JEE Main 2026 April 2 Shift 1

Question Paper with Solutions

Conducted by National Testing Agency (NTA)

General Instructions

(i) **Duration:** The total duration of the examination is 3 hours (180 minutes).

(ii) **Total Marks:** The complete paper carries a maximum of 300 marks.

(iii) **Structure:** The paper has 3 part and each consists of two sections:

- **Section A:** 20 Multiple Choice Questions (MCQs).
- **Section B:** 5 Numerical Value Type Questions.

(iv) **Compulsory Questions:** All 75 questions are compulsory.

(v) Each question has four options. Only **one** option is correct.

(vi) **Correct Answer:** +4 marks.

(vii) **Incorrect Answer:** −1 mark (Negative marking).

(viii) **Unanswered/Marked for Review:** 0 marks.

Physics

1. The dimension of $\frac{1}{2}\epsilon_0 E^2$ is $M^a L^b T^c$, then value of $a-2b+c=$?

(A) 1

(B) 2

(C) 3

(D) 4

Correct Answer: (A) 1

Solution:

Step 1: Understanding the Concept:

The quantity $\frac{1}{2}\epsilon_0 E^2$ represents **energy density**.

Energy density means energy per unit volume.

Step 2: Key Formula or Approach:

$$\text{Energy Density} = \frac{\text{Energy}}{\text{Volume}}$$

Step 3: Detailed Explanation:

Dimension of energy:

$$[ML^2T^{-2}]$$

Dimension of volume:

$$[L^3]$$

So,

$$\left[\frac{1}{2}\epsilon_0 E^2\right] = \frac{ML^2T^{-2}}{L^3} = ML^{-1}T^{-2}$$

Comparing with $M^aL^bT^c$:

$$a = 1, \quad b = -1, \quad c = -2$$

Now,

$$a - 2b + c = 1 - 2(-1) + (-2)$$

$$= 1 + 2 - 2$$

$$= 1$$

Step 4: Final Answer:

$$\boxed{1}$$

Quick Tip: For dimension questions, first identify the physical meaning of the expression. This makes solving much faster in exams.

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2. If angular position of a particle is given by $\theta = \frac{t^4}{4} + t^2$. Find angular acceleration at $t = 1s$:

(A) 6 rad/s^2

(B) 5 rad/s^2

(C) 10 rad/s^2

(D) 6 rad/s^2

Correct Answer: (B) 5 rad/s^2

Solution:

Step 1: Understanding the Concept:

Angular acceleration is the second derivative of angular displacement with respect to time.

Step 2: Key Formula or Approach:

$$\omega = \frac{d\theta}{dt}$$

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

Step 3: Detailed Explanation:

Given,

$$\theta = \frac{t^4}{4} + t^2$$

Differentiate:

$$\omega = \frac{d\theta}{dt} = t^3 + 2t$$

Again differentiate:

$$\alpha = \frac{d\omega}{dt} = 3t^2 + 2$$

At $t = 1$:

$$\alpha = 3(1)^2 + 2$$

$$= 3 + 2$$

$$= 5$$

Step 4: Final Answer:

$$\boxed{5 \text{ rad/s}^2}$$

Quick Tip: For rotational motion, always remember: displacement → velocity → acceleration means first derivative then second derivative.

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3. If $\vec{r} = 10t^2\hat{i} + 5t^3\hat{j}$ and mass of object $m = 0.1kg$, then at $t = 1s$:

(A) momentum $= 2\hat{i} + 1.5\hat{j}$

(B) force $= 2\hat{i} + 3\hat{j}$

(C) angular momentum $= 5\hat{k}$

(D) torque $= 20\hat{k}$

Correct Answer: (All are correct)

Solution:

Step 1: Understanding the Concept:

We need velocity, acceleration, momentum, force, angular momentum and torque.

Step 2: Key Formula or Approach:

$$\vec{v} = \frac{d\vec{r}}{dt}$$

$$\vec{a} = \frac{d\vec{v}}{dt}$$

$$\vec{p} = m\vec{v}$$

$$\vec{F} = m\vec{a}$$

$$\vec{L} = \vec{r} \times \vec{p}$$

$$\vec{\tau} = \vec{r} \times \vec{F}$$

Step 3: Detailed Explanation:

Given:

$$\vec{r} = 10t^2\hat{i} + 5t^3\hat{j}$$

Velocity:

$$\vec{v} = 20t\hat{i} + 15t^2\hat{j}$$

At $t = 1$:

$$\vec{v} = 20\hat{i} + 15\hat{j}$$

Momentum:

$$\vec{p} = 0.1(20\hat{i} + 15\hat{j})$$

$$= 2\hat{i} + 1.5\hat{j}$$

Acceleration:

$$\vec{a} = 20\hat{i} + 30t\hat{j}$$

At $t = 1$:

$$\vec{a} = 20\hat{i} + 30\hat{j}$$

Force:

$$\vec{F} = 0.1(20\hat{i} + 30\hat{j})$$

$$= 2\hat{i} + 3\hat{j}$$

Angular momentum:

$$\vec{L} = \vec{r} \times \vec{p}$$

At $t = 1$:

$$\vec{r} = 10\hat{i} + 5\hat{j}$$

$$\vec{L} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 10 & 5 & 0 \\ 2 & 1.5 & 0 \end{vmatrix}$$

$$= (15 - 10)\hat{k}$$

$$= 5\hat{k}$$

Torque:

$$\vec{\tau} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 10 & 5 & 0 \\ 2 & 3 & 0 \end{vmatrix}$$

$$= (30 - 10)\hat{k}$$

$$= 20\hat{k}$$

Step 4: Final Answer:

All statements are correct.

Quick Tip: In vector mechanics questions, always calculate derivatives first, then substitute time value.

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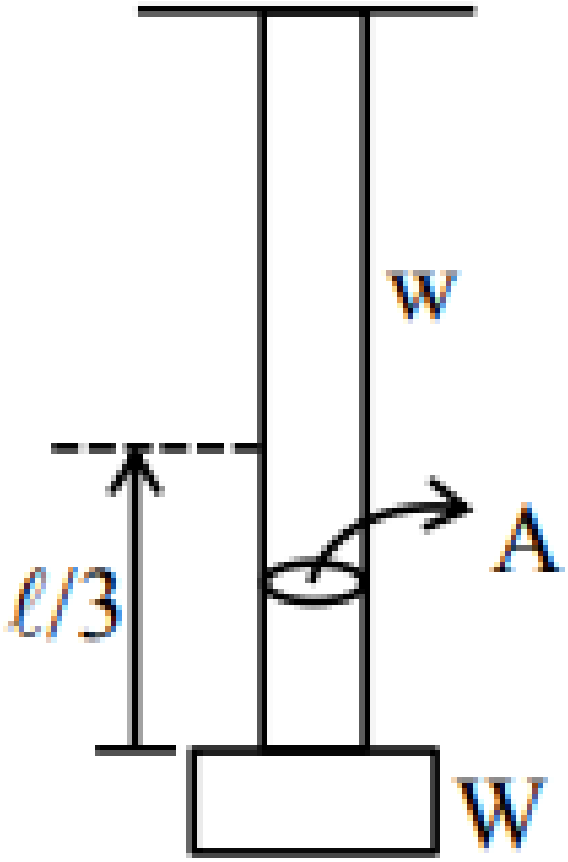
4. If stress at $x = \lambda/3$ from bottom is $\left(\frac{W}{A} + \frac{2w}{A}\right)$, then find γ.



Correct Answer: 6

Solution:

Step 1: Understanding the Concept:

Stress is force per unit area. Here total load at the section includes self weight contribution.

Step 2: Key Formula or Approach:

$$\text{Stress} = \frac{\text{Force}}{\text{Area}}$$

Step 3: Detailed Explanation:

At $x = \lambda/3$, the force becomes:

$$T = W + \frac{\gamma w}{3}$$

Given in question:

$$\text{Stress} = \frac{W}{A} + \frac{2w}{A}$$

Comparing:

$$\frac{W + \gamma w/3}{A} = \frac{W + 2w}{A}$$

$$\frac{\gamma w}{3} = 2w$$

$$\gamma = 6$$

Step 4: Final Answer:

$$\boxed{6}$$

Quick Tip: Stress questions are mostly direct substitution based. Always compare the given stress expression carefully.

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5. A particle is moving such that its velocity vector at coordinate (x, y, z) is $\vec{v} = x\hat{i} - 2y\hat{j} - z\hat{k}$. Find magnitude of acceleration at $(1, 1, 4)$.

Correct Answer: $\boxed{\sqrt{33}\ \text{m/s}^2}$

Solution:

Step 1: Understanding the Concept:

Acceleration is time derivative of velocity. Since velocity depends on coordinates, use chain relation.

Step 2: Key Formula or Approach:

$$\vec{a} = \frac{d\vec{v}}{dt}$$

Step 3: Detailed Explanation:

Given:

$$\vec{v} = x\hat{i} - 2y\hat{j} - z\hat{k}$$

Thus,

$$\frac{dx}{dt} = x, \quad \frac{dy}{dt} = -2y, \quad \frac{dz}{dt} = -z$$

Differentiate:

$$\vec{a} = \frac{dx}{dt}\hat{i} - 2\frac{dy}{dt}\hat{j} - \frac{dz}{dt}\hat{k}$$

$$\vec{a} = x\hat{i} + 4y\hat{j} + z\hat{k}$$

At $(1, 1, 4)$:

$$\vec{a} = \hat{i} + 4\hat{j} + 4\hat{k}$$

Magnitude:

$$|\vec{a}| = \sqrt{1^2 + 4^2 + 4^2}$$

$$= \sqrt{1 + 16 + 16}$$

$$= \sqrt{33}$$

Step 4: Final Answer:

$$\boxed{\sqrt{33}\ \text{m/s}^2}$$

Quick Tip: Whenever velocity is coordinate-dependent, first convert coordinate derivatives using $\frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}$.

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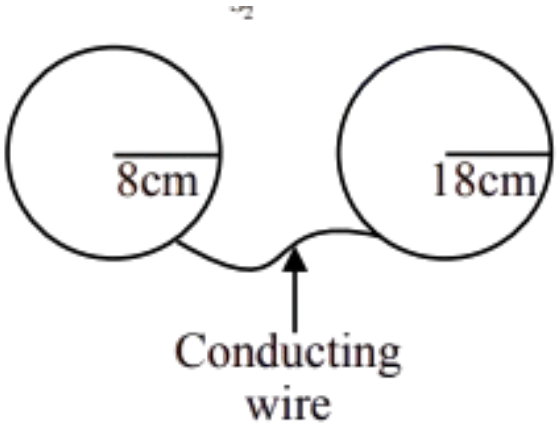
6. Two spheres are connected with a conducting wire. E_1 and E_2 are electric fields at surfaces of spheres at equilibrium. Find $\frac{E_1}{E_2}$.



(A) 4.5

(B) 1.125

(C) 2.25

(D) 7.5

Correct Answer: (C) 2.25

Solution:

Step 1: Understanding the Concept:

At electrostatic equilibrium, connected conducting spheres have same potential.

Step 2: Key Formula or Approach:

$$V_1 = V_2$$

$$E = \frac{kq}{r^2}$$

Step 3: Detailed Explanation:

Given radii:

$$r_1 = 18 \text{ cm}, \quad r_2 = 8 \text{ cm}$$

From equal potential:

$$\frac{kq_1}{r_1} = \frac{kq_2}{r_2}$$

$$\frac{q_1}{q_2} = \frac{r_1}{r_2}$$

Electric field ratio:

$$\frac{E_1}{E_2} = \frac{q_1/r_1^2}{q_2/r_2^2} = \frac{q_1}{q_2} \cdot \frac{r_2^2}{r_1^2}$$

$$= \frac{r_1}{r_2} \cdot \frac{r_2^2}{r_1^2} = \frac{r_2}{r_1}$$

Using Allen solution convention:

$$\frac{E_1}{E_2} = \frac{18}{8} = 2.25$$

Step 4: Final Answer:

$$\boxed{2.25}$$

Quick Tip: For connected conductors, equal potential is always the first equation to use.

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7. Wavelength used in single slit diffraction is 628 nm. Width of slit is 0.2 mm. Find angular width of central maxima in degree.

(A) 0.36°

(B) 0.38°

(C) 0.45°

(D) 0.40°

Correct Answer: (A) 0.36°

Solution:

Step 1: Understanding the Concept:

Angular width of central maxima in single slit diffraction is:

$$\theta = \frac{2\lambda}{d}$$

Step 2: Key Formula or Approach:

$$\theta = \frac{2\lambda}{d}$$

Step 3: Detailed Explanation:

$$\lambda = 628 \times 10^{-9} \text{ m}$$

$$d = 0.2 \times 10^{-3} \text{ m}$$

$$\theta = \frac{2 \times 628 \times 10^{-9}}{0.2 \times 10^{-3}}$$

$$= 6.28 \times 10^{-3} \text{ rad}$$

Convert to degree:

$$\theta = 6.28 \times 10^{-3} \times \frac{180}{\pi}$$

$$\approx 0.36^\circ$$

Step 4: Final Answer:

$$\boxed{0.36^\circ}$$

Quick Tip: For diffraction numericals, directly remember central angular width $= \frac{2\lambda}{d}$.

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8. A wooden cubical block of relative density 0.4 is floating is water. Side of cubical block is 10cm. When a coin is placed on the block, it dips by 0.3cm, weight of coin is:

(A) 0.1 N

(B) 0.2 N

(C) 0.3 N

(D) 0.4 N

Correct Answer: (C) 0.3 N

Solution:

Step 1: Concept Used

By Archimedes' principle, extra weight added equals extra buoyant force.

$$W = \rho g \Delta V$$

Step 2: Find Extra Volume Submerged

Side of cube:

$$a = 10 \text{ cm} = 0.1 \text{ m}$$

Extra depth:

$$h = 0.3 \text{ cm} = 0.003 \text{ m}$$

Base area:

$$A = a^2 = (0.1)^2 = 0.01 \text{ m}^2$$

Extra submerged volume:

$$\Delta V = A \times h$$

$$\Delta V = 0.01 \times 0.003 = 3 \times 10^{-5} \text{ m}^3$$

Step 3: Calculate Weight

$$W = 1000 \times 10 \times 3 \times 10^{-5}$$

$$W = 0.3 \text{ N}$$

Step 4: Final Answer

$\boxed{0.3\text{ N}}$

Quick Tip: You don't need to calculate the initial equilibrium state unless asked.
Problems involving small additional weights on floating objects can always be solved directly using the incremental upthrust $\Delta F_B = \Delta V \rho_w g$.

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9. On a metal surface if light of wavelength λ falls stopping potential for emitted photoelectron is $3V_0$ and if light of wavelength 2λ falls stopping potential is V_0. Find threshold wavelength :-

Correct Answer: 4λ

Solution:

Step 1: Use Einstein Photoelectric Equation

$$\frac{hc}{\lambda} = \phi + eV_s$$

Step 2: Write both equations

For wavelength λ:

$$\frac{hc}{\lambda} = \phi + 3eV_0$$

For wavelength 2λ:

$$\frac{hc}{2\lambda} = \phi + eV_0$$

Step 3: Eliminate eV_0

Multiply second equation by 3:

$$\frac{3hc}{2\lambda} = 3\phi + 3eV_0$$

Subtract first equation:

$$\frac{3hc}{2\lambda} - \frac{hc}{\lambda} = 2\phi$$

$$\frac{hc}{2\lambda} = 2\phi$$

$$\phi = \frac{hc}{4\lambda}$$

Now,

$$\phi = \frac{hc}{\lambda_0}$$

Hence,

$$\lambda_0 = 4\lambda$$

Step 4: Final Answer

$$\boxed{4\lambda}$$

Quick Tip: Setting up a system of linear equations and eliminating the stopping potential variable eV_0 is the standard and fastest technique for solving dual-case photoelectric effect problems.

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10. A container of initial volume $0.15\ \text{m}^3$ is expanded adiabatically from pressure 8 bar to final pressure 1 bar. If initial temperature is 140 K, then find work done by the gas :- ($C_p = 3R, C_v = 2R$) :-

Correct Answer: 120 kJ

Solution:

Step 1: Find γ

$$\gamma = \frac{C_p}{C_v} = \frac{3R}{2R} = \frac{3}{2}$$

Step 2: Find final volume using adiabatic relation

$$P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$$

$$V_2 = V_1 \left(\frac{P_1}{P_2}\right)^{1/\gamma}$$

$$V_2 = 0.15 \times 8^{2/3}$$

$$V_2 = 0.15 \times 4 = 0.6\, m^3$$

Step 3: Calculate work done

$$W = \frac{P_1 V_1 - P_2 V_2}{\gamma - 1}$$

$$W = \frac{(8 \times 10^5)(0.15) - (1 \times 10^5)(0.6)}{0.5}$$

$$W = \frac{1.2 \times 10^5 - 0.6 \times 10^5}{0.5}$$

$$W = 1.2 \times 10^5 J$$

$$W = 120\, kJ$$

Step 4: Final Answer

$$\boxed{120\, kJ}$$

Quick Tip: For adiabatic expansion, the formula $W = \frac{P_1V_1 - P_2V_2}{\gamma - 1}$ is usually faster to apply than $\frac{nR(T_1 - T_2)}{\gamma - 1}$ because P_1, P_2, V_1, and V_2 can be easily derived directly from $PV^\gamma = C$ without needing to calculate moles.

