JEE Main 2023 Mathematics Question Paper Jan 29 Shift 2

Time Allowed :3 Hour	**Maximum Marks :**120	**Total Questions :**30

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The test is of 3 hours duration.
2. The question paper consists of 30 questions. The maximum marks are 120.
3. The question paper consists of topics from Mathematics, having two sections.
(i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and –1 mark for wrong answer.
(ii) Section-B: This section contains 10 questions. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and –1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

1. Let S = {w1, w2,.......} be the sample space associated to a random experiment. Let $P(w_n) = \frac{P(w_{n-1})}{2}$, $n \geq 2$. Let $A = \{2k + 3l, k, l \in \mathbb{N}\}$ and $B = \{w_n : n \in A\}$. Then P(B) is equal to

(A) $\frac{1}{32}$

(B) $\frac{1}{64}$

(C) $\frac{3}{32}$

(D) $\frac{1}{16}$

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2. The statement B $\Leftrightarrow ((\sim A) \vee B)$ is equivalent to:

(A) $B \Rightarrow (A \wedge B)$
(B) $A \Rightarrow (A \vee B)$
(C) $A \Rightarrow B$
(D) $A \Leftrightarrow B$

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3. The number of 3 digit numbers, that are divisible by either 3 or 4 but not divisible by 48, is

(A) 472
(B) 432
(C) 507
(D) 400

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4. Consider a function $f : \mathbb{N} \to \mathbb{R}$, satisfying $f(1) + 2f(2) + 3f(3) + \cdots + xf(x) = x(x+1)f(x); x \geq 2$ with $f(1) = 1$. Then $\frac{1}{f(2022)} + \frac{1}{f(2028)}$ is equal to

(A) 8100
(B) 8200
(C) 8000
(D) 8400

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5. Let K be the sum of the coefficients of the odd powers of x in the expansion of $(1+x)^{99}$. Let a be the middle term in the expansion of $\left(2 + \frac{1}{\sqrt{2}}\right)^{200}$. If $\frac{^{200}C_{99} \cdot K}{a} = \frac{2^l m}{n}$, where m and n are odd numbers, then the ordered pair (l, n) is

(A) (51, 99)
(B) (50, 101)
(C) (50, 51)
(D) (51, 101)

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6. The shortest distance between the lines $\frac{x-1}{2} = \frac{y+8}{-7} = \frac{z-4}{5}$ and $\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-6}{-3}$ is

(A) $3\sqrt{3}$
(B) $2\sqrt{3}$
(C) $5\sqrt{3}$
(D) $4\sqrt{3}$

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7. The value of the integral $\int \frac{x^4+1}{x^6+1} dx$ is

(A) $\tan^{-1} 2 - \tan^{-1} 8 + \frac{\pi}{3}$
(B) $\tan^{-1} 2 + \tan^{-1} 8 - \frac{\pi}{3}$
(C) $\tan^{-1} 1 + \tan^{-1} 8 - \frac{\pi}{3}$
(D) $\tan^{-1} 1 - \tan^{-1} 8 + \frac{\pi}{3}$

8. Let f and g be twice differentiable functions on $\mathbb{R}$ such that
$f''(x) = g''(x) + 6x$
$f'(1) = 4g'(1) - 3 = 9$
$f(2) = 3g(2) = 12$.
Then which of the following is NOT true?

(A) If $-1 < x < 2$, then $f(x) - g(x) < 8$
(B) $f'(x) - g'(x) < 6$ for $-1 < x < 1$
(C) $g(-2) - f(-2) = 20$
(D) There exists $x_0 \in (1, 3/2)$ such that $f(x_0) = g(x_0)$

9. Let R be a relation defined on $\mathbb{N}$ as a R b if 2a + 3b is a multiple of 5, a, b $\in \mathbb{N}$. Then R is

(A) transitive but not symmetric
(B) an equivalence relation
(C) not reflexive
(D) symmetric but not transitive

