JEE Main 2023 Jan 29 Shift 1 Mathematics Question Paper

Total Time Allowed : 60 minutes	Maximum Marks : 100	Total Questions : 30	Questions to be answered :30

General Instructions

Read the following instructions very carefully and strictly follow them:

(1) The test is of 1 hour duration.
(2) The question paper consists of 30 questions. The maximum marks are 100.
(3) This is the question paper of Mathematics having 30 questions in each part of equal weightage.
(4) Each part has two sections.
(i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and –1 mark for wrong answer.
(ii) Section-B: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and –1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

Section-A

Question 1: The domain of

$$f(x) = \frac{\log_{x+1}(x-2)}{e^{2\log_x x} - (2x+3)},\ x \in R$$

is:

(1) $R - \{-1, 3\}$
(2) $(2, \infty) - \{3\}$
(3) $(-1, \infty) - \{3\}$
(4) $R - \{3\}$

Question 2: Let

$$f(x) = \frac{x^2 + 2x + 1}{x + 1}.$$

Then:

(1) $f(x)$ is many-one in $(-\infty, -1)$
(2) $f(x)$ is many-one in $(1, \infty)$
(3) $f(x)$ is one-one in $[1, \infty)$ but not in $(-\infty, \infty)$
(4) $f(x)$ is one-one in $(-\infty, \infty)$

Question 3: For two non-zero complex numbers z_1 and z_2, if

$$\text{Re}(z_1 z_2) = 0 \quad \text{and} \quad \text{Re}(z_1 + z_2) = 0,$$

then which of the following are possible?

(A) $\text{Im}(z_1) > 0$ and $\text{Im}(z_2) > 0$
(B) $\text{Im}(z_1) < 0$ and $\text{Im}(z_2) > 0$
(C) $\text{Im}(z_1) > 0$ and $\text{Im}(z_2) < 0$
(D) $\text{Im}(z_1) < 0$ and $\text{Im}(z_2) < 0$

Question 4: Let $\lambda \neq 0$ be a real number. Let α, β be the roots of the equation

$$14x^2 - 3\lambda x + 3\lambda = 0,$$

and α, γ be the roots of the equation

$$35x^2 - 53x + 4\lambda = 0.$$

Then $\frac{3\alpha}{\beta}$ and $\frac{4\alpha}{\gamma}$ are the roots of the equation:

(1) $7x^2 + 245x - 250 = 0$
(2) $7x^2 - 245x + 250 = 0$
(3) $49x^2 - 245x + 250 = 0$
(4) $49x^2 + 245x + 250 = 0$

Question 5: Consider the following system of equations:

$$\alpha x + 2y + z = 1$$

$$2\alpha x + 3y + z = 1$$

$$3x + \alpha y + 2z = \beta$$

For some $\alpha, \beta \in R$. Then which of the following is NOT correct:

(1) It has no solution if $\alpha = -1$ and $\beta \neq 2$.
(2) It has no solution for $\alpha = -1$ and for all $\beta \in R$.
(3) It has no solution for $\alpha = 3$ and for all $\beta \neq 2$.
(4) It has a solution for all $\alpha \neq -1$ and $\beta = 2$.

Question 6: Let α and β be real numbers. Consider a 3×3 matrix A such that:

$$A^2 = 3A + \alpha I,$$
$$A^4 = 21A + \beta I.$$

Then:

(1) $\alpha = 1$
(2) $\alpha = 4$
(3) $\beta = 8$
(4) $\beta = -8$

Question 7: Let $x = 2$ be a root of the equation $x^2 + px + q = 0$ and

$$f(x) = \begin{cases} \frac{1-\cos\left(x^2-4px+q-8q+16\right)}{(x-2p)^2}, & x \neq 2p, \\ 0, & x = 2p. \end{cases}$$

Then

$$\lim_{x \to 2p} [f(x)],$$

where $[\cdot]$ denotes the greatest integer function, is:

(1) 2
(2) 1
(3) 0
(4) -1

Question 8: Let

$$f(x) = x + \frac{a}{\pi^2 - 4} \sin x + \frac{b}{\pi^2 - 4} \cos x, \quad x \in R$$

be a function which satisfies

$$f(x) = x + \int_0^{\pi/2} \sin(x+y) f(y)\, dy.$$

Then $(a + b)$ is equal to:

(1) $-\pi(\pi + 2)$
(2) $-2\pi(\pi + 2)$
(3) $-2\pi(\pi - 2)$
(4) $-\pi(\pi - 2)$

