JEE Main 2026 April 2 Shift 2

Question Paper with Solution

Conducted by National Testing Agency (NTA)

General Instructions

(i) **Duration:** The total duration of the examination is 3 hours (180 minutes).

(ii) **Total Marks:** The complete paper carries a maximum of 300 marks.

(iii) **Structure:** The paper has 3 parts and each consists of two sections:

- **Section A:** 20 Multiple Choice Questions (MCQs).
- **Section B:** 5 Numerical Value Type Questions.

(iv) **Compulsory Questions:** All 75 questions are compulsory.

(v) Each question has four options. Only **one** option is correct.

(vi) Correct Answer: +4 marks.

(vii) **Incorrect Answer:** −1 mark (Negative marking).

(viii) **Unanswered/Marked for Review:** 0 marks.

Mathematics Section - A

1. Let α, β be the roots of the equation $x^2 - 3x + r = 0$, and $\frac{\alpha}{2}, 2\beta$ be the roots of the equation $x^2 + 3x + r = 0$. If the roots of the equation $x^2 + 6x = m$ are $2\alpha + \beta + 2r$ and $\alpha - 2\beta - \frac{r}{2}$, then m is equal to:

(A) -135

(B) -567

(C) 135

(D) 567

Correct Answer: (D) 567

Solution:

Step 1: Understanding the Concept:

The sum and product of the roots of a quadratic equation $ax^2 + bx + c = 0$ are given by $-b/a$ and c/a, respectively.

We apply these relations to the given quadratic equations to find the values of α, β, and r.

Step 2: Key Formula or Approach:

For the first equation $x^2 - 3x + r = 0$, the sum of roots is $\alpha + \beta = 3$ and the product is $\alpha\beta = r$.

For the second equation $x^2 + 3x + r = 0$, the sum of roots is $\frac{\alpha}{2} + 2\beta = -3$ and the product is $\left(\frac{\alpha}{2}\right)(2\beta) = r$.

Step 3: Detailed Explanation:

The product of roots condition for the second equation yields $\alpha\beta = r$, which is consistent with the first equation.

Now we solve the system of linear equations for the sum of roots.

$$\alpha + \beta = 3$$

$$\frac{\alpha}{2} + 2\beta = -3 \implies \alpha + 4\beta = -6$$

Subtracting the first equation from the second yields a new relation.

$$3\beta = -9 \implies \beta = -3$$

Substituting $\beta = -3$ into the first equation gives the value of α.

$$\alpha - 3 = 3 \implies \alpha = 6$$

Now we find r by multiplying the roots.

$$r = \alpha\beta = (6)(-3) = -18$$

Next, we calculate the roots R_1 and R_2 for the final equation $x^2 + 6x - m = 0$.

$$R_1 = 2\alpha + \beta + 2r = 2(6) + (-3) + 2(-18) = 12 - 3 - 36 = -27$$

$$R_2 = \alpha - 2\beta - \frac{r}{2} = 6 - 2(-3) - \left(\frac{-18}{2}\right) = 6 + 6 + 9 = 21$$

The sum of these roots is $R_1 + R_2 = -27 + 21 = -6$, which matches the coefficient of x in $x^2 + 6x - m = 0$.

The product of the roots is given by the constant term $-m$.

$$R_1 R_2 = -m$$

$$(-27)(21) = -m$$

$$-567 = -m \implies m = 567$$

Step 4: Final Answer:

The value of m is 567.

Quick Tip: When given multiple equations with shared parameters, use the sum and product of roots simultaneously to quickly isolate the variables.

Always rewrite equations like $x^2 + 6x = m$ into standard form $x^2 + 6x - m = 0$ to correctly identify the constant term.

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2. Let the circles $C_1 : |z| = r$ and $C_2 : |z - 3 - 4i| = 5, z \in \mathbb{C}$, be such that C_2 lies within C_1. If z_1 moves on C_1, z_2 moves on C_2 and $\min |z_1 - z_2| = 2$, then $\max |z_1 - z_2|$ is equal to:

(A) 12

(B) 17

(C) 22

(D) 24

Correct Answer: (C) 22

Solution:

Step 1: Understanding the Concept:

The distance between two complex numbers z_1 and z_2 is geometrically represented by $|z_1 - z_2|$. For two circles where one lies completely inside the other, the minimum and maximum distances between points on the two circles occur along the line connecting their centers.

Step 2: Key Formula or Approach:

Let C_1 have center $O_1(0,0)$ and radius r.

Let C_2 have center $O_2(3,4)$ and radius $R = 5$.

The distance between their centers is $d = |3+4i| = \sqrt{3^2+4^2} = 5$.

Since C_2 lies completely inside C_1, the minimum distance is $r-(d+R)$ and the maximum distance is $r+d+R$.

Step 3: Detailed Explanation:

Given that the minimum distance is 2, we can set up the equation for r.

$$\min|z_1 - z_2| = r-(d+R) = 2$$

Substitute the known values $d = 5$ and $R = 5$ into the equation.

$$r-(5+5) = 2$$

$$r-10 = 2 \implies r = 12$$

Now we need to find the maximum possible distance between a point on C_1 and a point on C_2.

$$\max|z_1 - z_2| = r+(d+R)$$

$$\max|z_1 - z_2| = 12+(5+5) = 12+10 = 22$$

Step 4: Final Answer:

The maximum distance $\max|z_1 - z_2|$ is 22.

Quick Tip: For nested circles, the minimum distance is $R_{outer} - (d + R_{inner})$ and the maximum distance is $R_{outer} + d + R_{inner}$, where d is the distance between their centers.

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3. If the system of equations

$x + 5y + 6z = 4$

$2x + 3y + 4z = 7$

$x + 6y + az = b$

has infinitely many solutions, then the point (a, b) lies on the line

(A) $y - x = 3$

(B) $x - y = 3$

(C) $x + y = 11$

(D) $x + y = 12$

Correct Answer: (B) $x - y = 3$

Solution:

Step 1: Understanding the Concept:

A system of three linear equations has infinitely many solutions if the determinant of the coefficient matrix is zero and the equations are linearly dependent.

Alternatively, we can eliminate one variable and force the resulting two equations in two variables to be identical.

Step 2: Key Formula or Approach:

We will eliminate x from the given system to form two identical equations in y and z.

Equation 1 is $x + 5y + 6z = 4$.

Equation 2 is $2x + 3y + 4z = 7$.

Equation 3 is $x + 6y + az = b$.

Step 3: Detailed Explanation:

Perform the operation (Equation 2) - 2× (Equation 1) to eliminate x.

$$(2x + 3y + 4z) - 2(x + 5y + 6z) = 7 - 2(4)$$

$$-7y - 8z = -1 \implies 7y + 8z = 1$$

This can be written as $y + \frac{8}{7}z = \frac{1}{7}$.

Now, perform (Equation 3) - (Equation 1) to eliminate x again.

$$(x + 6y + az) - (x + 5y + 6z) = b - 4$$

$$y + (a - 6)z = b - 4$$

For the system to have infinitely many solutions, these two newly formed equations must represent the same line.

By comparing the coefficients of z and the constant terms, we get the values of a and b.

$$a - 6 = \frac{8}{7} \implies a = 6 + \frac{8}{7} = \frac{50}{7}$$

$$b - 4 = \frac{1}{7} \implies b = 4 + \frac{1}{7} = \frac{29}{7}$$

Now we have the coordinates of the point $(a, b) = \left(\frac{50}{7}, \frac{29}{7}\right)$.

We check which option satisfies this point by finding $a - b$.

$$a - b = \frac{50}{7} - \frac{29}{7} = \frac{21}{7} = 3$$

Thus, the point satisfies the line equation $x - y = 3$.

Step 4: Final Answer:

The point (a, b) lies on the line $x - y = 3$.

Quick Tip: Instead of using Cramer's rule which can be lengthy, eliminating one variable and comparing the remaining equations is often much faster for "infinitely many solutions" problems.

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4. Let $a_1, a_2, a_3, \ldots$ be an A.P. and $g_1 = a_1, g_2, g_3, \ldots$ be an increasing G.P. If $a_1 = a_2 + g_2 = 1$ and $a_3 + g_3 = 4$, then $a_{10} + g_5$ is equal to:

(A) 81

(B) 76

(C) 62

(D) 55

Correct Answer: (D) 55

Solution:

Step 1: Understanding the Concept:

An Arithmetic Progression (A.P.) has a common difference d, so $a_n = a + (n-1)d$.

A Geometric Progression (G.P.) has a common ratio r, so $g_n = a \cdot r^{n-1}$.

We can use the given equations to find the first term a, the difference d, and the ratio r.

Step 2: Key Formula or Approach:

Let the first term be $a_1 = g_1 = a$.

From the problem, $a_1 = 1$, so $a = 1$.

The first given condition is $a_2 + g_2 = 1$.

The second given condition is $a_3 + g_3 = 4$.

Step 3: Detailed Explanation:

Substitute the general forms into the first condition.

$$(a+d) + ar = 1$$

Since $a = 1$, this simplifies nicely.

$$(1+d) + r = 1 \implies d + r = 0 \implies d = -r$$

Now, substitute the general forms into the second condition.

$$(a+2d) + ar^2 = 4$$

Substitute $a = 1$ and $d = -r$.

$$1 + 2(-r) + r^2 = 4$$

$$r^2 - 2r - 3 = 0$$

Factoring the quadratic equation yields the possible values for r.

$$(r-3)(r+1)=0$$

This gives $r=3$ or $r=-1$.

Since the G.P. is increasing and $g_1=1>0$, the common ratio must be greater than 1.

Therefore, we select $r=3$.

This means $d=-r=-3$.

We need to find the value of $a_{10}+g_5$.

Calculate the 10th term of the A.P.

$$a_{10}=a+9d=1+9(-3)=1-27=-26$$

Calculate the 5th term of the G.P.

$$g_5=a\cdot r^4=1\cdot(3)^4=81$$

Finally, compute the required sum.

$$a_{10}+g_5=-26+81=55$$

Step 4: Final Answer:

The sum $a_{10}+g_5$ is 55.

Quick Tip: Pay attention to keywords like "increasing G.P.".

Since the first term is positive, an increasing G.P. requires a common ratio $r>1$.

This instantly helps in discarding the negative root.

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5. The sum $\frac{1^3}{1}+\frac{1^3+2^3}{1+3}+\frac{1^3+2^3+3^3}{1+3+5}+\ldots$ up to 8 terms, is:

(A) 70

(B) 71

(C) 72

(D) 73

Correct Answer: (B) 71

Solution:

Step 1: Understanding the Concept:

We first need to find the general n-th term T_n of the given series.

The numerator of the n-th term is the sum of the cubes of the first n natural numbers.

The denominator is the sum of the first n odd numbers.

Step 2: Key Formula or Approach:

The formula for the numerator sum is $1^3 + 2^3 + \cdots + n^3 = \left(\frac{n(n+1)}{2}\right)^2 = \frac{n^2(n+1)^2}{4}$.

The formula for the denominator sum is $1 + 3 + 5 + \cdots + (2n-1) = n^2$.

Thus, the general term is given by $T_n = \frac{n^2(n+1)^2/4}{n^2} = \frac{(n+1)^2}{4}$.

Step 3: Detailed Explanation:

We need the sum up to 8 terms, which is $S_8 = \sum_{n=1}^{8} T_n$.

$$S_8 = \sum_{n=1}^{8} \frac{(n+1)^2}{4} = \frac{1}{4}\sum_{n=1}^{8}(n+1)^2$$

To evaluate this efficiently, let $k = n+1$.

As n goes from 1 to 8, k goes from 2 to 9.

$$S_8 = \frac{1}{4}\sum_{k=2}^{9} k^2$$

We know the sum of squares formula is $\sum_{k=1}^{m} k^2 = \frac{m(m+1)(2m+1)}{6}$.

For $m = 9$, we calculate the total sum.

$$\sum_{k=1}^{9} k^2 = \frac{9 \times 10 \times 19}{6} = 15 \times 19 = 285$$

Since our sum starts from $k = 2$, we subtract the $k = 1$ term which is $1^2 = 1$.

$$\sum_{k=2}^{9} k^2 = 285 - 1 = 284$$

Now, calculate the final sum S_8.

$$S_8 = \frac{1}{4} \times 284 = 71$$

Step 4: Final Answer:

The sum up to 8 terms is 71.

Quick Tip: Memorizing standard series sums is crucial for competitive exams.
The sum of the first n odd numbers is always n^2, and the sum of cubes is the square of the sum of the first n integers.

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6. If for $3 \le r \le 30$, $\binom{30}{30-r} + 3\binom{30}{31-r} + 3\binom{30}{32-r} + \binom{30}{33-r} = \binom{m}{r}$, then m equals:

(A) 31
(B) 32
(C) 33
(D) 34

Correct Answer: (C) 33

Solution:

Step 1: Understanding the Concept:

We use the symmetric property of binomial coefficients which states $\binom{n}{k} = \binom{n}{n-k}$.
This property allows us to simplify the given expression into a much more recognizable form.

Step 2: Key Formula or Approach:

Apply the symmetry property to each term individually.
$\binom{30}{30-r} = \binom{30}{r}$
$\binom{30}{31-r} = \binom{30}{r-1}$
$\binom{30}{32-r} = \binom{30}{r-2}$
$\binom{30}{33-r} = \binom{30}{r-3}$
The given expression simplifies to the following sum.

$$S = \binom{30}{r} + 3\binom{30}{r-1} + 3\binom{30}{r-2} + \binom{30}{r-3}$$

Step 3: Detailed Explanation:

Notice that the coefficients $1, 3, 3, 1$ are exactly the binomial coefficients of $(1+x)^3$.

This suggests finding the coefficient of x^r in a product of two polynomials.

Consider the algebraic identity:

$$(1+x)^{30} \cdot (1+x)^3 = (1+x)^{33}$$

Let's find the coefficient of x^r on both sides of this identity.

On the left side, we expand both binomials.

$$\left(\cdots + \binom{30}{r}x^r + \binom{30}{r-1}x^{r-1} + \ldots\right) \times \left(\binom{3}{0} + \binom{3}{1}x + \binom{3}{2}x^2 + \binom{3}{3}x^3\right)$$

The term with x^r is obtained by multiplying pairs that give a combined degree of r.

$$\binom{30}{r}\binom{3}{0} + \binom{30}{r-1}\binom{3}{1} + \binom{30}{r-2}\binom{3}{2} + \binom{30}{r-3}\binom{3}{3}$$

Substitute the known values of $\binom{3}{k}$.

$$= \binom{30}{r}(1) + \binom{30}{r-1}(3) + \binom{30}{r-2}(3) + \binom{30}{r-3}(1)$$

This matches our simplified expression S perfectly.

On the right side of the identity $(1+x)^{33}$, the coefficient of x^r is simply $\binom{33}{r}$.

Therefore, we can equate the two representations.

$$\binom{33}{r} = \binom{m}{r} \implies m = 33$$

Step 4: Final Answer:

The value of m is 33.

Quick Tip: Vandermonde's Identity, $\sum \binom{m}{k}\binom{n}{r-k} = \binom{m+n}{r}$, is highly useful for solving sums of products of binomial coefficients.

Recognizing the sequence $1, 3, 3, 1$ as $\binom{3}{k}$ is the key here.

7. Let p_n denote the total number of triangles formed by joining the vertices of an n-side regular polygon. If $p_{n+1} - p_n = 66$, then the sum of all distinct prime divisors of n is:

(A) 7

(B) 8

(C) 5

(D) 6

Correct Answer: (C) 5

Solution:

Step 1: Understanding the Concept:

To form a triangle, we need to choose 3 distinct vertices from the n vertices of the polygon. The number of ways to do this is given by the combination formula $\binom{n}{3}$.

Step 2: Key Formula or Approach:

The total number of triangles formed is $p_n = \binom{n}{3}$.

We are given the mathematical relation:

$$p_{n+1} - p_n = 66$$

Substituting the combination formula into this relation yields:

$$\binom{n+1}{3} - \binom{n}{3} = 66$$

Step 3: Detailed Explanation:

Using Pascal's Identity $\binom{n}{k} + \binom{n}{k-1} = \binom{n+1}{k}$, we can rewrite the left side.

$$\binom{n+1}{3} - \binom{n}{3} = \binom{n}{2}$$

So, we have a simplified equation.

$$\binom{n}{2} = 66$$

Expand the combination formula for further solving.

$$\frac{n(n-1)}{2} = 66 \implies n(n-1) = 132$$

We need to find two consecutive positive integers whose product is 132.

Since $11 \times 12 = 132$, we can determine that $n = 12$.

The prime factorization of 12 is $2^2 \times 3$.

The distinct prime divisors of 12 are 2 and 3.

The sum of these distinct prime divisors is $2 + 3 = 5$.

Step 4: Final Answer:

The sum of all distinct prime divisors of n is 5.

Quick Tip: Pascal's identity $\binom{n+1}{r} - \binom{n}{r} = \binom{n}{r-1}$ simplifies consecutive binomial terms instantly. Always look for this pattern to avoid expanding factorials manually.

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8. A man throws a fair coin repeatedly. He gets 10 points for each head he throws and 5 points for each tail he throws. If the probability that he gets exactly 30 points is $\frac{m}{n}$, $\gcd(m, n) = 1$, then $m + n$ is equal to:

(A) 53

(B) 55

(C) 107

(D) 105

Correct Answer: (C) 107

Solution:

Step 1: Understanding the Concept:

Let's simplify the points system by dividing everything by 5.

So, Heads (H) gives 2 points, and Tails (T) gives 1 point.

We now need the probability of obtaining exactly 6 points in this scaled system.

Step 2: Key Formula or Approach:

The man can reach exactly 6 points in multiple sequences of coin tosses.

Let's list all possible combinations of Heads and Tails that sum to exactly 6 points.

- 0 Heads, 6 Tails: $0(2) + 6(1) = 6$
- 1 Head, 4 Tails: $1(2) + 4(1) = 6$
- 2 Heads, 2 Tails: $2(2) + 2(1) = 6$
- 3 Heads, 0 Tails: $3(2) + 0(1) = 6$

Step 3: Detailed Explanation:

Now, we calculate the probability for each valid combination using permutations.

Case 1: 6 Tails in a total of 6 tosses.

The number of ways is $\frac{6!}{6!} = 1$.

The probability is $1 \times \left(\frac{1}{2}\right)^6 = \frac{1}{64}$.

Case 2: 1 Head, 4 Tails in a total of 5 tosses.

The number of ways is $\frac{5!}{1!4!} = 5$.

The probability is $5 \times \left(\frac{1}{2}\right)^5 = \frac{5}{32} = \frac{10}{64}$.

Case 3: 2 Heads, 2 Tails in a total of 4 tosses.

The number of ways is $\frac{4!}{2!2!} = 6$.

The probability is $6 \times \left(\frac{1}{2}\right)^4 = \frac{6}{16} = \frac{24}{64}$.

Case 4: 3 Heads, 0 Tails in a total of 3 tosses.

The number of ways is $\frac{3!}{3!} = 1$.

The probability is $1 \times \left(\frac{1}{2}\right)^3 = \frac{1}{8} = \frac{8}{64}$.

The total probability of scoring exactly 30 points (or scaled 6 points) is the sum of these probabilities.

$$P = \frac{1}{64} + \frac{10}{64} + \frac{24}{64} + \frac{8}{64} = \frac{43}{64}$$

We are given that this probability is $\frac{m}{n} = \frac{43}{64}$.

Since 43 is a prime number, $\gcd(43, 64) = 1$ is satisfied.

Therefore, we deduce that $m = 43$ and $n = 64$.

Finally, we calculate $m+n = 43+64 = 107$.

Step 4: Final Answer:

The value of $m+n$ is 107.

Quick Tip: To simplify probability state-space problems like this, always scale down the values by dividing by their GCD.

It makes listing the valid combinations much easier and significantly less prone to arithmetic errors.

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9. The mean and variance of n observations are 8 and 16, respectively. If the sum of the first $(n-1)$ observations is 48 and the sum of squares of the first $(n-1)$ observations is 496, then the value of n is:

(A) 21

(B) 16

(C) 13

(D) 7

Correct Answer: (D) 7

Solution:

Step 1: Understanding the Concept:

For n observations $x_1, x_2, \ldots, x_n$, the mean is $\bar{x} = \frac{\sum x_i}{n}$ and the variance is $\sigma^2 = \frac{\sum x_i^2}{n} - \bar{x}^2$.

We are given partial sums for the first $(n-1)$ terms, which allows us to find expressions for the n-th term x_n and its square x_n^2.

Step 2: Key Formula or Approach:

Using the given mean $\bar{x} = 8$, we get $\sum_{i=1}^{n} x_i = 8n$.

Using the given variance $\sigma^2 = 16$, we get $\frac{\sum_{i=1}^{n} x_i^2}{n} - 8^2 = 16$.

This simplifies to $\sum_{i=1}^{n} x_i^2 = 80n$.

Step 3: Detailed Explanation:

We are given the sum of the first $(n-1)$ observations.

$$\sum_{i=1}^{n-1} x_i = 48$$

This allows us to write the n-th observation in terms of n.

$$x_n = \sum_{i=1}^{n} x_i - \sum_{i=1}^{n-1} x_i = 8n - 48$$

We are also given the sum of squares of the first $(n-1)$ observations.

$$\sum_{i=1}^{n-1} x_i^2 = 496$$

This allows us to write the square of the n-th observation.

$$x_n^2 = \sum_{i=1}^{n} x_i^2 - \sum_{i=1}^{n-1} x_i^2 = 80n - 496$$

Equating the square of x_n from the first relation to the second relation gives a quadratic equation.

$$(8n-48)^2 = 80n - 496$$

Factor out constants to heavily simplify the calculation.

$$[8(n-6)]^2 = 16(5n-31)$$

$$64(n^2 - 12n + 36) = 16(5n-31)$$

Divide both sides by 16.

$$4(n^2 - 12n + 36) = 5n - 31$$

$$4n^2 - 48n + 144 - 5n + 31 = 0$$

$$4n^2 - 53n + 175 = 0$$

Now, solve this quadratic equation for n.

$$n = \frac{53 \pm \sqrt{53^2 - 4(4)(175)}}{2(4)}$$

$$n = \frac{53 \pm \sqrt{2809 - 2800}}{8} = \frac{53 \pm \sqrt{9}}{8} = \frac{53 \pm 3}{8}$$

This yields two mathematical possibilities.

$n = \frac{56}{8} = 7$ or $n = \frac{50}{8} = 6.25$.

Since the number of observations n must be a positive integer, we conclude that $n = 7$.

Step 4: Final Answer:

The value of n is 7.

Quick Tip: Always simplify large algebraic equations by factoring out common terms before expanding squares.

It drastically reduces calculation errors when applying the quadratic formula.

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10. Let a circle pass through the origin and its centre be the point of intersection of two mutually perpendicular lines $x + (k-1)y + 3 = 0$ and $2x + k^2y - 4 = 0$. If the line $x - y + 2 = 0$ intersects the circle at the points A and B, then $(AB)^2$ is equal to:

(A) 10

(B) 27

(C) 18

(D) 34

Correct Answer: (C) 18

Solution:

Step 1: Understanding the Concept:

Two lines are geometrically perpendicular if the product of their slopes is -1.

After finding k, we can find the intersection of the two lines to determine the center of the circle C.

The distance from the center C to the origin $(0,0)$ directly gives the radius r.
Using the perpendicular distance d from C to the given secant line, we can find the length of the chord AB.

Step 2: Key Formula or Approach:
First, identify the slopes of the given lines.
For $L_1 : x+(k-1)y+3=0$, the slope is $m_1 = \frac{-1}{k-1}$.
For $L_2 : 2x+k^2y-4=0$, the slope is $m_2 = \frac{-2}{k^2}$.
The condition for perpendicular lines is $m_1m_2 = -1$.
The length of a chord is given by Pythagoras theorem: $AB = 2\sqrt{r^2-d^2}$.

Step 3: Detailed Explanation:
Apply the perpendicularity condition.

$$\left(\frac{-1}{k-1}\right)\left(\frac{-2}{k^2}\right) = -1$$

$$\frac{2}{k^2(k-1)} = -1 \implies k^3-k^2+2=0$$

By inspection of small integers, $k=-1$ satisfies the equation since $(-1)^3-(-1)^2+2 = -1-1+2=0$.
Substitute $k=-1$ back into the equations of the lines to get their concrete forms.
$L_1 : x-2y+3=0$
$L_2 : 2x+y-4=0 \implies y=4-2x$
Substitute y from L_2 into L_1 to find the intersection.

$$x-2(4-2x)+3=0 \implies x-8+4x+3=0 \implies 5x=5 \implies x=1$$

$$y=4-2(1)=2$$

So, the center of the circle is $C(1,2)$.
Since the circle passes through the origin $(0,0)$, the radius squared is the squared distance to the origin.

$$r^2 = (1-0)^2+(2-0)^2 = 1+4 = 5$$

Now, calculate the perpendicular distance d from the center $C(1,2)$ to the line $x-y+2=0$.

$$d = \frac{|1(1)-1(2)+2|}{\sqrt{1^2+(-1)^2}} = \frac{|1|}{\sqrt{2}} = \frac{1}{\sqrt{2}}$$

The length of the chord squared can now be computed.

$$(AB)^2 = (2\sqrt{r^2-d^2})^2 = 4(r^2-d^2)$$

Substitute the values $r^2 = 5$ and $d^2 = \frac{1}{2}$ into the expression.

$$(AB)^2 = 4\left(5-\frac{1}{2}\right) = 4\left(\frac{9}{2}\right) = 18$$

Step 4: Final Answer:

The value of $(AB)^2$ is 18.

Quick Tip: The length of a chord intersected by a line on a circle is always $2\sqrt{r^2-d^2}$. You do not need to find the actual intersection points A and B; just use the center's perpendicular distance d and the circle's radius r.

