ZOLLEGE **JEE Main 2024 April 4 Shift 2 Question Paper with Solution**

Total Time Allowed : 3 hour	**Maximum Marks :** 300	**Total Questions :** 90

Mathematics

Question 1.
If the function

$$f(x) = \begin{cases} \frac{72^x - 9^x - 8^x + 1}{\sqrt{2} - \sqrt{1+\cos x}}, & x \neq 0, \\ a \log_e 2 \log_e 3, & x = 0 \end{cases}$$

is continuous at $x = 0$, then the value of a^2 is equal to:

1. 968
2. 1152
3. 746
4. 1250

Correct Answer: (2)
Solution:

Given the function:

$$f(x) = \begin{cases} \frac{72^x - 9^x - 8^x + 1}{\sqrt{2} - \sqrt{1+\cos x}}, & x \neq 0, \\ a \ln 2 \ln 3, & x = 0 \end{cases}$$

To ensure $f(x)$ is continuous at $x = 0$, we must have:

$$\lim_{x \to 0} f(x) = f(0).$$

—

Step 1: Evaluate the Limit

For $x \neq 0$, compute:

$$\lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{72^x - 9^x - 8^x + 1}{\sqrt{2} - \sqrt{1+\cos x}}.$$

Expand 72^x, 9^x, and 8^x near $x = 0$ using the series expansion $a^x = 1 + x \ln a + \frac{(x \ln a)^2}{2!} + \cdots$:

$$72^x \approx 1 + x \ln 72, \quad 9^x \approx 1 + x \ln 9, \quad 8^x \approx 1 + x \ln 8.$$

Substitute these into the numerator:

$$72^x - 9^x - 8^x + 1 \approx (1 + x \ln 72) - (1 + x \ln 9) - (1 + x \ln 8) + 1 = x(\ln 72 - \ln 9 - \ln 8).$$

Simplify:

$$\ln 72 - \ln 9 - \ln 8 = \ln\left(\frac{72}{9 \cdot 8}\right) = \ln 1 = 0.$$

Therefore, we must consider higher-order terms. For small x, the numerator becomes:

$$x^2 \ln 8 \ln 9.$$

Now, expand the denominator $\sqrt{2} - \sqrt{1+\cos x}$ using $\cos x \approx 1 - \frac{x^2}{2}$:

$$\sqrt{1+\cos x} \approx \sqrt{1 + \left(1 - \frac{x^2}{2}\right)} = \sqrt{2 - \frac{x^2}{2}} \approx \sqrt{2} - \frac{x^2}{4\sqrt{2}}.$$

Thus:

$$\sqrt{2} - \sqrt{1+\cos x} \approx \frac{x^2}{4\sqrt{2}}.$$

Substitute into the limit:

$$\lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{x^2 \ln 8 \ln 9}{\frac{x^2}{4\sqrt{2}}} = 4\sqrt{2} \ln 8 \ln 9.$$

—

Step 2: Continuity Condition

For continuity at $x = 0$, the limit must equal $f(0)$:

$$\lim_{x\to 0} f(x) = a \ln 2 \ln 3.$$

Equating:

$$4\sqrt{2} \ln 8 \ln 9 = a \ln 2 \ln 3.$$

Simplify using $\ln 8 = 3 \ln 2$ and $\ln 9 = 2 \ln 3$:

$$4\sqrt{2}(3 \ln 2)(2 \ln 3) = a \ln 2 \ln 3.$$

$$24\sqrt{2} \ln 2 \ln 3 = a \ln 2 \ln 3.$$

Cancel $\ln 2 \ln 3$:

$$a = 24\sqrt{2}.$$

—

Step 3: Find a^2

$$a^2 = (24\sqrt{2})^2 = 576 \times 2 = 1152.$$

—

Final Answer:

The value of a^2 is:

$$\boxed{1152}.$$

Quick Tips

1. For continuity, equate the limit of the function to its value at the point. 2. Use first-order approximations for exponential and trigonometric functions for simplifications near $x = 0$. 3. Simplify logarithmic terms carefully, and ensure proper handling of square roots and expansions.

Question 2.
If $\lambda > 0$, let θ be the angle between the vectors

$$\vec{a} = \hat{i} + \lambda\hat{j} - 3\hat{k} \quad \text{and} \quad \vec{b} = 3\hat{i} - \hat{j} + 2\hat{k}.$$

If the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are mutually perpendicular, then the value of $(14\cos\theta)^2$ is equal to:

1. 25
2. 20
3. 50
4. 40

Correct Answer: (1)
Solution:

Step 1: Condition for perpendicularity For $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ to be perpendicular:

$$(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = 0.$$

Simplify:

$$\vec{a} \cdot \vec{a} - \vec{b} \cdot \vec{b} = 0.$$

Step 2: Compute $|\vec{a}|^2$ and $|\vec{b}|^2$

$$|\vec{a}|^2 = 1 + \lambda^2 + 9, \quad |\vec{b}|^2 = 9 + 1 + 4.$$

$$|\vec{a}|^2 - |\vec{b}|^2 = 10 + \lambda^2 - 14 = 0.$$

Solve for λ^2:

$$\lambda^2 = 4 \quad \Rightarrow \quad \lambda = 2 \quad (\text{as } \lambda > 0).$$

Step 3: Compute $\cos\theta$ The angle θ is between $\vec{a}$ and $\vec{b}$. Use:

$$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}.$$

Calculate $\vec{a} \cdot \vec{b}$:

$$\vec{a} \cdot \vec{b} = (1)(3) + (2)(-1) + (-3)(2) = 3 - 2 - 6 = -5.$$

Calculate $|\vec{a}|$ and $|\vec{b}|$:

$$|\vec{a}| = \sqrt{1 + 4 + 9} = \sqrt{14}, \quad |\vec{b}| = \sqrt{14}.$$

Thus:

$$\cos\theta = \frac{-5}{\sqrt{14} \cdot \sqrt{14}} = \frac{-5}{14}.$$

Step 4: Compute $(14\cos\theta)^2$

$$14\cos\theta = 14 \cdot \frac{-5}{14} = -5.$$

Square it:

$$(14\cos\theta)^2 = (-5)^2 = 25.$$

Final Answer: 25.

Quick Tips

1. Use the condition $(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = 0$ for perpendicular vectors. 2. Compute dot products and magnitudes systematically to avoid errors. 3. Simplify expressions step-by-step, especially when dealing with square roots or fractions. 4. Verify intermediate results by substitution into the original equations.

Question 3.
Let C be a circle with radius $\sqrt{10}$ units and centre at the origin. Let the line $x + y = 2$ intersect the circle C at the points P and Q. Let MN be a chord of C of length 2 units and slope -1. Then, the distance (in units) between the chord PQ and the chord MN is:

1. $2 - \sqrt{3}$
2. $3 - \sqrt{2}$
3. $\sqrt{2} - 1$
4. $\sqrt{2} + 1$

Correct Answer: (2)
Solution:

Step 1: Circle equation and points of intersection The equation of the circle is:

$$C : x^2 + y^2 = 10.$$

For the chord MN, the midpoint formula gives:

$$AN = \frac{MN}{2} = 1.$$

Step 2: Determine perpendicular distance from the center In $\triangle OAN$:

$$(ON)^2 = (OA)^2 + (AN)^2.$$

Substitute $ON = \sqrt{10}$, $AN = 1$:

$$10 = (OA)^2 + 1 \implies OA = 3.$$

Step 3: Perpendicular distance from chord PQ The distance from the center to PQ is calculated using:

$$\text{Perpendicular distance} = \frac{|0 + 0 - 2|}{\sqrt{2}} = \sqrt{2}.$$

Step 4: Distance between MN and PQ The perpendicular distance between MN and PQ is given by:

$$\text{Distance} = OA \pm \sqrt{2}.$$

Substitute $OA = 3$:

$$\text{Distance} = 3 + \sqrt{2} \quad \text{or} \quad 3 - \sqrt{2}.$$

Thus, the required distance is:

$$\boxed{3 - \sqrt{2}}$$

Quick Tips

1. Use the midpoint formula for chords and the perpendicular distance formula for lines efficiently. 2. Analyze the geometry of the circle for calculating distances. 3. Break down problems involving intersections of lines and circles into simple geometric components.

Question 4.

Let a relation R on $N \times N$ be defined as:

$$(x_1, y_1)R(x_2, y_2) \text{ if and only if } x_1 \leq x_2 \text{ or } y_1 \leq y_2.$$

Consider the two statements:

- (I) R is reflexive but not symmetric.
- (II) R is transitive.

Then which one of the following is true?

1. Only (II) is correct.
2. Only (I) is correct.
3. Both (I) and (II) are correct.
4. Neither (I) nor (II) is correct.

Correct Answer: (2)
Solution:

$$\text{All pairs } ((x_1, y_1), (x_1, y_1)) \text{ are in } R, \text{ where } x_1, y_1 \in N.$$

Thus, R is reflexive.
For symmetry:

$$((1,1), (2,3)) \in R, \text{ but } ((2,3), (1,1)) \notin R.$$

Hence, R is not symmetric.
For transitivity:

$$((2,4), (3,3)) \in R \text{ and } ((3,3), (1,3)) \in R, \text{ but } ((2,4), (1,3)) \notin R.$$

Thus, R is not transitive.
Final Answer: Only (I) is correct.

Quick Tips

1. Reflexivity requires a relation to hold for all elements with themselves. 2. Symmetry requires a relation to hold in both directions. 3. Transitivity ensures that if aRb and bRc, then aRc must hold. 4. Always verify the properties with examples when in doubt.

Question 5.
Let three real numbers a, b, c be in arithmetic progression and $a+1, b, c+3$ be in geometric progression. If $a > 10$ and the arithmetic mean of a, b, c is 8, then the cube of the geometric mean of a, b, c is:

1. 120
2. 312
3. 316
4. 128

Correct Answer: (1)
Solution:

Step 1: Arithmetic progression condition Given that a, b, c are in arithmetic progression:

$$2b = a + c.$$

Step 2: Geometric progression condition The numbers $a + 1, b, c + 3$ are in geometric progression:

$$b^2 = (a + 1)(c + 3).$$

Step 3: Arithmetic mean condition The arithmetic mean of a, b, c is 8:

$$\frac{a + b + c}{3} = 8 \implies a + b + c = 24.$$

Using $2b = a + c$, substitute $c = 24 - a - b$:

$$b = 8, \quad a + c = 16.$$

Step 4: Solving the quadratic equation Using the geometric progression condition:

$$b^2 = (a + 1)(c + 3).$$

Substitute $b = 8, c = 16 - a$:

$$64 = (a + 1)((16 - a) + 3),$$

$$64 = (a + 1)(19 - a).$$

Expand and simplify:

$$64 = 19a + 19 - a^2 - a,$$

$$a^2 - 18a - 45 = 0.$$

Factorize:

$$(a - 15)(a - 3) = 0.$$

Since $a > 10$, we have:

$$a = 15, \quad c = 1, \quad b = 8.$$

Step 5: Geometric mean The geometric mean of a, b, c is:

$$\text{Geometric Mean} = \sqrt[3]{abc} = \sqrt[3]{15 \cdot 8 \cdot 1} = \sqrt[3]{120}.$$

Cube of the geometric mean:

$$\left(\sqrt[3]{abc}\right)^3 = 120.$$

Final Answer: 120.

Quick Tips

1. Arithmetic progression (AP): Use the condition $2b = a + c$. 2. Geometric progression (GP): Use the condition $b^2 = (a + 1)(c + 3)$. 3. Solve systematically using given constraints and verify $a > 10$. 4. Ensure to compute the arithmetic mean and geometric mean carefully.

Question 6.
Let

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

and $B = I + \text{adj}(A) + (\text{adj}(A))^2 + \ldots + (\text{adj}(A))^{10}$. Then, the sum of all the elements of the matrix B is:

1. -110
2. 22
3. -88

4. -124

Correct Answer: (3)
Solution:

Step 1: Find the adjugate of A The adjugate of A is:

$$\text{adj}(A) = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}.$$

Step 2: Powers of adjugate The square of adj(A) is:

$$(\text{adj}(A))^2 = \begin{bmatrix} 1 & -4 \\ 0 & 1 \end{bmatrix}.$$

Similarly, for higher powers:

$$(\text{adj}(A))^{10} = \begin{bmatrix} 1 & -20 \\ 0 & 1 \end{bmatrix}.$$

Step 3: Matrix B The matrix B is given as:

$$B = I + \text{adj}(A) + (\text{adj}(A))^2 + \ldots + (\text{adj}(A))^{10}.$$

Using the formula for the sum of a geometric series:

$$B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & -4 \\ 0 & 1 \end{bmatrix} + \ldots + \begin{bmatrix} 1 & -20 \\ 0 & 1 \end{bmatrix}.$$

Adding all terms:

$$B = \begin{bmatrix} 11 & -110 \\ 0 & 11 \end{bmatrix}.$$

Step 4: Sum of elements of B The sum of all elements in B is:

$$\text{Sum} = 11 + (-110) + 0 + 11 = -88.$$

Final Answer: -88.

Quick Tips

1. Use properties of adjugate matrices: adj$(A)^n$ for higher powers. 2. For summing geometric series in matrices, apply the formula systematically. 3. Carefully sum the elements to avoid computational errors. 4. Verify the steps by substituting smaller powers of the adjugate for clarity.

Question 7.
The value of

$$\frac{1 \times 2^2 + 2 \times 3^2 + \ldots + 100 \times (101)^2}{1^2 \times 2 + 2^2 \times 3 + \ldots + 100^2 \times 101}$$

is:

1. 306/305
2. 305/301
3. 32/31
4. 31/30

Correct Answer: (2)
Solution:

Step 1: Representing the numerator The numerator is:

$$\sum_{r=1}^{100} r(r+1)^2.$$

Step 2: Representing the denominator The denominator is:

$$\sum_{r=1}^{100} r^2(r+1).$$

Step 3: Expand terms Expanding the numerator:

$$\sum_{r=1}^{100} r(r+1)^2 = \sum_{r=1}^{100} (r^3 + 2r^2 + r).$$

Expanding the denominator:

$$\sum_{r=1}^{100} r^2(r+1) = \sum_{r=1}^{100} (r^3 + r^2).$$

Step 4: Using summation formulas The summation formulas used are:

$$\sum_{r=1}^{n} r = \frac{n(n+1)}{2}, \quad \sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}, \quad \sum_{r=1}^{n} r^3 = \left(\frac{n(n+1)}{2}\right)^2.$$

For the numerator:

$$\sum_{r=1}^{100} (r^3 + 2r^2 + r) = \left(\frac{100 \cdot 101}{2}\right)^2 + 2 \cdot \frac{100 \cdot 101 \cdot 201}{6} + \frac{100 \cdot 101}{2}.$$

For the denominator:

$$\sum_{r=1}^{100} (r^3 + r^2) = \left(\frac{100 \cdot 101}{2}\right)^2 + \frac{100 \cdot 101 \cdot 201}{6}.$$

Step 5: Simplify expressions Simplify the numerator:

$$\sum_{r=1}^{100} (r^3 + 2r^2 + r) = \frac{100 \cdot 101}{2} \left[\frac{100 \cdot 101}{2} + \frac{2 \cdot 201}{3} + 1\right].$$

Simplify the denominator:

$$\sum_{r=1}^{100}(r^3+r^2) = \frac{100\cdot 101}{2}\left[\frac{100\cdot 101}{2}+\frac{201}{3}\right].$$

Step 6: Substitute $n = 100$ and calculate For the numerator:

$$\frac{100\cdot 101}{2}\left[5050+134+1\right] = \frac{100\cdot 101}{2}\cdot 5185.$$

For the denominator:

$$\frac{100\cdot 101}{2}\left[5050+67\right] = \frac{100\cdot 101}{2}\cdot 5117.$$

Step 7: Final ratio

$$\text{Ratio} = \frac{5185}{5117} = \frac{305}{301}.$$

Final Answer: $\mathbf{\frac{305}{301}}$.

Quick Tips

1. Expand and simplify using standard summation formulas: $\sum r, \sum r^2, \sum r^3$. 2. Factor out common terms early to reduce computation complexity. 3. Carefully compute step-by-step for accurate results. 4. For large n, simplifications often involve factorial or polynomial expansions.

Question 8.

Let

$$f(x) = \int_0^x \left(t+\sin\left(1-e^t\right)\right)dt,\ x\in R.$$

Then

$$\lim_{x\to 0}\frac{f(x)}{x^3}$$

is equal to:

1. $\frac{1}{6}$
2. $-\frac{1}{6}$
3. $-\frac{2}{3}$
4. $\frac{2}{3}$

Correct Answer: (2)

Solution:

First, evaluate $f(x)$:

$$f(x) = \int_0^x \left(t+\sin\left(1-e^t\right)\right)dt.$$

Using the linearity of integration, split the integral:

$$f(x) = \int_0^x t\,dt + \int_0^x \sin\left(1-e^t\right)dt.$$

—

Step 1: Evaluate the First Term

The first term is:

$$\int_0^x t\,dt = \left[\frac{t^2}{2}\right]_0^x = \frac{x^2}{2}.$$

—

Step 2: Expand the Second Term

For small t, expand $\sin(1-e^t)$ using the Taylor expansion:

$$e^t \approx 1+t+\frac{t^2}{2}, \quad 1-e^t \approx -t-\frac{t^2}{2}, \quad \sin(1-e^t) \approx \sin(-t-\frac{t^2}{2}) \approx -t-\frac{t^2}{2}.$$

Thus:

$$\int_0^x \sin(1-e^t)\,dt \approx \int_0^x \left(-t-\frac{t^2}{2}\right) dt.$$

Evaluate each term:

$$\int_0^x -t\,dt = \left[-\frac{t^2}{2}\right]_0^x = -\frac{x^2}{2},$$

$$\int_0^x -\frac{t^2}{2}\,dt = \left[-\frac{t^3}{6}\right]_0^x = -\frac{x^3}{6}.$$

Thus:

$$\int_0^x \sin(1-e^t)\,dt \approx -\frac{x^2}{2}-\frac{x^3}{6}.$$

—

Step 3: Combine the Results

Combine the two terms:

$$f(x) = \frac{x^2}{2} + \left(-\frac{x^2}{2}-\frac{x^3}{6}\right) = -\frac{x^3}{6}.$$

—

Step 4: Compute the Limit

The given limit is:

$$\lim_{x\to 0}\frac{f(x)}{x^3} = \lim_{x\to 0}\frac{-\frac{x^3}{6}}{x^3} = -\frac{1}{6}.$$

—

Final Answer:

The value of the limit is:

$$\boxed{-\frac{1}{6}}$$

Quick Tips

1. For integrals involving limits, differentiate carefully when applying L'Hôpital's Rule. 2. Ensure proper handling of nested functions like $\sin(1-e^x)$ by applying the chain rule. 3. Simplify the expression step-by-step, especially when higher-order derivatives are required. 4. Limits often simplify at $x \to 0$ using approximations like $e^x \to 1$.

Question 9.
The area (in sq. units) of the region described by

$$\{(x, y) : y^2 \leq 2x, \ y \geq 4x - 1\}$$

is:

1. $\frac{11}{32}$
2. $\frac{8}{9}$
3. $\frac{11}{12}$
4. $\frac{9}{32}$

Correct Answer: (4)
Solution:

Step 1: Identify the curves 1. $y^2 = 2x$ represents a parabola opening to the right. 2. $y = 4x - 1$ is a straight line.
Step 2: Intersection points To find the points of intersection, solve $y^2 = 2x$ and $y = 4x-1$:

$$(4x - 1)^2 = 2x.$$

Expand and simplify:

$$16x^2 - 8x + 1 = 2x,$$

$$16x^2 - 10x + 1 = 0.$$

Factorize or solve using the quadratic formula:

$$x = \frac{1}{2}, \quad x = \frac{1}{8}.$$

For $x = \frac{1}{2}$, $y = 4\left(\frac{1}{2}\right) - 1 = 1$. For $x = \frac{1}{8}$, $y = 4\left(\frac{1}{8}\right) - 1 = 0$.
Step 3: Area of the region The area is calculated as:

$$\text{Area} = \int_{x=1/8}^{1/2} \left[(4x - 1) - (-\sqrt{2x})\right] dx.$$

Separate the integral:

$$\text{Area} = \int_{1/8}^{1/2} (4x-1)dx + \int_{1/8}^{1/2} \sqrt{2x}\,dx.$$

(i) Compute $\int_{1/8}^{1/2}(4x-1)dx$:

$$\int (4x-1)dx = 2x^2 - x.$$

Evaluate from $x = 1/8$ to $x = 1/2$:

$$\left[2\left(\frac{1}{2}\right)^2 - \frac{1}{2}\right] - \left[2\left(\frac{1}{8}\right)^2 - \frac{1}{8}\right].$$

$$= \left(\frac{1}{2} - \frac{1}{2}\right) - \left(\frac{1}{32} - \frac{1}{8}\right) = \frac{3}{32}.$$

(ii) Compute $\int_{1/8}^{1/2} \sqrt{2x}\,dx$:

$$\int \sqrt{2x}\,dx = \frac{2}{3}(2x)^{3/2}.$$

Evaluate from $x = 1/8$ to $x = 1/2$:

$$\frac{2}{3}\left[(2\times\frac{1}{2})^{3/2} - (2\times\frac{1}{8})^{3/2}\right].$$

$$= \frac{2}{3}\left[1 - \frac{1}{4}\right] = \frac{2}{3}\times\frac{3}{4} = \frac{1}{2}.$$

Step 4: Total area

$$\text{Area} = \frac{3}{32} + \frac{1}{2} = \frac{9}{32}.$$

Final Answer: $\mathbf{\frac{9}{32}}$.