Question 9: Let

$$A = \{(x, y) \in R^2 : y \geq 0, 2x \leq y \leq \sqrt{4 - (x-1)^2}\}$$

$$B = \{(x, y) \in R^2 : 0 \leq y \leq \min\{2x, \sqrt{4 - (x-1)^2}\}\}.$$

Then the ratio of the area of A to the area of B is:

(1) $\frac{\pi-1}{\pi+1}$
(2) $\frac{\pi}{\pi-1}$
(3) $\frac{\pi}{\pi+1}$
(4) $\frac{\pi+1}{\pi-1}$

Question 10: Let Δ be the area of the region

$$\{(x, y) \in R^2 : x^2 + y^2 \leq 21, y^2 \leq 4x, x \geq 1\}.$$

Then

$$\frac{1}{2}\left(\Delta - 21\sin^{-1}\left(\frac{2}{\sqrt{7}}\right)\right)$$

is equal to:

(1) $2\sqrt{3} - \frac{1}{3}$
(2) $\sqrt{3} - \frac{2}{3}$
(3) $2\sqrt{3} - \frac{2}{3}$
(4) $\sqrt{3} - \frac{4}{3}$

Question 11: A light ray emits from the origin making an angle $30°$ with the positive x-axis. After getting reflected by the line $x + y = 1$, if this ray intersects the x-axis at Q, then the abscissa of Q is:

(1) $\frac{2}{\sqrt{3}-1}$
(2) $\frac{2}{3+\sqrt{3}}$
(3) $\frac{2}{3-\sqrt{3}}$
(4) $\frac{\sqrt{3}}{2(\sqrt{3}+1)}$

Question 12: Let B and C be the two points on the line $y + x = 0$ such that B and C are symmetric with respect to the origin. Suppose A is a point on $y - 2x = 2$ such that $\triangle ABC$ is an equilateral triangle. Then, the area of $\triangle ABC$ is:

(1) $\sqrt{3}$
(2) $2\sqrt{3}$
(3) $\frac{8}{\sqrt{3}}$
(4) $\frac{10}{\sqrt{3}}$

Question 13: Let the tangents at the points $A(4, -11)$ and $B(8, -5)$ on the circle $x^2 + y^2 - 3x + 10y - 15 = 0$ intersect at the point C. Then the radius of the circle, whose center is C and the line joining A and B is its tangent, is equal to:

(1) $\frac{3\sqrt{3}}{4}$
(2) $2\sqrt{13}$
(3) 13
(4) $\frac{2\sqrt{13}}{3}$

Question 14: Let (x) denote the greatest integer. Consider the function $f(x) = \max\{x^2, 1 + [x]\}$, where $[x]$ denotes the greatest integer $\leq x$. Then the value of the integral

$$\int_0^2 f(x)\, dx$$

is:

(1) $\frac{5}{3} + 4\sqrt{2}$
(2) $\frac{8}{3} + 4\sqrt{2}$
(3) $\frac{1}{3} + 5\sqrt{2}$
(4) $\frac{4}{3} + 5\sqrt{2}$

Question 15: If the vectors $\mathbf{a} = 4\hat{i} + 2\hat{j} + 4\hat{k}$, $\mathbf{b} = \lambda\hat{i} - \mu\hat{j} - 2\hat{k}$, and $\mathbf{c} = 2\hat{i} + 3\hat{j}$ are coplanar, and the projection of a on b is 54 units, then the sum of all possible values of $\lambda + \mu$ is equal to:

(1) 0
(2) 6
(3) 24
(4) 18

Question 16: Fifteen football players of a club are given 15 T-shirts with their names written on the back. If the players pick up the T-shirts randomly, then the probability that at least 3 players pick the correct T-shirt is:

(1) $\frac{5}{24}$
(2) $\frac{2}{15}$
(3) $\frac{1}{6}$
(4) $\frac{5}{36}$

Question 17: Let

$$f(\theta) = 3\left(\sin^2\left(\frac{3\pi}{2} - \theta\right) + \sin^4\left(\frac{3\pi}{2} + \theta\right)\right) - 2(1 - \sin^2 2\theta),$$

and

$$S = \{\theta \in [0, \pi] : f'(\theta) = -\frac{\sqrt{3}}{2}\}.$$

If $4\beta = \sum_{\theta \in S} \theta$, then $f(\beta)$ is equal to:

(1) $\frac{11}{8}$
(2) $\frac{5}{4}$
(3) $\frac{9}{8}$
(4) $\frac{3}{2}$

Question 18: If p, q, and r are three propositions, then which of the following combinations of truth values of p, q, and r makes the logical expression

$$\{(p \vee q) \wedge ((\neg p) \vee r)\} \rightarrow ((\neg q) \vee r)$$

false?

