(\frac{10}{13}\right) = 2 - \frac{50}{13} = -\frac{24}{13}$.
Sum of abscissas $= 2 - \frac{24}{13} = \frac{26-24}{13} = \frac{2}{13}$.

Step 4: Final Answer:
Required value $= 13 \times \frac{2}{13} = 2$.

Quick Tip

The reflection of a point (h, k) about the line $y = x + c$ is simply $(k - c, h + c)$. Using such geometric properties for standard lines can avoid complex coordinate geometry formulas.

---

25. If $\int_0^1 4\cot^{-1}(1 - 2x + 4x^2)\,dx = a\tan^{-1}(2) - b\log_e(5)$, where $a, b \in \mathbf{N}$, then $(2a + b)$ is equal to ________.

Correct Answer: 9

Solution:

Step 1: Understanding the Concept:
We simplify the integrand using the identity $\cot^{-1} u = \tan^{-1}\left(\frac{1}{u}\right)$ and the formula $\tan^{-1} A - \tan^{-1} B = \tan^{-1}\left(\frac{A-B}{1+AB}\right)$.

Step 2: Key Formula or Approach:
Integrand: $4\cot^{-1}(1 - 2x + 4x^2) = 4\tan^{-1}\left(\frac{1}{1+4x^2-2x}\right) = 4\tan^{-1}\left(\frac{1}{1+2x(2x-1)}\right)$

Write the numerator as $2x - (2x-1)$:

$$= 4\left[\tan^{-1}(2x) - \tan^{-1}(2x-1)\right]$$

Step 3: Detailed Explanation:
The integral is $I = 4\int_0^1 \tan^{-1}(2x)\,dx - 4\int_0^1 \tan^{-1}(2x-1)\,dx$.
For the second integral, let $f(x) = \tan^{-1}(2x-1)$.
$f(1-x) = \tan^{-1}(2(1-x)-1) = \tan^{-1}(1-2x) = -\tan^{-1}(2x-1) = -f(x)$.
Using $\int_0^a f(x)\,dx = 0$ if $f(x) = -f(a-x)$, the second integral is 0.
So, $I = 4\int_0^1 \tan^{-1}(2x)\,dx$. Let $2x = t \implies dx = \frac{dt}{2}$.

$$I = 4\int_0^2 \tan^{-1} t \cdot \frac{dt}{2} = 2\int_0^2 \tan^{-1} t\,dt$$

Integrating by parts ($u = \tan^{-1} t, dv = dt$):

$$I = 2\left[t\tan^{-1} t - \int \frac{t}{1+t^2}\,dt\right]_0^2 = 2\left[t\tan^{-1} t - \frac{1}{2}\log_e(1+t^2)\right]_0^2$$

$$I = 2\left[2\tan^{-1} 2 - \frac{1}{2}\log_e 5 - 0\right] = 4\tan^{-1} 2 - \log_e 5$$

Comparing with $a\tan^{-1}(2) - b\log_e(5)$, we find $a = 4$ and $b = 1$.

Step 4: Final Answer:
$2a + b = 2(4) + 1 = 9$.

Quick Tip

For integrals of inverse trig functions, check for symmetry properties ($\int_0^a f(x)dx$) first. If the integrand is odd about the midpoint of the interval, the integral is zero, saving significant time.

Physics Section A

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26. Consider two identical metallic spheres of radius R each having charge Q and mass m. Their centers have an initial separation of $4R$. Both the spheres are given an initial speed of u towards each other. The minimum value of u, so that they can just touch each other is :

(Take $k = \frac{1}{4\pi\epsilon_0}$ and assume $kQ^2 > Gm^2$ where G is the Gravitational constant)

(A) $\sqrt{\frac{kQ^2}{4mR}\left(1 - \frac{Gm^2}{kQ^2}\right)}$

(B) $\sqrt{\frac{kQ^2}{2mR}\left(1 - \frac{Gm^2}{kQ^2}\right)}$

(C) $\sqrt{\frac{kQ^2}{2mR}\left(1 - \frac{Gm^2}{2kQ^2}\right)}$

(D) $\sqrt{\frac{kQ^2}{4mR}\left(1 + \frac{Gm^2}{kQ^2}\right)}$

Correct Answer: (A) $\sqrt{\frac{kQ^2}{4mR}\left(1 - \frac{Gm^2}{kQ^2}\right)}$

Solution:

Step 1: Understanding the Concept:
The spheres move towards each other under the influence of electrostatic repulsion and gravitational attraction.
The condition to "just touch" means that the spheres reach a separation of $2R$ (sum of radii) with zero relative velocity.
Since there are no non-conservative forces, the total mechanical energy of the system is conserved.

Step 2: Key Formula or Approach:
By the Law of Conservation of Energy: $U_i + K_i = U_f + K_f$.
Electrostatic Potential Energy: $U_e = \frac{kQ_1Q_2}{r}$.
Gravitational Potential Energy: $U_g = -\frac{Gm_1m_2}{r}$.
Net Potential Energy: $U = \frac{kQ^2 - Gm^2}{r}$.

Step 3: Detailed Explanation:
Initial state: separation $r_i = 4R$, speed of each sphere is u.
Initial Kinetic Energy: $K_i = 2 \times \frac{1}{2}mu^2 = mu^2$.
Initial Potential Energy: $U_i = \frac{kQ^2 - Gm^2}{4R}$.
Final state (just touching): separation $r_f = 2R$, final speed $v_f = 0$.
Final Kinetic Energy: $K_f = 0$.
Final Potential Energy: $U_f = \frac{kQ^2 - Gm^2}{2R}$.
Applying conservation of energy:

$$mu^2 + \frac{kQ^2 - Gm^2}{4R} = \frac{kQ^2 - Gm^2}{2R}$$

$$mu^2 = \frac{kQ^2 - Gm^2}{2R} - \frac{kQ^2 - Gm^2}{4R}$$

$$mu^2 = \frac{kQ^2 - Gm^2}{4R}$$

$$u^2 = \frac{kQ^2 - Gm^2}{4mR}$$

$$u = \sqrt{\frac{kQ^2}{4mR}\left(1 - \frac{Gm^2}{kQ^2}\right)}$$

Step 4: Final Answer:
The minimum value of the initial speed u is $\sqrt{\frac{kQ^2}{4mR}\left(1 - \frac{Gm^2}{kQ^2}\right)}$.

Quick Tip

When both gravity and electrostatics act between identical bodies, the net force behaves like a single inverse-square law with an effective constant $(kQ^2 - Gm^2)$. Energy conservation is the standard way to solve "just touching" or "minimum approach" problems.

---

27. Surface tension of two liquids (having same densities), T_1 and T_2, are measured using capillary rise method utilizing two tubes with inner radii of r_1 and r_2 where $r_1 > r_2$. The measured liquid heights in these tubes are h_1 and h_2 respectively. [Ignore the weight of the liquid about the lowest point of meniscus]. The heights h_1 and h_2 and surface tensions T_1 and T_2 satisfy the relation :

(A) $h_1 > h_2$ and $T_1 < T_2$
(B) $h_1 = h_2$ and $T_1 = T_2$
(C) $h_1 < h_2$ and $T_1 = T_2$
(D) $h_1 > h_2$ and $T_1 = T_2$

Correct Answer: (C) $h_1 < h_2$ and $T_1 = T_2$

Solution:

Step 1: Understanding the Concept:
The capillary rise h is the height to which a liquid rises in a narrow tube due to surface tension. It depends on the surface tension of the liquid, the radius of the tube, and the density of the liquid.

Step 2: Key Formula or Approach:
The capillary rise formula is given by:

$$h = \frac{2T\cos\theta}{r\rho g}$$

where T is surface tension, r is tube radius, ρ is density, and θ is the contact angle.

Step 3: Detailed Explanation:
If we assume the two liquids have the same surface tension ($T_1 = T_2$) and same densities ($\rho_1 = \rho_2$), and assume the contact angle θ is similar (usually taken as zero for pure water in glass):
The height of rise is inversely proportional to the radius of the capillary tube: $h \propto \frac{1}{r}$.
Given that $r_1 > r_2$:

$$\frac{h_1}{h_2} = \frac{r_2}{r_1}$$

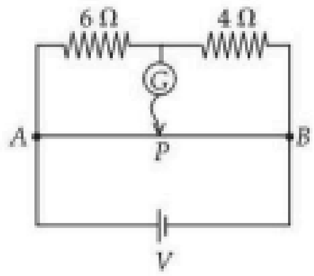
Since $r_1 > r_2$, it must follow that $h_1 < h_2$.
Thus, the heights satisfy $h_1 < h_2$ given that the surface tensions are equal ($T_1 = T_2$).

Step 4: Final Answer:
The relation is $h_1 < h_2$ and $T_1 = T_2$.

Quick Tip
In capillary rise, the product $h \times r$ is constant for a given liquid ($h_1 r_1 = h_2 r_2$). Therefore, a wider tube (larger r) always results in a lower liquid rise (smaller h).

---

28. The total length of potentiometer wire AB is 50 cm in the arrangement as shown in figure. If P is the point where the galvanometer shows zero reading then the length AP is _______ cm.



(A) 30
(B) 25
(C) 15
(D) 20

Correct Answer: (A) 30

Solution:

Step 1: Understanding the Concept:
The circuit represents a balanced bridge condition in a potentiometer setup.
When the galvanometer shows zero reading, the potential at point P on the wire must equal the potential between the resistors.
This is equivalent to the Wheatstone bridge principle.

Step 2: Key Formula or Approach:
For a balanced condition: $\frac{R_1}{R_2} = \frac{L_1}{L_2}$.
Here, $R_1 = 6\Omega$, $R_2 = 4\Omega$.
Let $AP = x$ and $PB = 50 - x$.

Step 3: Detailed Explanation:
The potential drops across the resistors and the wire segments must be proportional to their resistances (and thus lengths for the uniform wire).
At balance point P:

$$\frac{6}{x} = \frac{4}{50 - x}$$

Cross-multiplying the terms:

$$6(50 - x) = 4x$$

$$300 - 6x = 4x$$

$$10x = 300$$

$$x = 30 \text{ cm}$$

Step 4: Final Answer:
The length AP is 30 cm.

> **Quick Tip**
>
> In a balanced bridge setup on a wire, the ratio of the resistors equals the ratio of the corresponding wire segments. Always identify the segments starting from the common potential terminal (usually the positive terminal).

29. Keeping the significant figures in view, the sum of the physical quantities 52.01 **m,** 153.2 **m and** 0.123 **m is :**

(A) 205.33 m
(B) 205.333 m
(C) 205 m
(D) 205.3 m

Correct Answer: (D) 205.3 m

Solution:

Step 1: Understanding the Concept:
When adding or subtracting physical quantities, the final result should be rounded off to the same number of decimal places as the quantity with the least number of decimal places.

Step 2: Key Formula or Approach:
Identify the decimal places in each term:
52.01 m $\rightarrow$ 2 decimal places.
153.2 m $\rightarrow$ 1 decimal place.
0.123 m $\rightarrow$ 3 decimal places.

Step 3: Detailed Explanation:
Calculate the arithmetic sum:

$$52.01 + 153.2 + 0.123 = 205.333 \text{ m}$$

Now, apply the rule for significant figures in addition:
The quantity with the least number of decimal places is 153.2 m (1 decimal place).
Therefore, the final result must be rounded to 1 decimal place.
Rounding 205.333 to one decimal place gives 205.3.

Step 4: Final Answer:
The sum is 205.3 m.

Quick Tip

For addition/subtraction, look at decimal places. For multiplication/division, look at total significant figures. In competitive exams, always perform the full sum first and round off at the very last step.

---

30. Two cars A **and** B **each of mass** 10^3 **kg are moving on parallel tracks separated by a distance of** 10 **m, in same direction with speeds** 72 **km/h and** 36 **km/h. The**

magnitude of angular momentum of car A with respect to car B is ________ $\mathbf{J \cdot s}$.

(A) 3×10^5
(B) 10^5
(C) 3.6×10^5
(D) 2×10^5

Correct Answer: (B) 10^5

Solution:

Step 1: Understanding the Concept:
Angular momentum of a particle A with respect to B is defined as $\vec{L}_{A/B} = \vec{r}_{A/B} \times \vec{p}_{A/B}$.
For parallel motion, the magnitude is given by $L = m \times v_{\text{rel}} \times d_\perp$, where $d_\perp$ is the perpendicular distance between the tracks.

Step 2: Key Formula or Approach:
Relative velocity $v_{A/B} = v_A - v_B$.
Angular momentum $L = m \times |v_{A/B}| \times d$.

Step 3: Detailed Explanation:
Convert speeds to m/s:
$v_A = 72$ km/h $= 72 \times \frac{5}{18} = 20$ m/s.
$v_B = 36$ km/h $= 36 \times \frac{5}{18} = 10$ m/s.
Relative speed: $v_{\text{rel}} = |20 - 10| = 10$ m/s.
Mass $m = 10^3$ kg.
Perpendicular distance $d = 10$ m.
Magnitude of angular momentum:

$$L = m \times v_{\text{rel}} \times d$$

$$L = 10^3 \times 10 \times 10$$

$$L = 10^5 \text{ kg} \cdot \text{m}^2/\text{s}$$

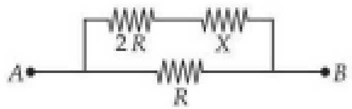
Step 4: Final Answer:
The magnitude of angular momentum is 10^5 J $\cdot$ s.

Quick Tip

For straight-line relative motion, angular momentum is simply the product of relative linear momentum and the perpendicular separation. It is independent of the relative longitudinal position of the cars.

---

31. Two known resistances of R Ω and $2R$ Ω and one unknown resistance X Ω are connected in a circuit as shown in the figure. If the equivalent resistance between points A and B in the circuit is X Ω, then the value of X is ________ Ω.