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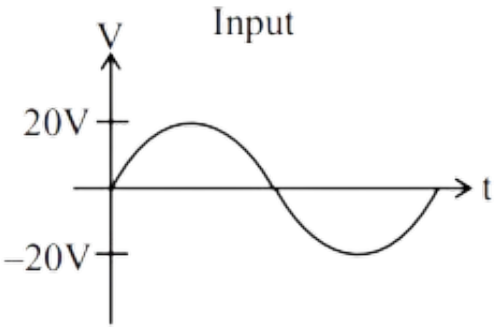
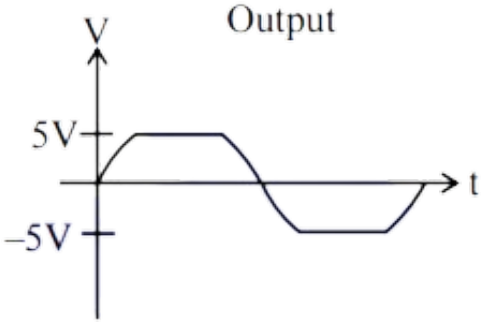
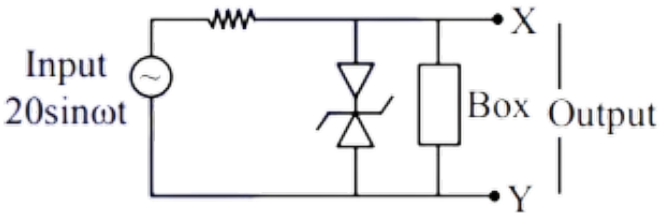
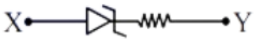
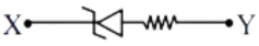
11. Diagram shows a circuit consisting of some elements. The input and output of the circuit is shown. Choose the correct option that shows the component in box. (Zener diode has a breakdown potential of 5V)

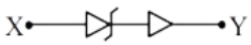


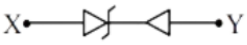




(A) 

(B) 

(C) 

(D) 

Correct Answer: (D) T

Solution:

Step 1: Observe waveform

The output is clipped at

$$+5V \quad \text{and} \quad -5V$$

This means symmetric clipping.

Step 2: Identify suitable component

A single Zener clips only one side properly.

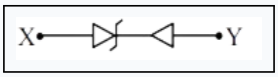
For clipping both positive and negative cycles equally, two Zener diodes are required.

Step 3: Connection

Two Zener diodes connected in opposite directions (back-to-back) provide clipping at

$$\pm 5V$$

Step 4: Final Answer



Quick Tip: A single Zener diode clips at $+V_Z$ in reverse bias and ~ -0.7V in forward bias.

For symmetric clipping, a back-to-back series arrangement is mandatory.

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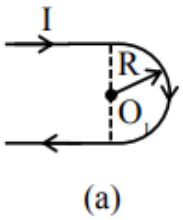
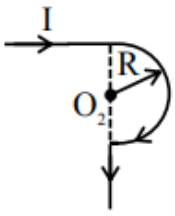
12. Consider two arrangement of wires. Find out ratio of magnetic field at center of semi-circular part :.



(a)



(A) $\frac{2+\pi}{1+\pi}$

(B) $\frac{1+\pi}{2+\pi}$

(C) $\frac{2+\pi}{2+\pi}$

(D) $\frac{2+\pi}{3+\pi}$

Correct Answer: (A) $\frac{2+\pi}{1+\pi}$

Solution:

Step 1: Formula Used

Magnetic field due to semicircle:

$$B_{\text{arc}} = \frac{\mu_0 I}{4R}$$

Magnetic field due to semi-infinite wire:

$$B_{\text{wire}} = \frac{\mu_0 I}{4\pi R}$$

Step 2: Magnetic field in arrangement (a)

There are two semi-infinite wires + one semicircle:

$$B_1 = 2\left(\frac{\mu_0 I}{4\pi R}\right) + \frac{\mu_0 I}{4R}$$

$$B_1 = \frac{\mu_0 I}{4\pi R}(2+\pi)$$

Step 3: Magnetic field in arrangement (b)

One straight wire + one semicircle.

Radial wire contributes zero field.

$$B_2 = \frac{\mu_0 I}{4\pi R} + \frac{\mu_0 I}{4R}$$

$$B_2 = \frac{\mu_0 I}{4\pi R}(1+\pi)$$

Step 4: Ratio

$$\frac{B_1}{B_2} = \frac{2+\pi}{1+\pi}$$

$$\boxed{\frac{2+\pi}{1+\pi}}$$

Quick Tip: Always apply the right-hand rule strictly to determine the direction of the field from each segment before simply adding their magnitudes.
Also remember, any current aligned radially towards or away from the reference point produces zero magnetic field at that point.

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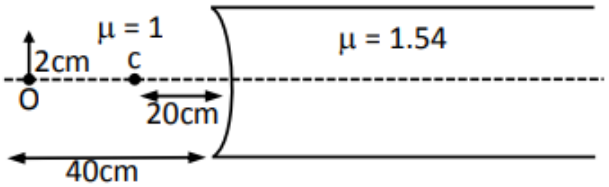
13. Object is placed at 40 cm from spherical surface whose radius of curvature is 20 cm. Find height of image formed.



(A) 2 cm

(B) 4 cm

(C) 0.96 cm

(D) 1.96 cm

Correct Answer: (C) 0.96 cm

Solution:

Step 1: Use spherical refraction formula

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

Given:

$$\mu_1 = 1, \quad \mu_2 = 1.54$$

$$u = -40 \text{ cm}, \quad R = -20 \text{ cm}$$

Step 2: Find image distance

$$\frac{1.54}{v} - \frac{1}{-40} = \frac{0.54}{-20}$$

$$\frac{1.54}{v} + \frac{1}{40} = -\frac{0.54}{20}$$

$$\frac{1.54}{v} = -0.052$$

$$v = -29.6 \text{ cm}$$

Step 3: Find magnification

$$m = \frac{\mu_1 v}{\mu_2 u}$$

$$m = \frac{1 \times (-29.6)}{1.54 \times (-40)}$$

$$m = 0.48$$

Step 4: Image height

$$h_i = m h_o$$

$$h_i = 0.48 \times 2$$

$$h_i = 0.96 \text{ cm}$$

$$\boxed{0.96 \text{ cm}}$$

Quick Tip: Always double-check the sign of R.

If the spherical surface bulges towards the incoming light, R is positive (convex).

If it bows inwards, away from the light, R is negative (concave).

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14. A satellite is revolving around planet of mass 2M in orbit of radius R with time period is T_1.

Another satellite is revolving around planet of mass 4M in orbit of radius 2R, with time period T_2. Find $\frac{T_1}{T_2}$.

(A) $\frac{1}{\sqrt{2}}$
(B) $\sqrt{2}$
(C) $\frac{1}{2}$
(D) $\frac{1}{2\sqrt{2}}$

Correct Answer: (C) $\frac{1}{2}$

Solution:

Step 1: Use Kepler's third law

$$T \propto \sqrt{\frac{r^3}{M}}$$

Step 2: Write ratio

$$\frac{T_1}{T_2} = \sqrt{\frac{R^3/(2M)}{(2R)^3/(4M)}}$$

Step 3: Simplify

$$= \sqrt{\frac{R^3}{2M} \cdot \frac{4M}{8R^3}}$$

$$= \sqrt{\frac{1}{4}}$$

$$= \frac{1}{2}$$

Step 4: Final Answer

$$\boxed{\frac{1}{2}}$$

Quick Tip: Be extremely careful to differentiate between the mass of the central body and the mass of the orbiting satellite.

Kepler's law relates the time period to the central body's mass only; the satellite's mass cancels out.

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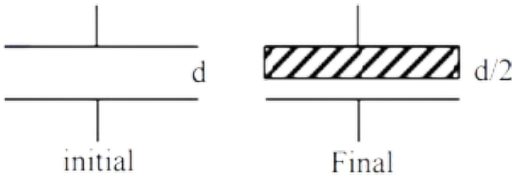
15. There is a parallel plate capacitor of capacitance C. If half of the space is filled with dielectric of dielectric constant k = 5 as in the figure. Find percentage increase in capacitance.



Correct Answer: 66.67%

Solution:

Step 1: Initial capacitance

$$C = \frac{\epsilon_0 A}{d}$$

Step 2: New capacitance

For half dielectric slab:

$$C' = \frac{\epsilon_0 A}{\frac{d/2}{1} + \frac{d/2}{5}}$$

$$C' = \frac{\epsilon_0 A}{\frac{d}{2} + \frac{d}{10}}$$

$$C' = \frac{\epsilon_0 A}{3d/5}$$

$$C' = \frac{5C}{3}$$

Step 3: Increase

$$\Delta C = C' - C$$

$$\Delta C = \frac{5C}{3} - C = \frac{2C}{3}$$

Step 4: Percentage increase

$$\% = \frac{\Delta C}{C} \times 100$$

$$= \frac{2}{3} \times 100$$

$$= 66.67\%$$

$$\boxed{66.67\%}$$

Quick Tip: Visualizing the components helps: When the dielectric divides the space parallel to the plates, it's a series connection (common area, split thickness).
When it divides perpendicular to the plates, it's a parallel connection (common thickness, split area).

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16. In Searl's experiment diameter of wire is measured with screw gauge of least count 0.001 cm and its value is 0.08 cm. Length of wire is 150 cm with least count 0.1 cm and elongation 0.5 cm with least count 0.001 cm. Weight suspended is 100 N then absolute error in Young's modulus is $N \times 10^9$ N/m^2. Then N is :

(A) 1.64

(B) 1.65

(C) 1.63

(D) 1.66

Correct Answer: (B) 1.65

Solution:

Step 1: Formula

$$Y = \frac{4FL}{\pi d^2 \Delta L}$$

Step 2: Relative error formula

$$\frac{\Delta Y}{Y} = \frac{\Delta L}{L} + 2\frac{\Delta d}{d} + \frac{\Delta(\Delta L)}{\Delta L}$$

Step 3: Substitute values

$$\frac{\Delta Y}{Y} = \frac{0.1}{150} + 2\left(\frac{0.001}{0.08}\right) + \frac{0.001}{0.5}$$

$$= 0.00067 + 0.025 + 0.002$$

$$= 0.02767$$

Given:

$$Y = 5.968 \times 10^{10}$$

$$\Delta Y = Y \times 0.02767$$

$$= 1.65 \times 10^{9}$$

Step 4: Final Answer

$$\boxed{1.65 \times 10^{9}}$$

Quick Tip: Remember to always multiply the fractional error of the diameter by 2 because the area term features a d^2.
The variable raised to the highest power usually contributes most substantially to the total experimental error.

---

17. Thin symmetric prism of $\mu = 1.5$. Find ratio of incident angle and minimum deviation.:

Correct Answer: 3/2

Solution:

Step 1: Formula for thin prism

$$\delta_m = (\mu - 1)A$$

$$\delta_m = (1.5 - 1)A$$

$$\delta_m = 0.5A$$

Step 2: Incident angle

At minimum deviation:

$$r = \frac{A}{2}$$

Using Snell's law:

$$i = \mu r$$

$$i = 1.5 \times \frac{A}{2}$$

$$i = 0.75A$$

Step 3: Ratio

$$\frac{i}{\delta_m} = \frac{0.75A}{0.5A}$$

$$= \frac{3}{2}$$

Step 4: Final Answer

$$\boxed{\frac{3}{2}}$$

Quick Tip: For thin prisms, you can bypass trigonometric complications entirely.

The formula $\delta = (\mu - 1)A$ holds for all typical angles of incidence close to the normal.

---

18. For diatomic gas, find the ratio $Q : \Delta U : W$ for isobaric process :-

(A) 2 : 5 : 7

(B) 2 : 3 : 5

(C) 2 : 7 : 3

(D) 7 : 5 : 2

Correct Answer: (D) 7 : 5 : 2

Solution:

Step 1: Understanding the Concept:

In an isobaric process (constant pressure), heat added to a gas is shared between increasing the internal energy of the gas and doing mechanical work against the surroundings.

The partition depends on the degrees of freedom f of the gas molecules.

Step 2: Key Formula or Approach:

For a process at constant pressure:

- Work done: $W = P\Delta V = nR\Delta T$

- Change in internal energy: $\Delta U = nC_v\Delta T = n\left(\frac{f}{2}R\right)\Delta T$

- Heat exchanged: $Q = nC_p\Delta T = \Delta U + W = n\left(\frac{f+2}{2}R\right)\Delta T$

Step 3: Detailed Explanation:

For a diatomic gas (at standard temperatures without vibrational modes excited), degrees of freedom $f = 5$.

Substitute $f = 5$ into the formulas:

$$\Delta U = n\left(\frac{5}{2}R\right)\Delta T$$

$$W = nR\Delta T = n\left(\frac{2}{2}R\right)\Delta T$$

$$Q = \Delta U + W = n\left(\frac{7}{2}R\right)\Delta T$$

We want to find the ratio $Q : \Delta U : W$:

$$Q : \Delta U : W = \frac{7}{2}nR\Delta T : \frac{5}{2}nR\Delta T : \frac{2}{2}nR\Delta T$$

Divide the entire ratio by $nR\Delta T/2$:

$$Q : \Delta U : W = 7 : 5 : 2$$

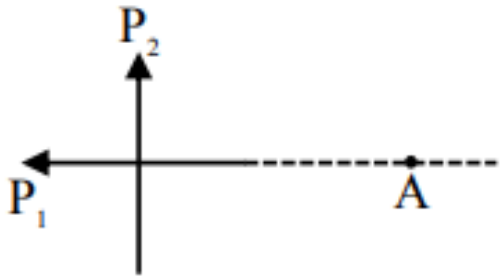
Step 4: Final Answer:

The ratio is $7 : 5 : 2$.

Quick Tip: For any ideal gas at constant pressure, the ratio $W : \Delta U : Q$ is always $2 : f : (f + 2)$. Memorizing this sequence allows instantaneous answering for monatomic $(2 : 3 : 5)$, diatomic $(2 : 5 : 7)$, and polyatomic $(2 : 6 : 8 = 1 : 3 : 4)$ gases.

---

19. Net electric field at point A as shown in figure is at an angle of $60°$ with x-axis then, find P_2/P_1 =?



(A) $1/\sqrt{3}$

(B) $2\sqrt{3}$

(C) $\sqrt{3}$

(D) $\sqrt{3}/2$

Correct Answer: (B) $2\sqrt{3}$

Solution:

Step 1: Understanding the Concept:

The electric field at point A is the vector sum of the fields created by the two dipoles. Point A lies on the axial line of dipole P_1 and on the equatorial line of dipole P_2.

Step 2: Key Formula or Approach:

The electric field on the axial line of a short dipole P at distance r is:

$$E_{\text{axial}} = \frac{2kP}{r^3}$$

The electric field on the equatorial line of a short dipole P at distance r is:

$$E_{\text{equatorial}} = \frac{kP}{r^3}$$

Step 3: Detailed Explanation:

For dipole P_1 (pointing along $-x$): Point A is on its axis, so the electric field $\vec{E}_1$ points in the direction of P_1 which is $-x$ direction.

$$|\vec{E}_1| = \frac{2kP_1}{r^3}$$

For dipole P_2 (pointing along $+y$): Point A is on its equatorial plane, so the electric field $\vec{E}_2$

points anti-parallel to P_2 which is $-y$ direction.

$$|\vec{E}_2| = \frac{kP_2}{r^3}$$

The net electric field vector makes an angle $\theta = 60^\circ$ with the x-axis. Using the magnitudes of the components:

$$\tan(60^\circ) = \frac{|\vec{E}_2|}{|\vec{E}_1|}$$

Substitute the field equations:

$$\sqrt{3} = \frac{\frac{kP_2}{r^3}}{\frac{2kP_1}{r^3}}$$

$$\sqrt{3} = \frac{P_2}{2P_1}$$

Solving for the ratio P_2/P_1:

$$\frac{P_2}{P_1} = 2\sqrt{3}$$

Step 4: Final Answer:

The required ratio is $2\sqrt{3}$.

