JEE Main 2023 Mathematics Question Paper Apr 8 Shift 2

Time Allowed :3 Hour	**Maximum Marks :**120	**Total Questions :**30

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The test is of 3 hours duration.
2. The question paper consists of 30 questions. The maximum marks are 120.
3. The question paper consists of topics from Mathematics, having two sections.
(i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and –1 mark for wrong answer.
(ii) Section-B: This section contains 10 questions. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and –1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

1. Let $A = \{1, 2, 3, 4, 5, 6, 7\}$. Then the relation $R = \{(x, y) \in A \times A : x + y = 7\}$ is

(A) reflexive but neither symmetric nor transitive
(B) symmetric but neither reflexive nor transitive
(C) transitive but neither symmetric nor reflexive
(D) an equivalence relation

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2. Let $A = \left\{\theta \in (0, 2\pi) : \frac{1+2i\sin\theta}{1-i\sin\theta}\right.$ is purely imaginary$\left.\right\}$. Then the sum of the elements in A is

(A) 2π
(B) 3π
(C) 4π
(D) π

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3. Let S be the set of all values of $\theta \in [-\pi, \pi]$ for which the system of linear equations
$x + y + \sqrt{3}z = 0$
$-x + (\tan\theta)y + \sqrt{7}z = 0$
$x + y + (\tan\theta)z = 0$
has non-trivial solution. Then $\frac{120}{\pi}\sum_{\theta\in S}\theta$ is equal to

(A) 10
(B) 20
(C) 30
(D) 40

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4. If $A = \begin{bmatrix} 1 & 5 \\ \lambda & 10 \end{bmatrix}$, $A^{-1} = \alpha A + \beta I$ and $\alpha + \beta = -2$, then $4\alpha^2 + \beta^2 + \lambda^2$ is equal to:

(A) 10
(B) 12
(C) 14
(D) 19

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5. If the number of words, with or without meaning, which can be made using all the letters of the word MATHEMATICS in which C and S do not come together, is $(6!)k$, then k is equal to

(A) 945
(B) 1890
(C) 2835
(D) 5670

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6. $25^{190} - 19^{190} - 8^{190} + 2^{190}$ is divisible by

(A) both 14 and 34
(B) 14 but not 34
(C) 34 but not 14
(D) neither 14 nor 34

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7. The absolute difference of the coefficients of x^{10} and x^7 in the expansion of $\left(2x^2 + \frac{1}{2x}\right)^{11}$ is equal to

(A) $11^3 - 11$
(B) $12^3 - 12$
(C) $10^3 - 10$
(D) $13^3 - 13$

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8. Let a_n be the n^{th} term of the series $5 + 8 + 14 + 23 + 35 + 50 + \ldots$ and $S_n = \sum_{k=1}^{n} a_k$. Then $S_{30} - a_{40}$ is equal to

(A) 11260
(B) 11280
(C) 11290
(D) 11310

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9. If $\alpha > \beta > 0$ are the roots of the equation $ax^2+bx+1 = 0$, and $\lim_{x\to\frac{1}{\alpha}} \left(\frac{1-\cos(x^2+bx+a)}{2(1-\alpha x)^2}\right)^{\frac{1}{2}} = \frac{1}{k}\left(\frac{1}{\beta} - \frac{1}{\alpha}\right)$, then k is equal to

(A) α
(B) β
(C) 2α
(D) 2β

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10. The integral $\int \left[\left(\frac{x}{2}\right)^x + \left(\frac{2}{x}\right)^x\right] \log_2 x \, dx$ is equal to

(A) $\left(\frac{x}{2}\right)^x + \left(\frac{2}{x}\right)^x + C$
(B) $\left(\frac{x}{2}\right)^x - \left(\frac{2}{x}\right)^x + C$
(C) $\left(\frac{x}{2}\right)^x \log_2\left(\frac{x}{2}\right) + C$
(D) $\left(\frac{2}{x}\right)^x \log_2\left(\frac{2}{x}\right) + C$

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11. Let O be the origin and OP and OQ be the tangents to the circle $x^2 + y^2 - 6x + 4y + 8 = 0$ at the points P and Q on it. If the circumcircle of the triangle OPQ passes through the point $(\alpha, \frac{1}{2})$, then a value of α is

(A) $\frac{1}{2}$
(B) 1
(C) $\frac{3}{2}$
(D) $\frac{5}{2}$

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12. Let $A(0,1), B(1,1)$ and $C(1,0)$ be the mid-points of the sides of a triangle with incentre at the point D. If the focus of the parabola $y^2 = 4ax$ passing through D is $(\alpha + \beta\sqrt{2}, 0)$, where α and β are rational numbers, then $\frac{\alpha}{\beta^2}$ is equal to

(A) $\frac{9}{2}$
(B) 6
(C) 8
(D) 12

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13. Let P be the plane passing through the line $\frac{x-1}{1} = \frac{y-2}{-3} = \frac{z+5}{7}$ and the point $(2, 4, -3)$. If the image of the point $(-1, 3, 4)$ in the plane P is (α, β, γ), then $\alpha + \beta + \gamma$ is equal to

(A) 9
(B) 10
(C) 11
(D) 12

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14. For $a, b \in \mathbb{Z}$ and $|a - b| \leq 10$, let the angle between the plane $P : ax + y - z = b$ and the line $l : x - 1 = a - y = z + 1$ be $\cos^{-1}\left(\frac{1}{3}\right)$. If the distance of the point $(6, -6, 4)$ from the plane P is $3\sqrt{6}$, then $a^4 + b^2$ is equal to