(A) R
(B) $(\sqrt{3}-1)R$
(C) $2(\sqrt{3}-1)R$
(D) $(\sqrt{3}+1)R$

Correct Answer: (B) $(\sqrt{3}-1)R$

Solution:

Step 1: Understanding the Concept:
The diagram shows a combination of resistors between points A and B.
We can simplify the network by identifying series and parallel combinations.

Step 2: Key Formula or Approach:
From the diagram, the branch containing $2R$ and X are in series, and this combination is in parallel with R.
The total equivalent resistance $R_{eq} = X$.

Step 3: Detailed Explanation:
Resistance of the top branch $= 2R + X$.
Resistance of the bottom branch $= R$.
Equivalent resistance X is given by the parallel formula:

$$\frac{1}{X} = \frac{1}{2R+X} + \frac{1}{R}$$

$$\frac{1}{X} = \frac{R+2R+X}{R(2R+X)}$$

$$R(2R+X) = X(3R+X)$$

$$2R^2 + RX = 3RX + X^2$$

$$X^2 + 2RX - 2R^2 = 0$$

Solve for X using the quadratic formula:

$$X = \frac{-2R \pm \sqrt{(2R)^2 - 4(1)(-2R^2)}}{2}$$

$$X = \frac{-2R \pm \sqrt{4R^2 + 8R^2}}{2}$$

$$X = \frac{-2R \pm \sqrt{12R^2}}{2}$$

$$X = \frac{-2R + 2R\sqrt{3}}{2} = (\sqrt{3} - 1)R$$

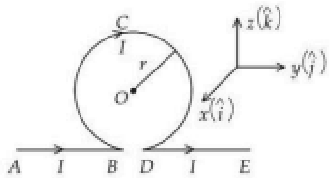
(We discard the negative root as resistance cannot be negative).

Step 4: Final Answer:
The value of X is $(\sqrt{3} - 1)R$.

> 💡 Quick Tip
>
> When the equivalent resistance is given as one of the unknown parameters in a network, it usually leads to a quadratic equation. Always select the positive root for resistance values.

---

32. An infinitely long straight wire carrying current I is bent in a planar shape as shown in the diagram. The radius of the circular part is r. The magnetic field at the centre O of the circular loop is :



(A) $\frac{\mu_0 I}{2\pi r}(\pi - 1)\hat{i}$
(B) $\frac{\mu_0 I}{2\pi r}(\pi + 1)\hat{i}$
(C) $-\frac{\mu_0 I}{2\pi r}(\pi - 1)\hat{i}$
(D) $-\frac{\mu_0 I}{2\pi r}(\pi + 1)\hat{i}$

Correct Answer: (C) $-\frac{\mu_0 I}{2\pi r}(\pi - 1)\hat{i}$

Solution:

Step 1: Understanding the Concept:
The magnetic field at the center O is the vector sum of fields produced by the circular loop and the long straight wire.
Using Right Hand Thumb Rule, we determine the directions.

Step 2: Key Formula or Approach:
Field of a circular loop at center: $B_{\text{loop}} = \frac{\mu_0 I}{2r}$.
Field of an infinite straight wire: $B_{\text{wire}} = \frac{\mu_0 I}{2\pi r}$.

Step 3: Detailed Explanation:
Based on the coordinate axes provided (z up, y right, x out):
Assume the loop is in the yz plane and the wire is parallel to z?
Looking at the diagram and options (which use $\hat{i}$):
Let's assume the current in the loop produces a field into the page ($-\hat{i}$) and the straight part produces a field out of the page ($+\hat{i}$).
Magnitude of loop field: $B_1 = \frac{\mu_0 I}{2r}$.
Magnitude of wire field: $B_2 = \frac{\mu_0 I}{2\pi r}$.
Net field $\vec{B} = \vec{B}_1 + \vec{B}_2 = \left(\frac{\mu_0 I}{2\pi r} - \frac{\mu_0 I}{2r}\right)\hat{i}$.
Taking common factor $\frac{\mu_0 I}{2\pi r}$:

$$\vec{B} = \frac{\mu_0 I}{2\pi r}(1 - \pi)\hat{i} = -\frac{\mu_0 I}{2\pi r}(\pi - 1)\hat{i}$$

Step 4: Final Answer:
The magnetic field is $-\frac{\mu_0 I}{2\pi r}(\pi - 1)\hat{i}$.

Quick Tip
For superimposed magnetic fields, always treat each component (straight wire, circular arc) separately. Use the Right-Hand Rule carefully: thumb along current and fingers curl towards the point of interest.

---

33. The energy of an electron in an orbit of the Bohr's atom is $-0.04E_0$ eV where E_0 is the ground state energy. If L is the angular momentum of the electron in this

orbit and h is the Planck's constant, then $\frac{2\pi L}{h}$ is ________ :

(A) 4
(B) 6
(C) 5
(D) 2

Correct Answer: (C) 5

Solution:

Step 1: Understanding the Concept:
In Bohr's model, the energy of an electron in the n^{th} orbit is $E_n = \frac{E_1}{n^2}$, where E_1 is the ground state energy (E_0).
The angular momentum L is quantized as $L = \frac{nh}{2\pi}$.

Step 2: Key Formula or Approach:
Energy relation: $E_n = \frac{E_0}{n^2}$.
Angular momentum: $L = \frac{nh}{2\pi} \implies \frac{2\pi L}{h} = n$.

Step 3: Detailed Explanation:
Given: $E_n = -0.04E_0$.
Comparing with the standard formula $E_n = \frac{E_0}{n^2}$:

$$\frac{E_0}{n^2} = 0.04E_0$$

$$\frac{1}{n^2} = 0.04 = \frac{4}{100}$$

$$n^2 = \frac{100}{4} = 25$$

$$n = 5$$

Now, substituting n into the angular momentum relation:

$$\frac{2\pi L}{h} = n = 5$$

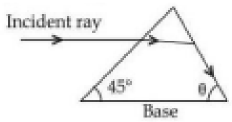
Step 4: Final Answer:
The value of $\frac{2\pi L}{h}$ is 5.

Quick Tip

In Bohr's model, the orbital number n can be directly found from the energy ratio: $n = \sqrt{E_{\text{ground}}/E_{\text{orbit}}}$. Once n is known, the quantized angular momentum follows immediately as n units of $h/2\pi$.

---

34. As shown in the diagram, when the incident ray is parallel to base of the prism, the emergent ray grazes along the second surface. If refractive index of the material of prism is $\sqrt{2}$, the angle θ of prism is :



(A) 90°
(B) 45°
(C) 75°
(D) 60°

Correct Answer: (C) 75°

Solution:

Step 1: Understanding the Concept:
The ray enters from air into the prism and then undergoes refraction such that the emergent ray grazes the surface ($i_e = 90°$).
This implies the ray strikes the second face at the critical angle.

Step 2: Key Formula or Approach:
Snell's Law: $\mu_1 \sin i = \mu_2 \sin r$.
Critical angle: $\sin \theta_c = \frac{1}{\mu}$.
Prism angle formula: $A = r_1 + r_2$.

Step 3: Detailed Explanation:
Given refractive index $\mu = \sqrt{2}$.
For the second surface, grazing emergence means $r_2 = \theta_c$:

$$\sin r_2 = \frac{1}{\mu} = \frac{1}{\sqrt{2}} \implies r_2 = 45°$$

The incident ray is horizontal and the prism base angle is 45°.
The angle between the horizontal ray and the first surface (inclined at 45°) is 45°.
So, the angle of incidence $i_1 = 45°$.
Using Snell's Law at the first surface:

$$1 \cdot \sin(45°) = \sqrt{2} \cdot \sin r_1$$

$$\frac{1}{\sqrt{2}} = \sqrt{2}\sin r_1 \implies \sin r_1 = \frac{1}{2} \implies r_1 = 30^\circ$$

The prism angle θ is:

$$\theta = r_1 + r_2 = 30^\circ + 45^\circ = 75^\circ$$

Step 4: Final Answer:
The angle of the prism θ is 75°.

> Quick Tip
>
> "Grazing along the surface" is a keyword for critical angle. Always remember that for symmetric prism orientations or horizontal incident rays, the geometry of the base angles directly relates to the angle of incidence at the first face.

---

35. Given below are two statements :
Statement I : In a Young's double slit experiment, the angular separation of fringes will increase as the screen is moved away from the plane of the slits.
Statement II : In a Young's double slit experiment, the angular separation of fringes will increase when monochromatic source is replaced by another monochromatic source of higher wavelength.
In the light of the above statements, choose the correct answer from the options given below :

(A) Both Statement I and Statement II are true
(B) Both Statement I and Statement II are false
(C) Statement I is true but Statement II is false
(D) Statement I is false but Statement II is true

Correct Answer: (D) Statement I is false but Statement II is true

Solution:

Step 1: Understanding the Concept:
In YDSE, the fringe width W is given by $W = \frac{\lambda D}{d}$, where D is the screen distance and d is the slit separation.
The angular fringe width θ is defined as the angle subtended by a fringe at the slits.

Step 2: Key Formula or Approach:
Angular separation $\theta = \frac{W}{D} = \frac{\lambda}{d}$.

Step 3: Detailed Explanation:
Analysis of Statement I:
The angular separation $\theta = \frac{\lambda}{d}$.
It depends only on wavelength λ and slit separation d.
It is independent of the distance D of the screen from the slits.
Therefore, moving the screen away does not change the angular separation.
Statement I is False.

Analysis of Statement II:
The angular separation $\theta = \frac{\lambda}{d}$ is directly proportional to the wavelength λ.
If the monochromatic source is replaced by one with a higher wavelength, θ will increase.
Statement II is True.

Step 4: Final Answer:
Statement I is false but Statement II is true.

Quick Tip

Always distinguish between linear fringe width (W, depends on screen distance D) and angular fringe width (θ, independent of D). If the problem mentions "angular separation", don't involve the screen distance in your calculations.

---

36. The kinetic energy of a simple harmonic oscillator is oscillating with angular frequency of 176 rad/s. The frequency of this simple harmonic oscillator is ________ Hz. $\left[\textbf{take } \pi = \frac{22}{7}\right]$

(A) 28
(B) 176
(C) 14
(D) 88

Correct Answer: (C) 14

Solution:

Step 1: Understanding the Concept:
In Simple Harmonic Motion (SHM), while the displacement and velocity oscillate with a frequency f, the energy (both kinetic and potential) oscillates with a frequency $2f$.
This is because kinetic energy peaks twice per full oscillation cycle (at both the positive and negative velocity peaks).

Step 2: Key Formula or Approach:
Let ω_{osc} be the angular frequency of the oscillator.
The kinetic energy $K = \frac{1}{2}mv^2 \propto \sin^2(\omega_{\text{osc}}t) \propto (1 - \cos(2\omega_{\text{osc}}t))$.

So, $\omega_{\text{energy}} = 2\omega_{\text{osc}}$.

Step 3: Detailed Explanation:
Given angular frequency of kinetic energy $\omega_{\text{energy}} = 176$ rad/s.

$$2\omega_{\text{osc}} = 176$$

$$\omega_{\text{osc}} = 88 \text{ rad/s}$$

Frequency f of the oscillator:

$$f = \frac{\omega_{\text{osc}}}{2\pi}$$

$$f = \frac{88}{2 \times \frac{22}{7}}$$

$$f = \frac{88 \times 7}{44} = 2 \times 7 = 14 \text{ Hz}$$

Step 4: Final Answer:
The frequency of the simple harmonic oscillator is 14 Hz.

Quick Tip

Energy in SHM is quadratic in terms of displacement/velocity. Mathematically, $\sin^2(\theta)$ oscillates at twice the frequency of $\sin(\theta)$. In exams, just divide the energy frequency by 2 to get the oscillator frequency.

---

37. A body of mass 2 kg is moving along x-direction such that its displacement as function of time is given by $x(t) = at^2 + \beta t + \gamma m$, where $a = 1$ m/s^2, $\beta = 1$ m/s and $\gamma = 1$ m. The work done on the body during the time interval $t = 2$ s to $t = 3$ s, is ________ J.

(A) 42
(B) 24
(C) 12
(D) 49

Correct Answer: (B) 24

Solution:

Step 1: Understanding the Concept:
Work done on a body is equal to the change in its kinetic energy (Work-Energy Theorem). We need to find the velocities at the initial and final times.

Step 2: Key Formula or Approach:
Velocity $v(t) = \frac{dx}{dt}$.
Work Done $W = \Delta KE = \frac{1}{2}m(v_f^2 - v_i^2)$.

Step 3: Detailed Explanation:
Displacement $x(t) = 1t^2 + 1t + 1$.
Differentiating with respect to time to get velocity:

$$v(t) = \frac{d}{dt}(t^2 + t + 1) = 2t + 1$$

Velocity at $t_i = 2$ s:

$$v_i = 2(2) + 1 = 5 \text{ m/s}$$

Velocity at $t_f = 3$ s:

$$v_f = 2(3) + 1 = 7 \text{ m/s}$$

Mass $m = 2$ kg.
Work Done W:

$$W = \frac{1}{2} \times 2 \times (7^2 - 5^2)$$

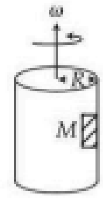
$$W = 49 - 25 = 24 \text{ J}$$

Step 4: Final Answer:
The work done on the body is 24 J.

Quick Tip

Using Work-Energy Theorem is often faster than integrating force over displacement, especially when the mass is constant and displacement is given as a simple polynomial of time. Just find initial and final speeds!

