



General Instructions

- (i) **Duration:** The total duration of the examination is 1 hours (60 minutes).
- (ii) **Total Marks:** The complete Mathematics paper carries a maximum of 100 marks.
- (iii) **Structure:** The paper consists of two sections:
 - **Section A:** 20 Multiple Choice Questions (MCQs).
 - **Section B:** 5 Numerical Value Type Questions.
- (iv) **Compulsory Questions:** All 25 questions are compulsory.
- (v) Each question has four options. Only **one** option is correct.
- (vi) **Correct Answer:** +4 marks.
- (vii) **Incorrect Answer:** –1 mark (Negative marking).
- (viii) **Unanswered/Marked for Review:** 0 marks.

Mathematics Section - A

1. Let $[\cdot]$ denote the greatest integer function. If the domain of the function $f(x) = \cos^{-1}\left(\frac{4x+2[x]}{3}\right)$ is $[\alpha, \beta]$, then $12(\alpha + \beta)$ is equal to:

- (A) 6
- (B) 8
- (C) 9
- (D) 4

2. If the set of all solutions of $|x^2 + x - 9| = |x| + |x^2 - 9|$ is $[\alpha, \beta] \cup [\gamma, \infty)$, then $(\alpha^2 + \beta^2 + \gamma^2)$ is equal to:

- (A) 9
 - (B) 18
 - (C) 36
 - (D) 72
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3. Let z be a complex number such that $|z + 2| = |z - 2|$ and $\arg\left(\frac{z+3}{z-i}\right) = \frac{\pi}{4}$. Then $|z|^2$ is equal to:

- (A) 9
 - (B) 4
 - (C) 5
 - (D) 1
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4. The number of functions $f : \{1, 2, 3, 4\} \rightarrow \{a, b, c\}$, which are not onto, is:

- (A) 48
 - (B) 45
 - (C) 51
 - (D) 35
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5. Let $S = \left\{ A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} : a, b, c, d \in \{0, 1, 2, 3, 4\} \text{ and } A^2 - 4A + 3I = 0 \right\}$ be a set of 2×2 matrices. Then the number of matrices in S , for which the sum of the diagonal elements is equal to 4, is:

- (A) 20
 - (B) 17
 - (C) 21
 - (D) 19
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6. Let $A = \begin{bmatrix} 1 & 1 & 2 \\ -2 & 0 & 1 \\ 1 & 3 & 5 \end{bmatrix}$. Then the sum of all elements of the matrix $\text{adj}(\text{adj}(2(\text{adj}A)^{-1}))$ is equal to:

- (A) 3
- (B) 4
- (C) -4
- (D) -3

7. The first term of an A.P. of 30 non-negative terms is $\frac{10}{3}$. If the sum of this A.P. is the cube of its last term, then its common difference is:

- (A) $\frac{5}{87}$
- (B) $\frac{25}{83}$
- (C) $\frac{15}{29}$
- (D) $\frac{5}{29}$

8. The number of ways, of forming a queue of 4 boys and 3 girls such that all the girls are not together, is:

- (A) 5040
- (B) 3050
- (C) 3410
- (D) 4320

9. Let the smallest value of $k \in \mathbb{N}$, for which the coefficient of x^3 in $(1+x)^3 + (1+x)^4 + (1+x)^5 + \dots + (1+x)^{99} + (1+kx)^{100}$, $x \neq 0$, is $(43n + \frac{101}{4}) \binom{100}{3}$ for some $n \in \mathbb{N}$, be p . Then the value of $p+n$ is:

- (A) 10
- (B) 11

(C) 12

(D) 13

10. Suppose that the mean and median of the non-negative numbers $21, 8, 17, a, 51, 103, b, 13, 67, (a > b)$, are 40 and 21, respectively. If the mean deviation about the median is 26, then $2a$ is equal to:

(A) 109

(B) 117

(C) 161

(D) 131

11. Let the line $L_1 : x + 3 = 0$ intersect the lines $L_2 : x - y = 0$ and $L_3 : 3x + y = 0$ at the points A and B, respectively. Let the bisector of the obtuse angle between the lines L_2 and L_3 intersect the line L_1 at the point C. Then $BC^2 : AC^2$ is equal to:

(A) 5:1

(B) 1:5

(C) 2:3

(D) 3:2

12. Let the vertex A of a triangle ABC be $(1, 2)$, and the mid-point of the side AB be $(5, -1)$. If the centroid of this triangle is $(3, 4)$ and its circumcenter is (α, β) , then $21(\alpha + \beta)$ is equal to:

(A) 309

(B) 403

(C) 497

(D) 524

13. Suppose that two chords, drawn from the point $(1, 2)$ on the circle $x^2 + y^2 + x - 3y = 0$ are bisected by the y-axis. If the other ends of these chords are R and S, and the mid point of the line segment RS is (α, β) , then $6(\alpha + \beta)$ is equal to:

- (A) 1
 - (B) 3
 - (C) 4
 - (D) 6
-

14. A line with direction ratios $1, -1, 2$ intersects the lines $\frac{x}{2} = \frac{y}{3} = \frac{z+1}{3}$ and $\frac{x+1}{-1} = \frac{y-2}{1} = \frac{z}{4}$ at the points P and Q, respectively. If the length of the line segment PQ is α , then $225\alpha^2$ is equal to:

- (A) 1024
 - (B) 1014
 - (C) 1104
 - (D) 1204
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15. The square of the distance of the point $(-2, -8, 6)$ from the line $\frac{x-1}{1} = \frac{y-1}{2} = \frac{z}{-1}$ along the line $\frac{x+5}{1} = \frac{y+5}{-1} = \frac{z}{2}$ is equal to:

- (A) 3
 - (B) 6
 - (C) 8
 - (D) 12
-

16. If $y = \tan^{-1} \left(\frac{3 \cos x - 4 \sin x}{4 \cos x + 3 \sin x} \right) + 2 \tan^{-1} \left(\frac{x}{1 + \sqrt{1-x^2}} \right)$, then $\frac{dy}{dx}$ at $x = \frac{\sqrt{3}}{2}$ is equal to:

- (A) 3
 - (B) -1
 - (C) 1
 - (D) 2
-

17. Let f be a real polynomial of degree n such that $f(x) = f'(x)f''(x)$, for all $x \in \mathbb{R}$. If $f(0) = 0$, then $36 \left(f'(2) + f''(2) + \int_0^2 f(x) dx \right)$ is equal to:

- (A) 42
(B) 46
(C) 56
(D) 66
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18. The area of the region $\{(x, y) : y \leq \pi - |x|, y \leq |x \sin x|, y \geq 0\}$ is:

- (A) $1 + \frac{\pi^2}{8}$
(B) $2 + \frac{\pi^2}{4}$
(C) $\frac{\pi^2}{8} - 1$
(D) $4 + \frac{\pi^2}{2}$
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19. Let $\int_{-2}^2 (|\sin x| + [x \sin x]) dx = 2(3 - \cos 2) + \beta$, where $[\cdot]$ is the greatest integer function. Then $\beta \sin\left(\frac{\beta}{2}\right)$ equals:

- (A) 1
(B) 2
(C) 4
(D) 8
-

20. Let $y = y(x)$ be the solution of the differential equation $\frac{dy}{dx} = (1 + x + x^2)(1 - y + y^2)$, $y(0) = \frac{1}{2}$. Then $(2y(1) - 1)$ is equal to:

- (A) $\sqrt{3} \tan\left(\frac{11\sqrt{3}}{6}\right)$
(B) $\frac{\sqrt{3}}{2} \tan\left(\frac{11\sqrt{3}}{12}\right)$
(C) $\sqrt{3} \tan\left(\frac{11\sqrt{3}}{12}\right)$
(D) $\frac{\sqrt{3}}{2} \tan\left(\frac{11\sqrt{3}}{6}\right)$
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Mathematics Section - B

21. A coin is tossed 8 times. If the probability that exactly 4 heads appear in the first six tosses and exactly 3 heads appear in the last five tosses is p , then $96p$ is equal to _____.

22. Consider the parabola $P : y^2 = 4kx$ and the ellipse $E : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. Let the line segment joining the points of intersection of P and E, be their latus rectums. If the eccentricity of E is e , then $e^2 + 2\sqrt{2}$ is equal to _____.

23. If $A = \frac{\sin 3^\circ}{\cos 9^\circ} + \frac{\sin 9^\circ}{\cos 27^\circ} + \frac{\sin 27^\circ}{\cos 81^\circ}$ and $B = \tan 81^\circ - \tan 3^\circ$, then $\frac{B}{A}$ is equal to _____.

24. Let $\vec{a}_k = (\tan \theta_k)\hat{i} + \hat{j}$ and $\vec{b}_k = \hat{i} - (\cot \theta_k)\hat{j}$, where $\theta_k = \frac{2^{k-1}\pi}{2^n+1}$, for some $n \in \mathbb{N}, n > 5$. Then the value of $\frac{\sum_{k=1}^n |\vec{a}_k|^2}{\sum_{k=1}^n |\vec{b}_k|^2}$ is _____.

25. The number of points, at which the function $f(x) = \max\{6x, 2+3x^2\} + |x-1| \cos \left| x^2 - \frac{1}{4} \right|, x \in (-\pi, \pi)$, is not differentiable, is _____.