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11. Let O be the origin, and P and Q be two points on the rectangular hyperbola $xy = 12$ such that the mid point of the line segment PQ is $\left(\frac{1}{2}, -\frac{1}{2}\right)$. Then the area of the triangle OPQ equals:

(A) $\frac{3}{2}$

(B) $\frac{5}{2}$

(C) $\frac{7}{2}$

(D) $\frac{9}{2}$

Correct Answer: (C) $\frac{7}{2}$

Solution:

Step 1: Understanding the Concept:

For a conic section, the equation of a chord with a given midpoint (x_1, y_1) is given by $T = S_1$. After finding the equation of the line PQ, we can solve it simultaneously with the curve $xy = 12$ to find the exact coordinates of points P and Q.

Finally, we use the determinant method to find the area of triangle OPQ.

Step 2: Key Formula or Approach:

The equation of the hyperbola is $S \equiv xy - 12 = 0$.

The midpoint is $(x_1, y_1) = \left(\frac{1}{2}, -\frac{1}{2}\right)$.

The chord formula is $T = S_1$, which translates to $\frac{xy_1 + yx_1}{2} - 12 = x_1 y_1 - 12$.

Step 3: Detailed Explanation:

Substitute the coordinates of the midpoint into the chord formula.

$$\frac{x(-1/2) + y(1/2)}{2} = (1/2)(-1/2)$$

$$\frac{-x + y}{4} = \frac{-1}{4}$$

Multiply by 4 to get the equation of the line PQ.

$$-x + y = -1 \implies y = x - 1$$

Now, substitute $y = x - 1$ into the hyperbola equation $xy = 12$ to find points P and Q.

$$x(x-1) = 12$$

$$x^2 - x - 12 = 0$$

Factor the quadratic equation.

$$(x-4)(x+3) = 0$$

This gives the x-coordinates of P and Q as $x = 4$ and $x = -3$.

Using $y = x - 1$, the corresponding y-coordinates are $y = 3$ and $y = -4$.

So, the points are $P(4,3)$ and $Q(-3,-4)$.

The area of triangle OPQ with the origin $O(0,0)$ is given by the formula $\frac{1}{2}|x_1 y_2 - x_2 y_1|$.

$$\text{Area} = \frac{1}{2}|(4)(-4) - (-3)(3)|$$

$$\text{Area} = \frac{1}{2}|-16 + 9|$$

$$\text{Area} = \frac{1}{2}|-7| = \frac{7}{2}$$

Step 4: Final Answer:

The area of the triangle OPQ equals $\frac{7}{2}$.

Quick Tip: The chord formula $T = S_1$ is a massive time-saver for any conic section when the midpoint of a chord is known.
For $xy = c^2$, T is simply $\frac{xy_1+yx_1}{2}$.

---

12. Let the parabola $y = x^2 + px + q$ passing through the point $(1, -1)$ be such that the distance between its vertex and the x-axis is minimum. Then the value of $p^2 + q^2$ is:

(A) 2

(B) 4

(C) 5

(D) 8

Correct Answer: (B) 4

Solution:

Step 1: Understanding the Concept:

The vertex of a parabola given by $y = ax^2 + bx + c$ is located at $x = \frac{-b}{2a}$.
The distance from the vertex to the x-axis is simply the absolute value of the y-coordinate of the vertex.
We are given that the parabola passes through a specific point, which gives us a relation between p and q.

Step 2: Key Formula or Approach:

For the parabola $y = x^2 + px + q$, the x-coordinate of the vertex is $x_v = \frac{-p}{2}$.
The y-coordinate of the vertex is $y_v = \left(\frac{-p}{2}\right)^2 + p\left(\frac{-p}{2}\right) + q = \frac{-p^2}{4} + q$.
The condition that the parabola passes through $(1, -1)$ means $-1 = 1^2 + p(1) + q$.

Step 3: Detailed Explanation:

From the passing point condition, we can express q in terms of p.

$$-1 = 1 + p + q \implies p + q = -2 \implies q = -2 - p$$

Substitute this expression for q into the y-coordinate of the vertex.

$$y_v = \frac{-p^2}{4} + (-2 - p) = -\left(\frac{p^2}{4} + p + 2\right)$$

The distance from the vertex to the x-axis is $D = |y_v|$.

$$D = \left|\frac{p^2}{4} + p + 2\right|$$

To minimize this distance, we complete the square for the quadratic expression inside the absolute value.

$$D = \frac{1}{4}\left|p^2 + 4p + 8\right|$$

$$D = \frac{1}{4}\left|(p^2 + 4p + 4) + 4\right|$$

$$D = \frac{1}{4}\left|(p + 2)^2 + 4\right|$$

Since $(p + 2)^2 \geq 0$, the minimum value of the expression inside the absolute value occurs when $(p + 2)^2 = 0$.

This happens when $p = -2$.

With $p = -2$, we find the corresponding value of q.

$$q = -2 - (-2) = 0$$

We need to find the value of $p^2 + q^2$.

$$p^2 + q^2 = (-2)^2 + 0^2 = 4 + 0 = 4$$

Step 4: Final Answer:

The value of $p^2 + q^2$ is 4.

Quick Tip: To find the minimum or maximum of a quadratic function quickly, completing the square is often faster and less prone to sign errors than using calculus (derivatives).

---

13. Let $P = \{\theta \in [0, 4\pi] : \tan^2\theta \neq 1\}$ and $S = \{a \in \mathbb{Z} : 2(\cos^8\theta - \sin^8\theta)\sec 2\theta = a^2, \theta \in P\}$. Then $n(S)$ is:

(A) 0

(B) 1

(C) 2

(D) 3

Correct Answer: (A) 0

Solution:

Step 1: Understanding the Concept:

We are dealing with a trigonometric identity that needs simplification.

After simplifying the expression $2(\cos^8\theta - \sin^8\theta)\sec 2\theta$, we will find its range of values over the allowed domain of θ.

Since a must be an integer, a^2 must be a perfect square integer that falls within this range.

Step 2: Key Formula or Approach:

We use the difference of squares factorization: $x^4 - y^4 = (x^2 - y^2)(x^2 + y^2)$.

Apply this to $\cos^8\theta - \sin^8\theta = (\cos^4\theta - \sin^4\theta)(\cos^4\theta + \sin^4\theta)$.

Further, $\cos^4\theta - \sin^4\theta = (\cos^2\theta - \sin^2\theta)(\cos^2\theta + \sin^2\theta)$.

Recall that $\cos^2\theta + \sin^2\theta = 1$ and $\cos^2\theta - \sin^2\theta = \cos 2\theta$.

Step 3: Detailed Explanation:

Let's simplify the given expression for a^2.

$$a^2 = 2(\cos^8\theta - \sin^8\theta)\sec 2\theta$$

$$a^2 = 2(\cos^4\theta - \sin^4\theta)(\cos^4\theta + \sin^4\theta)\left(\frac{1}{\cos 2\theta}\right)$$

$$a^2 = 2(\cos^2\theta - \sin^2\theta)(\cos^2\theta + \sin^2\theta)(\cos^4\theta + \sin^4\theta)\left(\frac{1}{\cos 2\theta}\right)$$

Substitute the basic identities.

$$a^2 = 2(\cos 2\theta)(1)(\cos^4\theta + \sin^4\theta)\left(\frac{1}{\cos 2\theta}\right)$$

Since $\theta \in P \implies \tan^2\theta \neq 1 \implies \cos 2\theta \neq 0$, we can safely cancel $\cos 2\theta$.

$$a^2 = 2(\cos^4\theta + \sin^4\theta)$$

We can rewrite $\cos^4\theta + \sin^4\theta$ by completing the square.

$$\cos^4\theta + \sin^4\theta = (\cos^2\theta + \sin^2\theta)^2 - 2\sin^2\theta\cos^2\theta = 1 - \frac{1}{2}(4\sin^2\theta\cos^2\theta) = 1 - \frac{1}{2}\sin^2 2\theta$$

Substitute this back into the equation for a^2.

$$a^2 = 2\left(1 - \frac{1}{2}\sin^2 2\theta\right) = 2 - \sin^2 2\theta$$

Now we determine the range of this expression.

We know that $0 \le \sin^2 2\theta \le 1$.

However, the domain condition $\tan^2\theta \neq 1$ means $\theta \neq \frac{\pi}{4}, \frac{3\pi}{4}, \ldots$, which implies $2\theta \neq \frac{\pi}{2}, \frac{3\pi}{2}, \ldots$.

Therefore, $\sin^2 2\theta$ can never be exactly equal to 1.

So the range of $\sin^2 2\theta$ is $[0, 1)$.

This means the range of a^2 is $2 - [0, 1) = (1, 2]$.

We need $a \in \mathbb{Z}$, which means a^2 must be a perfect square integer.

The only integer in the interval $(1, 2]$ is 2.

If $a^2 = 2$, then $a = \pm\sqrt{2}$, which are not integers.

Since there is no integer a that satisfies the equation, the set S is empty.

Step 4: Final Answer:

The number of elements $n(S)$ is 0.

Quick Tip: Always strictly evaluate domain constraints.
Here, $\tan^2\theta \neq 1$ subtly removes the possibility of $a^2 = 1$, restricting the interval to $(1, 2]$ and leaving no perfect squares available.

---

14. Let the vectors $\vec{a} = -\hat{i} + \hat{j} + 3\hat{k}$ and $\vec{b} = \hat{i} + 3\hat{j} + \hat{k}$. For some $\lambda, \mu \in \mathbb{R}$, let $\vec{c} = \lambda\vec{a} + \mu\vec{b}$. If $\vec{c} \cdot (3\hat{i} - 6\hat{j} + 2\hat{k}) = 10$ and $\vec{c} \cdot (\hat{i} + \hat{j} + \hat{k}) = -2$, then $|\vec{c}|^2$ is equal to:

(A) 8
(B) 12
(C) 14
(D) 15

Correct Answer: (B) 12

Solution:

Step 1: Understanding the Concept:
We are given $\vec{c}$ as a linear combination of two known vectors $\vec{a}$ and $\vec{b}$.
By applying the distributive property of the dot product, we can generate a system of two linear equations in terms of the unknown scalars λ and μ.
Once λ and μ are found, we can construct $\vec{c}$ and compute its squared magnitude.

Step 2: Key Formula or Approach:
The dot product distributes over vector addition: $(\lambda\vec{a} + \mu\vec{b}) \cdot \vec{v} = \lambda(\vec{a} \cdot \vec{v}) + \mu(\vec{b} \cdot \vec{v})$.
Let $\vec{v}_1 = 3\hat{i} - 6\hat{j} + 2\hat{k}$ and $\vec{v}_2 = \hat{i} + \hat{j} + \hat{k}$.
The given conditions are $\vec{c} \cdot \vec{v}_1 = 10$ and $\vec{c} \cdot \vec{v}_2 = -2$.

Step 3: Detailed Explanation:
First, calculate the dot products of $\vec{a}$ and $\vec{b}$ with $\vec{v}_1$.

$$\vec{a} \cdot \vec{v}_1 = (-1)(3) + (1)(-6) + (3)(2) = -3 - 6 + 6 = -3$$

$$\vec{b} \cdot \vec{v}_1 = (1)(3) + (3)(-6) + (1)(2) = 3 - 18 + 2 = -13$$

Substitute these into the first condition.

$$\lambda(-3) + \mu(-13) = 10 \implies -3\lambda - 13\mu = 10 \quad \text{— (Eq 1)}$$

Next, calculate the dot products of $\vec{a}$ and $\vec{b}$ with $\vec{v}_2$.

$$\vec{a} \cdot \vec{v}_2 = (-1)(1) + (1)(1) + (3)(1) = -1 + 1 + 3 = 3$$

$$\vec{b} \cdot \vec{v}_2 = (1)(1) + (3)(1) + (1)(1) = 1 + 3 + 1 = 5$$

Substitute these into the second condition.

$$\lambda(3) + \mu(5) = -2 \implies 3\lambda + 5\mu = -2 \quad \text{— (Eq 2)}$$

Now, solve the linear system by adding (Eq 1) and (Eq 2).

$$(-3\lambda - 13\mu) + (3\lambda + 5\mu) = 10 - 2$$

$$-8\mu = 8 \implies \mu = -1$$

Substitute $\mu = -1$ into (Eq 2) to find λ.

$$3\lambda + 5(-1) = -2 \implies 3\lambda = 3 \implies \lambda = 1$$

Now that we have λ and μ, we construct the vector $\vec{c}$.

$$\vec{c} = (1)\vec{a} + (-1)\vec{b} = \vec{a} - \vec{b}$$

$$\vec{c} = (-\hat{i} + \hat{j} + 3\hat{k}) - (\hat{i} + 3\hat{j} + \hat{k}) = -2\hat{i} - 2\hat{j} + 2\hat{k}$$

Finally, calculate the squared magnitude $|\vec{c}|^2$.

$$|\vec{c}|^2 = (-2)^2 + (-2)^2 + 2^2 = 4 + 4 + 4 = 12$$

Step 4: Final Answer:

The value of $|\vec{c}|^2$ is 12.

Quick Tip: Pre-calculating the dot products of the basis vectors ($\vec{a}$ and $\vec{b}$) with the target vectors prevents messy algebra and reduces the problem to a simple 2×2 linear system.

---

15. Let the point A be the foot of perpendicular drawn from the point $P(a, b, 0)$ on the line $\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-\alpha}{3}$. If the midpoint of the line segment PA is $\left(0, \frac{3}{4}, -\frac{1}{4}\right)$, then the value of $a^2+b^2+\alpha^2$ is equal to:

(A) 1

(B) 2

(C) 6

(D) 9

Correct Answer: (A) 1

Solution:

Step 1: Understanding the Concept:

We are given a point P and the midpoint of the segment connecting P to its foot of perpendicular A on a line.

Using the midpoint formula, we can express the coordinates of A in terms of the unknowns a and b.

Since A is the foot of the perpendicular, it must lie on the given line, and the vector $\vec{PA}$ must be orthogonal to the line's direction vector.

Step 2: Key Formula or Approach:

The midpoint M of points $P(x_1, y_1, z_1)$ and $A(x_2, y_2, z_2)$ is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$.

The direction vector of the line is $\vec{d} = 2\hat{i} + \hat{j} + 3\hat{k}$.

The orthogonality condition is $\vec{PA} \cdot \vec{d} = 0$.

Step 3: Detailed Explanation:

Let the coordinates of point A be (x_2, y_2, z_2).

Using the given midpoint $\left(0, \frac{3}{4}, -\frac{1}{4}\right)$, we can set up equations.

$$\frac{a+x_2}{2}=0 \implies x_2=-a$$

$$\frac{b+y_2}{2}=\frac{3}{4} \implies y_2=\frac{3}{2}-b$$

$$\frac{0+z_2}{2}=-\frac{1}{4} \implies z_2=-\frac{1}{2}$$

So, point A is $\left(-a, \frac{3}{2}-b, -\frac{1}{2}\right)$.

Since A lies on the line $\frac{x-1}{2}=\frac{y-2}{1}=\frac{z-\alpha}{3}$, we substitute its coordinates.

$$\frac{-a-1}{2}=\frac{\frac{3}{2}-b-2}{1}=\frac{-\frac{1}{2}-\alpha}{3}=k$$

Simplify the middle term.

$$\frac{-a-1}{2}=-\frac{1}{2}-b \implies -a-1=-1-2b \implies a=2b$$

Now, form the vector $\vec{PA}$.

$$\vec{PA}=(-a-a)\hat{i}+\left(\frac{3}{2}-b-b\right)\hat{j}+\left(-\frac{1}{2}-0\right)\hat{k}=-2a\hat{i}+\left(\frac{3}{2}-2b\right)\hat{j}-\frac{1}{2}\hat{k}$$

Apply the orthogonality condition $\vec{PA}\cdot\vec{d}=0$.

$$2(-2a)+1\left(\frac{3}{2}-2b\right)+3\left(-\frac{1}{2}\right)=0$$

$$-4a+\frac{3}{2}-2b-\frac{3}{2}=0$$

$$-4a-2b=0 \implies 2a+b=0 \implies b=-2a$$

We now have a system: $a=2b$ and $b=-2a$.

Substitute b into the first equation: $a=2(-2a) \implies a=-4a \implies 5a=0 \implies a=0$.

Since $a=0$, it follows that $b=0$.

Now substitute a and b back into the line equation to find α.

$$\frac{0-1}{2}=\frac{-\frac{1}{2}-\alpha}{3}$$

$$-\frac{1}{2}=\frac{-1-2\alpha}{6}$$

$$-3=-1-2\alpha \implies 2\alpha=2 \implies \alpha=1$$

Finally, compute the required sum.

$$a^2 + b^2 + \alpha^2 = 0^2 + 0^2 + 1^2 = 1$$

Step 4: Final Answer:

The value of $a^2 + b^2 + \alpha^2$ is 1.

Quick Tip: When a point lies on a line, equating the parametric fractions immediately gives you relations between variables.

Always use the orthogonality condition ($\vec{PA} \cdot \vec{d} = 0$) to generate the final required equation.

---

16. Two adjacent sides of a parallelogram PQRS are given by $\vec{PQ} = \hat{j} + \hat{k}$ and $\vec{PS} = \hat{i} - \hat{j}$. If the side PS is rotated about the point P by an acute angle α in the plane of the parallelogram so that it becomes perpendicular to the side PQ, then $\sin^2\left(\frac{5\alpha}{2}\right) - \sin^2\left(\frac{\alpha}{2}\right)$ is equal to:

(A) $\frac{1}{2}$
(B) $\frac{\sqrt{3}}{2}$
(C) $\frac{\sqrt{3}}{4}$
(D) $\frac{2\sqrt{3}}{5}$

Correct Answer: (B) $\frac{\sqrt{3}}{2}$

Solution:

Step 1: Understanding the Concept:

We first need to find the initial angle between the adjacent sides $\vec{PQ}$ and $\vec{PS}$ using the dot product.

The problem states that side PS is rotated by an acute angle α until it becomes completely perpendicular to PQ.

This implies the new angle between the vectors is 90°.

Step 2: Key Formula or Approach:

The angle θ between two vectors $\vec{u}$ and $\vec{v}$ is given by $\cos\theta = \frac{\vec{u}\cdot\vec{v}}{|\vec{u}||\vec{v}|}$.
The trigonometric identity $\sin^2 A - \sin^2 B = \sin(A-B)\sin(A+B)$ will be used to simplify the final expression.

Step 3: Detailed Explanation:

Let's calculate the initial angle θ between $\vec{PQ}$ and $\vec{PS}$.

$$\vec{PQ} = 0\hat{i} + 1\hat{j} + 1\hat{k} \implies |\vec{PQ}| = \sqrt{0^2 + 1^2 + 1^2} = \sqrt{2}$$

$$\vec{PS} = 1\hat{i} - 1\hat{j} + 0\hat{k} \implies |\vec{PS}| = \sqrt{1^2 + (-1)^2 + 0^2} = \sqrt{2}$$

$$\cos\theta = \frac{(0)(1) + (1)(-1) + (1)(0)}{\sqrt{2}\sqrt{2}} = \frac{-1}{2}$$

Since $\cos\theta = -1/2$, the initial angle is $\theta = 120^\circ$.

The vector is rotated by an acute angle α to make the new angle 90°.

The new angle is $\theta \pm \alpha = 90^\circ$.

So, $120^\circ \pm \alpha = 90^\circ$.

Since α must be an acute angle ($\alpha < 90^\circ$), we take $\alpha = 120^\circ - 90^\circ = 30^\circ$.

Now we evaluate the required trigonometric expression.

$$E = \sin^2\left(\frac{5\alpha}{2}\right) - \sin^2\left(\frac{\alpha}{2}\right)$$

Substitute $\alpha = 30^\circ$.

$$E = \sin^2(75^\circ) - \sin^2(15^\circ)$$

Apply the identity $\sin^2 A - \sin^2 B = \sin(A-B)\sin(A+B)$ with $A = 75^\circ$ and $B = 15^\circ$.

$$E = \sin(75^\circ - 15^\circ)\sin(75^\circ + 15^\circ)$$

$$E = \sin(60^\circ)\sin(90^\circ)$$

We know that $\sin(60^\circ) = \frac{\sqrt{3}}{2}$ and $\sin(90^\circ) = 1$.

$$E = \left(\frac{\sqrt{3}}{2}\right)(1) = \frac{\sqrt{3}}{2}$$

Step 4: Final Answer:

The value of the expression is $\frac{\sqrt{3}}{2}$.

Quick Tip: Whenever you see a difference of squares of sine functions, immediately use the identity $\sin^2 A - \sin^2 B = \sin(A-B)\sin(A+B)$.

It turns complex fractional angles into standard, easy-to-evaluate angles.

---

17. The value of $\int_0^{20\pi}(\sin^4 x + \cos^4 x)dx$ is equal to:

(A) $\frac{15\pi}{2}$

(B) 25π

(C) 15π

(D) $\frac{25\pi}{2}$

Correct Answer: (C) 15π

Solution:

Step 1: Understanding the Concept:

The integrand $f(x) = \sin^4 x + \cos^4 x$ is a periodic function.

We need to determine its fundamental period and use the property of definite integrals for periodic functions: $\int_0^{nT} f(x)dx = n\int_0^T f(x)dx$.

Step 2: Key Formula or Approach:

The period of both $\sin^4 x$ and $\cos^4 x$ is π, but their sum actually has a shorter period.

Observe that $f(x+\pi/2) = \sin^4(x+\pi/2) + \cos^4(x+\pi/2) = \cos^4 x + \sin^4 x = f(x)$.

Thus, the fundamental period T is $\frac{\pi}{2}$.

Wallis' formula for integrals of the form $\int_0^{\pi/2}\sin^n x dx = \int_0^{\pi/2}\cos^n x dx = \frac{(n-1)(n-3)...}{n(n-2)...}\cdot\frac{\pi}{2}$ (for even n).

Step 3: Detailed Explanation:

Since the period is $T = \frac{\pi}{2}$, we can rewrite the upper limit 20π as $40\times\frac{\pi}{2}$.

$$I = \int_0^{40\cdot\frac{\pi}{2}} (\sin^4 x + \cos^4 x)dx = 40\int_0^{\pi/2} (\sin^4 x + \cos^4 x)dx$$

Using Wallis' formula for $n = 4$, we evaluate the integrals of $\sin^4 x$ and $\cos^4 x$ separately.

$$\int_0^{\pi/2} \sin^4 x dx = \frac{3\cdot 1}{4\cdot 2}\cdot\frac{\pi}{2} = \frac{3\pi}{16}$$

Similarly, by symmetry:

$$\int_0^{\pi/2} \cos^4 x dx = \frac{3\pi}{16}$$

Now, substitute these back into the total integral.

$$I = 40\left(\frac{3\pi}{16} + \frac{3\pi}{16}\right) = 40\left(\frac{6\pi}{16}\right)$$

Simplify the fraction.

$$I = 40\left(\frac{3\pi}{8}\right) = 5\cdot 3\pi = 15\pi$$

Step 4: Final Answer:

The value of the integral is 15π.

Quick Tip: Always check for the smallest period of trigonometric sums before integrating.
Using Wallis' integrals $\int_0^{\pi/2} \sin^n x dx$ is the fastest way to evaluate powers of sine and cosine over $[0, \pi/2]$.

---

18. Let $f(x)$ be a polynomial of degree 5, and have extrema at $x = 1$ and $x = -1$. If $\lim_{x\to 0}\left(\frac{f(x)}{x^3}\right) = -5$, then $f(2) - f(-2)$ is equal to:

(A) 0

(B) 50

(C) 92

(D) 112

Correct Answer: (D) 112

Solution:

Step 1: Understanding the Concept:

The limit condition tells us about the lowest degree terms of the polynomial.

For the limit $\lim_{x\to 0}\frac{f(x)}{x^3}$ to exist and equal a non-zero finite value, the polynomial $f(x)$ must not have any x^0, x^1, or x^2 terms.

The extrema conditions tell us that the derivative $f'(x)$ evaluates to zero at specific points.

Step 2: Key Formula or Approach:

Let the 5th degree polynomial be $f(x) = ax^5 + bx^4 + cx^3 + dx^2 + ex + f$.

Due to the limit condition $\lim_{x\to 0}\frac{f(x)}{x^3} = -5$, we must have $f = 0$, $e = 0$, $d = 0$, and $c = -5$.

So, $f(x) = ax^5 + bx^4 - 5x^3$.

For extrema at $x = \pm 1$, we use $f'(1) = 0$ and $f'(-1) = 0$.

Step 3: Detailed Explanation:

Find the first derivative of the simplified polynomial.

$$f'(x) = 5ax^4 + 4bx^3 - 15x^2$$

Apply the condition $f'(1) = 0$.

$$5a(1)^4 + 4b(1)^3 - 15(1)^2 = 0 \implies 5a + 4b - 15 = 0 \quad \text{— (Eq 1)}$$

Apply the condition $f'(-1) = 0$.

$$5a(-1)^4 + 4b(-1)^3 - 15(-1)^2 = 0 \implies 5a - 4b - 15 = 0 \quad \text{— (Eq 2)}$$

Add (Eq 1) and (Eq 2) to eliminate b.

$$10a - 30 = 0 \implies a = 3$$

Subtract (Eq 2) from (Eq 1) to find b.

$$8b = 0 \implies b = 0$$

Now we have the full equation for the polynomial.

$$f(x) = 3x^5 - 5x^3$$

We need to calculate $f(2) - f(-2)$.

First, calculate $f(2)$.

$$f(2) = 3(2)^5 - 5(2)^3 = 3(32) - 5(8) = 96 - 40 = 56$$

Since $f(x)$ only has odd powers, it is an odd function, meaning $f(-x) = -f(x)$.

So, $f(-2) = -f(2) = -56$.

Finally, compute the difference.

$$f(2) - f(-2) = 56 - (-56) = 112$$

Step 4: Final Answer:

The value of $f(2) - f(-2)$ is 112.