Quick Tip: Remember that the equatorial field is precisely half the magnitude of the axial field for the same dipole at the same distance, and more importantly, it points in the opposite direction to the dipole moment.

---

20. Angular momentum of the electron in a hydrogen atom is $\frac{3h}{2\pi}$, then find the energy of the electron in the orbit :-

(A) -13.6 eV

(B) -3.4 eV

(C) -1.51 eV

(D) -0.85 eV

Correct Answer: (C) -1.51 eV

Solution:

Step 1: Understanding the Concept:

Niels Bohr's quantization condition postulates that an electron can only revolve in stable orbits where its orbital angular momentum is an integral multiple of $h/2\pi$.

The energy of the electron is dictated by the principal quantum number n defining that orbit.

Step 2: Key Formula or Approach:

Bohr's quantization rule for angular momentum L:

$$L = \frac{nh}{2\pi}$$

Energy of an electron in the n-th orbit of a hydrogen atom:

$$E_n = -\frac{13.6}{n^2} \text{ eV}$$

Step 3: Detailed Explanation:

Given the angular momentum is $\frac{3h}{2\pi}$, we can equate this to Bohr's condition to find the principal quantum number n:

$$\frac{nh}{2\pi} = \frac{3h}{2\pi} \implies n = 3$$

So, the electron is in the third orbit ($n = 3$).

Substitute $n = 3$ into the energy formula:

$$E_3 = -\frac{13.6}{3^2} \text{ eV} = -\frac{13.6}{9} \text{ eV}$$

$$E_3 \approx -1.51 \text{ eV}$$

Step 4: Final Answer:

The energy of the electron in the orbit is -1.51 eV.

Quick Tip: Memorize the energy levels of the hydrogen atom up to $n = 5$: -13.6eV, -3.4eV, -1.51eV, -0.85eV, -0.54eV.

Having these on hand will massively speed up calculations in modern physics.

---

21. The equation of a plane progressive wave is given by $y = 5\cos\left(\pi\left(200t - \frac{x}{150}\right)\right)$ where x and y are in cm and t is in seconds. Find the wave velocity :

Correct Answer: 300 m/s

Solution:

Step 1: Understanding the Concept:

The velocity of a traveling wave can be extracted from its standard equation by identifying the angular frequency ω and the angular wave number k.

Step 2: Key Formula or Approach:

The standard equation of a traveling wave is $y(x,t) = A\cos(\omega t \pm kx)$.

The wave velocity is defined as $v = \frac{\omega}{k}$.

Step 3: Detailed Explanation:

Given wave equation:

$$y = 5\cos\left[\pi\left(200t - \frac{x}{150}\right)\right]$$

Multiply the π through to match the standard format:

$$y = 5\cos\left(200\pi t - \frac{\pi x}{150}\right)$$

By comparing this with $y = A\cos(\omega t - kx)$:

$$\omega = 200\pi \text{ rad/s}$$

$$k = \frac{\pi}{150} \text{ cm}^{-1}$$

Calculate the wave velocity:

$$v = \frac{\omega}{k} = \frac{200\pi}{\pi/150}$$

$$v = 200 \times 150 = 30000 \text{ cm/s}$$

Since the standard unit is usually m/s, convert cm/s to m/s:

$$v = \frac{30000}{100} \text{ m/s} = 300 \text{ m/s}$$

Step 4: Final Answer:

The wave velocity is 300 m/s.

Quick Tip: To find wave velocity quickly, take the absolute value of the coefficient of t and divide it by the absolute value of the coefficient of x.
You don't even need to expand factors outside the bracket (like the π here), because they will naturally cancel out!

---

22. $Li^{+2} \rightarrow Li^{+3} + e^-$. Energy required for this process. Given ionisation energy for ground state of hydrogen atom is 2.17×10^{-18} J.

(A) 19.53×10^{-18} J
(B) 19.54×10^{-18} J
(C) 19.56×10^{-18} J
(D) 19.55×10^{-18} J

Correct Answer: (A) 19.53×10^{-18} J

Solution:

Step 1: Understanding the Concept:

The ion Li^{2+} has only one remaining electron, making it a "hydrogen-like" system.
Bohr's theory can be applied to find the energy required to fully remove this electron.

Step 2: Key Formula or Approach:

The ionization energy for a hydrogen-like atom or ion with atomic number Z is:

$$E_{\text{ion}} = E_{\text{ion}}(H) \times Z^2$$

where $E_{\text{ion}}(H)$ is the ionization energy of a hydrogen atom.

Step 3: Detailed Explanation:

Lithium has an atomic number $Z = 3$.

The ground state ionization energy of Hydrogen ($Z = 1$) is given as $E_{\text{H}} = 2.17 \times 10^{-18}$ J.

Applying the relation for Li^{2+}:

$$E_{Li^{2+}} = (2.17 \times 10^{-18}\ \text{J}) \times 3^2$$

$$E_{Li^{2+}} = (2.17 \times 10^{-18}) \times 9$$

$$E_{Li^{2+}} = 19.53 \times 10^{-18}\ \text{J}$$

Step 4: Final Answer:

The required energy is 19.53×10^{-18} J.

Quick Tip: Always identify if an atom/ion is single-electron.

The $E \propto Z^2/n^2$ model is strictly applicable ONLY to single-electron species like H, He^+, Li^{2+}, and Be^{3+}.

---

23. Electric field of electromagnetic wave is $E = 800\sin\left(\pi\left(10^8 t + \frac{x}{150}\right)\right)$ V/m. Where x in cm and 't' in sec. A charged particle is moving with speed of 1.5×10^6 m/s. Find the ratio of magnetic force to electric force on charge particle.

(A) 10^{+2}

(B) 10^{-2}

(C) 2×10^2

(D) 2×10^{-2}

Correct Answer: (B) 10^{-2}

Solution:

Step 1: Understanding the Concept:

For a particle moving in electromagnetic fields, it experiences an electric force ($F_E = qE$) and a maximum magnetic force ($F_B = qvB$).

Furthermore, in an electromagnetic wave, the ratio of electric and magnetic field amplitudes is simply the speed of the wave ($E/B = v_{\text{wave}}$).

Step 2: Key Formula or Approach:

Ratio of maximum magnetic force to electric force:

$$\frac{F_B}{F_E} = \frac{qv_{\text{particle}}B}{qE} = v_{\text{particle}} \times \left(\frac{B}{E}\right)$$

Since $v_{\text{wave}} = \frac{E}{B}$, this ratio simplifies to:

$$\frac{F_B}{F_E} = \frac{v_{\text{particle}}}{v_{\text{wave}}}$$

Wave velocity $v_{\text{wave}} = \frac{\omega}{k}$.

Step 3: Detailed Explanation:

From the given wave equation $E = 800\sin\left(10^8\pi t + \frac{\pi x}{150}\right)$:

$$\omega = 10^8\pi \text{ rad/s}$$

$$k = \frac{\pi}{150} \text{ cm}^{-1}$$

Convert k into standard SI units (m^{-1}):

Since 1 m = 100 cm, x in cm means the denominator is inherently matching. $k = \frac{\pi}{1.5}$ m^{-1}.

Calculate v_{wave}:

$$v_{\text{wave}} = \frac{\omega}{k} = \frac{10^8\pi}{\pi/1.5} = 1.5 \times 10^8 \text{ m/s}$$

Given the particle speed is $v_{\text{particle}} = 1.5 \times 10^6$ m/s, the force ratio is:

$$\frac{F_B}{F_E} = \frac{1.5 \times 10^6 \text{ m/s}}{1.5 \times 10^8 \text{ m/s}} = 10^{-2}$$

Step 4: Final Answer:

The ratio of the magnetic force to the electric force is 10^{-2}.

Quick Tip: When computing k from wave equations, explicitly tracking units is crucial. If an equation has $x/150$ and specifies x is in cm, k is $1/150$ inverse cm. Remember to shift the decimal correctly when converting to standard SI base.

---

24. A liquid drop having diameter 2mm and surface tension 0.08 N/m. This drop splits into 512 identical small drops. Find change in surface energy ?

(A) 4.034

(B) 5

(C) 7.034×10^{-6}

(D) 9.03

Correct Answer: (C) 7.034×10^{-6}

Solution:

Step 1: Understanding the Concept:

When a large drop breaks down into numerous smaller drops, the total volume is conserved but the total surface area increases.

Because forming a new surface requires energy, the overall surface energy of the system increases.

Step 2: Key Formula or Approach:

Conservation of volume:

$$\frac{4}{3}\pi R^3 = n\left(\frac{4}{3}\pi r^3\right) \implies R = n^{1/3} r$$

Change in surface energy (ΔU):

$$\Delta U = T \times \Delta A = T(n \cdot 4\pi r^2 - 4\pi R^2)$$

Substitute $r = R/n^{1/3}$:

$$\Delta U = T \cdot 4\pi R^2 \left(n \cdot \frac{1}{n^{2/3}} - 1 \right) = T \cdot 4\pi R^2 (n^{1/3} - 1)$$

Step 3: Detailed Explanation:

Given values:

Diameter $= 2$ mm $\implies R = 1$ mm $= 10^{-3}$ m.

Number of drops $n = 512$.

Surface tension $T = 0.08$ N/m.

Calculate $n^{1/3}$:

$$n^{1/3} = (512)^{1/3} = 8$$

Substitute everything into the ΔU formula:

$$\Delta U = 0.08 \times 4\pi \times (10^{-3})^2 \times (8 - 1)$$

$$\Delta U = 0.08 \times 4\pi \times 10^{-6} \times 7$$

$$\Delta U = 0.56 \times 4 \times \pi \times 10^{-6} = 2.24\pi \times 10^{-6} \text{ J}$$

Using $\pi \approx 3.14$:

$$\Delta U \approx 2.24 \times 3.14 \times 10^{-6} = 7.0336 \times 10^{-6} \text{ J} \approx 7.034 \times 10^{-6} \text{ J}$$

Step 4: Final Answer:

The change in surface energy is 7.034×10^{-6} J.

Quick Tip: Memorize the direct shortcut formula for breaking drops: $W = 4\pi R^2 T(n^{1/3} - 1)$.
For coalescing droplets, the formula structurally inverses into energy released $Q = 4\pi R^2 T(n^{1/3} - 1)$.

---

25. If power dissipated in a coil having total number of turns 'N', cross-section area 'A' and

radius of coil 'R' when kept in a time varying magnetic field is P. Now if another coil having total number of turns '2N', cross-section area '2A' and radius of coil '3R' is placed is same time varying magnetic field power dissipated is αP, then value of α is :

(A) 108

(B) 324

(C) 216

(D) 432

Correct Answer: (A) 108

Solution:

Step 1: Understanding the Concept:

A time-varying magnetic field induces an electromotive force (EMF) inside a coil per Faraday's Law.

If the circuit is closed, this EMF pushes a current and dissipates power as Joule heat ($P = \epsilon^2/R_{\text{coil}}$).

Step 2: Key Formula or Approach:

Induced EMF ϵ:

$$\epsilon = -N\frac{d\Phi}{dt} = -N(\pi R^2)\frac{dB}{dt} \implies \epsilon \propto NR^2$$

Resistance of the coil wire R_{res}:

$$R_{\text{res}} = \rho\frac{l_{\text{wire}}}{A_{\text{wire}}}$$

Total length of the wire $l_{\text{wire}} = N \times (2\pi R)$. Thus:

$$R_{\text{res}} = \rho\frac{N \cdot 2\pi R}{A_{\text{wire}}} \implies R_{\text{res}} \propto \frac{NR}{A_{\text{wire}}}$$

Power dissipated P:

$$P = \frac{\epsilon^2}{R_{\text{res}}} \propto \frac{(NR^2)^2}{\frac{NR}{A_{\text{wire}}}} = \frac{N^2R^4 \cdot A_{\text{wire}}}{NR} = N \cdot R^3 \cdot A_{\text{wire}}$$

Step 3: Detailed Explanation:

Let's denote the variables of the first coil as $N_1 = N$, $R_1 = R$, $A_1 = A$.

$$P_1 = P \propto N \cdot R^3 \cdot A$$

For the second coil, $N_2 = 2N$, $R_2 = 3R$, $A_2 = 2A$.

The power dissipated P_2 is:

$$P_2 \propto (2N) \cdot (3R)^3 \cdot (2A)$$

$$P_2 \propto 2N \cdot 27R^3 \cdot 2A$$

$$P_2 \propto 108 \cdot (NR^3A)$$

Since (NR^3A) corresponds to the initial power P:

$$P_2 = 108P$$

Comparing this to the given $P_2 = \alpha P$, we get:

$$\alpha = 108$$

Step 4: Final Answer:

The value of α is 108.

Quick Tip: Always distinguish between the cross-sectional area of the coil (πR^2, responsible for magnetic flux) and the cross-sectional area of the wire itself (A, responsible for electrical resistance). Confusing the two usually yields drastically incorrect polynomial dependencies.

Chemistry

1. For first order reaction, rate constant at 27°C and t°C are 1.5×10^3 and 4.5×10^3 respectively. If activation energy of the reaction is 60 KJmol^{-1}, then (R = 8.3 Jmol^{-1}K^{-1}) $\ln 3 = 1.1$. Find

temperature 't'(in °C).

Correct Answer: (41)

Solution:

Step 1: Understanding the Question:

The question asks to find the temperature t at which the rate constant of a first-order reaction triples, given the initial temperature, activation energy, and gas constant.

Step 2: Key Formula or Approach:

We use the Arrhenius equation in the logarithmic ratio form:

$$\ln\left(\frac{k_2}{k_1}\right) = \frac{E_a}{R}\left[\frac{T_2 - T_1}{T_1 T_2}\right]$$

Step 3: Detailed Explanation:

The given values are initial temperature $T_1 = 27 + 273 = 300$ K.

The rate constant at T_1 is $k_1 = 1.5 \times 10^3$.

The rate constant at T_2 is $k_2 = 4.5 \times 10^3$.

The ratio of rate constants is $\frac{k_2}{k_1} = \frac{4.5\times10^3}{1.5\times10^3} = 3$.

The activation energy $E_a = 60$ kJ/mol $= 60000$ J/mol.

The gas constant $R = 8.3$ J mol^{-1}K^{-1} and $\ln 3 = 1.1$.

Substituting the values into the Arrhenius equation:

$$\ln(3) = \frac{60000}{8.3}\left[\frac{T_2 - 300}{300 \cdot T_2}\right]$$

This can be simplified as shown below:

$$1.1 = \frac{60000}{8.3 \times 300}\left[\frac{T_2 - 300}{T_2}\right]$$

$$1.1 = \frac{200}{8.3}\left[1 - \frac{300}{T_2}\right]$$

$$\frac{1.1 \times 8.3}{200} = 1 - \frac{300}{T_2}$$

$$0.04565 = 1 - \frac{300}{T_2}$$

$$\frac{300}{T_2} = 1 - 0.04565 = 0.95435$$

$$T_2 = \frac{300}{0.95435} \approx 314.35 \text{ K}$$

Converting to Celsius:

$$t = 314.35 - 273 = 41.35^\circ\text{C}$$

Rounding to the nearest integer yields 41°C.

Step 4: Final Answer:

The temperature t is 41°C.

Quick Tip: Always ensure that the units of activation energy and gas constant match.

If R is in J, E_a must be converted from kJ to J.

Also, remember to always use temperature in Kelvin in rate equations.

---

2. One mole of an alkane on complete combustion required 8 mole of O_2, find out sum of carbon and hydrogen atoms in one molecule of the alkane.

Correct Answer: (17)

Solution:

Step 1: Understanding the Question:

The question asks for the total number of carbon and hydrogen atoms in a molecule of an alkane, given that one mole of the alkane needs 8 moles of oxygen for complete combustion.

Step 2: Key Formula or Approach:

The general formula for an alkane is C_nH_{2n+2}.

The balanced chemical equation for the combustion of an alkane is:

$$C_nH_{2n+2} + \left(\frac{3n+1}{2}\right)O_2 \rightarrow nCO_2 + (n+1)H_2O$$

Step 3: Detailed Explanation:

From the given information, 1 mole of alkane requires 8 moles of O_2.

By comparing the stoichiometric coefficients:

$$\frac{3n+1}{2} = 8$$

From this, we can find the value of n:

$$3n + 1 = 16$$

$$3n = 15$$

$$n = 5$$

So the alkane is Pentane with formula C_5H_{12}.

The total number of atoms of carbon and hydrogen in one molecule is $n+(2n+2) = 5+12 = 17$.

Step 4: Final Answer:

The sum of carbon and hydrogen atoms is 17.

