JEE Main 2023 Jan 25 Shift 1 Mathematics Question Paper

Time Allowed :3 Hour	**Maximum Marks :**300	**Total Questions :**90

General Instructions

Read the following instructions very carefully and strictly follow them:
(A) The test is of 3 hours duration.
(B) The question paper consists of 90 questions. The maximum marks are 300.
(C) There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage.
(D) Each part (subject) has two sections.
(i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and –1 mark for wrong answer.
(ii) Section-B: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and –1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

1. Let M be the maximum value of the product of two positive integers when their sum is 66. Let the sample space $S = \{x \in Z : (66 - x)x \geq \frac{5}{9}M\}$ and the event $A = \{x \in S : x$ is a multiple of $3\}$. Then $P(A)$ is equal to:

1. $\frac{15}{44}$
2. $\frac{1}{3}$
3. $\frac{7}{22}$
4. $\frac{1}{2}$

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2. Let $\vec{a}, \vec{b}, \vec{c}$ be three non-zero vectors such that $\vec{b} \cdot \vec{c} = 0$ and $\vec{a} \times \vec{b} = \frac{\vec{b} - \vec{c}}{2}$. If $\vec{d}$ is a vector such that $\vec{b} \cdot \vec{d} = \vec{a} \cdot \vec{b}$, then $(\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d})$ is equal to:

1. 1
2. $\frac{1}{4}$
3. 2
4. $\frac{1}{2}$

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3. Let $y = y(x)$ be the solution curve of the differential equation

$$\frac{dy}{dx} = \frac{y}{x}(1 + xy^2(1 + \log x)), \quad x > 0,\ y(1) = 3.$$

Then $\frac{y^2(x)}{9}$ is equal to:

1. $\frac{x^2}{5-2x^3(2+\log x^3)}$
2. $\frac{x^2}{2x^3(2+\log x^3)-3}$
3. $\frac{x^2}{3x^3(1+\log x^2)-2}$
4. $\frac{x^2}{7-3x^3(2+\log x^2)}$

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4. The value of

$$\lim_{n\to\infty} \frac{1 + 2 - 3 + 4 + 5 - 6 + \ldots + (3n-2) + (3n-1) - 3n}{\sqrt{2n^4 + 4n + 3} - \sqrt{n^4 + 5n + 4}}$$

is:

1. $\frac{\sqrt{2}+1}{2}$
2. $3(\sqrt{2} + 1)$
3. $\frac{3}{2}(\sqrt{2} + 1)$
4. $\frac{3}{2}\sqrt{2}$

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5. The points of intersection of the line $ax+by = 0$, $(a \neq b)$ and the circle $x^2+y^2-2x = 0$ are $A(\alpha, 0)$ and $B(1, \beta)$. The image of the circle with AB as a diameter in the line $x + y + 2 = 0$ is:

1. $x^2 + y^2 + 5x + 5y + 12 = 0$
2. $x^2 + y^2 + 3x + 5y + 8 = 0$
3. $x^2 + y^2 + 3x + 3y + 4 = 0$
4. $x^2 + y^2 - 5x - 5y + 12 = 0$

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6. The mean and variance of the marks obtained by the students in a test are 10 and 4 respectively. Later, the marks of one of the students is increased from 8 to 12. If the new mean of the marks is 10.2, then their new variance is equal to:

1. 4.04
2. 4.08
3. 3.96
4. 3.92

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7. Let

$$y(x) = (1+x)(1+x^2)(1+x^4)(1+x^8)(1+x^{16}).$$

Then $y' - y''$ at $x = -1$ is equal to:

1. 976
2. 464
3. 496
4. 944

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8. The vector $\vec{a} = -\hat{i} + 2\hat{j} + \hat{k}$ is rotated through a right angle, passing through the y-axis in its way, and the resulting vector is $\vec{b}$. Then the projection of $3\vec{a} + \sqrt{2}\vec{b}$ on $\vec{c} = 5\hat{i} + 4\hat{j} + 3\hat{k}$ is:

