JEE Main 2023 Mathematics Question Paper Apr 8 Shift 1

Time Allowed :3 Hour	**Maximum Marks :**120	**Total Questions :**30

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The test is of 3 hours duration.
2. The question paper consists of 30 questions. The maximum marks are 120.
3. The question paper consists of topics from Mathematics, having two sections.
(i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and –1 mark for wrong answer.
(ii) Section-B: This section contains 10 questions. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and –1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

1. Let the number of elements in sets A and B be five and two respectively. Then the number of subsets of $A \times B$ each having at least 3 and at most 6 elements is:

(A) 792
(B) 772
(C) 752
(D) 782

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2. If for $z = \alpha + i\beta$, $|z+2| = z + 4(1+i)$, then $\alpha+\beta$ and $\alpha\beta$ are the roots of the equation:

(A) $x^2 + 3x - 4 = 0$
(B) $x^2 + 7x + 12 = 0$
(C) $x^2 + 2x - 3 = 0$
(D) $x^2 + x - 12 = 0$

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3. Let α, β, γ be the three roots of the equation $x^3 + bx + c = 0$. If $\beta\gamma = 1 = -\alpha$, then $b^3 + 2c^3 - 3\alpha^3 - 6\beta^3 - 8\gamma^3$ is equal to:

(A) 169/8
(B) 155/8
(C) 19

(D) 21

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4. Let $A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$**. If** $|\mathbf{adj}(\mathbf{adj}(2A))| = (16)^n$**, then** n **is equal to:**

(A) 10
(B) 8
(C) 12
(D) 9

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5. Let $P = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$, $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ **and** $Q = PAP^T$**. If** $P^T Q^{2007} P = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$**, then** $2a + b - 3c - 4d$ **equal to:**

(A) 2004
(B) 2005
(C) 2006
(D) 2007

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6. The number of arrangements of the letters of the word "INDEPENDENCE" in which all the vowels always occur together is:

(A) 14800
(B) 33600
(C) 16800
(D) 18000

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7. The number of ways, in which 5 girls and 7 boys can be seated at a round table so that no two girls sit together, is:

(A) $126(5!)^2$
(B) $7(360)^2$
(C) $7(720)^2$
(D) 720

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8. If the coefficients of three consecutive terms in the expansion of $(1+x)^n$ are in the ratio $1:5:20$, then the coefficient of the fourth term is:

(A) 1827
(B) 3654
(C) 5481
(D) 2436

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9. Let $S_k = \frac{1+2+...+k}{k}$ and $\sum_{j=1}^{n} S_j^2 = \frac{n}{A}(Bn^2+Cn+D)$, where $A, B, C, D \in \mathbb{N}$ and A has least value. Then:

(A) $A+B+C+D$ is divisible by 5
(B) $A+B$ is divisible by D
(C) $A+B=5(D-C)$
(D) $A+C+D$ is not divisible by B

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10. $\lim_{x\to 0}\left(\frac{(1-\cos^2(3x))\sin^3(4x)}{\cos^2(4x)(\log_e(2x+1))^5}\right)$ **is equal to:**

(A) 9
(B) 15
(C) 18
(D) 24

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11. Let $I(x) = \int \frac{(x+1)}{x(1+xe^x)^2}dx, x>0$. If $\lim_{x\to\infty} I(x) = 0$, then $I(1)$ is equal to:

(A) $\frac{e+2}{e+1} - \log_e(e+1)$
(B) $\frac{e+2}{e+1} + \log_e(e+1)$
(C) $\frac{e+1}{e+2} - \log_e(e+1)$
(D) $\frac{e+1}{e+2} + \log_e(e+1)$

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12. The area of the region $\{(x,y): x^2 \leq y \leq 8-x^2, y \leq 7\}$ is:

(A) 18
(B) 20
(C) 21
(D) 24

13. Let $C(\alpha, \beta)$ be the circumcenter of the triangle formed by the lines
$4x + 3y = 69$,
$4y - 3x = 17$, **and**
$x + 7y = 61$.
Then $(\alpha - \beta)^2 + \alpha + \beta$ is equal to

(A) 15
(B) 16
(C) 17
(D) 18

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14. Let R be the focus of the parabola $y^2 = 20x$ and the line $y = mx + c$ intersect the parabola at two points P and Q. Let the point $G(10, 10)$ be the centroid of the triangle PQR. If $c - m = 6$, then $(PQ)^2$ is

(A) 296
(B) 325
(C) 317
(D) 346

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15. If the equation of the plane containing the line $x+2y+3z-4=0=2x+y-z+5$ and perpendicular to the plane $\vec{r} = (\vec{i}-\vec{j})+\lambda(\vec{i}+\vec{j}+\vec{k})+\mu(\vec{i}-2\vec{j}+3\vec{k})$ is $ax+by+cz=4$, then $(a-b+c)$ is equal to