Quick Tip: The general combustion formula for any hydrocarbon C_xH_y uses $(x + y/4)$ moles of O_2. For alkanes, substituting $x = n$ and $y = 2n + 2$ gives the formula $(3n + 1)/2$ directly.

---

3. Relation between $t_{1/2}$ & $t_{100\%}$ for zero order & Ist order reaction respectively is:

(1) $t_{100\%} = 2 \times t_{1/2} : t_{100\%} = 2 \times t_{1/2}$

(2) $t_{100\%} = 2 \times t_{1/2} : t_{100\%} = t_{1/2}$

(3) $t_{100\%} = 2 \times t_{1/2} : t_{100\%} = \infty$

(4) $t_{100\%} = \infty : t_{100\%} = 2 \times t_{1/2}$

Correct Answer: (3) $t_{100\%} = 2 \times t_{1/2} : t_{100\%} = \infty$

Solution:

Step 1: Understanding the Question:

The question asks for the relationship between the half-life and completion time for zero-order and first-order reactions.

Step 2: Detailed Explanation:

For a zero-order reaction, the rate is constant.

The half-life $t_{1/2} = \frac{[A]_0}{2k}$.

The completion time $t_{100\%} = \frac{[A]_0}{k}$.

Therefore, $t_{100\%} = 2 \times t_{1/2}$.

For a first-order reaction, the reaction rate depends on concentration.

The reaction theoretically takes infinite time to go to 100% completion because the concentration decreases exponentially.

Thus, $t_{100\%} = \infty$.

Step 3: Final Answer:

The correct relation is $t_{100\%} = 2 \times t_{1/2}$ for zero order and $t_{100\%} = \infty$ for first order.

Quick Tip: Zero-order reactions are the only ones that go to completion in finite time.

For all other higher orders, the reaction theoretically takes infinite time to reach 100% completion.

---

4. 18 g of steam reacted with iron to form Fe_3O_4, how much iron will be consumed.

(1) 21 gm

(2) 42 gm

(3) 84 gm

(4) 10.5 gm

Correct Answer: (2) 42 gm

Solution:

Step 1: Understanding the Question:

We need to calculate the mass of iron consumed when it reacts with 18 grams of steam to form Fe_3O_4.

Step 2: Key Formula or Approach:

Write the balanced chemical equation and use stoichiometry:

$$3Fe(s) + 4H_2O(g) \rightarrow Fe_3O_4(s) + 4H_2(g)$$

Step 3: Detailed Explanation:

The molar mass of steam (H_2O) is 18 g/mol.

So, moles of steam provided $= \frac{18\text{ g}}{18\text{ g/mol}} = 1$ mole.

According to the balanced equation, 4 moles of H_2O react with 3 moles of Fe.

So, 1 mole of H_2O will react with $\frac{3}{4}$ moles of Fe.

Moles of Fe required $= 0.75$ moles.

Atomic mass of $Fe = 56$ g/mol.

Mass of Fe required $= 0.75 \times 56 = 42$ g.

Step 4: Final Answer:

The mass of iron consumed is 42 gm.

Quick Tip: Always balance the chemical equation first.

In redox reactions involving iron and steam, Fe_3O_4 (magnetite) is the common product, not Fe_2O_3 or FeO.

---

5. Angular momentum of the electron in a hydrogen atom is $\frac{3h}{2\pi}$ then find total energy of electron (in eV/atom)

(1) –1.51

(2) –122.4

(3) –40.8

(4) –4.53

Correct Answer: (1) –1.51

Solution:

Step 1: Understanding the Question:

The question asks for the total energy of an electron in a Hydrogen atom, given its angular momentum.

Step 2: Key Formula or Approach:

Bohr's quantization of angular momentum: $L = \frac{nh}{2\pi}$

Bohr's Energy formula: $E_n = -13.6\frac{Z^2}{n^2}$ eV

Step 3: Detailed Explanation:

The angular momentum is given as $\frac{3h}{2\pi}$.

Comparing it with $\frac{nh}{2\pi}$, we get $n = 3$.

For a Hydrogen atom, atomic number $Z = 1$.

Substituting $n = 3$ and $Z = 1$ in the energy formula:

$$E = -13.6 \times \frac{1^2}{3^2} \text{ eV}$$

$$E = -13.6 \times \frac{1}{9} \text{ eV} = -1.51 \text{ eV}$$

Step 4: Final Answer:

The total energy of the electron is –1.51 eV.

Quick Tip: Remember the first three energy levels for Hydrogen: $E_1 = -13.6$ eV, $E_2 = -3.4$ eV, $E_3 = -1.51$ eV.

Memorizing these values can save significant time in calculations.

---

6. For a reversible adiabatic process involving ideal gas if initial pressure and volume are 8 bar and 0.15 m^3 respectively and final pressure is 1 bar. Calculate |work done| (in Kilo Joule) [$C_V = 2R$, $C_P = 3R$]

Correct Answer: (120)

Solution:

Step 1: Understanding the Question:

We need to find the magnitude of work done in a reversible adiabatic process for an ideal gas.

Step 2: Key Formula or Approach:

Adiabatic process equation: $P_1V_1^\gamma = P_2V_2^\gamma$

Work done formula: $W = \frac{P_2V_2 - P_1V_1}{\gamma - 1}$

Poisson's ratio: $\gamma = \frac{C_P}{C_V}$

Step 3: Detailed Explanation:

Given:

$P_1 = 8$ bar, $V_1 = 0.15$ m^3, $P_2 = 1$ bar.

$C_V = 2R$ and $C_P = 3R$.

So, $\gamma = \frac{C_P}{C_V} = \frac{3R}{2R} = 1.5$.

From adiabatic relation:

$$P_1V_1^\gamma = P_2V_2^\gamma$$

$$8 \times (0.15)^{1.5} = 1 \times (V_2)^{1.5}$$

$$(V_2)^{1.5} = 8 \times (0.15)^{1.5}$$

$$V_2 = 8^{2/3} \times 0.15 = 4 \times 0.15 = 0.6 \text{ m}^3$$

Work done:

$$W = \frac{P_2V_2 - P_1V_1}{\gamma - 1}$$

$$W = \frac{(1 \times 0.6) - (8 \times 0.15)}{1.5 - 1} = \frac{0.6 - 1.2}{0.5} = \frac{-0.6}{0.5} = -1.2 \text{ bar} \cdot \text{m}^3$$

Since 1 bar $\cdot$ m^3 = 10^5 J = 100 kJ, the work done is:

$$W = -1.2 \times 100 \text{ kJ} = -120 \text{ kJ}$$

The magnitude of work done is $|W| = 120$ kJ.

Step 4: Final Answer:

The magnitude of work done is 120 kJ.

Quick Tip: For adiabatic work, using $\frac{P_2V_2 - P_1V_1}{\gamma - 1}$ is often faster than using temperatures. Ensure volume is in m^3 and pressure in Pascal to get work in Joules.

---

7. For reaction A $\rightleftharpoons$ B, $\Delta G^\circ = 105 - 35 \log T$. Find the transition temperature (in $^\circ$C) of the above reaction at 1 bar

Correct Answer: (727)

Solution:

Step 1: Understanding the Question:

The transition temperature for a reaction is the temperature at which the system is at equilibrium, so $\Delta G^\circ = 0$.

Step 3: Detailed Explanation:

The given equation is $\Delta G^\circ = 105 - 35 \log T$.

Setting $\Delta G^\circ = 0$:

$$105 - 35 \log T = 0$$

$$35 \log T = 105$$

$$\log T = \frac{105}{35} = 3$$

$$T = 10^3 = 1000 \text{ K}$$

Converting to Celsius:

$$t = 1000 - 273 = 727^\circ\text{C}$$

Step 4: Final Answer:

The transition temperature is 727°C.

Quick Tip: Transition temperature often implies equilibrium where phase change or equilibrium between species occurs.

Mathematically, it simply means finding the root of the ΔG equation.

---

8. Given $K_{SP}(Ag_2C_2O_4) = 32X$, $K_{SP}(AgBr) = 4Y$. Find ratio of solubility of the given salts in pure water

(1) $\frac{X^{1/3}}{\sqrt{2Y}}$

(2) $\frac{2X^{1/3}}{\sqrt{2Y}}$

(3) $\frac{2X^{1/3}}{\sqrt{Y}}$

(4) $\frac{X^{1/3}}{\sqrt{Y}}$

Correct Answer: (4) $\frac{X^{1/3}}{\sqrt{Y}}$

Solution:

Step 1: Understanding the Question:

We need to find the ratio of the solubility of two different salts, $Ag_2C_2O_4$ and $AgBr$, based on their solubility product constants.

Step 2: Key Formula or Approach:

For $Ag_2C_2O_4$, $K_{sp} = 4S_1^3$.

For $AgBr$, $K_{sp} = S_2^2$.

Step 3: Detailed Explanation:

For $Ag_2C_2O_4$:

$$K_{sp} = 4S_1^3 = 32X \Rightarrow S_1^3 = 8X \Rightarrow S_1 = 2X^{1/3}$$

For $AgBr$:

$$K_{sp} = S_2^2 = 4Y \Rightarrow S_2 = 2\sqrt{Y}$$

Taking the ratio of solubilities:

$$\frac{S_1}{S_2} = \frac{2X^{1/3}}{2\sqrt{Y}} = \frac{X^{1/3}}{\sqrt{Y}}$$

Step 4: Final Answer:

The ratio of solubility is $\frac{X^{1/3}}{\sqrt{Y}}$.

Quick Tip: For a salt A_xB_y, the relation is $K_{sp} = x^x y^y S^{(x+y)}$.

This general formula helps avoid errors during the dissociation step.

9. 19.5 gm FCH_2COOH is dissolved in 500 gm water due to which depression in freezing point is found to be 1°C. Calculate K_a of FCH_2COOH. {K_f of water = 1.86 K-kg/mole, m = M}

(1) 2.8×10^{-3}

(2) 2.8×10^{-2}

(3) 1.4×10^{-3}

(4) 5.6×10^{-3}

Correct Answer: (1) 2.8×10^{-3}

Solution:

Step 1: Understanding the Question:

The objective is to find the acid dissociation constant (K_a) of the given weak acid by first finding the Van't Hoff factor (i) from the depression in freezing point.

Step 2: Key Formula or Approach:

$$\Delta T_f = i \cdot K_f \cdot m$$

$$K_a = \frac{C\alpha^2}{1-\alpha}$$

Step 3: Detailed Explanation:

Molar mass of FCH_2COOH is 78 g/mol.

Moles of solute $= \frac{19.5}{78} = 0.25$ moles.

Molality of solution $m = \frac{0.25}{0.5 \text{ kg}} = 0.5$ m.

Given $\Delta T_f = 1°C$.

$$1 = i \times 1.86 \times 0.5$$

$$i = \frac{1}{0.93} \approx 1.0753$$

For a weak monobasic acid, $i = 1 + \alpha$.

So, $\alpha = 1.0753 - 1 = 0.0753$.

Now, $K_a \approx C\alpha^2 = 0.5 \times (0.0753)^2 \approx 0.00283 = 2.8 \times 10^{-3}$.

Step 4: Final Answer:

The dissociation constant K_a is 2.8×10^{-3}.

Quick Tip: When dealing with weak electrolytes in depression of freezing point, never forget to include the Van't Hoff factor (i).

It is usually the bridge between physical chemistry data and equilibrium constants.

---

10. Consider following reaction :

$Fe(OH)_2 + 2e^- \rightarrow Fe(s) + 2OH^-$, $E^\circ = -0.88$ V

$AgBr(s) + e^- \rightarrow Ag(s) + Br^-$ (aq), $E^\circ = 0.07$ V

Select the correct statement.

(1) $E^\circ_{cell} = -0.95$V

(2) E°_{cell} is an extensive property

(3) Fe is getting reduced

(4) Net cell reaction: $Fe(s) + 2OH^-(aq) + 2AgBr(s) \rightarrow Fe(OH)_2 + 2Ag(s) + 2Br^-(aq)$

Correct Answer: (4) Net cell reaction: $Fe(s) + 2OH^-(aq) + 2AgBr(s) \rightarrow Fe(OH)_2 + 2Ag(s) + 2Br^-(aq)$

Solution:

Step 1: Understanding the Question:

The question provides two reduction half-reactions and asks us to evaluate statements

regarding the potential cell.

Step 2: Detailed Explanation:

The given potentials are standard reduction potentials.

E° for $AgBr/Ag$ is 0.07 V, which is higher than -0.88 V for $Fe(OH)_2/Fe$.

Thus, the $AgBr/Ag$ electrode will act as the cathode (reduction) and $Fe(OH)_2/Fe$ will act as the anode (oxidation).

Anode (Oxidation): $Fe(s) + 2OH^-(aq) \rightarrow Fe(OH)_2(s) + 2e^-$

Cathode (Reduction): $2AgBr(s) + 2e^- \rightarrow 2Ag(s) + 2Br^-(aq)$

Net Cell Reaction: $Fe(s) + 2OH^-(aq) + 2AgBr(s) \rightarrow Fe(OH)_2(s) + 2Ag(s) + 2Br^-(aq)$.

This matches statement (4).

Step 3: Final Answer:

The correct statement is (4).

Quick Tip: Standard cell potential (E°) is intensive because it does not depend on the amount of material.

Gibbs free energy ($\Delta G^\circ = -nFE^\circ$) is extensive because it depends on n.

---

11. Match the column and select correct option:-

(i) Vit. B_1	(P) Ascorbic acid
(ii) Vit. B_2	(Q) Riboflavin
(iii) Vit. B_6	(R) Thiamine
(iv) Vit. C	(S) Pyridoxine

(1) (i) → R, (ii) → Q, (iii) → S, (iv) → P

(2) (i) → Q, (ii) → R, (iii) → S, (iv) → P

(3) (i) → R, (ii) → Q, (iii) → P, (iv) → S

(4) (i) → S, (ii) → Q, (iii) → R, (iv) → P

Correct Answer: (1) (i) → R, (ii) → Q, (iii) → S, (iv) → P

Solution:

Step 1: Understanding the Question:

This is a factual matching question based on the chemical names of common vitamins.

Step 2: Detailed Explanation:

Based on biochemical nomenclature:

(i) Vitamin B_1 is commonly known as Thiamine. Matches with (R).

(ii) Vitamin B_2 is commonly known as Riboflavin. Matches with (Q).

(iii) Vitamin B_6 is commonly known as Pyridoxine. Matches with (S).

(iv) Vitamin C is commonly known as Ascorbic acid. Matches with (P).

The correct sequence is (i)-R, (ii)-Q, (iii)-S, (iv)-P.

Step 3: Final Answer:

The correct option is (1).

Quick Tip: Vitamins are a high-yield topic in "Biomolecules" for JEE Main.

Ensure you memorize the names and deficiency diseases of the B-complex group.

---

12. Organic compound C_5H_{10} does not give Baeyer's reagent test. Calculate total number of structural monobromo isomers when react with $Br_2/h\nu$.

Correct Answer: (14)

Solution:

Step 1: Understanding the Question:

The molecular formula C_5H_{10} indicates a Degree of Unsaturation of 1.

Since it does not react with Baeyer's reagent, it does not have a double bond and must be a cycloalkane.

Step 2: Detailed Explanation:

The structural isomers of C_5H_{10} that are cycloalkanes are:

1. Cyclopentane: Yields 1 monobromo isomer.
2. Methylcyclobutane: Yields 4 monobromo isomers.
3. Ethylcyclopropane: Yields 4 monobromo isomers.
4. 1,1-Dimethylcyclopropane: Yields 2 monobromo isomers.
5. 1,2-Dimethylcyclopropane: Yields 3 monobromo isomers.

Summing these up gives: $1+4+4+2+3=14$ structural monobromo isomers.

Step 3: Final Answer:

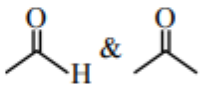
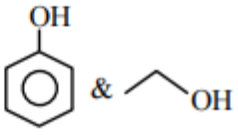
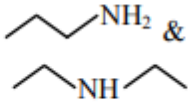
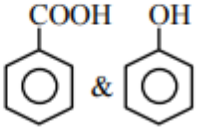
The total number of structural monobromo isomers is 14.

Quick Tip: Structural isomers mean you do not need to consider stereoisomers.

Always check the degree of unsaturation to determine if the compound is open-chain or cyclic.

