# JEE Main 2024 Feb 1 (Shift 2) Questions

| **Total Time Allowed : ** 180 minutes | **Maximum Marks :** 300 | **Total Questions :** 90 | **Questions to be answered :** 75 |
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**Q1.** Let $f(x) = |2x^2 + 5|x| - 3|, x \in R$. If $m$ and $n$ denote the number of points where $f$ is not continuous and not differentiable respectively, then $m + n$ is equal to:

1. 5
2. 2
3. 0
4. 3

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**Q2.** Let $\alpha$ and $\beta$ be the roots of the equation $px^2 + qx - r = 0$, where $p \neq 0$. If $p, q, r$ be the consecutive terms of a non-constant G.P and $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{3}{4}$, then the value of $(\alpha - \beta)^2$ is:

1. $\frac{80}{9}$
2. 9
3. $\frac{20}{3}$
4. 8

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**Q3.** The number of solutions of the equation $4\sin^2 x - 4\cos^3 x + 9 - 4\cos x = 0$, $x \in [-2\pi, 2\pi]$ is:

1. 1
2. 3
3. 2
4. 0

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**Q4.** The value of $\int_0^1 (2x^3 - 3x^2 - x + 1)^3\, dx$ is equal to:

1. 0
2. 1
3. 2
4. -1

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**Q5.** Let $P$ be a point on the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$. Let the line passing through $P$ and parallel to the $y$-axis meet the circle $x^2+y^2=9$ at point $Q$ such that $P$ and $Q$ are on the same side of the $x$-axis. Then, the eccentricity of the locus of the point $R$ on $PQ$ such that $PR:RQ=4:3$ as $P$ moves on the ellipse, is:

1. $\frac{11}{19}$
2. $\frac{13}{21}$
3. $\frac{\sqrt{139}}{23}$
4. $\frac{\sqrt{13}}{7}$

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**Q6.** Let $m$ and $n$ be the coefficients of the seventh and thirteenth terms respectively in the expansion of

$$\left(\frac{1}{3x^3}+\frac{1}{2x^3}\right)^{18}$$

Then $\left(\frac{n}{m}\right)^{\frac{1}{3}}$ is:

1. $\frac{4}{9}$
2. $\frac{1}{9}$
3. $\frac{1}{4}$
4. $\frac{9}{4}$

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**Q7.** Let $\alpha$ be a non-zero real number. Suppose $f:R\to R$ is a differentiable function such that $f(0)=2$ and

$$\lim_{x\to\infty} f(x)=1.$$

If $f'(x)=\alpha f(x)+3$, for all $x\in R$, then $f(-\log_2 2)$ is equal to:

1. 3
2. 5
3. 9
4. 7

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**Q8.** Let $P$ and $Q$ be the points on the line:

$$\frac{x+3}{8}=\frac{y-4}{2}=\frac{z+1}{2}$$

which are at a distance of 6 units from the point $R(1,2,3)$. If the centroid of the triangle $PQR$ is $(\alpha,\beta,\gamma)$, then $\alpha^2+\beta^2+\gamma^2$ is:

1. 26
2. 36
3. 18
4. 24

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**Q9.** Consider a $\triangle ABC$ where $A(1,2,3)$, $B(-2,8,0)$, and $C(3,6,7)$. If the angle bisector of $\angle BAC$ meets the line $BC$ at $D$, then the length of the projection of the vector $\overrightarrow{AD}$ on the vector $\overrightarrow{AC}$ is:

1. $\frac{37}{2\sqrt{38}}$
2. $\frac{\sqrt{38}}{2}$
3. $\frac{39}{2\sqrt{38}}$
4. $\sqrt{19}$

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**Q10.** Let $S_n$ denote the sum of the first $n$ terms of an arithmetic progression. If $S_{10} = 390$ and the ratio of the tenth and the fifth terms is $15:7$, then $S_{15} - S_5$ is equal to:

1. 800
2. 890
3. 790
4. 690

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**Q11.** If

$$\int_0^{\frac{\pi}{3}} \cos^4 x\, dx = a\pi + b\sqrt{3},$$

where $a$ and $b$ are rational numbers, then $9a + 8b$ is equal to:

1. 2
2. 1
3. 3
4. $\frac{3}{2}$

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**Q12.** If $z$ is a complex number such that $|z| \geq 1$, then the minimum value of

$$\left| z + \frac{1}{2}(3 + 4i) \right|$$

is:

1. $\frac{5}{2}$
2. 2
3. 3
4. $\frac{3}{2}$

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**Q13.** If the domain of the function

$$f(x) = \frac{\sqrt{x^2 - 25}}{4 - x^2} + \log_{10}(x^2 + 2x - 15)$$

is $(-\infty, \alpha) \cup [\beta, \infty)$, then $\alpha^2 + \beta^3$ is equal to:

1. 140
2. 175
3. 150
4. 125

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**Q14.** Consider the relations $R_1$ and $R_2$ defined as $aR_1b$ if and only if $a^2 + b^2 = 1$ for all $a, b \in R$ and $(a, b)R_2(c, d)$ if and only if $a + d = b + c$ for all $(a, b), (c, d) \in N \times N$. Then:

1. Only $R_1$ is an equivalence relation
2. Only $R_2$ is an equivalence relation
3. $R_1$ and $R_2$ both are equivalence relations
4. Neither $R_1$ nor $R_2$ is an equivalence relation

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**Q15.** If the mirror image of the point $P(3, 4, 9)$ in the line

$$\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-2}{1}$$

is $(\alpha, \beta, \gamma)$, then $14(\alpha + \beta + \gamma)$ is:

1. 102
2. 138
3. 108
4. 132

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**Q16.** Let

$$f(x) = \begin{cases} x-1, & \text{if } x \text{ is even} \\ 2x, & \text{if } x \text{ is odd} \end{cases}, \quad x \in N.$$

