# JEE Main 2023 Mathematics Question Paper Jan 24 Shift 2

| **Time Allowed :**3 Hour | **Maximum Marks :**120 | **Total Questions :**30 |
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**General Instructions**

**Read the following instructions very carefully and strictly follow them:**

1. The test is of 3 hours duration.
2. The question paper consists of 30 questions. The maximum marks are 120.
3. The question paper consists of topics from Mathematics, having two sections.
   (i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and –1 mark for wrong answer.
   (ii) Section-B: This section contains 10 questions. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and –1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

**1. The equations of the sides AB and AC of a triangle ABC are $(\lambda+1)x+\lambda y=4$ and $\lambda x+(1-\lambda)y+\lambda=0$ respectively. Its vertex A is on the y - axis and its orthocentre is (1, 2). The length of the tangent from the point C to the part of the parabola $y^2=6x$ in the first quadrant is :**

(A) 4
(B) 2
(C) $2\sqrt{2}$
(D) $\sqrt{6}$

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**2. Let p and q be two statements. Then $\sim(p\wedge(p\Rightarrow\sim q))$ is equivalent to**

(A) $p\vee((\sim p)\wedge q)$
(B) $(\sim p)\vee q$
(C) $p\vee(p\wedge q)$
(D) $p\vee(p\wedge(\sim q))$

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**3. Let $y=y(x)$ be the solution of the differential equation $(x^2-3y^2)dx+3xydy=0, y(1)=1$. Then $6y^2(e)$ is equal to**

(A) $\frac{3}{2}e^2$
(B) $e^2$

(C) $2e^2$
(D) $3e^2$

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**4. The number of real solutions of the equation $3(x^2+\frac{1}{x^2})-2(x+\frac{1}{x})+5=0$, is**

(A) 3
(B) 4
(C) 0
(D) 2

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**5. If $f(x)=\frac{2^{2x}}{2^{2x}+2}, x\in\mathbb{R}$, then $f\left(\frac{1}{2023}\right)+f\left(\frac{2}{2023}\right)+\cdots+f\left(\frac{2022}{2023}\right)$ is equal to**

(A) 2010
(B) 2011
(C) 1011
(D) 1010

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**6. If the system of equations**
$x+2y+3z=3$
$4x+3y-4z=4$
$8x+4y-\lambda z=9+\mu$
**has infinitely many solutions, then the ordered pair $(\lambda,\mu)$ is equal to :**

(A) $\left(-\frac{72}{5},\frac{21}{5}\right)$
(B) $\left(-\frac{72}{5},-\frac{21}{5}\right)$
(C) $\left(\frac{72}{5},\frac{21}{5}\right)$
(D) $\left(\frac{72}{5},-\frac{21}{5}\right)$

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**7. The locus of the mid points of the chords of the circle $C_i:(x-4)^2+(y-5)^2=4$ which subtend an angle $\theta_i$ at the centre of the circle $C_i$, is a circle of radius $r_i$. If $\theta_1=\frac{\pi}{3},\theta_3=\frac{2\pi}{3}$ and $r_1^2=r_2^2+r_3^2$, then $\theta_2$ is equal to**

(A) $\frac{\pi}{4}$
(B) $\frac{\pi}{2}$
(C) $\frac{3\pi}{4}$
(D) $\frac{\pi}{6}$

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**8. Let the plane containing the line of intersection of the planes $P_1: x+(\lambda+4)y+z=1$ and $P_2: 2x+y+z=2$ pass through the points $(0,1,0)$ and $(1,0,1)$. Then the distance**

**of the point $(2\lambda, \lambda, -\lambda)$ from the plane $P_2$ is**

(A) $2\sqrt{6}$
(B) $5\sqrt{6}$
(C) $3\sqrt{6}$
(D) $4\sqrt{6}$

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**9. Let the six numbers $a_1, a_2, a_3, a_4, a_5, a_6$ be in A.P. and $a_1 + a_3 = 10$. If the mean of these six numbers is $\frac{19}{2}$ and their variance is $\sigma^2$, then $8\sigma^2$ is equal to :**

(A) 200
(B) 210
(C) 220
(D) 105

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**10. The set of all values of $a$ for which $\lim_{x \to a}([x-5]-[2x+2]) = 0$, where $[x]$ denotes the greatest integer less than or equal to $x$ is equal to**

(A) $(-7.5, -6.5]$
(B) $[-7.5, -6.5)$
(C) $(-7.5, -6.5)$
(D) $[-7.5, -6.5]$

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**11. If $f(x) = x^3 - x^2 f'(1) + x f''(2) - f'''(3), x \in \mathbb{R}$, then**

(A) $f(1) + f(2) + f(3) = f(0)$
(B) $f(3) - f(2) = f(1)$
(C) $3f(1) + f(2) = f(3)$
(D) $2f(0) - f(1) + f(3) = f(2)$

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**12. Let $f(x)$ be a function such that $f(x+y) = f(x)f(y)$ for all $x, y \in \mathbb{N}$. If $f(1) = 3$ and $\sum_{k=1}^{n} f(k) = 3279$, then the value of $n$ is**

(A) 8
(B) 7
(C) 9
(D) 6

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**13. The value of $\left(\frac{1+\sin\frac{2\pi}{9}+i\cos\frac{2\pi}{9}}{1+\sin\frac{2\pi}{9}-i\cos\frac{2\pi}{9}}\right)^3$ is**

(A) $\frac{1}{2}(1-i\sqrt{3})$
(B) $-\frac{1}{2}(\sqrt{3}-i)$
(C) $-\frac{1}{2}(1-i\sqrt{3})$
(D) $\frac{1}{2}(\sqrt{3}+i)$