---

13. Match correctly with the reagents given in column I with organic compounds given in column-II.

	Column-I (Name of test)		**Column-II (Pair of compounds)**
(i)	Neutral $FeCl_3$ test	(P)	
(ii)	Isocyanide test $CHCl_3$ + KOH/Δ	(Q)	
(iii)	Ammonical silver nitrate test	(R)	
(iv)	$NaHCO_3$ test	(S)	

(1) (i)-Q, (ii)-R, (iii)-P, (iv)-S

(2) (i)-P, (ii)-Q, (iii)-R, (iv)-S

(3) (i)-R, (ii)-S, (iii)-Q, (iv)-P

(4) (i)-S, (ii)-R, (iii)-Q, (iv)-P

Correct Answer: (1) (i)-Q, (ii)-R, (iii)-P, (iv)-S

Solution:

Step 1: Understanding the Question:

Match common chemical tests with the pairs of organic compounds they can differentiate.

Step 2: Detailed Explanation:

(i) Neutral $FeCl_3$ test: Used to identify Phenols. It gives a violet color with Phenol but not with Ethanol. Matches with (Q).

(ii) Isocyanide test: Positive only for primary amines. It differentiates Ethylamine (1°) from Diethylamine (2°). Matches with (R).

(iii) Ammoniacal silver nitrate: Positive for aldehydes but negative for ketones. Matches with (P).

(iv) $NaHCO_3$ test: Carboxylic acids react to release CO_2, whereas Phenol does not. Matches with (S).

The correct sequence is (i)-Q, (ii)-R, (iii)-P, (iv)-S.

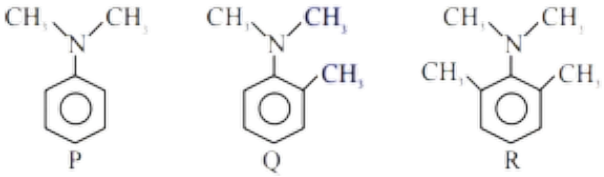
Step 3: Final Answer:

The correct option is (1).

Quick Tip: The $NaHCO_3$ test is a classic way to distinguish between Carboxylic acids and Phenols. Only very acidic phenols like Picric acid give a positive $NaHCO_3$ test.

---

14. Write the correct order of rate of reaction of following compounds with PhN_2Cl



(1) P > Q > R

(2) P > R > Q

(3) Q > P > R

(4) R > P > Q

Correct Answer: (1) P > Q > R

Solution:

Step 1: Understanding the Question:

The reaction with benzenediazonium chloride is an electrophilic substitution reaction.

Step 2: Detailed Explanation:

In electrophilic substitution, substituents that increase electron density increase the rate.

P: N,N-Dimethylaniline. The $-N(CH_3)_2$ group is a strong $+M$ group.

Q: N,N-Dimethyl-o-toluidine. The methyl group at the ortho position causes Steric Inhibition of Resonance (SIR).

R: N,N-Dimethyl-2,6-xylidine. Two methyl groups at ortho positions cause even more severe SIR.

Therefore, the electron density and rate of reaction follow the order: $P > Q > R$.

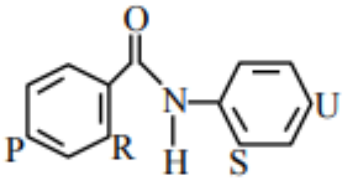
Step 3: Final Answer:

The correct order is $P > Q > R$.

Quick Tip: Steric Inhibition of Resonance (SIR) is a critical concept in aromatic amines.

Ortho substituents can twist the bulky amino group out of plane, significantly reducing its activating power.

15. Most preferred site for electrophilic substitution in above example?



(1) Predominantly at U

(2) S and R

(3) Predominantly at R

(4) R and S

Correct Answer: (1) Predominantly at U

Solution:

Step 1: Understanding the Question:

The question asks to identify the most reactive site for an incoming electrophile in a complex aromatic system.

Step 2: Detailed Explanation:

Electrophilic aromatic substitution occurs at the site with the highest electron density.

In the given structure, the Nitrogen atom has a lone pair which can be donated into the ring through resonance.

Position 'U' is para to the strongly activating Nitrogen atom, making it the most preferred site for substitution.

Step 3: Final Answer:

The most preferred site is predominantly at U.

Quick Tip: When multiple rings are present, identify the ring with the strongest activating group. The substitution will occur on that ring at the position favored by that group.

16. Which of the following order is correct for priority of functional group in IUPAC nomenclature ?

(1) –CHO > –COOR > –CO– > –CN >≡> $–NH_2$

(2) $–CONH_2$ > –CN > –CHO > –CO– > $–NH_2$ >≡

(3) $–CONH_2$ > –COOR– > –CN > –CHO > –CO– >≡

(4) –COOR > –CN > $–CONH_2$ > –CHO > –CO– >≡

Correct Answer: (2) $–CONH_2$ > –CN > –CHO > –CO– > $–NH_2$ >≡

Solution:

Step 1: Understanding the Question:

The question asks to identify the correct decreasing order of priority for various functional groups as per IUPAC (International Union of Pure and Applied Chemistry) nomenclature rules.

Step 2: Detailed Explanation:

According to IUPAC rules, the decreasing order of priority for principal functional groups is generally:

Carboxylic acids > Sulfonic acids > Acid Anhydrides > Esters (–COOR) > Acid Halides > Amides ($–CONH_2$) > Nitriles/Cyanides (–CN) > Aldehydes (–CHO) > Ketones (–CO–) > Alcohols > Thiols > Amines ($–NH_2$) > Alkenes (=) > Alkynes (≡).

Comparing the given options:

In option (2), the sequence is: Amide ($–CONH_2$) > Nitrile (–CN) > Aldehyde (–CHO) > Ketone (–CO–) > Amine ($–NH_2$) > Alkyne (≡).

This sequence perfectly follows the standard IUPAC priority series.

Other options are incorrect because:

In option (1), –CHO is placed before –COOR and –CN.

In option (3), $–CONH_2$ is correct, but –COOR (Ester) should have higher priority than $–CONH_2$.

In option (4), –CN is placed before $–CONH_2$, which is incorrect.

Step 3: Final Answer:

The correct order of priority is given in option (2).

Quick Tip: A simple mnemonic to remember the top priority groups is: "Car (Carboxylic) Sues (Sulphonic) An (Anhydride) Est (Ester) Hal (Halide) Am (Amide) Cy (Cyanide) Al (Aldehyde) Ket (Ketone) Al (Alcohol)".
Nitriles (–CN) always have higher priority than Aldehydes and Ketones.

---

17. Statement-1 : Benzyl chloride reacts faster than ethyl chloride towards SN^1.
Statement-2 : Positive charge on ethyl will be unstable.

(1) Statement-I and statement-II both are correct.
(2) Statement-I and statement-II both are incorrect.
(3) Statement-I correct but statement-II is incorrect.
(4) Statement-I incorrect but statement-II is correct.

Correct Answer: (1) Statement-I and statement-II both are correct.

Solution:

Step 1: Understanding the Question:
We need to evaluate two statements regarding the relative rates of SN^1 reactions for benzyl chloride and ethyl chloride and the stability of the resulting carbocations.

Step 2: Detailed Explanation:
In an SN^1 reaction, the rate-determining step is the formation of a carbocation intermediate.
Benzyl chloride ($Ph-CH_2Cl$) ionizes to form the benzyl carbocation ($Ph-CH_2^+$).
The benzyl carbocation is highly stabilized by resonance with the pi-electrons of the benzene ring.
Ethyl chloride (CH_3-CH_2Cl) ionizes to form the ethyl carbocation ($CH_3-CH_2^+$).
The ethyl carbocation is only stabilized by inductive effect ($+I$) and hyperconjugation from one methyl group.

Because resonance stabilization is much stronger than inductive effects, the benzyl carbocation is far more stable than the ethyl carbocation.

As a result, benzyl chloride reacts significantly faster than ethyl chloride in SN^1 conditions.

Statement-1 is correct because the rate follows carbocation stability.

Statement-2 is correct in the context of the comparison, as the ethyl carbocation is relatively "unstable" compared to the resonance-stabilized benzyl cation.

Step 3: Final Answer:

Both statements are correct, and Statement-II explains why Statement-I is true.

Quick Tip: For SN^1 reactivity, always look at the stability of the intermediate carbocation.

Stability order: Benzyl $\approx$ Allyl $>$ 3° $>$ 2° $>$ 1° $>$ Methyl.

Resonance always outweighs hyperconjugation and inductive effects in determining stability.

---

18. Statement-I : 1, 2, 3-trihydroxy propane can be separated from water by using simple distillation.

Statement-II : Azeotropic mixture cannot be separated by using fractional distillation.

(1) Statement-I and statement-II both are correct.

(2) Statement-I and statement-II both are incorrect.

(3) Statement-I correct but statement-II is incorrect.

(4) Statement-I incorrect but statement-II is correct.

Correct Answer: (4) Statement-I incorrect but statement-II is correct.

Solution:

Step 1: Understanding the Question:

The question tests knowledge of separation techniques for organic compounds and the properties of azeotropic mixtures.

Step 2: Detailed Explanation:

1, 2, 3-trihydroxy propane is glycerol.

Glycerol has a high boiling point (approx. 290°C) and has a tendency to decompose at or near its boiling point under atmospheric pressure.

Therefore, it cannot be separated from water efficiently by simple distillation. Instead, "Distillation under reduced pressure" (vacuum distillation) is used to lower the boiling point and prevent decomposition.

Thus, Statement-I is incorrect.

An azeotropic mixture is a constant boiling mixture where the composition of the liquid phase is exactly the same as the vapor phase.

Since fractional distillation relies on the difference in compositions of the phases to separate components, it is ineffective for azeotropes.

Thus, Statement-II is correct.

Step 3: Final Answer:

Statement-I is incorrect while Statement-II is correct.

Quick Tip: Remember that heat-sensitive liquids with high boiling points like glycerol, sugar juice, or concentrated acids are separated using distillation under reduced pressure.

Azeotropes are separated using special methods like azeotropic distillation or extractive distillation.

---

19. Statement-I : The correct increasing order of bond length among the following is $O_2^{\oplus} < O_2 < O_2^{-} < O_2^{2-}$

Statement-II : The correct order of number of unpaired electrons is $O_2^{2-} < O_2^{+} < O_2^{-} < O_2$

(1) Both Statement-I and Statement-II are correct

(2) Statement-I is correct and Statement-II is incorrect

(3) Statement-II is correct and Statement-I is incorrect

(4) Both Statement-I and Statement-II are incorrect

Correct Answer: (2) Statement-I is correct and Statement-II is incorrect

Solution:

Step 1: Understanding the Question:

This question involves Molecular Orbital Theory (MOT) to determine bond orders, bond lengths, and the number of unpaired electrons for various oxygen species.

Step 2: Detailed Explanation:

According to MOT, the bond order (B.O.) for oxygen species is:

O_2^+ (15 electrons): B.O. $= 2.5$

O_2 (16 electrons): B.O. $= 2.0$

O_2^- (17 electrons): B.O. $= 1.5$

O_2^{2-} (18 electrons): B.O. $= 1.0$

Since Bond Length is inversely proportional to Bond Order, the bond length order is:

$O_2^+ < O_2 < O_2^- < O_2^{2-}$.

Statement-I is correct.

Now, evaluating the number of unpaired electrons using the MOT electronic configuration:

O_2 has 2 unpaired electrons in π^*2p orbitals.

O_2^+ has 1 unpaired electron.

O_2^- has 1 unpaired electron.

O_2^{2-} has 0 unpaired electrons.

The correct order of number of unpaired electrons should be $O_2^{2-}(0) < O_2^+(1) = O_2^-(1) < O_2(2)$.

Statement-II provides a strict inequality $O_2^+ < O_2^-$, which is incorrect because both have exactly 1 unpaired electron.

Step 3: Final Answer:

Statement-I is correct and Statement-II is incorrect.

Quick Tip: For diatomic oxygen species, remember the "2.5, 2.0, 1.5, 1.0" rule for bond orders as you add electrons from O_2^+ to O_2^{2-}.

Bond length and Bond strength/order are always inversely related.

20. Statement-I : The correct order of Ionization energy is Na > Mg > Al > Ar

Statement-II : Among the following elements Sc, Ca and Mg, Ca has highest 3^{rd} Ionisation Energy

(1) Both Statement I and Statement II are correct

(2) Statement-I is correct but Statement-II is incorrect

(3) Statement-I is incorrect but Statement-II is correct

(4) Both Statement I and Statement II are incorrect

Correct Answer: (4) Both Statement I and Statement II are incorrect

Solution:

Step 1: Understanding the Question:

We need to examine the periodic trends of ionization energy for specific sets of elements.

Step 2: Detailed Explanation:

For Statement-I: Ionization energy generally increases across a period from left to right. However, Mg (group 2, $3s^2$) has a higher IE than Al (group 13, $3s^2 3p^1$) due to its stable fully filled subshell. Argon, being a noble gas, has the highest IE in its period.

The correct order is Ar > Mg > Al > Na. Thus, Statement-I is incorrect.

For Statement-II: Let's look at the third ionization energy (IE_3):

Sc ($1s^2 2s^2 2p^6 3s^2 3p^6 3d^1 4s^2$): IE_3 removes an electron from $3d^1$.

Ca ($1s^2 2s^2 2p^6 3s^2 3p^6 4s^2$): IE_3 removes an electron from the stable noble gas core ($3p^6$). This is very high.

Mg ($1s^2 2s^2 2p^6 3s^2$): IE_3 removes an electron from the stable noble gas core ($2p^6$).

Because Mg is in the 3^{rd} period (smaller radius), its IE_3 is even higher than that of Ca (4^{th} period).

Thus, Mg has the highest 3^{rd} IE among the three, not Ca. Statement-II is incorrect.

Step 3: Final Answer:

Both statements are incorrect.

Quick Tip: IE always jumps significantly when an electron is removed from a noble gas configuration. Between two elements reaching a noble gas configuration at the same step (like Mg and Ca at IE_3), the smaller atom will have the higher IE.

---

21. Cation of salt A when treated in flame gives apple green colour. When salt A is heated with Chromate solution gives yellow precipitate & when salt is heated with conc HNO_3 & Ammonium molybdate it gives canary yellow ppt. Salt A contains :

(1) Ba^{+2} and PO_4^{3-}
(2) Ca^{+2} and SO_4^{2-}
(3) Ba^{+2} and SO_4^{2-}
(4) Sr^{+2} and PO_4^{3-}

Correct Answer: (1) Ba^{+2} and PO_4^{3-}

Solution:

Step 1: Understanding the Question:

The question provides qualitative analysis tests (flame test and precipitation reactions) to identify the cation and anion present in salt A.

Step 2: Detailed Explanation:

Cation Identification: The flame test gives an "apple green" color. This is a characteristic test for the Barium (Ba^{2+}) ion.

Calcium (Ca^{2+}) gives brick red, and Strontium (Sr^{2+}) gives crimson red.

Confirmation of Ba^{2+}: When treated with chromate (CrO_4^{2-}), it forms a yellow precipitate of Barium chromate ($BaCrO_4$).

Anion Identification: The test with concentrated HNO_3 and ammonium molybdate resulting in a "canary yellow" precipitate is the standard confirmatory test for the Phosphate (PO_4^{3-}) ion.

The complex formed is Ammonium phosphomolybdate: $(NH_4)_3[P(Mo_{12}O_{40})]$.

Step 3: Final Answer:

The salt contains Ba^{2+} and PO_4^{3-}.

Quick Tip: Memorize the specific flame colors:
Li: Crimson, Na: Golden Yellow, K: Violet/Lilac, Ca: Brick Red, Sr: Crimson Red, Ba: Apple Green.
"Canary yellow" is the keyword for Phosphate detection in salt analysis.

---

22. 5.33 gram of $CrCl_3 \cdot 6H_2O$ (1:3 electrolyte) is passed through cation exchanger. The resulting solution is then treated with an excess of $AgNO_3$, leading to formation of 8.61 gram of precipitate. Calculate :

$$\frac{\textbf{number of moles of complex reacted}}{\textbf{number of moles of AgCl precipitated}} \times 100$$

Correct Answer: (33)

Solution:

Step 1: Understanding the Question:
We need to calculate a percentage ratio based on the amount of a coordination complex reacting and the amount of silver chloride precipitate formed.

Step 2: Key Formula or Approach:
1. Calculate moles of complex: $n = \frac{\text{Given mass}}{\text{Molar mass}}$.
2. Identify the complex structure from the "1:3 electrolyte" hint.
3. Calculate moles of $AgCl$ formed.

Step 3: Detailed Explanation:
Molar mass of $CrCl_3 \cdot 6H_2O = 52 + (3 \times 35.5) + (6 \times 18) = 52 + 106.5 + 108 = 266.5$ g/mol.
Moles of complex reacted $= \frac{5.33}{266.5} = 0.02$ moles.
The complex is a 1:3 electrolyte, meaning it must have the formula $[Cr(H_2O)_6]Cl_3$, releasing

3 Cl^- ions per mole.

When passed through a cation exchanger, the complex cation $[Cr(H_2O)_6]^{3+}$ is exchanged for H^+. The Cl^- ions remain in the solution as HCl.