(1) $p = \mathrm{T}$, $q = \mathrm{F}$, $r = \mathrm{T}$
(2) $p = \mathrm{T}$, $q = \mathrm{T}$, $r = \mathrm{F}$
(3) $p = \mathrm{F}$, $q = \mathrm{T}$, $r = \mathrm{F}$
(4) $p = \mathrm{T}$, $q = \mathrm{F}$, $r = \mathrm{F}$

Question 19: Three rotten apples are accidentally mixed with seven good apples, and four apples are drawn one by one without replacement. Let the random variable X denote the number of rotten apples. If μ and σ^2 represent the mean and variance of X, respectively, then $10(\mu^2 + \sigma^2)$ is equal to:

(1) 20
(2) 250
(3) 25
(4) 30

Question 20: Let $y = f(x)$ be the solution of the differential equation

$$y(x+1)\,dx - x^2\,dy = 0, \quad y(1) = e.$$

Then $\lim_{x \to 0^+} f(x)$ is equal to:

(1) 0
(2) $\frac{1}{e}$
(3) e^2
(4) $\frac{2}{e}$

Section-B

Question 21: Let the coordinates of one vertex of $\triangle ABC$ be $A(0, 2, \alpha)$ and the other two vertices lie on the line

$$\frac{x+\alpha}{5} = \frac{y-1}{2} = \frac{z+4}{3}.$$

For $\alpha \in Z$, if the area of $\triangle ABC$ is 21 sq. units and the line segment BC has length $2\sqrt{21}$ units, then α^2 is equal to——.

Question 22: Let the equation of the plane P containing the line

$$\frac{y-x}{2} = \frac{z+8}{10} = t$$

be $ax+by+3z = 2(a+b)$, and the distance of the plane P from the point $(1, 27, 7)$ be c. Then $a^2+b^2+c^2$ is equal to——.

Question 23: Suppose f is a function satisfying $f(x+y) = f(x) + f(y)$ for all $x, y \in N$ and $f(1) = \frac{1}{5}$. If

$$\sum_{n=1}^{m} \frac{f(n)}{n(n+1)(n+2)} = \frac{1}{12},$$

then m is equal to———.

Question 24: Let $a_1, a_2, a_3, \ldots$ be a geometric progression (GP) of increasing positive numbers. If the product of the fourth and sixth terms is 9 and the sum of the fifth and seventh terms is 24, then $a_1a_9 + a_2a_4a_9 + a_5 + a_7$ is equal to———.

Question 25: Let a, b, and c be three non-zero, non-coplanar vectors. Let the position vectors of four points A, B, C, and D be $\mathbf{a} - \mathbf{b} + \mathbf{c}$, $\lambda\mathbf{a} - 3\mathbf{b} + 4\mathbf{c}$, $-\mathbf{a} + 2\mathbf{b} - 3\mathbf{c}$, and $2\mathbf{a} + 4\mathbf{b} + 6\mathbf{c}$ respectively. If $\overrightarrow{AB}$, $\overrightarrow{AC}$, and $\overrightarrow{AD}$ are coplanar, then λ is———-.

Question 26: If all the six-digit numbers $x_1x_2x_3x_4x_5x_6$ with $0 < x_1 < x_2 < x_3 < x_4 < x_5 < x_6$ are arranged in increasing order, then the sum of the digits in the 72-th number is?

Question 27: Let $f : R \to R$ be a differentiable function that satisfies the relation

$$f(x+y) = f(x) + f(y) - 1, \quad \forall x, y \in R.$$

If $f'(0) = 2$, then $|f(-2)|$ is equal to———-.

Question 28: If the coefficient of x^9 in $\left(ax^3 + \frac{1}{\beta x}\right)^{11}$ and the coefficient of x^{-9} in $\left(ax - \frac{1}{\beta x^3}\right)^{11}$ are equal, then $(\alpha\beta)^2$ is equal to——.

Question 29: Let the coefficients of three consecutive terms in the binomial expansion of $(1+2x)^n$ be in the ratio $2:5:8$. Then the coefficient of the term, which is in the middle of these three terms, is ?

Question 30: Five-digit numbers are formed using the digits $\{1, 2, 3, 5, 7\}$ with repetitions allowed, and are written in descending order with serial numbers.

For example, the number 77777 has serial number 1. Then the serial number of 35337 is——.