Quick Tip: A limit of the form $\lim_{x\to 0} \frac{f(x)}{x^n} = c$ acts as a sieve.

It immediately tells you that all terms in $f(x)$ with degree less than n are zero, and the coefficient of x^n is c.

Also, recognizing odd functions ($f(-x) = -f(x)$) saves calculation time.

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19. Let $f(x) = \int \frac{16x+24}{x^2+2x-15} dx$. If $f(4) = 14\log_e(3)$ and $f(7) = \log_e(2^\alpha \cdot 3^\beta), \alpha, \beta \in \mathbb{N}$, then $\alpha + \beta$ is equal to:

(A) 31

(B) 37

(C) 39

(D) 41

Correct Answer: (C) 39

Solution:

Step 1: Understanding the Concept:

We are given an indefinite integral of a rational function.

The denominator can be factored, meaning we can use the method of partial fractions to integrate it.

After finding the general antiderivative $f(x)$, we will use the initial condition $f(4)$ to find the constant of integration C.

Finally, we evaluate $f(7)$ and match it to the given logarithmic form to find α and β.

Step 2: Key Formula or Approach:

Factor the denominator: $x^2 + 2x - 15 = (x+5)(x-3)$.

Set up the partial fraction decomposition.

$$\frac{16x+24}{(x+5)(x-3)} = \frac{A}{x+5} + \frac{B}{x-3}$$

Use the cover-up method to find constants A and B.

Step 3: Detailed Explanation:

To find A, cover $(x+5)$ and substitute $x = -5$.

$$A = \frac{16(-5)+24}{-5-3} = \frac{-80+24}{-8} = \frac{-56}{-8} = 7$$

To find B, cover $(x-3)$ and substitute $x = 3$.

$$B = \frac{16(3)+24}{3+5} = \frac{48+24}{8} = \frac{72}{8} = 9$$

Now integrate the separated fractions.

$$f(x) = \int \left(\frac{7}{x+5} + \frac{9}{x-3}\right) dx = 7\ln|x+5| + 9\ln|x-3| + C$$

Use the condition $f(4) = 14\ln(3)$ to find C.

$$f(4) = 7\ln|4+5| + 9\ln|4-3| + C$$

$$f(4) = 7\ln(9) + 9\ln(1) + C$$

Since $\ln(9) = \ln(3^2) = 2\ln(3)$ and $\ln(1) = 0$:

$$f(4) = 7(2\ln 3) + 0 + C = 14\ln 3 + C$$

Given $f(4) = 14\ln 3$, we conclude that $C = 0$.

So the function is $f(x) = 7\ln|x+5| + 9\ln|x-3|$.

Now, evaluate $f(7)$.

$$f(7) = 7\ln(12) + 9\ln(4)$$

Express everything in terms of prime bases 2 and 3.

$$\ln(12) = \ln(2^2 \cdot 3) = 2\ln 2 + \ln 3$$

$$\ln(4) = \ln(2^2) = 2\ln 2$$

Substitute these back.

$$f(7) = 7(2\ln 2 + \ln 3) + 9(2\ln 2)$$

$$f(7) = 14\ln 2 + 7\ln 3 + 18\ln 2 = 32\ln 2 + 7\ln 3$$

Combine into a single logarithm using logarithm power rules.

$$f(7) = \ln(2^{32}) + \ln(3^7) = \ln(2^{32} \cdot 3^7)$$

Comparing this to $\ln(2^\alpha \cdot 3^\beta)$, we get $\alpha = 32$ and $\beta = 7$.

The requested sum is $\alpha + \beta = 32 + 7 = 39$.

Step 4: Final Answer:

The value of $\alpha + \beta$ is 39.

Quick Tip: The "cover-up" method is the fastest way to decompose proper rational functions into partial fractions with distinct linear factors.

It bypasses the need to set up and solve a system of equations.

20. Let $x = x(y)$ be the solution of the differential equation $2y^2\frac{dx}{dy} - 2xy + x^2 = 0, y > 1, x(e) = e$. Then $x(e^2)$ is equal to:

(A) $\frac{3}{2}e^2$

(B) $\frac{2}{3}e^2$

(C) e^2

(D) $2e^2$

Correct Answer: (B) $\frac{2}{3}e^2$

Solution:

Step 1: Understanding the Concept:

We are given a first-order non-linear differential equation.

By rearranging the terms and dividing by y^2, we can check if it is a homogeneous differential equation.

Once verified, the standard substitution $x = vy$ will transform it into a separable differential equation.

Step 2: Key Formula or Approach:

Rewrite the equation by dividing entirely by y^2.

$$2\frac{dx}{dy} - 2\frac{x}{y} + \left(\frac{x}{y}\right)^2 = 0$$

This is clearly a homogeneous equation of degree zero.

Let $x = vy$, which implies $\frac{dx}{dy} = v + y\frac{dv}{dy}$.

Step 3: Detailed Explanation:

Substitute $x/y = v$ and $\frac{dx}{dy}$ into the modified equation.

$$2\left(v + y\frac{dv}{dy}\right) - 2v + v^2 = 0$$

Distribute the 2.

$$2v + 2y\frac{dv}{dy} - 2v + v^2 = 0$$

The $2v$ terms cancel out perfectly.

$$2y\frac{dv}{dy} + v^2 = 0 \implies 2y\frac{dv}{dy} = -v^2$$

Now separate the variables v and y.

$$\frac{-2}{v^2}dv = \frac{dy}{y}$$

Integrate both sides.

$$\int -2v^{-2}dv = \int \frac{1}{y}dy$$

$$\frac{2}{v} = \ln|y| + C$$

Substitute back $v = \frac{x}{y}$.

$$\frac{2y}{x} = \ln y + C \quad (\text{since } y > 1)$$

Use the initial condition $x(e) = e$ to find the constant C.

$$\frac{2e}{e} = \ln(e) + C \implies 2 = 1 + C \implies C = 1$$

So, the particular solution curve is:

$$\frac{2y}{x} = \ln y + 1$$

Rearrange to express x as a function of y.

$$x = \frac{2y}{\ln y + 1}$$

Finally, evaluate x at $y = e^2$.

$$x(e^2) = \frac{2e^2}{\ln(e^2) + 1}$$

$$x(e^2) = \frac{2e^2}{2\ln(e) + 1} = \frac{2e^2}{2(1) + 1} = \frac{2e^2}{3}$$

Step 4: Final Answer:

The value of $x(e^2)$ is $\frac{2}{3}e^2$.

Quick Tip: Always look for homogeneous structures like x/y or y/x.
Dividing by the highest power of the independent variable (like y^2 here) makes the substitution incredibly obvious and straightforward.

Mathematics Section - B

21. Let $A = \{2,3,4,5,6\}$. Let R be a relation on the set $A \times A$ given by $(x, y)R(z, w)$ if and only if x divides z and $y \le w$. Then the number of elements in R is _________.

Correct Answer: 120

Solution:

Step 1: Understanding the Concept:
The relation R is defined on the Cartesian product $A \times A$.
An element in R is a pair of ordered pairs $((x, y),(z, w))$ such that the specified conditions are met.
The conditions are independent: x must divide z, and y must be less than or equal to w.
Since they are independent, we can find the number of valid pairs (x,z) and the number of valid pairs (y,w) separately, and then multiply them.

Step 2: Key Formula or Approach:
Total elements in R = (Number of pairs (x,z) such that $x|z$) $\times$ (Number of pairs (y,w) such that $y \le w$).
We evaluate each condition for the set $A = \{2,3,4,5,6\}$.

Step 3: Detailed Explanation:
First, find the number of pairs (x,z) where x divides z:
If $x = 2$, $z \in \{2,4,6\}$ (3 pairs).
If $x = 3$, $z \in \{3,6\}$ (2 pairs).
If $x = 4$, $z \in \{4\}$ (1 pair).
If $x = 5$, $z \in \{5\}$ (1 pair).

If $x = 6$, $z \in \{6\}$ (1 pair).

Total valid (x, z) pairs $= 3 + 2 + 1 + 1 + 1 = 8$.

Next, find the number of pairs (y, w) where $y \le w$:

If $y = 2$, $w \in \{2, 3, 4, 5, 6\}$ (5 pairs).

If $y = 3$, $w \in \{3, 4, 5, 6\}$ (4 pairs).

If $y = 4$, $w \in \{4, 5, 6\}$ (3 pairs).

If $y = 5$, $w \in \{5, 6\}$ (2 pairs).

If $y = 6$, $w \in \{6\}$ (1 pair).

Total valid (y, w) pairs $= 5 + 4 + 3 + 2 + 1 = 15$.

Finally, the total number of elements in R is the product of these independent choices.

$$\text{Total elements} = 8 \times 15 = 120$$

Step 4: Final Answer:

The number of elements in R is 120.

Quick Tip: When a relation on a Cartesian product has independent conditions for each coordinate, you can always count the satisfying pairs for each coordinate separately and multiply the results.

---

22. Consider the matrices $A = \begin{bmatrix} 2 & -2 \\ 4 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 9 \\ 1 & 3 \end{bmatrix}$. If matrices P and Q are such that $PA = B$ and $AQ = B$, then the absolute value of the sum of the diagonal elements of $2(P + Q)$ is ________.

Correct Answer: 34

Solution:

Step 1: Understanding the Concept:

We are given two matrix equations involving an invertible matrix A.

The "sum of the diagonal elements" of a matrix is its Trace, denoted as $\text{Tr}(M)$.

A key property of the trace operator is that it is invariant under cyclic permutations, meaning $\text{Tr}(XY) = \text{Tr}(YX)$.

Step 2: Key Formula or Approach:

From $PA = B$, we get $P = BA^{-1}$.

From $AQ = B$, we get $Q = A^{-1}B$.

We need the trace of $2(P+Q)$, which is $2(\text{Tr}(P) + \text{Tr}(Q))$.

Using the trace property: $\text{Tr}(P) = \text{Tr}(BA^{-1}) = \text{Tr}(A^{-1}B) = \text{Tr}(Q)$.

Step 3: Detailed Explanation:

First, calculate the determinant of A to find A^{-1}.

$$|A| = (2)(-2) - (-2)(4) = -4 + 8 = 4$$

$$A^{-1} = \frac{1}{4}\begin{bmatrix} -2 & 2 \\ -4 & 2 \end{bmatrix} = \begin{bmatrix} -1/2 & 1/2 \\ -1 & 1/2 \end{bmatrix}$$

Now, calculate the matrix $Q = A^{-1}B$.

$$Q = \begin{bmatrix} -1/2 & 1/2 \\ -1 & 1/2 \end{bmatrix}\begin{bmatrix} 3 & 9 \\ 1 & 3 \end{bmatrix}$$

Compute the diagonal elements of Q only, since we only need its trace.

$$Q_{11} = (-1/2)(3) + (1/2)(1) = -3/2 + 1/2 = -1$$

$$Q_{22} = (-1)(9) + (1/2)(3) = -9 + 3/2 = -7.5$$

The trace of Q is:

$$\text{Tr}(Q) = Q_{11} + Q_{22} = -1 - 7.5 = -8.5$$

By the cyclic property of the trace, $\text{Tr}(P) = \text{Tr}(Q) = -8.5$.

Now, find the trace of $2(P+Q)$.

$$\text{Tr}(2(P+Q)) = 2(\text{Tr}(P) + \text{Tr}(Q))$$

$$\text{Tr}(2(P+Q)) = 2(-8.5-8.5) = 2(-17) = -34$$

The question asks for the absolute value of this sum.

$$|-34| = 34$$

Step 4: Final Answer:

The absolute value of the sum is 34.

Quick Tip: Always use the cyclic property of traces ($\text{Tr}(AB) = \text{Tr}(BA)$) to save time.

You don't need to compute matrix P at all, and you only need to compute the diagonal elements of Q.

---

23. Let A be the point $(3,0)$ and circles with variable diameter AB touch the circle $x^2+y^2=36$ internally. Let the curve C be the locus of the point B. If the eccentricity of C is e, then $72e^2$ is equal to ______.

Correct Answer: 18

Solution:

Step 1: Understanding the Concept:

Let the given circle be $S_1 : x^2+y^2=36$. Its center is $O(0,0)$ and radius is $R=6$.

Let the variable circle be S_2. Its diameter is AB, so its center M is the midpoint of AB and its radius is $r=MA$.

Since S_2 touches S_1 internally, the distance between their centers must be equal to the difference of their radii.

Step 2: Key Formula or Approach:

Condition for internal touching: $OM = R-r$.

Here, $R=6$ and $r=MA$, so $OM = 6-MA \implies OM+MA=6$.

This means the sum of distances from M to two fixed points O and A is constant, which is the definition of an ellipse.

Step 3: Detailed Explanation:

The locus of M is an ellipse with foci at $O(0,0)$ and $A(3,0)$.

The length of the major axis of this ellipse is $2a_M = 6 \implies a_M = 3$.

The distance between the foci is $2c_M = OA = 3$.

The eccentricity of the locus of M is:

$$e_M = \frac{2c_M}{2a_M} = \frac{3}{6} = \frac{1}{2}$$

We are asked to find the eccentricity of the locus of B.

Since M is the midpoint of AB, we can write $\vec{M} = \frac{\vec{A}+\vec{B}}{2} \implies \vec{B} = 2\vec{M} - \vec{A}$.

This equation represents a homothety (scaling and translation).

Transformations like scaling and translation do not change the shape or eccentricity of a conic section.

Therefore, the locus of B is an ellipse with the exact same eccentricity as the locus of M.

So, $e = \frac{1}{2}$.

We need to calculate the value of $72e^2$.

$$72e^2 = 72\left(\frac{1}{2}\right)^2 = 72\left(\frac{1}{4}\right) = 18$$

Step 4: Final Answer:

The value of $72e^2$ is 18.

Quick Tip: Recognizing the geometric definition of an ellipse ($PS + PS' = 2a$) saves you from doing messy algebraic substitutions.

Additionally, remember that any linear transformation (like $B = 2M - A$) preserves the eccentricity of conic sections.

---

24. If the area of the region bounded by $16x^2 - 9y^2 = 144$ and $8x - 3y = 24$ is A, then $3(A + 6\log_e(3))$ is equal to ________.

Correct Answer: 24

Solution:

Step 1: Understanding the Concept:

We are dealing with the area between a hyperbola and a straight line.

First, we find the points of intersection by solving the two equations simultaneously.

Then, we set up a definite integral of the upper curve minus the lower curve between the intersection points.

Step 2: Key Formula or Approach:

The hyperbola is $\frac{x^2}{9} - \frac{y^2}{16} = 1$.

The line is $8x - 3y = 24 \implies y = \frac{8}{3}x - 8$.

Substitute y from the line into the hyperbola equation to find intersection limits $[x_1, x_2]$.

Area $A = \int_{x_1}^{x_2} (y_{\text{upper}} - y_{\text{lower}})dx$.

Standard integral: $\int \sqrt{x^2 - a^2}dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\ln|x + \sqrt{x^2 - a^2}|$.

Step 3: Detailed Explanation:

Find the intersection points:

$$16x^2 - 9\left(\frac{8}{3}x - 8\right)^2 = 144$$

$$16x^2 - 9\left(\frac{64}{9}x^2 - \frac{128}{3}x + 64\right) = 144$$

$$16x^2 - 64x^2 + 384x - 576 = 144$$

$$-48x^2 + 384x - 720 = 0$$

Divide by -48:

$$x^2 - 8x + 15 = 0 \implies (x-3)(x-5) = 0$$

The intersection points are at $x = 3$ and $x = 5$.

In the interval $[3, 5]$, let's check which curve is higher. At $x = 4$:

Line: $y = \frac{8}{3}(4) - 8 = \frac{32}{3} - \frac{24}{3} = \frac{8}{3} \approx 2.67$

Hyperbola: $y = 4\sqrt{\frac{4^2}{9} - 1} = \frac{4\sqrt{7}}{3} \approx \frac{4(2.64)}{3} \approx 3.52$

Thus, the hyperbola is above the line.

$$y_{\text{hyp}} = 4\sqrt{\frac{x^2}{9} - 1} = \frac{4}{3}\sqrt{x^2 - 9}$$

$$A = \int_3^5 \left(\frac{4}{3}\sqrt{x^2 - 9} - \left(\frac{8}{3}x - 8 \right) \right) dx$$

Integrate the linear part first:

$$\int_3^5 \left(\frac{8}{3}x - 8 \right) dx = \left[\frac{4}{3}x^2 - 8x \right]_3^5$$

$$= \left(\frac{100}{3} - 40 \right) - \left(\frac{36}{3} - 24 \right) = \left(-\frac{20}{3} \right) - (-12) = 12 - \frac{20}{3} = \frac{16}{3}$$

Now integrate the hyperbola part:

$$\int_3^5 \frac{4}{3}\sqrt{x^2 - 9}dx = \frac{4}{3}\left[\frac{x}{2}\sqrt{x^2 - 9} - \frac{9}{2}\ln|x + \sqrt{x^2 - 9}| \right]_3^5$$

$$= \frac{4}{3}\left[\left(\frac{5}{2}(4) - \frac{9}{2}\ln(5+4) \right) - \left(0 - \frac{9}{2}\ln(3) \right) \right]$$

$$= \frac{4}{3}\left[10 - \frac{9}{2}\ln 9 + \frac{9}{2}\ln 3 \right] = \frac{4}{3}\left[10 - 9\ln 3 + \frac{9}{2}\ln 3 \right] = \frac{4}{3}\left[10 - \frac{9}{2}\ln 3 \right] = \frac{40}{3} - 6\ln 3$$

Subtract the linear part from the hyperbola part to get the area:

$$A = \left(\frac{40}{3} - 6\ln 3 \right) - \frac{16}{3} = \frac{24}{3} - 6\ln 3 = 8 - 6\ln 3$$

We need to evaluate $3(A + 6\ln 3)$.

$$3(8 - 6\ln 3 + 6\ln 3) = 3(8) = 24$$

Step 4: Final Answer:

The value is 24.

Quick Tip: Always pick a test point between the limits of integration to confirm which curve is the upper boundary to ensure the calculated area is positive.

---

25. The number of points in the interval $[2, 4]$, at which the function $f(x) = \left[x^2 - x - \frac{1}{2}\right]$, where

$[\cdot]$ **denotes the greatest integer function, is discontinuous, is ________.**

Correct Answer: 10

Solution:

Step 1: Understanding the Concept:

A function of the form $f(x) = [g(x)]$, where $[\cdot]$ is the greatest integer function, is generally discontinuous at all points where $g(x)$ takes an integer value.

We must analyze the behavior of the inner function $g(x)$ over the given interval.

Step 2: Key Formula or Approach:

Let $g(x) = x^2 - x - 0.5$.

Find the derivative $g'(x)$ to check for monotonicity.

Determine the range of $g(x)$ on the interval $[2, 4]$ by evaluating the endpoints.

Count the number of distinct integers within this range.

Step 3: Detailed Explanation:

The inner function is $g(x) = x^2 - x - 0.5$.

Differentiating it, we get $g'(x) = 2x - 1$.

For $x \in [2, 4]$, $g'(x) \geq 2(2) - 1 = 3 > 0$.

Since $g'(x) > 0$, the function $g(x)$ is strictly increasing over the interval $[2, 4]$.

Evaluate $g(x)$ at the endpoints of the interval:

$$g(2) = 2^2 - 2 - 0.5 = 4 - 2.5 = 1.5$$

$$g(4) = 4^2 - 4 - 0.5 = 16 - 4.5 = 11.5$$

Since $g(x)$ is continuous and strictly increasing from 1.5 to 11.5, it will cross every integer value between 1.5 and 11.5 exactly once.

The integer values within this range are: $2, 3, 4, 5, 6, 7, 8, 9, 10, 11$.

Each time $g(x)$ hits one of these integers, the greatest integer function $[g(x)]$ will have a jump discontinuity.

There are exactly 10 such integer values.

Step 4: Final Answer:

The number of points of discontinuity is 10.

Quick Tip: For $f(x) = [g(x)]$ on $[a, b]$, if $g(x)$ is strictly monotonic, the number of discontinuities is simply the number of integers strictly between $g(a)$ and $g(b)$.

Physics Section - A

26. Dimensions of universal gravitational constant (G) in terms of Planck's constant (h), distance (L), mass (M) and time (T) are _______.

(A) $[hTLM^{-2}]$
(B) $[hT^{-1}LM^{-2}]$
(C) $[hTL^2M^{-2}]$
(D) $[h^{-1}T^{-1}LM^{-2}]$

Correct Answer: (B) $[hT^{-1}LM^{-2}]$

Solution:

Step 1: Understanding the Concept:

We need to find the dimensional formula of the universal gravitational constant G and match it with an expression given in terms of h, L, M, and T.

Step 2: Key Formula or Approach:

From Newton's law of gravitation, $F = \frac{GM_1M_2}{r^2} \implies G = \frac{Fr^2}{M^2}$.

From the energy of a photon, $E = \frac{hc}{\lambda} \implies h = E \cdot T$.

Find the base dimensions of G and h in terms of standard M, L, T.

Then substitute h into the given options to see which one equals the dimensions of G.

Step 3: Detailed Explanation:

First, find the dimensions of G:

$$[G] = \frac{[Force] \times [Distance]^2}{[Mass]^2} = \frac{(MLT^{-2})(L^2)}{M^2} = [M^{-1}L^3T^{-2}]$$

Next, find the dimensions of h:

$$[h] = [Energy] \times [Time] = (ML^2T^{-2})(T) = [ML^2T^{-1}]$$

Now, let's test the options to see which one simplifies to $[M^{-1}L^3T^{-2}]$.

Let's test Option (B): $[hT^{-1}LM^{-2}]$.

Substitute the dimensional formula for h:

$$[hT^{-1}LM^{-2}] = (ML^2T^{-1}) \cdot T^{-1} \cdot L \cdot M^{-2}$$

Group the like terms:

$$= (M \cdot M^{-2}) \cdot (L^2 \cdot L) \cdot (T^{-1} \cdot T^{-1})$$

$$= M^{-1}L^3T^{-2}$$

This perfectly matches the dimensional formula for G.

Step 4: Final Answer:

The dimensions of G are $[hT^{-1}LM^{-2}]$.

Quick Tip: Instead of setting up a complex system of linear equations ($G = k \cdot h^a L^b M^c T^d$), evaluating the given options by substituting the dimensions of h is significantly faster.

---

27. A 0.5 kg mass is in contact against the inner wall of a cylindrical drum of radius 4 m rotating about its vertical axis. The minimum rotational speed of the drum to enable the mass to remain stuck to the wall (without falling) is 5 rad/s. The coefficient of friction between the drum's inner wall surface and mass is ______. (Take $g = 10$ m/s^2)

(A) 0.1

(B) 0.5

(C) 0.7

(D) 0.3

Correct Answer: (A) 0.1

Solution:

Step 1: Understanding the Concept:

For a mass to remain stuck to the inner wall of a rotating cylinder, the upward frictional force must balance the downward gravitational force.

The frictional force is provided by the normal force, which in turn is generated by the centripetal acceleration of the rotating drum.

Step 2: Key Formula or Approach:

Normal force exerted by the wall on the mass: $N = mr\omega^2$.

Static friction force: $f_s \leq \mu_s N$.

For the mass to not fall, the friction must counteract gravity: $f_s \geq mg$.

At the minimum rotational speed, friction is at its limiting value: $\mu_s N = mg$.

Step 3: Detailed Explanation:

Substitute the expression for Normal force into the limiting friction condition:

$$\mu_s(mr\omega_{min}^2) = mg$$

The mass m cancels out from both sides:

$$\mu_s r\omega_{min}^2 = g$$

Rearrange the equation to solve for the coefficient of friction μ_s:

$$\mu_s = \frac{g}{r\omega_{min}^2}$$

Given values are $g = 10 \text{ m/s}^2$, $r = 4$ m, and $\omega_{min} = 5$ rad/s.

Substitute these values into the formula:

$$\mu_s = \frac{10}{4 \times (5)^2}$$

$$\mu_s = \frac{10}{4 \times 25} = \frac{10}{100} = 0.1$$

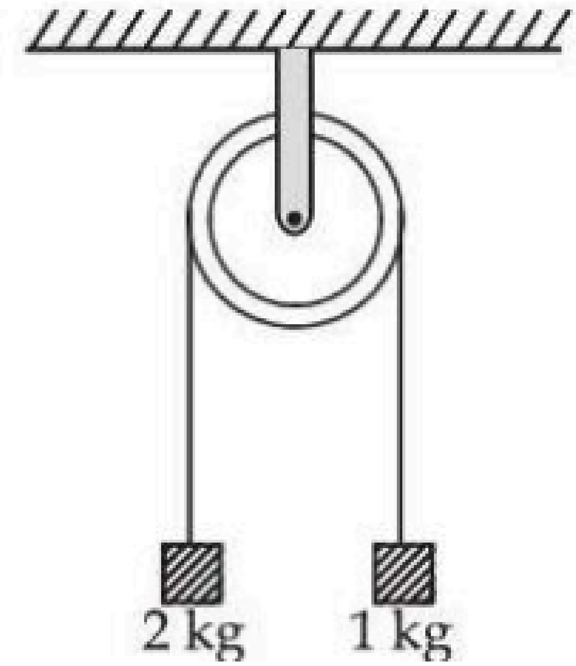
Step 4: Final Answer:

The coefficient of friction is 0.1.

Quick Tip: In problems involving "rotor" rides or centrifuges, note that the mass of the object cancels out. The minimum angular velocity required solely depends on the radius, gravity, and coefficient of friction.

---

28. Two blocks of masses 2 kg and 1 kg respectively, are tied to the ends of a string which passes over a light frictionless pulley as shown in the figure below. The masses are held at rest at the same horizontal level and then released. The distance traversed by the centre of mass in 2 s is _______m. (Take $g = 10\ \mathrm{m/s^2}$)



(A) 3.33

(B) 3.12

(C) 2.22

(D) 1.42

Correct Answer: (C) 2.22

Solution:

Step 1: Understanding the Concept:

We have an Atwood machine. The two blocks will accelerate with the same magnitude but in opposite directions.