If for some $a \in N$, $f(f(a)) = 21$, then

$$\lim_{x \to a} \left\{ \frac{x^3}{a} - \left\lfloor \frac{x}{a} \right\rfloor \right\},$$

where $\lfloor t \rfloor$ denotes the greatest integer less than or equal to $t$, is equal to:

1. 121
2. 144
3. 169
4. 225

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**Q17.** Let the system of equations

$$x + 2y + 3z = 5, \quad 2x + 3y + z = 9, \quad 4x + 3y + \lambda z = \mu$$

have an infinite number of solutions. Then $\lambda + 2\mu$ is equal to:

1. 28
2. 17
3. 22
4. 15

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**Q18.** Consider 10 observations $x_1, x_2, \ldots, x_{10}$ such that

$$\sum_{i=1}^{10}(x_i - \alpha) = 2 \quad \text{and} \quad \sum_{i=1}^{10}(x_i - \beta)^2 = 40,$$

where $\alpha$ and $\beta$ are positive integers. Let the mean and the variance of the observations be $\frac{6}{5}$ and $\frac{84}{25}$ respectively. The value of $\frac{\beta}{\alpha}$ is equal to:

1. 2
2. $\frac{3}{2}$
3. $\frac{5}{2}$
4. 1

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**Q19.** Let Ajay will not appear in JEE exam with probability $p = \frac{2}{7}$, while both Ajay and Vijay will appear in the exam with probability $q = \frac{1}{5}$. Then the probability that Ajay will appear in the exam and Vijay will not appear is:

1. $\frac{9}{35}$
2. $\frac{18}{35}$
3. $\frac{24}{35}$
4. $\frac{3}{35}$

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**Q20.** Let the locus of the midpoints of the chords of the circle $x^2 + (y-1)^2 = 1$ drawn from the origin intersect the line $x + y = 1$ at $P$ and $Q$. Then, the length of $PQ$ is:

1. $\frac{1}{\sqrt{2}}$
2. $\sqrt{2}$
3. $\frac{1}{2}$
4. 1

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**Q21.** If three successive terms of a G.P. with common ratio $r$ ($r > 1$) are the lengths of the sides of a triangle and $\lfloor r \rfloor$ denotes the greatest integer less than or equal to $r$, then $3\lfloor r \rfloor + \lfloor -r \rfloor$ is equal to:

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**Q22.** Let $A = I_2 - MM^T$, where $M$ is a real matrix of order $2 \times 1$ such that the relation $M^T M = I_1$ holds. If $\lambda$ is a real number such that the relation $AX = \lambda X$ holds for some non-zero real matrix $X$ of order $2 \times 1$, then the sum of squares of all possible values of $\lambda$ is equal to:

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**Q23.** Let $f : (0, \infty) \to R$ and

$$F(x) = \int_0^x t f(t)\, dt.$$

If $F(x^2) = x^4 + x^5$, then

$$\sum_{r=1}^{12} f(r^2)$$

is equal to:

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**Q24.** If

$$y = \frac{(\sqrt{x}+1)\left(x^2 - \sqrt{x}\right)}{x\sqrt{x} + x + \sqrt{x}} + \frac{1}{15}\left(3\cos^2 x - 5\right)\cos^3 x,$$

then $96y'\left(\frac{\pi}{6}\right)$ is equal to:

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**Q25.** Let

$$\vec{a} = \hat{i} + \hat{j} + \hat{k}, \quad \vec{b} = -\hat{i} - 8\hat{j} + 2\hat{k}, \quad \text{and} \quad \vec{c} = 4\hat{i} + c_2\hat{j} + c_3\hat{k}$$

be three vectors such that

$$\vec{b} \times \vec{a} = \vec{c} \times \vec{a}.$$

If the angle between the vector $\vec{c}$ and the vector $3\hat{i} + 4\hat{j} + \hat{k}$ is $\theta$, then the greatest integer less than or equal to $\tan^2\theta$ is:

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**Q26.** The lines $L_1, L_2, \ldots, L_{20}$ are distinct. For $n = 1, 2, 3, \ldots, 10$, all the lines $L_{2n-1}$ are parallel to each other, and all the lines $L_{2n}$ pass through a given point $P$. The maximum number of points of intersection of pairs of lines from the set $\{L_1, L_2, \ldots, L_{20}\}$ is equal to:

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**Q27.** Three points $O(0,0)$, $P(a, a^2)$, and $Q(-b, b^2)$ $(a > 0, b > 0)$ are on the parabola $y = x^2$. Let $S_1$ be the area of the region bounded by the line $PQ$ and the parabola, and $S_2$ be the area of the triangle $OPQ$. If the minimum value of $\frac{S_1}{S_2}$ is $\frac{m}{n}$ where $\gcd(m, n) = 1$, then $m + n$ is equal to:

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**Q28.** The sum of squares of all possible values of $k$, for which the area of the region bounded by the parabolas

$$2y^2 = kx \quad \text{and} \quad ky^2 = 2(y - x)$$

is maximum, is equal to:

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**Q29.** If

$$\frac{dx}{dy} = \frac{1 + x - y^2}{y}, \quad x(1) = 1,$$

then $5x(2)$ is equal to:

1. 5

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**Q30.** Let $\triangle ABC$ be an isosceles triangle in which $A$ is at $(-1, 0)$, $\angle A = \frac{2\pi}{3}$, $AB = AC$, and $B$ is on the positive $x$-axis. If $BC = 4\sqrt{3}$ and the line $BC$ intersects the line $y = x+3$ at $(\alpha, \beta)$, then $\frac{\beta^4}{\alpha^2}$ is:

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