1. $3\sqrt{2}$
2. 1
3. $\sqrt{6}$
4. $2\sqrt{3}$

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9. The minimum value of the function

$$f(x) = \int_0^2 e^{|k-t|} dt$$

is:

1. $2(e-1)$
2. $2e-1$
3. 2
4. $e(e-1)$

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10. Consider the lines L_1 and L_2 given by

$$L_1 : \frac{x-1}{2} = \frac{y-3}{2} = \frac{z-2}{2}, \quad L_2 : \frac{x-2}{1} = \frac{y-2}{2} = \frac{z-3}{3}.$$

A line L_3 having direction ratios $1, -1, -2$ intersects L_1 and L_2 at the points P and Q respectively. Then the length of line segment PQ is:

1. $2\sqrt{6}$
2. $3\sqrt{2}$
3. $4\sqrt{3}$
4. 4

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11. Let $x = 2$ be a local minima of the function

$$f(x) = 2x^4 - 18x^2 + 8x + 12, \quad x \in (-4, 4).$$

If M is the local maximum value of the function $f(x)$ in $(-4, 4)$, then M is:

1. $12\sqrt{6} - \frac{33}{2}$
2. $12\sqrt{6} - \frac{31}{2}$
3. $18\sqrt{6} - \frac{33}{2}$
4. $18\sqrt{6} - \frac{31}{2}$

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12. Let $z_1 = 2 + 3i$ and $z_2 = 3 + 4i$. The set

$$S = \{z \in C : |z - z_1|^2 - |z - z_2|^2 = |z_1 - z_2|^2\}$$

represents a:

1. straight line with sum of its intercepts on the coordinate axes 14
2. hyperbola with the length of the transverse axis 7
3. straight line with the sum of its intercepts on the coordinate axes equals -18
4. hyperbola with eccentricity 2

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13. The distance of the point $(6, -2\sqrt{2})$ from the common tangent $y = mx + c$, $m > 0$, of the curves $x = 2y^2$ and $x = 1 + y^2$ is:

1. $\frac{1}{3}$
2. 5
3. $\frac{14}{3}$
4. $5\sqrt{3}$

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14. Let S_1 and S_2 be respectively the sets of all $a \in R - \{0\}$ for which the system of linear equations:

$$\begin{aligned} ax + 2ay - 3az &= 1, \\ (2a+1)x + (2a+3)y + (a+1)z &= 2, \\ (3a+5)x + (a+5)y + (a+2)z &= 3, \end{aligned}$$

has unique solution and infinitely many solutions. Then:

1. $n(S_1) = 2$ and S_2 is an infinite set
2. S_1 is an infinite set and $n(S_2) = 2$
3. $S_1 = \emptyset$ and $S_2 = R - \{0\}$
4. $S_1 = R - \{0\}$ and $S_2 = \emptyset$

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15. Let $f(x) = \int \frac{2x}{x^2+1}(x^2+3)\,dx$. If $f(3) = \frac{1}{2}(\log_e 5 - \log_e 6)$, then $f(4)$ is equal to:

1. $\frac{1}{2}(\log_e 17 - \log_e 19)$
2. $\log_e 17 - \log_e 18$
3. $\frac{1}{2}(\log_e 19 - \log_e 17)$
4. $\log_e 19 - \log_e 17$

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16. The statement $(p \wedge (\sim q)) \Rightarrow (p \Rightarrow (\sim q))$ is:

1. Equivalent to $(\sim p) \vee (\sim q)$
2. A tautology
3. Equivalent to $p \vee q$
4. A contradiction

17. Let $f : (0, 1) \to R$ be a function defined by

$$f(x) = \frac{1}{1 - e^{-x}},$$

and

$$g(x) = (f(-x) - f(x)).$$

Consider two statements:

(I) g is an increasing function in $(0, 1)$,

(II) g is one-one in $(0, 1)$.