10. If the tangent at a point P on the parabola $y^2 = 3x$ is parallel to the line $x + 2y = 1$ and the tangents at the points Q and R on the ellipse $\frac{x^2}{4} + \frac{y^2}{1} = 1$ are perpendicular to the line $x - y = 2$, then the area of the triangle PQR is:

(A) $\frac{5}{\sqrt{3}}$
(B) $5\sqrt{3}$
(C) $3\sqrt{5}$
(D) $\frac{9}{\sqrt{5}}$

11. If $\vec{a} = \hat{i} + 2\hat{k}$, $\vec{b} = \hat{i} + \hat{j} + \hat{k}$, $\vec{c} = 7\hat{i} - 3\hat{j} + 4\hat{k}$, $\vec{r} \times \vec{b} + \vec{b} \times \vec{c} = \vec{0}$ and $\vec{r} \cdot \vec{a} = 0$. Then $\vec{r} \cdot \vec{c}$ is equal to

(A) 30
(B) 32
(C) 36
(D) 34

12. If the lines $\frac{x-1}{2} = \frac{y-2}{1} = \frac{z+3}{2}$ and $\frac{x-a}{1} = \frac{y+2}{-3} = \frac{z-2}{1}$ intersect at the point P, then the distance of the point P from the plane $z = a$ is:

(A) 10
(B) 22
(C) 28
(D) 16

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13. The value of the integral $\int_{1/2}^{2} \frac{\tan^{-1} x}{x} dx$ is equal to

(A) $\frac{1}{2} \log_e 2$
(B) $\frac{\pi}{4} \log_e 2$
(C) $\pi \log_e 2$
(D) $\frac{\pi}{2} \log_e 2$

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14. The plane $2x - y + z = 4$ intersects the line segment joining the points A(a, -2, 4) and B(2, b, -3) at the point C in the ratio 2:1 and the distance of the point C from the origin is $\sqrt{5}$. If $ab < 0$ and P is the point (a-b, b, 2b-a) then CP^2 is equal to

(A) $\frac{16}{3}$

(B) $\frac{17}{3}$

(C) $\frac{73}{3}$

(D) $\frac{97}{3}$

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15. The area of the region $A = \{(x, y) : |\cos x - \sin x| \leq y \leq \sin x, 0 \leq x \leq \frac{\pi}{2}\}$ is

(A) $\sqrt{5} - 2\sqrt{2} + 1$
(B) $1 - \frac{1}{\sqrt{2}}$
(C) $\sqrt{2} - 1$
(D) $\sqrt{5} + 2\sqrt{2} - 4.5$

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16. The letters of the word OUGHT are written in all possible ways and these words are arranged as in a dictionary, in a series. Then the serial number of the word TOUGH is

(A) 79
(B) 86
(C) 84
(D) 89

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17. Let $\vec{a} = 4\hat{i} + 3\hat{j}$ and $\vec{\beta} = 3\hat{i} - 4\hat{j} + 5\hat{k}$. If $\vec{c}$ is a vector such that $\vec{c} \cdot (\vec{a} \times \vec{\beta}) + 25 = 0$, $\vec{c} \cdot (\hat{i} + \hat{j} + \hat{k}) = 4$, and projection of $\vec{c}$ on $\vec{a}$ is 1, then the projection of $\vec{c}$ on $\vec{\beta}$ equals

(A) $\frac{5}{\sqrt{2}}$
(B) $\frac{1}{\sqrt{2}}$
(C) $\frac{3}{\sqrt{2}}$
(D) $\frac{7}{\sqrt{2}}$

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18. The set of all values of λ for which the equation $\cos^2(2x) - 2\sin^2(x) - 2\cos^2(x) = \lambda$ has a real solution x, is

(A) $[1, \frac{3}{2}]$
(B) $[\frac{1}{2}, 1]$
(C) $[\frac{3}{2}, 2]$
(D) $[-2, -1]$