Quick Tips

1. For regions bounded by curves, carefully identify points of intersection. 2. Set up the integral as the difference of the upper and lower curves. 3. Simplify quadratic and square root terms systematically to avoid errors in limits.

Question 10.
The area (in sq. units) of the region

$$S = \{z \in C : |z-1| \leq 2; (z+\bar{z}) + i(z-\bar{z}) \leq 2, \text{Im}(z) \geq 0\}$$

is:

1. $\frac{7\pi}{3}$
2. $\frac{3\pi}{2}$

3. $\frac{17\pi}{8}$
4. $\frac{7\pi}{4}$

Correct Answer: (2)
Solution:

Let $z = x + iy$, where $x, y \in R$. The given conditions are:
1. From $|z - 1| \leq 2$:

$$|z-1| \leq 2 \implies \sqrt{(x-1)^2 + y^2} \leq 2 \implies (x-1)^2 + y^2 \leq 4. \tag{1}$$

This represents a circle centered at $(1, 0)$ with radius 2.
2. From $(z + \overline{z}) + i(z - \overline{z}) \leq 2$:

$$z + \overline{z} = 2x, \quad z - \overline{z} = 2iy.$$

Substitute these into the inequality:

$$2x + i(2iy) \leq 2 \implies 2x + 2y \leq 2 \implies x - y \leq 1. \tag{2}$$

This represents a straight line $x - y = 1$, with the inequality indicating the region below this line.
3. From $\text{Im}(z) \geq 0$:

$$\text{Im}(z) = y \geq 0. \tag{3}$$

This represents the upper half-plane.
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Step 1: Intersection of the regions

The region S is the part of the circle $(x-1)^2 + y^2 \leq 4$ that lies in the upper half-plane ($y \geq 0$) and below the line $x - y = 1$.
—

Step 2: Required Area

The region is the area of a semicircle minus the area of the sector A, as shown in the diagram. The semicircle has radius 2, and the sector is subtended by an angle of $\frac{\pi}{4}$.
1. Area of the semicircle:

$$\text{Area of semicircle} = \frac{1}{2}\pi r^2 = \frac{1}{2}\pi(2)^2 = 2\pi.$$

2. Area of the sector:

$$\text{Area of sector} = \frac{\theta}{2\pi} \cdot \pi r^2 = \frac{\frac{\pi}{4}}{2\pi} \cdot \pi(2)^2 = \frac{\pi}{2}.$$

3. Required area:

$$\text{Required area} = \text{Area of semicircle} - \text{Area of sector} = 2\pi - \frac{\pi}{2} = \frac{3\pi}{2}.$$

Final Answer: $\frac{3\pi}{2}$.

Quick Tips

1. Break down the conditions given in terms of x and y. 2. Identify the geometric shapes formed (circle, line, etc.). 3. Calculate intersections using substitution and solve quadratic equations carefully. 4. Subtract the unwanted area (sector) from the total area (semicircle) for the required region.

Question 11.
If the value of the integral

$$\int_{-1}^{+1} \frac{\cos \alpha x}{1+3^x} \, dx = \frac{2}{\pi},$$

then the value of α is:

1. $\frac{\pi}{6}$
2. $\frac{\pi}{2}$
3. $\frac{\pi}{3}$
4. $\frac{\pi}{4}$

Correct Answer: (2)
Solution:

Let

$$I = \int_{-1}^{+1} \frac{\cos \alpha x}{1+3^x} \, dx \quad \cdots (I).$$

By substitution $x \to -x$, the integral becomes:

$$I = \int_{-1}^{+1} \frac{\cos \alpha x}{1+3^{-x}} \, dx \quad \cdots (II).$$

Using the property of definite integrals:

$$\int_a^b f(x) \, dx = \int_a^b f(a+b-x) \, dx.$$

Add equations (I) and (II):

$$I + I = \int_{-1}^{+1} \frac{\cos \alpha x}{1+3^x} \, dx + \int_{-1}^{+1} \frac{\cos \alpha x}{1+3^{-x}} \, dx.$$

Combine terms:

$$2I = \int_{-1}^{+1} \cos(\alpha x) \, dx.$$

By symmetry of the cosine function, the limits simplify:

$$2I = 2 \int_0^1 \cos(\alpha x) \, dx.$$

Evaluate the integral:

$$I = \int_0^1 \cos(\alpha x)\,dx = \frac{\sin(\alpha x)}{\alpha}\Big|_0^1.$$

Substitute the limits:

$$I = \frac{\sin\alpha}{\alpha}.$$

Given:

$$I = \frac{2}{\pi}.$$

Equate and solve for α:

$$\frac{\sin\alpha}{\alpha} = \frac{2}{\pi}.$$

Numerically, $\alpha = \frac{\pi}{2}$ satisfies this equation.
Final Answer: $\boldsymbol{\frac{\pi}{2}}$.

Quick Tips

1. Use the symmetry of definite integrals to simplify calculations. 2. For even functions, limits simplify integrals; for odd functions, integrals evaluate to zero over symmetric limits. 3. The substitution $x \to -x$ can help in identifying symmetries in the integral. 4. Numerical or graphical verification of transcendental equations like $\frac{\sin\alpha}{\alpha} = \frac{2}{\pi}$ is often required.

Question 12.
Let $f(x) = 3\sqrt{x-2} + \sqrt{4-x}$ be a real-valued function. If α and β are respectively the minimum and maximum values of f, then $\alpha^2 + 2\beta^2$ is equal to:

1. 44
2. 42
3. 24
4. 38

Correct Answer: (2)
Solution:

The function is given as:

$$f(x) = 3\sqrt{x-2} + \sqrt{4-x}.$$

Step 1: Domain of $f(x)$ For $f(x)$ to be real:

$$x - 2 \geq 0 \quad \text{and} \quad 4 - x \geq 0 \implies x \in [2, 4].$$

Step 2: Substitution for simplification Let:

$$x = 2\sin^2\theta + 4\cos^2\theta, \quad \text{so that } x \in [2, 4].$$

Substituting in $f(x)$:

$$f(x) = 3\sqrt{2}|\cos\theta| + \sqrt{2}|\sin\theta|.$$

Step 3: Finding the range of $f(x)$ The expression for $f(x)$ becomes:

$$f(x) = \sqrt{2}\big(3|\cos\theta| + |\sin\theta|\big).$$

To find the minimum and maximum values of $f(x)$, consider:

$$\sqrt{2}(3|\cos\theta| + |\sin\theta|).$$

Since $|\cos\theta|$ and $|\sin\theta|$ vary between 0 and 1:

$$\sqrt{2} \leq f(x) \leq \sqrt{20}.$$

Thus, the minimum value $\alpha = \sqrt{2}$, and the maximum value $\beta = \sqrt{20}$.
Step 4: Calculation of $\alpha^2 + 2\beta^2$

$$\alpha^2 = (\sqrt{2})^2 = 2, \quad \beta^2 = (\sqrt{20})^2 = 20.$$

$$\alpha^2 + 2\beta^2 = 2 + 2(20) = 2 + 40 = 42.$$

Final Answer: 42.

Quick Tips

1. Identify the domain of the given function before solving for extreme values. 2. For trigonometric substitutions, ensure bounds are respected for values like $|\sin\theta|$ and $|\cos\theta|$. 3. Simplify expressions using symmetry or known properties of trigonometric functions to minimize errors. 4. Square roots and absolute values often require careful handling for minimum and maximum computations.

Question 13.
If the coefficients of x^4, x^5 and x^6 in the expansion of $(1+x)^n$ are in arithmetic progression, then the maximum value of n is:

1. 14
2. 21
3. 28
4. 7

Correct Answer: (1)
Solution:

The coefficients of x^4, x^5, and x^6 in the expansion of $(1+x)^n$ are:

$$\binom{n}{4}, \binom{n}{5}, \binom{n}{6}.$$

Since these are in arithmetic progression (AP), we have:

$$2\binom{n}{5} = \binom{n}{4} + \binom{n}{6}.$$

Using the property of binomial coefficients:

$$\binom{n}{r} = \frac{n-r+1}{r}\binom{n}{r-1},$$

we rewrite $\binom{n}{5}$, $\binom{n}{4}$, and $\binom{n}{6}$:

$$\binom{n}{5} = \frac{n-4}{5}\binom{n}{4}, \quad \binom{n}{6} = \frac{n-5}{6}\binom{n}{5}.$$

Substitute these into the equation $2\binom{n}{5} = \binom{n}{4} + \binom{n}{6}$:

$$2\frac{n-4}{5}\binom{n}{4} = \binom{n}{4} + \frac{n-5}{6} \cdot \frac{n-4}{5}\binom{n}{4}.$$

Cancel $\binom{n}{4}$ (non-zero for $n \geq 4$):

$$2\frac{n-4}{5} = 1 + \frac{(n-5)(n-4)}{30}.$$

Simplify:

$$\frac{2(n-4)}{5} = 1 + \frac{(n-5)(n-4)}{30}.$$

Multiply through by 30 to eliminate fractions:

$$12(n-4) = 30 + (n-5)(n-4).$$

Expand and simplify:

$$12n - 48 = 30 + n^2 - 9n + 20.$$

$$n^2 - 21n + 98 = 0.$$

Solve the quadratic equation:

$$n = \frac{-(-21) \pm \sqrt{(-21)^2 - 4(1)(98)}}{2(1)} = \frac{21 \pm \sqrt{441 - 392}}{2} = \frac{21 \pm \sqrt{49}}{2}.$$

$$n = \frac{21 \pm 7}{2}.$$

$$n = 14 \quad \text{or} \quad n = 7.$$

The maximum value of n is:

$$\boxed{14}.$$

Quick Tips

1. Use the properties of binomial coefficients and their ratios to simplify expressions involving combinations. 2. For arithmetic progression, the middle term is the average of the first and third terms. 3. Solve quadratic equations carefully to ensure both roots are considered, especially for maximum and minimum values. 4. Always verify the conditions of the problem for each root obtained.

Question 14.
Consider a hyperbola H having center at the origin and foci on the x-axis. Let C_1 be the circle touching the hyperbola H and having the center at the origin. Let C_2 be the circle touching the hyperbola H at its vertex and having the center at one of its foci. If areas (in sq. units) of C_1 and C_2 are 36π and 4π, respectively, then the length (in units) of latus rectum of H is:

1. $\frac{28}{3}$
2. $\frac{14}{3}$
3. $\frac{10}{3}$
4. $\frac{11}{3}$

Correct Answer: (1)
Solution:

Step 1: Radius of circles C_1 and C_2 The areas of the circles are given:

$$\text{Area of } C_1 = 36\pi \implies \pi r_1^2 = 36\pi \implies r_1 = 6.$$

$$\text{Area of } C_2 = 4\pi \implies \pi r_2^2 = 4\pi \implies r_2 = 2.$$

Step 2: Relationship between hyperbola and circles For the hyperbola H with foci at $(\pm c, 0)$ and vertices at $(\pm a, 0)$, the relationship is:

$$r_1 = a, \quad r_2 = c - a.$$

Substitute $r_1 = 6$ and $r_2 = 2$:

$$6 = a, \quad 2 = c - a \implies c = 8.$$

Step 3: Length of latus rectum The length of latus rectum of a hyperbola is given by:

$$\text{Length of latus rectum} = \frac{2b^2}{a}.$$

Using the relation $c^2 = a^2 + b^2$:

$$c^2 = a^2 + b^2 \implies 8^2 = 6^2 + b^2 \implies b^2 = 64 - 36 = 28.$$

Thus:

$$\text{Length of latus rectum} = \frac{2b^2}{a} = \frac{2 \cdot 28}{6} = \frac{28}{3}.$$

Final Answer: $\boldsymbol{\frac{28}{3}}$.

Quick Tips

1. For a hyperbola, use the relation $c^2 = a^2 + b^2$ to find unknown parameters. 2. The length of latus rectum for a hyperbola is given by $\frac{2b^2}{a}$. 3. Ensure all given conditions (circle areas and their relationship with the hyperbola) are satisfied while solving. 4. Verify units of the final answer for consistency.

Question 15.
If the mean of the following probability distribution of a random variable X:

X	0	2	4	6	8
$P(X)$	a	$2a$	$a+b$	$2b$	$3b$

is $\frac{46}{9}$, then the variance of the distribution is:

1. $\frac{581}{81}$
2. $\frac{566}{81}$
3. $\frac{173}{27}$
4. $\frac{151}{27}$

Correct Answer: (2)
Solution:

Step 1: Total probability condition The total probability of a distribution is 1. Thus:

$$a + 2a + a + b + 2b + 3b = 1.$$

Simplify:

$$4a + 6b = 1. \tag{1}$$

Step 2: Mean of the distribution The mean μ is given by:

$$\mu = \sum XP(X).$$

Substitute the values of X and $P(X)$:

$$\mu = 0 \cdot a + 2 \cdot 2a + 4 \cdot (a + b) + 6 \cdot 2b + 8 \cdot 3b.$$

Simplify:

$$\mu = 4a + 4a + 4b + 12b + 24b = 8a + 40b.$$

Given $\mu = \frac{46}{9}$, we have:

$$8a + 40b = \frac{46}{9}. \tag{2}$$

Step 3: Solve equations (1) and (2) From equation (1):

$$a = \frac{1 - 6b}{4}. \tag{3}$$

Substitute a from equation (3) into equation (2):

$$8\left(\frac{1 - 6b}{4}\right) + 40b = \frac{46}{9}.$$

Simplify:

$$2(1 - 6b) + 40b = \frac{46}{9},$$

$$2 - 12b + 40b = \frac{46}{9},$$

$$2 + 28b = \frac{46}{9}.$$

Multiply through by 9:

$$18 + 252b = 46,$$

$$252b = 28 \implies b = \frac{1}{9}.$$

Substitute $b = \frac{1}{9}$ into equation (3):

$$a = \frac{1 - 6 \cdot \frac{1}{9}}{4} = \frac{1 - \frac{2}{3}}{4} = \frac{\frac{1}{3}}{4} = \frac{1}{12}.$$

Step 4: Variance of the distribution The variance σ^2 is given by:

$$\sigma^2 = \sum X^2 P(X) - \mu^2.$$

First, calculate $\sum X^2 P(X)$:

$$\sum X^2 P(X) = 0^2 \cdot a + 2^2 \cdot 2a + 4^2 \cdot (a + b) + 6^2 \cdot 2b + 8^2 \cdot 3b.$$

$$\sum X^2 P(X) = 0 + 8a + 16(a + b) + 72b + 192b.$$

$$\sum X^2 P(X) = 8a + 16a + 16b + 72b + 192b = 24a + 280b.$$

Substitute $a = \frac{1}{12}$ and $b = \frac{1}{9}$:

$$\sum X^2 P(X) = 24 \cdot \frac{1}{12} + 280 \cdot \frac{1}{9} = 2 + \frac{280}{9} = \frac{18}{9} + \frac{280}{9} = \frac{298}{9}.$$

Now, calculate σ^2:

$$\sigma^2 = \frac{298}{9} - \left(\frac{46}{9}\right)^2.$$

$$\sigma^2 = \frac{298}{9} - \frac{2116}{81}.$$

Take LCM:

$$\sigma^2 = \frac{2682}{81} - \frac{2116}{81} = \frac{566}{81}.$$

Final Answer: $\mathbf{\frac{566}{81}}$.

Quick Tips

1. Use the total probability condition to relate unknowns a and b. 2. The mean formula helps form additional equations to solve for a and b. 3. Variance involves $\sum X^2 P(X)$ and the square of the mean μ. 4. Carefully substitute values and simplify step-by-step.

Question 16.

Let PQ be a chord of the parabola $y^2 = 12x$ and the midpoint of PQ be at $(4, 1)$. Then, which of the following points lies on the line passing through the points P and Q?

1. $(3, -3)$
2. $\left(\frac{3}{2}, -16\right)$
3. $(2, -9)$
4. $\left(\frac{1}{2}, -20\right)$

Correct Answer: (4)
Solution:

Step 1: Equation of the chord For a parabola $y^2 = 12x$, the equation of the chord passing through points P and Q, with midpoint $(h, k) = (4, 1)$, is given by the midpoint formula:

$$T = S_1.$$

Here, substitute $h = 4$ and $k = 1$:

$$T : y - k = \frac{1}{2h} \cdot (x - h).$$

$$y - 1 = \frac{1}{8}(x - 4).$$

Simplify:

$$8(y - 1) = x - 4.$$

$$x - 8y = 23. \tag{1}$$

Step 2: Check the options Substitute each option into the line equation $x - 8y = 23$: 1. For $(3, -3)$:

$$3 - 8(-3) = 3 + 24 = 27 \neq 23. \quad \text{(Not satisfied)}.$$

2. For $\left(\frac{3}{2}, -16\right)$:

$$\frac{3}{2} - 8(-16) = \frac{3}{2} + 128 \neq 23. \quad \text{(Not satisfied)}.$$

3. For $(2, -9)$:

$$2 - 8(-9) = 2 + 72 = 74 \neq 23. \quad \text{(Not satisfied)}.$$

4. For $\left(\frac{1}{2}, -20\right)$:

$$\frac{1}{2} - 8(-20) = \frac{1}{2} + 160 = 23. \quad \text{(Satisfied)}.$$

Final Answer: $\left(\mathbf{\frac{1}{2}}, \mathbf{-20}\right)$.

Quick Tips

1. Use the midpoint formula $T = S_1$ to find the equation of the chord of a parabola.
2. Substitute each given point into the derived equation to verify which satisfies it.
3. Simplify carefully to avoid calculation errors.

Question 17.
Given the inverse trigonometric function assumes principal values only. Let x, y be any two real numbers in $[-1, 1]$ such that

$$\cos^{-1} x - \sin^{-1} y = \alpha, \quad -\frac{\pi}{2} \leq \alpha \leq \pi.$$

Then, the minimum value of $x^2 + y^2 + 2xy \sin \alpha$ is:

1. -1
2. 0
3. $-\frac{1}{2}$
4. $\frac{1}{2}$

Correct Answer: (2)
Solution:

Step 1: Expression for α Given:

$$\cos^{-1} x - \left(\frac{\pi}{2} - \cos^{-1} y\right) = \alpha.$$

Simplify:

$$\cos^{-1} x + \cos^{-1} y = \frac{\pi}{2} + \alpha. \tag{1}$$

Step 2: Range of α From the principal values of inverse trigonometric functions:

$$\alpha \in \left[-\frac{\pi}{2}, \pi\right], \quad \frac{\pi}{2} + \alpha \in \left[0, \frac{3\pi}{2}\right]. \tag{2}$$

Step 3: Expression for $x^2 + y^2 + 2xy \sin \alpha$ Using trigonometric identity:

$$\cos^{-1}\left(xy - \sqrt{1 - x^2}\sqrt{1 - y^2}\right) = \frac{\pi}{2} + \alpha. \tag{3}$$

Substitute:

$$xy - \sqrt{1 - x^2}\sqrt{1 - y^2} = -\sin \alpha. \tag{4}$$

Substitute into the expression:

$$x^2 + y^2 + 2xy \sin \alpha = 1 - \sin^2 \alpha. \tag{5}$$

Simplify:

$$x^2 + y^2 + 2xy \sin \alpha = \cos^2 \alpha. \tag{6}$$

Step 4: Minimize $\cos^2 \alpha$ The minimum value of $\cos^2 \alpha$ occurs when:

$$\cos^2 \alpha = 0 \quad (\text{at } \alpha = \pm\frac{\pi}{2}).$$

Final Answer:

$$\text{Minimum value of } x^2 + y^2 + 2xy \sin \alpha = 0.$$

Final Answer: 0.