Then:

1. Only (I) is true
2. Only (II) is true
3. Neither (I) nor (II) is true
4. Both (I) and (II) are true

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18. The distance of the point P(4, 6, -2) from the line passing through the point $(-3, 2, 3)$ and parallel to a line with direction ratios 3, 3, -1 is equal to:

1. 3
2. $\sqrt{6}$
3. $2\sqrt{3}$
4. $\sqrt{14}$

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19. Let $x, y, z > 1$ and

$$A = \begin{bmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 2 & \log_y z \\ \log_z x & \log_z y & 3 \end{bmatrix}.$$

Then adj(adjA^2) is equal to:

1. 6^4
2. 2^8
3. 4^8
4. 2^4

20. If a_r is the coefficient of x^{10-r} in the binomial expansion of $(1+x)^{10}$, then

$$\sum_{r=1}^{10} r^3 \left(\frac{a_r}{a_{r-1}}\right)^2 \text{ is equal to:}$$

1. 4895
2. 1210
3. 5445
4. 3025

21: Number of Non-Empty Subsets with Sum Divisible by 3
Problem: Let $S = \{1, 2, 3, 5, 7, 10, 11\}$. The number of non-empty subsets of S such that the sum of their elements is divisible by 3 is __.

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22. For some $a, b, c \in N$, let $f(x) = ax - 3$ and $g(x) = x^b + c$, $x \in R$. If $(f \circ g)^{-1}(x) = \left(\frac{x-7}{2}\right)^{1/3}$, then $(f \circ g)(ac) + (g \circ f)(b)$ is equal to __.

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23. The vertices of a hyperbola H are $(\pm 6, 0)$ and its eccentricity is $\frac{\sqrt{5}}{2}$. Let N be the normal to H at a point in the first quadrant and parallel to the line $\sqrt{2}x + y = 2\sqrt{2}$. If d is the length of the line segment of N between H and the y-axis, then d^2 is equal to __.

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24. Let $S = \left\{\alpha : \log_2\left(9^{2\alpha-4} + 13\right) - \log_2\left(\frac{5}{2} \cdot 3^{2\alpha-4} + 1\right) = 2\right\}$. Then the maximum value of β for which the equation

$$x^2 - 2\left(\sum_{\alpha \in S} \alpha\right)x + \sum_{\alpha \in S}(\alpha+1)^2\beta = 0$$

has real roots, is __.

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25. The constant term in the expansion of $\left(2x + \frac{1}{x^7} + 3x^2\right)^5$ is __.

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26. Let A_1, A_2, A_3 be the three A.P. with the same common difference d and having their first terms as $A, A+1, A+2$, respectively. Let a, b, c be the 7th, 9th, and 17th terms of A_1, A_2, A_3, respectively, such that

$$\begin{vmatrix} a & 7 & 1 \\ 2b & 17 & 1 \\ c & 17 & 1 \end{vmatrix} + 70 = 0.$$

If $a = 29$, then the sum of the first 20 terms of an AP whose first term is $c - a - b$ and common difference is $\frac{d}{12}$, is equal to __.

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27. If the sum of all the solutions of

$$\tan^{-1}\left(\frac{2x}{1-x^2}\right) + \cot^{-1}\left(\frac{1-x^2}{2x}\right) = \frac{\pi}{3},$$

where $-1 < x < 1, x \neq 0$, is $\alpha - \frac{4}{\sqrt{3}}$, then α is equal to __.

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28. Let the equation of the plane passing through the line

$$x - 2y - z - 5 = 0 \quad \text{and} \quad x + y + 3z - 5 = 0,$$

and parallel to the line

$$x + y + 2z - 7 = 0 \quad \text{and} \quad 2x + 3y + z - 2 = 0,$$

be $ax + by + cz = 65$. Then the distance of the point (a, b, c) from the plane $2x + 2y - z + 16 = 0$ is __.

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29. Let x and y be distinct integers where $1 \leq x \leq 25$ and $1 \leq y \leq 25$. Then, the number of ways of choosing x and y, such that $x + y$ is divisible by 5, is __.

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30. It the area enclosed by the parabolas $P_1 : 2y = 5x^2$ and $P_2 : x^2 - y + 6 = 0$ is equal to the area enclosed by P_1 and $y = \alpha x, \alpha > 0$, then α^3 is equal to __.
