JEE Main 2023 Mathematics Question Paper Jan 31 Shift 1

Time Allowed :3 Hour	Maximum Marks :120	Total Questions :30

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The test is of 3 hours duration.
2. The question paper consists of 30 questions. The maximum marks are 120.
3. The question paper consists of topics from Mathematics, having two sections.
(i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and –1 mark for wrong answer.
(ii) Section-B: This section contains 10 questions. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and –1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

1. For all z $\in$ C on the curve C$_1$: —z— = 4, let the locus of the point z + $\frac{1}{z}$ be the curve C$_2$. Then :

(A) the curves C_1 and C_2 intersect at 4 points
(B) the curve C_1 lies inside C_2
(C) the curves C_1 and C_2 intersect at 2 points
(D) the curve C_2 lies inside C_1

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2. The value of $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{(2+3\sin x)}{\sin x(1+\cos x)}dx$ is equal to

(A) $-2+3\sqrt{3}+\log_e\sqrt{3}$
(B) $\frac{10}{3}+\sqrt{3}+\log_e\sqrt{3}$
(C) $\frac{10}{3}-\sqrt{3}-\log_e\sqrt{3}$
(D) $\frac{10}{3}-\sqrt{3}+\log_e\sqrt{3}$

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3. Let a differentiable function f satisfy f(x) + $\int_3^x \frac{f(t)}{t}dt = \sqrt{x+1}$, x $\geq$ 3. Then 12f(8) is equal to :

(A) 34
(B) 1

(C) 19
(D) 17

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4. If the domain of the function f(x) = $\frac{[x]}{1+x^2}$, where [x] is greatest integer $\leq$ x, is [2,6), then its range is

(A) $(\frac{5}{37}, \frac{2}{5}] \cup \{\frac{9}{29}, \frac{27}{109}, \frac{18}{89}, \frac{9}{53}\}$

(B) $(\frac{5}{37}, \frac{2}{5}]$

(C) $(\frac{5}{26}, \frac{2}{5}]$

(D) $(\frac{5}{37}, \frac{5}{26}] \cup (\frac{2}{13}, \frac{4}{17}] \cup (\frac{3}{17}, \frac{3}{10}] \cup (\frac{1}{5}, \frac{2}{5}]$

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5. Let R be a relation on N$\times$N defined by (a, b)R(c, d) if and only if ad(b - c) = bc(a - d). Then R is

(A) reflexive and symmetric but not transitive
(B) transitive but neither reflexive nor symmetric
(C) symmetric and transitive but not reflexive
(D) symmetric but neither reflexive nor transitive

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6. Let y = f(x) represent a parabola with focus $(\frac{1}{2}, 0)$ and directrix y = $-\frac{1}{2}$. Then S = $\{x \in \mathbb{R} : \tan^{-1}(\sqrt{f(x)}) + \sin^{-1}(\sqrt{f(x)+1}) = \frac{\pi}{2}\}$:

(A) contains exactly one element
(B) is an infinite set
(C) contains exactly two elements
(D) is an empty set

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7. Let $y = f(x) = \sin^3\left(\frac{\pi}{3}\cos\left(\frac{\pi}{3\sqrt{2}}(-4x^3+5x^2+1)^{3/2}\right)\right)$. Then, at x = 1

(A) $2y' + 3\pi^2 y = 0$
(B) $\sqrt{2}y' - 3\pi^2 y = 0$
(C) $y' + 3\pi^2 y = 0$
(D) $2y' + \sqrt{3}\pi^2 y = 0$

8. If the sum and product of four positive consecutive terms of a G.P., are 126 and 1296, respectively, then the sum of common ratios of all such GPs is

(A) $\frac{9}{2}$
(B) 3
(C) 7
(D) 14

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9. For the system of linear equations
$\mathbf{x + y + z = 6}$
$\alpha\mathbf{x} + \beta\mathbf{y} + \mathbf{7z = 3}$
$\mathbf{x + 2y + 3z = 14}$
which of the following is NOT true?