38. A large drum having radius R is spinning around its axis with angular velocity ω, as shown in figure. The minimum value of ω so that a body of mass M remains stuck to the inner wall of the drum, taking the coefficient of friction between the drum surface and mass M as μ, is :



(A) $\sqrt{\frac{\mu g}{R}}$
(B) $\sqrt{\frac{g}{\mu R}}$
(C) $\sqrt{\frac{2g}{\mu R}}$
(D) $\sqrt{\frac{g}{2\mu R}}$

Correct Answer: (B) $\sqrt{\frac{g}{\mu R}}$

Solution:

Step 1: Understanding the Concept:
For the block to remain stuck, the downward gravitational force must be balanced by the upward static frictional force.
The normal force required for friction is provided by the centripetal force from the drum's rotation.

Step 2: Key Formula or Approach:
Normal Force $N = M\omega^2 R$.
Limiting friction $f_s = \mu N$.
Equilibrium condition: $f_s \geq Mg$.

Step 3: Detailed Explanation:
The forces acting on the block in the rotating frame:
1. Weight Mg downwards.
2. Frictional force f upwards.
3. Normal reaction $N = M\omega^2 R$ towards the center (centripetal force).
For the block not to slide down:

$$f = Mg$$

Since maximum possible friction is μN:

$$\mu N \geq Mg$$

$$\mu(M\omega^2 R) \geq Mg$$

$$\omega^2 \geq \frac{g}{\mu R}$$

The minimum angular velocity $\omega_{\min}$ is:

$$\omega_{\min} = \sqrt{\frac{g}{\mu R}}$$

Step 4: Final Answer:
The minimum value of ω is $\sqrt{\frac{g}{\mu R}}$.

> Quick Tip
>
> In vertical circular setups (like a rotor), remember that friction supports the weight while the normal force acts as the centripetal force. The critical condition for "not sliding" is always $\mu \times$ Centripetal Force $\geq$ Weight.

---

39. A capacitor C is first charged fully with potential difference of V_0 and disconnected from the battery. The charged capacitor is connected across an inductor having inductance L. In t s 25% of the initial energy in the capacitor is transferred to the inductor. The value of t is _______ s.

(A) $\frac{\pi\sqrt{LC}}{2}$
(B) $\frac{\pi\sqrt{LC}}{6}$
(C) $\frac{\pi\sqrt{LC}}{3}$
(D) $\frac{\pi\sqrt{LC}}{2}$

Correct Answer: (B) $\frac{\pi\sqrt{LC}}{6}$

Solution:

Step 1: Understanding the Concept:
In an LC circuit, energy oscillates between the capacitor and the inductor.
The total energy is conserved: $U_{\text{total}} = U_C + U_L$.

Step 2: Key Formula or Approach:
Initial energy $U_0 = \frac{1}{2}CV_0^2$.
Charge as a function of time: $q(t) = Q_0 \cos(\omega t)$, where $\omega = \frac{1}{\sqrt{LC}}$.

Energy in capacitor: $U_C = \frac{q^2}{2C} = U_0 \cos^2(\omega t)$.

Step 3: Detailed Explanation:
If 25% of initial energy is transferred to the inductor, then 75% of the energy remains in the capacitor.

$$U_C = 0.75U_0 = \frac{3}{4}U_0$$

Substituting the formula for U_C:

$$U_0 \cos^2(\omega t) = \frac{3}{4}U_0$$

$$\cos^2(\omega t) = \frac{3}{4}$$

$$\cos(\omega t) = \frac{\sqrt{3}}{2}$$

The first time this happens is at:

$$\omega t = \frac{\pi}{6}$$

Since $\omega = \frac{1}{\sqrt{LC}}$:

$$\frac{t}{\sqrt{LC}} = \frac{\pi}{6} \implies t = \frac{\pi\sqrt{LC}}{6}$$

Step 4: Final Answer:
The value of t is $\frac{\pi\sqrt{LC}}{6}$.

Quick Tip
Energy distribution in an LC circuit follows trigonometric squares. 25

---

40. A spherical body of radius r and density σ falls freely through a viscous liquid having density ρ and viscosity η and attains a terminal velocity v_0. Estimated maximum error in the quantity η is : (Ignore errors associated with σ, ρ and g, gravitational acceleration)

(1) $2\left[\frac{\Delta r}{r} - \frac{\Delta v_0}{v_0}\right]$
(2) $2\left[\frac{\Delta r}{r} + \frac{\Delta v_0}{v_0}\right]$

(3) $\frac{2\Delta r}{r} + \frac{\Delta v_0}{v_0}$
(4) $2\frac{\Delta r}{r} + \frac{\Delta v_0}{v_0}$

Correct Answer: (4) $2\frac{\Delta r}{r} + \frac{\Delta v_0}{v_0}$

Solution:

Step 1: Understanding the Concept:
Terminal velocity occurs when viscous drag equals net weight.
The formula for viscosity is derived from Stokes' Law and buoyancy.
Error propagation rules are applied to the resulting functional relationship.

Step 2: Key Formula or Approach:
Terminal Velocity $v_0 = \frac{2}{9}\frac{r^2(\sigma-\rho)g}{\eta}$.
Rearranging for viscosity: $\eta = \frac{2}{9}\frac{r^2(\sigma-\rho)g}{v_0}$.

Step 3: Detailed Explanation:
From the relation $\eta = K\frac{r^2}{v_0}$ (where K includes constants without error):
Taking natural logarithm on both sides:

$$\ln \eta = \ln K + 2\ln r - \ln v_0$$

Differentiating to find relative errors:

$$\frac{d\eta}{\eta} = 2\frac{dr}{r} - \frac{dv_0}{v_0}$$

The maximum estimated error (absolute fractional error) is the sum of the magnitudes of individual errors:

$$\left(\frac{\Delta\eta}{\eta}\right)_{\max} = 2\frac{\Delta r}{r} + \frac{\Delta v_0}{v_0}$$

Step 4: Final Answer:
The estimated maximum error in η is $2\frac{\Delta r}{r} + \frac{\Delta v_0}{v_0}$.

Quick Tip

For any formula $X = A^a B^b$, the maximum relative error is $|a|\frac{\Delta A}{A} + |b|\frac{\Delta B}{B}$. In terminal velocity, since $\eta \propto r^2$, the radius error contribution is doubled.

---

41. A river of width 200 m is flowing from west to east with a speed of 18 km/h. A boat, moving with speed of 36 km/h in still water, is made to travel one-round

trip (bank to bank of the river). Minimum time taken by the boat for this journey and also the displacement along the river bank are _______ and _______ respectively.

(A) 20 s and 100 m
(B) 40 s and 100 m
(C) 40 s and 0 m
(D) 40 s and 200 m

Correct Answer: (D) 40 s and 200 m

Solution:

Step 1: Understanding the Concept:
The boat crosses a river in minimum time when it steers perpendicular to the river flow (directly towards the opposite bank). During the crossing, the river flow causes a "drift" along the direction of the river bank. A "one-round trip" involves crossing the river and returning to the original bank.

Step 2: Key Formula or Approach:
1. Convert units to SI: 1 km/h $= \frac{5}{18}$ m/s.
2. Minimum time to cross: $t = \frac{w}{v_b}$, where w is width and v_b is boat speed in still water.
3. Drift along the bank: $x = v_r \times t$, where v_r is river speed.

Step 3: Detailed Explanation:
River width $w = 200$ m.
River speed $v_r = 18 \times \frac{5}{18} = 5$ m/s.
Boat speed in still water $v_b = 36 \times \frac{5}{18} = 10$ m/s.

For minimum time, the boat must steer directly across the river in both directions.
Time taken to cross $(t_1) = \frac{w}{v_b} = \frac{200}{10} = 20$ s.
Time taken to return $(t_2) = \frac{w}{v_b} = \frac{200}{10} = 20$ s.
Total time for the one-round trip $= t_1 + t_2 = 20 + 20 = 40$ s.

During both the crossing and the return trip, the boat is being drifted downstream by the river flow at 5 m/s.
Drift during crossing $(x_1) = v_r \times t_1 = 5 \times 20 = 100$ m (downstream).
Drift during return $(x_2) = v_r \times t_2 = 5 \times 20 = 100$ m (further downstream).
Total displacement along the river bank $= x_1 + x_2 = 100 + 100 = 200$ m.

Step 4: Final Answer:
The minimum time for the journey is 40 s and the displacement along the river bank is 200 m.

Quick Tip

For minimum time to cross, steering direction is always perpendicular to the bank. Note that a round trip bank-to-bank does not mean returning to the same starting point; the river drift acts in the same direction during both legs of the trip.

---

42. The r.m.s. speed of oxygen molecules at $47°$C is equal to that of the hydrogen molecules kept at ________ °C. (Mass of oxygen molecule/mass of hydrogen molecule = 32/2)

(A) -20
(B) -253
(C) -235
(D) -100

Correct Answer: (B) -253

Solution:

Step 1: Understanding the Concept:
The root mean square (r.m.s.) speed of gas molecules depends on the absolute temperature (T) and the molar mass (M) of the gas.

Step 2: Key Formula or Approach:
The formula for r.m.s. speed is:

$$v_{rms} = \sqrt{\frac{3RT}{M}}$$

where R is the universal gas constant.

Step 3: Detailed Explanation:
Given that $v_{rms}(\mathrm{O_2}) = v_{rms}(\mathrm{H_2})$, we have:

$$\sqrt{\frac{3RT_{\mathrm{O_2}}}{M_{\mathrm{O_2}}}} = \sqrt{\frac{3RT_{\mathrm{H_2}}}{M_{\mathrm{H_2}}}}$$

Squaring both sides and simplifying:

$$\frac{T_{\mathrm{O_2}}}{M_{\mathrm{O_2}}} = \frac{T_{\mathrm{H_2}}}{M_{\mathrm{H_2}}}$$

Temperature of Oxygen in Kelvin, $T_{\mathrm{O_2}} = 47 + 273 = 320$ K.
Mass ratio is $M_{\mathrm{O_2}} : M_{\mathrm{H_2}} = 32 : 2$.

$$\frac{320}{32} = \frac{T_{\mathrm{H_2}}}{2}$$

$$10 = \frac{T_{\mathrm{H_2}}}{2} \implies T_{\mathrm{H_2}} = 20 \text{ K}$$

Converting temperature to Celsius:

$$t_{H_2} = 20 - 273 = -253°C$$

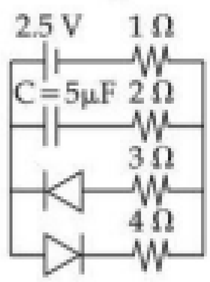
Step 4: Final Answer:
The temperature is $-253°C$.

Quick Tip

Always perform calculations for gas speeds using absolute temperature in Kelvin ($T = t + 273$). The ratio T/M must be constant for two gases to have the same r.m.s. speed.

---

43. The charge stored by the capacitor C in the given circuit in the steady state is ________ μC.



(A) 10
(B) 7.5
(C) 5
(D) 12.5

Correct Answer: (D) 12.5

Solution:

Step 1: Understanding the Concept:
In steady state of a DC circuit, a capacitor acts as an open circuit (infinite resistance), meaning no current flows through its branch. Also, an ideal diode conducts only when forward biased.

Step 2: Key Formula or Approach:
1. In steady state, current in the capacitor branch is zero.
2. Potential difference across the capacitor branch is determined by parallel combinations.
3. Charge stored $Q = C \times V$.

Step 3: Detailed Explanation:
The circuit consists of four parallel branches connected across a 2.5 V source:

1. A 1 Ω resistor.
2. A branch with $C = 5\ \mu$F and a 2 Ω resistor.
3. A branch with a forward-biased diode and a 3 Ω resistor.
4. A branch with a reverse-biased diode and a 4 Ω resistor.

In steady state:
- No current flows through the second branch because of the capacitor. Thus, the potential drop across the 2 Ω resistor is zero.
- The whole branch (Capacitor + 2 Ω resistor) is in parallel with the 2.5 V battery.
- Therefore, the potential difference across the capacitor is $V_c = 2.5\text{ V} - 0\text{ V} = 2.5\text{ V}$.

Calculating charge:

$$Q = C \times V_c = 5\ \mu\text{F} \times 2.5\text{ V} = 12.5\ \mu\text{C}$$

Step 4: Final Answer:
The charge stored is 12.5 μC.

> Quick Tip
>
> In steady state, just remove the capacitor branch to find potentials at the terminals, then re-calculate the potential difference for $Q = CV$. Remember that an ideal diode in reverse bias behaves like an open switch.

---

44. A battery with EMF E and internal resistance r is connected across a resistance R. The power consumption in R will be maximum when :

(A) $R = r$
(B) $R = r/2$
(C) $R = \sqrt{2}r$
(D) $R = 2r$

Correct Answer: (A) $R = r$

Solution:

Step 1: Understanding the Concept:
This is a standard application of the Maximum Power Transfer Theorem, which states that maximum power is dissipated in a load resistance when its value equals the internal resistance of the power source.

Step 3: Detailed Explanation:
The current in the circuit is given by Ohm's law:

$$I = \frac{E}{R + r}$$

The power dissipated in the external resistor R is:

$$P = I^2 R = \left(\frac{E}{R + r}\right)^2 R = \frac{E^2 R}{(R + r)^2}$$

To find the condition for maximum power, we differentiate P with respect to R and set it to zero ($\frac{dP}{dR} = 0$):

$$\frac{dP}{dR} = E^2 \left[\frac{(R + r)^2(1) - R \cdot 2(R + r)}{(R + r)^4}\right] = 0$$

$$(R + r)^2 - 2R(R + r) = 0$$

$$(R + r)[(R + r) - 2R] = 0$$

Since $R + r \neq 0$, we have:

$$R + r - 2R = 0 \implies R = r$$

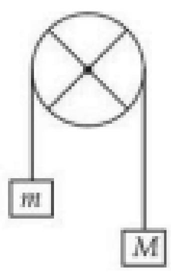
Step 4: Final Answer:
Power consumption is maximum when $R = r$.