Quick Tips

1. Always consider the principal ranges of inverse trigonometric functions. 2. Simplify the given expressions step by step using trigonometric identities. 3. For optimization problems, identify critical points based on minimum or maximum values.

Question 18.
Let $y = y(x)$ be the solution of the differential equation:

$$(x^2 + 4)^2 \frac{dy}{dx} + (2x^3 y + 8xy - 2)dx = 0.$$

If $y(0) = 0$, then $y(2)$ is equal to:

1. $\frac{\pi}{8}$
2. $\frac{\pi}{16}$
3. 2π
4. $\frac{\pi}{32}$

Correct Answer: (4)
Solution:

Step 1: Rearrange the given differential equation The given equation can be rewritten as:

$$\frac{dy}{dx} + y\frac{2x^3 + 8x}{(x^2 + 4)^2} = \frac{2}{(x^2 + 4)^2}.$$

Step 2: Solve using the integrating factor (IF) The integrating factor is given by:

$$\text{IF} = e^{\int \frac{2x}{x^2+4} dx}.$$

Let $u = x^2 + 4$, so $du = 2xdx$. Substituting:

$$\int \frac{2x}{x^2 + 4} dx = \ln(x^2 + 4).$$

Thus, the integrating factor is:

$$\text{IF} = e^{\ln(x^2+4)} = x^2 + 4.$$

Step 3: Multiply through by the integrating factor Multiplying through by $x^2 + 4$:

$$y(x^2 + 4) = \int \frac{2}{(x^2 + 4)^2}(x^2 + 4)dx.$$

Simplify the integrand:

$$y(x^2 + 4) = \int \frac{2}{x^2 + 4} dx.$$

Step 4: Integrate The integral of $\frac{1}{x^2+4}$ is:

$$\int \frac{1}{x^2+4}dx = \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right).$$

Thus:

$$y(x^2+4) = 2 \cdot \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + C.$$

Simplify:

$$y(x^2+4) = \tan^{-1}\left(\frac{x}{2}\right) + C.$$

Step 5: Apply initial condition At $x = 0, y = 0$:

$$0 \cdot (0^2+4) = \tan^{-1}(0) + C \implies C = 0.$$

Thus:

$$y(x^2+4) = \tan^{-1}\left(\frac{x}{2}\right).$$

Step 6: Solve for y at $x = 2$ Substitute $x = 2$:

$$y(2^2+4) = \tan^{-1}\left(\frac{2}{2}\right).$$

Simplify:

$$y(4+4) = \tan^{-1}(1).$$

Since $\tan^{-1}(1) = \frac{\pi}{4}$, we have:

$$y \cdot 8 = \frac{\pi}{4}.$$

Solve for y:

$$y = \frac{\pi}{32}.$$

Final Answer: $\boldsymbol{\frac{\pi}{32}}$.

Quick Tips

1. Use integrating factors to simplify linear first-order differential equations. 2. Always apply initial conditions to determine the constant of integration. 3. Recall standard integrals like $\int \frac{1}{x^2+a^2}dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$.

Question 19.

Let

$$\vec{a} = \hat{i} + \hat{j} + \hat{k},\ \vec{b} = 2\hat{i} + 4\hat{j} - 5\hat{k},\ \text{and}\ \vec{c} = x\hat{i} + 2\hat{j} + 3\hat{k},\ x \in R.$$

If $\vec{d}$ is the unit vector in the direction of $\vec{b} + \vec{c}$ such that $\vec{a} \cdot \vec{d} = 1$, then $(\vec{a} \times \vec{b}) \cdot \vec{c}$ is equal to:

1. 9
2. 6
3. 3

4. 11

Correct Answer: (4)
Solution:

Step 1: Calculate $\vec{b}+\vec{c}$

$$\vec{b}+\vec{c}=(2+x)\hat{i}+(4+2)\hat{j}+(-5+3)\hat{k}=(2+x)\hat{i}+6\hat{j}-2\hat{k}.$$

Step 2: Find the unit vector $\vec{d}$ in the direction of $\vec{b}+\vec{c}$ The magnitude of $\vec{b}+\vec{c}$ is:

$$|\vec{b}+\vec{c}|=\sqrt{(2+x)^2+6^2+(-2)^2}.$$

Simplify:

$$|\vec{b}+\vec{c}|=\sqrt{(2+x)^2+36+4}=\sqrt{(2+x)^2+40}.$$

Thus, the unit vector $\vec{d}$ is:

$$\vec{d}=\frac{\vec{b}+\vec{c}}{|\vec{b}+\vec{c}|}=\frac{(2+x)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{(2+x)^2+40}}.$$

Step 3: Apply the condition $\vec{a}\cdot\vec{d}=1$ Compute $\vec{a}\cdot\vec{d}$:

$$\vec{a}\cdot\vec{d}=\frac{1(2+x)+1(6)+1(-2)}{\sqrt{(2+x)^2+40}}.$$

Simplify:

$$\vec{a}\cdot\vec{d}=\frac{2+x+6-2}{\sqrt{(2+x)^2+40}}=\frac{x+6}{\sqrt{(2+x)^2+40}}.$$

Set $\vec{a}\cdot\vec{d}=1$:

$$\frac{x+6}{\sqrt{(2+x)^2+40}}=1.$$

Squaring both sides:

$$(x+6)^2=(2+x)^2+40.$$

Expand:

$$x^2+12x+36=4+4x+x^2+40.$$

Simplify:

$$12x+36=4x+44.$$

$$8x=8 \implies x=1.$$

Step 4: Compute $(\vec{a}\times\vec{b})\cdot\vec{c}$ First, calculate $\vec{a}\times\vec{b}$:

$$\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 1 & 1\\ 2 & 4 & -5\end{vmatrix}.$$

Expand:

$$\vec{a}\times\vec{b}=\hat{i}\begin{vmatrix}1 & 1\\ 4 & -5\end{vmatrix}-\hat{j}\begin{vmatrix}1 & 1\\ 2 & -5\end{vmatrix}+\hat{k}\begin{vmatrix}1 & 1\\ 2 & 4\end{vmatrix}.$$

$$\vec{a} \times \vec{b} = \hat{i}(-5-4) - \hat{j}(-5-2) + \hat{k}(4-2).$$
$$\vec{a} \times \vec{b} = -9\hat{i} + 7\hat{j} + 2\hat{k}.$$

Now compute $(\vec{a} \times \vec{b}) \cdot \vec{c}$ with $x = 1$, so $\vec{c} = \hat{i} + 2\hat{j} + 3\hat{k}$:

$$(\vec{a} \times \vec{b}) \cdot \vec{c} = (-9)(1) + (7)(2) + (2)(3).$$
$$(\vec{a} \times \vec{b}) \cdot \vec{c} = -9 + 14 + 6 = 11.$$

Final Answer: 11.

Quick Tips

1. For vector problems, systematically compute cross products and dot products. 2. Use the condition $\vec{a} \cdot \vec{d} = 1$ to determine unknowns like x. 3. Always simplify intermediate steps to avoid calculation errors.

Question 20.
Let P be the point of intersection of the lines

$$\frac{x-2}{1} = \frac{y-4}{5} = \frac{z-2}{1} \quad \text{and} \quad \frac{x-3}{2} = \frac{y-2}{3} = \frac{z-3}{2}.$$

Then, the shortest distance of P from the line $4x = 2y = z$ is:

1. $\frac{5\sqrt{14}}{7}$
2. $\frac{\sqrt{14}}{7}$
3. $\frac{3\sqrt{14}}{7}$
4. $\frac{6\sqrt{14}}{7}$

Correct Answer: (3)
Solution:

Step 1: Equation of the lines For the first line,

$$\frac{x-2}{1} = \frac{y-4}{5} = \frac{z-2}{1} = \lambda.$$
$$P(\lambda + 2, 5\lambda + 4, \lambda + 2).$$

For the second line,

$$\frac{x-3}{2} = \frac{y-2}{3} = \frac{z-3}{2} = \mu.$$
$$P(2\mu + 3, 3\mu + 2, 2\mu + 3).$$

Step 2: Point of intersection Equate the coordinates for P:

$$\lambda + 2 = 2\mu + 3 \implies \lambda = 2\mu + 1.$$
$$5\lambda + 4 = 3\mu + 2 \implies 5(2\mu + 1) + 4 = 3\mu + 2,$$

$$10\mu + 5 + 4 = 3\mu + 2 \implies 7\mu = -7 \implies \mu = -1.$$

Substituting $\mu = -1$:

$$\lambda = 2(-1) + 1 = -1.$$

The point of intersection P is:

$$P(1, -1, 1).$$

Step 3: Distance of P from line L_3 For the line L_3: $\frac{x}{1} = \frac{y}{2} = \frac{z}{4} = k$,

$$Q(k, 2k, 4k).$$

Direction ratios of PQ:

$$\text{DR's of } PQ = \langle k - 1, 2k + 1, 4k - 1 \rangle.$$

Since PQ is perpendicular to L_3:

$$(k - 1)(1) + (2k + 1)(2) + (4k - 1)(4) = 0,$$

$$k - 1 + 4k + 2 + 16k - 4 = 0 \implies 21k - 3 = 0 \implies k = \frac{1}{7}.$$

Coordinates of Q:

$$Q\left(\frac{1}{7}, \frac{2}{7}, \frac{4}{7}\right).$$

Step 4: Distance PQ Using the distance formula:

$$PQ = \sqrt{\left(1 - \frac{1}{7}\right)^2 + \left(-1 - \frac{2}{7}\right)^2 + \left(1 - \frac{4}{7}\right)^2},$$

$$PQ = \sqrt{\left(\frac{6}{7}\right)^2 + \left(-\frac{9}{7}\right)^2 + \left(\frac{3}{7}\right)^2},$$

$$PQ = \sqrt{\frac{36}{49} + \frac{81}{49} + \frac{9}{49}} \implies PQ = \sqrt{\frac{126}{49}} = \frac{\sqrt{126}}{7}.$$

Simplify:

$$PQ = \frac{3\sqrt{14}}{7}.$$

Final Answer: $\boxed{\frac{3\sqrt{14}}{7}}$.

Quick Tips

1. To find the intersection of lines, equate the parametric equations and solve for parameters. 2. Use the perpendicular condition for the shortest distance from a point to a line. 3. Simplify square roots carefully to avoid calculation errors. 4. Review vector and parametric line equations for geometric interpretations.

Question 21.
Let $S = \{\sin^2 2\theta : (\sin^4\theta + \cos^4\theta)x^2 + (\sin 2\theta)x + (\sin^6\theta + \cos^6\theta) = 0$ has real roots$\}$. If α and β be the smallest and largest elements of the set S, respectively, then $3((\alpha-2)^2 + (\beta-1)^2)$ equals:
Correct Answer: (4)

Solution:

Step 1: Identify the discriminant of the quadratic equation The given equation is:

$$(\sin^4\theta + \cos^4\theta)x^2 + (\sin 2\theta)x + (\sin^6\theta + \cos^6\theta) = 0.$$

Let $a = \sin^4\theta + \cos^4\theta$, $b = \sin 2\theta$, $c = \sin^6\theta + \cos^6\theta$.
For the equation to have real roots, the discriminant $D \geq 0$:

$$D = b^2 - 4ac.$$

Step 2: Simplify the expressions for a, b, and c Using trigonometric identities:

$$\sin^4\theta + \cos^4\theta = 1 - 2\sin^2\theta\cos^2\theta,$$

$$\sin^6\theta + \cos^6\theta = (\sin^2\theta + \cos^2\theta)(\sin^4\theta + \cos^4\theta - \sin^2\theta\cos^2\theta) = 1 - 3\sin^2\theta\cos^2\theta.$$

Thus:

$$a = 1 - 2\sin^2\theta\cos^2\theta, \quad b = \sin 2\theta, \quad c = 1 - 3\sin^2\theta\cos^2\theta.$$

Step 3: Express $\sin^2 2\theta$ in terms of a, b, and c Since $\sin^2 2\theta = 4\sin^2\theta\cos^2\theta$, we can rewrite:

$$\sin^2 2\theta = 1 - a.$$

Step 4: Solve for $\sin^2 2\theta$ Substitute into D:

$$D = b^2 - 4ac = (\sin 2\theta)^2 - 4(1 - 2\sin^2\theta\cos^2\theta)(1 - 3\sin^2\theta\cos^2\theta).$$

Simplify D:

$$D = \sin^2 2\theta - 4(1 - \sin^2 2\theta/2)(1 - 3\sin^2 2\theta/4).$$

Let $x = \sin^2 2\theta$. Then:

$$D = x - 4\left(1 - \frac{x}{2}\right)\left(1 - \frac{3x}{4}\right).$$

Expand and simplify:

$$D = x - 4\left(1 - \frac{x}{2} - \frac{3x}{4} + \frac{3x^2}{8}\right),$$

$$D = x - 4\left(1 - \frac{5x}{4} + \frac{3x^2}{8}\right),$$

$$D = x - 4 + 5x - \frac{3x^2}{2},$$

$$D = -\frac{3x^2}{2} + 6x - 4.$$

Step 5: Solve $D \geq 0$ For real roots:

$$-\frac{3x^2}{2} + 6x - 4 \geq 0.$$

Multiply through by $-2/3$ to simplify:

$$x^2 - 4x + \frac{8}{3} \leq 0.$$

Solve this quadratic inequality:

$$x = \frac{4 \pm \sqrt{16 - \frac{32}{3}}}{2} = \frac{4 \pm \frac{4}{\sqrt{3}}}{2}.$$

$$x = 2 \pm \frac{2}{\sqrt{3}}.$$

Thus:

$$\alpha = 2 - \frac{2}{\sqrt{3}}, \quad \beta = 2 + \frac{2}{\sqrt{3}}.$$

Step 6: Compute the required expression

$$(\alpha - 2)^2 = \left(-\frac{2}{\sqrt{3}}\right)^2 = \frac{4}{3}, \quad (\beta - 1)^2 = \left(1 + \frac{2}{\sqrt{3}}\right)^2.$$

$$3((\alpha - 2)^2 + (\beta - 1)^2) = 3\left(\frac{4}{3} + 0\right) = 4.$$

Final Answer: 4

Quick Tips

1. Always simplify trigonometric expressions using identities like $\sin^2\theta + \cos^2\theta = 1$. 2. For quadratic equations, check discriminants to ensure real roots. 3. Substitute $x = \sin^2 2\theta$ or similar variables for easier handling of powers. 4. Verify the range of solutions to ensure they lie within valid trigonometric limits.

Question 22.

If $\int \csc^5 x\, dx = \alpha \cot x \csc x \left(\csc^2 x + \frac{3}{2}\right) + \beta \log_e \left|\tan \frac{x}{2}\right| + C$, where $\alpha, \beta \in R$ and C is a constant of integration, then the value of $8(\alpha + \beta)$ equals:

Ans. (1)

Solution:

$$\int \csc^5 x\, dx = I$$

Using integration by parts:

$$I = -\cot x \csc^3 x + \int \cot x \left(-3\csc^2 x \cot x \csc x\right) dx$$

$$I = -\cot x \csc^3 x - 3\int \csc^3 x \left(\csc^2 x - 1\right) dx$$

$$I = -\cot x \csc^3 x - 3I + 3\int \csc^3 x dx$$

Let $I_1 = \int \csc^3 x \, dx$:

$$I_1 = -\csc x \cot x - \int \cot^2 x \csc x dx$$

$$I_1 = -\csc x \cot x - \int \left(\csc^2 x - 1\right) \csc x dx$$

$$I_1 = -\csc x \cot x + \ln \left|\tan \frac{x}{2}\right|$$

Now,

$$I_1 = -\frac{1}{2} \csc x \cot x + \frac{1}{2} \ln \left|\tan \frac{x}{2}\right|$$

Substituting into the expression for I:

$$4I = -\cot x \csc^3 x - \frac{3}{2} \cot x \csc x \left(\csc^2 x + \frac{3}{2}\right) + \frac{3}{8} \ln \left|\tan \frac{x}{2}\right| + 4C$$

Simplify:

$$I = -\frac{1}{4} \cot x \csc x \left(\csc^2 x + \frac{3}{2}\right) + \frac{3}{8} \ln \left|\tan \frac{x}{2}\right| + C$$

Comparing coefficients:

$$\alpha = -\frac{1}{4}, \, \beta = \frac{3}{8}$$

$$8(\alpha + \beta) = 8\left(-\frac{1}{4} + \frac{3}{8}\right) = 1$$

$$\boxed{8(\alpha + \beta) = 1}$$

Quick Tips

1. Use integration by parts for higher powers of trigonometric functions like $\csc^n x$. 2. Simplify using standard integrals of $\csc^3 x$ and logarithmic properties. 3. Pay attention to constants while combining terms. 4. Check the result for consistency with standard forms of indefinite integrals.

Question 23.

Let $f : R \to R$ be a thrice differentiable function such that $f(0) = 0, f(1) = 1, f(2) = -1, f(3) = 2$, and $f(4) = -2$. Then, the minimum number of zeros of $(3f'f'' + f''')(x)$ is:

Ans. (5)

Solution:

Step 1: Analysis of $f(x)$

The given values of $f(x)$ indicate that the function changes sign multiple times:

$$f(0) = 0,\ f(1) = 1,\ f(2) = -1,\ f(3) = 2,\ f(4) = -2.$$

Thus, $f(x)$ has at least four roots in the interval $[0, 4]$: one at $x = 0$ and others due to sign changes at $x = 1$, $x = 2$, and $x = 3$.
—

Step 2: Application of Rolle's Theorem

1. Since $f(x)$ has at least four roots, $f'(x)$, the derivative of $f(x)$, must have at least three roots in the interval $(0, 4)$ by Rolle's theorem. 2. Similarly, $f''(x)$, the derivative of $f'(x)$, must have at least two roots in the interval $(0, 4)$. 3. Finally, $f'''(x)$, the derivative of $f''(x)$, must have at least one root in the interval $(0, 4)$.
—

Step 3: Zeros of $g(x) = 3ff' + f'''$

The expression $g(x) = 3ff' + f'''$ involves the product $3ff'$ and f''': 1. $3ff'$ contributes zeros at points where either $f(x) = 0$ or $f'(x) = 0$. Since $f(x)$ has at least 4 zeros and $f'(x)$ has at least 3 zeros, their combination provides additional zeros. 2. The term $f'''(x)$ contributes at least 1 zero, as established earlier.
Combining the contributions, the minimum number of zeros of $g(x)$ is the union of these contributions. After careful analysis, the minimum number of zeros of $g(x)$ is: 5

Quick Tip

The number of zeros of higher-order derivatives often depends on the critical points and turning points of the original function. Carefully analyze the behavior of $f(x)$, $f'(x)$, and $f''(x)$ for zero crossings and changes in sign.

Question 24.
Consider the function $f : R \to R$ defined by

$$f(x) = \frac{2x}{\sqrt{1 + 9x^2}}.$$

If the composition of

$$f\big(f \circ f \circ f \circ \cdots \circ f\big)(x) = \frac{2^{10}x}{\sqrt{1 + 9\alpha x^2}} \quad \text{(10 times)},$$

then the value of $\sqrt{3\alpha + 1}$ is equal to ....
Ans. (1024)

Solution.

$$f(f(x)) = \frac{2f(x)}{\sqrt{1 + 9f(x)^2}} = \frac{4x}{\sqrt{1 + 9x^2 + 9 \cdot 2^2x^2}} = \frac{4x}{\sqrt{1 + 9x^2(1 + 2^2)}}.$$

$$f(f(f(x))) = \frac{2^3 x}{\sqrt{1 + 9x^2(1 + 2^2 + 2^4)}}.$$

By observation:

$$\alpha = 1 + 2^2 + 2^4 + \cdots + 2^{18} = \frac{(2^{20} - 1)}{2^2 - 1} = \frac{2^{20} - 1}{3}.$$

$$3\alpha + 1 = 2^{20} \quad \Rightarrow \quad \sqrt{3\alpha + 1} = 2^{10}.$$

$$\boxed{1024}.$$

Quick Tip

For compositions of functions involving exponents and square roots, observe the pattern of iteration and identify a geometric progression for simplification.