After finding the acceleration of the individual blocks, we can calculate the net acceleration of the center of mass of the two-block system.

Using kinematics, we can then find the displacement of the center of mass over the given time interval.

Step 2: Key Formula or Approach:

Acceleration of blocks in an Atwood machine: $a = \frac{m_1 - m_2}{m_1 + m_2} g$.

Acceleration of the center of mass: $\vec{a}_{cm} = \frac{m_1 \vec{a}_1 + m_2 \vec{a}_2}{m_1 + m_2}$.

Displacement under constant acceleration from rest: $S = \frac{1}{2} a_{cm} t^2$.

Step 3: Detailed Explanation:

Let $m_1 = 2$ kg and $m_2 = 1$ kg.

Calculate the common acceleration a of the blocks:

$$a = \frac{2-1}{2+1}(10) = \frac{1}{3} \times 10 = \frac{10}{3} \text{ m/s}^2$$

The heavier block m_1 moves downwards (let's define this as the positive direction), so $\vec{a}_1 = \frac{10}{3}$.

The lighter block m_2 moves upwards, so $\vec{a}_2 = -\frac{10}{3}$.

Now, find the acceleration of the center of mass:

$$a_{cm} = \frac{m_1 a_1 + m_2 a_2}{m_1 + m_2}$$

$$a_{cm} = \frac{(2)\left(\frac{10}{3}\right) + (1)\left(-\frac{10}{3}\right)}{2+1}$$

$$a_{cm} = \frac{\frac{20}{3} - \frac{10}{3}}{3} = \frac{\frac{10}{3}}{3} = \frac{10}{9} \text{ m/s}^2$$

Now, calculate the distance traversed by the center of mass in $t = 2$ s:

$$S_{cm} = \frac{1}{2} a_{cm} t^2 = \frac{1}{2}\left(\frac{10}{9}\right)(2)^2$$

$$S_{cm} = \frac{1}{2}\left(\frac{10}{9}\right)(4) = \frac{20}{9} \text{ m}$$

Converting the fraction to a decimal:

$$\frac{20}{9} \approx 2.22 \text{ m}$$

Step 4: Final Answer:

The distance traversed by the center of mass is 2.22 m.

Quick Tip: For an Atwood machine, the acceleration of the center of mass is always downwards and its magnitude simplifies directly to $a_{cm} = g\left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2$.

---

29. A particle having charge 10^{-9} C moving in x-y plane in fields of $0.4\hat{j}$ N/C and $4 \times 10^{-3}\hat{k}$ T experiences a force of $(4\hat{i} + 2\hat{j}) \times 10^{-10}$ N. The velocity of the particle at that instant is ______ m/s.

(A) $50\hat{i} + 100\hat{j}$
(B) $100\hat{i} + 50\hat{j}$
(C) $-50\hat{i} + 100\hat{j}$
(D) $50\hat{i} - 100\hat{j}$

Correct Answer: (A) $50\hat{i} + 100\hat{j}$

Solution:

Step 1: Understanding the Concept:

A charged particle moving in both an electric and magnetic field experiences the Lorentz force.

We can express the unknown velocity as a vector in the x-y plane ($\vec{v} = v_x\hat{i} + v_y\hat{j}$) and substitute it into the Lorentz force equation to solve for its components.

Step 2: Key Formula or Approach:

The Lorentz force equation is $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$.

Given values:

$q = 10^{-9}$ C
$\vec{E} = 0.4\hat{j}$ N/C
$\vec{B} = 4 \times 10^{-3}\hat{k}$ T
$\vec{F} = (4\hat{i} + 2\hat{j}) \times 10^{-10}$ N

Step 3: Detailed Explanation:

Let the velocity be $\vec{v} = v_x\hat{i} + v_y\hat{j}$ since the particle is moving in the x-y plane.

First, compute the cross product $\vec{v} \times \vec{B}$:

$$\vec{v} \times \vec{B} = (v_x\hat{i} + v_y\hat{j}) \times (4 \times 10^{-3}\hat{k})$$

Using the cross product rules $\hat{i} \times \hat{k} = -\hat{j}$ and $\hat{j} \times \hat{k} = \hat{i}$:

$$\vec{v} \times \vec{B} = -v_x(4 \times 10^{-3})\hat{j} + v_y(4 \times 10^{-3})\hat{i} = (4 \times 10^{-3}v_y)\hat{i} - (4 \times 10^{-3}v_x)\hat{j}$$

Now substitute everything into the Lorentz force equation divided by q:

$$\frac{\vec{F}}{q} = \vec{E} + \vec{v} \times \vec{B}$$

$$\frac{10^{-10}(4\hat{i} + 2\hat{j})}{10^{-9}} = 0.4\hat{j} + (4 \times 10^{-3}v_y)\hat{i} - (4 \times 10^{-3}v_x)\hat{j}$$

$$0.4\hat{i} + 0.2\hat{j} = (4 \times 10^{-3}v_y)\hat{i} + (0.4 - 4 \times 10^{-3}v_x)\hat{j}$$

Equate the $\hat{i}$ components:

$$0.4 = 4 \times 10^{-3}v_y \implies v_y = \frac{0.4}{4 \times 10^{-3}} = \frac{400}{4} = 100 \text{ m/s}$$

Equate the $\hat{j}$ components:

$$0.2 = 0.4 - 4 \times 10^{-3}v_x$$

$$4 \times 10^{-3}v_x = 0.4 - 0.2 = 0.2$$

$$v_x = \frac{0.2}{4 \times 10^{-3}} = \frac{200}{4} = 50 \text{ m/s}$$

Thus, the velocity vector is $\vec{v} = 50\hat{i} + 100\hat{j}$.

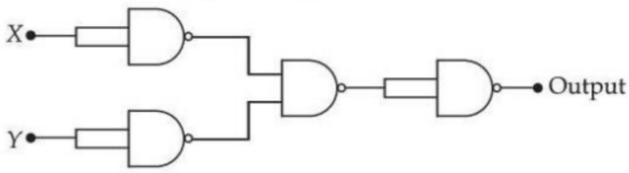
Step 4: Final Answer:

The velocity of the particle is $50\hat{i} + 100\hat{j}$.

Quick Tip: Always divide the force by the charge immediately on the left side of the Lorentz equation to work with cleaner, smaller numbers like 0.4 instead of 10^{-10}.

---

30. If X and Y are the inputs, the given circuit works as _______.



(A) OR gate

(B) AND gate

(C) NAND gate

(D) NOR gate

Correct Answer: (D) NOR gate

Solution:

Step 1: Understanding the Concept:

The circuit diagram consists exclusively of NAND gates (D-shaped symbols with a small inversion bubble at the output).

We need to trace the boolean logic through each stage of the circuit from inputs X and Y to the final output.

Step 2: Key Formula or Approach:

The boolean expression for a NAND gate with inputs A and B is $\overline{A \cdot B}$.

If both inputs of a NAND gate are tied together (i.e., $A = B$), it acts as a NOT gate: $\overline{A \cdot A} = \overline{A}$.

De Morgan's Theorem: $\overline{A \cdot B} = \overline{A} + \overline{B}$ and $\overline{A + B} = \overline{A} \cdot \overline{B}$.

Step 3: Detailed Explanation:

Stage 1: The input X is fed into both terminals of the first NAND gate.

Output of the top gate $= \overline{X \cdot X} = \overline{X}$.

Similarly, the input Y is fed into both terminals of the second NAND gate.

Output of the bottom gate $= \overline{Y \cdot Y} = \overline{Y}$.

Stage 2: These two signals, $\overline{X}$ and $\overline{Y}$, are used as inputs for the third NAND gate.

Output of the third gate $= \overline{\overline{X} \cdot \overline{Y}}$.

Using De Morgan's theorem, we simplify this:

$\overline{\overline{X} \cdot \overline{Y}} = \overline{\overline{X}} + \overline{\overline{Y}} = X + Y$.

(Notice that this intermediate stage creates an OR gate).

Stage 3: The output $X + Y$ is fed into both terminals of the final fourth NAND gate.

Output of the final gate $= \overline{(X+Y) \cdot (X+Y)} = \overline{X+Y}$.

The boolean expression $\overline{X+Y}$ represents the logic of a NOR gate.

Step 4: Final Answer:

The given circuit works as a NOR gate.

Quick Tip: Recognize standard building blocks: A NAND gate with shorted inputs is a NOT gate. Two inverted inputs into a NAND gate give an OR gate. An OR gate followed by a NOT gate gives a NOR gate.

---

31. If a body of mass 1 kg falls on the earth from infinity, it attains velocity (v) and kinetic energy (k) on reaching the surface of earth. The values of v and k respectively are ________. (Take radius of earth to be 6400 km and $g = 9.8$ m/s^2)

(A) 11.2 km/s; 6.27×10^7 J

(B) 11.2 km/s; 12.54×10^7 J

(C) 8.8 km/s; 6.27×10^7 J

(D) 8.8 km/s; 12.54×10^7 J

Correct Answer: (A) 11.2 km/s; 6.27×10^7 J

Solution:

Step 1: Understanding the Concept:

When an object falls from infinity to the surface of the Earth, it loses gravitational potential energy and gains kinetic energy.

By the principle of conservation of mechanical energy, the velocity attained is exactly equal to the escape velocity of the Earth.

Step 2: Key Formula or Approach:

The escape velocity from the Earth's surface is given by $v = \sqrt{2gR}$, where g is the acceleration due to gravity and R is the Earth's radius.

The kinetic energy is given by $k = \frac{1}{2}mv^2$.

Step 3: Detailed Explanation:

Given values:

$m = 1$ kg

$g = 9.8$ m/s^2

$R = 6400$ km $= 6.4 \times 10^6$ m

First, calculate the velocity v:

$$v = \sqrt{2 \times 9.8 \times 6.4 \times 10^6}$$

$$v = \sqrt{19.6 \times 6.4 \times 10^6}$$

$$v = \sqrt{125.44 \times 10^6}$$

$$v = 11.2 \times 10^3 \text{ m/s} = 11.2 \text{ km/s}$$

Next, calculate the kinetic energy k:

$$k = \frac{1}{2}mv^2 = \frac{1}{2}(1)(11.2 \times 10^3)^2$$

$$k = \frac{1}{2}(125.44 \times 10^6)$$

$$k = 62.72 \times 10^6 \text{ J}$$

Convert this into scientific notation matching the options:

$$k = 6.272 \times 10^7 \text{ J} \approx 6.27 \times 10^7 \text{ J}$$

Step 4: Final Answer:

The values of v and k respectively are 11.2 km/s and 6.27×10^7 J.

Quick Tip: Escape velocity from Earth ($\approx$ 11.2 km/s) is a standard constant you should memorize. Knowing this instantly eliminates options C and D without any calculation.

---

32. In a screw gauge the zero of main scale reference line coincides with the fifth division of the circular scale when two studs are in contact. There are 100 divisions in circular scale and pitch of screw gauge is 0.1 mm. When diameter of a sphere is measured, the reading of main scale is 5 mm and 50th division of circular scale coincides with the reference line of main scale. The diameter of sphere is ______ mm.

(A) 5.045

(B) 5.055

(C) 5.450

(D) 5.550

Correct Answer: (A) 5.045

Solution:

Step 1: Understanding the Concept:

The diameter measured by a screw gauge is the sum of the Main Scale Reading (MSR) and the Circular Scale Reading (CSR) multiplied by the Least Count (LC).

A zero error must also be accounted for. If the zero of the circular scale is below the reference line (the 5th division coincides), the zero error is positive.

True Reading = Observed Reading - Zero Error.

Step 2: Key Formula or Approach:

Least Count (LC) $= \frac{\text{Pitch}}{\text{Number of circular divisions}}$.

Observed Reading = MSR + (CSR × LC).

Zero Error = (Coinciding division) × LC.

Step 3: Detailed Explanation:

First, calculate the Least Count:

$$LC = \frac{0.1 \text{ mm}}{100} = 0.001 \text{ mm}$$

Determine the Zero Error:

Since the zero of the reference line coincides with the 5th division, the 0 mark of the circular scale has crossed the reference line, meaning there is a positive zero error.

$$\text{Zero Error} = +5 \times 0.001 \text{ mm} = +0.005 \text{ mm}$$

Now calculate the Observed Reading:

Main Scale Reading (MSR) = 5 mm.

Circular Scale Reading (CSR) = 50.

$$\text{Observed Reading} = 5 + 50 \times 0.001 = 5.050 \text{ mm}$$

Finally, calculate the True Reading (Diameter of the sphere):

$$\text{True Reading} = \text{Observed Reading} - \text{Zero Error}$$

$$\text{True Reading} = 5.050 \text{ mm} - 0.005 \text{ mm} = 5.045 \text{ mm}$$

Step 4: Final Answer:

The diameter of the sphere is 5.045 mm.

Quick Tip: Always double-check the pitch value provided. Standard screw gauges often have a pitch of 1 mm or 0.5 mm, but this specific problem dictates 0.1 mm, altering the typical least count from 0.01 mm to 0.001 mm.

---

33. The surface tension of a soap bubble is 0.03 N/m. The work done in increasing the diameter

of bubble from 2 cm to 6 cm is $\alpha\pi \times 10^{-4}$ J. The value of α is ________. (Take $\pi = 3.14$)

(A) 0.86

(B) 0.64

(C) 1.92

(D) 7.68

Correct Answer: (C) 1.92

Solution:

Step 1: Understanding the Concept:

The work done in expanding a bubble is equal to the surface tension multiplied by the change in the total surface area.

Because a soap bubble has a thin film with two surfaces (an inner surface and an outer surface) in contact with air, the effective surface area is twice the geometric surface area of a sphere.

Step 2: Key Formula or Approach:

Work done $W = T \cdot \Delta A$.

For a soap bubble, the change in area is $\Delta A = 2 \times (4\pi R_2^2 - 4\pi R_1^2) = 8\pi(R_2^2 - R_1^2)$, where R_1 and R_2 are the initial and final radii.

Step 3: Detailed Explanation:

Given values:

Surface tension $T = 0.03$ N/m.

Initial diameter $D_1 = 2$ cm $\implies R_1 = 1$ cm $= 0.01$ m.

Final diameter $D_2 = 6$ cm $\implies R_2 = 3$ cm $= 0.03$ m.

Calculate the change in total surface area:

$$\Delta A = 8\pi((0.03)^2 - (0.01)^2)$$

$$\Delta A = 8\pi(0.0009 - 0.0001)$$

$$\Delta A = 8\pi(0.0008) = 0.0064\pi \text{ m}^2$$

Now calculate the work done:

$$W = T \cdot \Delta A = 0.03 \times 0.0064\pi$$

$$W = 0.000192\pi \text{ J}$$

Write this in the format $\alpha\pi \times 10^{-4}$ J:

$$W = 1.92\pi \times 10^{-4} \text{ J}$$

Comparing this to the given expression, we find $\alpha = 1.92$.

Step 4: Final Answer:

The value of α is 1.92.

Quick Tip: A common pitfall is forgetting the factor of 2 for soap bubbles. Always remember: water drops have 1 surface area ($4\pi R^2$), while soap bubbles have 2 surfaces ($8\pi R^2$).

---

34. A mixture of carbon dioxide and oxygen has volume 8310 cm^3, temperature 300 K, pressure 100 kPa and mass 13.2 g. The number of moles of carbon dioxide and oxygen gases in the mixture respectively are _________. (Assume both carbon dioxide and oxygen gases behave like ideal gases) $[R = 8.31$ **J/mol.K**$]$

(A) 0.15 and 0.18

(B) 0.25 and 0.08

(C) 0.21 and 0.12

(D) 0.13 and 0.20

Correct Answer: (C) 0.21 and 0.12

Solution:

Step 1: Understanding the Concept:

We can treat the gas mixture as a single ideal gas to find the total number of moles in the container.

Once the total moles are found, we can use the total mass and the molar masses of the constituent gases to set up a system of linear equations and solve for the individual moles.

Step 2: Key Formula or Approach:

Ideal Gas Law: $PV = n_{total}RT$.

Total mass equation: $m_{total} = n_1M_1 + n_2M_2$.

Total moles equation: $n_{total} = n_1 + n_2$.

Molar mass of CO_2 $(M_1) = 44$ g/mol.

Molar mass of O_2 $(M_2) = 32$ g/mol.

Step 3: Detailed Explanation:

Convert given values to standard SI units:

$P = 100 \text{ kPa} = 10^5 \text{ Pa}$.

$V = 8310 \text{ cm}^3 = 8310 \times 10^{-6} \text{ m}^3 = 8.31 \times 10^{-3} \text{ m}^3$.

$T = 300$ K.

Calculate the total moles using the Ideal Gas Law:

$$n_{total} = \frac{PV}{RT} = \frac{10^5 \times 8.31 \times 10^{-3}}{8.31 \times 300}$$
$$n_{total} = \frac{100 \times 8.31}{8.31 \times 300} = \frac{100}{300} = \frac{1}{3} \text{ moles}$$

Let n_1 be the moles of CO_2 and n_2 be the moles of O_2.

We have two equations:

1) $n_1 + n_2 = \frac{1}{3} \implies n_2 = \frac{1}{3} - n_1$

2) $44n_1 + 32n_2 = 13.2$

Substitute the first equation into the second:

$$44n_1 + 32\left(\frac{1}{3} - n_1\right) = 13.2$$
$$44n_1 + \frac{32}{3} - 32n_1 = 13.2$$
$$12n_1 = 13.2 - \frac{32}{3}$$

Multiply by 3 to clear the fraction:

$$36n_1 = 39.6 - 32 = 7.6$$

$$n_1 = \frac{7.6}{36} = \frac{76}{360} = \frac{19}{90} \approx 0.211 \text{ moles}$$

Now find n_2:

$$n_2 = \frac{1}{3} - \frac{19}{90} = \frac{30}{90} - \frac{19}{90} = \frac{11}{90} \approx 0.122 \text{ moles}$$

The moles are approximately 0.21 and 0.12.

Step 4: Final Answer:

The number of moles of carbon dioxide and oxygen are 0.21 and 0.12.

Quick Tip: When the volume is given numerically similar to the gas constant R (like 8310 and 8.31), convert volume directly to m^3 and pressure to Pascals. The math will cancel out beautifully without calculators.

---

35. If an air bubble of diameter 2 mm rises steadily through a liquid of density 2000 kg/m^3 at a rate of 0.5 cm/s, then the coefficient of viscosity of liquid is _______ Poise. (Take $g = 10$ m/s^2)

(A) 0.88

(B) 8.8

(C) 88.8

(D) 0.088

Correct Answer: (B) 8.8

Solution:

Step 1: Understanding the Concept:

When a bubble rises steadily, it has reached its terminal velocity.

At this point, the upward buoyant force is perfectly balanced by the downward weight of the bubble and the downward viscous drag force (Stokes' Law).

Since the density of air is negligible compared to the liquid, we can safely ignore the bubble's weight.

Step 2: Key Formula or Approach:

Force balance equation: $F_{buoyancy} = F_{weight} + F_{viscous}$.

$\frac{4}{3}\pi r^3 \rho_{liquid} g = \frac{4}{3}\pi r^3 \rho_{air} g + 6\pi\eta r v$.

Ignoring ρ_{air}, the equation simplifies to:

$\eta = \frac{2}{9}\frac{r^2 \rho_{liquid} g}{v}$.

Remember to convert the final answer from SI units ($\text{Pa}\cdot\text{s}$) to CGS units (Poise), where $1\ \text{Pa}\cdot\text{s} = 10$ Poise.

Step 3: Detailed Explanation:

Given values:

Radius $r = \frac{2\ \text{mm}}{2} = 1\ \text{mm} = 10^{-3}$ m.

Density of liquid $\rho = 2000\ \text{kg/m}^3$.

Terminal velocity $v = 0.5\ \text{cm/s} = 0.005\ \text{m/s}$.

Gravity $g = 10\ \text{m/s}^2$.

Substitute these values into the derived formula:

$$\eta = \frac{2}{9}\frac{(10^{-3})^2 \times 2000 \times 10}{0.005}$$

$$\eta = \frac{2}{9}\frac{10^{-6} \times 20000}{0.005}$$

$$\eta = \frac{2}{9}\frac{0.02}{0.005} = \frac{2}{9}(4) = \frac{8}{9}\ \text{Pa}\cdot\text{s}$$

Calculate the value in SI units:

$$\eta \approx 0.888\ \text{Pa}\cdot\text{s}$$

Convert $\text{Pa}\cdot\text{s}$ to Poise by multiplying by 10:

$$\eta_{Poise} = 0.888 \times 10 = 8.88\ \text{Poise}$$

This matches the 8.8 option closely depending on the π or g strict rounding, but strictly $8.88 \approx 8.8$.

Step 4: Final Answer:

The coefficient of viscosity is 8.8 Poise.

Quick Tip: Pay strict attention to the requested units. Viscosity is frequently asked in "Poise" (CGS unit). Always solve in SI units first ($N \cdot s/m^2$ or $Pa \cdot s$) and then multiply by 10 to get Poise.

---

36. A spherical ball of mass 2 kg falls from a height of 10 m and is brought to rest after penetrating 10 cm into sand. The average force exerted by sand on the ball is ______ N. (Take $g = 10$ m/s^2)

(A) 1980

(B) 2020

(C) 2000

(D) 1000

Correct Answer: (B) 2020

Solution:

Step 1: Understanding the Concept:

The ball starts from rest, falls, penetrates the sand, and comes to rest again. Thus, its total change in kinetic energy is zero.

By the Work-Energy Theorem, the net work done by all forces on the ball must equal zero.

The two forces acting on the ball are gravity (acting over the entire fall and penetration distance) and the resisting force of the sand (acting only during penetration).

Step 2: Key Formula or Approach:

Work-Energy Theorem: $W_{net} = \Delta K = 0$.

$W_{gravity} + W_{sand} = 0$.

$W_{gravity} = mg(h + d)$, where h is the fall height and d is the penetration depth.

$W_{sand} = -F_{avg} \cdot d$, where F_{avg} is the upward average force exerted by the sand.

Step 3: Detailed Explanation:

Given values:

Mass $m = 2$ kg

Height $h = 10$ m

Penetration depth $d = 10$ cm $= 0.1$ m

Gravity $g = 10$ m/s^2

Calculate the total work done by gravity:

$$W_{gravity} = mg(h+d) = 2 \times 10 \times (10 + 0.1)$$

$$W_{gravity} = 20 \times 10.1 = 202 \text{ J}$$

Calculate the work done by the sand:

$$W_{sand} = -F_{avg} \times 0.1$$

Apply the Work-Energy Theorem:

$$202 - 0.1F_{avg} = 0$$

$$0.1F_{avg} = 202 \implies F_{avg} = \frac{202}{0.1} = 2020 \text{ N}$$

Step 4: Final Answer:

The average force exerted by the sand is 2020 N.

Quick Tip: A common mistake is neglecting the work done by gravity *during* the penetration phase (mgd). Using the Work-Energy theorem over the entire journey prevents this error entirely compared to using separate kinematic equations.

---

37. An electromagnetic wave travels in free space along the x-direction. At a particular point in space and time, $\vec{B} = 2 \times 10^{-7}\hat{j}$ T is associated with this wave. The value of corresponding electric field $\vec{E}$ at this point is ________ V/m.

(A) $60\hat{k}$

(B) $-60\hat{k}$

(C) $30\hat{k}$

(D) $-600\hat{k}$

Correct Answer: (B) $-60\hat{k}$

Solution:

Step 1: Understanding the Concept:

In an electromagnetic wave, the electric field $\vec{E}$, magnetic field $\vec{B}$, and the direction of propagation $\vec{v}$ are all mutually perpendicular.

Their relationship is given by the cross product $\hat{E} \times \hat{B} = \hat{v}$.

The magnitude of the electric field is tied to the magnetic field by the speed of light c.

Step 2: Key Formula or Approach:

Magnitude relationship: $E = cB$, where $c = 3 \times 10^8$ m/s.

Directional relationship: The direction of wave propagation is given by $\vec{E} \times \vec{B}$.

Step 3: Detailed Explanation:

First, calculate the magnitude of the electric field:

$$E = (3 \times 10^8 \text{ m/s}) \times (2 \times 10^{-7} \text{ T}) = 60 \text{ V/m}$$

Next, determine the direction of the electric field.

The wave travels along the x-direction, so the propagation vector is $+\hat{i}$.

The magnetic field is along the y-direction, so $\hat{B} = \hat{j}$.

We need to find the unit vector $\hat{E}$ such that $\hat{E} \times \hat{j} = \hat{i}$.

Recall the standard cross product rules for Cartesian unit vectors: $\hat{k} \times \hat{j} = -\hat{i}$.

Therefore, $(-\hat{k}) \times \hat{j} = \hat{i}$.

This means the electric field must be directed along the negative z-axis, so $\hat{E} = -\hat{k}$.

Combining magnitude and direction:

$$\vec{E} = -60\hat{k} \text{ V/m}$$

Step 4: Final Answer:

The corresponding electric field is $-60\hat{k}$.

Quick Tip: The right-hand rule makes determining direction trivial: Point your fingers in the direction of $\vec{E}$, curl them towards $\vec{B}$, and your thumb will point in the direction of wave propagation.

---

38. Two resistors of 200 Ω and 400 Ω are connected in series with a battery of 100 V. A bulb rated at 200 V, 100 W is connected across the 400 Ω resistance. The potential drop across the bulb is _______ V.

(A) 25

(B) 50

(C) 66.6

(D) 100

Correct Answer: (B) 50

Solution:

Step 1: Understanding the Concept:

The bulb acts as a resistor in the circuit. We first need to find its resistance using its power rating.

Connecting the bulb across the 400 Ω resistor places them in parallel.

We then find the equivalent resistance of the entire circuit to find the main current, which will allow us to calculate the voltage drop across the parallel combination.