Quick Tip

The Maximum Power Transfer Theorem is a recurring concept. At the condition $R = r$, the efficiency of the power transfer is 50% because half the energy is lost in the internal resistance.

---

45. The pulley shown in figure is made using a thin rim and two rods of length equal to diameter of the rim. The rim and each rod have a mass of M. Two blocks of mass of M and m are attached to two ends of a light string passing over the pulley, which is hinged to rotate freely in vertical plane about its center. The magnitudes of the acceleration experienced by the blocks is ________ (assume no slipping of string on pulley).



(A) $\frac{(M-m)g}{\left[\frac{13}{6}\right]M+m}$
(B) $\frac{(M-m)g}{\left[\frac{8}{3}\right]M+m}$
(C) $\frac{(M-m)g}{2M+m}$
(D) $\frac{(M-m)g}{M+m}$

Correct Answer: (B) $\frac{(M-m)g}{\left[\frac{8}{3}\right]M+m}$

Solution:

Step 1: Understanding the Concept:
This problem involves rotational and translational dynamics. The driving force is the difference in weights, and the total inertia includes both the masses and the moment of inertia of the non-ideal pulley.

Step 2: Key Formula or Approach:
1. Acceleration $a = \frac{\text{Net Driving Force}}{\text{Total Equivalent Mass}}$ where Equivalent Mass of pulley $= \frac{I}{R^2}$.
2. Moment of inertia of a rim: $I_{rim} = MR^2$.
3. Moment of inertia of a rod about its center: $I_{rod} = \frac{1}{12}ML^2$.

Step 3: Detailed Explanation:
First, calculate the total moment of inertia (I) of the pulley. It consists of a rim of mass M and radius R, and two rods of length $2R$ and mass M each.

$$I_{rim} = MR^2$$

For one rod of length $L = 2R$:

$$I_{rod} = \frac{1}{12}M(2R)^2 = \frac{4}{12}MR^2 = \frac{1}{3}MR^2$$

Total I for the pulley (rim + 2 rods):

$$I = MR^2 + 2 \times \left(\frac{1}{3}MR^2\right) = MR^2 + \frac{2}{3}MR^2 = \frac{5}{3}MR^2$$

Now, using the equivalent mass approach:

$$a = \frac{(M-m)g}{M + m + \frac{I}{R^2}}$$

Substitute the value of I:

$$a = \frac{(M-m)g}{M + m + \frac{5/3MR^2}{R^2}} = \frac{(M-m)g}{M + m + \frac{5}{3}M}$$

Combine terms involving M:

$$a = \frac{(M-m)g}{\left(1 + \frac{5}{3}\right)M + m} = \frac{(M-m)g}{\left(\frac{8}{3}\right)M + m}$$

Step 4: Final Answer:
The magnitude of acceleration is $\frac{(M-m)g}{\left[\frac{8}{3}\right]M+m}$.

💡 Quick Tip

For a compound object, the total moment of inertia is the sum of moments of its individual parts. Always check the axis of rotation—here the hinged center is the CM for the rim and both rods.

Physics Section B

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46. An electromagnetic wave of frequency 100 MHz propagates through a medium of conductivity $\sigma = 10$ mho/m. The ratio of maximum conduction current density to maximum displacement current density is ________. [Take $\frac{1}{4\pi\epsilon_0} = 9 \times 10^9$ Nm^2/C^2]

Correct Answer: 1800

Solution:

Step 1: Understanding the Concept:
In a conducting medium, an electromagnetic wave generates both a conduction current and a displacement current.
The conduction current density is given by $J_c = \sigma E$, and the displacement current density is given by $J_d = \epsilon_0 \frac{\partial E}{\partial t}$ (assuming the medium has permittivity of free space).
For a harmonic electric field $E = E_0 \sin(\omega t)$, the maximum densities are compared.

Step 2: Key Formula or Approach:
The ratio of the maximum values is given by:

$$\text{Ratio} = \frac{J_{c(\max)}}{J_{d(\max)}} = \frac{\sigma E_0}{\epsilon_0 \omega E_0} = \frac{\sigma}{\epsilon_0 \omega}$$

where $\omega = 2\pi f$.

Step 3: Detailed Explanation:
Given:
$f = 100$ MHz $= 10^8$ Hz $\implies \omega = 2\pi \times 10^8$ rad/s.
$\sigma = 10$ mho/m.
From $\frac{1}{4\pi\epsilon_0} = 9 \times 10^9$, we have $\epsilon_0 = \frac{1}{36\pi \times 10^9}$ C^2/(Nm^2).

Substituting the values:

$$\text{Ratio} = \frac{10}{\left(\frac{1}{36\pi \times 10^9}\right) \times (2\pi \times 10^8)}$$

$$\text{Ratio} = \frac{10 \times 36\pi \times 10^9}{2\pi \times 10^8}$$

$$\text{Ratio} = 10 \times 18 \times \frac{10^9}{10^8}$$

$$\text{Ratio} = 180 \times 10 = 1800$$

Step 4: Final Answer:
The ratio of maximum conduction current density to maximum displacement current density is 1800.

> Quick Tip
>
> The ratio $\frac{\sigma}{\epsilon\omega}$ is a dimensionless quantity that characterizes the medium. For high ratios, the medium behaves like a conductor, while for low ratios, it behaves like a dielectric.

---

47. A diatomic gas ($\gamma = 1.4$) does 100 J of work when it is expanded isobarically. Then the heat given to the gas ________ J.

Correct Answer: 350

Solution:

Step 1: Understanding the Concept:
In an isobaric process (constant pressure), the heat supplied (Q) is used to change the internal energy (ΔU) and perform work (W).
The heat supplied is $Q = nC_p\Delta T$ and the work done is $W = P\Delta V = nR\Delta T$.

Step 2: Key Formula or Approach:
The relationship between heat and work in an isobaric process is:

$$\frac{Q}{W} = \frac{nC_p\Delta T}{nR\Delta T} = \frac{C_p}{R}$$

Using the relation $C_p = \frac{\gamma R}{\gamma - 1}$, the ratio becomes:

$$\frac{Q}{W} = \frac{\gamma}{\gamma - 1}$$

Step 3: Detailed Explanation:
Given:
$W = 100$ J.
$\gamma = 1.4$.
Calculating the heat supplied:

$$Q = W\left(\frac{\gamma}{\gamma - 1}\right)$$

$$Q = 100 \times \left(\frac{1.4}{1.4 - 1}\right)$$

$$Q = 100 \times \frac{1.4}{0.4}$$

$$Q = 100 \times \frac{14}{4}$$

$$Q = 100 \times 3.5 = 350 \text{ J}$$

Step 4: Final Answer:
The heat given to the gas is 350 J.

Quick Tip

For a diatomic gas in an isobaric process, the energy distribution is always fixed: 7 units of heat are supplied, 5 units increase internal energy, and 2 units perform work. Since work is 100 J (2 units), one unit is 50 J, and total heat is $7 \times 50 = 350$ J.

---

48. In a Young's double slit experiment set up, the two slits are kept 0.4 mm apart and screen is placed at 1 m from slits. If a thin transparent sheet of thickness 20 μm is introduced in front of one of the slits then center bright fringe shifts by 20 mm on the screen. The refractive index of transparent sheet is given by $\frac{\alpha}{10}$, where

α **is** ________.

Correct Answer: 14

Solution:

Step 1: Understanding the Concept:
In Young's Double Slit Experiment, placing a transparent sheet of thickness t and refractive index μ in the path of one of the slits introduces an additional optical path difference of $(\mu-1)t$. This causes the entire fringe pattern, including the central bright fringe, to shift by a distance Δy on the screen.

Step 2: Key Formula or Approach:
The formula for the lateral shift of the fringe pattern is given by:

$$\Delta y = \frac{D}{d}(\mu - 1)t$$

Where:
Δy is the shift of the central fringe.
D is the distance between the slits and the screen.
d is the separation between the two slits.
μ is the refractive index of the sheet.
t is the thickness of the sheet.

Step 3: Detailed Explanation:
From the question, we have the following data:
Slit separation, $d = 0.4$ mm $= 0.4 \times 10^{-3}$ m.
Screen distance, $D = 1$ m.
Thickness of the sheet, $t = 20\ \mu$m $= 20 \times 10^{-6}$ m.
Shift of the fringe, $\Delta y = 20$ mm $= 20 \times 10^{-3}$ m.
The refractive index is given as $\mu = \frac{\alpha}{10}$.
Substituting the values into the shift formula:

$$20 \times 10^{-3} = \frac{1}{0.4 \times 10^{-3}} \times (\mu - 1) \times (20 \times 10^{-6})$$

$$20 \times 10^{-3} = 2500 \times (\mu - 1) \times 20 \times 10^{-6}$$

$$20 \times 10^{-3} = 50,000 \times 10^{-6} \times (\mu - 1)$$

$$20 \times 10^{-3} = 50 \times 10^{-3} \times (\mu - 1)$$

Dividing both sides by 50×10^{-3}:

$$\frac{20}{50} = \mu - 1$$

$$0.4 = \mu - 1$$

$$\mu = 1.4$$

Given $\mu = \frac{\alpha}{10}$, we compare:

$$\frac{\alpha}{10} = 1.4$$

$$\alpha = 14$$

.

Step 4: Final Answer:
The value of α is 14.

> **Quick Tip**
>
> The shift of the central fringe is directly proportional to the thickness of the sheet and the factor $(\mu - 1)$. Always convert all dimensions to standard SI units (meters) to avoid calculation errors with powers of ten.

---

49. A particle having electric charge 3×10^{-19} C and mass 6×10^{-27} kg is accelerated by applying an electric potential of 1.21 V. Wavelength of the matter wave associated with the particle is $\alpha \times 10^{-12}$ m. The value of α is ________. (Take Planck's constant $= 6.6 \times 10^{-34}$ J.s)

Correct Answer: 10

Solution:

Step 1: Understanding the Concept:
According to de-Broglie's hypothesis, any moving particle has a wavelength associated with it, known as the de-Broglie wavelength.
When a charged particle is accelerated through a potential difference, it gains kinetic energy,

which in turn determines its wavelength.

Step 2: Key Formula or Approach:
The de-Broglie wavelength (λ) is given by:

$$\lambda = \frac{h}{p}$$

Where p is the momentum. Since Kinetic Energy $K = qV = \frac{p^2}{2m}$, we have $p = \sqrt{2mqV}$.
Therefore:

$$\lambda = \frac{h}{\sqrt{2mqV}}$$

Step 3: Detailed Explanation:
Given values:
Charge $q = 3 \times 10^{-19}$ C.
Mass $m = 6 \times 10^{-27}$ kg.
Potential difference $V = 1.21$ V.
Planck's constant $h = 6.6 \times 10^{-34}$ J.s.
Substituting these values into the formula:

$$\lambda = \frac{6.6 \times 10^{-34}}{\sqrt{2 \times (6 \times 10^{-27}) \times (3 \times 10^{-19}) \times 1.21}}$$

Simplify the expression inside the square root:

$$\text{Denominator} = \sqrt{36 \times 10^{-46} \times 1.21}$$

$$\text{Denominator} = \sqrt{36} \times \sqrt{10^{-46}} \times \sqrt{1.21}$$

$$\text{Denominator} = 6 \times 10^{-23} \times 1.1$$

$$\text{Denominator} = 6.6 \times 10^{-23} \text{ kg} \cdot \text{m/s}$$

.
Now, calculate the wavelength:

$$\lambda = \frac{6.6 \times 10^{-34}}{6.6 \times 10^{-23}}$$

$$\lambda = 10^{-11} \text{ m}$$

The question asks for the answer in the form $\alpha \times 10^{-12}$ m:

$$\lambda = 10 \times 10^{-12} \text{ m}$$

By comparison, $\alpha = 10$.

Step 4: Final Answer:
The value of α is 10.

> Quick Tip
>
> While solving for $\sqrt{2mqV}$, look for perfect squares. Here, $2 \times 6 \times 3 = 36$ and $1.21 = (1.1)^2$, which simplifies the calculation significantly without needing a calculator.

---

50. The terminal velocity of a metallic ball of radius 6 mm in a viscous fluid is 20 cm/s. The terminal velocity of another ball of same material and having radius 3 mm in the same fluid will be ________ cm/s.

Correct Answer: 5

Solution:

Step 1: Understanding the Concept:
Terminal velocity is the constant velocity reached by an object falling through a viscous fluid when the net force on it becomes zero.
This happens when the downward force of gravity is perfectly balanced by the upward buoyant force and the viscous drag force.

Step 2: Key Formula or Approach:
The terminal velocity (v_t) of a spherical object is given by Stokes' Law:

$$v_t = \frac{2}{9}\frac{r^2(\rho - \sigma)g}{\eta}$$

Where:
r is the radius of the ball.
ρ is the density of the ball.
σ is the density of the fluid.
η is the coefficient of viscosity.
For the same material and the same fluid, all terms except r^2 are constant.
Thus, $v_t \propto r^2$.

Step 3: Detailed Explanation:
Let v_1 and r_1 be the initial terminal velocity and radius.
Let v_2 and r_2 be the new terminal velocity and radius.
From the proportionality $v_t \propto r^2$, we can write:

$$\frac{v_2}{v_1} = \left(\frac{r_2}{r_1}\right)^2$$

Given:
$r_1 = 6$ mm
$v_1 = 20$ cm/s
$r_2 = 3$ mm
Substituting these values:

$$\frac{v_2}{20} = \left(\frac{3}{6}\right)^2$$

$$\frac{v_2}{20} = \left(\frac{1}{2}\right)^2$$

$$\frac{v_2}{20} = \frac{1}{4}$$

$$v_2 = \frac{20}{4} = 5 \text{ cm/s}$$

.