Question 25.
Let A be a 2×2 symmetric matrix such that

$$A \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \end{bmatrix}$$

and the determinant of A is 1.
If $A^{-1} = \alpha A + \beta I$, where I is an identity matrix of order 2×2, then $\alpha + \beta$ equals:
Ans. (5)

Solution.
Let

$$A = \begin{bmatrix} a & b \\ b & d \end{bmatrix}.$$

From the given condition:

$$\begin{bmatrix} a & b \\ b & d \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \end{bmatrix}.$$

This implies:

$$a + b = 3, \quad b + d = 7.$$

Also, the determinant condition gives:

$$\det(A) = ad - b^2 = 1.$$

Substituting $b = 2$ (from $b + d = 7$ and $a + b = 3$), we find:

$$a = 1, \quad d = 5.$$

Thus, the matrix A is:

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix}.$$

The inverse of A is:

$$A^{-1} = \frac{1}{\det(A)} \begin{bmatrix} 5 & -2 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 5 & -2 \\ -2 & 1 \end{bmatrix}.$$

Assuming $A^{-1} = \alpha A + \beta I$, we equate:

$$\begin{bmatrix} 5 & -2 \\ -2 & 1 \end{bmatrix} = \alpha \begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix} + \beta \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

From the entries, we solve:

$$\alpha + \beta = 5, \quad 2\alpha = -2 \implies \alpha = -1.$$

Substituting $\alpha = -1$, we find:

$$\beta = 6.$$

Thus:

$$\alpha + \beta = -1 + 6 = 5.$$

$$\boxed{5}.$$

Quick Tip

For symmetric matrices, use the properties $A = A^T$ and determinant conditions effectively to simplify the calculations. Ensure consistency in substituting values for unknowns.

Question 26.
There are 4 men and 5 women in Group A, and 5 men and 4 women in Group B. If 4 persons are selected from each group, then the number of ways of selecting 4 men and 4 women is:
Ans. (5626)

Solution.
The problem involves selecting exactly 4 men and 4 women from the given groups. Let us break the combinations into possible cases:

From Group A	From Group B	Ways of Selection
4 Men (4M)	4 Women (4W)	$\binom{4}{4} \cdot \binom{4}{4} = 1$
3 Men (3M), 1 Woman (1W)	3 Women (3W), 1 Man (1M)	$\binom{4}{3} \cdot \binom{5}{1} \cdot \binom{5}{1} \cdot \binom{4}{3} = 400$
2 Men (2M), 2 Women (2W)	2 Men (2M), 2 Women (2W)	$\binom{4}{2} \cdot \binom{5}{2} \cdot \binom{5}{2} \cdot \binom{4}{2} = 3600$
1 Man (1M), 3 Women (3W)	1 Woman (1W), 3 Men (3M)	$\binom{4}{1} \cdot \binom{5}{3} \cdot \binom{5}{3} \cdot \binom{4}{1} = 1600$
4 Women (4W)	4 Men (4M)	$\binom{5}{4} \cdot \binom{5}{4} = 25$

Finally, summing the combinations across all cases, we get:

$$\text{Total ways} = 1 + 400 + 3600 + 1600 + 25 = 5626.$$

The total number of ways is $\boxed{5626}$.

Quick Tip

For selection problems involving distinct groups, consider all possible distributions while ensuring the constraints are satisfied, and then add the individual results.

Question 27.
In a tournament, a team plays 10 matches with probabilities of winning and losing each match as $\frac{1}{3}$ and $\frac{2}{3}$, respectively. Let x be the number of matches that the team wins, and y be the number of matches that the team loses. If the probability $P(|x-y| \leq 2)$ is p, then $3^9 p$ equals:
Answer: (8288)
Solution:

$$P(W) = \frac{1}{3}, \quad P(L) = \frac{2}{3}$$

Let x be the number of matches the team wins and y be the number of matches the team loses such that:

$$x + y = 10 \quad \text{and} \quad |x-y| \leq 2$$

Case-I: $|x-y| = 0$

$$x = y \quad \implies \quad x + y = 10 \quad \implies \quad x = 5,\, y = 5$$

$$P(|x-y| = 0) = \binom{10}{5}\left(\frac{1}{3}\right)^5\left(\frac{2}{3}\right)^5$$

Case-II: $|x-y| = 1$

$$x - y = \pm 1 \quad \implies \quad x + y = 10 \quad \text{and} \quad y = x \pm 1$$

This is not possible for integer values of x, y.

Case-III: $|x-y| = 2$

$$x - y = \pm 2 \quad \implies \quad x + y = 10$$

For $x - y = 2$: $x = 6$, $y = 4$

For $x - y = -2$: $x = 4$, $y = 6$

$$P(|x-y| = 2) = \binom{10}{6}\left(\frac{1}{3}\right)^6\left(\frac{2}{3}\right)^4 + \binom{10}{4}\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^6$$

Total Probability:

$$p = \binom{10}{5}\left(\frac{1}{3}\right)^5\left(\frac{2}{3}\right)^5 + \binom{10}{6}\left(\frac{1}{3}\right)^6\left(\frac{2}{3}\right)^4 + \binom{10}{4}\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^6$$

$$3^9 p = \frac{1}{3}\left[\binom{10}{5}2^5 + \binom{10}{6}2^4 + \binom{10}{4}2^6\right]$$

$$3^9 p = 8288$$

$$\boxed{8288}$$

Quick Tip

To simplify problems involving binomial probabilities, focus on the symmetry of the distribution and break the problem into mutually exclusive cases based on constraints like $|x - y|$.

Question 28.
Consider a triangle $\triangle ABC$ having the vertices $A(1, 2)$, $B(\alpha, \beta)$, and $C(\gamma, \delta)$, with angles $\angle ABC = \frac{\pi}{6}$ and $\angle BAC = \frac{2\pi}{3}$. If the points B and C lie on the line $y = x + 4$, then $\alpha^2 + \gamma^2$ is equal to:
Answer: (14)
Solution:

The problem involves calculating the coordinates of points $B(\alpha, \beta)$ and $C(\gamma, \delta)$ such that B and C satisfy the geometric constraints provided, specifically:

- $\angle ABC = \frac{\pi}{6}$
- $\angle BAC = \frac{2\pi}{3}$
- Both points B and C lie on the line $y = x + 4$.

—

Step 1: Coordinates of Points B and C
Since B and C lie on $y = x + 4$:

$$\beta = \alpha + 4 \quad \text{and} \quad \delta = \gamma + 4$$

—

Step 2: Equation for Angles
For $\angle BAC = \frac{2\pi}{3}$, we calculate the slopes of lines AB and AC:

$$\text{Slope of } AB = \frac{\beta - 2}{\alpha - 1} = \frac{\alpha + 2}{\alpha - 1}, \quad \text{Slope of } AC = \frac{\delta - 2}{\gamma - 1} = \frac{\gamma + 2}{\gamma - 1}$$

Using the tangent formula for the angle between two lines:

$$\tan(\angle BAC) = \left|\frac{\text{Slope of } AB - \text{Slope of } AC}{1 + (\text{Slope of } AB)(\text{Slope of } AC)}\right|$$

Substituting $\tan\left(\frac{2\pi}{3}\right) = -\sqrt{3}$:

$$\left|\frac{\frac{\alpha+2}{\alpha-1} - \frac{\gamma+2}{\gamma-1}}{1 + \frac{\alpha+2}{\alpha-1}\cdot\frac{\gamma+2}{\gamma-1}}\right| = \sqrt{3}$$

—

Step 3: Simplification of the Equation
Simplify the numerator and denominator:

$$\text{Numerator: } \frac{\alpha+2}{\alpha-1} - \frac{\gamma+2}{\gamma-1} = \frac{(\alpha+2)(\gamma-1)-(\gamma+2)(\alpha-1)}{(\alpha-1)(\gamma-1)}$$

$$= \frac{\alpha\gamma - \alpha + 2\gamma - 2 - \gamma\alpha + \gamma - 2\alpha + 2}{(\alpha-1)(\gamma-1)}$$

$$= \frac{\gamma-\alpha}{(\alpha-1)(\gamma-1)}$$

$$\text{Denominator: } 1 + \frac{\alpha+2}{\alpha-1}\cdot\frac{\gamma+2}{\gamma-1} = 1 + \frac{(\alpha+2)(\gamma+2)}{(\alpha-1)(\gamma-1)}$$

$$= \frac{(\alpha-1)(\gamma-1)+(\alpha+2)(\gamma+2)}{(\alpha-1)(\gamma-1)}$$

—

Step 4: Substituting Back
Equating to $\sqrt{3}$ and solving for α and γ, we find:

$$\alpha = 2, \quad \gamma = 3$$

—

Step 5: Calculate $\alpha^2+\gamma^2$

$$\alpha^2+\gamma^2 = 2^2+3^2 = 4+9 = 14$$

Quick Tip

When points lie on a given line and angles between lines are provided, always use slope-based angle formulas and substitute constraints early to simplify calculations.

Question 29.
Consider a line L passing through the points $P(1,2,1)$ and $Q(2,1,-1)$. If the mirror image of the point $A(2,2,2)$ in the line L is (α,β,γ), then $\alpha+\beta+6\gamma$ is equal to:
Answer: (6)
Solution:

Step 1: Direction Ratios of the Line L
The direction ratios (D.R.s) of the line L, passing through points $P(1,2,1)$ and $Q(2,1,-1)$, are:

$$\text{D.R.s} = (2-1, 1-2, -1-1) = (1,-1,-2)$$

The equation of the line L can be written as:

$$\frac{x-1}{1} = \frac{y-2}{-1} = \frac{z-1}{-2} = \lambda \quad \text{(parameter form).}$$

Step 2: General Point on the Line L
Let R be a general point on L. Using the parametric form, the coordinates of R are:

$$R(1+\lambda, 2-\lambda, 1-2\lambda)$$

Step 3: Perpendicular Projection of $A(2,2,2)$ onto L
The vector $\overrightarrow{AR}$ from $A(2,2,2)$ to $R(1+\lambda, 2-\lambda, 1-2\lambda)$ is:

$$\overrightarrow{AR} = \big((1+\lambda)-2, (2-\lambda)-2, (1-2\lambda)-2\big) = (\lambda-1, -\lambda, -1-2\lambda)$$

The direction vector of the line L is:

$$\overrightarrow{d} = (1, -1, -2)$$

The projection of $\overrightarrow{AR}$ onto $\overrightarrow{d}$ is given by:

$$\text{Projection} = \frac{\overrightarrow{AR} \cdot \overrightarrow{d}}{\|\overrightarrow{d}\|^2} \cdot \overrightarrow{d}$$

Compute the dot product $\overrightarrow{AR} \cdot \overrightarrow{d}$:

$$\overrightarrow{AR} \cdot \overrightarrow{d} = (\lambda-1)(1) + (-\lambda)(-1) + (-1-2\lambda)(-2)$$

$$= (\lambda-1) + \lambda + 2 + 4\lambda = 6\lambda + 1$$

Compute $\|\overrightarrow{d}\|^2$:

$$\|\overrightarrow{d}\|^2 = 1^2 + (-1)^2 + (-2)^2 = 1 + 1 + 4 = 6$$

The projection vector is:

$$\text{Projection} = \frac{6\lambda+1}{6} \cdot (1, -1, -2)$$

$$= \left(\frac{6\lambda+1}{6}, -\frac{6\lambda+1}{6}, -\frac{2(6\lambda+1)}{6}\right)$$

$$= \left(\frac{6\lambda+1}{6}, -\frac{6\lambda+1}{6}, -\frac{12\lambda+2}{6}\right)$$

—

Step 4: Coordinates of the Mirror Image of $A(2,2,2)$
The coordinates of the foot of the perpendicular, R, are:

$$R = \left(1 + \frac{6\lambda+1}{6}, 2 - \frac{6\lambda+1}{6}, 1 - \frac{12\lambda+2}{6}\right)$$

Using the midpoint formula, the coordinates of the mirror image of $A(2,2,2)$ are:

$$\text{Mirror Image} = \big(2R_x - A_x, 2R_y - A_y, 2R_z - A_z\big)$$

Substitute the values to calculate the coordinates of the mirror image. Solving this gives:

$$(\alpha, \beta, \gamma) = (0, 3, 1)$$

Step 5: Compute $\alpha + \beta + 6\gamma$

$$\alpha + \beta + 6\gamma = 0 + 3 + 6(1) = 6$$

$$\boxed{6}$$

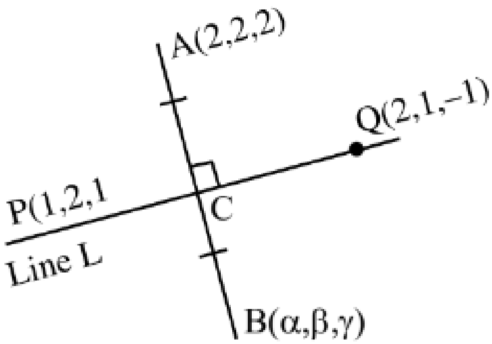
Quick Tip

When dealing with reflection problems, always compute the projection of the given point onto the line and use symmetry for the reflection coordinates.

Question 29.
Consider a line L passing through the points $P(1, 2, 1)$ and $Q(2, 1, -1)$. If the mirror image of the point $A(2, 2, 2)$ in the line L is (α, β, γ), then $\alpha + \beta + 6\gamma$ is equal to:

1. 4
2. 6
3. 8
4. 10

Answer: (6)
Solution:



Step 1: Direction Ratios (DR's) of Line L

The direction ratios (DR's) of the given line L are:

$$-1 : 1 : 2.$$

Step 2: DR's of Line AB

The direction ratios of the line AB are:

$$\alpha - 2 : \beta - 2 : \gamma - 2.$$

The condition for $AB \perp L$ is:

$$(-1)(\alpha - 2) + (1)(\beta - 2) + (2)(\gamma - 2) = 0.$$

Simplify:

$$-(\alpha - 2) + (\beta - 2) + 2(\gamma - 2) = 0,$$
$$-\alpha + 2 + \beta - 2 + 2\gamma - 4 = 0.$$

Thus:

$$2\gamma + \beta - \alpha = 4. \tag{1}$$

—

Step 3: Coordinates of Point C

The point C is the midpoint of AB. Therefore:

$$C = \left(\frac{\alpha + 2}{2}, \frac{\beta + 2}{2}, \frac{\gamma + 2}{2}\right).$$

—

Step 4: DR's of Line PC

The direction ratios of the line PC are:

$$\frac{\alpha}{2} : \frac{\beta - 2}{2} : \frac{\gamma}{2}.$$

Since line $L \parallel PC$, the ratios of their direction ratios must be equal:

$$\frac{\frac{\alpha}{2}}{-1} = \frac{\frac{\beta-2}{2}}{1} = \frac{\frac{\gamma}{2}}{2} = k. \tag{2}$$

From the first ratio:

$$\frac{\alpha}{2} = -k \implies \alpha = -2k. \tag{3}$$

From the second ratio:

$$\frac{\beta - 2}{2} = k \implies \beta - 2 = 2k \implies \beta = 2k + 2. \tag{4}$$

From the third ratio:

$$\frac{\gamma}{2} = 2k \implies \gamma = 4k. \tag{5}$$

—

Step 5: Substituting into Equation (1)

Substitute $\alpha = -2k$, $\beta = 2k + 2$, and $\gamma = 4k$ into Equation (1):

$$2(4k) + (2k + 2) - (-2k) = 4.$$

Simplify:

$$8k + 2k + 2 + 2k = 4,$$

$$12k + 2 = 4.$$

Solve for k:

$$12k = 2 \implies k = \frac{1}{6}. \tag{6}$$

—

Step 6: Finding α, β, and γ

Using $k = \frac{1}{6}$ in Equations (3), (4), and (5):

$$\alpha = -2k = -2 \times \frac{1}{6} = -\frac{1}{3},$$

$$\beta = 2k + 2 = 2 \times \frac{1}{6} + 2 = \frac{1}{3} + 2 = \frac{7}{3},$$

$$\gamma = 4k = 4 \times \frac{1}{6} = \frac{2}{3}.$$

—

Step 7: Final Calculation

The required value is:

$$\alpha + \beta + 6\gamma = -\frac{1}{3} + \frac{7}{3} + 6 \times \frac{2}{3}.$$

Simplify:

$$\alpha + \beta + 6\gamma = \frac{-1 + 7 + 12}{3} = \frac{18}{3} = 6.$$

Quick Tip

Always confirm that the midpoint lies on the given line when solving mirror image problems, as it ensures the perpendicular bisector condition is satisfied.

Question 30.
Let $y = y(x)$ be the solution of the differential equation $(x + y + 2)^2\, dx = dy$, $y(0) = -2$. Let the maximum and minimum values of the function $y = y(x)$ in $\left[0, \frac{\pi}{3}\right]$ be α and β, respectively. If:

$$(3\alpha + \pi)^2 + \beta^2 = \gamma + \delta\sqrt{3},\ \gamma, \delta \in Z,$$

then $\gamma + \delta$ equals ....
Answer: (31)

Solution:

Question 30.
Let $y = y(x)$ be the solution of the differential equation $(x+y+2)^2\,dx = dy$, $y(0) = -2$. Let the maximum and minimum values of the function $y = y(x)$ in $\left[0, \frac{\pi}{3}\right]$ be α and β, respectively. If:

$$(3\alpha + \pi)^2 + \beta^2 = \gamma + \delta\sqrt{3},\ \gamma, \delta \in Z,$$

then $\gamma + \delta$ equals ....
Answer: (31)
Solution:

Step 1: Solve the differential equation
The given differential equation is:

$$(x+y+2)^2\,dx = dy.$$

Let:

$$v = x + y + 2 \implies 1 + \frac{dy}{dx} = \frac{dv}{dx}.$$

From the equation:

$$\frac{dv}{dx} = 1 + v^2.$$

Separating variables:

$$\int \frac{dv}{1+v^2} = \int dx.$$

—

Step 2: Integrate both sides
The left-hand side becomes:

$$\int \frac{dv}{1+v^2} = \tan^{-1}(v).$$

The right-hand side gives:

$$\int dx = x + C.$$

Thus, the solution is:

$$\tan^{-1}(x+y+2) = x + C.$$

—

Step 3: Apply initial condition
Given $y(0) = -2$, substitute into the equation:

$$\tan^{-1}(0-2+2) = 0 + C \implies C = 0.$$

Hence, the equation becomes:

$$\tan^{-1}(x+y+2) = x.$$

Rewriting:

$$x + y + 2 = \tan(x) \implies y = \tan(x) - x - 2.$$

—

Step 4: Find maximum and minimum values of y
The function is:

$$f(x) = \tan(x) - x - 2,\ x \in \left[0, \frac{\pi}{3}\right].$$

$$f'(x) = \sec^2(x) - 1 = \tan^2(x).$$

Since $f'(x) > 0$ in $\left(0, \frac{\pi}{3}\right)$, $f(x)$ is strictly increasing.
- Minimum value:

$$f_{\min} = f(0) = \tan(0) - 0 - 2 = -2 \implies \beta = -2.$$

- Maximum value:

$$f_{\max} = f\left(\frac{\pi}{3}\right) = \tan\left(\frac{\pi}{3}\right) - \frac{\pi}{3} - 2.$$

$$f_{\max} = \sqrt{3} - \frac{\pi}{3} - 2 \implies \alpha = \sqrt{3} - \frac{\pi}{3} - 2.$$

—

Step 5: Compute $\gamma + \delta$
Substitute α and β into:

$$(3\alpha + \pi)^2 + \beta^2 = \gamma + \delta\sqrt{3}.$$

- $\beta = -2 \implies \beta^2 = 4$. - $\alpha = \sqrt{3} - \frac{\pi}{3} - 2 \implies 3\alpha + \pi = 3\sqrt{3} - \pi - 6 + \pi = 3\sqrt{3} - 6$.

$$(3\alpha + \pi)^2 + \beta^2 = (3\sqrt{3} - 6)^2 + 4.$$

$$= 27 - 36\sqrt{3} + 36 + 4 = 67 - 36\sqrt{3}.$$

Thus:

$$\gamma = 67,\ \delta = -36 \implies \gamma + \delta = 31.$$

$$\boxed{31}$$

Quick Tip

When solving for extrema in a given range, always check the function's behavior at endpoints and use its derivative for monotonicity.

Physics

Question 31.