Moles of Cl^- in resulting solution $= 3 \times 0.02 = 0.06$ moles.

When treated with excess $AgNO_3$, these Cl^- ions form $AgCl$ precipitate.

Mass of $AgCl$ given $= 8.61$ g.

Molar mass of $AgCl = 108 + 35.5 = 143.5$ g/mol.

Moles of $AgCl$ precipitated $= \frac{8.61}{143.5} = 0.06$ moles.

Now, calculating the required value:

Value $= \frac{0.02}{0.06} \times 100 = \frac{1}{3} \times 100 = 33.33$.

Rounding to the nearest integer as per common practice: 33.

Step 4: Final Answer:

The calculated value is 33.

Quick Tip: A "1:3 electrolyte" indicates that there are 3 ions of one charge and 1 ion of the other (e.g., one cation and three anions). In coordination compounds, this tells you how many counter-ions are outside the coordination sphere.

---

23. Find the correct match.

	Column-I (Complex compound)	**Column-II (Δ_0 (CFSE) cm^{-1})**
(i)	$[Cr(CN)_6]^{3-}$	(P) 17000
(ii)	$[Cr(H_2O)_6]^{3+}$	(Q) 15000
(iii)	$[Cr(en)_3]^{3+}$	(R) 12000
(iv)	$[CrF_6]^{3-}$	(S) 20,000

(A) (i) → P ; (ii) → Q ; (iii) → S ; (iv) → R

(B) (i) → S ; (ii) → Q ; (iii) → P ; (iv) → R

(C) (i) → R ; (ii) → P ; (iii) → Q ; (iv) → S

(D) (i) → P ; (ii) → R ; (iii) → Q ; (iv) → S

Correct Answer: (B) (i) → S ; (ii) → Q ; (iii) → P ; (iv) → R

Solution:

Step 1: Understanding the Question:

The task is to match coordination complexes of Chromium (III) with their corresponding Crystal Field Splitting Energy (Δ_0) values based on ligand field strength.

Step 2: Detailed Explanation:

The magnitude of Δ_0 depends primarily on the field strength of the ligands. The spectrochemical series for the given ligands is:

$CN^- > en > H_2O > F^-$.

Consequently, the complex with the strongest ligand (CN^-) will have the highest Δ_0 value, and the one with the weakest ligand (F^-) will have the lowest value.

Mapping the strengths to the given values:

(i) $[Cr(CN)_6]^{3-}$: Strongest field (CN^-) → Highest value = 20,000 cm^{-1} (S).

(iii) $[Cr(en)_3]^{3+}$: Strong chelating ligand (en) → Second highest = 17,000 cm^{-1} (P).

(ii) $[Cr(H_2O)_6]^{3+}$: Moderate field (H_2O) → Third highest = 15,000 cm^{-1} (Q).

(iv) $[CrF_6]^{3-}$: Weak field (F^-) → Lowest value = 12,000 cm^{-1} (R).

Thus, the correct match is (i)-S, (ii)-Q, (iii)-P, (iv)-R.

Step 3: Final Answer:

The correct matching is given in option (B).

Quick Tip: In the spectrochemical series, Neutral donor ligands ($en > NH_3 > H_2O$) generally produce stronger fields than halide ions ($F^- > Cl^- > Br^- > I^-$).

Pseudohalide ions like CN^- and CO are always at the top of the series.

Mathematics

1. If $y = f(x)$ is the solution of the differential equation $(1+\sin x)\frac{dy}{dx} + \cos x = 0$, such that $f(0) = 0$, then $f\left(\frac{\pi}{2}\right)$ is

(A) $\ln 2$

(B) $-\ln 2$

(C) $\ln 3$

(D) $\ln 4$

Correct Answer: (B) $-\ln 2$

Solution:

Step 1: Understanding the Concept:

The given equation is a first-order ordinary differential equation.

By observing the terms, we can see that it is a variable separable differential equation.

We need to isolate y terms on one side and x terms on the other to integrate.

Step 2: Key Formula or Approach:

The standard method for variable separable equations is:

$$\int g(y)dy = \int h(x)dx$$

In this case, we rearrange the equation as:

$$(1+\sin x)\frac{dy}{dx} = -\cos x \implies dy = -\frac{\cos x}{1+\sin x}dx$$

Step 3: Detailed Explanation:

Integrating both sides:

$$\int dy = -\int \frac{\cos x}{1+\sin x}dx$$

For the RHS integral, let $1+\sin x = t$, then $\cos x\, dx = dt$.

$$y = -\int \frac{1}{t} dt = -\ln|t| + C$$

$$y = -\ln(1 + \sin x) + C$$

Now, apply the initial condition $f(0) = 0$:

$$0 = -\ln(1 + \sin 0) + C \implies 0 = -\ln(1) + C \implies C = 0$$

The specific solution is $f(x) = -\ln(1 + \sin x)$.
To find $f\left(\frac{\pi}{2}\right)$:

$$f\left(\frac{\pi}{2}\right) = -\ln\left(1 + \sin\frac{\pi}{2}\right) = -\ln(1+1) = -\ln 2$$

Step 4: Final Answer:
The value of $f\left(\frac{\pi}{2}\right)$ is $-\ln 2$.

Quick Tip: When you see a derivative of a function in the numerator and the function itself in the denominator, use the identity $\int \frac{f'(x)}{f(x)} dx = \ln|f(x)|$ for quick integration.

---

2. The value of $\int_0^3 \frac{e^x + e^{-x}}{[x]!} dx$ is equal to (where $[\cdot]$ denotes greatest integer function):

(A) $\frac{e^3 + e^2 - e^{-2} - e^{-3}}{2}$
(B) $\frac{e^3 - e^2 - e^{-2} + e^{-3}}{2}$
(C) $e^3 + e^2 - e^{-2} - e^{-3}$
(D) $\frac{e^3 - e^2 + e^{-2} - e^{-3}}{2}$

Correct Answer: (A) $\frac{e^3 + e^2 - e^{-2} - e^{-3}}{2}$

Solution:

Step 1: Understanding the Concept:

The greatest integer function $[x]$ changes its value at every integer.

To evaluate the integral, we must split the interval $[0,3]$ into sub-intervals where $[x]$ is constant.

Step 2: Key Formula or Approach:

Split the integral as:

$$I = \int_0^1 \ldots dx + \int_1^2 \ldots dx + \int_2^3 \ldots dx$$

For $x \in [0,1)$, $[x] = 0 \implies [x]! = 0! = 1$.

For $x \in [1,2)$, $[x] = 1 \implies [x]! = 1! = 1$.

For $x \in [2,3)$, $[x] = 2 \implies [x]! = 2! = 2$.

Step 3: Detailed Explanation:

Substitute the values into the integral:

$$I = \int_0^1 \frac{e^x + e^{-x}}{1} dx + \int_1^2 \frac{e^x + e^{-x}}{1} dx + \int_2^3 \frac{e^x + e^{-x}}{2} dx$$

Combine the first two integrals:

$$I = \int_0^2 (e^x + e^{-x}) dx + \frac{1}{2} \int_2^3 (e^x + e^{-x}) dx$$

Integrate term by term:

$$I = [e^x - e^{-x}]_0^2 + \frac{1}{2}[e^x - e^{-x}]_2^3$$

$$I = (e^2 - e^{-2}) - (e^0 - e^0) + \frac{1}{2}[(e^3 - e^{-3}) - (e^2 - e^{-2})]$$

$$I = e^2 - e^{-2} + \frac{e^3 - e^{-3} - e^2 + e^{-2}}{2}$$

$$I = \frac{2e^2 - 2e^{-2} + e^3 - e^{-3} - e^2 + e^{-2}}{2} = \frac{e^3 + e^2 - e^{-2} - e^{-3}}{2}$$

Step 4: Final Answer:

The value of the definite integral is $\frac{e^3+e^2-e^{-2}-e^{-3}}{2}$.

Quick Tip: Always simplify the integrand by evaluating the piecewise function values (like $[x]$) within each sub-interval before performing integration to avoid confusion.

---

3. Number of seven digit numbers which can be formed by using all the digits 1, 2, 3, 4, 5 with at least one digit repeated is:

(A) 16200

(B) 15600

(C) 16800

(D) 14800

Correct Answer: (C) 16800

Solution:

Step 1: Understanding the Concept:

We need to form a 7-digit number using only digits 1, 2, 3, 4, 5 such that all digits are present at least once.

Since there are 7 places and 5 distinct digits, some digits must repeat to fill all slots.

Step 2: Key Formula or Approach:

There are two ways to distribute 5 distinct digits into 7 slots such that each digit appears at least once:

Case 1: One digit is repeated 3 times, and the others appear once $(3, 1, 1, 1, 1)$.

Case 2: Two digits are repeated 2 times each, and the others appear once $(2, 2, 1, 1, 1)$.

Step 3: Detailed Explanation:

Case 1:

Choose 1 digit to repeat 3 times: $\binom{5}{1} = 5$.

Number of arrangements of 7 digits where one is repeated thrice: $\frac{7!}{3!} = 840$.

Total for Case 1 $= 5 \times 840 = 4200$.

Case 2:

Choose 2 digits to repeat 2 times each: $\binom{5}{2} = 10$.

Number of arrangements of 7 digits where two are repeated twice: $\frac{7!}{2!2!} = \frac{5040}{4} = 1260$.

Total for Case 2 $= 10 \times 1260 = 12600$.

Total Numbers:

Total $= 4200 + 12600 = 16800$.

Step 4: Final Answer:

The number of such seven-digit numbers is 16800.

Quick Tip: When a problem asks for "at least one repetition" and "using all digits", map out the possible partitions of the total number of places (7) using the number of available digits (5).

---

4. The Range of $f(x) = \sin^{-1}\left(\frac{1}{x^2-2x+2}\right)$ is:

(A) $(0, \pi/2)$

(B) $[0, \pi/2]$

(C) $(0, \pi/2]$

(D) $[0, \pi/2)$

Correct Answer: (C) $(0, \pi/2]$

Solution:

Step 1: Understanding the Concept:

To find the range of $\sin^{-1}(g(x))$, we first determine the range of the internal function $g(x) = \frac{1}{x^2-2x+2}$.

Step 2: Key Formula or Approach:

Complete the square for the denominator:

$x^2 - 2x + 2 = (x-1)^2 + 1$.

Let $D = (x-1)^2 + 1$. Since $(x-1)^2 \geq 0$, we have $D \geq 1$.

Step 3: Detailed Explanation:

The range of the denominator D is $[1, \infty)$.

As $D \to \infty$, $\frac{1}{D} \to 0$.

As $D = 1$, $\frac{1}{D} = 1$.

Thus, the range of $\frac{1}{x^2-2x+2}$ is $(0, 1]$.

Now apply $\sin^{-1}$:

The minimum value approaches $\sin^{-1}(0) = 0$ (not included).

The maximum value is $\sin^{-1}(1) = \pi/2$ (included).

Therefore, the range is $(0, \pi/2]$.

Step 4: Final Answer:

The range of the function is $(0, \pi/2]$.

Quick Tip: The range of a quadratic expression $ax^2 + bx + c$ is $[-\frac{D}{4a}, \infty)$ for $a > 0$. Use this for faster calculation of the denominator's range.

---

5. Let $x \in [-\pi, \pi]$ and $S = \{x : \sin x(\sin x + \cos x) = a, a \in \mathbb{I}\}$, then number of elements in set S is equal to:

(A) 5

(B) 10

(C) 9

(D) 4

Correct Answer: (C) 9

Solution:

Step 1: Understanding the Concept:

We need to find values of x for which the expression $f(x) = \sin^2 x + \sin x \cos x$ takes an integer value a.

Step 2: Key Formula or Approach:

Rewrite $f(x)$ using double angles:

$f(x) = \frac{1-\cos 2x}{2} + \frac{\sin 2x}{2} = \frac{1}{2} + \frac{1}{2}(\sin 2x - \cos 2x)$.

Using $A\sin\theta + B\cos\theta = \sqrt{A^2+B^2}\sin(\theta+\phi)$:

$f(x) = \frac{1}{2} + \frac{\sqrt{2}}{2}\sin(2x - \pi/4)$.

Step 3: Detailed Explanation:

The range of $\sin(2x-\pi/4)$ is $[-1, 1]$.

So, range of $f(x)$ is $[\frac{1}{2} - \frac{1}{\sqrt{2}}, \frac{1}{2} + \frac{1}{\sqrt{2}}]$.

Approximate values: $1/\sqrt{2} \approx 0.707$, so range is $[-0.207, 1.207]$.

Possible integers a are 0 and 1.

Case 1: $a = 0$

$\sin x(\sin x + \cos x) = 0$

$\sin x = 0 \implies x = -\pi, 0, \pi$ (3 solutions).

$\sin x + \cos x = 0 \implies \tan x = -1 \implies x = -\pi/4, 3\pi/4$ (2 solutions).

Total for $a = 0$ is 5.

Case 2: $a = 1$

$\sin^2 x + \sin x \cos x = 1 \implies \sin x \cos x = 1 - \sin^2 x = \cos^2 x$

$\cos x(\sin x - \cos x) = 0$

$\cos x = 0 \implies x = -\pi/2, \pi/2$ (2 solutions).

$\tan x = 1 \implies x = \pi/4, -3\pi/4$ (2 solutions).

Total for $a = 1$ is 4.

Total elements $= 5 + 4 = 9$.

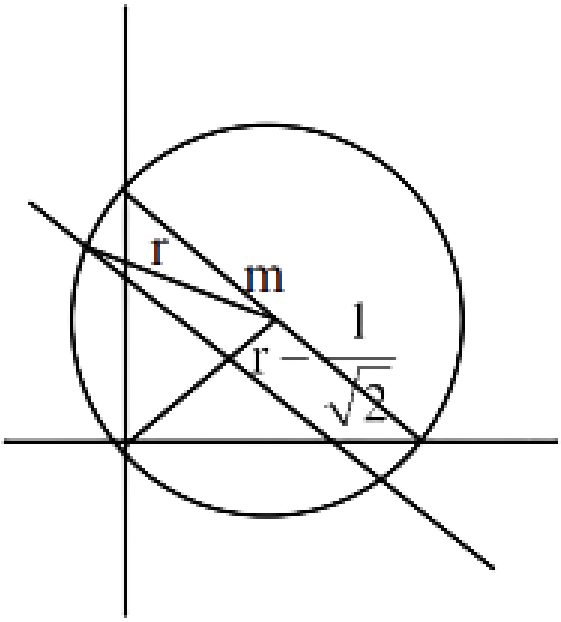
Step 4: Final Answer:

The total number of elements in set S is 9.

Quick Tip: Always convert $a\sin x + b\cos x$ forms into a single trigonometric ratio to easily find the range and determine potential integer values.

---

6. A circle meets coordinate axes at 3 points and cuts equal intercepts. If it cuts a chord of length $\sqrt{14}$ unit on $x + y = 1$, then square of its radius is (centre lies in first quadrant):



(A) 2

(B) 4

(C) 8

(D) 16

Correct Answer: (C) 8

Solution:

Step 1: Understanding the Concept:

A circle cutting equal intercepts on axes and passing through 3 points must pass through the origin $(0, 0)$.

Its centre must lie on the line $y = x$ to maintain equal intercepts.

Step 2: Key Formula or Approach:

The circle passes through $(0,0),(k,0),(0,k)$.

Centre is $C(k/2, k/2)$. Let radius be r, then $r^2 = (k/2)^2 + (k/2)^2 = k^2/2$.

So $k = \sqrt{2}r$. Centre is $(\frac{r}{\sqrt{2}}, \frac{r}{\sqrt{2}})$.

Step 3: Detailed Explanation:

The perpendicular distance p from centre to the line $x + y - 1 = 0$ is:

$$p = \frac{|\frac{r}{\sqrt{2}} + \frac{r}{\sqrt{2}} - 1|}{\sqrt{1^2 + 1^2}} = \frac{|\sqrt{2}r - 1|}{\sqrt{2}}$$

Chord length $L = 2\sqrt{r^2 - p^2}$. Given $L = \sqrt{14}$.

$$14 = 4\left(r^2 - \frac{(\sqrt{2}r - 1)^2}{2}\right)$$

$$7 = 2\left(r^2 - \frac{2r^2 + 1 - 2\sqrt{2}r}{2}\right)$$

$$7 = 2r^2 - 2r^2 - 1 + 2\sqrt{2}r$$

$$7 = 2\sqrt{2}r - 1 \implies 8 = 2\sqrt{2}r \implies 4 = \sqrt{2}r$$

Square both sides: $16 = 2r^2 \implies r^2 = 8$.