(A) For every point $(\alpha,\beta) \neq (7,7)$ on the line x - 2y + 7 = 0, the system has infinitely many solutions
(B) If $\alpha = \beta = 7$, then the system has no solution
(C) There is a unique point (α,β) on the line x + 2y + 18 = 0 for which the system has infinitely many solutions
(D) If $\alpha = \beta$ and $\alpha \neq 7$, then the system has a unique solution

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10. Let $\alpha \in (0,1)$ and $\beta = \log_e(1-\alpha)$. Let $P_n(x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots + (-1)^{n-1}\frac{x^n}{n}$, $x \in (0,1)$. Then the integral $\int_0^\alpha \frac{t^{50}}{1-t}dt$ is equal to

(A) $P_{50}(\alpha) - \beta$
(B) $\beta - P_{50}(\alpha)$
(C) $\beta + P_{50}(\alpha)$
(D) $-(\beta + P_{50}(\alpha))$

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11. If the maximum distance of normal to the ellipse $\frac{x^2}{4} + \frac{y^2}{b^2} = 1$, b < 2, from the origin is 1, then the eccentricity of the ellipse is :

(A) $\frac{1}{\sqrt{2}}$
(B) $\frac{\sqrt{3}}{2}$
(C) $\frac{1}{2}$
(D) $\frac{\sqrt{5}}{3}$

12. A bag contains 6 balls. Two balls are drawn from it at random and both are found to be black. The probability that the bag contains at least 5 black balls is

(A) $\frac{2}{7}$

(B) $\frac{5}{7}$

(C) $\frac{5}{6}$

(D) $\frac{3}{7}$

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13. If $\sin^{-1}\frac{\alpha}{17} + \cos^{-1}\frac{4}{5} - \tan^{-1}\frac{77}{36} = 0,\ 0 < \alpha < 13,$ **then** $\sin^{-1}(\sin\alpha) + \cos^{-1}(\cos\alpha)$ **is equal to**

(A) π
(B) $16 - 5\pi$
(C) 16
(D) 0

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14. A wire of length 20 m is to be cut into two pieces. A piece of length l_1 **is bent to make a square of area** A_1**, and the other piece of length** l_2 **is made into a circle of area** A_2**. If** $2A_1 + 3A_2$ **is minimum then** $(\pi l_1) : l_2$ **is equal to :**

(A) 3:1
(B) 4:1
(C) 1:6
(D) 6:1

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15. Let A = $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 12 & -3 \end{pmatrix}$**. Then the sum of the diagonal elements of the matrix** $(A+I)^{11}$ **is equal to :**

(A) 4097
(B) 4094
(C) 2050
(D) 6144

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16. Let a circle C_1 be obtained on rolling the circle $x^2+y^2-4x-6y+11=0$ upwards 4 units on the tangent T to it at the point (3, 2). Let C_2 be the image of C_1 in T. Let A and B be the centres of circles C_1 and C_2 respectively, and M and N be respectively the feet of perpendiculars drawn from A and B on the x-axis. Then the area of the trapezium AMNB is:

(A) $4(1+\sqrt{2})$
(B) $3+2\sqrt{2}$
(C) $2(1+\sqrt{2})$
(D) $2(2+\sqrt{2})$

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17. Let $\vec{a}=2\hat{i}+\hat{j}+\hat{k}$ and $\vec{b},\vec{c}$ be two nonzero vectors such that $|\vec{a}+\vec{b}+\vec{c}|=|\vec{a}+\vec{b}-\vec{c}|$ and $\vec{b}\cdot\vec{c}=0$. Consider the following two statements :
(i) $|\vec{a}+\lambda\vec{c}|\geq|\vec{a}|$ for all $\lambda\in\mathbb{R}$
(ii) $\vec{a}$ and $\vec{c}$ are always parallel.
Then,