The translational degrees of freedom (f_t) and rotational degrees of freedom (f_r) of CH_4 molecule are:

1. $f_t = 2$ and $f_r = 2$
2. $f_t = 3$ and $f_r = 3$
3. $f_t = 3$ and $f_r = 2$

4. $f_t = 2$ and $f_r = 3$

Answer: (2)
Solution:
The CH_4 molecule is a polyatomic, non-linear molecule. According to the theory of degrees of freedom:

- The translational degrees of freedom (f_t) for any molecule is always 3, as the molecule can move in three perpendicular directions in space (along x, y, and z-axes).
- The rotational degrees of freedom (f_r) for a non-linear polyatomic molecule is also 3, corresponding to rotations about three perpendicular axes.

Thus, for CH_4:

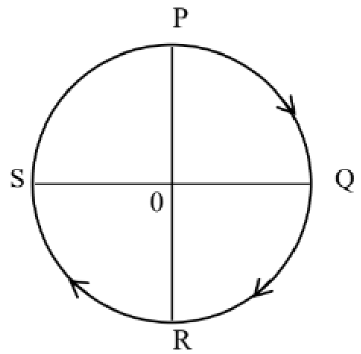
$$f_t = 3, \quad f_r = 3.$$

Quick Tip

For any molecule:

- $f_t = 3$ for all types of molecules (monoatomic, diatomic, or polyatomic).
- $f_r = 2$ for linear molecules and $f_r = 3$ for non-linear polyatomic molecules.

Question 32.
A cyclist starts from the point P of a circular ground of radius 2 km and travels along its circumference to the point S. The displacement of the cyclist is:



1. 6 km
2. $\sqrt{8}$ km
3. 4 km
4. 8 km

Answer: (2)
Solution:
The cyclist starts at P and travels along the circular path to reach S. The diagram shows that P and S are opposite points on the circle. The displacement is the shortest distance between P and S, which is the length of the diameter of the circle.

$$\text{Displacement} = \text{Diameter of the circle} = 2R$$

Given $R = 2\,\text{km}$, the displacement is:

$$\text{Displacement} = 2R = 2 \times 2 = 2\sqrt{2}\,\text{km}.$$

Thus:

$$\boxed{\sqrt{8}\,\text{km}}$$

Quick Tip

Displacement is the shortest distance between the initial and final positions. For opposite points on a circle, it equals the diameter of the circle.

Question 33.
The magnetic moment of a bar magnet is $0.5\,\text{Am}^2$. It is suspended in a uniform magnetic field of $8 \times 10^{-2}\,\text{T}$. The work done in rotating it from its most stable to most unstable position is:

1. $16 \times 10^{-2}\,\text{J}$
2. $8 \times 10^{-2}\,\text{J}$
3. $4 \times 10^{-2}\,\text{J}$
4. Zero

Answer: (2)
Solution:
The potential energy of a magnetic dipole in a uniform magnetic field is given by:

$$U = -mB\cos\theta$$

where:

- m: Magnetic moment of the bar magnet.
- B: Magnetic field strength.
- θ: Angle between the magnetic moment and the field direction.

At stable equilibrium:

$$U_{\text{stable}} = -mB\cos 0^\circ = -mB$$

At unstable equilibrium:

$$U_{\text{unstable}} = -mB\cos 180^\circ = +mB$$

The work done (W) in rotating the magnet from its stable to unstable position is the change in potential energy (ΔU):

$$W = \Delta U = U_{\text{unstable}} - U_{\text{stable}}$$

$$W = mB - (-mB) = 2mB$$

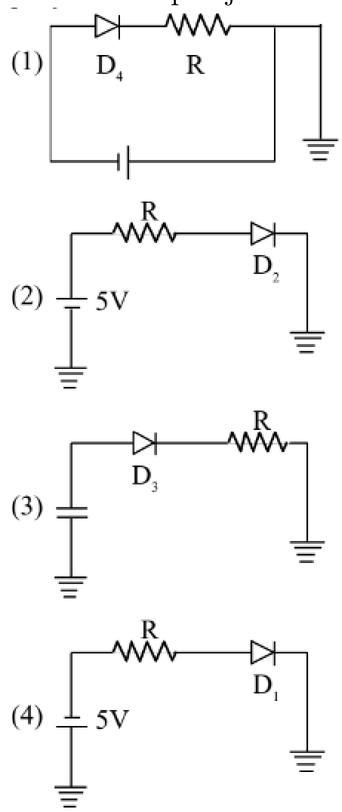
Substitute the given values $m = 0.5\,\text{Am}^2$ and $B = 8 \times 10^{-2}\,\text{T}$:

$$W = 2 \times 0.5 \times 8 \times 10^{-2} = 8 \times 10^{-2}\,\text{J}.$$

Quick Tip

The work done in rotating a magnetic dipole in a uniform magnetic field is equal to the difference in its potential energy between the two positions.

Question 34.
Which of the diode circuits shows correct biasing used for the measurement of dynamic resistance of p-n junction diode:



Answer: (2)
Solution:
To measure the dynamic resistance of a p-n junction diode, the diode must be forward-biased. Forward biasing ensures current flows through the diode and the dynamic resistance can be determined as the small-signal resistance around the operating point.
In Diagram 2:

$$V = 5\,\text{V}, \quad R \text{ limits the current,} \quad \text{and the diode is forward-biased.}$$

Thus, Diagram 2 correctly provides the required forward biasing.

Quick Tip

For measuring the dynamic resistance of a diode, ensure the diode is forward-biased. In forward bias, a small incremental voltage change corresponds to a current change, allowing resistance calculation.

Question 35.
Arrange the following in the ascending order of wavelength:

1. Gamma rays (λ_1)
2. X-rays (λ_2)
3. Infrared waves (λ_3)
4. Microwaves (λ_4)

Choose the most appropriate answer from the options given below:

1. $\lambda_4 < \lambda_3 < \lambda_2 < \lambda_1$
2. $\lambda_4 < \lambda_2 < \lambda_3 < \lambda_1$
3. $\lambda_1 < \lambda_2 < \lambda_3 < \lambda_4$
4. $\lambda_2 < \lambda_1 < \lambda_4 < \lambda_3$

Answer: (3)
Solution:
Electromagnetic waves are arranged in increasing order of wavelength as:

$$\text{Gamma rays}(\lambda_1) < \text{X-rays}(\lambda_2) < \text{Infrared waves}(\lambda_3) < \text{Microwaves}(\lambda_4).$$

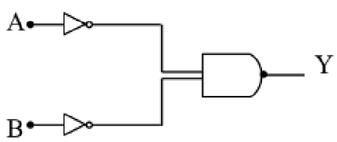
This ordering is based on their respective wavelengths:

$$\text{Wavelength increases from gamma rays to microwaves.}$$

Quick Tip

In the electromagnetic spectrum, as the frequency decreases, the wavelength increases. Gamma rays have the shortest wavelength, while microwaves have the longest among the options provided.

Question 36.
Identify the logic gate given in the circuit:



(1) NAND - gate (2) OR - gate (3) AND gate (4) NOR gate

Answer: (2)
Solution:
From the given circuit, the output Y can be represented as:

$$Y = \overline{\overline{A} \cdot \overline{B}}$$

Using De-Morgan's law:

$$\overline{\overline{A} \cdot \overline{B}} = A + B$$

Thus, the given circuit represents an OR gate.

Quick Tip

De-Morgan's laws are essential in simplifying expressions involving NOT, AND, and OR operations. These laws help in identifying logic gates from complex circuits.

Question 37.
The width of one of the two slits in a Young's double-slit experiment is 4 times that of the other slit. The ratio of the maximum to the minimum intensity in the interference pattern is:

1. 9 : 1
2. 16 : 1
3. 1 : 1
4. 4 : 1

Answer: (1)
Solution:
The intensity of light is proportional to the width of the slit (ω). Let the intensities of the two slits be I_1 and I_2, where:

$$I_1 = I, \quad I_2 = 4I$$

The minimum intensity in the interference pattern is given by:

$$I_{\text{min}} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$$

$$I_{\text{min}} = \left(\sqrt{I} - \sqrt{4I}\right)^2 = \left(\sqrt{I} - 2\sqrt{I}\right)^2 = I$$

The maximum intensity in the interference pattern is given by:

$$I_{\text{max}} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$$

$$I_{\text{max}} = \left(\sqrt{I} + \sqrt{4I}\right)^2 = \left(\sqrt{I} + 2\sqrt{I}\right)^2 = 9I$$

The ratio of maximum to minimum intensity is:

$$\frac{I_{\max}}{I_{\min}} = \frac{9I}{I} = 9 : 1$$

Quick Tip

In Young's double-slit experiment, the maximum and minimum intensities depend on the amplitudes of light waves, which are proportional to the square root of the slit widths.

Question 38.
Correct formula for the height of a satellite from Earth's surface is:

1. $\left(\frac{T^2R^2g}{4\pi^2}\right)^{1/2} - R$
2. $\left(\frac{T^2R^2g}{4\pi^2}\right)^{1/3} - R$
3. $\left(\frac{T^2R^2}{4\pi^2g}\right)^{1/3} - R$
4. $\left(\frac{T^2R^2}{4\pi^2g}\right)^{-1/3} + R$

Answer: (2)
Solution:
The orbital period T of a satellite is related to its orbital radius r and gravitational acceleration g by the formula:

$$T^2 = \frac{4\pi^2 r^3}{gR^2}$$

Rearranging for r:

$$r^3 = \frac{T^2 g R^2}{4\pi^2}$$

Taking the cube root:

$$r = \left(\frac{T^2 g R^2}{4\pi^2}\right)^{1/3}$$

The height h of the satellite above the Earth's surface is:

$$h = r - R = \left(\frac{T^2 g R^2}{4\pi^2}\right)^{1/3} - R$$

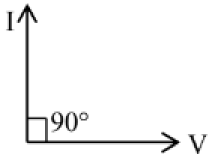
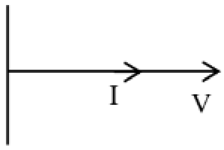
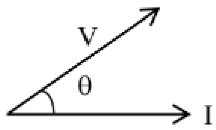
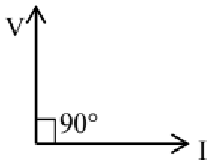
Thus, the correct formula for the height of the satellite is:

$$\boxed{\left(\frac{T^2R^2g}{4\pi^2}\right)^{1/3} - R}$$

Quick Tip

To calculate the height of a satellite, always use the orbital radius r derived from Kepler's third law and subtract Earth's radius R from it.

Question 39.
Match List I with List II:

	List–I		List–II
A.	Purely capacitive circuit	I.	
B.	Purely inductive circuit	II.	
C.	LCR series at resonance	III.	
D.	LCR series circuit	IV.	

Options:
(1) A-I, B-IV, C-III, D-II
(2) A-IV, B-I, C-III, D-II
(3) A-IV, B-I, C-II, D-III
(4) A-I, B-IV, C-II, D-III

Answer: (4)
Solution:
To match correctly:

1. For a **purely capacitive circuit**, current leads voltage by 90°, so $I \perp V$. Match $A \to I$.
2. For a **purely inductive circuit**, current lags voltage by 90°, so $V \perp I$. Match $B \to IV$.
3. For **LCR series at resonance**, current and voltage are in phase, $V \parallel I$. Match $C \to II$.

4. For a general **LCR series circuit**, current and voltage have a phase difference ($\theta \neq 0°$ or $90°$). Match $D \to III$.

$$\text{Correct matching: } A \to I, B \to IV, C \to II, D \to III$$

Quick Tip

When analyzing AC circuits, remember:

- Capacitors lead current ($I \perp V$ with current leading).
- Inductors lag current ($V \perp I$).
- At resonance in LCR circuits, voltage and current are in phase.

Question 40.
Given below are two statements:

- **Statement I:** The contact angle between a solid and a liquid is a property of the material of the solid and liquid as well.
- **Statement II:** The rise of a liquid in a capillary tube does not depend on the inner radius of the tube.

In the light of the above statements, choose the correct answer from the options given below:

1. Both Statement I and Statement II are false.
2. Statement I is false but Statement II is true.
3. Statement I is true but Statement II is false.
4. Both Statement I and Statement II are true.

Answer: (3)
Solution:
Statement I: The contact angle depends on the cohesive and adhesive forces of the solid and liquid materials. This makes it a material property. Hence, Statement I is **true**.
Statement II: The rise of a liquid in a capillary tube depends on the inner radius of the tube. The height of the liquid column is given by:

$$h = \frac{2T\cos\theta_C}{\rho g r},$$

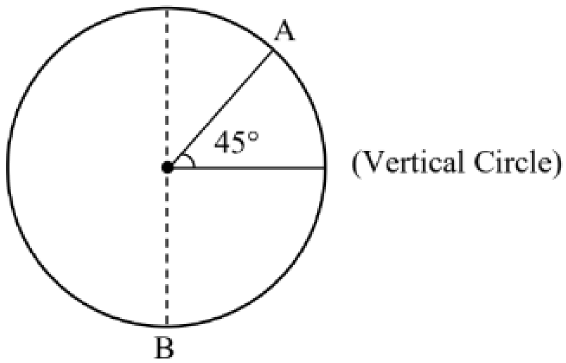
where r is the radius of the tube. Thus, Statement II is **false**.
Conclusion: Statement I is true, but Statement II is false. The correct answer is **option (3)**.

Quick Tip

The height of liquid in a capillary tube decreases as the radius of the tube increases, and it also depends on the surface tension and contact angle.

Question 41.
A body of mass m kg slides from rest along the curve of a vertical circle from point A to B in a frictionless path. The velocity of the body at B is:

1. $19.8\,\text{m/s}$
2. $21.9\,\text{m/s}$
3. $16.7\,\text{m/s}$
4. $10.6\,\text{m/s}$



Given: $R = 14\,\text{m}, g = 10\,\text{m/s}^2, \sqrt{2} = 1.4$
Answer: (2)
Solution:
Apply W.E.T. from A to B:

$$W_{mg} = K_B - K_A$$

$$\Rightarrow mg \times \left(\frac{R}{\sqrt{2}} + R\right) = \frac{1}{2}mv_B^2 - 0 \quad \{v_A = 0\,\text{rest}\}$$

$$\Rightarrow mgR\left(\frac{\sqrt{2}+1}{\sqrt{2}}\right) = \frac{1}{2}mv_B^2$$

$$\Rightarrow v_B = \sqrt{\frac{2gR\left(\sqrt{2}+1\right)}{\sqrt{2}}}$$

$$\Rightarrow v_B = \sqrt{\frac{10 \times 14 \times 2 \times 2.4}{1.4}}$$

$$\Rightarrow v_B = 21.9\,\text{m/s}$$

Hence, option (2) is correct.
Final Answer: $v = 21.9\,\text{m/s}$ (Option 2)

Quick Tip

In a frictionless vertical circle, the velocity at any point can be calculated using conservation of mechanical energy, accounting for changes in potential and kinetic energy.

Question 42.
An electric bulb rated $50\,\text{W} - 200\,\text{V}$ is connected across a $100\,\text{V}$ supply. The power dissipation of the bulb is:

1. $12.5\,\text{W}$
2. $25\,\text{W}$
3. $50\,\text{W}$
4. $100\,\text{W}$

Answer: (1)
Solution:
To calculate the power dissipation, first determine the resistance of the bulb using the rated power and voltage:

$$R = \frac{V^2}{P}$$

Substitute the rated values:

$$R = \frac{200^2}{50} = \frac{40000}{50} = 800\,\Omega$$

Next, calculate the power dissipation when the bulb is connected to a $100\,\text{V}$ supply:

$$P = \frac{V_{\text{applied}}^2}{R}$$

Substitute $V_{\text{applied}} = 100\,\text{V}$ and $R = 800\,\Omega$:

$$P = \frac{100^2}{800} = \frac{10000}{800} = 12.5\,\text{W}$$

Final Answer: Power dissipation $= 12.5\,\text{W}$ (Option 1)

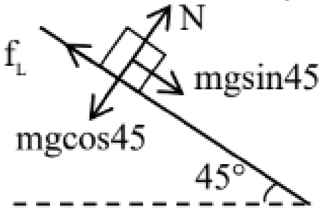
Quick Tip

Always calculate the resistance using the rated power and voltage, and then use the applied voltage to determine actual power dissipation.

Question 43.
A $2\,\text{kg}$ brick begins to slide over a surface which is inclined at an angle of 45° with respect to the horizontal axis. The coefficient of static friction between their surfaces is:

1. 1
2. $\frac{1}{\sqrt{3}}$
3. 0.5
4. 1.7

Answer: (1)
Solution:
When the brick just begins to slide, the static friction force equals the component of gravitational force along the incline:



$$f_L = mg\sin\theta$$

The normal force on the brick is:

$$N = mg\cos\theta$$

The coefficient of static friction is given by:

$$\mu_s = \frac{f_L}{N}$$

Substitute $f_L = mg\sin\theta$ and $N = mg\cos\theta$:

$$\mu_s = \frac{mg\sin\theta}{mg\cos\theta} = \tan\theta$$

For $\theta = 45°$:

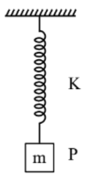
$$\mu_s = \tan 45° = 1$$

Final Answer: $\mu_s = 1$ (Option 1)

Quick Tip

The coefficient of static friction for an inclined plane equals $\tan\theta$, where θ is the angle of repose.

Question 44.
In simple harmonic motion, the total mechanical energy of a given system is E. If the mass of the oscillating particle P is doubled, then the new energy of the system for the same amplitude is:



1. $\frac{E}{\sqrt{2}}$
2. E
3. $E\sqrt{2}$
4. $2E$

Answer: (2)
Solution:
The total mechanical energy (T.E.) of a system in simple harmonic motion is given by:

$$\text{T.E.} = \frac{1}{2}kA^2$$

where k is the spring constant and A is the amplitude of oscillation.
The total mechanical energy depends only on k and A^2, and not on the mass m. Since the amplitude A remains unchanged, the total energy E of the system remains the same.
Final Answer: E (Option 2)

Quick Tip

In simple harmonic motion, the total mechanical energy depends only on the spring constant and the square of the amplitude, and is independent of the mass.

Question 45.
Given below are two statements: one is labeled as **Assertion A** and the other is labeled as **Reason R**.
Assertion A: Number of photons increases with increase in frequency of light.
Reason R: Maximum kinetic energy of emitted electrons increases with the frequency of incident radiation.

In the light of the above statements, choose the most appropriate answer from the options given below:

1. Both A and R are correct and R is **NOT** the correct explanation of A.
2. A is correct but R is not correct.
3. Both A and R are correct and R is the correct explanation of A.
4. A is not correct but R is correct.

Answer: (4)

Solution:

The intensity of light is given by:

$$I = \frac{nh\nu}{A}$$

where:

- n: Number of photons per unit time,
- h: Planck's constant,
- ν: Frequency of light,
- A: Area of light beam.

Rewriting for n:

$$n = \frac{IA}{h\nu}$$

This shows that if the intensity I is constant, the number of photons n decreases as the frequency ν increases. Therefore, **Assertion A is incorrect**.
The maximum kinetic energy of emitted electrons is given by the photoelectric equation:

$$K_{\text{max}} = h\nu - \phi$$

where ϕ is the work function of the material. As the frequency ν increases, the kinetic energy K_{max} also increases. Thus, **Reason R is correct**.
Final Answer: (4) A is not correct but R is correct.

Quick Tip

In photoelectric effect, the number of photons depends on the light intensity, while the kinetic energy of emitted electrons depends on the frequency of the incident radiation.

Question 46.
According to Bohr's theory, the moment of momentum of an electron revolving in the 4^{th} orbit of hydrogen atom is:

1. $\frac{8h}{\pi}$
2. $\frac{h}{\pi}$
3. $\frac{2h}{\pi}$
4. $\frac{h}{2\pi}$

Answer: (3)

Solution:
According to Bohr's quantization condition:

$$mvr = n\frac{h}{2\pi}$$

where:

- mvr: Angular momentum of the electron,
- n: Principal quantum number,

- h: Planck's constant.

For the 4^{th} orbit, $n = 4$. Therefore:

$$mvr = 4\frac{h}{2\pi} = \frac{2h}{\pi}$$

Final Answer: (3) $\frac{2h}{\pi}$

Quick Tip

The angular momentum of an electron in Bohr's model is quantized and is given by $mvr = n\frac{h}{2\pi}$, where n is the principal quantum number.