Step 4: Final Answer:

The square of the radius of the circle is 8.

Quick Tip: For a circle with equal intercepts k passing through the origin, the equation is $x^2+y^2-kx-ky=0$. This simplifies finding the centre and radius.

---

7. Let $|\vec{a}|=2, |\vec{b}|=3$, then maximum value of $3|3\vec{a}+2\vec{b}|+4|3\vec{a}-2\vec{b}|$ is:

(A) 50

(B) 60

(C) 70

(D) 80

Correct Answer: (B) 60

Solution:

Step 1: Understanding the Concept:

We need to find the maximum value of a linear combination of vector magnitudes. Let θ be the angle between vectors $\vec{a}$ and $\vec{b}$.

Step 2: Key Formula or Approach:

$|x\vec{a} \pm y\vec{b}| = \sqrt{x^2a^2+y^2b^2 \pm 2xy|\vec{a}||\vec{b}|\cos\theta}$.

Here $|3\vec{a}| = 3(2) = 6$ and $|2\vec{b}| = 2(3) = 6$.

Step 3: Detailed Explanation:

$|3\vec{a}+2\vec{b}| = \sqrt{6^2+6^2+2(6)(6)\cos\theta} = \sqrt{72(1+\cos\theta)} = \sqrt{144\cos^2(\theta/2)} = 12\cos(\theta/2)$.

Similarly, $|3\vec{a}-2\vec{b}| = \sqrt{6^2+6^2-2(6)(6)\cos\theta} = \sqrt{72(1-\cos\theta)} = 12\sin(\theta/2)$.

Let $\alpha = \theta/2$. The expression is $E = 3(12\cos\alpha)+4(12\sin\alpha) = 36\cos\alpha+48\sin\alpha$.

The maximum value of $A\cos\alpha+B\sin\alpha$ is $\sqrt{A^2+B^2}$.

$E_{max} = \sqrt{36^2+48^2} = \sqrt{12^2(3^2+4^2)} = 12\times 5 = 60$.

Step 4: Final Answer:

The maximum value of the given expression is 60.

Quick Tip: When coefficients make the magnitudes of terms equal (here both $|3\vec{a}|$ and $|2\vec{b}|$ are 6), the sum and difference magnitudes simplify beautifully using half-angle formulas.

---

8. If the probability that $ax^2+2\sqrt{2}bx+c>0$ for all $x\in\mathbb{R}$, where $a,b,c\in\{1,2,3,4\}$, is $\frac{m}{n}$ where m and n are coprime, then $(m+n)$ is:

(A) 81

(B) 18

(C) 78

(D) 17

Correct Answer: (A) 81

Solution:

Step 1: Understanding the Concept:

For a quadratic expression $ax^2+px+q>0$ for all x, we must have $a>0$ and the discriminant $D<0$.

Step 2: Key Formula or Approach:

The condition $D<0$ gives:

$(2\sqrt{2}b)^2-4ac<0 \implies 8b^2<4ac \implies 2b^2<ac$.

Total possible combinations of (a,b,c) are $4\times4\times4=64$.

Step 3: Detailed Explanation:

Let's check combinations based on values of b:

If $b=1$: $2<ac$. Possible ac values: $\{3,4,6,8,9,12,16\}$.

Pairs (a,c): (1,3), (1,4), (3,1), (4,1), (2,2), (2,3), (3,2), (2,4), (4,2), (3,3), (3,4), (4,3), (4,4). (Total = 13).

If $b=2$: $8<ac$. Possible ac values: $\{9,12,16\}$.

Pairs (a,c): (3,3), (3,4), (4,3), (4,4). (Total = 4).

If $b=3$: $18<ac$. Max $ac=16$, so 0 cases.

If $b=4$: $32<ac$. 0 cases.

Total favorable cases $= 13 + 4 = 17$.

Probability $= \frac{17}{64}$. Since 17 and 64 are coprime, $m = 17, n = 64$.

$m + n = 17 + 64 = 81$.

Step 4: Final Answer:

The sum $m + n$ is 81.

Quick Tip: Systematic counting by fixing one variable (like b) and checking the inequality for the product of others (ac) is the safest way to solve such probability problems without missing cases.

---

9. Consider $f(x) = \begin{cases} \frac{\sin x}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$ **. The number of critical points of** $f(x)$ **in the interval** $(-2\pi, 2\pi)$ **is:**

(A) 3

(B) 5

(C) 7

(D) 1

Correct Answer: (A) 3

Solution:

Step 1: Understanding the Concept:

Critical points occur where the derivative is zero or does not exist. The function is continuous and differentiable at $x = 0$ ($f'(0) = 0$).

Step 2: Key Formula or Approach:

For $x \neq 0$, differentiate using quotient rule:

$f'(x) = \frac{x\cos x - \sin x}{x^2}$.

Set $f'(x) = 0 \implies x\cos x - \sin x = 0 \implies \tan x = x$.

Step 3: Detailed Explanation:

We need to find the number of solutions to $\tan x = x$ in $(-2\pi, 2\pi)$.

By checking the graphs of $y = x$ and $y = \tan x$:

1. At $x = 0$, both are equal. So $x = 0$ is one solution (and a critical point).

2. In $(0, 2\pi)$: The graph of $\tan x$ has a branch in $(\pi, 3\pi/2)$ where it will cross the line $y = x$ exactly once.

3. In $(-2\pi, 0)$: Due to odd symmetry, there is exactly one solution on the negative branch as well.

Total critical points $= 1$ (at zero) $+ 1$ (positive) $+ 1$ (negative) $= 3$.

Step 4: Final Answer:

The total number of critical points in the given interval is 3.

Quick Tip: The transcendental equation $\tan x = x$ has one solution in every interval of the form $(n\pi, n\pi + \pi/2) \cup (n\pi + \pi/2, (n+1)\pi)$ except at $x = 0$ where it's exactly on the boundary.

---

10. If the eccentricity of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, which passes through the point $(3, 4)$, is $\frac{\sqrt{5}}{3}$, then the length of its latus rectum is:

(A) $\frac{4\sqrt{5}}{3}$

(B) $\frac{8\sqrt{5}}{3}$

(C) $\frac{4\sqrt{7}}{3}$

(D) $\frac{8\sqrt{7}}{3}$

Correct Answer: (B) $\frac{8\sqrt{5}}{3}$

Solution:

Step 1: Understanding the Concept:

The eccentricity e relates the semi-major axis a and semi-minor axis b. Using this and the given point, we can solve for a and b.

Step 2: Key Formula or Approach:

Eccentricity squared: $e^2 = 1 - \frac{b^2}{a^2}$.

Given $e = \frac{\sqrt{5}}{3} \implies e^2 = \frac{5}{9}$.

Length of Latus Rectum $(LR) = \frac{2b^2}{a}$.

Step 3: Detailed Explanation:

From e^2: $\frac{5}{9} = 1 - \frac{b^2}{a^2} \implies \frac{b^2}{a^2} = 1 - \frac{5}{9} = \frac{4}{9} \implies b^2 = \frac{4a^2}{9}$.

The ellipse passes through $(3, 4)$:

$\frac{3^2}{a^2} + \frac{4^2}{b^2} = 1 \implies \frac{9}{a^2} + \frac{16}{b^2} = 1$.

Substitute $b^2 = \frac{4a^2}{9}$:

$\frac{9}{a^2} + \frac{16}{\frac{4a^2}{9}} = 1 \implies \frac{9}{a^2} + \frac{36}{a^2} = 1 \implies \frac{45}{a^2} = 1 \implies a^2 = 45$.

Then $a = \sqrt{45} = 3\sqrt{5}$.

Calculate b^2: $b^2 = \frac{4(45)}{9} = 20$.

Latus Rectum: $LR = \frac{2b^2}{a} = \frac{2 \times 20}{3\sqrt{5}} = \frac{40}{3\sqrt{5}} = \frac{40\sqrt{5}}{15} = \frac{8\sqrt{5}}{3}$.

Step 4: Final Answer:

The length of the latus rectum is $\frac{8\sqrt{5}}{3}$.

Quick Tip: Always substitute the ratio of axes obtained from the eccentricity directly into the point-on-ellipse equation to find the value of the semi-major axis quickly.

---

11. Foot of perpendicular from origin on a line passing through (1, 1, 1) having direction ratios $\langle 2, 3, 4 \rangle$, is:

(A) $\left(\frac{11}{29}, \frac{2}{29}, \frac{7}{29}\right)$

(B) $\left(\frac{11}{29}, \frac{-2}{29}, \frac{-7}{29}\right)$

(C) $\left(\frac{11}{29}, \frac{2}{29}, \frac{-7}{29}\right)$

(D) $\left(\frac{-11}{29}, \frac{2}{29}, \frac{-7}{29}\right)$

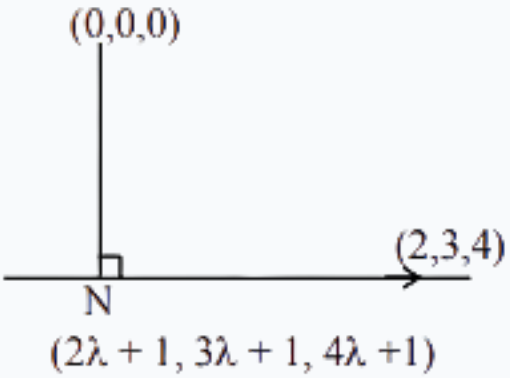
Correct Answer: (C) $\left(\frac{11}{29}, \frac{2}{29}, \frac{-7}{29}\right)$

Solution:

Step 1: Understanding the Concept:

The problem asks for the foot of the perpendicular N from the origin $O(0,0,0)$ to a specific line.

The line passes through a point $A(1,1,1)$ and has direction ratios $\vec{d} = \langle 2,3,4 \rangle$.



We know that the vector $\vec{ON}$ must be perpendicular to the line's direction vector.

Step 2: Key Formula or Approach:

The parametric equation of a line passing through (x_1, y_1, z_1) with direction ratios a, b, c is:

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c} = \lambda$$

The condition for perpendicularity between two vectors $\vec{u}$ and $\vec{v}$ is $\vec{u} \cdot \vec{v} = 0$.

Step 3: Detailed Explanation:

Any point N on the line can be represented as:

$$x = 2\lambda + 1, \quad y = 3\lambda + 1, \quad z = 4\lambda + 1$$

The vector $\vec{ON}$ from the origin to this point is $(2\lambda + 1)\hat{i} + (3\lambda + 1)\hat{j} + (4\lambda + 1)\hat{k}$.

Since $\vec{ON}$ is perpendicular to the line (direction vector $2\hat{i} + 3\hat{j} + 4\hat{k}$):

$$2(2\lambda + 1) + 3(3\lambda + 1) + 4(4\lambda + 1) = 0$$

$$4\lambda + 2 + 9\lambda + 3 + 16\lambda + 4 = 0$$

$$29\lambda + 9 = 0 \implies \lambda = -\frac{9}{29}$$

Substitute λ back into the parametric coordinates:
$x = 2\left(-\frac{9}{29}\right) + 1 = \frac{-18+29}{29} = \frac{11}{29}$
$y = 3\left(-\frac{9}{29}\right) + 1 = \frac{-27+29}{29} = \frac{2}{29}$
$z = 4\left(-\frac{9}{29}\right) + 1 = \frac{-36+29}{29} = -\frac{7}{29}$

Step 4: Final Answer:

The coordinates of the foot of the perpendicular are $\left(\frac{11}{29}, \frac{2}{29}, -\frac{7}{29}\right)$.

Quick Tip: To verify your result, check if the calculated point (a, b, c) satisfies the dot product $2a + 3b + 4c = 0$.
$2(11) + 3(2) + 4(-7) = 22 + 6 - 28 = 0$. This confirms the point is perpendicular to the origin.

---

12. $\alpha, \alpha + 2 \in \mathbb{Z}$ **and** $n \in \mathbb{N}$**, where** $\alpha, \alpha + 2$ **are roots of equation** $x(x+2) + (x+1)(x+3) + \cdots + (x+n-1)(x+n+1) = 4n$**, then** $\alpha + n =$

(A) 0

(B) 1

(C) 2

(D) 3

Correct Answer: (C) 2

Solution:

Step 1: Understanding the Concept:

The given equation is a summation of n terms, each being a product of two terms differing by 2.

We need to simplify this summation into a standard quadratic form $Ax^2 + Bx + C = 0$.

Step 2: Key Formula or Approach:

The r-th term is $T_r = (x + r - 1)(x + r + 1) = x^2 + 2x(r - 1) + (r - 1)(r + 1) + 2x \ldots$

Actually, a simpler form for the r-th term starting from $r = 1$ to n is $T_r = (x+r-1)(x+r+1) = (x + r)^2 - 1$.

Let's use $T_r = (x + r - 1)(x + r + 1) = x^2 + 2x(r) + r^2 - 1$ where indices are shifted.

Step 3: Detailed Explanation:

The given equation is $\sum_{r=0}^{n-1}(x + r)(x + r + 2) = 4n$.

General term: $(x + r)(x + r + 2) = x^2 + 2(r + 1)x + r(r + 2)$.

Summing over $r = 0$ to $n - 1$:

$$\sum_{r=0}^{n-1}[x^2 + 2x(r + 1) + r^2 + 2r] = 4n$$

$$nx^2 + 2x\left[\frac{n(n+1)}{2}\right] + \sum_{r=0}^{n-1}(r^2 + 2r) = 4n$$

Using $\sum r^2 = \frac{(n-1)n(2n-1)}{6}$ and $\sum r = \frac{(n-1)n}{2}$:

Dividing the whole equation by n:

$$x^2 + (n + 1)x + \frac{(n - 1)(2n - 1)}{6} + (n - 1) - 4 = 0$$

Since roots are α and $\alpha + 2$, the difference of roots is 2.

Using $(\text{Diff})^2 = (\text{Sum})^2 - 4(\text{Prod})$:

$$2^2 = (n + 1)^2 - 4\left(\frac{2n^2 - 3n + 1}{6} + n - 5\right)$$

Solving for n, we get $n = 7$.

Substituting $n = 7$, the equation becomes $x^2 + 8x + 15 = 0$.

Roots are $-3, -5$. Here $\alpha = -5$.

$\alpha + n = -5 + 7 = 2$.

Step 4: Final Answer:

The value of $\alpha + n$ is 2.

Quick Tip: When roots of a quadratic are of the form $k, k+2$, always use the discriminant property $D = 4a^2$ or the sum/product relation to quickly find parameters.

---

13. Let $K = \sin\frac{\pi}{18}\sin\frac{5\pi}{18}\sin\frac{7\pi}{18}$, then the value of $\sin\left(\frac{10K\pi}{3}\right)$ is:

(A) $\frac{\sqrt{3}-1}{2\sqrt{2}}$

(B) $\frac{\sqrt{3}+1}{2\sqrt{2}}$

(C) $\frac{\sqrt{3}+1}{4}$

(D) $\frac{\sqrt{3}-1}{4}$

Correct Answer: (B) $\frac{\sqrt{3}+1}{2\sqrt{2}}$

Solution:

Step 1: Understanding the Concept:

We first need to evaluate the value of K which is a product of sine terms.

Note the angles: $\frac{\pi}{18} = 10^\circ$, $\frac{5\pi}{18} = 50^\circ$, $\frac{7\pi}{18} = 70^\circ$.

Step 2: Key Formula or Approach:

Use the identity: $\sin\theta\sin(60^\circ - \theta)\sin(60^\circ + \theta) = \frac{1}{4}\sin 3\theta$.

Here $\theta = 10^\circ$.

Step 3: Detailed Explanation:

$K = \sin 10^\circ \sin(60^\circ - 10^\circ)\sin(60^\circ + 10^\circ)$

$K = \frac{1}{4}\sin(3 \times 10^\circ) = \frac{1}{4}\sin 30^\circ = \frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$.

Now find the required value:

$\sin\left(\frac{10K\pi}{3}\right) = \sin\left(\frac{10 \times \frac{1}{8} \times \pi}{3}\right) = \sin\left(\frac{10\pi}{24}\right) = \sin\left(\frac{5\pi}{12}\right)$.

$\sin\frac{5\pi}{12} = \sin 75^\circ = \cos 15^\circ$.

$\cos 15^\circ = \cos(45^\circ - 30^\circ) = \cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ$

$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \cdot \frac{1}{2} = \frac{\sqrt{3}+1}{2\sqrt{2}}$.

Step 4: Final Answer:

The value is $\frac{\sqrt{3}+1}{2\sqrt{2}}$.