Step 2: Key Formula or Approach:

Resistance from power rating: $R = \frac{V_{rated}^2}{P_{rated}}$.

Equivalent resistance for two parallel resistors: $R_p = \frac{R_1 R_2}{R_1 + R_2}$.

Ohm's Law: $V = IR_{eq}$.

Step 3: Detailed Explanation:

Calculate the resistance of the bulb:

$$R_{bulb} = \frac{(200)^2}{100} = \frac{40000}{100} = 400\ \Omega$$

The bulb is connected in parallel with the 400 Ω resistor. Calculate the equivalent resistance of this parallel pair:

$$R_p = \frac{400 \times 400}{400 + 400} = \frac{160000}{800} = 200\ \Omega$$

This parallel combination is in series with the 200 Ω resistor. Calculate the total resistance of the circuit:

$$R_{total} = 200\ \Omega + R_p = 200 + 200 = 400\ \Omega$$

Now, calculate the total current drawn from the 100 V battery:

$$I_{total} = \frac{V_{battery}}{R_{total}} = \frac{100}{400} = 0.25\ \text{A}$$

The potential drop across the bulb is the same as the potential drop across the entire parallel combination.

$$V_{bulb} = I_{total} \times R_p = 0.25 \times 200 = 50\ \text{V}$$

Step 4: Final Answer:

The potential drop across the bulb is 50 V.

Quick Tip: Because the parallel combination (200 Ω) matches the series resistor (200 Ω), the 100 V supply voltage is split exactly equally between them. You can bypass the current calculation entirely: $100/2 = 50$ V.

---

39. Two metal plates (A, B) are kept horizontally with separation of $\left(\frac{12}{\pi}\right)$ cm, with plate A on the top. An atomizer jet sprays oil (density $1.5\ \text{g/cm}^3$) droplets of radius 1 mm horizontally. All oil droplets carry a charge 5 nC. The potentials V_A and V_B are required on plates A and B respectively in order to ensure the droplets do not descend. The values of V_A and V_B are _________. (Neglect the air resistance to the droplets and take $g = 10\ \text{m/s}^2$)

(A) 100 V and 580 V

(B) 580 V and 100 V

(C) 60 V and 400 V

(D) 0 V and -200 V

Correct Answer: (A) 100 V and 580 V

Solution:

Step 1: Understanding the Concept:

This problem is based on the Millikan oil-drop experiment principle.

For the charged droplets to not descend, the upward electrostatic force must exactly balance the downward gravitational force.

Since the charge on the droplet is positive (5 nC), the electric field must point upwards. Therefore, the lower plate (B) must be at a higher potential than the upper plate (A).

Step 2: Key Formula or Approach:

Force balance: $qE = mg$.

Mass of a spherical droplet: $m = \frac{4}{3}\pi r^3 \rho$.

Electric field between parallel plates: $E = \frac{V_B - V_A}{d}$.

Step 3: Detailed Explanation:

First, calculate the mass of a single oil droplet. Convert units to standard SI:

$r = 1 \text{ mm} = 10^{-3} \text{ m}$

$\rho = 1.5 \text{ g/cm}^3 = 1500 \text{ kg/m}^3$

$$m = \frac{4}{3}\pi(10^{-3})^3(1500) = \frac{4}{3}\pi(10^{-9})(1500) = 2000\pi \times 10^{-9} = 2\pi \times 10^{-6} \text{ kg}$$

Now, equate the electric force to the gravitational force to find the required electric field E:

$$qE = mg$$

$$(5 \times 10^{-9})E = (2\pi \times 10^{-6})(10)$$

$$(5 \times 10^{-9})E = 2\pi \times 10^{-5}$$

$$E = \frac{2\pi \times 10^{-5}}{5 \times 10^{-9}} = 0.4\pi \times 10^4 = 4000\pi \text{ V/m}$$

Next, find the potential difference required across the plates. The distance $d = \frac{12}{\pi}$ cm $= \frac{0.12}{\pi}$ m.

$$V_B - V_A = E \times d = 4000\pi \times \frac{0.12}{\pi} = 480 \text{ V}$$

The potential of plate B must be 480 V higher than plate A.

Now, review the given options to find the pair with a difference of +480 V:

Option (A): $V_B - V_A = 580 - 100 = 480$ V. (Matches)

Option (B): $V_B - V_A = 100 - 580 = -480$ V.

Option (C): $V_B - V_A = 400 - 60 = 340$ V.

Option (D): $V_B - V_A = -200 - 0 = -200$ V.

Step 4: Final Answer:

The values of V_A and V_B are 100 V and 580 V.

Quick Tip: Always establish the direction of the required electric field first. Positive charges need an upward field to levitate, meaning the bottom plate must be positive (higher potential) relative to the top plate.

---

40. Two point charges $8\ \mu$C and $-2\ \mu$C are located at $x = 2$ cm and $x = 4$ cm, respectively on the x-axis. The ratio of electric flux due to these charges through two spheres of radii 3 cm and 5 cm with their centers at the origin is ________.

(A) 4 : 1

(B) 3 : 4

(C) 4 : 3

(D) 4 : 5

Correct Answer: (C) 4 : 3

Solution:

Step 1: Understanding the Concept:

According to Gauss's Law, the total electric flux through a closed surface is directly proportional to the net electric charge enclosed by that surface.

Charges outside the surface do not contribute to the net flux.

Step 2: Key Formula or Approach:

Gauss's Law: $\Phi = \frac{q_{enclosed}}{\epsilon_0}$.

Identify which charges are located inside each of the given spheres.

Step 3: Detailed Explanation:

Let's analyze the first sphere:

Radius $R_1 = 3$ cm, centered at the origin.

The charge $q_1 = 8\ \mu$C is at $x = 2$ cm, which is inside the sphere $(2 < 3)$.

The charge $q_2 = -2\ \mu$C is at $x = 4$ cm, which is outside the sphere $(4 > 3)$.

So, the net charge enclosed by the first sphere is $q_{encl,1} = 8\ \mu$C.

The flux through the first sphere is $\Phi_1 = \frac{8\ \mu C}{\epsilon_0}$.

Now, let's analyze the second sphere:

Radius $R_2 = 5$ cm, centered at the origin.

Both the charge $q_1 = 8\ \mu$C (at $x = 2$ cm) and the charge $q_2 = -2\ \mu$C (at $x = 4$ cm) are inside this sphere $(2 < 5$ and $4 < 5)$.

So, the net charge enclosed by the second sphere is $q_{encl,2} = 8\ \mu C + (-2\ \mu C) = 6\ \mu C$.

The flux through the second sphere is $\Phi_2 = \frac{6\ \mu C}{\epsilon_0}$.

Finally, find the ratio of the two fluxes:

$$\text{Ratio} = \frac{\Phi_1}{\Phi_2} = \frac{8\ \mu C/\epsilon_0}{6\ \mu C/\epsilon_0} = \frac{8}{6} = \frac{4}{3}$$

Step 4: Final Answer:

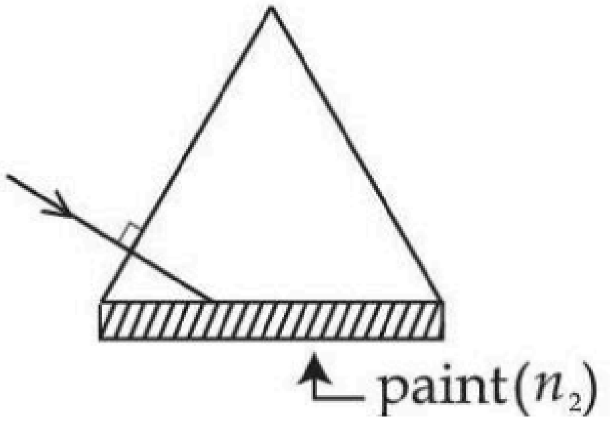
The ratio of electric flux is $4 : 3$.

Quick Tip: Gauss's Law relies strictly on *net enclosed charge*. The position of the charges inside the boundary does not change the total flux, so you only need to determine if a coordinate is less than the radius.

---

41. One side of an equilateral prism is painted by a transparent material of refractive index

n_2. The refractive index of prism is 1.6. The minimum value of n_2 required for total internal reflection from painted face is _________.



(A) $3\sqrt{3}/1.6$

(B) $\sqrt{3}$

(C) $3.2/\sqrt{3}$

(D) $4\sqrt{3}/5$

Correct Answer: (D) $4\sqrt{3}/5$

Solution:

Step 1: Understanding the Concept:

For a light ray to undergo Total Internal Reflection (TIR) at an interface, it must travel from a denser medium to a rarer medium.

The angle of incidence must be greater than or equal to the critical angle ($i \geq \theta_c$).

Based on the geometric setup of an equilateral prism, a horizontally entering ray hits the second face at a specific angle of incidence, allowing us to find the required refractive index.

Step 2: Key Formula or Approach:

The critical angle is given by $\sin\theta_c = \frac{n_2}{n_1}$, where $n_1 = 1.6$.

For TIR to occur, the condition is $\sin i \geq \sin\theta_c \implies \sin i \geq \frac{n_2}{n_1}$.

Rearranging gives the maximum possible value for n_2 to allow TIR: $n_2 \leq n_1 \sin i$.

Step 3: Detailed Explanation:

The prism is equilateral, meaning all its angles are 60°.

The light ray is incident normally on the first face, meaning it passes through without any deviation.

It then hits the second face. The normal to the second face makes a 30° angle with the horizontal.

By geometry, the angle of incidence i at the second face is 60°.

Apply the TIR condition:

$$\sin(60^\circ) \geq \frac{n_2}{1.6}$$

Substitute the value of $\sin(60^\circ)$:

$$\frac{\sqrt{3}}{2} \geq \frac{n_2}{1.6}$$

Solve for n_2:

$$n_2 \leq 1.6 \times \frac{\sqrt{3}}{2}$$

$$n_2 \leq 0.8\sqrt{3}$$

Convert the decimal to a fraction to match the options:

$$0.8 = \frac{8}{10} = \frac{4}{5}$$

So, $n_2 \leq \frac{4\sqrt{3}}{5}$.

The limiting (or maximum boundary) value for n_2 is $\frac{4\sqrt{3}}{5}$.

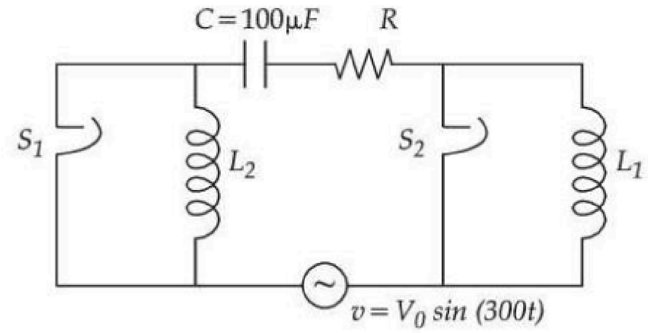
Step 4: Final Answer:

The required limiting value of n_2 is $\frac{4\sqrt{3}}{5}$.

Quick Tip: Pay close attention to geometry when a ray enters normally. In a prism with angle A, a normally incident ray always hits the opposite face at an angle of incidence exactly equal to A.

---

42. The figure given below shows an LCR series circuit with two switches S_1 and S_2. When switch S_1 is closed keeping S_2 open, the phase difference (ϕ) between the current and source voltage is 30° and phase difference is 60° when S_2 is closed keeping S_1 open. The value of $(3L_1 - L_2)$ is ________ H. (Given $C = 100\mu$F, $v = V_0 \sin(300t)$)



(A) $\frac{9}{2}$

(B) $\frac{2}{9}$

(C) $\frac{1}{3}$

(D) 3

Correct Answer: (B) $\frac{2}{9}$

Solution:

Step 1: Understanding the Concept:

The circuit has inductors, a capacitor, and a resistor in series, but switches can short out certain components.

Closing S_1 shorts out L_2, leaving L_1, C, R in the circuit.

Closing S_2 shorts out L_1, leaving L_2, C, R in the circuit.

We use the phase angle formula for series LCR circuits to establish equations for both cases.

Step 2: Key Formula or Approach:

The phase difference ϕ in a series LCR circuit is given by $\tan\phi = \frac{|X_L - X_C|}{R}$.

Assuming the circuit remains capacitive ($X_C > X_L$), we use $\tan\phi = \frac{X_C - X_L}{R}$.

(Note: Assuming it is inductive yields the exact same final result for the required expression).

Step 3: Detailed Explanation:

Case 1: S_1 closed, S_2 open. L_2 is shorted.

The phase difference is 30°.

$$\tan 30^\circ = \frac{X_C - X_{L1}}{R} \implies \frac{1}{\sqrt{3}} = \frac{X_C - X_{L1}}{R}$$

$$X_{L1} = X_C - \frac{R}{\sqrt{3}} \quad \text{— (Eq 1)}$$

Case 2: S_2 closed, S_1 open. L_1 is shorted.

The phase difference is 60°.

$$\tan 60^\circ = \frac{X_C - X_{L2}}{R} \implies \sqrt{3} = \frac{X_C - X_{L2}}{R}$$

$$X_{L2} = X_C - R\sqrt{3} \quad \text{— (Eq 2)}$$

We are asked to find the value of $3L_1 - L_2$.

Since $X_L = \omega L$, we can write $3L_1 - L_2 = \frac{3X_{L1} - X_{L2}}{\omega}$.

Let's compute $3X_{L1} - X_{L2}$ using Eq 1 and Eq 2:

$$3X_{L1} - X_{L2} = 3\left(X_C - \frac{R}{\sqrt{3}}\right) - \left(X_C - R\sqrt{3}\right)$$

$$3X_{L1} - X_{L2} = 3X_C - \sqrt{3}R - X_C + \sqrt{3}R = 2X_C$$

The unknown resistance R cancels out perfectly!

Now, calculate X_C:

Given $\omega = 300$ rad/s and $C = 100\mu\text{F} = 10^{-4}$ F.

$$X_C = \frac{1}{\omega C} = \frac{1}{300 \times 10^{-4}} = \frac{10^4}{300} = \frac{100}{3}\ \Omega$$

Finally, evaluate the required expression:

$$3L_1 - L_2 = \frac{2X_C}{\omega} = \frac{2 \times (100/3)}{300} = \frac{200}{900} = \frac{2}{9}\ \text{H}$$

Step 4: Final Answer:

The value of $(3L_1 - L_2)$ is $\frac{2}{9}$.

Quick Tip: When a problem asks for a specific linear combination of variables (like $3L_1 - L_2$) and fails to provide a seemingly necessary constant (like R), it is a massive hint that the missing constant will algebraically cancel out during evaluation.

---

43. A circular current loop of radius R is placed inside square loop of side length L (L » R) such that they are co-planar and their centers coincide. The permeability of free space is μ_0. The mutual inductance between circular loop and square loop is ________.

(A) $2\sqrt{2}\mu_0 L^2/R$

(B) $\sqrt{2}\mu_0 L^2/R$

(C) $\sqrt{2}\mu_0 R^2/L$

(D) $2\sqrt{2}\mu_0 R^2/L$

Correct Answer: (D) $2\sqrt{2}\mu_0 R^2/L$

Solution:

Step 1: Understanding the Concept:

Mutual inductance is the ratio of the magnetic flux linked with one circuit to the current in the other circuit.

Because $L \gg R$, we can assume the magnetic field produced by the large square loop is practically uniform over the entire small area of the inner circular loop.

Step 2: Key Formula or Approach:

The magnetic field at the center of a square loop of side L carrying current I is derived using the Biot-Savart law: $B = \frac{\mu_0 I}{4\pi d}(\sin\theta_1 + \sin\theta_2)$ for each of the 4 sides.

Magnetic flux $\Phi = B \times A_{circle}$.

Mutual inductance $M = \frac{\Phi}{I}$.

Step 3: Detailed Explanation:

For the square loop, the perpendicular distance from the center to any side is $d = L/2$.

The angles made by the ends of a side at the center are $\theta_1 = 45^\circ$ and $\theta_2 = 45^\circ$.

The magnetic field due to one side of the square at the center is:

$$B_{side} = \frac{\mu_0 I}{4\pi(L/2)}(\sin 45^\circ + \sin 45^\circ) = \frac{\mu_0 I}{2\pi L}\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) = \frac{\mu_0 I}{2\pi L}\left(\frac{2}{\sqrt{2}}\right) = \frac{\sqrt{2}\mu_0 I}{\pi L}$$

Since there are 4 identical sides, the total magnetic field at the center is:

$$B_{total} = 4 \times B_{side} = 4\left(\frac{\sqrt{2}\mu_0 I}{\pi L}\right) = \frac{4\sqrt{2}\mu_0 I}{\pi L}$$

Wait, let's re-evaluate this carefully.

$B_{side} = \frac{\mu_0 I}{4\pi(L/2)}(\sin 45^\circ + \sin 45^\circ) = \frac{2\mu_0 I}{4\pi L}(\sqrt{2}) = \frac{\sqrt{2}\mu_0 I}{2\pi L}$.

Total field $B_{total} = 4 \times \frac{\sqrt{2}\mu_0 I}{2\pi L} = \frac{2\sqrt{2}\mu_0 I}{\pi L}$.

(Correction applied: The factor of 2 was initially misplaced).

Now, calculate the magnetic flux through the small circular loop of area $A = \pi R^2$:

$$\Phi = B_{total} \times A = \left(\frac{2\sqrt{2}\mu_0 I}{\pi L}\right)(\pi R^2) = \frac{2\sqrt{2}\mu_0 R^2 I}{L}$$

Finally, the mutual inductance M is the flux per unit current:

$$M = \frac{\Phi}{I} = \frac{2\sqrt{2}\mu_0 R^2}{L}$$

Step 4: Final Answer:

The mutual inductance is $2\sqrt{2}\mu_0 R^2/L$.

Quick Tip: Always use the "large to small" approach for mutual inductance when dealing with vastly different sizes: compute the B-field from the larger loop and multiply by the area of the smaller loop. The reciprocity theorem ensures $M_{12} = M_{21}$.

---

44. The binding energy per nucleon of $^{209}_{83}$Bi is ________ MeV.
Take $m(^{209}_{83}\text{Bi}) = 208.980388$ u, $m_p = 1.007825$ u, $m_n = 1.008665$ u, 1 u $= 931$ MeV/c^2

(A) 7.48

(B) 7.84

(C) 8.79

(D) 6.94

Correct Answer: (B) 7.84

Solution:

Step 1: Understanding the Concept:

The binding energy is the energy required to disassemble a nucleus into its constituent protons and neutrons.

It is calculated by finding the mass defect (the difference between the sum of the masses of individual nucleons and the actual mass of the nucleus) and converting it to energy. Finally, dividing by the total number of nucleons gives the binding energy per nucleon.

Step 2: Key Formula or Approach:

Number of protons $Z = 83$, Number of neutrons $N = A - Z = 209 - 83 = 126$.

Mass defect: $\Delta m = Zm_p + Nm_n - M_{nucleus}$.

Binding Energy: $BE = \Delta m \times 931$ MeV.

Binding Energy per nucleon: $BE/A = BE/209$.

Step 3: Detailed Explanation:

Calculate the total mass of the individual constituent nucleons:

Mass of 83 protons = 83 × 1.007825 u = 83.649475 u.

Mass of 126 neutrons = 126 × 1.008665 u = 127.091790 u.

Total constituent mass = 83.649475 + 127.091790 = 210.741265 u.

Now, subtract the actual mass of the Bismuth nucleus to find the mass defect:

$$\Delta m = 210.741265 \text{ u} - 208.980388 \text{ u} = 1.760877 \text{ u}$$

Convert the mass defect into energy in MeV:

$$BE = 1.760877 \text{ u} \times 931 \text{ MeV/u} = 1639.376487 \text{ MeV}$$

Calculate the binding energy per nucleon by dividing by $A = 209$:

$$\frac{BE}{A} = \frac{1639.376487}{209} \approx 7.8439 \text{ MeV/nucleon}$$

This rounds to 7.84 MeV.

Step 4: Final Answer:

The binding energy per nucleon is 7.84 MeV.

Quick Tip: Always maintain at least 6 decimal places during mass defect calculations to prevent severe rounding errors before multiplying by 931.

45. The equation of motion of a particle is given by $x = a\sin(50t + \pi/3)$ cm. The particle will come to rest at time t_1 and it will have zero acceleration at time t_2. The t_1 and t_2 respectively are ________.

(A) $\frac{\pi}{300}$ s, $\frac{\pi}{75}$ s
(B) $\frac{\pi}{75}$ s, $\frac{\pi}{300}$ s
(C) $\frac{\pi}{300}$ s, $\frac{\pi}{25}$ s
(D) $\frac{\pi}{50}$ s, $\frac{\pi}{100}$ s

Correct Answer: (A) $\frac{\pi}{300}$ s, $\frac{\pi}{75}$ s

Solution:

Step 1: Understanding the Concept:

The given equation describes Simple Harmonic Motion (SHM).

The particle comes to rest when its velocity is zero (at the extreme positions).

The particle has zero acceleration when it passes through the mean position.

We find velocity and acceleration by taking the first and second derivatives of the position function with respect to time.

Step 2: Key Formula or Approach:

Velocity $v = \frac{dx}{dt}$.

Acceleration $A = \frac{dv}{dt}$.

Set $v = 0$ to find t_1 and $A = 0$ to find t_2.

Step 3: Detailed Explanation:

Given $x = a\sin(50t + \pi/3)$.

Differentiate to find velocity:

$$v = \frac{d}{dt}[a\sin(50t + \pi/3)] = 50a\cos(50t + \pi/3)$$

The particle comes to rest when $v = 0$:

$$50a\cos(50t_1 + \pi/3) = 0 \implies \cos(50t_1 + \pi/3) = 0$$

The first positive time this occurs is when the phase equals $\pi/2$:

$$50t_1 + \frac{\pi}{3} = \frac{\pi}{2}$$
$$50t_1 = \frac{\pi}{2} - \frac{\pi}{3} = \frac{\pi}{6}$$
$$t_1 = \frac{\pi}{6 \times 50} = \frac{\pi}{300} \text{ s}$$

Now, differentiate velocity to find acceleration:

$$A = \frac{d}{dt}[50a\cos(50t + \pi/3)] = -2500a\sin(50t + \pi/3)$$

The acceleration is zero when $A = 0$:

$$-2500a\sin(50t_2 + \pi/3) = 0 \implies \sin(50t_2 + \pi/3) = 0$$

The first positive time this occurs is when the phase equals π:

$$50t_2 + \frac{\pi}{3} = \pi$$
$$50t_2 = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$$
$$t_2 = \frac{2\pi}{3 \times 50} = \frac{2\pi}{150} = \frac{\pi}{75} \text{ s}$$

Step 4: Final Answer:

The times t_1 and t_2 respectively are $\frac{\pi}{300}$ s, $\frac{\pi}{75}$ s.

Quick Tip: In SHM, velocity is zero at extremes (phase $= \pi/2, 3\pi/2$) and acceleration is zero at the mean position (phase $= \pi, 2\pi$). Setting the phase equal to these standard angles gives the fastest solution.

Physics Section - B

46. In a Young's double slit experiment, the intensity at some point on the screen is found

to be $\frac{3}{4}$ times of the maximum of the interference pattern. The path difference between the interfering waves at this point is $\frac{\lambda}{x}$ where λ is wavelength of the incident light. The value of x is ________.

Correct Answer: 6

Solution:

Step 1: Understanding the Concept:

In an interference pattern, the resultant intensity at any point on the screen depends on the phase difference between the two interfering waves.

Once the phase difference is found, it can be directly converted into the corresponding path difference.

Step 2: Key Formula or Approach:

The intensity formula is $I = I_{max} \cos^2\left(\frac{\phi}{2}\right)$, where ϕ is the phase difference.

The relation between phase difference and path difference is $\phi = \frac{2\pi}{\lambda}\Delta x$.

Step 3: Detailed Explanation:

Given that the intensity is $I = \frac{3}{4}I_{max}$.

Substitute this into the intensity formula:

$$\frac{3}{4}I_{max} = I_{max}\cos^2\left(\frac{\phi}{2}\right)$$

$$\cos^2\left(\frac{\phi}{2}\right) = \frac{3}{4}$$

Taking the square root of both sides:

$$\cos\left(\frac{\phi}{2}\right) = \frac{\sqrt{3}}{2}$$

The principal angle whose cosine is $\frac{\sqrt{3}}{2}$ is $30°$ or $\frac{\pi}{6}$ radians.

$$\frac{\phi}{2} = \frac{\pi}{6} \implies \phi = \frac{\pi}{3} \text{ radians}$$

Now, convert the phase difference to a path difference Δx:

$$\phi = \frac{2\pi}{\lambda}\Delta x$$

$$\frac{\pi}{3} = \frac{2\pi}{\lambda}\Delta x$$

$$\Delta x = \frac{\lambda}{3 \times 2} = \frac{\lambda}{6}$$

Comparing this to the given format $\frac{\lambda}{x}$, we find that $x = 6$.

Step 4: Final Answer:

The value of x is 6.

Quick Tip: Remember the standard intensity fractions: $I = I_{max}$ at $\Delta x = 0$, $I = I_{max}/2$ at $\Delta x = \lambda/4$, and $I = 3I_{max}/4$ at $\Delta x = \lambda/6$. Memorizing these saves calculation time.

---

47. Using Bohr's model, calculate the ratio of the magnetic fields generated due to the motion of the electrons in the 2^{nd} and 4^{th} orbits of hydrogen atom ________.

Correct Answer: 32

Solution:

Step 1: Understanding the Concept:

An electron orbiting a nucleus behaves like a circular current loop.

This loop generates a magnetic field at the center of the orbit.

By expressing the magnetic field in terms of the principal quantum number n, we can easily find the ratio for different orbits.

Step 2: Key Formula or Approach:

The magnetic field at the center of a circular current loop is $B = \frac{\mu_0 I}{2r}$.

The equivalent current is $I = \frac{e}{T} = \frac{ev}{2\pi r}$.