(A) 25
(B) 32
(C) 48
(D) 85

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15. The area of the quadrilateral $ABCD$ with vertices $A(2, 1, 1), B(1, 2, 5), C(-2, -3, 5)$ and $D(1, -6, -7)$ is equal to

(A) 48
(B) $8\sqrt{38}$
(C) $9\sqrt{38}$
(D) 54

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16. Let the vectors $\vec{u}_1 = \hat{i} + \hat{j} + a\hat{k}, \vec{u}_2 = \hat{i} + b\hat{j} + \hat{k}$ and $\vec{u}_3 = c\hat{i} + \hat{j} + \hat{k}$ be coplanar. If the vectors $\vec{v}_1 = (a + b)\hat{i} + c\hat{j} + a\hat{k}, \vec{v}_2 = a\hat{i} + (b + c)\hat{j} + a\hat{k}$ and $\vec{v}_3 = a\hat{i} + b\hat{j} + (c + a)\hat{k}$ are also coplanar, then $6(a + b + c)$ is equal to

(A) 0
(B) 4
(C) 6
(D) 12

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17. If the probability that the random variable X takes values x is given by $P(X = x) = k(x + 1)3^{-x}, x = 0, 1, 2, 3, \ldots,$ where k is a constant, then $P(X \geq 2)$ is equal to

(A) $\frac{7}{18}$
(B) $\frac{11}{18}$

(C) $\frac{7}{27}$
(D) $\frac{20}{27}$

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18. Let the mean and variance of 12 observations be $\frac{9}{2}$ and 4 respectively. Later on, it was observed that two observations were considered as 9 and 10 instead of 7 and 14 respectively. If the correct variance is $\frac{m}{n}$, where m and n are coprime, then $m+n$ is equal to

(A) 314
(B) 315
(C) 316
(D) 317

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19. The value of $36(4\cos^2 9^\circ - 1)(4\cos^2 27^\circ - 1)(4\cos^2 81^\circ - 1)(4\cos^2 243^\circ - 1)$ is

(A) 18
(B) 27
(C) 36
(D) 54

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20. The negation of $(p \wedge (\sim q)) \vee (\sim p)$ is equivalent to

(A) $p \vee (q \vee (\sim p))$
(B) $p \wedge q$
(C) $p \wedge (\sim q)$
(D) $p \wedge (q \wedge (\sim p))$

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21. If the domain of the function $\log_e\left(\frac{6x^2+5x+1}{2x-1}\right) + \cos^{-1}\left(\frac{2x^2-3x+4}{3x-5}\right)$ is $(\alpha, \beta) \cup (\gamma, \delta]$, then $18(\alpha^2+\beta^2+\gamma^2+\delta^2)$ is equal to _____.

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22. Let m and n be the numbers of real roots of the quadratic equations $x^2-12x+[x]+31=0$ and $x^2-5|x+2|-4=0$ respectively, where $[x]$ denotes the greatest integer $\leq x$. Then m^2+mn+n^2 is equal to _____.

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23. Let $R = \{a, b, c, d, e\}$ and $S = \{1, 2, 3, 4\}$. Total number of onto functions $f : R \to S$ such that $f(a) \neq 1$, is equal to _____.

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24. Let $0 < z < y < x$ be three real numbers such that $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ are in an arithmetic progression and $x, \sqrt{2}y, z$ are in a geometric progression. If $xy + yz + zx = \frac{3}{\sqrt{2}}xyz$, then $3(x + y + z)^2$ is equal to _____.

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25. Let k and m be positive real numbers such that the function
$f(x) = \begin{cases} 3x^2 + k\sqrt{x} + 1, & 0 < x < 1 \\ mx^2 + k, & x \geq 1 \end{cases}$ **is differentiable for all $x > 0$. Then $\frac{8f'(8)}{f'\left(\frac{1}{8}\right)}$ is equal to _____.**

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26. Let $[t]$ denote the greatest integer function. If $\int_0^{2.4} [x^2] dx = \alpha + \beta\sqrt{2} + \gamma\sqrt{3} + \delta\sqrt{5}$, then $\alpha + \beta + \gamma + \delta$ is equal to _____.

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27. Let the area enclosed by the lines $x - y = 2, y = 0$ and the curve $f(x) = \min\{x^2 + \frac{3}{4}, 1 + |x|\}$ where $[x]$ denotes the greatest integer $\leq x$, be A. Then the value of $12A$ is _____.

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28. Let the solution curve $x = x(y), 0 < y < \frac{\pi}{2}$, of the differential equation $(\log_e(\cos y))^2 \cos^2 y dx - (1 + 3x \log_e(\cos y)) \sin y dy = 0$ satisfy $x\left(\frac{\pi}{3}\right) = \frac{1}{2 \log_e 2}$. If $x\left(\frac{\pi}{6}\right) = \frac{1}{\log_e m - \log_e n}$, where m and n are coprime, then mn is equal to _____.

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29. The ordinates of the points P and Q on the parabola with focus (3, 0) and directrix $x = -3$ are in the ratio 3:1. If R(α, β) is the point of intersection of the tangents to the parabola at P and Q, then $\frac{\beta^2}{\alpha}$ is equal to _____.

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30. Let P_1 be the plane $3x - y - 7z = 11$ and P_2 be the plane passing through the points (2, -1, 0), (2, 0, -1), and (5, 1, 1). If the foot of the perpendicular drawn

from the point (7, 4, -1) on the line of intersection of the planes P_1 and P_2 is $(\alpha, \beta, \gamma$, then $\alpha + \beta + \gamma$ is equal to _____.
