



General Instructions

- (i) **Duration:** The total duration of the examination is 1 hours (60 minutes).
- (ii) **Total Marks:** The complete Mathematics paper carries a maximum of 100 marks.
- (iii) **Structure:** The paper consists of two sections:
 - **Section A:** 20 Multiple Choice Questions (MCQs).
 - **Section B:** 5 Numerical Value Type Questions.
- (iv) **Compulsory Questions:** All 25 questions are compulsory.
- (v) Each question has four options. Only **one** option is correct.
- (vi) **Correct Answer:** +4 marks.
- (vii) **Incorrect Answer:** –1 mark (Negative marking).
- (viii) **Unanswered/Marked for Review:** 0 marks.

Mathematics Section - A

1. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined as $f(x) = \frac{2x^2 - 3x + 2}{3x^2 + x + 3}$. Then f is:

- (A) both one-one and onto
- (B) one-one but not onto
- (C) onto but not one-one
- (D) neither one-one nor onto

2. Consider the quadratic equation $(n^2 - 2n + 2)x^2 - 3x + (n^2 - 2n + 2)^2 = 0$, $n \in \mathbb{R}$. Let α be the minimum value of the product of its roots and β be the maximum value of the sum of its roots. Then the sum of the first six terms of the G.P, whose first term is α and the common ratio is $\frac{\alpha}{\beta}$, is:

- (A) $\frac{61}{37}$
(B) $\frac{121}{81}$
(C) $\frac{364}{243}$
(D) $\frac{1093}{729}$
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3. Let $S = \{z \in \mathbb{C} : z^2 + \sqrt{6}iz - 3 = 0\}$. Then $\sum_{z \in S} z^8$ is equal to:

- (A) 162
(B) 184
(C) 262
(D) 324
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4. The sum of all possible values of $\theta \in [0, 2\pi]$, for which the system of equations :

$$x \cos 3\theta - 8y - 12z = 0$$

$$x \cos 2\theta + 3y + 3z = 0$$

$$x + y + 3z = 0$$

has a non-trivial solution, is equal to :

- (A) π
(B) 2π
(C) 3π
(D) 4π
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5. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 9 & 3 & 1 \end{bmatrix}$ and $B = [b_{ij}]$, $1 \leq i, j \leq 3$. If $B = A^{99} - I$, then the value of $\frac{b_{31} - b_{21}}{b_{32}}$ is:

- (A) 99
 - (B) 199
 - (C) 149
 - (D) 159
-

6. The sum $1 + \frac{1}{2}(1^2 + 2^2) + \frac{1}{3}(1^2 + 2^2 + 3^2) + \dots$ upto 10 terms is equal to:

- (A) 130
 - (B) 155
 - (C) $\frac{315}{2}$
 - (D) $\frac{325}{2}$
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7. A building has ground floor and 10 more floors. Nine persons enter in a lift at the ground floor. The lift goes up to the 10th floor. The number of ways, in which any 4 persons exit at a floor and the remaining 5 persons exit at a different floor, if the lift does not stop at the first and the second floors, is equal to :

- (A) 2184
 - (B) 3064
 - (C) 7056
 - (D) 11340
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8. Let the mean and the variance of seven observations $2, 4, \alpha, 8, \beta, 12, 14$, $\alpha < \beta$, be 8 and 16 respectively. Then the quadratic equation whose roots are $3\alpha + 2$ and $2\beta + 1$ is :

- (A) $x^2 - 35x + 306 = 0$
- (B) $x^2 - 41x + 420 = 0$
- (C) $x^2 - 45x + 506 = 0$
- (D) $x^2 - 37x + 342 = 0$

9. A bag contains 6 blue and 6 green balls. Pairs of balls are drawn without replacement until the bag is empty. The probability that each drawn pair consists of one blue ball and one green ball is:

- (A) $\frac{63}{925}$
(B) $\frac{17}{231}$
(C) $\frac{16}{231}$
(D) $\frac{64}{925}$
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10. Let C be a circle having centre in the first quadrant and touching the x -axis at a distance of 3 units from the origin. If the circle C has an intercept of length $6\sqrt{3}$ on y -axis, then the length of the chord of the circle C on the line $x - y = 3$ is:

- (A) 8
(B) 6
(C) $6\sqrt{2}$
(D) $8\sqrt{2}$
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11. The eccentricity of an ellipse E with centre at the origin O is $\frac{\sqrt{3}}{2}$ and its directrices are $x = \pm \frac{4\sqrt{6}}{3}$. Let $H : \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ be a hyperbola whose eccentricity is equal to the length of semi-major axis of E , and whose length of latus rectum is equal to the length of minor axis of E . Then the distance between the foci of H is :

- (A) $\frac{4\sqrt{2}}{\sqrt{7}}$
(B) $\frac{4\sqrt{2}}{7}$
(C) $\frac{4}{\sqrt{7}}$
(D) $\frac{8}{7}$
-

12. Let $x = -9$ be a directrix of an ellipse E , whose centre is at the origin and eccentricity is $\frac{1}{3}$. Let $P(\alpha, 0)$, $\alpha > 0$, be a focus of E and AB be a chord passing through P . Then the locus of the mid point of AB is :