Substituting I gives $B = \frac{\mu_0 ev}{4\pi r^2}$.

From Bohr's model, velocity $v \propto \frac{1}{n}$ and radius $r \propto n^2$.

Step 3: Detailed Explanation:

Substitute the proportionalities of v and r into the magnetic field equation:

$$B \propto \frac{v}{r^2}$$

$$B \propto \frac{(1/n)}{(n^2)^2}$$

$$B \propto \frac{1/n}{n^4} = \frac{1}{n^5}$$

So, the magnetic field is inversely proportional to the fifth power of the principal quantum number.

We need the ratio of the magnetic fields in the 2nd ($n_1 = 2$) and 4th ($n_2 = 4$) orbits.

$$\frac{B_2}{B_4} = \frac{(n_4)^5}{(n_2)^5}$$

$$\frac{B_2}{B_4} = \left(\frac{4}{2}\right)^5$$

$$\frac{B_2}{B_4} = (2)^5 = 32$$

Step 4: Final Answer:

The ratio of the magnetic fields is 32.

Quick Tip: For Bohr model proportionalities, memorizing $r \propto n^2/Z$, $v \propto Z/n$, and $E \propto Z^2/n^2$ allows you to quickly derive secondary proportionalities like $T \propto n^3/Z^2$ and $B \propto Z^3/n^5$.

---

48. 5 moles of unknown gas is heated at constant volume from 10 °C to 20 °C. The molar specific heat of this gas at constant pressure $c_p = 8$ cal/mol.$^\circ$C and $R = 8.36$ J/mol.$^\circ$C. The change in the internal energy of the gas is _________ calorie.

Correct Answer: 300

Solution:

Step 1: Understanding the Concept:

The change in internal energy of an ideal gas depends only on its temperature change and molar specific heat at constant volume (c_v), regardless of the process path.

We must ensure all values are in consistent units (calories) before applying the formula.

Step 2: Key Formula or Approach:

Mayer's relation: $c_p - c_v = R$.

Change in internal energy: $\Delta U = nc_v\Delta T$.

Mechanical equivalent of heat: $1 \text{ calorie} = 4.18 \text{ Joules}$.

Step 3: Detailed Explanation:

First, convert the universal gas constant R from Joules to calories.

$$R = 8.36 \text{ J/mol.}^\circ\text{C} = \frac{8.36}{4.18} \text{ cal/mol.}^\circ\text{C} = 2 \text{ cal/mol.}^\circ\text{C}$$

Next, calculate the molar specific heat at constant volume c_v using Mayer's relation.

$$c_v = c_p - R = 8 \text{ cal/mol.}^\circ\text{C} - 2 \text{ cal/mol.}^\circ\text{C} = 6 \text{ cal/mol.}^\circ\text{C}$$

Now, identify the given parameters for the process:

Number of moles $n = 5$.

Temperature change $\Delta T = 20^\circ\text{C} - 10^\circ\text{C} = 10^\circ\text{C}$.

Calculate the change in internal energy:

$$\Delta U = nc_v\Delta T$$

$$\Delta U = 5 \times 6 \times 10$$

$$\Delta U = 300 \text{ calories}$$

Step 4: Final Answer:

The change in the internal energy is 300 calorie.

Quick Tip: Always check the units of R and specific heats. Examiners frequently mix Joules and Calories in the same problem to test your attention to dimensional consistency.

---

49. If sunlight is focused on a paper using convex lens, it starts burning the paper in shortest time when the lens is kept at 30 cm above the paper. If the radius of curvature of the lens is 60 cm then the refractive index of the lens material is $\frac{\alpha}{10}$. The value of α is _________.

Correct Answer: 20

Solution:

Step 1: Understanding the Concept:

Sunlight consists of parallel rays from infinity. When a convex lens focuses parallel rays, they converge exactly at the principal focus.

Since the burning is fastest when focused perfectly, the distance between the lens and the paper is the focal length f.

We can use the Lens Maker's Formula to relate the focal length, radii of curvature, and the refractive index.

Step 2: Key Formula or Approach:

Lens Maker's Formula: $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$.

For an equiconvex lens (implied when only one radius is given), $R_1 = R$ and $R_2 = -R$.

Step 3: Detailed Explanation:

From the problem description, the focal length is $f = 30$ cm.

The radius of curvature is given as $R = 60$ cm. So $R_1 = 60$ cm and $R_2 = -60$ cm.

Apply the Lens Maker's Formula:

$$\frac{1}{30} = (\mu - 1)\left(\frac{1}{60} - \frac{1}{-60}\right)$$

$$\frac{1}{30} = (\mu - 1)\left(\frac{1}{60} + \frac{1}{60}\right)$$

$$\frac{1}{30} = (\mu - 1)\left(\frac{2}{60}\right)$$

$$\frac{1}{30} = \frac{\mu - 1}{30}$$

Multiply both sides by 30:

$$1 = \mu - 1 \implies \mu = 2$$

The problem states the refractive index is $\frac{\alpha}{10}$.

Equating the values:

$$\frac{\alpha}{10} = 2 \implies \alpha = 20$$

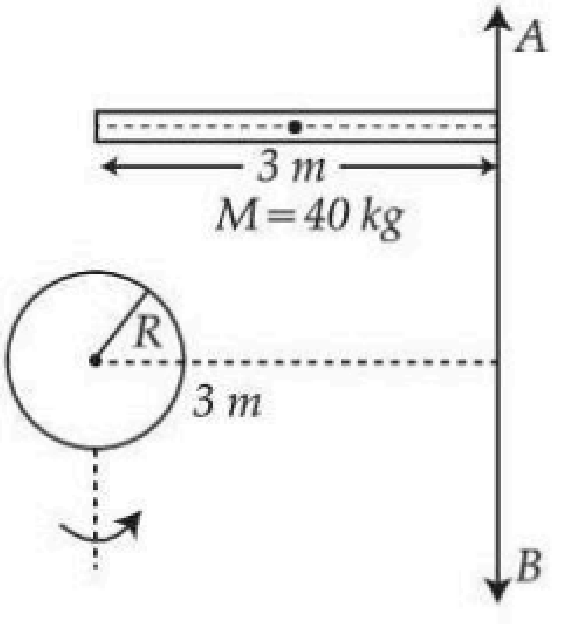
Step 4: Final Answer:

The value of α is 20.

Quick Tip: When a problem simply states "the radius of curvature of the lens", safely assume it refers to a symmetric equiconvex or equiconcave lens where $|R_1| = |R_2| = R$.

---

50. Moment of inertia about an axis AB for a rod of mass 40 kg and length 3 m is same as that of a solid sphere of mass of 10 kg and radius R about an axis parallel to AB axis with separation of 3 m as shown in figure below. The value of R is given as $\sqrt{\frac{\alpha}{2}}$. The value of α is ________.



Correct Answer: 15

Solution:

Step 1: Understanding the Concept:

The total moment of inertia of the solid sphere about the axis AB relies on the parallel axis theorem, as it rotates about an axis separated from its center of mass.

We equate this to the moment of inertia of the rod rotating about one of its ends.

Step 2: Key Formula or Approach:

Moment of inertia of a rod about its end: $I_{rod} = \frac{ML^2}{3}$.

Moment of inertia of a solid sphere about its center of mass: $I_{cm} = \frac{2}{5}mR^2$.

Parallel axis theorem: $I_{sphere} = I_{cm} + md^2$.

Equate the two moments of inertia: $I_{rod} = I_{sphere}$.

Step 3: Detailed Explanation:

First, calculate the moment of inertia of the rod.

Mass of rod $M = 40$ kg, Length $L = 3$ m.

$$I_{rod} = \frac{40 \times (3)^2}{3} = 40 \times 3 = 120 \text{ kg m}^2$$

Next, set up the expression for the moment of inertia of the solid sphere.

Mass of sphere $m = 10$ kg, distance from axis $d = 3$ m.

$$I_{sphere} = \frac{2}{5}mR^2 + md^2$$

$$I_{sphere} = \frac{2}{5}(10)R^2 + 10(3)^2$$

$$I_{sphere} = 4R^2 + 90$$

Now, equate the two moments of inertia:

$$4R^2 + 90 = 120$$

$$4R^2 = 120 - 90 = 30$$

$$R^2 = \frac{30}{4} = \frac{15}{2}$$

Take the square root to find R:

$$R = \sqrt{\frac{15}{2}}$$

The problem states $R = \sqrt{\frac{\alpha}{2}}$. By comparing, we get:

$$\alpha = 15$$

Step 4: Final Answer:

The value of α is 15.

Quick Tip: Always double-check which axis the object is rotating around. For a rod, $ML^2/12$ is for the center, but $ML^2/3$ is for the edge. Misidentifying the axis is the most common error in rigid body dynamics.

Chemistry Section - A

51. The ratio of mass percentage (w/w) of C : H in a hydrocarbon is 12 : 1. It has two carbon atoms. The weight (in g) of $CO_2(g)$ formed when 3.38 g of this hydrocarbon is completely burnt in oxygen is : (Given : Molar mass in g mol^{-1} C : 12, H : 1, O : 16)

(A) 5.68

(B) 11.44

(C) 22.74

(D) 17.05

Correct Answer: (B) 11.44

Solution:

Step 1: Understanding the Concept:

We first determine the empirical formula of the hydrocarbon using the given mass ratio of Carbon to Hydrogen.

Using the fact that it contains exactly two carbon atoms, we find its exact molecular formula and molar mass.

Finally, we apply stoichiometry to the combustion reaction to find the mass of CO_2 produced.

Step 2: Key Formula or Approach:

Moles $= \frac{\text{Given Mass}}{\text{Atomic/Molar Mass}}$.

Balanced combustion equation: $C_xH_y + (x + \frac{y}{4})O_2 \rightarrow xCO_2 + \frac{y}{2}H_2O$.

Step 3: Detailed Explanation:

The given mass ratio of C : H is 12 : 1.

Calculate the mole ratio by dividing by their respective atomic masses:

Moles of C $= \frac{12}{12} = 1$.

Moles of H $= \frac{1}{1} = 1$.

The empirical formula is CH.

Since the molecule contains exactly two carbon atoms, its molecular formula must be C_2H_2 (Ethyne).

The molar mass of C_2H_2 is $(2 \times 12) + (2 \times 1) = 26$ g/mol.

The amount of hydrocarbon burned is 3.38 g. Calculate the moles of C_2H_2:

$$\text{Moles of } C_2H_2 = \frac{3.38}{26} = 0.13 \text{ mol}$$

Write the balanced combustion equation:

$$C_2H_2 + \frac{5}{2}O_2 \rightarrow 2CO_2 + H_2O$$

From the stoichiometry, 1 mole of C_2H_2 produces 2 moles of CO_2.

So, 0.13 moles of C_2H_2 will produce:

$$\text{Moles of } CO_2 = 2 \times 0.13 = 0.26 \text{ mol}$$

Now, calculate the mass of the produced CO_2 (Molar mass of $CO_2 = 44$ g/mol):

$$\text{Mass of } CO_2 = 0.26 \text{ mol} \times 44 \text{ g/mol} = 11.44 \text{ g}$$

Step 4: Final Answer:

The weight of CO_2 formed is 11.44 g.

Quick Tip: To avoid writing the full balanced combustion equation, apply the Principle of Atomic Conservation (POAC) on Carbon: Moles of C atoms in reactant = Moles of C atoms in product. $2 \times n_{C_2H_2} = 1 \times n_{CO_2}$.

---

52. The first and second ionization constants of a weak dibasic acid H_2A are 8.1×10^{-8} and 1.0×10^{-13} respectively. 0.1 mol of H_2A was dissolved in 1L of 0.1 M HCl solution. The concentration of HA^- in the resultant solution is :

(A) 0.1 M

(B) 9.53×10^{-6} M

(C) 8.1×10^{-8} M

(D) 1.0×10^{-13} M

Correct Answer: (C) 8.1×10^{-8} M

Solution:

Step 1: Understanding the Concept:

The dissociation of the weak acid H_2A occurs in a solution already containing a strong acid (HCl).

Due to the common ion effect, the presence of the strong acid heavily suppresses the dissociation of the weak acid.

We can assume the concentration of H^+ is determined entirely by the HCl, and the concentration of un-ionized H_2A remains equal to its initial concentration.

Step 2: Key Formula or Approach:

The first dissociation step is: $H_2A \rightleftharpoons H^+ + HA^-$.

The equilibrium constant expression is: $K_{a1} = \frac{[H^+][HA^-]}{[H_2A]}$.
Given values: $K_{a1} = 8.1 \times 10^{-8}$, $[HCl] = 0.1$ M, Initial $[H_2A] = 0.1$ M.

Step 3: Detailed Explanation:

The strong acid completely dissociates:

$HCl \rightarrow H^+ + Cl^-$

So, the initial concentration of H^+ is 0.1 M.

Let x be the amount of H_2A that dissociates. At equilibrium:

$[H_2A] = 0.1 - x \approx 0.1$ M (since x is extremely small).

$[H^+] = 0.1 + x \approx 0.1$ M (common ion effect).

$[HA^-] = x$.

Substitute these equilibrium concentrations into the K_{a1} expression:

$$8.1 \times 10^{-8} = \frac{(0.1) \cdot [HA^-]}{0.1}$$

The 0.1 terms cancel out perfectly:

$$[HA^-] = 8.1 \times 10^{-8} \text{ M}$$

(Note: The second dissociation step involving K_{a2} consumes a negligible amount of HA^- due to its extremely small constant 1.0×10^{-13}, so this approximation holds true).

Step 4: Final Answer:

The concentration of HA^- is 8.1×10^{-8} M.

Quick Tip: When a weak acid is placed in a strong acid of the same concentration, the concentration of the first conjugate base $[HA^-]$ is almost exactly equal to K_{a1}.

---

53. SF_4 is isostructural with :

A. BrF_4^-

B. CH_4

C. IF_4^+

D. XeF_4

E. XeO_2F_2

Choose the correct answer from the options given below :

(A) C Only

(B) C and E Only

(C) A and D Only

(D) B and E Only

Correct Answer: (B) C and E Only

Solution:

Step 1: Understanding the Concept:

Two molecules are isostructural if they possess the exact same molecular geometry (shape).

We determine the shape using VSEPR theory by counting the number of bond pairs (bp) and lone pairs (lp) on the central atom.

Step 2: Key Formula or Approach:

Steric number (SN) = (Number of valence electrons on central atom + number of monovalent atoms - cationic charge + anionic charge) / 2.

Find SN and the number of lone pairs to determine hybridization and geometry.

Step 3: Detailed Explanation:

First, analyze the reference molecule SF_4:

Sulfur (S) has 6 valence electrons. It forms 4 single bonds with F.

Remaining electrons = 2 (1 lone pair).

Total electron domains = 4 (bp) + 1 (lp) = 5. Hybridization is sp^3d.

Geometry with 1 lone pair in a trigonal bipyramidal arrangement is "See-saw".

Now analyze the options:

A. BrF_4^-: Bromine (Br) has 7 valence e, +1 for negative charge = 8 e.

4 bonds with F means 4 remaining e (2 lone pairs).

SN = 4 (bp) + 2 (lp) = 6 (sp^3d^2). Geometry is "Square planar".

B. CH_4: Carbon (C) has 4 valence e. 4 bonds, 0 lone pairs.

SN = 4 (bp). Geometry is "Tetrahedral".

C. IF_4^+: Iodine (I) has 7 valence e, -1 for positive charge = 6 e.

4 bonds with F means 2 remaining e (1 lone pair).

SN = 4 (bp) + 1 (lp) = 5 (sp^3d). Geometry is "See-saw". (Matches!)

D. XeF_4: Xenon (Xe) has 8 valence e.

4 bonds with F means 4 remaining e (2 lone pairs).

SN = 4 (bp) + 2 (lp) = 6 (sp^3d^2). Geometry is "Square planar".

E. XeO_2F_2: Xenon (Xe) has 8 valence e.

It forms 2 double bonds with O (using 4 e) and 2 single bonds with F (using 2 e). Total 6 e used.

Remaining e = 2 (1 lone pair).

Electron domains = 4 (sigma bonds) + 1 (lp) = 5 (sp^3d). Geometry is "See-saw". (Matches!)

Both IF_4^+ and XeO_2F_2 have a see-saw structure identical to SF_4.

Step 4: Final Answer:

The correct choice is C and E Only.

Quick Tip: To quickly find isostructural species, identify the number of lone pairs. If two species have the same steric number AND the same number of lone pairs, they are virtually always isostructural.

---

54. Gas 'A' undergoes change from state 'X' to state 'Y'. In this process, the heat absorbed and work done by the gas is 10 J and 18 J respectively. Now gas is brought back to state 'X' by another process during which 6 J of heat is evolved. In the reverse process of 'Y' to 'X',

(A) 18 J of the work is done by the gas 'A'.

(B) 2 J of the work is done by the gas 'A'.

(C) 12 J of the work is done on the gas 'A' by the surrounding.

(D) 14 J of the work is done on the gas 'A' by the surrounding.

Correct Answer: (D) 14 J of the work is done on the gas 'A' by the surrounding.

Solution:

Step 1: Understanding the Concept:

Internal energy (U) is a state function. This means the change in internal energy for a round trip (from X to Y and back to X) is exactly zero.

We apply the First Law of Thermodynamics to both the forward and reverse processes to find the unknown work value.

Step 2: Key Formula or Approach:

First Law of Thermodynamics (IUPAC convention): $\Delta U = q + w$.

Here, q is positive if heat is absorbed, negative if evolved.

w is positive if work is done ON the gas, negative if work is done BY the gas.

For a cyclic process: $\Delta U_{X\to Y} = -\Delta U_{Y\to X}$.

Step 3: Detailed Explanation:

Analyze the forward process ($X \to Y$):

Heat absorbed, $q_1 = +10$ J.

Work done BY the gas means it expanded, so $w_1 = -18$ J.

Calculate the change in internal energy:

$$\Delta U_{X\to Y} = q_1 + w_1 = 10\text{ J} + (-18\text{ J}) = -8\text{ J}$$

Analyze the reverse process ($Y \to X$):

Since internal energy is a state function:

$$\Delta U_{Y\to X} = -\Delta U_{X\to Y} = -(-8\text{ J}) = +8\text{ J}$$

Heat is evolved in this step, so $q_2 = -6$ J.

Apply the First Law to find the work w_2:

$$\Delta U_{Y\to X} = q_2 + w_2$$

$$8\text{ J} = -6\text{ J} + w_2$$

$$w_2 = 8\text{ J} + 6\text{ J} = +14\text{ J}$$

Since w_2 is positive, it means 14 J of work is done ON the gas by the surroundings.

Step 4: Final Answer:

14 J of the work is done on the gas 'A' by the surrounding.

Quick Tip: Strictly adhere to IUPAC sign conventions in Chemistry: Work done BY the system is negative ($-w$), Work done ON the system is positive ($+w$). This contrasts with standard Physics conventions.

---

55. Solution A is prepared by dissolving 1 g of a protein (molar mass = 50000 g mol^{-1}) in 0.5 L of water at 300 K. Its osmotic pressure is x bar. Solution B is made by dissolving 2 g of same protein in 1 L of water at 300 K. Osmotic pressure of solution B is y bar. Entire solution of A is mixed with entire solution of B at same temperature. The osmotic pressure of resultant solution is z bar. x, y and z respectively are : (R = 0.083 L bar mol^{-1} K^{-1})

(A) $9.96 \times 10^{-4}; 9.96 \times 10^{-4}; 9.96 \times 10^{-4}$

(B) $9.96 \times 10^{-4}; 9.96 \times 10^{-4}; 19.92 \times 10^{-4}$

(C) $4.98 \times 10^{-4}; 4.98 \times 10^{-4}; 9.96 \times 10^{-4}$

(D) $4.98 \times 10^{-4}; 4.98 \times 10^{-4}; 4.98 \times 10^{-4}$

Correct Answer: (A) $9.96 \times 10^{-4}; 9.96 \times 10^{-4}; 9.96 \times 10^{-4}$

Solution:

Step 1: Understanding the Concept:

Osmotic pressure depends directly on the molar concentration (Molarity) of the solution.

If two solutions have the identical concentration and temperature, they will have identical osmotic pressures.

Furthermore, mixing two solutions of the exact same concentration will result in a final mixture with that same constant concentration.

Step 2: Key Formula or Approach:

Molarity $C = \frac{\text{moles of solute}}{\text{Volume of solution in L}} = \frac{\text{mass/Molar mass}}{V}$.

Osmotic pressure $\pi = CRT$.

Step 3: Detailed Explanation:

Analyze Solution A:

Mass $m_A = 1$ g, Volume $V_A = 0.5$ L.

Concentration $C_A = \frac{1/50000}{0.5} = \frac{2}{50000} = 4 \times 10^{-5}$ mol/L.

Calculate the osmotic pressure x:

$$x = \pi_A = C_A RT = (4 \times 10^{-5}) \times 0.083 \times 300$$

$$x = 1200 \times 10^{-5} \times 0.083 = 99.6 \times 10^{-5} = 9.96 \times 10^{-4} \text{ bar}$$

Analyze Solution B:

Mass $m_B = 2$ g, Volume $V_B = 1$ L.

Concentration $C_B = \frac{2/50000}{1} = \frac{2}{50000} = 4 \times 10^{-5}$ mol/L.

Since $C_A = C_B$ and temperature is the same, the osmotic pressure is identical:

$$y = \pi_B = 9.96 \times 10^{-4} \text{ bar}$$

Analyze the Mixed Solution:

Total mass $= 1 \text{ g} + 2 \text{ g} = 3$ g.

Total volume $= 0.5 \text{ L} + 1 \text{ L} = 1.5$ L.

Concentration $C_{mix} = \frac{3/50000}{1.5} = \frac{2}{50000} = 4 \times 10^{-5}$ mol/L.

Since the final concentration remains identical, the final osmotic pressure z is also identical.

$$z = \pi_{mix} = 9.96 \times 10^{-4} \text{ bar}$$

Step 4: Final Answer:

The values are $9.96 \times 10^{-4}; 9.96 \times 10^{-4}; 9.96 \times 10^{-4}$.

Quick Tip: Before doing any complex calculations, always compare the ratio of mass to volume for both solutions. If $m_1/v_1 = m_2/v_2$, the concentrations are equal, meaning mixing them changes nothing about intensive properties like concentration or osmotic pressure.

---

56. At 25°C, 20.0 mL of 0.2 M weak monoprotic acid HX is titrated against 0.2 M NaOH. The pH of the solution (a) at the start of the titration (when NaOH has not been added) and (b) when 10 mL of NaOH is added respectively, are :

Given : $K_a = 5 \times 10^{-4}$, **pK**$_a = 3.3$, $\alpha \ll 1$

(A) 0.7, 2.0

(B) 2.0, 3.3

(C) 1.1, 2.2

(D) 3.0, 2.2

Correct Answer: (B) 2.0, 3.3

Solution:

Step 1: Understanding the Concept:

Part (a) is a simple weak acid pH calculation before any base is added.

Part (b) describes a partial neutralization. Since we add exactly half the volume of base needed to fully neutralize the acid, we create an equimolar buffer solution of the weak acid and its conjugate base.

Step 2: Key Formula or Approach:

For a weak acid with $\alpha \ll 1$: $[\mathrm{H}^+] = \sqrt{K_a \cdot C}$ and $\mathrm{pH} = \frac{1}{2}(\mathrm{pK}_a - \log C)$.

For a buffer solution, Henderson-Hasselbalch equation: $\mathrm{pH} = \mathrm{pK}_a + \log \frac{[\mathrm{Salt}]}{[\mathrm{Acid}]}$.

Step 3: Detailed Explanation:

(a) At the start of the titration:

Concentration of HX, $C = 0.2$ M.

Calculate the $[\mathrm{H}^+]$ concentration:

$$[\mathrm{H}^+] = \sqrt{K_a \cdot C} = \sqrt{5 \times 10^{-4} \times 0.2} = \sqrt{1.0 \times 10^{-4}} = 10^{-2}\ \mathrm{M}$$

$$pH = -\log(10^{-2}) = 2.0$$

(b) After adding 10 mL of 0.2 M NaOH:

Initial millimoles of HX = 20.0 mL × 0.2 M = 4.0 mmol.

Millimoles of NaOH added = 10.0 mL × 0.2 M = 2.0 mmol.

The strong base reacts completely with the weak acid:

$HX + NaOH \rightarrow NaX + H_2O$

Remaining HX = 4.0 − 2.0 = 2.0 mmol.

Formed NaX (salt) = 2.0 mmol.

Since the millimoles of the weak acid and its conjugate base are equal ([Salt] = [Acid]), this is the half-equivalence point.

Using the Henderson-Hasselbalch equation:

$$pH = pK_a + \log\left(\frac{2.0}{2.0}\right) = 3.3 + \log(1) = 3.3 + 0 = 3.3$$

Step 4: Final Answer:

The pH values are 2.0 and 3.3 respectively.

Quick Tip: At the exact half-equivalence point of any weak acid-strong base titration, the pH is always perfectly equal to the pK_a of the weak acid. Identifying this saves you from doing buffer math.

---

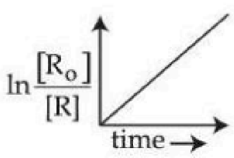
57. Consider the reaction aX → bY, for which the rate constant at 30°C is 1×10^{-3} mol^{-1} L s^{-1}. Which of the following statements are true ?

A. When concentration of 'X' is increased to four times, the rate of reaction becomes 16 times.

B. The reaction is a second order reaction.

C. The half-life period is independent of the concentration of X.

D. Decomposition of N_2O_5 is an example of the above reaction.

E. 

is valid for the above reaction.

Choose the correct answer from the options given below :

(A) A and B Only

(B) A, B and C Only

(C) A, B, D and E Only

(D) C and D Only

Correct Answer: (A) A and B Only

Solution:

Step 1: Understanding the Concept:

The order of a chemical reaction can be deduced directly from the units of its rate constant (k).