(A) both (i) and (ii) are correct
(B) only (ii) is correct
(C) neither (i) nor (ii) is correct
(D) only (i) is correct

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18. (S1) $(p\Rightarrow q)\vee(p\wedge(\sim q))$ is a tautology
(S2) $((\sim p)\Rightarrow(\sim q))\wedge((\sim p)\wedge q)$ is a contradiction. Then

(A) only (S1) is correct
(B) both (S1) and (S2) are correct
(C) both (S1) and (S2) are wrong
(D) only (S2) is correct

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19. The number of real roots of the equation $\sqrt{x^2-4x+3}+\sqrt{x^2-9}=\sqrt{4x^2-14x+6}$, is :

(A) 0
(B) 3
(C) 1
(D) 2

20. Let the shortest distance between the lines L: $\frac{x-5}{-2} = \frac{y-\lambda}{0} = \frac{z+\lambda}{1}, \lambda \geq 0$ and $L_1 : x+1 = y-1 = 4-z$ be $2\sqrt{6}$. If (α, β, γ) lies on L, then which of the following is NOT possible?

(A) $\alpha + 2\gamma = 24$
(B) $2\alpha - \gamma = 9$
(C) $\alpha - 2\gamma = 19$
(D) $2\alpha + \gamma = 7$

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21. If the variance of the frequency distribution

x_i	2	3	4	5	6	7	8
Frequency f_i	3	6	16	α	9	5	6

is 3, then α is equal to.........

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22. Number of 4-digit numbers that are less than or equal to 2800 and either divisible by 3 or by 11, is equal to.........

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23. Let for $x \in \mathbb{R}$, $f(x) = \frac{x+|x|}{2}$ and $g(x) = \begin{cases} x, & x < 0 \\ x^2, & x \geq 0 \end{cases}$. Then area bounded by the curve y = (fog)(x) and the lines y = 0, 2y - x = 15 is equal to..........

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24. Let 5 digit numbers be constructed using the digits 0, 2, 3, 4, 7, 9 with repetition allowed, and are arranged in ascending order with serial numbers. Then the serial number of the number 42923 is........

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25. Let the line L: $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-3}{1}$ intersect the plane 2x + y + 3z = 16 at the point P. Let the point Q be the foot of perpendicular from the point R(1, -1, -3) on the line L. If α is the area of triangle PQR, then α^2 is equal to........

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26. Let θ be the angle between the planes $P_1 : \vec{r}\cdot(\hat{i}+\hat{j}+2\hat{k}) = 9$ and $P_2 : \vec{r}\cdot(2\hat{i}-\hat{j}+\hat{k}) = 15$. Let L be the line that meets P_2 at the point (4, -2, 5) and makes an angle θ with

the normal of P_2. If α is the angle between L and P_2, then $(\tan^2\theta)(\cot^2\alpha)$ is equal to...........

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27. Let $\alpha > 0$, be the smallest number such that the expansion of $(x^{2/3} + \frac{2}{x^3})^{30}$ has a term $\beta x^{-\alpha}, \beta \in \mathbb{N}$. Then α is equal to.......

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28. The remainder on dividing 5^{99} by 11 is.......

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29. Let $a_1, a_2, ..., a_n$ be in A.P. If $a_5 = 2a_7$ and $a_{11} = 18$,
then $12\left(\frac{1}{\sqrt{a_{10}}+\sqrt{a_{11}}} + \frac{1}{\sqrt{a_{11}}+\sqrt{a_{12}}} + \cdots + \frac{1}{\sqrt{a_{17}}+\sqrt{a_{18}}}\right)$ is equal to..........

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30. Let $\vec{a}$ and $\vec{b}$ be two vectors such that $|\vec{a}| = \sqrt{14}, |\vec{b}| = \sqrt{6}$ and $|\vec{a} \times \vec{b}| = \sqrt{48}$. Then $(\vec{a} \cdot \vec{b})^2$ is equal to........