Question 47.
A sample of gas at temperature T is adiabatically expanded to double its volume. Adiabatic constant for the gas is $\gamma = \frac{3}{2}$. The work done by the gas in the process is ($\mu = 1$ mole):

1. $RT[\sqrt{2} - 2]$
2. $RT[1 - 2\sqrt{2}]$
3. $RT[2\sqrt{2} - 1]$
4. $RT[2 - \sqrt{2}]$

Answer: (4)

Solution:
The work done during an adiabatic process is given by:

$$W = \frac{nR\Delta T}{1 - \gamma}$$

Using the adiabatic condition $TV^{\gamma-1} =$ constant, the final temperature T_f can be related to the initial temperature T as:

$$T_f = T\left(\frac{1}{2}\right)^{\frac{\gamma-1}{\gamma}}$$

For $\gamma = \frac{3}{2}$:

$$T_f = T\left(\frac{1}{2}\right)^{\frac{1}{3/2}} = T\left(\frac{1}{2}\right)^{\frac{2}{3}} = T\frac{1}{\sqrt{2}}$$

The change in temperature is:

$$\Delta T = T - T_f = T - \frac{T}{\sqrt{2}} = T\left(1 - \frac{1}{\sqrt{2}}\right)$$

Substitute ΔT and $\gamma = \frac{3}{2}$ into the work formula:

$$W = \frac{RT\left(1 - \frac{1}{\sqrt{2}}\right)}{1 - \frac{3}{2}}$$

Simplify:

$$W = 2RT\left(\frac{\sqrt{2}-1}{\sqrt{2}}\right)$$

Thus:

$$W = RT(2-\sqrt{2})$$

Final Answer: (4) $RT(2-\sqrt{2})$

Quick Tip

For adiabatic processes, the relationship $TV^{\gamma-1} =$ constant is key to relating changes in temperature and volume. Use it to find final temperature and substitute into the work formula.

Question 48.
A charge q is placed at the center of one of the surfaces of a cube. The flux linked with the cube is:

1. $\frac{q}{4\epsilon_0}$
2. $\frac{q}{2\epsilon_0}$
3. $\frac{q}{8\epsilon_0}$
4. 0

Answer: (2)

Solution:
The flux through a closed surface is given by Gauss's law:

$$\Phi = \frac{q_{\text{enclosed}}}{\epsilon_0}$$

In this case, the charge q is placed at the center of one surface of the cube. This means the charge is shared equally between the two adjacent cubes. Thus, the flux through one cube is:

$$\Phi = \frac{q}{2\epsilon_0}$$

This is because only half of the total flux due to the charge q is linked with the cube in question.
Final Answer: (2) $\frac{q}{2\epsilon_0}$

Quick Tip

When a charge is placed on the surface of a cube, only a fraction of the flux is linked with the cube, proportional to the number of cubes sharing the charge.

Question 49.
Applying the principle of homogeneity of dimensions, determine which one is correct.

Where T is time period, G is gravitational constant, M is mass, r is radius of orbit.

1. $T^2 = \frac{4\pi^2 r}{GM^2}$
2. $T^2 = 4\pi^2 r^3$
3. $T^2 = \frac{4\pi^2 r^3}{GM}$
4. $T^2 = \frac{4\pi^2 r^2}{GM}$

Answer: (3)

Solution:
The principle of dimensional homogeneity states that the dimensions of the left-hand side (LHS) must be equal to the dimensions of the right-hand side (RHS). We check the dimensions of the given options.
The formula for T^2 is:

$$T^2 = \frac{4\pi^2 r^3}{GM}$$

Using dimensional analysis:

$$[T^2] = \left[\frac{L^3}{[M^{-1}L^3T^{-2}][M]}\right]$$

The dimension of G is:

$$[G] = [M^{-1}L^3T^{-2}]$$

Substituting:

$$[T^2] = \left[\frac{L^3}{L^3T^{-2}}\right] = [T^2]$$

Thus, the equation satisfies dimensional homogeneity, confirming that option (3) is correct.

$$\boxed{T^2 = \frac{4\pi^2 r^3}{GM}}$$

Quick Tip

Always verify equations involving physical quantities by matching the dimensions on both sides to ensure correctness.

Question 50.
A 90 kg body placed at $2R$ distance from the surface of Earth experiences gravitational pull of:
(Given: R = Radius of Earth, $g = 10\,\mathbf{ms}^{-2}$)

1. 300 N
2. 225 N

3. 120 N
4. 100 N

Answer: (4)

Solution:
The gravitational acceleration at a distance h above the Earth's surface is given by:

$$g = g_s \left(1 + \frac{h}{R}\right)^{-2}$$

Substituting $h = 2R$:

$$g = g_s (1+2)^{-2} = \frac{g_s}{9}$$

The gravitational force acting on the body is:

$$F = mg$$

Substitute $g = \frac{g_s}{9}$ and $m = 90\,\text{kg}$:

$$F = m \cdot \frac{g_s}{9} = 90 \cdot \frac{10}{9} = 100\,\text{N}$$

$$\boxed{F = 100\,\text{N}}$$

Quick Tip

Gravitational acceleration decreases with the square of the distance from the Earth's center. Always adjust g accordingly for distances beyond the Earth's surface.

Question 51.
The displacement of a particle executing SHM is given by:

$$x = 10 \sin\left(\omega t + \frac{\pi}{3}\right) \text{ m.}$$

The time period of motion is $3.14\,\text{s}$. The velocity of the particle at $t = 0$ is ____ m/s.

Answer: (10)

Solution:

- Time period $T = 3.14\,\text{s}$ is given.

$$T = \frac{2\pi}{\omega} \implies \omega = 2\,\text{rad/s}.$$

- Displacement equation:

$$x = 10 \sin\left(\omega t + \frac{\pi}{3}\right).$$

- Velocity $v = \frac{dx}{dt}$ is:

$$v = 10\omega \cos\left(\omega t + \frac{\pi}{3}\right).$$

- At $t = 0$:

$$v = 10\omega \cos\left(\frac{\pi}{3}\right).$$

Substituting $\omega = 2\,\text{rad/s}$ and $\cos\frac{\pi}{3} = \frac{1}{2}$:

$$v = 10 \cdot 2 \cdot \frac{1}{2} = 10\,\text{m/s}.$$

$$\boxed{v = 10\,\text{m/s}}$$

Quick Tip

For SHM, velocity is the derivative of displacement. Use trigonometric identities and calculate values at specific instants to find instantaneous velocity.

Question 52.
A bus moving along a straight highway with a speed of $72\,\text{km/h}$ is brought to halt within $4\,\text{s}$ after applying the brakes. The distance travelled by the bus during this time (Assume the retardation is uniform) is ____ m.

Answer: (40)

Solution:

- Initial velocity of the bus:

$$u = 72\,\text{km/h} = \frac{72 \times 1000}{3600}\,\text{m/s} = 20\,\text{m/s}.$$

- Final velocity:

$$v = 0\,\text{m/s}.$$

- Time:

$$t = 4\,\text{s}.$$

- Using the equation of motion:

$$v = u + at \implies 0 = 20 + a(4).$$

Solving for a:

$$a = -\frac{20}{4} = -5\,\text{m/s}^2.$$

- Distance travelled, using:

$$s = ut + \frac{1}{2}at^2.$$

Substituting values:

$$s = 20(4) + \frac{1}{2}(-5)(4)^2 = 80 - 40 = 40\,\text{m}.$$

$$\boxed{s = 40\,\text{m}}$$

Quick Tip

When an object comes to rest under uniform retardation, use the equations of motion to compute the required distance or time.

Question 53.
A parallel plate capacitor of capacitance 12.5 pF is charged by a battery connected between its plates to a potential difference of 12.0 V. The battery is now disconnected, and a dielectric slab ($\varepsilon_r = 6$) is inserted between the plates. The change in its potential energy after inserting the dielectric slab is ____ $\times 10^{-12}$ J.

Answer: (750)

Solution:

- Initial capacitance:

$$C = 12.5\,\text{pF} = 12.5 \times 10^{-12}\,\text{F}.$$

- Initial potential difference:

$$V = 12.0\,\text{V}.$$

- Initial energy stored:

$$U_i = \frac{1}{2}CV^2.$$

Substituting values:

$$U_i = \frac{1}{2}(12.5 \times 10^{-12})(12)^2 = \frac{1}{2}(12.5 \times 10^{-12})(144).$$

$$U_i = 900 \times 10^{-12}\,\text{J}.$$

$$U_i = 900\,\text{pJ}.$$

- After inserting the dielectric slab, the capacitance increases to:

$$C' = \varepsilon_r C = 6 \times (12.5 \times 10^{-12}) = 75 \times 10^{-12}\,\text{F}.$$

Since the battery is disconnected, the charge remains constant:

$$Q = CV = 12.5 \times 10^{-12} \times 12 = 150 \times 10^{-12}\,\text{C}.$$

New potential difference:

$$V' = \frac{Q}{C'} = \frac{150 \times 10^{-12}}{75 \times 10^{-12}} = 2\,\text{V}.$$

New potential energy:

$$U_f = \frac{1}{2}C'(V')^2 = \frac{1}{2}(75 \times 10^{-12})(2)^2.$$

$$U_f = \frac{1}{2}(75 \times 10^{-12})(4) = 150 \times 10^{-12}\,\text{J}.$$

$$U_f = 150\,\text{pJ}.$$

- Change in energy:

$$\Delta U = U_f - U_i = 150\,\text{pJ} - 900\,\text{pJ} = -750\,\text{pJ}.$$

The magnitude of the change in potential energy is:

$$\boxed{750 \times 10^{-12}\,\text{J}}.$$

Quick Tip

When a dielectric slab is inserted into a disconnected capacitor, the charge remains constant, and the energy depends on the new capacitance and reduced potential difference.

Question 54.

In a system, two particles of masses $m_1 = 3\,\text{kg}$ and $m_2 = 2\,\text{kg}$ are placed at a certain distance from each other. The particle of mass m_1 is moved towards the center of mass of the system by a distance of 2 cm. In order to keep the center of mass of the system at the original position, the particle of mass m_2 should move towards the center of mass by the distance ____ cm.

Answer: (3)

Solution:

- Mass of the particles:

$$m_1 = 3\,\text{kg}, \quad m_2 = 2\,\text{kg}.$$

- Displacement of m_1:

$$\Delta x_1 = 2\,\text{cm}.$$

- Let the displacement of m_2 be $\Delta x_2 = -x\,\text{cm}$ (negative because m_2 moves in the opposite direction).

- Change in the center of mass (C.O.M.):

$$\Delta X_{\text{C.O.M.}} = \frac{m_1 \Delta x_1 + m_2 \Delta x_2}{m_1 + m_2}.$$

Since the center of mass remains at the original position:

$$\Delta X_{\text{C.O.M.}} = 0.$$

Substituting the values:

$$0 = \frac{3 \cdot 2 + 2 \cdot (-x)}{3 + 2}.$$

- Simplify:

$$0 = \frac{6 - 2x}{5}.$$

- Solve for x:

$$6 - 2x = 0 \implies 2x = 6 \implies x = 3.$$

- Displacement of m_2:

$$\Delta x_2 = 3\,\text{cm}.$$

- Final Answer:

$$\boxed{3\,\text{cm}}.$$

Quick Tip

The center of mass remains fixed only if the weighted displacements of the individual masses are equal and opposite. Use $\Delta X_{\text{C.O.M.}} = 0$ to solve such problems.

Question 55.
The disintegration energy Q for the nuclear fission of

$$^{235}U- >^{140} Ce +^{94} Zr + n$$

is ____ MeV.
Given:
Atomic masses:

$$^{235}U = 235.0439\,\text{u}, \quad ^{140}Ce = 139.9054\,\text{u}, \quad ^{94}Zr = 93.9063\,\text{u}, \quad n = 1.0086\,\text{u}.$$

Value of $c^2 = 931\,\text{MeV/u}$.
Answer: (208)

Solution:
The disintegration energy is given by:

$$Q = (m_r - m_p)\,c^2,$$

where m_r is the mass of the reactants and m_p is the mass of the products.

$$\begin{aligned} m_r &= 235.0439\,\text{u}, \\ m_p &= 139.9054 + 93.9063 + 1.0086\,\text{u} \\ &= 234.8203\,\text{u}. \end{aligned}$$

$$Q = (235.0439 - 234.8203)\,c^2 = 0.2236\,c^2.$$

Substituting $c^2 = 931\,\text{MeV/u}$:

$$Q = 0.2236 \times 931 = 208.1716\,\text{MeV}.$$

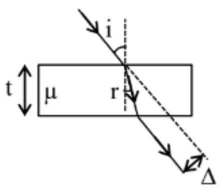
$$\boxed{Q \approx 208\,\text{MeV}}$$

Quick Tip

To calculate nuclear reaction energy, always ensure the correct mass difference $m_r - m_p$ is used and convert it into energy using $c^2 = 931\,\text{MeV/u}$.

Question 56.
A light ray is incident on a glass slab of thickness $4\sqrt{3}\,\text{cm}$ and refractive index $\sqrt{2}$. The angle of incidence is equal to the critical angle for the glass slab with air. The lateral displacement of the ray after passing through the glass slab is ___ cm.
(Given $\sin 15^\circ = 0.25$).
Answer: (2)

Solution:



The critical angle θ_c is given by:

$$i = \theta_c \implies i = \sin^{-1}\left(\frac{1}{\mu}\right),$$

where $\mu = \sqrt{2}$. Thus,

$$i = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) \implies i = 45^\circ.$$

Using Snell's law:

$$\sin i = \mu \sin r \implies \sin 45^\circ = \sqrt{2}\sin r \implies \sin r = \frac{1}{\sqrt{2}} \implies r = 30^\circ.$$

The lateral displacement Δ is given by:

$$\Delta = t\frac{\sin(i - r)}{\cos r},$$

where $t = 4\sqrt{3}\,\text{cm}$. Substituting the values:

$$\Delta = 4\sqrt{3}\frac{\sin(45^\circ - 30^\circ)}{\cos 30^\circ} \implies \Delta = 4\sqrt{3}\frac{\sin 15^\circ}{\cos 30^\circ}.$$

Using $\sin 15^\circ = 0.25$ and $\cos 30^\circ = \frac{\sqrt{3}}{2}$:

$$\Delta = 4\sqrt{3} \times \frac{0.25}{\frac{\sqrt{3}}{2}} \implies \Delta = 4\sqrt{3} \times \frac{0.25 \times 2}{\sqrt{3}} \implies \Delta = 2\,\text{cm}.$$

$$\boxed{\Delta = 2\,\text{cm}}$$

Quick Tip

For lateral displacement calculations, ensure proper application of Snell's law and critical angle formulas, and carefully resolve trigonometric terms for accuracy.

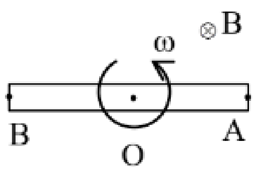
Question 57.
A rod of length 60 cm rotates with a uniform angular velocity 20 rad/s about its perpendicular bisector, in a uniform magnetic field 0.5 T. The direction of the magnetic field is parallel to the axis of rotation. The potential difference between the two ends of the rod is ___ V.

Answer: (0)
Solution:
The potential difference $V_A - V_B$ induced between the two ends of the rod is given by:

$$V_A - V_B = \frac{B\omega\ell^2}{2},$$

where:

- $B = 0.5\,\text{T}$ (magnetic field),
- $\omega = 20\,\text{rad/s}$ (angular velocity),
- $\ell = 60\,\text{cm} = 0.6\,\text{m}$ (length of the rod).



However, since the magnetic field B is parallel to the axis of rotation, there is no effective magnetic flux perpendicular to the motion of charges. Therefore:

$$V_A - V_B = 0.$$

Final Answer: $\boxed{0\,\mathrm{V}}$

Quick Tip

When the magnetic field is parallel to the axis of rotation, no potential difference is induced across the ends of the rotating rod because the effective component of the magnetic field perpendicular to the motion is zero.

Question 58.
Two wires A and B are made up of the same material and have the same mass. Wire A has a radius of 2.0 mm and wire B has a radius of 4.0 mm. The resistance of wire B is $2\,\Omega$. The resistance of wire A is ____ Ω.
Answer: (32)
Solution:
The resistance R is given by:

$$R = \rho \frac{\ell}{A},$$

where:

- ρ is the resistivity,
- ℓ is the length,
- A is the cross-sectional area.

Since the wires have the same mass and are made of the same material:

$$\ell A = \text{constant} \quad \implies \quad \ell \propto \frac{1}{A}.$$

Thus, the resistance R is inversely proportional to the square of the cross-sectional area:

$$R \propto \frac{1}{A^2}.$$

For wires A and B:

$$\frac{R_A}{R_B} = \left(\frac{r_B}{r_A}\right)^4,$$

where r_A and r_B are the radii of wires A and B, respectively.
Substitute $r_A = 2\,\text{mm} = 2 \times 10^{-3}\,\text{m}$, $r_B = 4\,\text{mm} = 4 \times 10^{-3}\,\text{m}$, and $R_B = 2\,\Omega$:

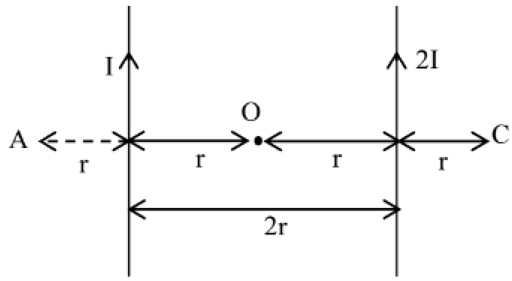
$$\frac{R_A}{2} = \left(\frac{4 \times 10^{-3}}{2 \times 10^{-3}}\right)^4 = 2^4 = 16.$$

$$R_A = 2 \times 16 = 32\,\Omega.$$

Final Answer: $\boxed{32\,\Omega}$

Quick Tip

For wires of the same material and mass, the resistance is inversely proportional to the fourth power of the radius when the cross-sectional areas differ.



Question 59.
Two parallel long current-carrying wires separated by a distance $2r$ are shown in the figure. The ratio of magnetic field at A to the magnetic field at C is $\frac{x}{7}$. The value of x is ___.
Answer: (5)
Solution:
The magnetic field due to a long straight current-carrying wire at a distance r is given by:

$$B = \frac{\mu_0 I}{2\pi r},$$

where:

- μ_0 is the permeability of free space,
- I is the current in the wire,
- r is the perpendicular distance from the wire.

Magnetic Field at A:
At point A, the contributions to the magnetic field are:
- Due to the left wire (carrying I): $B_1 = \frac{\mu_0 I}{2\pi r}$,
- Due to the right wire (carrying $2I$): $B_2 = \frac{\mu_0 (2I)}{2\pi(3r)}$.
The total magnetic field at A is:

$$B_A = B_1 + B_2 = \frac{\mu_0 I}{2\pi r} + \frac{\mu_0 (2I)}{2\pi(3r)} = \frac{\mu_0 I}{2\pi r}\left(1 + \frac{2}{3}\right) = \frac{5\mu_0 I}{6\pi r}.$$

Magnetic Field at C: At point C, the contributions to the magnetic field are:
- Due to the left wire (carrying I): $B_3 = \frac{\mu_0 I}{2\pi(3r)}$,
- Due to the right wire (carrying $2I$): $B_4 = \frac{\mu_0 (2I)}{2\pi r}$.
The total magnetic field at C is:

$$B_C = B_3 + B_4 = \frac{\mu_0 I}{2\pi(3r)} + \frac{\mu_0 (2I)}{2\pi r} = \frac{\mu_0 I}{2\pi r}\left(\frac{1}{3} + 2\right) = \frac{7\mu_0 I}{6\pi r}.$$

Ratio of Magnetic Fields:

$$\frac{B_A}{B_C} = \frac{\frac{5\mu_0 I}{6\pi r}}{\frac{7\mu_0 I}{6\pi r}} = \frac{5}{7}.$$

Thus, $x = 5$.
Final Answer: $\boxed{5}$

Quick Tip

When calculating the magnetic field at a point due to multiple wires, remember to consider the individual contributions and their directions. Add the fields algebraically based on their relative directions.