Once the order is identified, all standard kinetic properties (rate law, half-life formula, and graphical relationships) are fixed.

Step 2: Key Formula or Approach:

General unit of rate constant: $M^{1-n}s^{-1}$ or $(\text{mol L}^{-1})^{1-n}s^{-1}$, where n is the order.

For 2nd order: Rate $= k[X]^2$ and $t_{1/2} = \frac{1}{k[X]_0}$.

Step 3: Detailed Explanation:

Analyze the given rate constant:

$k = 1 \times 10^{-3}\ \text{mol}^{-1}\ \text{L s}^{-1} = 1 \times 10^{-3}\ \text{M}^{-1}\text{s}^{-1}$.

Equating units: $1 - n = -1 \implies n = 2$. So, this is a second-order reaction.

Now evaluate the statements:

A. Rate $= k[X]^2$. If $[X] \to 4[X]$, Rate $\to k(4[X])^2 = 16k[X]^2$. The rate becomes 16 times. (TRUE)

B. As derived from the units, the reaction is a second-order reaction. (TRUE)

C. For a 2nd order reaction, $t_{1/2} = \frac{1}{k[X]_0}$. It is inversely dependent on the initial concentration, not independent. (FALSE)

D. The thermal decomposition of N_2O_5 is a well-known first-order reaction. (FALSE)

E. A plot of $\ln([R]_0/[R])$ vs time yielding a straight line represents the integrated rate law for a first-order reaction. For a 2nd order reaction, the linear plot is $1/[R]$ vs time. (FALSE)

Therefore, only statements A and B are correct.

Step 4: Final Answer:

The correct choice is A and B Only.

Quick Tip: Always memorize the generic formula for rate constant units: $(\text{concentration})^{1-n}(\text{time})^{-1}$. It allows instant identification of the reaction order without any experimental data tables.

---

58. The correct set that contains all kinds (basic, acidic, amphoteric and neutral) of oxides is :

(A) Na_2O, K_2O, Al_2O_3 and As_2O_3

(B) Al_2O_3, As_2O_3, CO and NO

(C) K_2O, Cl_2O_7, As_2O_3 and NO

(D) Na_2O, N_2O, Al_2O_3 and CO

Correct Answer: (C) K_2O, Cl_2O_7, As_2O_3 and NO

Solution:

Step 1: Understanding the Concept:

Oxides are classified based on their acid-base characteristics:

- Basic: Typically metal oxides of groups 1 2 (e.g., Na_2O, K_2O).

- Acidic: Typically non-metal oxides with high oxidation states (e.g., Cl_2O_7, SO_3).

- Amphoteric: Certain metal/metalloid oxides that react with both acids and bases (e.g., Al_2O_3, ZnO, As_2O_3).

- Neutral: Non-metal oxides that react with neither (strictly CO, NO, N_2O).

Step 2: Key Formula or Approach:

We must systematically evaluate each option to find the one containing exactly one of each class.

Step 3: Detailed Explanation:

Let's analyze the options:

Option (A): Na_2O (Basic), K_2O (Basic), Al_2O_3 (Amphoteric), As_2O_3 (Amphoteric).

Result: Missing Acidic and Neutral oxides.

Option (B): Al_2O_3 (Amphoteric), As_2O_3 (Amphoteric), CO (Neutral), NO (Neutral).

Result: Missing Basic and Acidic oxides.

Option (C): K_2O (Basic), Cl_2O_7 (Acidic), As_2O_3 (Amphoteric), NO (Neutral).

Result: Contains exactly one of each kind!

Option (D): Na_2O (Basic), N_2O (Neutral), Al_2O_3 (Amphoteric), CO (Neutral).

Result: Missing Acidic oxides.

Step 4: Final Answer:

The correct set is K_2O, Cl_2O_7, As_2O_3 and NO.

Quick Tip: There are only three neutral oxides you must strictly memorize for exams: CO, NO, and N_2O. If a set asks for a neutral oxide, it MUST contain one of these three.

---

59. Given below are two statements :

Statement I : The second ionization enthalpy of B, Al and Ga is in the order of B $>$ Al $>$ Ga.

Statement II : The correct order in terms of first ionization enthalpy is Si $<$ Ge $<$ Pb $<$ Sn.

In the light of the above statements, choose the correct answer from the options given below :

(A) Both Statement I and Statement II are true

(B) Both Statement I and Statement II are false

(C) Statement I is true but Statement II is false

(D) Statement I is false but Statement II is true

Correct Answer: (B) Both Statement I and Statement II are false

Solution:

Step 1: Understanding the Concept:

Ionization enthalpy generally decreases down a group due to increased atomic size. However, deviations occur in the p-block due to poor shielding by d and f-orbitals (d-block contraction and lanthanide contraction), increasing the effective nuclear charge (Z_{eff}) on valence electrons.

Step 2: Key Formula or Approach:

Evaluate Group 13 second IE anomalies (B, Al, Ga).

Evaluate Group 14 first IE anomalies (Si, Ge, Sn, Pb).

Step 3: Detailed Explanation:

Evaluate Statement I:

We are looking at the Second Ionization Energy (IE_2) of Group 13 elements.

B (Group 13, Period 2): B^+ is $1s^2 2s^2$. Removing an electron from the small $2s$ orbital requires very high energy.

Al (Group 13, Period 3): Al^+ is $[Ne]3s^2$.

Ga (Group 13, Period 4): Ga^+ is $[Ar]3d^{10}4s^2$.

Because the $3d$ electrons in Gallium shield the nucleus very poorly, the $4s$ electrons experience a significantly higher effective nuclear charge compared to Aluminum. Thus, it is harder to remove the $4s$ electron from Ga^+ than the $3s$ electron from Al^+.

The actual order of IE_2 is $B > Ga > Al$.

The statement claims $B > Al > Ga$, which is FALSE.

Evaluate Statement II:

We are looking at the First Ionization Energy (IE_1) of Group 14 elements.

The general trend is a decrease down the group: $C > Si > Ge > Sn$.

However, for Lead (Pb), the presence of filled $4f$ and $5d$ orbitals causes severe lanthanide contraction. The very poor shielding heavily increases Z_{eff}, making Pb's IE_1 slightly higher than Sn's.

The actual order of IE_1 is $C > Si > Ge > Pb > Sn$.

The statement claims $Si < Ge < Pb < Sn$, which asserts a completely reversed and incorrect trend.

Therefore, Statement II is FALSE.

Step 4: Final Answer:

Both Statement I and Statement II are false.

Quick Tip: Always watch out for Gallium (Ga) and Lead (Pb) in ionization energy trend questions. The poor shielding of d-orbitals makes Ga abnormally high compared to Al, and f-orbitals make Pb abnormally high compared to Sn.

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60. Given below are two statements :

Statement I : Among Zn, Mn, Sc and Cu, the energy required to remove the third valence electron is highest for Zn and lowest for Sc.

Statement II : The correct order of the following complexes in terms of CFSE is $[Co(H_2O)_6]^{2+} < [Co(H_2O)_6]^{3+} < [Co(en)_3]^{3+}$.

In the light of the above statements, choose the correct answer from the options given below :

(A) Both Statement I and Statement II are true

(B) Both Statement I and Statement II are false

(C) Statement I is true but Statement II is false

(D) Statement I is false but Statement II is true

Correct Answer: (A) Both Statement I and Statement II are true

Solution:

Step 1: Understanding the Concept:

Statement I relies on the stability of electronic configurations. The Third Ionization Energy (IE_3) involves removing an electron from a +2 cation. High stability of the +2 ion means a high IE_3.

Statement II relies on Crystal Field Stabilization Energy (CFSE). CFSE depends on the oxidation state of the central metal and the field strength of the ligands.

Step 2: Key Formula or Approach:

Write electronic configurations for the +2 ions to assess IE_3.

For CFSE (Δ_o): It increases with higher oxidation state of the central metal ion and with stronger field ligands (Spectrochemical series).

Step 3: Detailed Explanation:

Evaluate Statement I:

Sc: $[Ar]3d^14s^2 \implies Sc^{2+}$ is $[Ar]3d^1$. Removing the 3rd electron leaves a highly stable noble gas core (Ar). Thus, its IE_3 is exceptionally low (lowest in the 3d series).
Zn: $[Ar]3d^{10}4s^2 \implies Zn^{2+}$ is $[Ar]3d^{10}$. This is a completely filled, extremely stable d-subshell. Disrupting this full shell requires enormous energy. Thus, its IE_3 is exceptionally high (highest in the 3d series).
Statement I is TRUE.

Evaluate Statement II:
We compare the Crystal Field Splitting Energy (Δ_o) of three complexes.
1) $[Co(H_2O)_6]^{2+}$: Cobalt is in +2 oxidation state. H_2O is a weak field ligand.
2) $[Co(H_2O)_6]^{3+}$: Cobalt is in +3 oxidation state. A higher oxidation state strongly pulls the ligands closer, significantly increasing the splitting energy (Δ_o) compared to the +2 state, even with the same weak ligand.
3) $[Co(en)_3]^{3+}$: Cobalt is in +3 oxidation state, but Ethylenediamine (en) is a much stronger field ligand than H_2O (as per the spectrochemical series). This results in the highest splitting energy.
Therefore, the order of CFSE is: $[Co(H_2O)_6]^{2+} < [Co(H_2O)_6]^{3+} < [Co(en)_3]^{3+}$.
Statement II is TRUE.

Step 4: Final Answer:
Both Statement I and Statement II are true.

Quick Tip: A d^0, d^5, or d^{10} configuration signifies an impending massive jump in ionization energy. For CFSE, remember the priority: Oxidation state of central metal affects Δ_o more dramatically than moderate ligand changes.

61. **Which of the following complexes will show coordination isomerism ?**

A. $[Ag(NH_3)_2][Ag(CN)_2]$
B. $[Co(NH_3)_6][Cr(CN)_6]$
C. $[Co(NH_3)_6][Co(CN)_6]$
D. $[Fe(NH_3)_6][Co(CN)_6]$
E. $[Co(NH_3)_6][Fe(CN)_6]$

Choose the correct answer from the options given below :

(A) B, C and D Only

(B) B, D and E Only

(C) A, C and D Only

(D) C, D and E Only

Correct Answer: (B) B, D and E Only

Solution:

Step 1: Understanding the Concept:

Coordination isomerism arises from the interchange of ligands between the cationic and anionic entities of different metal ions present in a complex salt.

To form a valid coordination isomer, exchanging the ligands must result in a new pair of complex cation and complex anion without forming a neutral (non-salt) complex.

Step 2: Key Formula or Approach:

Analyze each given pair:

1) Ensure that swapping ligands doesn't result in a neutral molecule (which would stop it from being a complex salt).

2) Identify complexes with different metal centers, as they represent the clearest and most standard examples of coordination isomerism (e.g., swapping all ligands between two different metals gives a distinctly new compound).

Step 3: Detailed Explanation:

Let's evaluate the given complexes:

A. $[Ag(NH_3)_2][Ag(CN)_2]$: Exchanging one ligand gives $[Ag(NH_3)(CN)]$, which is a neutral molecule. Therefore, it cannot exist as a complex salt isomer.

B. $[Co(NH_3)_6][Cr(CN)_6]$: Exchanging all ligands gives $[Cr(NH_3)_6][Co(CN)_6]$, which is a clear coordination isomer. (Valid)

C. $[Co(NH_3)_6][Co(CN)_6]$: While it can form partial exchange isomers like $[Co(NH_3)_5(CN)][Co(NH_3)(CN)_5]$, standard textbook definitions heavily emphasize the interchange of ligands between different metal ions for classical coordination isomerism.

D. $[Fe(NH_3)_6][Co(CN)_6]$: Exchanging ligands gives $[Co(NH_3)_6][Fe(CN)_6]$. (Valid)

E. $[Co(NH_3)_6][Fe(CN)_6]$: This is exactly the coordination isomer of D. If D shows it, E also inherently represents a system capable of it. (Valid)

Since D and E are coordination isomers of each other, they must appear together in the correct option. The only option pairing D, E, and another valid complex with differing metals is B, D, and E.

Step 4: Final Answer:

The complexes that show coordination isomerism strictly are B, D, and E.

Quick Tip: If two options (like D and E) are literally coordination isomers of each other, they must always be grouped together in the correct multiple-choice option. Finding this pair immediately narrows down your choices.

---

62. Complete combustion of X g of an organic compound gave 0.25 g of CO_2 and 0.12 g of H_2O. If the % of carbon is 25% and of hydrogen is 4.89%, then X = _____ $\times 10^{-3}$ g. (Nearest integer) (Molar mass of C, H and O are 12, 1 and 16 g mol^{-1} respectively.)

(A) 273

(B) 27

(C) 2730

(D) 227

Correct Answer: (A) 273

Solution:

Step 1: Understanding the Concept:

In Liebig's combustion method, all the carbon in the organic compound is converted to CO_2, and all the hydrogen is converted to H_2O.

By knowing the mass of CO_2 produced, we can calculate the exact mass of carbon present. Using the given percentage of carbon, we can then find the total mass X of the original compound.

Step 2: Key Formula or Approach:

Mass of Carbon $= \frac{12}{44} \times$ Mass of CO_2.

Percentage of Carbon $= \left(\frac{\text{Mass of C}}{X}\right) \times 100$.

Rearranging gives $X = \frac{\text{Mass of C}}{\text{Percentage of C}} \times 100$.

Step 3: Detailed Explanation:

First, calculate the mass of carbon in the 0.25 g of CO_2 produced:

$$\text{Mass of C} = \frac{12}{44} \times 0.25 \text{ g}$$

$$\text{Mass of C} = \frac{3}{11} \times \frac{1}{4} = \frac{3}{44} \text{ g} \approx 0.06818 \text{ g}$$

We are given that the percentage of carbon in the original compound is 25%.

$$25 = \left(\frac{3/44}{X}\right) \times 100$$

$$\frac{1}{4} = \frac{3}{44X}$$

$$44X = 12 \implies X = \frac{12}{44} = \frac{3}{11} \text{ g}$$

Convert this value into decimal form:

$$X \approx 0.272727\ldots \text{ g}$$

The question asks for the answer in the format $X = \text{blank} \times 10^{-3}$ g.

$$0.2727 \text{ g} = 272.7 \times 10^{-3} \text{ g}$$

Rounding to the nearest integer, we get 273.

(Verification with Hydrogen: Mass of H $= \frac{2}{18} \times 0.12 = 0.0133$ g.

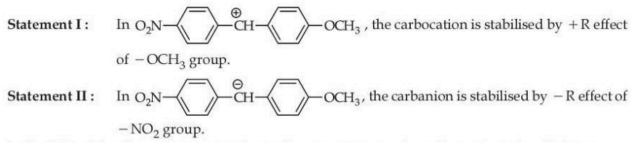
Step 4: Final Answer:

The value of X is 273×10^{-3} g.

Quick Tip: You only need to use one of the elemental percentages (Carbon or Hydrogen) to find the total mass. Doing the calculation with Carbon is mathematically cleaner because 25% simplifies nicely to 1/4.

---

63. Given below are two statements :



In the light of the above statements, choose the correct answer from the options given below :

(A) Both Statement I and Statement II are true

(B) Both Statement I and Statement II are false

(C) Statement I is true but Statement II is false

(D) Statement I is false but Statement II is true

Correct Answer: (A) Both Statement I and Statement II are true

Solution:

Step 1: Understanding the Concept:

Carbocations (electron-deficient species) are highly stabilized by electron-donating groups (+R or +I effects).

Carbanions (electron-rich species) are highly stabilized by electron-withdrawing groups (-R or -I effects).

We must evaluate the nature of the $-OCH_3$ and $-NO_2$ groups attached to the aromatic rings.

Step 2: Key Formula or Approach:

$-OCH_3$ has lone pairs on oxygen, allowing it to donate electron density into the ring via resonance (+R effect).

$-NO_2$ is strongly electronegative and has a formal positive charge on nitrogen, allowing it to withdraw electron density from the ring via resonance (-R effect).

Step 3: Detailed Explanation:

Evaluate Statement I:

The molecule features a central carbocation (CH^+). The para-methoxy group ($-OCH_3$) pushes electron density towards the carbocation through the benzene ring via the +R (resonance) effect. This delocalizes the positive charge and stabilizes the carbocation. Statement I is TRUE.

Evaluate Statement II:

The molecule features a central carbanion (CH^-). The para-nitro group ($-NO_2$) pulls the excess electron density away from the carbanion into its own oxygen atoms via the -R (resonance) effect. This delocalizes the negative charge and stabilizes the carbanion. Statement II is TRUE.

Step 4: Final Answer:

Both Statement I and Statement II are true.

Quick Tip: Always remember: "Like stabilizes like." Electron-donating groups (+R) stabilize positive charges, while electron-withdrawing groups (-R) stabilize negative charges.

---

64. The compound (X) on

(i) heating in the presence of anhydrous $AlCl_3$ and HCl gas gives 2,4-dimethyl pentane

(ii) aromatization gives toluene and

(iii) cyclisation gives methyl cyclohexane

The correct name of compound (X) is :

(A) Hept-2-ene

(B) Hept-1,3,5-triene

(C) Heptane

(D) Hept-2,4,6-triene

Correct Answer: (C) Heptane

Solution:

Step 1: Understanding the Concept:

We must identify a hydrocarbon that undergoes three specific standard reactions: Isomerization, Aromatization, and Cyclization.

These are hallmark reactions of straight-chain alkanes (paraffins) in the petroleum industry (reforming processes).

Step 2: Key Formula or Approach:

Isomerization: Normal alkanes heat with $AlCl_3/HCl$ to form branched alkanes.

Aromatization: Alkanes with 6 or more carbons heat over catalysts (Cr_2O_3, V_2O_5, Mo_2O_3) at high temp/pressure to form aromatic rings.

Carbon counting: The products dictate exactly how many carbon atoms are in the parent chain.

Step 3: Detailed Explanation:

Let's analyze the products to determine the carbon count of (X):

(i) 2,4-dimethyl pentane has 5 (pentane) $+2$ (dimethyl) $= 7$ carbon atoms.

(ii) Toluene ($C_6H_5CH_3$) has $6 + 1 = 7$ carbon atoms.

(iii) Methyl cyclohexane ($C_6H_{11}CH_3$) has $6 + 1 = 7$ carbon atoms.

Since all products contain 7 carbon atoms, the starting compound (X) must be a 7-carbon straight-chain alkane.

The presence of reagents like $AlCl_3/HCl$ for branching definitively points to an alkane. Alkenes and trienes would undergo addition reactions rather than standard skeletal isomerization under these specific conditions.

Therefore, compound (X) is strictly n-heptane.

Step 4: Final Answer:

The correct name of the compound is Heptane.

Quick Tip: Catalytic reforming (aromatization) converts n-hexane to benzene and n-heptane to toluene. Knowing this standard industrial reaction provides an instant shortcut to the answer.

---

65. Correct statements regarding alkyl halides (R-X) among the following are :

A. Alcohol being less polar solvent as compared to water, alcoholic KOH favours elimination reaction with R-X.

B. Order of reactivity towards S_N1 mechanism is $C_6H_5-CH_2-Cl > C_6H_5-CHCl-C_6H_5$.

C. Non substituted aryl halides exhibit properties similar to alkyl halides.

D. Vinyl chloride is an example of haloalkene and allyl chloride is an example of haloalkyne.

E. R-Cl can be prepared by reacting R-OH with $SOCl_2$ but Ar-Cl cannot be prepared by reacting Ar-OH with $SOCl_2$.

Choose the correct answer from the options given below :

(A) A, B and C Only

(B) B and D Only

(C) A and E Only

(D) D and E Only

Correct Answer: (C) A and E Only

Solution:

Step 1: Understanding the Concept:

Evaluate each statement based on standard principles of haloalkane and haloarene chemistry, specifically focusing on substitution vs. elimination conditions, carbocation stability, and phenol reactivity.

Step 2: Key Formula or Approach:

- Aqueous KOH favors Nucleophilic Substitution (S_N) to form alcohols. Alcoholic KOH favors Elimination (E2) to form alkenes.
- S_N1 reactivity depends solely on the stability of the intermediate carbocation.
- Aryl and vinyl halides are unreactive towards S_N due to partial double bond character from resonance.

Step 3: Detailed Explanation:

A. TRUE. Alcoholic KOH provides the ethoxide ion ($C_2H_5O^-$), which acts as a strong base rather than a nucleophile due to the less polar nature of the alcohol solvent, heavily favoring dehydrohalogenation (elimination).

B. FALSE. The carbocation formed from $C_6H_5-CHCl-C_6H_5$ is diphenylmethyl (Ph_2CH^+), which is stabilized by extensive resonance from two benzene rings. It is much more stable than the benzyl carbocation ($PhCH_2^+$). Thus, the reactivity order is reversed.

C. FALSE. Aryl halides are highly unreactive towards nucleophilic substitution compared to alkyl halides because the C-X bond acquires partial double bond character due to resonance with the benzene ring.

D. FALSE. Vinyl chloride ($CH_2 = CHCl$) is a haloalkene. However, allyl chloride ($CH_2 = CH-CH_2Cl$) is also a haloalkene, not a haloalkyne (which would require a triple bond).

E. TRUE. Aliphatic alcohols (R-OH) readily react with $SOCl_2$ to form alkyl chlorides. Phenols (Ar-OH) do not react with $SOCl_2$ because the C-O bond in phenol has partial double bond character and cannot be easily cleaved.

The only correct statements are A and E.

Step 4: Final Answer:

The correct choice is A and E Only.

Quick Tip: A classic distinction to memorize: Aq. KOH = Substitution (Alcohols). Alc. KOH = Elimination (Alkenes). Also, Phenols never undergo direct nucleophilic substitution on the C-OH bond.

---

66. An organic compound "x" where molar ratio of C, O and H are equal, on treatment with 50% KOH under reflux followed by acidification produced "y". The most likely structure of "y" is :

Molar mass of 'x' is 58 **g mol**$^{-1}$

(A) $CH_2 = CH - C(=O) - OH$

(B) $CH_3 - CH = CH - CH = O$

(C) $O = C(OH) - CH_2 - OH$

(D) $CH_3 - C(=O) - OH$

Correct Answer: (C) $O = C(OH) - CH_2 - OH$

Solution:

Step 1: Understanding the Concept:

We first determine the empirical and molecular formula of compound "x" using its molar ratios and molar mass.

Once the structure of "x" is identified, we react it with 50% KOH, which is the classic condition for the Cannizzaro reaction (for aldehydes lacking alpha-hydrogens).

Step 2: Key Formula or Approach:

If molar ratios are equal, the empirical formula is CHO.

Empirical mass $= 12(C) + 1(H) + 16(O) = 29$ g/mol.

$n = \frac{\text{Molar Mass}}{\text{Empirical Mass}} = \frac{58}{29} = 2.$

Molecular formula $= C_2H_2O_2$.

Step 3: Detailed Explanation:

The only logical stable structure for $C_2H_2O_2$ is Glyoxal: $OHC - CHO$.

Glyoxal is a dialdehyde with no alpha-hydrogens. When treated with concentrated (50%) base like KOH, it undergoes an intramolecular Cannizzaro reaction.

In an intramolecular Cannizzaro reaction, one aldehyde group is oxidized to a carboxylic acid salt, and the other is reduced to a primary alcohol.

Reaction:

$OHC - CHO + KOH \rightarrow HOCH_2 - COOK$ (Potassium glycolate)

Subsequent acidification replaces the potassium ion with a proton:

$HOCH_2 - COOK + H^+ \rightarrow HOCH_2 - COOH$ (Glycolic acid)

This structure matches option C: $O = C(OH) - CH_2 - OH$.

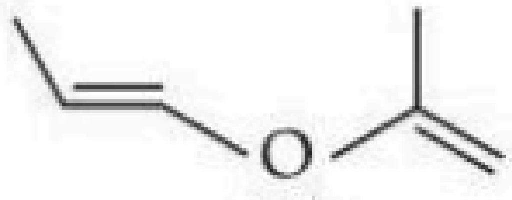
Step 4: Final Answer:

The structure of "y" is $O = C(OH) - CH_2 - OH$.

Quick Tip: Any aldehyde lacking α-hydrogens (like formaldehyde, benzaldehyde, or glyoxal) will immediately undergo the Cannizzaro reaction in the presence of concentrated base ($> 50\%$).

---

67. A molecule (X) with following structure under mild acidic condition is hydrolysed to produce (Y) and (Z). Identify the correct statements about (Y) and (Z).



(X)

A. Both (Y) and (Z) have same molar mass.

B. (Y) and (Z) can be distinguished from each other by $NaHCO_3$.

C. (Y) and (Z) react with HCN with same rates.

D. (Y) and (Z) undergo addition reaction with 2,4-DNP.

Choose the correct answer from the options given below :

(A) A, B and C Only

(B) B and C Only

(C) C and D Only

(D) A and B Only

Correct Answer: (D) A and B Only

Solution:

Step 1: Understanding the Concept:

The given molecule (X) is an ester: isopropyl acetate ($CH_3COOCH(CH_3)_2$).

Hydrolysis of an ester under acidic conditions breaks the ester bond to yield a carboxylic acid and an alcohol.

We need to identify these products and evaluate their physical and chemical properties against the given statements.

Step 2: Key Formula or Approach:

Ester Hydrolysis: R-COO-R' + $H_2O \xrightarrow{H^+}$ R-COOH + R'-OH.

Here, $R = CH_3$ and $R' = CH(CH_3)_2$.

Step 3: Detailed Explanation:

Perform the hydrolysis on isopropyl acetate:

$CH_3COOCH(CH_3)_2 + H_2O \rightarrow CH_3COOH$ (Acetic acid) + $CH_3CH(OH)CH_3$ (Isopropanol).

Let (Y) be Acetic acid and (Z) be Isopropanol (or vice versa).