Step 4: Final Answer:
The terminal velocity of the second ball is 5 cm/s.

Quick Tip

In terminal velocity problems where only the radius changes, remember that $v \propto r^2$. If the radius is halved, the velocity becomes one-fourth $(1/2^2)$. If the radius is doubled, the velocity becomes four times (2^2).

Chemistry Section A

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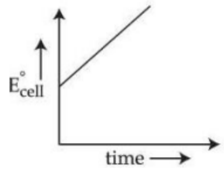
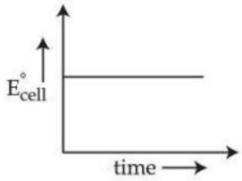
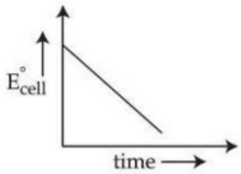
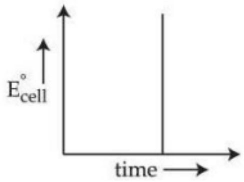
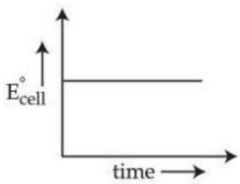
51. For a closed circuit Daniell cell, which of the following plots is the accurate one at a given temperature?

(A) 

(B) 

(C) 

(D) 

Correct Answer: (B) 

Solution:

Step 1: Understanding the Concept:
The Daniell cell is a type of electrochemical cell. E°_{cell} represents the standard cell potential, which is the potential difference between the two electrodes under standard conditions (1 M concentration, 1 bar pressure, and a specified temperature).

Step 2: Key Formula or Approach:
The standard cell potential is a constant property for a specific electrochemical reaction at a given temperature. It is calculated as:

$$E^\circ_{cell} = E^\circ_{cathode} - E^\circ_{anode}$$

Step 3: Detailed Explanation:
1. In a Daniell cell, the standard potential is approximately 1.1 V.
2. Unlike the cell potential (E_{cell}), which depends on the concentrations of ions and changes as the reaction progresses (calculated via the Nernst equation), the standard cell potential (E°_{cell}) remains constant.
3. At a fixed temperature, E°_{cell} does not vary with time as the battery discharges.

4. Therefore, a plot of E°_{cell} versus time will be a horizontal line, reflecting its constant nature.

Step 4: Final Answer:
The correct plot is the horizontal line shown in Option 2.

Quick Tip

Remember the difference between E_{cell} and E°_{cell}. The superscript $\circ$ denotes standard conditions, making it an intensive property and a constant for a given reaction at constant temperature.

---

52. Given below are four compounds :
(a) n-propyl chloride (b) iso-propyl chloride
(c) sec-butyl chloride (d) neo-pentyl chloride
Percentage of carbon in the one which exhibits optical isomerism is :

(A) 56
(B) 40
(C) 46
(D) 52

Correct Answer: (D) 52

Solution:

Step 1: Understanding the Concept:
Optical isomerism occurs in compounds that possess a chiral center (an atom, usually carbon, bonded to four different groups). We first identify which of the given alkyl chlorides is chiral.

Step 2: Key Formula or Approach:
Percentage of carbon $= \left(\frac{\text{Total mass of Carbon}}{\text{Molar mass of compound}}\right) \times 100$

Step 3: Detailed Explanation:
1. **n-propyl chloride:** $CH_3 - CH_2 - CH_2 - Cl$. No chiral center.
2. **iso-propyl chloride:** $(CH_3)_2CH - Cl$. No chiral center (two identical methyl groups).
3. **sec-butyl chloride:** $CH_3 - \overset{*}{C}H(Cl) - CH_2 - CH_3$. The marked carbon is chiral (bonded to H, Cl, methyl, and ethyl groups). This compound exhibits optical isomerism.
4. **neo-pentyl chloride:** $(CH_3)_3C - CH_2 - Cl$. No chiral center.

The compound is sec-butyl chloride, C_4H_9Cl.
Molecular weight $= (4 \times 12) + (9 \times 1) + 35.5 = 48 + 9 + 35.5 = 92.5$ g/mol.
Mass of Carbon $= 4 \times 12 = 48$ g.
Percentage of carbon $= \frac{48}{92.5} \times 100 \approx 51.89\%$.
Rounding to the nearest integer, we get 52%.

Step 4: Final Answer:
The percentage of carbon in sec-butyl chloride is 52.

> Quick Tip
>
> For butyl chlorides: n-butyl and isobutyl are primary, sec-butyl is secondary (and chiral), and tert-butyl is tertiary. Identifying the chiral center is the fastest way to start this problem.

---

53. Given below are two statements :
Statement I : Crystal Field Stabilization Energy (CFSE) of $[Cr(H_2O)_6]^{2+}$ is greater than that of $[Mn(H_2O)_6]^{2+}$.
Statement II : Potassium ferricyanide has a greater spin-only magnetic moment than sodium ferrocyanide.
In the light of the above statements, choose the correct answer from the options given below :

(A) Statement I is true but Statement II is false
(B) Statement I is false but Statement II is true
(C) Both Statement I and Statement II are true
(D) Both Statement I and Statement II are false

Correct Answer: (C) Both Statement I and Statement II are true

Solution:

Step 1: Understanding the Concept:
CFSE depends on the electronic configuration of the central metal ion and the field strength of the ligands. Magnetic moment (μ) is determined by the number of unpaired electrons (n) using the formula $\mu = \sqrt{n(n+2)}$ BM.

Step 2: Detailed Explanation:
Analysis of Statement I:
- $[Cr(H_2O)_6]^{2+}$: Cr^{2+} has $3d^4$ configuration. H_2O is a weak field ligand, so it is high spin: $t_{2g}^3 e_g^1$.
CFSE $= (3 \times -0.4\Delta_o) + (1 \times 0.6\Delta_o) = -0.6\Delta_o$.
- $[Mn(H_2O)_6]^{2+}$: Mn^{2+} has $3d^5$ configuration. H_2O is weak field: $t_{2g}^3 e_g^2$.
CFSE $= (3 \times -0.4\Delta_o) + (2 \times 0.6\Delta_o) = 0$.
Since $|-0.6\Delta_o| > 0$, Statement I is true.

Analysis of Statement II:
- Potassium ferricyanide, $K_3[Fe(CN)_6]$: Fe^{3+} is $3d^5$. CN^- is a strong field ligand, so it is low spin: $t_{2g}^5 e_g^0$. Number of unpaired electrons $(n) = 1$.
- Sodium ferrocyanide, $Na_4[Fe(CN)_6]$: Fe^{2+} is $3d^6$. CN^- is a strong field ligand, so it is low spin: $t_{2g}^6 e_g^0$. Number of unpaired electrons $(n) = 0$.

Ferricyanide has $n = 1$, while ferrocyanide has $n = 0$. Thus, ferricyanide has a greater magnetic moment. Statement II is true.

Step 3: Final Answer:
Both statements are true.

Quick Tip

For octahedral complexes with weak field ligands, d^5 always has a CFSE of zero. For strong field ligands, d^6 (Fe^{2+}, Co^{3+}) is often diamagnetic ($n = 0$).

---

54. The correct statements are :
A. Activation energy for enzyme catalysed hydrolysis of sucrose is lower than that of acid catalysed hydrolysis.
B. During denaturation, secondary and tertiary structures of a protein are destroyed but primary structure remains intact.
C. Nucleotides are joined together by glycosidic linkage between C_1 and C_4 carbons of the pentose sugar.
D. Quaternary structure of proteins represents overall folding of the polypeptide chain.
Choose the correct answer from the options given below :

(A) A, B and D Only
(B) A and B Only
(C) B and C Only
(D) A, C and D Only

Correct Answer: (B) A and B Only

Solution:

Step 1: Understanding the Concept:
This question tests knowledge of biocatalysis, protein structure/denaturation, and nucleic acid structure.

Step 2: Detailed Explanation:
Statement A: Enzymes are highly efficient biological catalysts that lower the activation energy (E_a) significantly more than inorganic or acid catalysts. Thus, enzyme-catalyzed hydrolysis of sucrose (by sucrase) has a lower E_a than acid-catalyzed hydrolysis. **Correct.**
Statement B: Denaturation involves the unfolding of proteins due to heat or pH changes, which breaks hydrogen bonds and disulfide bridges. This destroys the secondary, tertiary, and quaternary structures, but the sequence of amino acids (primary structure) remains undisturbed. **Correct.**
Statement C: Nucleotides in nucleic acids (DNA/RNA) are joined by phosphodiester linkages between the 5' phosphate of one nucleotide and the 3' hydroxyl of the pentose sugar of the next.

Glycosidic linkages connect the sugar to the nitrogenous base. **Incorrect.**
Statement D: The overall folding of a *single* polypeptide chain is represented by the **tertiary structure**. Quaternary structure refers to the spatial arrangement and assembly of *multiple* polypeptide subunits in a multi-subunit protein (like hemoglobin). **Incorrect.**

Note: If the options consider D as overall folding in a general sense, A, B, D might be a choice, but strictly speaking, D is the definition of tertiary structure. Looking at the provided answer key logic, A and B are the most solid facts.

Step 3: Final Answer:
The correct statements are A and B.

> Quick Tip
>
> Remember: Primary structure is just the "text" (amino acid sequence) and is covalent, so it is hard to break by denaturation. Glycosidic bonds are for sugars and nucleosides; phosphodiester bonds are for nucleotide polymers.

---

55. By usual analysis, 1.00 g of compound (X) gave 1.79 g of magnesium pyrophosphate. The percentage of phosphorus in compound (X) is : (nearest integer) (Given, molar mass in $g\,mol^{-1}: O = 16, Mg = 24, P = 31$)

(A) 20
(B) 40
(C) 30
(D) 50

Correct Answer: (D) 50

Solution:

Step 1: Understanding the Concept:
Quantitative analysis of phosphorus involves converting the phosphorus in an organic compound into magnesium pyrophosphate ($Mg_2P_2O_7$). We use the stoichiometry of this compound to find the mass of phosphorus.

Step 2: Key Formula or Approach:

$$\%P = \frac{62}{222} \times \frac{\text{Mass of } Mg_2P_2O_7}{\text{Mass of compound}} \times 100$$

Here, molar mass of $Mg_2P_2O_7 = (2 \times 24) + (2 \times 31) + (7 \times 16) = 48 + 62 + 112 = 222$ g/mol.
Mass of Phosphorus in 1 mole of pyrophosphate $= 2 \times 31 = 62$ g.

Step 3: Detailed Explanation:
1. Mass of compound (X) = 1.00 g.

2. Mass of $Mg_2P_2O_7$ formed = 1.79 g.
3. Mass of Phosphorus = $\left(\frac{62}{222}\right) \times 1.79$ g.
Mass of P $\approx 0.279 \times 1.79 \approx 0.4994$ g.
4. Percentage of P = $\left(\frac{0.4994}{1.00}\right) \times 100 = 49.94\%$.
5. Rounding to the nearest integer, we get 50.

Step 4: Final Answer:
The percentage of phosphorus in the compound is 50.

> Quick Tip
>
> The factor $\frac{62}{222}$ (or $\frac{31}{111}$) is specific to phosphorus analysis. Memorizing molar masses of common analytical precipitates like $BaSO_4$ (233), $AgCl$ (143.5), and $Mg_2P_2O_7$ (222) saves time.

---

56. Aqueous HCl reacts with $MnO_2(s)$ to form $MnCl_2(aq)$, $Cl_2(g)$ and $H_2O(l)$. What is the weight (in g) of Cl_2 liberated when 8.7 g of $MnO_2(s)$ is reacted with excess aqueous HCl solution?
(Given Molar mass in $g\,mol^{-1}$: $Mn = 55, Cl = 35.5, O = 16, H = 1$)

(A) 21.3
(B) 71
(C) 14.2
(D) 7.1

Correct Answer: (D) 7.1

Solution:

Step 1: Understanding the Concept:
This is a stoichiometry problem. We first write the balanced chemical equation for the reaction between MnO_2 and HCl.

Step 2: Key Formula or Approach:
Balanced Equation: $MnO_2 + 4HCl \rightarrow MnCl_2 + Cl_2 + 2H_2O$
Moles of $MnO_2 = \frac{\text{Given mass}}{\text{Molar mass}}$

Step 3: Detailed Explanation:
1. Molar mass of $MnO_2 = 55 + (2 \times 16) = 55 + 32 = 87$ g/mol.
2. Moles of MnO_2 used $= \frac{8.7 \text{ g}}{87 \text{ g/mol}} = 0.1$ mol.
3. According to the balanced equation, 1 mole of MnO_2 produces 1 mole of Cl_2.
4. Therefore, 0.1 mol of MnO_2 will produce 0.1 mol of Cl_2.
5. Molar mass of $Cl_2 = 2 \times 35.5 = 71$ g/mol.
6. Weight of Cl_2 = moles $\times$ molar mass = 0.1 mol $\times$ 71 g/mol = 7.1 g.

Step 4: Final Answer:
The weight of Cl_2 liberated is 7.1 g.

> Quick Tip
>
> This reaction is the standard laboratory method for the preparation of Chlorine gas. In stoichiometry, always ensure your reaction is balanced before calculating mole ratios.