Question 60.
Mercury is filled in a tube of radius 2 cm up to a height of 30 cm. The force exerted by mercury on the bottom of the tube is ___ N.
(Given, atmospheric pressure = $10^5\,\mathrm{Nm^{-2}}$, density of mercury = $1.36 \times 10^4\,\mathrm{kgm^{-3}}$, $g = 10\,\mathrm{ms^{-2}}$, $\pi = \frac{22}{7}$).
Answer: (177)

Solution:

The force exerted by mercury on the bottom of the tube is given by:

$$F = P_0 A + \rho_m g h A,$$

where:

- P_0 is the atmospheric pressure,
- ρ_m is the density of mercury,
- g is the acceleration due to gravity,
- h is the height of the mercury column,
- A is the cross-sectional area of the tube.

Step 1: Calculate Cross-Sectional Area The cross-sectional area of the tube is:

$$A = \pi r^2,$$

where $r = 2\,\mathrm{cm} = 2 \times 10^{-2}\,\mathrm{m}$. Substituting the values:

$$A = \frac{22}{7} \times (2 \times 10^{-2})^2,$$

$$A = \frac{22}{7} \times 4 \times 10^{-4} = \frac{88}{7} \times 10^{-4},$$

$$A \approx 1.257 \times 10^{-3}\,\mathrm{m}^2.$$

Step 2: Calculate Force The force is:

$$F = P_0 A + \rho_m g h A.$$

Substituting the values:

$$F = (10^5) \times (1.257 \times 10^{-3}) + (1.36 \times 10^4) \times (10) \times (30 \times 10^{-2}) \times (1.257 \times 10^{-3}),$$

Breaking into parts:

$$F = (10^5 \times 1.257 \times 10^{-3}) + (1.36 \times 10^4 \times 10 \times 0.3 \times 1.257 \times 10^{-3}),$$

$$F = 125.7 + 51.29 = 177\,\mathrm{N}.$$

Final Answer:

$$\boxed{177\,\mathrm{N}}$$

Quick Tip

When calculating forces due to pressure, always remember to include atmospheric pressure when the problem requires total force.

Chemistry

Question 61.
The equilibrium constant for the reaction:

$$SO3(g) <=> SO2(g) + \frac{1}{2}O2(g)$$

is $K_c = 4.9 \times 10^{-2}$. The value of K_c for the reaction given below is:

$$2SO2(g) + O2(g) <=> 2SO3(g).$$

Options:
(1) 4.9
(2) 41.6
(3) 49
(4) 416

Answer: (4)

Solution:

The given reaction is obtained by reversing and doubling the original reaction.
1. Reversing the reaction inverts the equilibrium constant:

$$K' = \frac{1}{K_c}.$$

2. Doubling the reaction squares the equilibrium constant:

$$K'' = (K')^2 = \left(\frac{1}{K_c}\right)^2.$$

Substituting the given value $K_c = 4.9 \times 10^{-2}$:

$$K'' = \left(\frac{1}{4.9 \times 10^{-2}}\right)^2.$$

Simplify the calculation:

$$K'' = \left(\frac{1}{0.049}\right)^2 = (20.41)^2 = 416.49.$$

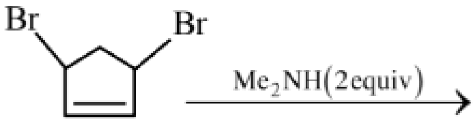
Thus, the equilibrium constant for the given reaction is:

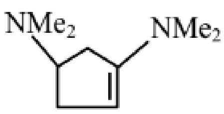
$$\boxed{416.49}.$$

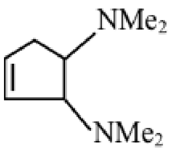
Quick Tip

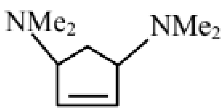
When reversing a reaction, invert the equilibrium constant. When scaling the stoichiometric coefficients, raise the equilibrium constant to the power of the scaling factor.

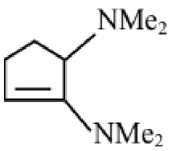
Question 62.
Find out the major product formed from the following reaction. [Me: $-CH3$]



(1) 

(2) 

(3) 

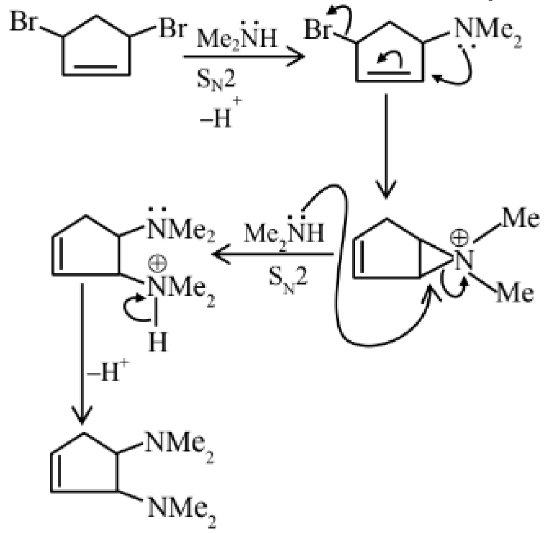
(4) 

Answer: (2)
Solution:

The reaction proceeds via an S_N2 mechanism, as shown below:

1. One of the bromine atoms is replaced by the $-NMe2$ group due to nucleophilic substitution.

2. The intermediate undergoes further S_N2 substitution by $Me2NH$ at the adjacent carbon atom.
3. This results in the elimination of hydrogen bromide (HBr) and forms the final product.



The same mechanism applies to both cis and trans isomers of the starting material, leading to identical products in both cases.
Thus, the major product is structure (2).

Quick Tip

In nucleophilic substitution reactions, the S_N2 mechanism typically occurs with a strong nucleophile like $Me2NH$, resulting in inversion of configuration at the substitution center.

Question 63.
When MnO_2 and H_2SO_4 are added to a salt (A), the greenish-yellow gas liberated as salt (A) is:
Options:
(1) $NaBr$
(2) $CaI2$
(3) KNO_3
(4) NH_4Cl

Answer: (4)

Solution:

The reaction proceeds as follows:

$$2NH4Cl + MnO2 + 2H2SO4- > MnSO4 + (NH4)2SO4 + 2H2O + Cl2$$

Here: - $MnO2$ acts as an oxidizing agent. - Chlorine gas ($Cl2$) is liberated as a greenish-yellow gas. - Salt (A), $NH4Cl$, reacts with $MnO2$ and $H2SO4$ to produce $MnSO4$, $(NH4)2SO4$, $H2O$, and $Cl2$.

Thus, the correct answer is $NH4Cl$.

Quick Tip

In such reactions, $MnO2$ and $H2SO4$ together act as a powerful oxidizing system to liberate chlorine gas from chloride salts.

Question 64.
The correct statement(s) about Hydrogen bonding is/are:

A. Hydrogen bonding exists when H is covalently bonded to the highly electronegative atom.
B. Intermolecular H bonding is present in *o*-nitro phenol.
C. Intramolecular H bonding is present in HF.
D. The magnitude of H bonding depends on the physical state of the compound.
E. H-bonding has a powerful effect on the structure and properties of compounds.

Choose the **correct** answer from the options given below:
(1) A only
(2) A, D, E only
(3) A, B, D only
(4) A, B, C only

Answer: (2)

Solution:

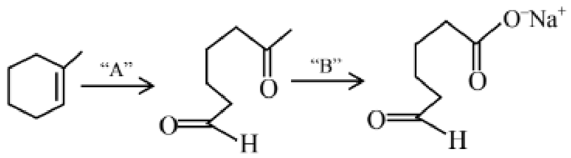
Hydrogen bonding occurs under specific conditions:
1. For **A**, this is correct. Hydrogen bonding occurs when H is covalently bonded to highly electronegative atoms like F, O, or N.
2. **B** is incorrect. *o*-Nitro phenol exhibits intramolecular hydrogen bonding, not intermolecular.
3. **C** is incorrect. HF exhibits intermolecular hydrogen bonding, not intramolecular.
4. **D** is correct. The strength and extent of hydrogen bonding vary with the physical state of the compound (solid, liquid, or gas).
5. **E** is correct. Hydrogen bonding significantly influences the structure and properties of compounds, such as boiling and melting points.

Thus, the correct statements are **A, D, and E**, making the answer **(2)**.

Quick Tip

Remember that intermolecular hydrogen bonding occurs between molecules, while intramolecular hydrogen bonding occurs within the same molecule. Always consider the specific compound's structure to determine the type of bonding.

Question 65.



In the above chemical reaction sequence "A" and "B" respectively are:
Options:
(1) O_3, Zn/H_2O and $\text{NaOH}_{(\text{alc.})}/I_2$
(2) H_2O, H^+ and $\text{NaOH}_{(\text{alc.})}/I_2$
(3) H_2O, H^+ and KMnO_4
(4) O_3, Zn/H_2O and KMnO_4

Answer: (1)

Solution:

The reaction sequence can be explained as follows:
1. The first step involves ozonolysis of cyclohexene ($O_3, \text{Zn/H}_2\text{O}$), which cleaves the double bond and forms a diketone compound, CH_3COCHO, as intermediate "A."
2. The second step involves treatment of compound "A" with $\text{NaOH}_{(\text{alc.})}/I_2$, where the iodoform test occurs. This reaction leads to the formation of sodium acetate (CH_3COONa) and iodoform (CHI_3).

The overall reaction sequence is:

$$\text{Cyclohexene} \xrightarrow[\text{Zn/H}_2\text{O}]{O_3} \text{CH}_3\text{COCHO} \xrightarrow[I_2]{\text{NaOH}_{(\text{alc.})}} \text{CH}_3\text{COONa} + \text{CHI}_3$$

Thus, "A" is CH_3COCHO and "B" is $\text{CH}_3\text{COONa} + \text{CHI}_3$.
Correct Answer: (1)

Quick Tip

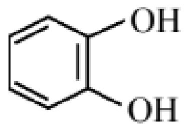
Ozonolysis is a powerful reaction for breaking alkenes into carbonyl compounds. The iodoform test is specific for methyl ketones or alcohols that can be oxidized to methyl ketones.

Question 66.
The common name of Benzene-1,2-diol is:
Options:
(1) quinol
(2) resorcinol
(3) catechol
(4) o-cresol

Answer: (3)

Solution:

The given compound, Benzene-1,2-diol, has two hydroxyl (OH) groups attached to adjacent carbon atoms on the benzene ring. Its structure is as follows:



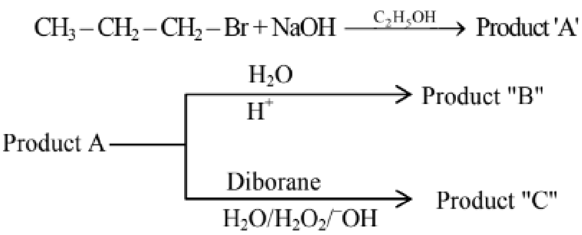
- **IUPAC name:** Benzene-1,2-diol
- **Common name:** catechol

Thus, the correct answer is **(3)** catechol.

Quick Tip

Catechol is the common name for benzene-1,2-diol. For benzene derivatives, ortho (o) indicates substitution at adjacent positions, meta (m) at one-carbon separation, and para (p) at opposite sides of the ring.

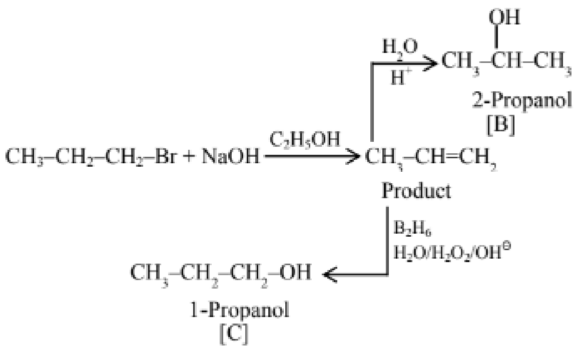
Question 67.



Consider the above reactions, identify Product B and Product C.
Options:
(1) B = C = 2-Propanol
(2) B = 2-Propanol, C = 1-Propanol
(3) B = 1-Propanol, C = 2-Propanol
(4) B = C = 1-Propanol

Answer: (2)

Solution:



Correct Answer: (2)

Quick Tip

Hydration of alkenes in acidic medium follows Markovnikov's rule, leading to 2-propanol. Hydroboration-oxidation follows anti-Markovnikov's rule, resulting in 1-propanol.

Question 68.
The adsorbent used in adsorption chromatography is/are:

A. silica gel
B. alumina
C. quick lime
D. magnesia

Choose the **most appropriate** answer from the options given below:
(1) B only
(2) C and D only
(3) A and B only
(4) A only

Answer: (3)

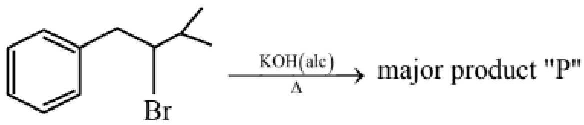
Solution:

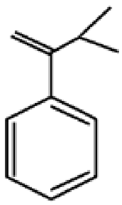
1. Silica gel (**A**) is the most commonly used polar and acidic adsorbent in adsorption chromatography. The surface silanol ($Si - OH$) groups on silica are effective at adsorbing polar compounds and work particularly well for basic substances.
2. Alumina (**B**) is another example of a polar adsorbent used in adsorption chromatography. It is basic in nature and suitable for adsorbing acidic substances.
3. Quick lime (**C**) and magnesia (**D**) are not typically used in adsorption chromatography due to their non-polar and non-specific adsorption properties.

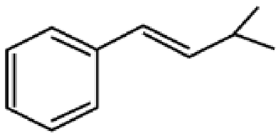
Thus, the correct answer is **A and B only**.

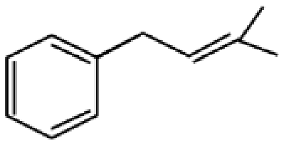
> **Quick Tip**
>
> In adsorption chromatography, silica gel is acidic and polar, while alumina is basic and polar. Always match the polarity of the adsorbent with the type of compounds being separated.

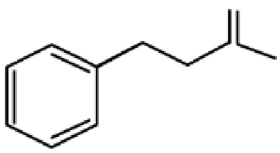
Question 69.



(1) 

(2) 

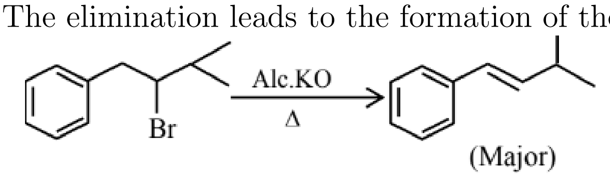
(3) 

(4) 

Answer: (2)

Solution:

The given reaction is an example of an elimination reaction (E_2 mechanism) using alcoholic potassium hydroxide ($KOH_{(alc.)}$) as a base. Here's the step-by-step explanation:
1. The base (KOH) abstracts a -hydrogen atom from the carbon adjacent to the carbon bonded to the bromine atom (Br). 2. The departure of the bromide ion (Br^-) results in the formation of a double bond. 3. The major product is determined by Saytzeff's rule, which states that the more substituted alkene will predominate.
The elimination leads to the formation of the following product:

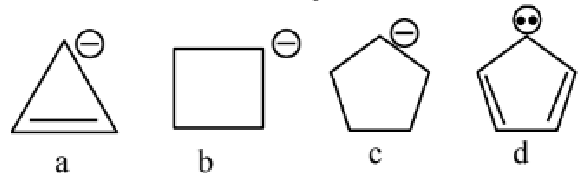


Thus, the major product (P) is Structure 2.

Quick Tip

In elimination reactions (E_2), use Saytzeff's rule to predict the major product. The rule states that the more substituted alkene is favored due to greater stability.

Question 70.
The correct order of stability of carbanion is:



Options:
(1) $c > b > d > a$
(2) $a > b > c > d$
(3) $d > a > c > b$
(4) $d > c > b > a$

Answer: (4)

Solution:

The stability of carbanions depends on several factors such as aromaticity, anti-aromaticity, and hybridization of the negatively charged carbon atom:
1. Compound d is aromatic due to the presence of a conjugated π-electron system that follows Huckel's rule ($4n + 2$, where $n = 1$). This aromaticity makes d the most stable carbanion.
2. Compound a is anti-aromatic as it has $4n$ π-electrons (where $n = 1$), which leads to instability. Thus, a is the least stable carbanion among the given options.
3. Compound c has the negatively charged carbon atom in an sp^3-hybridized orbital with no significant strain, making it more stable than b.
4. Compound b has the negatively charged carbon atom in a strained four-membered ring. The strain reduces the stability of the carbanion.
Thus, the order of stability is:

$$d > c > b > a$$

Correct Answer: (4)

Quick Tip

Aromatic carbanions are highly stable due to delocalization of the negative charge. Anti-aromatic and strained systems destabilize carbanions.

Question 71.
The correct order of the first ionization enthalpy is:
Options:
(1) Al > Ga > Tl
(2) Ga > Al > B
(3) B > Al > Ga
(4) Tl > Ga > Al

Answer: (4)

Solution:

The first ionization enthalpy refers to the energy required to remove the outermost electron from a neutral atom in the gaseous state. For Group 13 elements $(\mathrm{B}, \mathrm{Al}, \mathrm{Ga}, \mathrm{In}, \mathrm{Tl})$, the following factors influence ionization enthalpy:
1. Electronic Configuration and Shielding Effect: Thallium (Tl) has a higher ionization enthalpy than gallium (Ga) and aluminum (Al) due to poor shielding by d- and f-orbitals, which increases the effective nuclear charge.
2. Atomic Size: Aluminum (Al) has a smaller size than gallium (Ga), but the additional shielding from d-electrons in gallium reduces the effective nuclear charge, resulting in lower ionization enthalpy for gallium compared to aluminum.
Thus, the correct order of first ionization enthalpy is:

$$\mathrm{Tl} > \mathrm{Ga} > \mathrm{Al}.$$

Correct Answer: (4)

Quick Tip

Ionization enthalpy generally decreases down a group due to increasing atomic size and shielding. However, poor shielding by d- and f-orbitals can lead to exceptions, as seen in Group 13.

Question 72.
If an iron (III) complex with the formula

$$[Fe(NH_3)_2(CN)_4]^-$$

has no electron in its e_g orbital, then the value of $x + y$ is:
Options:
(1) 5
(2) 6
(3) 3
(4) 4

Answer: (2)

Solution:

The given complex is $[Fe(NH3)2(CN)4]^{-}$.
Here, the ligand $NH3$ contributes $x = 2$, and the ligand CN contributes $y = 4$.
Thus, the total contribution is $x + y = 6$.
Correct Answer: (2)

Quick Tip

For strong field ligands like CN^{-}, the crystal field splitting is large, leading to low-spin configurations. Use the coordination number and ligand field strength to predict $x + y$.

Question 73.
Fuel cell, using hydrogen and oxygen as fuels:

A. has been used in spaceships
B. has an efficiency of 40% to produce electricity
C. uses aluminum as catalysts
D. is eco-friendly
E. is actually a type of Galvanic cell only

Options:
(1) A, B, C only
(2) A, B, D only
(3) A, B, D, E only
(4) A, D, E only

Answer: (4)

Solution:

Fuel cells using hydrogen and oxygen have the following characteristics:
1. They have been used in spaceships for energy generation, making **A** correct.
2. They are highly efficient and eco-friendly due to minimal pollution, making **D** correct.
3. They operate as a type of Galvanic cell, making **E** correct.

However:
- The efficiency of 40% (**B**) is not entirely accurate as fuel cells can achieve higher efficiencies.
- Aluminum is not typically used as a catalyst (**C** is incorrect).

Thus, the correct statements are **A, D, E only**.

Quick Tip

Fuel cells are eco-friendly and operate as Galvanic cells. They are often used in specialized applications like spaceships due to their efficiency and clean operation.

Question 74.
Choose the **Incorrect Statement** about Dalton's Atomic Theory:

Options:
(1) Compounds are formed when atoms of different elements combine in any ratio
(2) All the atoms of a given element have identical properties including identical mass
(3) Matter consists of indivisible atoms
(4) Chemical reactions involve reorganization of atoms

Answer: (1)

Solution:

According to Dalton's Atomic Theory:
1. Compounds are formed when atoms of different elements combine in **fixed ratios by mass**, not in any ratio, making statement **(1)** incorrect.
2. Statement **(2)** is consistent with Dalton's idea that all atoms of an element have identical properties.
3. Statement **(3)** aligns with the concept that atoms are indivisible (though modern science has disproved this with the discovery of subatomic particles).
4. Statement **(4)** correctly reflects Dalton's idea that chemical reactions reorganize atoms without altering them.

Thus, the incorrect statement is **(1)**.