Now, evaluate the statements:

A. Molar mass of Acetic acid ($C_2H_4O_2$) $= 2(12) + 4(1) + 2(16) = 60$ g/mol.

Molar mass of Isopropanol (C_3H_8O) $= 3(12) + 8(1) + 16 = 60$ g/mol.

They have the exact same molar mass. Statement A is TRUE.

B. $NaHCO_3$ is a weak base used to test for carboxylic acids. Acetic acid will react to release effervescence of CO_2 gas. Isopropanol (an alcohol) will not react. They can be distinguished. Statement B is TRUE.

C. HCN reacts with carbonyl groups (aldehydes and ketones) via nucleophilic addition. Neither acetic acid nor isopropanol behaves as a typical aldehyde/ketone for HCN addition. Statement C is FALSE.

D. 2,4-Dinitrophenylhydrazine (Brady's reagent) is a test for aldehydes and ketones. Neither the carboxylic acid nor the alcohol will undergo addition with 2,4-DNP. Statement D is FALSE.

Therefore, only statements A and B are correct.

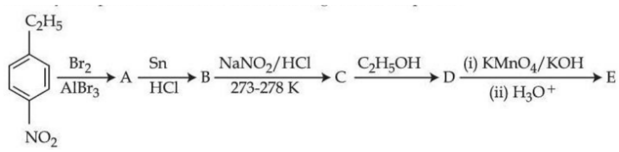
Step 4: Final Answer:

The correct choice is A and B Only.

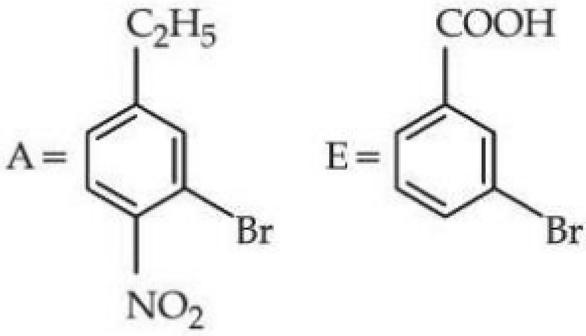
Quick Tip: It is a classic competitive exam trick to pair acetic acid and propanol (or isopropanol) because they coincidentally share the exact same molar mass (60 g/mol) despite belonging to entirely different functional groups.

---

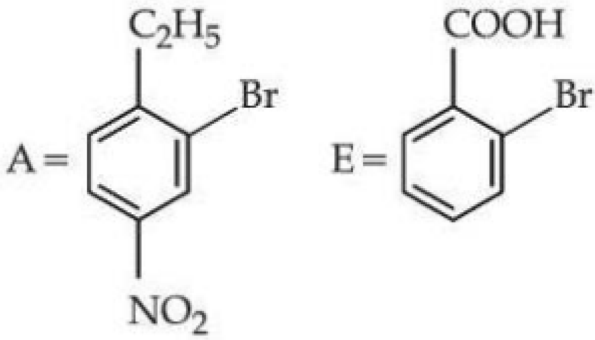
68. Identify compounds A and E in the following reaction sequence.



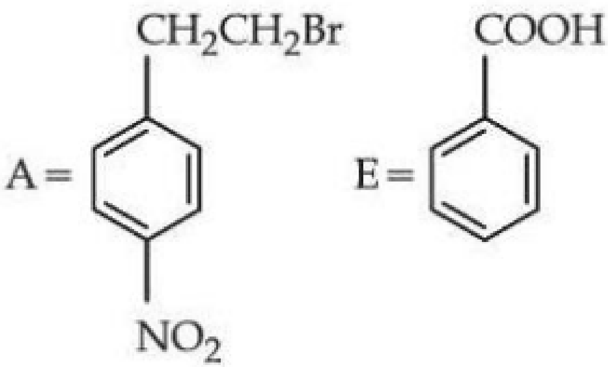
(A)



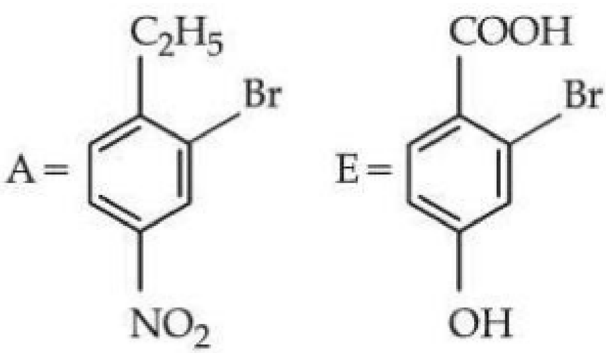
(B)



(C)



(D)



Correct Answer: (A)

Solution:

Step 1: Understanding the Concept:

We must trace a multi-step organic synthesis starting from p-ethylnitrobenzene.

The steps involve electrophilic aromatic substitution, reduction of a nitro group, diazotization, deamination, and strong oxidation of an alkyl side chain.

Step 2: Key Formula or Approach:

1. $Br_2/AlBr_3$: Electrophilic bromination. Ortho/para directing groups (alkyl) dominate over meta directing groups (nitro).
2. Sn/HCl: Reduces $-NO_2$ to $-NH_2$.
3. $NaNO_2/HCl$: Converts $-NH_2$ to diazonium $-N_2^+Cl^-$.
4. C_2H_5OH: Mild reducing agent that replaces the diazonium group with a Hydrogen atom.
5. $KMnO_4/H_3O^+$: Oxidizes any alkyl side chain with benzylic hydrogens directly into a $-COOH$ group.

Step 3: Detailed Explanation:

Start with p-ethylnitrobenzene. The ethyl group is at position 1, and the nitro group is at position 4.

Reaction 1: Bromination. The ethyl group is activating and ortho/para directing. The nitro group is deactivating and meta directing. Both groups direct the incoming Bromine to the exact same position: ortho to the ethyl group (which is meta to the nitro group).

Thus, Compound A is 2-bromo-1-ethyl-4-nitrobenzene.

Reaction 2: Sn/HCl reduces the $-NO_2$ group. Compound B is 4-amino-2-bromo-1-ethylbenzene.

Reaction 3: Diazotization. Compound C is the diazonium salt at position 4.

Reaction 4: Ethanol (C_2H_5OH) reduces the diazonium salt, removing the nitrogen group entirely and replacing it with H. Compound D is simply 2-bromo-1-ethylbenzene (or o-bromoethylbenzene).

Reaction 5: Strong oxidation with $KMnO_4$. The ethyl side chain is completely oxidized to a carboxylic acid group. The bromine atom is unaffected.

Thus, Compound E is 2-bromobenzoic acid.

Step 4: Final Answer:

Compound A is 2-bromo-1-ethyl-4-nitrobenzene and Compound E is 2-bromobenzoic acid.

Quick Tip: Ethanol (CH_3CH_2OH) and Hypophosphorous acid (H_3PO_2) are the two standard reagents used to completely remove a diazonium group from a benzene ring (deamination).

---

69. Identify the correct pair having amino acid (A) and the hormone (B) that is iodinated derivative of the amino acid (A).

(T and Y represent one letter code for amino acids)

Amino acid (A) Hormone (B)

(A) T Insulin

(B) T Thyroxine

(C) Y Thyroxine

(D) Y Insulin

Correct Answer: (C) Y Thyroxine

Solution:

Step 1: Understanding the Concept:

We must identify the amino acid that acts as the biological precursor for thyroid hormones and match it to its correct one-letter biochemical abbreviation.

Step 2: Key Formula or Approach:

The hormone produced by the thyroid gland that contains iodine is Thyroxine (T4).

Thyroxine is synthesized in the body through the iodination of the amino acid Tyrosine.

The one-letter codes for amino acids: Tyrosine is 'Y'. Threonine is 'T'.

Step 3: Detailed Explanation:

The hormone is explicitly described as an iodinated derivative. Insulin is a large peptide hormone and is not an iodinated derivative of a single amino acid. Thyroxine, however, is formed by the addition of iodine to the phenolic ring of Tyrosine.

Thus, Hormone (B) = Thyroxine.

The precursor amino acid (A) = Tyrosine.

Now, check the one-letter codes. Since 'T' is already taken by Threonine, Tyrosine is assigned the letter 'Y' (based on the second letter of its name to distinguish it).

Therefore, Amino acid (A) is Y.

Step 4: Final Answer:

The correct pair is Y and Thyroxine.

Quick Tip: One-letter amino acid codes are frequently tested. Remember the tricky ones: Y = Tyrosine, W = Tryptophan, F = Phenylalanine, D = Aspartic Acid, E = Glutamic Acid.

Chemistry Section - B

70. Among Fe^{2+}, Fe^{3+}, Cr^{2+} and Zn^{2+}, the ion that shows positive borax bead test and with highest ionisation enthalpy is :

(A) Fe^{2+}

(B) Zn^{2+}

(C) Cr^{2+}

(D) Fe^{3+}

Correct Answer: (D) Fe^{3+}

Solution:

Step 1: Understanding the Concept:

The borax bead test is a qualitative analytical method used to identify transition metals. Only ions with unpaired d-electrons (colored ions) give a positive test.

The ionization enthalpy refers to the energy required to remove an electron from the specific ion listed.

Step 2: Key Formula or Approach:
Check electron configurations for unpaired electrons to confirm the borax bead test.
Evaluate the stability of the electron configuration to determine which ion has the highest ionization enthalpy (i.e., which is hardest to oxidize further).

Step 3: Detailed Explanation:
First, evaluate the borax bead test constraint:
Zn^{2+} has a $[Ar]3d^{10}$ configuration. Since it has a completely filled d-subshell, it is diamagnetic and colorless, meaning it does not respond to the borax bead test. We can eliminate Zn^{2+}.
The remaining ions (Fe^{2+}, Fe^{3+}, Cr^{2+}) all have partially filled d-orbitals, are colored, and give positive borax bead tests.
Next, evaluate the ionization enthalpy constraint:
We must find the energy required to remove one electron from each ion:
- $Fe^{2+} \rightarrow Fe^{3+} + e^-$: Going from $3d^6$ to $3d^5$. Removing this electron achieves a highly stable half-filled state, so the energy required is relatively low.
- $Cr^{2+} \rightarrow Cr^{3+} + e^-$: Going from $3d^4$ to $3d^3$. The $3d^3$ state is exceptionally stable in aqueous/complex forms (half-filled t_{2g}), so this ionization is also relatively easy.
- $Fe^{3+} \rightarrow Fe^{4+} + e^-$: Fe^{3+} has a $[Ar]3d^5$ configuration. This is an exactly half-filled d-subshell, making it spherically symmetrical and exceptionally stable. Removing an electron from this perfectly stable d^5 core requires an enormous amount of energy.
Therefore, Fe^{3+} has the highest ionization enthalpy among the active candidates.

Step 4: Final Answer:
The ion is Fe^{3+}.

Quick Tip: Any ion with a d^5 (like Fe^{3+}, Mn^{2+}) or d^{10} configuration represents an energy "wall". Removing an electron from these states will always require the highest ionization energy among peers.

---

71. The surface of sodium metal is irradiated with radiation of wavelength x nm. The kinetic

energy of ejected electrons is 2.8×10^{-20} J. The work function of sodium is 2.3 eV. The value of x is _______ $\times 10^2$ nm. (Nearest integer)
(Given : $h = 6.6 \times 10^{-34}$ J s; 1 eV $= 1.6 \times 10^{-19}$ J; $c = 3.0 \times 10^8$ m s^{-1})

Correct Answer: 5

Solution:

Step 1: Understanding the Concept:

This problem uses Einstein's Photoelectric Equation, which states that the total energy of an incident photon is split into the work function (binding energy) of the metal and the maximum kinetic energy of the ejected electron.

Step 2: Key Formula or Approach:

Photoelectric equation: $E_{photon} = W + KE$.
Energy of a photon: $E = \frac{hc}{\lambda}$.
We must convert all energy values to a common unit (Joules) before solving for the wavelength λ.

Step 3: Detailed Explanation:

First, convert the work function W from eV to Joules:

$$W = 2.3 \text{ eV} \times (1.6 \times 10^{-19} \text{ J/eV}) = 3.68 \times 10^{-19} \text{ J}$$

The kinetic energy is given as:

$$KE = 2.8 \times 10^{-20} \text{ J} = 0.28 \times 10^{-19} \text{ J}$$

Calculate the total energy of the incident photon:

$$E = W + KE = (3.68 \times 10^{-19}) + (0.28 \times 10^{-19})$$

$$E = 3.96 \times 10^{-19} \text{ J}$$

Now, use the photon energy formula to find the wavelength λ:

$$E = \frac{hc}{\lambda} \implies \lambda = \frac{hc}{E}$$

Substitute the given constants:

$$hc = (6.6 \times 10^{-34} \text{ J s}) \times (3.0 \times 10^{8} \text{ m/s}) = 19.8 \times 10^{-26} \text{ J m}$$

$$\lambda = \frac{19.8 \times 10^{-26}}{3.96 \times 10^{-19}}$$

$$\lambda = 5 \times 10^{-7} \text{ m}$$

Convert the wavelength to nanometers ($1 \text{ m} = 10^9 \text{ nm}$):

$$\lambda = 5 \times 10^{-7} \times 10^{9} \text{ nm} = 500 \text{ nm}$$

The question asks for the value in the format $x \times 10^2$ nm.

$$500 \text{ nm} = 5 \times 10^2 \text{ nm}$$

Comparing this to the format, $x = 5$.

Step 4: Final Answer:

The value of x is 5.

Quick Tip: Always align the powers of 10 before adding numbers in scientific notation. Converting 2.8×10^{-20} to 0.28×10^{-19} prevents a very common alignment error during addition.

---

72. Consider the following gas phase reaction being carried out in a closed vessel at 25°C.

$$2A(g) \longrightarrow 4B(g) + C(g)$$

time (min)	total pressure of the system (mm Hg)
30	300
∞	600

The pressure of C(g) at 30 minutes time interval would be ________ mm Hg. (nearest integer)

Correct Answer: 20

Solution:

Step 1: Understanding the Concept:

We are dealing with a gas-phase reaction where we track the progress using the total pressure of the system.

At time $t = \infty$, the reaction is assumed to have gone to complete conversion. This allows us to calculate the initial pressure of the reactant.

Using the data at $t = 30$ min, we can construct an ICE (Initial, Change, Equilibrium) table to find the partial pressures of individual components.

Step 2: Key Formula or Approach:

Let initial pressure of A be P_0.

At $t = \infty$, $P_A = 0$, and the products B and C exert pressure based on stoichiometry.

At $t = 30$, set up the change in pressure as $-2x$ for A, $+4x$ for B, and $+x$ for C.

Total pressure = Sum of partial pressures.

Step 3: Detailed Explanation:

Let's analyze the state at $t = \infty$:

Initial moles of A gives 0 moles of A, and 4/2 moles of B and 1/2 moles of C.

So, if initial pressure is P_0, the final pressure is entirely due to products.

$$P_\infty = P_B + P_C = \left(\frac{4}{2}\right)P_0 + \left(\frac{1}{2}\right)P_0 = 2P_0 + 0.5P_0 = 2.5P_0$$

We are given $P_\infty = 600$ mm Hg.

$$2.5P_0 = 600 \implies P_0 = \frac{600}{2.5} = 240 \text{ mm Hg}$$

Now, set up the ICE table for $t = 30$ min:

$2A(g) \rightarrow 4B(g) + C(g)$

Initial: $240 \quad 0 \quad 0$

Change: $-2x \quad +4x \quad +x$

At 30m: $240 - 2x \quad 4x \quad x$

The total pressure at 30 minutes is the sum of these partial pressures:

$$P_{total} = (240 - 2x) + 4x + x = 240 + 3x$$

We are given $P_{total} = 300$ mm Hg at $t = 30$ min.

$$240 + 3x = 300$$

$$3x = 60 \implies x = 20 \text{ mm Hg}$$

The question asks for the partial pressure of C(g) at 30 minutes.

From the ICE table, $P_C = x$.

Therefore, $P_C = 20$ mm Hg.

Step 4: Final Answer:

The pressure of C(g) at 30 minutes is 20.

Quick Tip: The "time $= \infty$" data point in chemical kinetics always gives you the theoretical 100% yield state, which is the most reliable way to reverse-calculate the exact initial concentration or pressure of the reactants.

---

73. Consider the following two half-cell reactions along with the standard reduction potential given :

$$CO_2 + 6H^+ + 6e^- \longrightarrow CH_3OH + H_2O \qquad E^o_{red} = 0.02 \text{ V}$$

$$\frac{1}{2}O_2 + 2H^+ + 2e^- \longrightarrow H_2O \qquad E^o_{red} = 1.23 \text{ V}$$

A fuel cell was set up using the above two reactions such that the cell operates under the standard condition of 1 bar pressure and 298 K temperature. The fuel cell works with 80% efficiency. If the work derived from the cell using 1 mol of CH_3OH is used to compress an ideal gas isothermally against a constant pressure of 1 kPa, then the change in the volume of the gas, ΔV = ________ m^3. (nearest integer)

Given : F = 96500 C mol^{-1}

Correct Answer: 560

Solution:

Step 1: Understanding the Concept:

A fuel cell derives electrical work from a spontaneous chemical reaction. We first find the standard cell potential E°_{cell}.

Using this, we calculate the maximum possible thermodynamic work (ΔG°).

We factor in the cell's efficiency to find the actual usable work, and then apply this work to the physics formula for mechanical compression of a gas to find the volume change.

Step 2: Key Formula or Approach:

Overall Cell Potential: $E^\circ_{cell} = E^\circ_{\text{cathode}} - E^\circ_{\text{anode}}$.

Maximum electrical work: $W_{max} = -\Delta G^\circ = nFE^\circ_{cell}$.

Actual work: $W_{actual} = \text{Efficiency} \times W_{max}$.

Mechanical work of compression: $W = P_{ext}\Delta V$.

Step 3: Detailed Explanation:

Identify the anode and cathode based on standard reduction potentials. The reaction with the higher potential (1.23 V) will be the reduction (cathode). The other (0.02 V) will be reversed to oxidation (anode).

$$E^\circ_{cell} = 1.23 \text{ V} - 0.02 \text{ V} = 1.21 \text{ V}$$

The oxidation of 1 mole of CH_3OH involves 6 electrons, so $n = 6$.

Calculate the maximum theoretical work for 1 mole of methanol:

$$W_{max} = nFE^\circ_{cell} = 6 \times 96500 \text{ C} \times 1.21 \text{ V}$$

$$W_{max} = 579000 \times 1.21 = 700590 \text{ J}$$

The fuel cell operates at 80% efficiency, so the actual usable work is:

$$W_{actual} = 0.80 \times 700590 \text{ J} = 560472 \text{ J}$$

This work is used to compress a gas against an external pressure of 1 kPa = 1000 Pa.

$$W_{actual} = P_{ext}\Delta V$$

$$560472 \text{ J} = 1000 \text{ Pa} \times \Delta V$$

$$\Delta V = \frac{560472}{1000} = 560.472 \text{ m}^3$$

Rounding to the nearest integer gives 560.

Step 4: Final Answer:

The change in the volume of the gas is 560.

Quick Tip: For fuel cell thermodynamics, remember that n is the total number of electrons exchanged per mole of the primary fuel (here, methanol). Balancing the entire equation isn't necessary if you already see "6e-" in the methanol half-reaction.

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74. Number of paramagnetic ions among the following d- and f-block metal ions is ________.
$Mn^{2+}, Cu^{2+}, Zn^{2+}, Yb^{2+}, Sc^{3+}, La^{3+}, Gd^{3+}, Lu^{3+}, Ti^{4+}, Ce^{4+}$
(Atomic number of Mn = 25, Cu = 29, Zn = 30, Yb = 70, Sc = 21, La = 57, Gd = 64, Lu = 71, Ti = 22, Ce = 58)

Correct Answer: 3

Solution:

Step 1: Understanding the Concept:

An ion is paramagnetic if it possesses one or more unpaired electrons in its atomic orbitals.
An ion is diamagnetic if all of its electrons are paired (typically occurring when subshells are completely empty, like d^0, f^0, or completely full, like d^{10}, f^{14}).

Step 2: Key Formula or Approach:

Write out the electronic configuration for each neutral atom using the nearest noble gas core.
Remove electrons to form the ion (remember to remove from the outermost s-orbital first, then d, then f).
Count the number of unpaired electrons.

Step 3: Detailed Explanation:

Let's evaluate each ion:

1. $Mn^{2+}(Z = 25)$: Neutral is $[Ar]3d^5 4s^2$. Ion is $[Ar]3d^5$. 5 unpaired e → Paramagnetic.
2. $Cu^{2+}(Z = 29)$: Neutral is $[Ar]3d^{10} 4s^1$. Ion is $[Ar]3d^9$. 1 unpaired e → Paramagnetic.
3. $Zn^{2+}(Z = 30)$: Neutral is $[Ar]3d^{10} 4s^2$. Ion is $[Ar]3d^{10}$. 0 unpaired e → Diamagnetic.
4. $Yb^{2+}(Z = 70)$: Neutral is $[Xe]4f^{14} 6s^2$. Ion is $[Xe]4f^{14}$. 0 unpaired e → Diamagnetic.
5. $Sc^{3+}(Z = 21)$: Neutral is $[Ar]3d^1 4s^2$. Ion is $[Ar]3d^0$. 0 unpaired e → Diamagnetic.
6. $La^{3+}(Z = 57)$: Neutral is $[Xe]5d^1 6s^2$. Ion is $[Xe]4f^0 5d^0$. 0 unpaired e → Diamagnetic.
7. $Gd^{3+}(Z = 64)$: Neutral is $[Xe]4f^7 5d^1 6s^2$. Ion is $[Xe]4f^7$. 7 unpaired e → Paramagnetic.
8. $Lu^{3+}(Z = 71)$: Neutral is $[Xe]4f^{14} 5d^1 6s^2$. Ion is $[Xe]4f^{14}$. 0 unpaired e → Diamagnetic.
9. $Ti^{4+}(Z = 22)$: Neutral is $[Ar]3d^2 4s^2$. Ion is $[Ar]3d^0$. 0 unpaired e → Diamagnetic.
10. $Ce^{4+}(Z = 58)$: Neutral is $[Xe]4f^1 5d^1 6s^2$. Ion is $[Xe]4f^0$. 0 unpaired e → Diamagnetic.

The paramagnetic ions are Mn^{2+}, Cu^{2+}, and Gd^{3+}.

Total count is 3.

Step 4: Final Answer:

The number of paramagnetic ions is 3.

Quick Tip: For f-block elements, +3 is the most common oxidation state. Ions like La^{3+}, Lu^{3+}, and Ce^{4+} are explicitly stable because they achieve empty (f^0) or completely full (f^{14}) configurations, making them completely diamagnetic.

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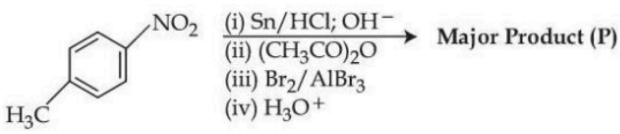
75. Consider the following reactions sequence



When the product (P) is subjected to Carius analysis using $AgNO_3$, 1.0 g of the product (P) will produce _________ g of the precipitate of AgBr. (Nearest Integer)

(Given : molar mass in g mol^{-1} C : 12, H : 1, O : 16, N : 14, Br : 80, Ag : 108)

Correct Answer: 1

Solution:

Step 1: Understanding the Concept:

We first decipher the organic reaction sequence to find the exact molecular structure of the final product (P).

Then, we apply the stoichiometry of the Carius method, which quantitatively converts all halogen atoms in an organic compound into a silver halide precipitate (AgBr).

Step 2: Key Formula or Approach:

Step (i): Reduction of nitro to amine.

Step (ii): Acetylation of amine for protection.

Step (iii): Electrophilic aromatic bromination.

Step (iv): Deprotection (hydrolysis) of the amide back to an amine.

Carius method stoichiometry: 1 mole of P (containing 1 Br) yields 1 mole of AgBr.

Mass of AgBr $= \frac{\text{Mass of P}}{\text{Molar Mass of P}} \times$ Molar Mass of AgBr.

Step 3: Detailed Explanation:

Trace the reactions starting from p-Nitrotoluene:

(i) Sn/HCl followed by OH^- reduces the p-NO_2 group to an $-NH_2$ group. The product is p-toluidine.

(ii) $(CH_3CO)_2O$ acetylates the amine to protect it, forming p-methylacetanilide. This dampens the extreme activating effect of the amine group to prevent poly-bromination.

(iii) $Br_2/AlBr_3$ is a bromination step. The $-NHCOCH_3$ group is highly activating and ortho/para directing. The $-CH_3$ group is weakly activating and ortho/para directing. The stronger $-NHCOCH_3$ group controls the direction. Since the para position is blocked by the methyl group, bromination occurs at the ortho position relative to the $-NHCOCH_3$ group.

(iv) H_3O^+ hydrolyzes the protective amide group back to an amine.

The final product (P) is 2-bromo-4-methylaniline.

The molecular formula of (P) is $C_6H_3(NH_2)(Br)(CH_3)$ which combines to C_7H_8BrN.

Calculate the molar mass of (P):

$M_P = (7 \times 12) + (8 \times 1) + 80 + 14 = 84 + 8 + 80 + 14 = 186$ g/mol.

Calculate the molar mass of AgBr:

$M_{AgBr} = 108 + 80 = 188$ g/mol.

According to the Carius method, 1 mole of P (186 g) yields 1 mole of AgBr (188 g).

For 1.0 g of P, the mass of AgBr produced is:

$$\text{Mass} = \frac{188}{186} \times 1.0 \text{ g} = 1.0107 \text{ g}$$

Rounding to the nearest integer, we get 1 g.

Step 4: Final Answer:

The mass of the precipitate produced is 1.

Quick Tip: Acetylation of an aniline derivative before halogenation is a classic technique to guarantee mono-halogenation. It effectively steps down the intense activating power of the $-NH_2$ group, preventing the formation of 2,4,6-tribromo products.