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**14. The number of integers, greater than 7000 that can be formed, using the digits 3, 5, 6, 7, 8 without repetition, is**

(A) 48
(B) 168
(C) 220
(D) 120

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**15.** $\int_{\frac{3\sqrt{2}}{4}}^{\frac{3\sqrt{3}}{4}} \frac{48}{\sqrt{9-4x^2}}dx$ **is equal to**

(A) $\frac{\pi}{6}$
(B) $\frac{\pi}{3}$
(C) $2\pi$
(D) $\frac{\pi}{2}$

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**16. Let** $\vec{a} = 4\hat{i} + 3\hat{j} + 5\hat{k}$ **and** $\vec{\beta} = \hat{i} + 2\hat{j} - 4\hat{k}$**. Let** $\vec{\beta}_1$ **be parallel to** $\vec{a}$ **and** $\vec{\beta}_2$ **be perpendicular to** $\vec{a}$**. If** $\vec{\beta} = \vec{\beta}_1 + \vec{\beta}_2$**, then the value of** $5\vec{\beta}_2 \cdot (\hat{i} + \hat{j} + \hat{k})$ **is**

(A) 6
(B) 9
(C) 11
(D) 7

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**17. Let A be a** $3 \times 3$ **matrix such that** $|\text{adj}(\text{adj } A)| = 12^4$**. Then** $|A^{-1}\text{adj } A|$ **is equal to**

(A) 12
(B) 1
(C) $\sqrt{6}$
(D) $2\sqrt{3}$

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**18. If the foot of the perpendicular drawn from** $(1, 9, 7)$ **to the line passing through the point** $(3, 2, 1)$ **and parallel to the planes** $x + 2y + z = 0$ **and** $3y - z = 3$ **is** $(\alpha, \beta, \gamma)$**,**

**then $\alpha+\beta+\gamma$ is equal to**

(A) 5
(B) 3
(C) 1
(D) $-1$

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**19. The number of square matrices of order 5 with entries from the set $\{0,1\}$, such that the sum of all the elements in each row is 1 and the sum of all the elements in each column is also 1, is**

(A) 225
(B) 125
(C) 150
(D) 120

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**20. If $({}^{30}C_1)^2+2({}^{30}C_2)^2+3({}^{30}C_3)^2+\cdots+30({}^{30}C_{30})^2=\frac{\alpha 60!}{(30!)^2}$, then $\alpha$ is equal to :**

(A) 60
(B) 15
(C) 10
(D) 30

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**21. If $\frac{1^3+2^3+3^3+\ldots \textbf{ up to } n \textbf{ terms}}{1\cdot 3+2\cdot 5+3\cdot 7+\ldots \textbf{ up to } n \textbf{ terms}}=\frac{9}{5}$, then the value of $n$ is ____**

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**22. Let $f$ be a differentiable function defined on $[0,\frac{\pi}{2}]$ such that $f(x)>0$ and $f(x)+\int_0^x f(t)\sqrt{1-(\log_e f(t))^2}dt=e, \forall x\in[0,\frac{\pi}{2}]$. Then $(6\log_e f(\frac{\pi}{6}))^2$ is equal to ____**

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**23. The equations of the sides AB, BC and CA of a triangle ABC are : $2x+y=0, x+py=21a, (a\neq 0)$ and $x-y=3$ respectively. Let $P(2,a)$ be the centroid of $\triangle ABC$. Then $(BC)^2$ is equal to ____**

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**24. If the shortest distance between the lines $\frac{x+\sqrt{6}}{2}=\frac{y-\sqrt{6}}{3}=\frac{z-\sqrt{6}}{4}$ and $\frac{x-\lambda}{3}=\frac{y-2\sqrt{6}}{4}=\frac{z+2\sqrt{6}}{5}$ is 6, then the square of sum of all possible values of $\lambda$ is ____**

**25. Let $S = \{\theta \in [0, 2\pi) : \tan(\pi\cos\theta) + \tan(\pi\sin\theta) = 0\}$. Then $\sum_{\theta\in S}\sin^2(\theta + \frac{\pi}{4})$ is equal to ____**

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**26. The minimum number of elements that must be added to the relation $R = \{(a, a), (b, b), (c, c), (d, d), (a, b), (b, c), (b, d)\}$ on the set $\{a, b, c, d\}$ so that it is an equivalence relation, is ____**

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**27. Three urns A, B and C contain 4 red, 6 black; 5 red, 5 black; and $\lambda$ red, 4 black balls respectively. One of the urns is selected at random and a ball is drawn. If the ball drawn is red and the probability that it is drawn from urn C is 0.4, then the square of the length of the side of the largest equilateral triangle, inscribed in the parabola $y^2 = \lambda x$ with one vertex at the vertex of the parabola, is ____**

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**28. Let $\vec{a} = \hat{i} + 2\hat{j} + \lambda\hat{k}, \vec{b} = 3\hat{i} - 5\hat{j} - \lambda\hat{k}, \vec{a}\cdot\vec{c} = 7, 2\vec{b}\cdot\vec{c} + 43 = 0$ and $\vec{a}\times\vec{c} = \vec{b}\times\vec{c}$. Then $|\vec{a}\cdot\vec{b}|$ is equal to ____**

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**29. Let the sum of the coefficients of the first three terms in the expansion of $(x - \frac{3}{x^2})^n, x \neq 0, n \in \mathbb{N}$, be 376. Then the coefficient of $x^4$ is ____**

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**30. If the area of the region bounded by the curves $y^2 - 2y = -x, x + y = 0$ is A, then 8 A is equal to ____**

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