



General Instructions

- (i) **Duration:** The total duration of the examination is 1 hours (60 minutes).
- (ii) **Total Marks:** The complete Mathematics paper carries a maximum of 100 marks.
- (iii) **Structure:** The paper consists of two sections:
 - **Section A:** 20 Multiple Choice Questions (MCQs).
 - **Section B:** 5 Numerical Value Type Questions.
- (iv) **Compulsory Questions:** All 25 questions are compulsory.
- (v) Each question has four options. Only **one** option is correct.
- (vi) **Correct Answer:** +4 marks.
- (vii) **Incorrect Answer:** –1 mark (Negative marking).
- (viii) **Unanswered/Marked for Review:** 0 marks.

Mathematics Section - A

1. For the function $f : [1, \infty) \rightarrow [1, \infty)$ defined by $f(x) = (x - 1)^4 + 1$, among the two statements:

(I) The set $S = \{x \in [1, \infty) : f(x) = f^{-1}(x)\}$ contains exactly two elements, and

(II) The set $S = \{x \in [1, \infty) : f(x) = f^{-1}(x + 1)\}$ is an empty set,

(A) only (I) is TRUE

(B) only (II) is TRUE

- (C) both (I) and (II) are TRUE
(D) neither (I) nor (II) is TRUE
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2. Let $S = \{z \in \mathbb{C} : z^2 + 4z + 16 = 0\}$. Then $\sum_{z \in S} |z + \sqrt{3}i|^2$ is equal to:

- (A) 42
(B) 23
(C) 27
(D) 38
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3. If the system of equations:

$$x + y + z = 5$$

$$x + 2y + 3z = 9$$

$$x + 3y + \lambda z = \mu$$

has infinitely many solutions, then the value of $\lambda + \mu$ is:

- (A) 16
(B) 18
(C) 19
(D) 21
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4. If $\alpha = 1$ and $\beta = 1 + i\sqrt{2}$, where $i = \sqrt{-1}$ are two roots of the equation $x^3 + ax^2 + bx + c = 0$, $a, b, c \in \mathbb{R}$, then $\int_{-1}^1 (x^3 + ax^2 + bx + c)dx$ is equal to:

- (A) -2
(B) -4
(C) -8
(D) -10
-

5. If the quadratic equation $(\lambda + 2)x^2 - 3\lambda x + 4\lambda = 0$, $\lambda \neq -2$, has two positive roots, then the number of possible integral values of λ is:

- (A) 1
 - (B) 2
 - (C) 3
 - (D) 4
-

6. Let $A = \begin{bmatrix} 1 & 2 & 7 \\ 4 & -2 & 8 \\ 3 & 8 & -7 \end{bmatrix}$ and $\det(A - \alpha I) = 0$, where α is a real number. If the largest possible value of α is p , then the circle $(x - p)^2 + (y - 2p)^2 = 320$, intersects the co-ordinate axes at

- (A) 1 point
 - (B) 2 points
 - (C) 3 points
 - (D) 4 points
-

7. Let $\alpha = \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots \infty$ and $\beta = \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \dots \infty$. Then the value of $(0.2)^{\log_{\sqrt{5}}(\alpha)} + (0.04)^{\log_5(\beta)}$ is equal to:

- (A) 4
 - (B) 5
 - (C) 8
 - (D) 25
-

8. For 10 observations $x_1, x_2, \dots, x_{10}$, if $\sum_{i=1}^{10} (x_i + 2)^2 = 180$ and $\sum_{i=1}^{10} (x_i - 1)^2 = 90$, then their standard deviation is:

- (A) 2
 - (B) $\sqrt{3}$
 - (C) $2\sqrt{2}$
 - (D) 3
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9. In the expansion of $\left(9x - \frac{1}{3\sqrt{x}}\right)^{18}$, $x > 0$, if the term independent of x is $(221)k$, then k is

equal to:

- (A) 84
 - (B) 78
 - (C) 168
 - (D) 198
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10. Let $P(3 \cos \alpha, 2 \sin \alpha)$, $\alpha \neq 0$, be a point on the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$. Q be a point on the circle $x^2 + y^2 - 14x - 14y + 82 = 0$ and R be a point on the line $x + y = 5$ such that the centroid of the triangle PQR is $(2 + \cos \alpha, 3 + \frac{2}{3} \sin \alpha)$. Then the sum of the ordinates of all possible points R is:

- (A) 6
 - (B) 2
 - (C) 4
 - (D) 8
-

11. Let $H : \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ be a hyperbola such that the distance between its foci is 6 and the distance between its directrices is $\frac{8}{3}$. If the line $x = \alpha$ intersects the hyperbola H at the points A and B such that the area of the triangle AOB is $4\sqrt{15}$, where O is the origin, then α^2 equals

- (A) 12
 - (B) 16
 - (C) 24
 - (D) 25
-

12. $\max_{0 \leq x \leq \pi} \left(16 \sin\left(\frac{x}{2}\right) \left| \cos^3\left(\frac{x}{2}\right) \right| \right)$ is equal to:

- (A) $\frac{3\sqrt{3}}{2}$
 - (B) $3\sqrt{3}$
 - (C) $4\sqrt{3}$
 - (D) $6\sqrt{3}$
-

13. The shortest distance between the lines

$$\vec{r} = \left(\frac{1}{3}\hat{i} + 2\hat{j} + \frac{8}{3}\hat{k}\right) + \lambda(2\hat{i} - 5\hat{j} + 6\hat{k})$$

and $\vec{r} = \left(-\frac{2}{3}\hat{i} - \frac{1}{3}\hat{k}\right) + \mu(\hat{j} - \hat{k})$, $\lambda, \mu \in \mathbb{R}$, is:

