



General Instructions

- (i) **Duration:** The total duration of the examination is 1 hours (60 minutes).
- (ii) **Total Marks:** The complete Mathematics paper carries a maximum of 100 marks.
- (iii) **Structure:** The paper consists of two sections:
 - **Section A:** 20 Multiple Choice Questions (MCQs).
 - **Section B:** 5 Numerical Value Type Questions.
- (iv) **Compulsory Questions:** All 25 questions are compulsory.
- (v) Each question has four options. Only **one** option is correct.
- (vi) **Correct Answer:** +4 marks.
- (vii) **Incorrect Answer:** –1 mark (Negative marking).
- (viii) **Unanswered/Marked for Review:** 0 marks.

Mathematics Section - A

1. Consider the relation R on the set $\{-2, -1, 0, 1, 2\}$ defined by $(a, b) \in R$ if and only if $1 + ab > 0$.

Then, among the statements :

I. The number of elements in R is 17

II. R is an equivalence relation

(A) Only I is true

(B) Only II is true

- (C) Both I and II are true
(D) Neither I nor II is true
-

2. The number of values of $z \in \mathbb{C}$, satisfying the equations

$|z - (4 + 8i)| = \sqrt{10}$ and $|z - (3 + 5i)| + |z - (5 + 11i)| = 4\sqrt{5}$, is :

- (A) 0
(B) 2
(C) 1
(D) 4
-

3. If the system of linear equations :

$$x + y + z = 6,$$

$$x + 2y + 5z = 10,$$

$$2x + 3y + \lambda z = \mu$$

has infinitely many solutions, then the value of $\lambda + \mu$ equals :

- (A) 12
(B) 16
(C) 22
(D) 28
-

4. Let $A = \begin{bmatrix} \alpha & 1 & 2 \\ 2 & 3 & 0 \\ 0 & 4 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -5\alpha & 0 \\ 0 & 4\alpha & -2\alpha \end{bmatrix} + \text{adj}(A)$. If $\det(B) = 66$, then $\det(\text{adj}(A))$ equals :

- (A) 289
(B) 361
(C) 441
(D) 529
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5. Let $\alpha = 3 + 4 + 8 + 9 + 13 + 14 + \dots$ upto 40 terms. If $(\tan \beta)^{\frac{\alpha}{1020}}$ is a root of the equation

$x^2 + x - 2 = 0$, $\beta \in (0, \frac{\pi}{2})$, then $\sin^2 \beta + 3 \cos^2 \beta$ is equal to :

- (A) 2
 - (B) $\frac{7}{4}$
 - (C) $\frac{5}{2}$
 - (D) $\frac{3}{2}$
-

6. A candidate has to go to the examination centre to appear in an examination. The candidate uses only one means of transportation for the entire distance out of bus, scooter and car. The probabilities of the candidate going by bus, scooter and car, respectively, are $\frac{2}{5}$, $\frac{1}{5}$ and $\frac{2}{5}$. The probabilities that the candidate reaches late at the examination centre are $\frac{1}{5}$, $\frac{1}{3}$ and $\frac{1}{4}$ if the candidate uses bus, scooter and car, respectively. Given that the candidate reached late at the examination centre, the probability that the candidate travelled by bus is :

- (A) $\frac{11}{37}$
 - (B) $\frac{12}{37}$
 - (C) $\frac{13}{37}$
 - (D) $\frac{14}{37}$
-

7. A set of four observations has mean 1 and variance 13. Another set of six observations has mean 2 and variance 1. Then, the variance of all these 10 observations is equal to :

- (A) 5.96
 - (B) 6.14
 - (C) 6.04
 - (D) 6.24
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8. If $26 \left(\frac{2^3}{3} ({}^{12}C_2) + \frac{2^5}{5} ({}^{12}C_4) + \frac{2^7}{7} ({}^{12}C_6) + \dots + \frac{2^{13}}{13} ({}^{12}C_{12}) \right) = 3^{13} - \alpha$, then α is equal to :