- (A) $9y^2 = 8x(1 - x)$
(B) $3y^2 = 4x(1 - x)$
(C) $9y^2 = 8x(x - 1)$
(D) $3y^2 = 4x(x - 1)$
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13. If $\sin(\tan^{-1}(x\sqrt{2})) = \cot(\sin^{-1}\sqrt{1-x^2})$, $x \in (0, 1)$, then the value of x is :

- (A) $\frac{1}{2}$
(B) $\frac{1}{3}$
(C) $\frac{2}{3}$
(D) $\frac{5}{8}$
-

14. The shortest distance between the lines

$$\frac{x-4}{1} = \frac{y-3}{2} = \frac{z-2}{-3}$$

and

$$\frac{x+2}{2} = \frac{y-6}{4} = \frac{z-5}{-5}$$

is :

- (A) $\frac{5\sqrt{6}}{6}$
(B) $2\sqrt{5}$
(C) $3\sqrt{5}$
(D) $4\sqrt{5}$
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15. Let $\vec{a} = 2\hat{i} + 3\hat{j} + 3\hat{k}$ and $\vec{b} = 6\hat{i} + 3\hat{j} + 3\hat{k}$. Then the square of the area of the triangle with adjacent sides determined by the vectors $(2\vec{a} + 3\vec{b})$ and $(\vec{a} - \vec{b})$ is :

- (A) 450
(B) 900
(C) 1800

(D) 2400

16. Let $\lim_{x \rightarrow 2} \frac{(\tan(x-2))(rx^2 + (p-2)x - 2p)}{(x-2)^2} = 5$ for some $r, p \in \mathbb{R}$. If the set of all possible values of q , such that the roots of the equation $rx^2 - px + q = 0$ lie in $(0, 2)$, be the interval $(\alpha, \beta]$, then $4(\alpha + \beta)$ equals :

(A) 11

(B) 13

(C) 17

(D) 21

17. Let $A = \begin{bmatrix} 1 & 3 & -1 \\ 2 & 1 & \alpha \\ 0 & 1 & -1 \end{bmatrix}$ be a singular matrix. Let $f(x) = \int_0^x (t^2 + 2t + 3)dt$, $x \in [1, \alpha]$. If M and m are respectively the maximum and the minimum values of f in $[1, \alpha]$, then $3(M - m)$ is equal to :

(A) 64

(B) 68

(C) 72

(D) 76

18. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be such that $f(xy) = f(x)f(y)$, for all $x, y \in \mathbb{R}$ and $f(0) \neq 0$. Let $g : [1, \infty) \rightarrow \mathbb{R}$ be a differentiable function such that

$$x^2 g(x) = \int_1^x (t^2 f(t) - t g(t)) dt.$$

Then $g(2)$ is equal to :

(A) $\frac{13}{8}$

(B) $\frac{11}{16}$

(C) $\frac{15}{32}$

(D) $\frac{17}{64}$

19. The area of the region $\{(x, y) : x^2 - 8x \leq y \leq -|x|\}$ is:

- (A) $\frac{343}{6}$
(B) $\frac{637}{6}$
(C) $\frac{437}{6}$
(D) $\frac{523}{6}$
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20. The value of the integral $\int_{-1}^1 \frac{x^3 + |x| + 1}{x^2 + 2|x| + 1} dx$ is equal to :

- (A) $3 \log_e 2$
(B) $2 \log_e 2$
(C) $5 \log_e 3$
(D) $3 \log_e 3$
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Mathematics Section - B

21. Let $R = \{(x, y) \in \mathbb{N} \times \mathbb{N} : \log_e(x + y) \leq 2\}$. Then the minimum number of elements, required to be added in R to make it a transitive relation, is _____.

22. If $(1 - x^3)^{10} = \sum_{r=0}^{10} a_r x^r (1 - x)^{30-2r}$, then $\frac{9a_9}{a_{10}}$ is equal to _____.

23. Let the line $x - y = 4$ intersect the circle $C : (x - 4)^2 + (y + 3)^2 = 9$ at the points Q and R. If $P(\alpha, \beta)$ is a point on C such that $PQ = PR$, then $(6\alpha + 8\beta)^2$ is equal to _____.

24. Let the image of the point $P(0, -5, 0)$ in the line $\frac{x-1}{2} = \frac{y}{1} = \frac{z+1}{-2}$ be the point R and the image of the point $Q(0, -1/2, 0)$ in the line $\frac{x-1}{1} = \frac{y+9}{4} = \frac{z+1}{1}$ be the point S. Then the square of the area

of the parallelogram PQRS is _____.

25. Let $f(x) = \begin{cases} x^3 + 8; & x < 0, \\ x^2 - 4; & x \geq 0, \end{cases}$ and $g(x) = \begin{cases} (x - 8)^{1/3}; & x < 0, \\ (x + 4)^{1/2}; & x \geq 0. \end{cases}$ Then the number of points, where the function $g \circ f$ is discontinuous, is _____.