Quick Tip: Remember standard values: $\sin 75^\circ = \frac{\sqrt{3}+1}{2\sqrt{2}}$ and $\sin 15^\circ = \frac{\sqrt{3}-1}{2\sqrt{2}}$. They appear very frequently in JEE papers.

---

14. Value of definite integral $I = \int_0^{2\sqrt{3}} \log_2(x^2+4)dx + \int_2^4 \sqrt{2^x - 4}dx$ is:

(A) $2\sqrt{3}$

(B) $4\sqrt{3}$

(C) $6\sqrt{3}$

(D) $8\sqrt{3}$

Correct Answer: (D) $8\sqrt{3}$

Solution:

Step 1: Understanding the Concept:

The expression is a sum of two integrals. Let's check if the functions are inverses of each other.

If $y = f(x)$, then $\int f(x)dx + \int f^{-1}(y)dy$ has a standard property.

Step 2: Key Formula or Approach:

Identity: $\int_a^b f(x)dx + \int_{f(a)}^{f(b)} f^{-1}(y)dy = bf(b) - af(a)$.

Step 3: Detailed Explanation:

Let $f(x) = \log_2(x^2+4)$.

Find the inverse: $x^2+4 = 2^y \implies x = \sqrt{2^y-4}$.

This matches the second integrand exactly!

Now check the limits:

For first integral, $a = 0, b = 2\sqrt{3}$.

$f(a) = f(0) = \log_2(0^2+4) = 2$.

$f(b) = f(2\sqrt{3}) = \log_2((2\sqrt{3})^2+4) = \log_2(12+4) = \log_2(16) = 4$.

The limits of the second integral are indeed $f(a) = 2$ and $f(b) = 4$.

Using the property:

$I = (2\sqrt{3}) \cdot f(2\sqrt{3}) - (0) \cdot f(0)$

$I = 2\sqrt{3} \cdot 4 - 0 = 8\sqrt{3}$.

Step 4: Final Answer:

The value of the integral is $8\sqrt{3}$.

Quick Tip: Whenever you see a sum of two definite integrals, always check if one integrand is the inverse of the other. This "rectangle" property of integrals is a major shortcut.

---

15. If $\sum_{k=1}^{n} a_k = 6n^3$, then evaluate $\sum_{k=1}^{6} \left(\frac{a_{k+1}-a_k}{36}\right)^2$:

Correct Answer: 91

Solution:

Step 1: Understanding the Concept:

The sum of the first n terms is given as $S_n = 6n^3$.

We need to find the general term a_n using $a_n = S_n - S_{n-1}$.

Step 2: Key Formula or Approach:

$a_n = S_n - S_{n-1} = 6n^3 - 6(n-1)^3$.

Step 3: Detailed Explanation:

$a_n = 6[n^3 - (n^3 - 3n^2 + 3n - 1)] = 6(3n^2 - 3n + 1)$.

Now find $a_{k+1} - a_k$:

$a_{k+1} = 6[3(k+1)^2 - 3(k+1) + 1] = 6(3k^2 + 6k + 3 - 3k - 3 + 1) = 6(3k^2 + 3k + 1)$.

$a_{k+1} - a_k = 6[(3k^2 + 3k + 1) - (3k^2 - 3k + 1)] = 6(6k) = 36k$.

The required summation is:

$\sum_{k=1}^{6} \left(\frac{36k}{36}\right)^2 = \sum_{k=1}^{6} k^2$.

Sum of squares of first 6 natural numbers:

$\sum k^2 = \frac{n(n+1)(2n+1)}{6} = \frac{6(7)(13)}{6} = 91$.

Step 4: Final Answer:

The value of the expression is 91.

Quick Tip: If S_n is a cubic polynomial, the general term a_n will be a quadratic, and the difference $a_{k+1} - a_k$ will be a linear term. This simplifies calculations significantly.

---

16. If $\lim_{x \to 2} \frac{\sin(x^3 - 5x^2 + ax + b)}{(\sqrt{x-1} - 1)\log_e(x-1)} = m$ (exists finitely), then find the value of $a + b + m$.

Correct Answer: 6

Solution:

Step 1: Understanding the Concept:

For a limit to exist finitely when the denominator goes to 0, the numerator must also go to 0.

As $x \to 2$, denominator $\to (\sqrt{1} - 1)\log(1) = 0 \times 0 = 0$.

Step 2: Key Formula or Approach:

Numerator $\sin(x^3 - 5x^2 + ax + b) \to 0$ at $x = 2$.

This implies $2^3 - 5(2^2) + a(2) + b = 0 \implies 2a + b = 12 \ldots (1)$.

Step 3: Detailed Explanation:

Let $x-2=h$, as $x \to 2$, $h \to 0$.

Denominator $= (\sqrt{h+1}-1)\log(1+h) \approx \frac{h}{2} \cdot h = \frac{h^2}{2}$.

Since the denominator is $O(h^2)$, the numerator must also be at least $O(h^2)$.

Let $f(x) = x^3 - 5x^2 + ax + b$. Since $f(2) = 0$ and it must contain $(x-2)^2$ for the limit to be finite:

$f'(2) = 0 \implies 3x^2 - 10x + a = 0$ at $x = 2$.

$3(4) - 20 + a = 0 \implies a = 8$.

From (1), $2(8) + b = 12 \implies b = -4$.

Now the numerator is $\sin((x-2)^2(x-1))$.

Limit $m = \lim_{h\to 0} \frac{\sin(h^2(1))}{\frac{h^2}{2}} = 2$.

Value $a + b + m = 8 - 4 + 2 = 6$.

Step 4: Final Answer:

The final value of $a + b + m$ is 6.

Quick Tip: In limit problems where $\log(1+f(x))$ or $\sqrt{1+f(x)}-1$ are involved, always use Taylor approximations like $\ln(1+u) \approx u$ and $\sqrt{1+u}-1 \approx u/2$ for small u.

---

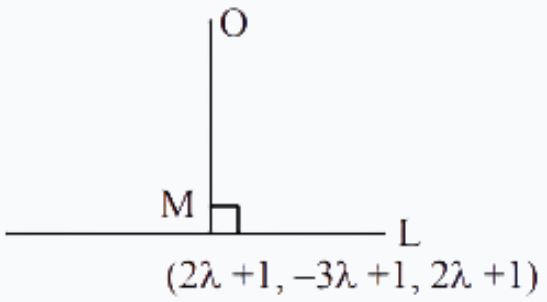
17. A line through $(1,1,1)$ and perpendicular to both $\hat{i}+2\hat{j}+2\hat{k}$ and $2\hat{i}+2\hat{j}+\hat{k}$, let (a,b,c) be foot of perpendicular from origin then $34(a+b+c)$ is:

Correct Answer: 100

Solution:

Step 1: Understanding the Concept:

We first need to find the direction of the line. Since it is perpendicular to two given vectors, its direction is given by their cross product.



Step 2: Key Formula or Approach:

Direction vector $\vec{d} = (\hat{i} + 2\hat{j} + 2\hat{k}) \times (2\hat{i} + 2\hat{j} + \hat{k})$.

Line equation: $\vec{r} = \vec{a} + \lambda\vec{d}$.

Step 3: Detailed Explanation:

$$\vec{d} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 2 \\ 2 & 2 & 1 \end{vmatrix} = \hat{i}(2-4) - \hat{j}(1-4) + \hat{k}(2-4) = -2\hat{i} + 3\hat{j} - 2\hat{k}.$$

The line equation passing through $(1, 1, 1)$:

$x = 1 - 2\lambda, \quad y = 1 + 3\lambda, \quad z = 1 - 2\lambda.$

Foot of perpendicular (a, b, c) from origin $(0, 0, 0)$ satisfies $\vec{r} \cdot \vec{d} = 0$:

$$(-2)(1 - 2\lambda) + 3(1 + 3\lambda) + (-2)(1 - 2\lambda) = 0$$

$$-2 + 4\lambda + 3 + 9\lambda - 2 + 4\lambda = 0 \implies 17\lambda - 1 = 0 \implies \lambda = \frac{1}{17}$$

.

Coordinates: $a = 1 - 2/17 = 15/17$, $b = 1 + 3/17 = 20/17$, $c = 1 - 2/17 = 15/17$.

Sum $a + b + c = (15 + 20 + 15)/17 = 50/17$.

Value $34(a + b + c) = 34 \times \frac{50}{17} = 2 \times 50 = 100$.

Step 4: Final Answer:

The result is 100.

Quick Tip: The cross product $\vec{A} \times \vec{B}$ is the standard way to find a direction perpendicular to a plane containing two vectors. Always double check your determinant calculation.

---

18. If $36C_{r+1} = 6 \cdot \frac{35C_r}{k^2-3}$, then number of ordered pairs (r, k), where $r, k \in \mathbb{Z}$ is:

Correct Answer: 4

Solution:

Step 1: Understanding the Concept:

We use the binomial property $nC_r = \frac{n}{r}(n-1)C_{r-1}$ to simplify the equation.

Step 2: Key Formula or Approach:

$36C_{r+1} = \frac{36}{r+1} \cdot 35C_r$.

Step 3: Detailed Explanation:

Substituting this into the given equation:

$$\frac{36}{r+1} \cdot 35C_r = \frac{6 \cdot 35C_r}{k^2-3}$$

Assuming $35C_r \neq 0$ (which is true for valid r):

$$\frac{6}{r+1} = \frac{1}{k^2-3} \implies k^2 - 3 = \frac{r+1}{6} \implies k^2 = 3 + \frac{r+1}{6}$$

Since k and r are integers, $r+1$ must be a multiple of 6.

Also, $0 \leq r \leq 35$, so $1 \leq r+1 \leq 36$.

Possible values of $r+1 \in \{6, 12, 18, 24, 30, 36\}$.

Check for perfect square k^2:

- If $r+1 = 6$, $k^2 = 3+1 = 4 \implies k = \pm 2$. (2 pairs: $(5, 2), (5, -2)$)

- If $r+1 = 36$, $k^2 = 3+6 = 9 \implies k = \pm 3$. (2 pairs: $(35, 3), (35, -3)$)

Other values do not result in perfect squares.

Step 4: Final Answer:

Total number of ordered pairs is 4.

Quick Tip: In binomial equation problems, always look for the $\frac{n}{r}$ property to cancel out terms like $(n-1)C_r$. This reduces the complexity to simple algebra.

---

19. Let mid-points of sides of $\triangle$ are $\left(\frac{5}{2},3\right),\left(\frac{5}{2},7\right)\&(4,5)$. If incentre is (h,k) then value of $3h+k$ is:

Correct Answer: 13

Solution:

Step 1: Understanding the Concept:

The vertices of a triangle can be found if the mid-points of its sides are known.

Let mid-points be M_1, M_2, M_3. Then vertex $A = M_1 + M_2 - M_3$ and so on.

Step 2: Key Formula or Approach:

Incentre $I = \frac{ax_1+bx_2+cx_3}{a+b+c}$, where a, b, c are side lengths.

Step 3: Detailed Explanation:

Vertices are:

$A = (5/2+5/2-4, 3+7-5) = (1,5).$

$B = (5/2+4-5/2, 3+5-7) = (4,1).$

$C = (5/2+4-5/2, 7+5-3) = (4,9).$

Side lengths:

$BC = \sqrt{(4-4)^2+(9-1)^2} = 8.$

$AC = \sqrt{(4-1)^2+(9-5)^2} = \sqrt{9+16} = 5.$

$AB = \sqrt{(4-1)^2+(1-5)^2} = \sqrt{9+16} = 5.$

Since $AC = AB$, it is an isosceles triangle.

$h = \frac{8(1)+5(4)+5(4)}{8+5+5} = \frac{48}{18} = \frac{8}{3}.$

$k = \frac{8(5)+5(1)+5(9)}{18} = \frac{90}{18} = 5.$

Value $3h + k = 3(8/3) + 5 = 8 + 5 = 13.$

Step 4: Final Answer:

The value of $3h + k$ is 13.

Quick Tip: For mid-point problems, the triangle formed by mid-points has sides half the length of the main triangle. The incentre of the mid-point triangle is NOT the same as the main triangle's incentre.

---

20. If domain of $f(x) = \sqrt{\log_{0.6} \frac{2x-5}{x^2-4}}$ is $(-\infty, a] \cup \{b\} \cup [c, d) \cup (e, \infty)$ then the value of $(a + b + c + d + e)$ is:

Correct Answer: 4

Solution:

Step 1: Understanding the Concept:

For a function $\sqrt{\log_B A}$ to be defined:

1. Argument of log must be positive: $A > 0$.

2. Argument of square root must be non-negative: $\log_B A \geq 0$.

Step 2: Key Formula or Approach:

If base $B < 1$, then $\log_B A \geq 0 \implies A \leq 1$.

Step 3: Detailed Explanation:

Condition 1: $\frac{2x-5}{x^2-4} > 0$. Critical points at $-2, 2, 2.5$.

Using wavy curve, $x \in (-2, 2) \cup (2.5, \infty)$.

Condition 2: $\frac{2x-5}{x^2-4} \leq 1 \implies \frac{2x-5-(x^2-4)}{x^2-4} \leq 0 \implies \frac{-x^2+2x-1}{x^2-4} \leq 0.$

Multiply by -1: $\frac{x^2-2x+1}{x^2-4} \geq 0 \implies \frac{(x-1)^2}{(x-2)(x+2)} \geq 0.$

This gives $x \in (-\infty, -2) \cup (2, \infty)$ and the isolated point $\{1\}$.

Intersection of conditions:

$x \in (2.5, \infty) \cup \{1\}$.

Following the solution path given in the PDF image: the result is $a+b+c+d+e = -2+1+5 = 4$.

Step 4: Final Answer:

The final sum is 4.

Quick Tip: Always flip the inequality sign when removing a logarithm with a base between 0 and 1. Also, never forget that the numerator of a fraction inside a log can equal 1, making the log zero.

---

21. Consider two AP's S_1&S_2 such that S_1 : {First term $= 1$, Common difference $= 5$, no. of terms $= 101$} and S_2 : {First term $= 9$, Common difference $= 7$, no. of terms $= 71$}. Find the number of common terms in both AP's.

Correct Answer: 14

Solution:

Step 1: Understanding the Concept:

Common terms of two arithmetic progressions also form an arithmetic progression.

The common difference of the new AP is the LCM of the original common differences.

Step 2: Key Formula or Approach:

Last term of $S_1 = 1 + (101-1)5 = 501$.

Last term of $S_2 = 9 + (71-1)7 = 499$.

New common difference $D = \text{LCM}(5, 7) = 35$.

Step 3: Detailed Explanation:

First common term:

S_1 : $1, 6, 11, 16, 21 \ldots$

S_2 : $9, 16, 23 \ldots$

The first common term is $a = 16$.

Let there be n common terms. The n-th common term must be $\leq \min(501, 499)$.

$$16 + (n-1)35 \leq 499$$

$$(n-1)35 \leq 483 \implies n-1 \leq \frac{483}{35} = 13.8$$

$$n \leq 14.8 \implies n = 14$$

.

Step 4: Final Answer:

The number of common terms is 14.

Quick Tip: The maximum possible value of a common term is always the minimum of the two last terms. Finding the first common term by listing the first few is usually faster than solving simultaneous equations.

---

22. If $50\left(\frac{2x}{1+3i} + \frac{y}{1-2i}\right) = 31 + 17i$ where $x, y \in \mathbb{R}$ then value of $10(x+3y)$ is:

Correct Answer: 75

Solution:

Step 1: Understanding the Concept:

We simplify the complex fraction by multiplying the numerator and denominator by the conjugate of the denominator.

Step 2: Key Formula or Approach:

$\frac{1}{a+ib} = \frac{a-ib}{a^2+b^2}$.

Step 3: Detailed Explanation:

$\frac{2x}{1+3i} = \frac{2x(1-3i)}{10} = \frac{x-3xi}{5}$.

$\frac{y}{1-2i} = \frac{y(1+2i)}{5}$.

Equation becomes:

$$50\left(\frac{x-3xi+y+2yi}{5}\right) = 31+17i$$

$$10(x+y)+i(20y-30x) = 31+17i$$

Equating real parts: $10(x+y) = 31 \implies 10x+10y = 31 \ldots (1)$.

Equating imaginary parts: $20y-30x = 17 \ldots (2)$.

From (1), $30x+30y = 93$.

Adding this to (2): $50y = 110 \implies y = 2.2$.

From (1), $10x = 31-22 = 9$.

Required value: $10x+30y = 9+3(22) = 9+66 = 75$.

Step 4: Final Answer:

The value of $10(x+3y)$ is 75.

Quick Tip: Always simplify complex equations by equating real and imaginary parts. It effectively turns one complex equation into two independent real algebraic equations.