- (A) 45
- (B) 48
- (C) 51

(D) 54

9. A person has three different bags and four different books. The number of ways, in which he can put these books in the bags so that no bag is empty, is :

(A) 18

(B) 36

(C) 39

(D) 72

10. If a straight line drawn through the point of intersection of the lines $4x + 3y - 1 = 0$ and $3x + 4y - 1 = 0$, meets the co-ordinate axes at the points P and Q, then the locus of the mid point of PQ is :

(A) $x + y - 7 = 0$

(B) $x + y - 14xy = 0$

(C) $2x + y + 14xy = 0$

(D) $x + 2y - 14xy = 0$

11. Let O be the vertex of the parabola $y^2 = 4x$ and its chords OP and OQ are perpendicular to each other. If the locus of the mid-point of the line segment PQ is a conic C, then the length of its latus rectum is :

(A) 1

(B) 2

(C) 4

(D) 8

12. Let $\alpha = 3 \sin^{-1} \left(\frac{6}{11} \right)$ and $\beta = 3 \cos^{-1} \left(\frac{4}{9} \right)$, where inverse trigonometric functions take only the principal values.

Given below are two statements :

Statement I : $\cos(\alpha + \beta) > 0$.

Statement II : $\cos(\alpha) < 0$.

In the light of the above statements, choose the correct answer from the options given below :

- (A) Both Statement I and Statement II are true
 - (B) Both Statement I and Statement II are false
 - (C) Statement I is true but Statement II is false
 - (D) Statement I is false but Statement II is true
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13. For the function $f(x) = e^{\sin|x|} - |x|$, $x \in \mathbb{R}$, consider the following statements :

Statement I : f is differentiable for all $x \in \mathbb{R}$.

Statement II : f is increasing in $(-\pi, -\frac{\pi}{2})$.

In the light of the above statements, choose the correct answer from the options given below :

- (A) Both Statement I and Statement II are true
 - (B) Both Statement I and Statement II are false
 - (C) Statement I is true but Statement II is false
 - (D) Statement I is false but Statement II is true
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14. Let $\vec{a} = 4\hat{i} - \hat{j} + 3\hat{k}$, $\vec{b} = 10\hat{i} + 2\hat{j} - \hat{k}$ and a vector $\vec{c}$ be such that $2(\vec{a} \times \vec{b}) + 3(\vec{b} \times \vec{c}) = \vec{0}$.

If $\vec{a} \cdot \vec{c} = 15$, then $\vec{c} \cdot (\hat{i} + \hat{j} - 3\hat{k})$ is equal to :

- (A) -6
 - (B) -5
 - (C) -4
 - (D) -3
-

15. Let the foot of perpendicular from the point $(\lambda, 2, 3)$ on the line $\frac{x-4}{1} = \frac{y-9}{2} = \frac{z-5}{1}$ be the point $(1, \mu, 2)$. Then the distance between the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6}$ and $\frac{x-\lambda}{2} = \frac{y-\mu}{3} = \frac{z+5}{6}$ is equal to :

- (A) $\frac{12}{7}$
- (B) $\frac{\sqrt{145}}{7}$
- (C) $\frac{\sqrt{146}}{7}$

(D) $\frac{\sqrt{143}}{7}$

16. The value of the integral $\int_0^2 \frac{\sqrt{x(x^2+x+1)}}{(\sqrt{x+1})(\sqrt{x^4+x^2+1})} dx$ is equal to :

(A) $\frac{1}{3} \log_e (3 - 2\sqrt{2})$

(B) $\frac{2}{3} \log_e (4 + \sqrt{2})$

(C) $\frac{2}{3} \log_e (3 + 2\sqrt{2})$

(D) $\frac{1}{3} \log_e (1 + 6\sqrt{2})$

17. Let $y = y(x)$ be the solution of the differential equation

$x\sqrt{1-x^2}dy + (y\sqrt{1-x^2} - x\cos^{-1}x)dx = 0, x \in (0, 1), \lim_{x \rightarrow 1^-} y(x) = 1$. Then $y\left(\frac{1}{2}\right)$ equals :