---

57. Match List - I with List - II.

	List - I Reagents		List - II Reaction Name (Involving aldehydes)
A.	H_2, Pd-$BaSO_4$	I.	Etard Reaction
B.	$SnCl_2$, HCl	II.	Rosenmund Reduction
C.	CrO_2Cl_2, CS_2	III.	Gatterman - Koch Reaction
D.	CO, HCl, Anhyd. $AlCl_3$	IV.	Stephen Reaction

Choose the **correct** answer from the options given below :

Choose the correct answer from the options given below :

(A) A-II, B-III, C-IV, D-I
(B) A-II, B-IV, C-I, D-III
(C) A-IV, B-I, C-II, D-III
(D) A-IV, B-III, C-I, D-II

Correct Answer: (B) A-II, B-IV, C-I, D-III

Solution:

Step 1: Understanding the Concept:
This question involves matching specific reagents with their well-known named organic reactions for the preparation of aldehydes.

Step 2: Detailed Explanation:
- **A.** $H_2, Pd - BaSO_4$: This is the Lindlar catalyst variant used in the **Rosenmund Reduction**. It reduces acyl chlorides to aldehydes. $\rightarrow$ **A-II**.
- **B.** $SnCl_2, HCl$: This reagent reduces nitriles to imines, which are then hydrolyzed to aldehydes. This is the **Stephen Reaction**. $\rightarrow$ **B-IV**.
- **C.** CrO_2Cl_2, CS_2: Chromyl chloride in carbon disulfide oxidizes the methyl group of toluene to a chromium complex, which on hydrolysis gives benzaldehyde. This is the **Etard Reaction**. $\rightarrow$ **C-I**.
- **D.** CO, HCl, **Anhyd.** $AlCl_3$: Carbon monoxide and HCl in the presence of anhydrous $AlCl_3$/CuCl react with benzene to form benzaldehyde. This is the **Gatterman-Koch Reaction**. $\rightarrow$ **D-III**.

Matching these results: A-II, B-IV, C-I, D-III.

Step 3: Final Answer:
The correct matching is A-II, B-IV, C-I, D-III.

> 💡 Quick Tip
>
> Aldehyde preparation reactions are high-yield topics. Remember: "Etard = Chromyl chloride", "Stephen = Tin chloride", and "Rosenmund = Hydrogen/Palladium".

---

58. Given below are two statements :
Statement I : The correct order in terms of bond dissociation enthalpy is $Cl_2 > Br_2 > F_2 > I_2$**.**
Statement II : The correct trend in the covalent character of the metal halides is $[SnCl_4 > SnCl_2]$**,** $[PbCl_4 > PbCl_2]$ **and** $[UF_4 > UF_6]$**.**
In the light of the above statements, choose the correct answer from the options given below :

(A) Both Statement I and Statement II are true
(B) Statement I is false but Statement II is true
(C) Statement I is true but Statement II is false
(D) Both Statement I and Statement II are false

Correct Answer: (C) Statement I is true but Statement II is false

Solution:

Step 1: Understanding the Concept:
Statement I deals with the anomalous bond dissociation enthalpy of Halogens. Statement II deals with Fajan's rules for covalent character.

Step 2: Detailed Explanation:
Analysis of Statement I:
The bond dissociation enthalpy of halogens follows the order: $Cl_2 > Br_2 > F_2 > I_2$. Normally, it should decrease down the group, but F_2 has a lower enthalpy than Cl_2 and Br_2 due to the small size of Fluorine and large lone pair-lone pair repulsions in the short $F - F$ bond. **Statement I is true.**

Analysis of Statement II:
According to Fajan's rules, covalent character increases with the increasing charge on the cation.
- For $SnCl_4$ and $SnCl_2$, Sn^{4+} has a higher charge than Sn^{2+}, so $SnCl_4$ is more covalent.
- For $PbCl_4$ and $PbCl_2$, Pb^{4+} has a higher charge than Pb^{2+}, so $PbCl_4$ is more covalent.
- For UF_4 and UF_6, U^{6+} has a higher charge than U^{4+}. Therefore, UF_6 is *more* covalent than UF_4.
The statement says $[UF_4 > UF_6]$ for covalent character, which is incorrect. **Statement II is false.**

Step 3: Final Answer:
Statement I is true but Statement II is false.

Quick Tip

Fajan's Rule simplified: More charge on cation + small cation + large anion = More Covalent Character. For halogens, F_2 is always the "exception" in bond energy due to its tiny size.

---

59. Consider the following spectral lines for atomic hydrogen :
A. First line of Paschen series
B. Second line of Balmer series
C. Third line of Paschen series
D. Fourth line of Brackett series
The correct arrangement of the above lines in ascending order of energy is :

(A) C ¡ D ¡ B ¡ A
(B) A ¡ B ¡ C ¡ D
(C) D ¡ C ¡ A ¡ B
(D) D ¡ A ¡ C ¡ B

Correct Answer: (D) D ¡ A ¡ C ¡ B

Solution:

Step 1: Understanding the Concept:
The energy of a spectral line in the hydrogen spectrum is given by the difference in energy between the two levels: $E \propto \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$.

Step 2: Key Formula or Approach:
Energy $(E) \propto R\left(\frac{1}{n_{lower}^2} - \frac{1}{n_{upper}^2}\right)$.

Step 3: Detailed Explanation:
Let's calculate the relative energy factor $k = \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$ for each:
- **A. First line Paschen**: $n = 4 \to n = 3$.
$k_A = \frac{1}{3^2} - \frac{1}{4^2} = \frac{1}{9} - \frac{1}{16} = \frac{7}{144} \approx 0.0486$.
- **B. Second line Balmer**: $n = 4 \to n = 2$.
$k_B = \frac{1}{2^2} - \frac{1}{4^2} = \frac{1}{4} - \frac{1}{16} = \frac{3}{16} = 0.1875$.
- **C. Third line Paschen**: $n = 6 \to n = 3$.
$k_C = \frac{1}{3^2} - \frac{1}{6^2} = \frac{1}{9} - \frac{1}{36} = \frac{3}{36} = 0.0833$.
- **D. Fourth line Brackett**: $n = 8 \to n = 4$.
$k_D = \frac{1}{4^2} - \frac{1}{8^2} = \frac{1}{16} - \frac{1}{64} = \frac{3}{64} \approx 0.0468$.

Ordering the values: $0.0468(D) < 0.0486(A) < 0.0833(C) < 0.1875(B)$.

Ascending order of energy: $D < A < C < B$.

Step 4: Final Answer:
The correct arrangement is D ¡ A ¡ C ¡ B.

> Quick Tip
>
> Generally, for hydrogen series energy: $Lyman > Balmer > Paschen > Brackett > Pfund$. Within a series, energy increases with the line number. Comparing Balmer to Paschen is easy, but comparing the start of one series to the end of another requires the $1/n^2$ calculation.

---

60. Decomposition of A is a first order reaction at T(K) and is given by $A(g) \rightarrow B(g) + C(g)$.
In a closed 1 L vessel, 1 bar A(g) is allowed to decompose at T(K). After 100 minutes, the total pressure was 1.5 bar. What is the rate constant (in min^{-1}) of the reaction? ($\log 2 = 0.3$)

(A) 6.9×10^{-4}
(B) 6.9×10^{-1}
(C) 6.9×10^{-2}
(D) 6.9×10^{-3}

Correct Answer: (D) 6.9×10^{-3}

Solution:

Step 1: Understanding the Concept:
For a first-order gaseous reaction, we can use partial pressures to calculate the rate constant k. Pressure changes as the moles of gas increase during decomposition.

Step 2: Key Formula or Approach:
Rate constant $k = \frac{2.303}{t} \log\left(\frac{P_0}{P_t}\right)$, where P_0 is initial pressure of A and P_t is pressure of A at time t.

Step 3: Detailed Explanation:
Reaction: $A(g) \rightarrow B(g) + C(g)$
At $t = 0$: 1 bar 0 0
At $t = 100$: $1 - x$ x x

Total pressure $P_{total} = (1 - x) + x + x = 1 + x$.
Given $P_{total} = 1.5$ bar at 100 min.
$1 + x = 1.5 \implies x = 0.5$ bar.
Pressure of A remaining at 100 min, $P_A = 1 - x = 1 - 0.5 = 0.5$ bar.

Calculating k:

$$k = \frac{2.303}{100} \log\left(\frac{1}{0.5}\right)$$
$$k = \frac{2.303}{100} \log(2)$$
$$k = \frac{2.303 \times 0.3}{100} = \frac{0.6909}{100} \approx 6.9 \times 10^{-3} \text{ min}^{-1}$$

Step 4: Final Answer:
The rate constant is 6.9×10^{-3} min^{-1}.

> **Quick Tip**
>
> For a reaction $A \to nB$, the relationship is $P_{total} = P_0 + (n-1)x$. In this case $n = 2$, so $x = P_{total} - P_0$. Always remember to plug in the remaining pressure of reactant, not the total pressure, into the log term.

---

61. The correct order of the rate of the reaction for the following reaction with respect to nucleophiles is :
$CH_3Br + Nu^{\ominus} \to CH_3Nu + Br^{\ominus}$

(A) $CH_3COO^- > PhO^- > {}^-OH > ClO_4^-$
(B) $ClO_4^- > CH_3COO^- > {}^-OH > PhO^-$
(C) ${}^-OH > PhO^- > CH_3COO^- > ClO_4^-$
(D) $PhO^- > {}^-OH > CH_3COO^- > ClO_4^-$

Correct Answer: (C) ${}^-OH > PhO^- > CH_3COO^- > ClO_4^-$

Solution:

Step 1: Understanding the Concept:
This is an S_N2 reaction on a primary alkyl halide. The rate depends on the nucleophilicity of the species. Nucleophilicity generally follows basicity when the attacking atom is the same (Oxygen in all these cases).

Step 2: Detailed Explanation:
1. Nucleophilicity correlates with the strength of the conjugate base. Stronger bases are generally better nucleophiles.
2. Let's compare the acid strengths of their conjugate acids:
- H_2O ($K_a \approx 10^{-14}$) $\to$ Weakest acid
- $PhOH$ (Phenol, $K_a \approx 10^{-10}$)
- CH_3COOH (Acetic acid, $K_a \approx 10^{-5}$)
- $HClO_4$ (Perchloric acid, $K_a \approx 10^{7}$) $\to$ Strongest acid
3. Basicity order of conjugate bases (reverse of acid strength):

$OH^- > PhO^- > CH_3COO^- > ClO_4^-$
4. Therefore, the nucleophilicity order in a polar protic solvent (like water) follows the same trend as basicity for these Oxygen-based anions.

Step 3: Final Answer:
The correct order is $^-OH > PhO^- > CH_3COO^- > ClO_4^-$.

> Quick Tip
>
> If the attacking atom is the same, just look for the strongest base. Strong bases have stable conjugate acids (high pKa). ClO_4^- is the conjugate base of a superacid, so it is an extremely poor nucleophile.

---

62. Consider the following data :
$\Delta_f H°$(methane, g) $= -X$ kJ mol^{-1}
Enthalpy of sublimation of graphite $= Y$ kJ mol^{-1}
Dissociation enthalpy of $H_2 = Z$ kJ mol^{-1}
The bond enthalpy of C - H bond is given by :

(A) $\frac{X+Y+4Z}{2}$
(B) $\frac{X+Y+2Z}{4}$
(C) $\frac{-X+Y+Z}{4}$
(D) $X + Y + Z$

Correct Answer: (B) $\frac{X+Y+2Z}{4}$

Solution:

Step 1: Understanding the Concept:
We use Hess's Law and the relationship between formation enthalpy and bond enthalpies. Bond enthalpy of C-H is the average energy required to break one C-H bond in methane.

Step 2: Key Formula or Approach:
$\Delta_f H°(CH_4, g) = \Delta H_{sub}(C, s) + 2\Delta H_{diss}(H_2) - 4$ (Bond Enthalpy of C-H)

Step 3: Detailed Explanation:
The formation of methane from its elements in standard states:

$$C(s, \text{graphite}) + 2H_2(g) \rightarrow CH_4(g) \qquad \Delta H = -X$$

Breaking this down into steps: 1. Sublimation of Carbon: $C(s) \rightarrow C(g) \qquad \Delta H_1 = Y$
2. Dissociation of Hydrogen: $2H_2(g) \rightarrow 4H(g) \qquad \Delta H_2 = 2 \times Z$
3. Formation of 4 C-H bonds: $C(g) + 4H(g) \rightarrow CH_4(g) \qquad \Delta H_3 = -4 \times BE(C - H)$

According to Hess's Law:

$$-X = Y + 2Z - 4 \times BE(C - H)$$

$$4 \times BE(C-H) = X + Y + 2Z$$
$$BE(C-H) = \frac{X+Y+2Z}{4}$$

Step 4: Final Answer:
The bond enthalpy of the C-H bond is $\frac{X+Y+2Z}{4}$.

Quick Tip

Remember: $\Delta H_{rxn} = \sum BE_{\text{reactants}} - \sum BE_{\text{products}}$. For formation, reactants must be in their **atomic** gaseous states. That is why sublimation and dissociation are added first.

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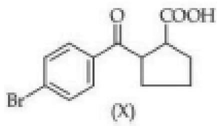
63. Given below are two statements :
Statement I : Compound (X), shown below, dissolves in $NaHCO_3$ solution and has two chiral carbon atoms.



Statement II : Compound (Y), shown below, has two carbons with sp^3 hybridization, one carbon with sp^2 and one carbon with sp hybridization.

In the light of the above statements, choose the correct answer from the options given below :

(A) Statement I is false but Statement II is true
(B) Both Statement I and Statement II are false
(C) Both Statement I and Statement II are true
(D) Statement I is true but Statement II is false

Correct Answer: (D) Statement I is true but Statement II is false

Solution:

Step 1: Understanding the Concept:
Statement I evaluates solubility based on acidic functional groups and identifies chiral centers. Statement II determines hybridization based on the bonding environment of carbon atoms.

Step 2: Detailed Explanation:
Analysis of Statement I:
- Compound (X) contains a carboxylic acid group ($-COOH$). Any organic compound with a $-COOH$ group is acidic enough to react with and dissolve in an aqueous solution of sodium bicarbonate ($NaHCO_3$).