(A) 18
(B) 20
(C) 22
(D) 24

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16. The shortest distance between the lines $\frac{x-4}{4} = \frac{y+2}{5} = \frac{z+3}{3}$ and $\frac{x-1}{3} = \frac{y-3}{4} = \frac{z-4}{2}$ is

(A) $6\sqrt{3}$
(B) $3\sqrt{6}$
(C) $2\sqrt{6}$
(D) $6\sqrt{2}$

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17. If the points with position vectors $\alpha\vec{i} + 10\vec{j} + 13\vec{k}$, $6\vec{i} + 11\vec{j} + 11\vec{k}$, $\frac{9}{2}\vec{i} + \beta\vec{j} - 8\vec{k}$ are collinear, then $(19\alpha - 6\beta)^2$ is equal to

(A) 16
(B) 25
(C) 36
(D) 49

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18. In a bolt factory, machines A, B and C manufacture respectively $20\%, 30\%$ and 50% of the total bolts. Of their output $3, 4$ and 2 percent are respectively defective bolts. A bolt is drawn at random from the product. If the bolt drawn is found the defective, then the probability that it is manufactured by the machine C is

(A) $\frac{2}{7}$
(B) $\frac{5}{14}$
(C) $\frac{3}{7}$
(D) $\frac{9}{28}$

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19. Let $f(x) = \frac{\sin x + \cos x - \sqrt{2}}{\sin x - \cos x}$, $x \in [0, \pi] - \{\frac{\pi}{4}\}$. Then $f\left(\frac{7\pi}{12}\right) f'\left(\frac{7\pi}{12}\right)$ is equal to

(A) $-\frac{2}{3}$
(B) $\frac{-1}{3\sqrt{3}}$
(C) $\frac{2}{3\sqrt{3}}$
(D) $\frac{2}{9}$

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20. Negation of $(p \Rightarrow q) \Rightarrow (q \Rightarrow p)$ is

(A) $p \vee (\sim q)$
(B) $(\sim p) \vee q$
(C) $q \wedge (\sim p)$
(D) $(\sim q) \wedge p$

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21. Let $A = \{0, 3, 4, 6, 7, 8, 9, 10\}$ and R be the relation defined on A such that $R = \{(x, y) \in A \times A : x - y$ is an odd positive integer or $x - y = 2\}$. The minimum number of elements that must be added to the relation R, so that it is a symmetric relation, is equal to ______.

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22. Let $[t]$ denote the greatest integer $\leq t$. If the constant term in the expansion of $\left(3x^2 - \frac{1}{2x^5}\right)^7$ is α, then $[\alpha]$ is equal to ______.

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23. If a_n is the greatest term in the sequence $a_n = \frac{n^3}{n^4+147}, n = 1, 2, 3, \ldots,$ then n is equal to ______.

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24. Let $[t]$ denote the greatest integer $\leq t$. Then $\frac{2}{\pi}\int_{\pi/6}^{5\pi/6}[8|\text{cosec } x| - 5|\cot x|]dx$ is equal to ______.

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25. If the solution curve of the differential equation $(y - 2\log_e x)dx + (x\log_e x^2)dy = 0, x > 1$ passes through the points $\left(e, \frac{4}{3}\right)$ and (e^4, α), then α is equal to ______.

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26. The largest natural number n such that 3^n divides $66!$ is ______.

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27. Consider a circle $C_1 : x^2 + y^2 - 4x - 2y = \alpha - 5$. Let its mirror image in the line $y = 2x + 1$ be another circle $C_2 : 5x^2 + 5y^2 - 10fx - 10gy + 36 = 0$. Let r be the radius of C_2. Then $\alpha + r$ is equal to ______.

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28. Let λ_1, λ_2 be the values of λ for which the points $\left(\frac{5}{2}, 1, \lambda\right)$ and $(-2, 0, 1)$ are at equal distance from the plane $2x + 3y - 6z + 7 = 0$. If $\lambda_1 > \lambda_2$, then the distance of the point $(\lambda_1 - \lambda_2, \lambda_2, \lambda_1)$ from the line $\frac{x-5}{1} = \frac{y-1}{2} = \frac{z+7}{2}$ is ______.

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29. Let $\vec{a} = 6\hat{i} + 9\hat{j} + 12\hat{k}, \vec{b} = \alpha\hat{i} + 11\hat{j} - 2\hat{k}$ and $\vec{c}$ be vectors such that $\vec{a} \times \vec{c} = \vec{a} \times \vec{b}$. If $\vec{a} \cdot \vec{c} = -12, \vec{c} \cdot (\hat{i} - 2\hat{j} + \hat{k}) = 5$, then $\vec{c} \cdot (\hat{i} + \hat{j} + \hat{k})$ is equal to ______.

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30. Let the mean and variance of 8 numbers $x, y, 10, 12, 6, 12, 4, 8$ be 9 and 9.25 respectively. If $x > y$, then $3x - 2y$ is equal to ______.