(A) $3 - \frac{\pi}{\sqrt{3}}$

(B) $4 - \sqrt{3}\pi$

(C) $4 - \frac{2\pi}{\sqrt{3}}$

(D) $3 - \frac{\pi}{2\sqrt{3}}$

18. Let $f : (1, \infty) \rightarrow \mathbb{R}$ be a function defined as $f(x) = \frac{x-1}{x+1}$. Let $f^{i+1}(x) = f(f^i(x)), i = 1, 2, \dots, 25$, where $f^1(x) = f(x)$. If $g(x) + f^{26}(x) = 0, x \in (1, \infty)$, then the area of the region bounded by the curves $y = g(x), 2y = 2x - 3, y = 0$ and $x = 4$ is :

(A) $\frac{1}{8} + \log_e 2$

(B) $\frac{1}{4} + \log_e 2$

(C) $\frac{5}{6} + 3\log_e 2$

(D) $\frac{5}{6} + \log_e 2$

19. Let $f(x) = \begin{cases} \frac{1}{3} & , x \leq \pi/2 \\ \frac{b(1-\sin x)}{(\pi-2x)^2} & , x > \pi/2 \end{cases}$. If f is continuous at $x = \pi/2$, then the value of $\int_0^{3b-6} |x^2 + 2x - 3| dx$ is :

- (A) 5
(B) 2
(C) 3
(D) 4

20. Let $\frac{x^2}{f(a^2+7a+3)} + \frac{y^2}{f(3a+15)} = 1$ represent an ellipse with major axis along y-axis, where f is a strictly decreasing positive function on $\mathbb{R}$. If the set of all possible values of a is $\mathbb{R} - [\alpha, \beta]$, then $\alpha^2 + \beta^2$ is equal to :

- (A) 28
(B) 40
(C) 61
(D) 24

Mathematics Section - B

21. The sum of squares of all the real solutions of the equation $\log_{(x+1)}(2x^2 + 5x + 3) = 4 - \log_{(2x+3)}(x^2 + 2x + 1)$ is equal to _____.

22. If $\int_{\pi/6}^{\pi/4} \left(\cot\left(x - \frac{\pi}{3}\right) \cot\left(x + \frac{\pi}{3}\right) + 1 \right) dx = \alpha \log_e(\sqrt{3} - 1)$, then $9\alpha^2$ is equal to _____.

23. Let a line L_1 pass through the origin and be perpendicular to the lines

$$L_2 : \vec{r} = (3+t)\hat{i} + (2t-1)\hat{j} + (2t+4)\hat{k} \text{ and}$$

$$L_3 : \vec{r} = (3+2s)\hat{i} + (3+2s)\hat{j} + (2+s)\hat{k}, t, s \in \mathbb{R}.$$

If $(a, b, c), a \in \mathbb{Z}$, is the point on L_3 at a distance of $\sqrt{17}$ from the point of intersection of L_1 and L_2 , then $(a+b+c)^2$ is equal to _____.

24. Consider the circle $C : x^2 + y^2 - 6x - 8y - 11 = 0$. Let a variable chord AB of the circle C subtend a right angle at the origin. If the locus of the foot of the perpendicular drawn from the origin on the chord AB is the circle $x^2 + y^2 - \alpha x - \beta y - \gamma = 0$, then $\alpha + \beta + 2\gamma$ is equal to _____.

25. Let f be a polynomial function such that $\log_2(f(x)) = (\log_2(2 + \frac{2}{3} + \frac{2}{9} + \dots \infty)) \cdot \log_3(1 + \frac{f(x)}{f(1/x)})$, $x > 0$ and $f(6) = 37$. Then $\sum_{n=1}^{10} f(n)$ is equal to _____.