- Checking chirality: Structure (X) has chiral centers in the side chain where carbons are bonded to four different groups (H, OH, Br, and the rest of the molecule). Two such centers are present. Thus, Statement I is true.

Analysis of Statement II:
- Compound (Y) is $(CH_3)_2CH - C \equiv C - CHO$.
- The two methyl group carbons (CH_3) and the methine carbon (CH) are sp^3 hybridized (3 carbons).
- The two carbons in the triple bond ($C \equiv C$) are sp hybridized.
- The aldehyde carbon (CHO) is sp^2 hybridized.
- Statement II says it has "two sp^3" carbons. In fact, it has three. Therefore, Statement II is false.

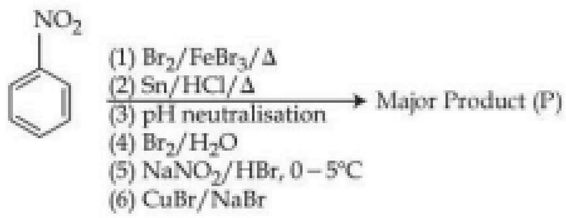
Step 3: Final Answer:
Statement I is true but Statement II is false.

Quick Tip

Solubility in $NaHCO_3$ is a standard test for $-COOH$ and nitro-substituted phenols.
For hybridization: Single bond (sp^3), Double bond (sp^2), Triple bond (sp).

---

64. Consider the reaction sequence starting from nitrobenzene:



The number of bromine atom(s) in the final product (P) will be:

(A) 3
(B) 5
(C) 6
(D) 1

Correct Answer: (B) 5

Solution:

Step 1: Understanding the Concept:
The problem involves a series of electrophilic aromatic substitutions, reduction, diazotization, and a Sandmeyer reaction. Key factors include the directing effects of functional groups (nitro is meta-directing, while amino is strongly activating and ortho/para directing).

Step 2: Detailed Explanation:

1. **Bromination of Nitrobenzene:** The first step uses $Br_2/FeBr_3$, which introduces a Br atom via electrophilic aromatic substitution. Since the $-NO_2$ group is meta-directing, the product is m-bromonitrobenzene. (1 Br atom).

2. **Reduction and Neutralization:** Sn/HCl reduces the nitro group to an amino group. After pH neutralization, m-bromoaniline is obtained.

3. **Bromination of Aniline:** The $-NH_2$ group is a very strong activator and directs to ortho and para positions. In m-bromoaniline (with $-NH_2$ at pos-1 and Br at pos-3), the activated positions are 2, 4, and 6. Bromine water (Br_2/H_2O) is a strong reagent that causes substitution at all available activated positions. This results in the addition of 3 more Br atoms. The product is 2,4,6-tribromo-3-bromoaniline. (Total 4 Br atoms).

4. **Diazotization and Sandmeyer Reaction:** The amino group is converted to a diazonium salt ($-N_2^+Br^-$) using $NaNO_2/HBr$ at low temperatures. Subsequently, treatment with $CuBr/NaBr$ (Sandmeyer reaction) replaces the diazonium group with a Br atom. (Adds 1 Br atom).

Total bromine atoms in the final product $P = 1$(from step 1)+3(from Step 3)+1(from step 6) = 5.

Step 3: Final Answer:
The number of bromine atoms in the final product (P) is 5.

> Quick Tip
>
> Remember that aniline is so reactive that bromination with aqueous bromine water leads to poly-substitution at all free ortho and para positions. Always track the initial substituents to avoid missing them in the final count.

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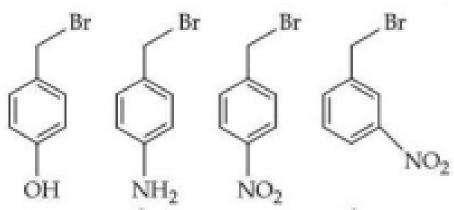
65. The correct order of reactivity of the following benzyl halides towards reaction with KCN is:



a b c d

(A) $b > a > c > d$
(B) $a > b > c > d$
(C) $a > b > d > c$
(D) $b > a > d > c$

Correct Answer: (D) $b > a > d > c$

Solution:

Step 1: Understanding the Concept:
The reaction with KCN involves nucleophilic substitution. Given the substituents and options, the reactivity order depends on the stability of the intermediate benzyl carbocation (suggesting an S_N1 mechanism) or the electronic environment of the benzylic carbon.

Step 2: Detailed Explanation:
The reactivity of benzyl halides towards nucleophilic substitution is enhanced by groups that stabilize the developing positive charge in the transition state.

- **(b) p-NH_2:** The amino group at the para position has a very strong $+M$ (mesomeric) effect, which provides maximum stabilization to the carbocation.

- **(a) p-OH:** The hydroxyl group also has a strong $+M$ effect, providing stabilization, but it is less than that of the amino group.

- **(d) m-NO_2:** At the meta position, the nitro group only exerts a $-I$ (inductive) effect. It destabilizes the transition state, but less so than when it's at the para position.

- **(c) p-NO_2:** At the para position, the nitro group exerts both $-I$ and a very strong $-M$ effect, causing the maximum destabilization of the benzylic center.

Thus, the decreasing order of reactivity is $b > a > d > c$.

Step 3: Final Answer:
The correct order of reactivity is $b > a > d > c$.

Quick Tip

For substituted benzyl halides, electron-donating groups ($+M > +I$) increase the rate of S_N1 reactions by stabilizing the carbocation, while electron-withdrawing groups decrease it. para-substituents have stronger mesomeric effects than meta-substituents.

---

66. Given below are some of the statements about Mn and Mn_2O_7. Identify the correct statements.
A. Mn forms the oxide Mn_2O_7 in which Mn is in its highest oxidation state.
B. Oxygen stabilizes the Mn in higher oxidation states by forming multiple bonds with Mn.
C. Mn_2O_7 is an ionic oxide.
D. The structure of Mn_2O_7 consists of one bridged oxygen.
Choose the correct answer from the options given below:

(A) A, B and D Only
(B) A, C and D Only
(C) A, B and C Only
(D) A, B, C and D

Correct Answer: (A) A, B and D Only

Solution:

Step 1: Understanding the Concept:
This question tests the properties of Manganese and its highest oxide (Mn_2O_7). We need to evaluate the oxidation state, the nature of bonding, and the molecular structure.

Step 2: Detailed Explanation:

- **Statement A:** Manganese can show various oxidation states from +2 to +7. In Mn_2O_7, the oxidation state of Mn is indeed +7, which is its maximum value. **(Correct)**

- **Statement B:** High oxidation states of transition metals are stabilized by highly electronegative atoms like Oxygen and Fluorine. Oxygen can form $p\pi - d\pi$ multiple bonds, which effectively stabilize high formal charges. **(Correct)**

- **Statement C:** According to Fajans' Rules, as the oxidation state increases, the covalent character of the oxide increases. Mn_2O_7 is a covalent, green oily liquid, not ionic. **(Incorrect)**

- **Statement D:** The structure of Mn_2O_7 consists of two MnO_4 tetrahedra that share a common vertex. Thus, it contains one $Mn - O - Mn$ bridge. **(Correct)**

Statements A, B, and D are correct.

Step 3: Final Answer:
The correct statements are A, B, and D only.

Quick Tip

Oxides of metals in very high oxidation states (> +4) are typically covalent and acidic. For example, Mn_2O_7 and CrO_3 are covalent and act as strong oxidizing agents.

---

67. On heating a mixture of common salt and $K_2Cr_2O_7$ in equal amount along with concentrated H_2SO_4 in a test tube, a gas is evolved. Formula of the gas evolved and oxidation state of the central metal atom in the gas respectively are:

(A) CrO_2Cl_2 and +6
(B) CrO_2Cl_2 and +5
(C) $Cr_2O_2Cl_2$ and +3
(D) $Cr_2O_2Cl_2$ and +6

Correct Answer: (A) CrO_2Cl_2 and +6

Solution:

Step 1: Understanding the Concept:
The description refers to the "Chromyl Chloride Test," which is used in qualitative analysis to detect the presence of chloride ions (Cl^-) in a sample.

Step 2: Key Formula or Approach:
The overall balanced equation for the reaction is:

$$K_2Cr_2O_7 + 4NaCl + 6H_2SO_4 \rightarrow 2CrO_2Cl_2 \uparrow + 2KHSO_4 + 4NaHSO_4 + 3H_2O$$

Step 3: Detailed Explanation:
When potassium dichromate ($K_2Cr_2O_7$) is heated with a chloride salt (like $NaCl$) and concentrated sulfuric acid (H_2SO_4), reddish-brown vapors of chromyl chloride (CrO_2Cl_2) are produced.
In CrO_2Cl_2, Oxygen has an oxidation state of -2 and Chlorine has -1. Let the oxidation state of Cr be x:

$$x + 2(-2) + 2(-1) = 0$$
$$x - 4 - 2 = 0 \implies x = +6$$

The oxidation state of the central metal atom (Chromium) is +6.

Step 4: Final Answer:
The gas is CrO_2Cl_2 and the oxidation state of Cr is +6.

Quick Tip

The Chromyl Chloride test is characteristic for chloride ions. Fluorides, bromides, and iodides do not give this test because they either don't form volatile oxyhalides or are oxidized to free halogens by dichromate.

---

68. Given below are two statements:
Statement I: The correct order in terms of atomic/ionic radii is $Al > Mg > Mg^{2+} > Al^{3+}$.
Statement II: The correct order in terms of the magnitude of electron gain enthalpy is $Cl > Br > S > O$.
In the light of the above statements, choose the correct answer from the options given below:

(A) Both Statement I and Statement II are false
(B) Both Statement I and Statement II are true
(C) Statement I is false but Statement II is true
(D) Statement I is true but Statement II is false

Correct Answer: (C) Statement I is false but Statement II is true

Solution:

Step 1: Understanding the Concept:
Periodic properties like atomic/ionic radii follow specific trends across periods and down groups. Cations are always smaller than their parent atoms. Electron gain enthalpy reflects the energy change when an atom accepts an electron.

Step 2: Detailed Explanation:
Analysis of Statement I:
Across a period (left to right), atomic radius decreases due to increasing nuclear charge. Thus, $Mg > Al$.
Cations are smaller than neutral atoms. For isoelectronic ions (Mg^{2+} and Al^{3+}), the radius decreases as the atomic number (positive charge) increases. Thus, $Mg^{2+} > Al^{3+}$.
The correct overall order is $Mg > Al > Mg^{2+} > Al^{3+}$. Statement I is **false**.

Analysis of Statement II:
Halogens (Group 17) have much more negative electron gain enthalpies than Group 16 elements. Magnitude-wise, $Cl, Br > S, O$.
In Group 17, $Cl > Br$. In Group 16, $S > O$ (Oxygen is exceptionally small, leading to electron-electron repulsion).
The given order $Cl > Br > S > O$ correctly reflects the magnitude of electron gain enthalpy. Statement II is **true**.

Step 3: Final Answer:
Statement I is false but Statement II is true.

> 💡 Quick Tip
>
> For isoelectronic species, radius decreases with an increase in atomic number. For electron gain enthalpy, Sulfur and Chlorine have higher magnitudes than Oxygen and Fluorine respectively due to the very small size of 2p elements causing inter-electronic repulsions.

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69. Match List - I with List - II.

	List - I Pair of Compounds		List - II Type of Isomers
A.	2-Methylpropene and but-1-ene	I.	Stereoisomers
B.	Cis-but-2-ene and trans-but-2-ene	II.	Position isomers
C.	2-Butanol and diethyl ether	III.	Chain isomers
D.	But-1-ene and but-2-ene	IV.	Functional group isomers

Choose the correct answer from the options given below:

(A) A-II, B-I, C-IV, D-III
(B) A-I, B-IV, C-III, D-II
(C) A-III, B-I, C-II, D-IV
(D) A-III, B-I, C-IV, D-II

Correct Answer: (D) A-III, B-I, C-IV, D-II

Solution:

Step 1: Understanding the Concept:
Isomerism describes compounds with the same molecular formula but different arrangements. It is divided into structural (connectivity differs) and stereo (spatial arrangement differs) isomerism.

Step 2: Detailed Explanation:

- **A:** 2-Methylpropene (isobutene) and but-1-ene are C_4H_8 isomers with different carbon skeletons (branched vs straight). These are **Chain isomers** (III).

- **B:** Cis and trans-but-2-ene differ only in the spatial orientation of groups around the double bond. These are geometrical isomers, a sub-type of **Stereoisomers** (I).

- **C:** 2-Butanol (an alcohol) and diethyl ether (an ether) both have the formula $C_4H_{10}O$. Since they have different functional groups, they are **Functional group isomers** (IV).

- **D:** But-1-ene and but-2-ene are straight-chain alkenes (C_4H_8) that differ only in the location of the double bond. These are **Position isomers** (II).

Matching sequence: A-III, B-I, C-IV, D-II.

Step 3: Final Answer:
The correct match is A-III, B-I, C-IV, D-II.

Quick Tip
To distinguish structural isomers: first check if functional groups differ, then check for chain branching, and finally for the position of a substituent or multiple bond.

---

70. The correct increasing order of $C-H(A)$, $C-O(B)$, $C=O(C)$ and $C\equiv N(D)$ bonds in terms of covalent bond length is:

(A) $A < D < C < B$
(B) $A < B < C < D$
(C) $D < C < B < A$
(D) $D < C < A < B$

Correct Answer: (A) $A < D < C < B$

Solution:

Step 1: Understanding the Concept:
Bond length is determined by the size of the bonded atoms and the bond order (number of electron pairs shared). Higher bond orders (Triple < Double < Single) result in shorter bond lengths.