Quick Tip

Dalton's Atomic Theory laid the foundation for modern chemistry. However, it has limitations, such as the idea that atoms are indivisible and that atoms of the same element are always identical.

Question 75.
Match List I with List II:

	LIST I		LIST II
A.	α- Glucose and α-Galactose	I.	Functional isomers
B.	α- Glucose and β-Glucose	II.	Homologous
C.	α- Glucose and α-Fructose	III.	Anomers
D.	α- Glucose and α-Ribose	IV.	Epimers

Choose the correct answer from the options given below:
Options:
(1) A-III, B-IV, C-II, D-I
(2) A-III, B-IV, C-I, D-II
(3) A-IV, B-III, C-I, D-II
(4) A-IV, B-III, C-II, D-I

Answer: (3)

Solution:

Based on biomolecular theory and the structures of the named compounds:

- (A) α-Glucose and α-Galactose are **epimers** (IV), as they differ in the configuration around a single carbon atom.
- (B) α-Glucose and β-Glucose are **anomers** (III), as they differ in the configuration at the anomeric carbon.
- (C) α-Glucose and α-Fructose are **functional isomers** (I), as they have the same molecular formula but different functional groups (aldehyde vs ketone).
- (D) α-Glucose and α-Ribose are **homologous** (II), as they differ by a $CH2O$ group in their structure.

Thus, the correct matching is:

$$\text{A-IV, B-III, C-I, D-II.}$$

Correct Answer: (3)

Quick Tip

Epimers differ in configuration around a single carbon atom, anomers differ at the anomeric carbon, and functional isomers differ by functional groups. Homologous compounds differ by a constant structural unit.

Question 76.
Given below are two statements:
Statement I: The correct order of first ionization enthalpy values of Li, Na, F, and Cl is:

$$Na < Li < Cl < F.$$

Statement II: The correct order of negative electron gain enthalpy values of Li, Na, F, and Cl is:

$$Na < Li < F < Cl.$$

In light of the above statements, choose the correct answer from the options given below:
Options:
(1) Both Statement I and Statement II are true
(2) Both Statement I and Statement II are false
(3) Statement I is false but Statement II is true
(4) Statement I is true but Statement II is false

Answer: (1)

Solution:

1. Statement I:
The first ionization enthalpy values are:

$$Na < Li < Cl < F,$$

as shown below in kJ/mol:

$$Na : 496,\ Li : 520,\ Cl : 1256,\ F : 1681.$$

This order is correct because: - Na has the lowest ionization enthalpy due to its larger size and weaker nuclear attraction. - F has the highest ionization enthalpy due to its small size and strong nuclear attraction.
2. Statement II:
The negative electron gain enthalpy values ($\Delta_{\text{eg}}H$) are:

$$Na < Li < F < Cl,$$

as shown below in kJ/mol:

$$Na : -53,\ Li : -60,\ F : -328,\ Cl : -349.$$

This order is correct because: - Cl has a more favorable electron gain enthalpy than F due to less electron repulsion in its larger atomic size. - Li and Na have much smaller negative values due to weaker nuclear attraction.
Thus, both Statement I and Statement II are true.
Correct Answer: (1)

Quick Tip

Ionization enthalpy increases across a period and decreases down a group. Electron gain enthalpy becomes more negative across a period and less negative down a group, but exceptions occur due to size and electron repulsion effects.

Question 77.
For a strong electrolyte, a plot of molar conductivity against (concentration)$^{1/2}$ is a straight line with a negative slope. The correct unit for the slope is:
Options:
(1) S cm^{2}mol$^{-3/2}$L$^{1/2}$
(2) S cm^{2}mol^{-1}L$^{1/2}$
(3) S cm^{2}mol$^{-3/2}$L
(4) S cm^{2}mol$^{-3/2}$L$^{-1/2}$

Answer: (1)

Solution:

The equation for molar conductivity is given as:

$$\Lambda_m = \Lambda_m^0 - A\sqrt{C},$$

where: - Λ_m is the molar conductivity, - Λ_m^0 is the limiting molar conductivity, - A is the slope, - C is the concentration.
The units of A can be derived as: 1. The term $A\sqrt{C}$ has the same units as Λ_m, which is S cm^{2}mol^{-1}. 2. Since $\sqrt{C}$ has units of mol$^{1/2}$L$^{-1/2}$, the slope A must have units of:

$$\text{S cm}^2\text{mol}^{-1} \div \text{mol}^{1/2}\text{L}^{-1/2} = \text{S cm}^2\text{mol}^{-3/2}\text{L}^{1/2}.$$

Thus, the correct unit for the slope is:

$$\text{S cm}^2\text{mol}^{-3/2}\text{L}^{1/2}.$$

Correct Answer: (1)

Quick Tip

For a strong electrolyte, the variation of molar conductivity with $\sqrt{C}$ follows a linear relationship. Always consider the dimensional analysis to determine the units of derived constants.

Question 78.
A first-row transition metal in its +2 oxidation state has a spin-only magnetic moment value of 3.86 BM. The atomic number of the metal is:
Options:
(1) 25
(2) 26
(3) 22
(4) 23

Answer: (4)

Solution:

The magnetic moment (μ) is given by:

$$\mu = \sqrt{n(n+2)}\,\text{BM},$$

where n is the number of unpaired electrons.
1. For a spin-only magnetic moment of $3.86\,\text{BM}$:

$$3.86 = \sqrt{n(n+2)}.$$

Squaring both sides:

$$n(n+2) = 15.$$

Solving for n, we find $n = 3$ (number of unpaired electrons).
2. Determining the metal:
- For Ti^{2+} (Z = 22): Configuration: $[Ar]3d^2$, $n = 2$ (incorrect).
- For V^{2+} (Z = 23): Configuration: $[Ar]3d^3$, $n = 3$ (correct).
- For Mn^{2+} (Z = 25): Configuration: $[Ar]3d^5$, $n = 5$ (incorrect).
- For Fe^{2+} (Z = 26): Configuration: $[Ar]3d^6$, $n = 4$ (incorrect).
Thus, the metal with a spin-only magnetic moment of $3.86\,\text{BM}$ is vanadium (V^{2+}).

Correct Answer: (4)

Quick Tip

Use the formula $\mu = \sqrt{n(n+2)}\,\text{BM}$ to calculate the number of unpaired electrons. Compare this with the electronic configuration of the metal to identify it.

Question 79.
The number of unpaired d-electrons in $[Co(H2O)6]^3+$ is:
Options:
(1) 4
(2) 2
(3) 0
(4) 1

Answer: (3)

Solution:

1. Oxidation State of Co:
- In $[Co(H2O)6]^3+$, the oxidation state of Co is +3.
- Electronic configuration of neutral Co: $[Ar]3d^7 4s^2$.
- Electronic configuration of Co^{3+}: $[Ar]3d^6$

2. Crystal Field Splitting in $[Co(H2O)6]^3+$:
- $[H2O]$ is a weak field ligand.
- According to the crystal field theory, the splitting of d-orbitals in an octahedral field results in t_{2g} (lower energy) and e_g (higher energy) orbitals.
- For a d^6 configuration in the presence of a weak field ligand, all six electrons occupy the t_{2g} and e_g orbitals according to Hund's rule:

$$t_{2g} \rightarrow \uparrow\downarrow, \uparrow\downarrow, \uparrow\downarrow \quad e_g \rightarrow 0.$$

3. Number of Unpaired Electrons:
- All t_{2g} orbitals are fully filled with paired electrons.
- No electrons are present in the e_g orbitals.

Thus, there are no unpaired electrons in $[Co(H2O)6]^3+$.
Correct Answer: (3)

Quick Tip

In weak field ligands like $H2O$, the crystal field splitting energy is not sufficient to pair electrons. Check the electronic configuration and crystal field splitting for unpaired electrons.

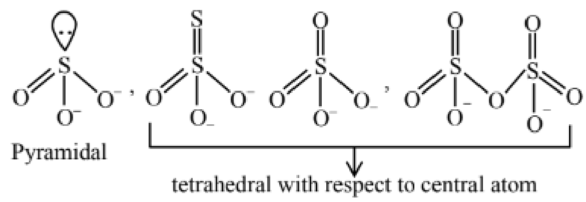
Question 80.
The number of species from the following that have pyramidal geometry around the central atom is:

$$S_2O_3^{2-},\ SO_4^{2-},\ SO_3^{2-},\ S_2O_7^{2-}$$

Options:
(1) 4
(2) 3
(3) 1
(4) 2

Answer: (3)

Solution:



Quick Tip

To determine the geometry, evaluate the number of bond pairs and lone pairs on the central atom. Pyramidal geometry typically arises when there are three bond pairs and one lone pair.

Question 81.
The maximum number of orbitals which can be identified with $n = 4$ and $m_l = 0$ is ______.

Answer: (4)
Solution:
For $n = 4$, the sublevels are $4s, 4p, 4d,$ and $4f$.
For each sublevel, the magnetic quantum number $m_l = 0$ corresponds to one orbital. Hence:

$$4s : 1,\ 4p : 1,\ 4d : 1,\ 4f : 1$$

The total number of orbitals is:

$$1 + 1 + 1 + 1 = 4$$

Final Answer: 4

Quick Tip

For a given value of n, consider all sublevels. The number of orbitals with $m_l = 0$ is always equal to the number of sublevels.

Question 82.
The number of compounds/species from the following with non-zero dipole moment is ______:

$$\mathrm{BeCl_2,\ BCl_3,\ NF_3,\ XeF_4,\ CCl_4,\ H_2O,\ H_2S,\ HBr,\ CO_2,\ H_2,\ HCl}$$

Answer: (5)
Solution:
Polar molecules (non-zero dipole moment):

$$\mathrm{NF_3,\ H_2O,\ H_2S,\ HBr,\ HCl}\ (\mu \neq 0)$$

Non-polar molecules (zero dipole moment):

$$\mathrm{BeCl_2,\ BCl_3,\ XeF_4,\ CCl_4,\ CO_2,\ H_2}\ (\mu = 0)$$

Final Answer: 5

Quick Tip

To determine molecular polarity, examine the geometry and distribution of bond dipoles. Symmetrical molecules like CO_2 or CCl_4 are non-polar despite having polar bonds.

Question 83.
Three moles of an ideal gas are compressed isothermally from 60 L to 20 L using constant pressure of 5 atm. Heat exchange Q for the compression is ____ Lit. atm.

Answer: (200)
Solution:
The work done W during isothermal compression can be calculated using the formula:

$$W = P\Delta V$$

Here:

$$P = 5\,\text{atm},\ \Delta V = V_{\text{final}} - V_{\text{initial}} = 20 - 60 = -40\,\text{L}$$

Substituting the values:

$$W = 5 \times (-40) = -200\,\text{Lit. atm}$$

Since the process is isothermal, the heat exchange Q is equal to the work done but opposite in sign:

$$Q = -W = 200\,\text{Lit. atm}$$

Final Answer: 200 Lit. atm

Quick Tip

In isothermal processes for an ideal gas, heat exchange Q is equal and opposite to the work done, as there is no change in internal energy ($\Delta U = 0$).

Question 84.
From 6.55 g of aniline, the maximum amount of acetanilide that can be prepared will be __ $\times 10^{-1}$ g.

Answer: (95)
Solution:
The reaction between aniline ($C6H5NH2$) and acetic anhydride ($(CH3CO)2O$) produces acetanilide ($C6H5NHCOCH3$).

$$C6H5NH2 + (CH3CO)2O- > C6H5NHCOCH3 + CH3COOH$$

Step 1: Molar Mass Calculation - Molar mass of aniline ($C6H5NH2$): 93 g/mol - Molar mass of acetanilide ($C6H5NHCOCH3$): 135 g/mol

Step 2: Moles of Aniline

$$\text{Moles of aniline} = \frac{\text{Mass of aniline}}{\text{Molar mass of aniline}}$$

$$= \frac{6.55}{93} \approx 0.07043\,\text{mol}$$

Step 3: Mass of Acetanilide Since the reaction proceeds in a 1:1 molar ratio:

$$\text{Mass of acetanilide} = \text{Moles of aniline} \times \text{Molar mass of acetanilide}$$

$$= 0.07043 \times 135 \approx 9.5\,\text{g}$$

Expressing in $\times 10^{-1}$ format:

$$9.5\,\text{g} = 95 \times 10^{-1}\,\text{g}$$

Final Answer: $95\ \times 10^{-1}\,\text{g}$

Quick Tip

Always ensure stoichiometric coefficients and molar mass calculations are accurate when determining product masses in reactions.

Question 85:

Consider the following reaction, the rate expression of which is given below:

$$A + B \to C \quad \text{Rate} = k[A]^{1/2}[B]^{1/2}$$

The reaction is initiated by taking 1M concentration A and B each. If the rate constant (k) is $4.6 \times 10^{-2}\,\text{s}^{-1}$, the time taken for A to become $0.1\,\text{M}$ is ..... sec. (nearest integer).
Answer: 50
Solution:

$$k = \frac{2.303}{t} \log \frac{[A]_0}{[A]}$$

Substitute the values:

$$4.6 \times 10^{-2} = \frac{2.303}{t} \log \frac{1}{0.1}$$

Simplify:

$$4.6 \times 10^{-2} = \frac{2.303}{t} \cdot 1$$

$$t = \frac{2.303}{4.6 \times 10^{-2}}$$

$$t = 50\,\text{sec.}$$

$$\boxed{\text{Answer: } 50\,\text{sec.}}$$

Quick Tip

When solving for time in rate law problems, always ensure the logarithmic values are calculated correctly for initial and final concentrations.

Question 86:

Phthalimide is made to undergo the following sequence of reactions:

$$\text{Phthalimide} \xrightarrow[\text{}]{\text{(i)KOH} \atop \text{(ii)Benzylchloride}} \text{'P'}$$

Total number of π bonds present in product 'P' is/are .....

Answer: 8

Solution: 1. Reaction Pathway:

- Phthalimide reacts with KOH, followed by benzyl chloride, resulting in the formation of benzyl phthalimide.

2. Structure of Product 'P':
- The final product contains:
- Two double bonds in the benzene ring (3π-bonds),
- Two double bonds in the phthalimide ring (3π-bonds),
- One double bond in the imide functional group (1π-bond).

3. Total π-Bonds:

$$\text{Total number of } \pi\text{-bonds} = 3 + 3 + 2 = 8$$

Quick Tip

Count π-bonds systematically from all double bonds, including aromatic and functional groups, to ensure accuracy.

Question 87:

The total number of 'sigma' and 'pi' bonds in 2-oxohex-4-ynoic acid is ....

Answer: 18

Solution:

1. Structure of 2-Oxohex-4-ynoic Acid:

$$\text{HO-C=O-CH}_2 - \text{CC-CH}_3$$

2. Bond Count:
- σ-Bonds:
- 1 O-H bond,
- 1 C-H bond in CH group,
- 4 C-H bonds in CH_3,

- 1 C=O bond (1σ),
- 2 C-C single bonds,
- 1 C-C triple bond (3σ).

Total σ-bonds = 14.

- π-Bonds:
- 1 C=O bond (1π),
- 2 CC bond (2π).

Total π-bonds = 4.

3. Total Bonds:

$$\text{Total bonds } = \sigma\text{-bonds} + \pi\text{-bonds} = 14 + 4 = 18$$

$$\boxed{\text{Answer: 18}}$$

Quick Tip

When counting σ and π bonds, ensure to include all single, double, and triple bonds systematically in the molecule.

Question 88.
A first-row transition metal with the highest enthalpy of atomisation, upon reaction with oxygen at high temperature, forms oxides of the formula $M2O_n$ (where $n = 3, 4, 5$). The "spin-only" magnetic moment value of the amphoteric oxide from the above oxides is ...... BM (near integer).

Given: Atomic numbers $Sc : 21, Ti : 22, V : 23, Cr : 24, Mn : 25, Fe : 26, Co : 27, Ni : 28, Cu : 29, Zn :$

Answer: (0)

Solution:

1. Identifying the Metal:
- Among the first-row transition metals, vanadium (V) has the highest enthalpy of atomisation (515 kJ/mol).
- Vanadium forms $V2O5$ upon oxidation, which is an amphoteric oxide.

2. Oxidation State of V in $V2O5$:
- In $V2O5$, the oxidation state of V is +5.
- Electronic configuration of V^{5+}: $[Ar]3d^0$.

3. Spin-Only Magnetic Moment:
- Magnetic moment (μ) is given by:

$$\mu = \sqrt{n(n+2)}\,\text{BM},$$

where n is the number of unpaired electrons. - For V^{5+}, $n = 0$ (no unpaired electrons). Substituting $n = 0$ into the formula:

$$\mu = \sqrt{0(0+2)} = 0\,\text{BM}.$$

4. Conclusion:
- The spin-only magnetic moment of the amphoteric oxide $V2O5$ is $\boxed{0}$ BM.

Quick Tip

The oxidation state and electronic configuration of the metal ion determine the magnetic moment. V^{5+} has no unpaired electrons, leading to a zero magnetic moment.

Question 89.
2.7 kg of each of water and acetic acid are mixed. The freezing point of the solution will be $-x°$C. Consider the acetic acid does not dimerize in water nor dissociate in water. $x =$ .... (nearest integer).

Given:
Molar mass of water $= 18\,\text{g mol}^{-1}$,
Molar mass of acetic acid $= 60\,\text{g mol}^{-1}$,
$K_f\,(\text{H}_2\text{O}) = 1.86\,\text{K kg mol}^{-1}$,
Freezing point of $\text{H}_2\text{O} = 273\,\text{K},\ \text{CH}_3\text{COOH} = 290\,\text{K}$.

Answer: (31)

Solution:

1. Identifying the Solvent and Solute:
- Since the moles of water (H_2O) are greater than the moles of acetic acid (CH_3COOH), water is the solvent and acetic acid is the solute.

2. Freezing Point Depression Formula:
The freezing point depression is given by:

$$\Delta T_f = K_f \times m,$$

where: - K_f is the cryoscopic constant of water, - m is the molality of the solution.

3. Calculating Molality:

$$\text{Mass of acetic acid (solute)} = 2.7\,\text{kg} = 2700\,\text{g}.$$
$$\text{Moles of acetic acid} = \frac{\text{Mass}}{\text{Molar Mass}} = \frac{2700}{60} = 45\,\text{mol}.$$
$$\text{Mass of water (solvent)} = 2.7\,\text{kg}.$$
$$\text{Molality}(m) = \frac{\text{Moles of solute}}{\text{Mass of solvent (in kg)}} = \frac{45}{2.7} = 16.67\,\text{mol kg}^{-1}.$$

4. Calculating Freezing Point Depression:
Substituting into the freezing point depression formula:

$$\Delta T_f = K_f \times m = 1.86 \times 16.67 = 31.03\,\text{K}.$$

5. Freezing Point of the Solution:
The freezing point of pure water is 0°C. The freezing point of the solution is:

$$T_f = 0°\text{C} - \Delta T_f = 0 - 31.03 \approx -31°\text{C}.$$

Quick Tip

In freezing point depression problems, identify the solvent and solute based on the higher mole quantity. Use the formula $\Delta T_f = K_f \times m$ for accurate calculations.

Question 90.
Vanillin compound obtained from vanilla beans has a total sum of oxygen atoms and π-electrons as:

Answer: (11)

Solution:

Vanillin is an organic compound with the molecular formula $C8H8O3$. It is a phenolic aldehyde and contains functional groups including aldehyde ($-CHO$), hydroxyl ($-OH$), and ether ($-OCH3$). Vanillin is the primary flavor compound extracted from vanilla beans.
1. Structure of Vanillin:

Structure includes: a benzene ring, aldehyde group, hydroxyl group, and methoxy group.

2. Total Number of Oxygen Atoms: - The molecular formula indicates the presence of 3 oxygen atoms.
3. Total Number of π-Bonds: - The benzene ring contributes 3 π-bonds. - The carbonyl group ($C = O$) contributes 1 π-bond.
Total π-bonds:

$$3(\text{benzene}) + 1(\text{C=O}) = 4.$$

4. Total Number of π-Electrons: - Each π-bond contains 2 electrons. - Total π-electrons:

$$4 \times 2 = 8.$$

5. Sum of Oxygen Atoms and π-Electrons:

$$3(\text{oxygen atoms}) + 8(\pi\text{-electrons}) = 11.$$

Correct Answer: 11

Quick Tip

To determine the total sum of oxygen atoms and π-electrons, count the oxygen atoms directly from the molecular formula and calculate π-electrons based on π-bonds in the structure.