- (A) $\sqrt{5}$
 - (B) 3
 - (C) $2\sqrt{3}$
 - (D) $\sqrt{15}$
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14. If $(2\alpha + 1, \alpha^2 - 3\alpha, \frac{\alpha-1}{2})$ is the image of $(\alpha, 2\alpha, 1)$ in the line $\frac{x-2}{3} = \frac{y-1}{2} = \frac{z}{1}$, then the possible value(s) of α is (are)

- (A) Only 3
 - (B) Only 3 and -1
 - (C) Only 3, 1/4 and -1
 - (D) Only 3 and 1/4
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15. Let $\hat{u}$ and $\hat{v}$ be unit vectors inclined at an acute angle such that $|\hat{u} \times \hat{v}| = \frac{\sqrt{3}}{2}$. If $\vec{A} = \lambda\hat{u} + \hat{v} + (\hat{u} \times \hat{v})$, then λ is equal to:

- (A) $\frac{4}{3}(\vec{A} \cdot \hat{u}) - \frac{2}{3}(\vec{A} \cdot \hat{v})$
 - (B) $\frac{2}{3}(\vec{A} \cdot \hat{u}) - \frac{1}{3}(\vec{A} \cdot \hat{v})$
 - (C) $\frac{4}{3}(\vec{A} \cdot \hat{u}) + \frac{2}{3}(\vec{A} \cdot \hat{v})$
 - (D) $(\vec{A} \cdot \hat{u}) - \frac{1}{2}(\vec{A} \cdot \hat{v})$
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16. Let for some $\alpha \in \mathbb{R}$, $f : \mathbb{R} \rightarrow \mathbb{R}$ be a function satisfying $f(x+y) = f(x) + 2y^2 + y + \alpha xy$ for all $x, y \in \mathbb{R}$. If $f(0) = -1$ and $f(1) = 2$, then the value of $\sum_{n=1}^5 (\alpha + f(n))$ is:

- (A) 110
- (B) 140
- (C) 150
- (D) 170

17. Let $A = \{(a, b, c) : a, b, c \text{ are non-negative integers and } a + b + 2c = 22\}$. Then $n(A)$ is equal to:

- (A) 121
 - (B) 124
 - (C) 144
 - (D) 169
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18. The area of the region bounded by the curves $x + 3y^2 = 0$ and $x + 4y^2 = 1$ is equal to:

- (A) $\frac{1}{3}$
 - (B) $\frac{2}{3}$
 - (C) $\frac{4}{3}$
 - (D) $\frac{5}{3}$
-

19. Let $y = y(x)$ be the solution of the differential equation: $\frac{dy}{dx} + \left(\frac{6x^2 + (3x^2 + 2x^3 + 4)e^{-2x}}{(x^3 + 2)(2 + e^{-2x})} \right) y = 2 + e^{-2x}$, $x \in (-1, 2)$, satisfying $y(0) = \frac{3}{2}$. If $y(1) = \alpha(2 + e^{-2})$, then α is equal to:

- (A) $\frac{13}{8}$
 - (B) $\frac{6}{13}$
 - (C) $\frac{12}{13}$
 - (D) $\frac{13}{12}$
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20. The integral $\int_0^1 \cot^{-1}(1 + x + x^2) dx$ is equal to:

- (A) $2 \tan^{-1} 2 + \frac{1}{2} \log_e \left(\frac{5}{4} \right) + \frac{\pi}{2}$
 - (B) $2 \tan^{-1} 2 + \frac{1}{2} \log_e \left(\frac{5}{4} \right) - \frac{\pi}{2}$
 - (C) $2 \tan^{-1} 2 - \frac{1}{2} \log_e \left(\frac{5}{4} \right) + \frac{\pi}{2}$
 - (D) $2 \tan^{-1} 2 - \frac{1}{2} \log_e \left(\frac{5}{4} \right) - \frac{\pi}{2}$
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21. From a month of 31 days, 3 different dates are selected at random. If the probability that these dates are in an increasing A.P is equal to a/b , where $a, b \in \mathbb{N}$ and $\gcd(a, b) = 1$, then $a + b$ is equal to _____

22. Let $f(x) = \begin{cases} e^{x-1}, & x < 0 \\ x^2 - 5x + 6, & x \geq 0 \end{cases}$ and $g(x) = f(|x|) + |f(x)|$. If the number of points where g is not continuous and is not differentiable are α and β respectively, then $\alpha + \beta$ is equal to _____.

23. Let A, B be points on the two half-lines $x - \sqrt{3}|y| = \alpha, \alpha > 0$ at a distance of α from their point of intersection P. The line segment AB meets the angle bisector of the given half-lines at the point Q. If $PQ = \frac{9}{2}$ and R is the radius of the circumcircle of $\triangle PAB$, then $\frac{\alpha^2}{R}$ is equal to _____

24. Let A, B and C be the vertices of a variable right angled triangle inscribed in the parabola $y^2 = 16x$. Let the vertex B containing the right angle be $(4, 8)$ and the locus of the centroid of $\triangle ABC$ be a conic C_0 . Then three times the length of latus rectum of C_0 is _____.

25. Let f be a twice differentiable function such that $f(x) = \int_0^x \tan(t-x)dt - \int_0^x f(t) \tan t dt, x \in (-\frac{\pi}{2}, \frac{\pi}{2})$. Then $f''(\frac{\pi}{6}) + 12f'(-\frac{\pi}{6}) + f(\frac{\pi}{6})$ is equal to _____.