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19. The set of all values of $t \in \mathbb{R}$, for which the matrix
$A = \begin{pmatrix} e^t & e^{-t}(\sin t - 2\cos t) & e^{-t}(-2\sin t - \cos t) \\ e^t & e^{-t}(2\sin t + \cos t) & e^{-t}(\sin t - 2\cos t) \\ e^t & e^{-t}\cos t & e^{-t}\sin t \end{pmatrix}$ **is invertible, is**

(A) $\{(2k+1)\frac{\pi}{2}, k \in \mathbb{Z}\}$
(B) $\mathbb{R}$
(C) $\{k\pi, k \in \mathbb{Z}\}$
(D) $\{k\pi + \frac{\pi}{4}, k \in \mathbb{Z}\}$

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20. Let $y = y(x)$ be the solution of the differential equation $x \log_e x \frac{dy}{dx} + y = x^2 \log_e x, (x > 1)$. If $y(2) = 2$, then $y(e)$ is equal to

(A) $\frac{2+e^2}{2}$
(B) $\frac{1+e^2}{2}$
(C) $\frac{1+e^2}{4}$

(D) $\frac{4+e^2}{4}$

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21. The total number of 4-digit numbers whose greatest common divisor with 54 is 2, is

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22. If the equation of the normal to the curve $y = \frac{x-a}{(x+b)(x-2)}$ **at the point** $(1, -3)$ **is** $x - 4y = 13$**, then the value of** $a + b$ **is equal to**

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23. Let $X = \{11, 12, 13, ..., 40, 41\}$ **and** $Y = \{61, 62, 63, ..., 90, 91\}$ **be the two sets of observations. If** $\bar{x}$ **and** $\bar{y}$ **are their respective means and** σ^2 **is the variance of all the observations in** $X \cup Y$**, then** $\bar{x} + \bar{y} - \sigma^2$ **is equal to**

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24. A triangle is formed by the tangents at the point (2, 2) on the curves $y^2 = 2x$ **and** $x^2 + y^2 = 4x$**, and the line** $x + y + 2 = 0$**. If r is the radius of its circumcircle, then** r^2 **is equal to**

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25. Let $a_1, a_2, ..., a_7$ **be the roots of the equation** $x^7 + 3x^5 - 13x^3 - 15x = 0$ **and** $|a_1| \geq |a_2| \geq ... \geq |a_7|$**. Then** $a_1a_2 - a_3a_4 + a_5a_6$ **is equal to**

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26. Let A be a symmetric matrix such that $A\begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$**. If the sum of the diagonal elements of A is s, then** $\frac{s^2}{2}$ **is equal to**

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27. Let $a_1 = b_1 = 1$ **and** $a_n = a_{n-1} + (n-1)$**,** $b_n = b_{n-1} + a_{n-1}$**,** $\forall n \geq 2$**. If** $S = \sum_{n=1}^{\infty} \frac{b_n}{2^n}$ **and** $T = \sum_{n=1}^{\infty} \frac{a_n}{2^{n-1}}$**, then** $2^7(2S - T)$ **is equal to**

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28. A circle with centre (2, 3) and radius 4 intersects the line $x + y = 3$ **at the points P and Q. If the tangents at P and Q intersect at the point** $S(\alpha, \beta)$**, then** $4\alpha - 7\beta$ **is**

equal to

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29. Let $\{a_k\}$ and $\{b_k\}$, $k \in \mathbb{N}$, be two G.P.s with common ratios r_1 and r_2 respectively such that $a_1 = b_1 = 4$ and $r_1 < r_2$. Let $c_k = a_k + b_k$, $k \in \mathbb{N}$. If $c_2 = 5$ and $c_3 = \frac{13}{4}$, then $\sum_{k=1}^{\infty} c_k - (12a_6 - 8b_4)$ is equal to

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30. Let $\alpha = 8 - 14i$, $A = \{z \in \mathbb{C} : |\frac{\alpha z - \bar{\alpha}\bar{z}}{z^2 - (\bar{z})^2 - 112i}| = 1\}$ and $B = \{z \in \mathbb{C} : |z + 3i| = 4\}$. Then $\sum_{z \in A \cap B}$(Re $z -$ Im z) is equal to

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