Step 2: Detailed Explanation:
Let's analyze the factors affecting these bond lengths:

1. **C-H (A):** Though it's a single bond, Hydrogen is the smallest possible atom. This results in a very short bond length (typically ~ 109 pm).

2. **C-O (B):** This is a single bond between two Period-2 atoms. It is the longest among the group (typically ~ 143 pm).

3. **C=O (C):** This is a double bond. Due to the higher bond order, it is shorter than the $C-O$ single bond (typically ~ 122 pm).

4. **C≡N (D):** This is a triple bond. Higher bond order makes it even shorter than a double bond (typically ~ 116 pm).

Comparing these, $C-H$ is the shortest due to atomic size, followed by the triple bond ($C \equiv N$), the double bond ($C = O$), and finally the single bond ($C - O$).
Increasing order: $A < D < C < B$.

Step 3: Final Answer:
The correct increasing order is $A < D < C < B$.

💡 Quick Tip

Bond Length $\propto$ (Atomic radii of bonded atoms) / (Bond Order). Always check bond order first; if orders are the same, use atomic sizes to decide.

Chemistry Section B

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71. The first and second ionization constants of H_2X are 2.5×10^{-8} and 1.0×10^{-13} respectively. The concentration of X^{2-} in 0.1 M H_2X solution is ______ $\times 10^{-15}$ M. (Nearest Integer)

Correct Answer: 100

Solution:

Step 1: Understanding the Concept:
A diprotic acid H_2X dissociates in two steps. The first dissociation provides the bulk of the hydrogen ion concentration, while the second dissociation determines the concentration of the dianion X^{2-}.
In a solution of a weak diprotic acid where $K_{a1} \gg K_{a2}$, the concentration of the dianion is approximately equal to the second dissociation constant (K_{a2}), regardless of the initial concentration of the acid.

Step 2: Key Formula or Approach:
For the first dissociation:
$$H_2X \rightleftharpoons H^+ + HX^- \quad (K_{a1})$$
Since it is a weak acid, $[H^+] \approx [HX^-]$.
For the second dissociation:
$$HX^- \rightleftharpoons H^+ + X^{2-} \quad (K_{a2})$$

The equilibrium expression is:

$$K_{a2} = \frac{[H^+][X^{2-}]}{[HX^-]}$$

Substituting $[H^+] \approx [HX^-]$, we get $[X^{2-}] \approx K_{a2}$.

Step 3: Detailed Explanation:
Given:
$K_{a1} = 2.5 \times 10^{-8}$
$K_{a2} = 1.0 \times 10^{-13}$
Initial concentration $C = 0.1$ M.

Calculating $[H^+]$ from the first dissociation:

$$[H^+] = \sqrt{K_{a1} \cdot C} = \sqrt{2.5 \times 10^{-8} \times 0.1} = \sqrt{25 \times 10^{-10}} = 5 \times 10^{-5} \text{ M}$$

Since K_{a2} is extremely small (10^{-13}), the further dissociation of HX^- into X^{2-} will not significantly change the $[H^+]$ or $[HX^-]$.
From the second dissociation equilibrium:

$$[X^{2-}] = \frac{K_{a2} \cdot [HX^-]}{[H^+]}$$

Using the approximation $[HX^-] \approx [H^+]$:

$$[X^{2-}] = K_{a2} = 1.0 \times 10^{-13} \text{ M}$$

The question asks for the concentration in terms of 10^{-15} M:

$$1.0 \times 10^{-13} = 100 \times 10^{-15} \text{ M}$$

Step 4: Final Answer:
The concentration of X^{2-} is 100×10^{-15} M. Thus, the integer is 100.

> Quick Tip
>
> For any weak polyprotic acid where the successive ionization constants differ by several orders of magnitude ($K_{a1} \gg K_{a2} \gg K_{a3}$), the concentration of the conjugate base formed in the second step is numerically equal to K_{a2}.

---

72. The osmotic pressure of a living cell is 12 atm at 300 K. The strength of sodium chloride solution that is isotonic with the living cell at this temperature is ______ gL^{-1}. (Nearest integer)
Given: $R = 0.08$ L atm $K^{-1}mol^{-1}$
Assume complete dissociation of NaCl
(Given: Molar mass of Na and Cl are 23 and 35.5 $gmol^{-1}$ respectively.)

Correct Answer: 15

Solution:

Step 1: Understanding the Concept:
Two solutions are said to be isotonic if they have the same osmotic pressure at the same temperature. Osmotic pressure (π) is a colligative property and depends on the number of particles in the solution. For electrolytes like NaCl, we must include the van't Hoff factor (i).

Step 2: Key Formula or Approach:
Osmotic pressure formula:

$$\pi = iCRT$$

where:
i = van't Hoff factor (For NaCl with complete dissociation, $i = 2$)
C = Molar concentration ($molL^{-1}$)
R = Gas constant (0.08 L atm $K^{-1}mol^{-1}$)
T = Temperature (300 K)

Step 3: Detailed Explanation:
Given osmotic pressure of the cell = 12 atm.
For an isotonic solution of NaCl:

$$\pi_{\text{NaCl}} = \pi_{\text{cell}} = 12 \text{ atm}$$

$$iCRT = 12$$

Substituting the values:

$$2 \times C \times 0.08 \times 300 = 12$$

$$C \times 48 = 12$$

$$C = \frac{12}{48} = 0.25 \text{ mol L}^{-1}$$

Strength of the solution is the mass of solute per liter of solution:

$$\text{Strength} = \text{Molarity} \times \text{Molar Mass}$$

Molar mass of NaCl $= 23 + 35.5 = 58.5$ $gmol^{-1}$.

$$\text{Strength} = 0.25 \times 58.5 = 14.625 \text{ g L}^{-1}$$

Rounding to the nearest integer, we get 15.

Step 4: Final Answer:
The strength of the NaCl solution is approximately 15 gL^{-1}.

Quick Tip

Isotonic solutions have equal effective molarity (osmolarity). Always multiply the molarity by the van't Hoff factor (i) when dealing with electrolytes to compare their osmotic effects correctly.

73. Identify the metal ions among Co^{2+}, Ni^{2+}, Fe^{2+}, V^{3+} and Ti^{2+} having a spin-only magnetic moment value more than 3.0 BM. The sum of unpaired electrons present in the high spin octahedral complexes formed by those metal ions is ______.

Correct Answer: 7

Solution:

Step 1: Understanding the Concept:
The spin-only magnetic moment (μ) is calculated based on the number of unpaired electrons (n). For $\mu > 3.0$ BM, the number of unpaired electrons must be 3 or more because:
If $n = 2$, $\mu = \sqrt{2(2+2)} = \sqrt{8} \approx 2.83$ BM
If $n = 3$, $\mu = \sqrt{3(3+2)} = \sqrt{15} \approx 3.87$ BM

Step 2: Key Formula or Approach:
Spin-only magnetic moment formula:

$$\mu = \sqrt{n(n+2)} \text{ BM}$$

We need to determine the electronic configuration of each ion in a high-spin octahedral field (t_{2g} and e_g).

Step 3: Detailed Explanation:
Let's analyze the configuration of each high-spin octahedral ion:

- **Co^{2+} ($3d^7$):** In high spin, the configuration is $t_{2g}^5 e_g^2$. Number of unpaired electrons (n) = 3. $\mu = \sqrt{15} \approx 3.87$ BM (> 3.0).
- **Ni^{2+} ($3d^8$):** The configuration is $t_{2g}^6 e_g^2$ (same for high/low spin). Number of unpaired electrons (n) = 2. $\mu = \sqrt{8} \approx 2.83$ BM (< 3.0).
- **Fe^{2+} ($3d^6$):** In high spin, the configuration is $t_{2g}^4 e_g^2$. Number of unpaired electrons (n) = 4. $\mu = \sqrt{24} \approx 4.90$ BM (> 3.0).
- **V^{3+} ($3d^2$):** The configuration is $t_{2g}^2 e_g^0$. Number of unpaired electrons (n) = 2. $\mu = \sqrt{8} \approx$ 2.83 BM (< 3.0).
- **Ti^{2+} ($3d^2$):** The configuration is $t_{2g}^2 e_g^0$. Number of unpaired electrons (n) = 2. $\mu = \sqrt{8} \approx$ 2.83 BM (< 3.0).

The ions with $\mu > 3.0$ BM are Co^{2+} and Fe^{2+}.
Sum of unpaired electrons = $n(Co^{2+}) + n(Fe^{2+}) = 3 + 4 = 7$.

Step 4: Final Answer:
The sum of unpaired electrons is 7.

Quick Tip

A quick shortcut for spin-only magnetic moment is: the value of μ is always "n.something", where n is the number of unpaired electrons. For example, if $n = 3$, $\mu \approx 3.8$ BM. Thus, for $\mu > 3.0$, n must be ≥ 3.

---

74. A substance 'X' (1.5 g) dissolved in 150 g of a solvent 'Y' (molar mass = 300 $gmol^{-1}$) led to an elevation of the boiling point by 0.5 K. The relative lowering in the vapour pressure of the solvent 'Y' is ______ $\times 10^{-2}$. (nearest integer)
Given: K_b of the solvent = 5.0 K kg mol^{-1}

Assume the solution to be dilute and no association or dissociation of X takes place in solution.

Correct Answer: 3

Solution:

Step 1: Understanding the Concept:
The elevation in boiling point (ΔT_b) and relative lowering of vapour pressure (RLVP) are both colligative properties. ΔT_b is used to find the molar mass of the solute, which is then used to calculate the mole fraction of the solute (which equals RLVP).

Step 2: Key Formula or Approach:
1. Elevation in boiling point: $\Delta T_b = K_b \times m$, where $m = \frac{w_X \times 1000}{M_X \times W_Y(g)}$.
2. Relative Lowering of Vapour Pressure (RLVP): $\frac{P^o - P}{P^o} = \chi_X = \frac{n_X}{n_X + n_Y} \approx \frac{n_X}{n_Y}$ (for dilute solutions).

Step 3: Detailed Explanation:
Step 1: Find the molar mass of X (M_X):
$\Delta T_b = 0.5$ K, $K_b = 5.0$, $w_X = 1.5$ g, $W_Y = 150$ g.

$$0.5 = 5.0 \times \frac{1.5 \times 1000}{M_X \times 150}$$
$$0.5 = \frac{5.0 \times 10}{M_X} = \frac{50}{M_X}$$
$$M_X = \frac{50}{0.5} = 100 \text{ g/mol}$$

Step 2: Calculate moles of X and solvent Y:
$n_X = \frac{1.5}{100} = 0.015$ mol
$n_Y = \frac{W_Y}{M_Y} = \frac{150}{300} = 0.5$ mol

Step 3: Calculate RLVP (χ_X):

$$\text{RLVP} = \frac{n_X}{n_X + n_Y} = \frac{0.015}{0.015 + 0.5} = \frac{0.015}{0.515} \approx 0.0291$$

In terms of 10^{-2}:

$$\text{RLVP} \approx 2.91 \times 10^{-2}$$

Nearest integer = 3.

Step 4: Final Answer:
The relative lowering in vapour pressure is 3×10^{-2}.

> **Quick Tip**
>
> For very dilute solutions, RLVP is approximately equal to the ratio of moles of solute to moles of solvent (n/N). This simplifies the calculation significantly without sacrificing accuracy in competitive exams.

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75. MX is a sparingly soluble salt that follows the given solubility equilibrium at 298 **K.**
$MX(s) \rightleftharpoons M^+(aq) + X^-(aq); K_{sp} = 10^{-5}$
If the standard reduction potential for $M^+(aq) + e^- \rightarrow M(s)$ **is** $E^o_{M^+/M} = 0.79V$**, then the value of the standard reduction potential for the metal/metal insoluble salt electrode** $E^o_{X^-/MX(s)/M}$ **is ______ mV. (nearest integer)**
Given: $\frac{2.303RT}{F} = 0.059V$

Correct Answer: 495

Solution:

Step 1: Understanding the Concept:
The standard reduction potential of a metal-metal insoluble salt electrode ($E^o_{X^-/MX/M}$) is related to the standard reduction potential of the metal ion ($E^o_{M^+/M}$) and the solubility product (K_{sp}) of the salt. This is because the concentration of M^+ is controlled by the concentration of X^- through the K_{sp} equilibrium.

Step 2: Key Formula or Approach:
The Nernst equation for the system at standard conditions ($[X^-] = 1$ M) yields the relationship:

$$E^o_{X^-/MX/M} = E^o_{M^+/M} + \frac{2.303RT}{nF} \log K_{sp}$$

For the given reaction, $n = 1$.

Step 3: Detailed Explanation: Consider the half-cell reaction:

$$MX(s) + e^- \rightleftharpoons M(s) + X^-(aq)$$

This can be broken down into: 1. $MX(s) \rightleftharpoons M^+(aq) + X^-(aq) \quad (K_{sp})$
2. $M^+(aq) + e^- \rightleftharpoons M(s) \quad (E^o_{M^+/M})$

Given: $E^o_{M^+/M} = 0.79V$
$K_{sp} = 10^{-5}$
$\frac{2.303RT}{F} = 0.059V$

Applying the formula:

$$E^o = 0.79 + 0.059\log(10^{-5})$$
$$E^o = 0.79 + 0.059 \times (-5)$$
$$E^o = 0.79 - 0.295$$
$$E^o = 0.495V$$

To convert to mV:

$$0.495V = 495 \text{ mV}$$

Step 4: Final Answer:
The standard reduction potential is 495 mV.

Quick Tip

The presence of a common ion (X^-) decreases the concentration of M^+, which according to Le Chatelier's principle and the Nernst equation, always reduces the reduction potential of the M^+/M couple. Therefore, E^o for the insoluble salt electrode is always less than the E^o for the metal ion.