

JEE Main 2023 Jan 30 Shift 1 Question Paper

Total Time Allowed : 180 minutes	Maximum Marks : 300	Total Questions : 90	Questions to be answered : 90
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General Instructions

Read the following instructions very carefully and strictly follow them:

- (1) The test is of 3 hours duration.
- (2) The question paper consists of 90 questions. The maximum marks are 300.
- (3) There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage.
- (4) Each part (subject) has two sections.
 - (i) **Section-A:** This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and –1 mark for wrong answer.
 - (ii) **Section-B:** This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and –1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

Physics**Section A**

Question 1: The charge flowing in a conductor changes with time as

$$Q(t) = \alpha t - \beta t^2 + \gamma t^3,$$

where α, β, γ are constants. The minimum value of current is:

Options:

- (1) $\alpha - \frac{3\beta^2}{\gamma}$
- (2) $\alpha - \frac{\gamma^2}{3\beta}$
- (3) $\beta - \frac{\alpha^2}{3\gamma}$
- (4) $\alpha - \frac{\beta^2}{3\gamma}$

Question 2: The pressure (P) and temperature (T) relationship of an ideal gas obeys the equation $PT^2 = \text{constant}$. The volume expansion coefficient of the gas will be:

Options:

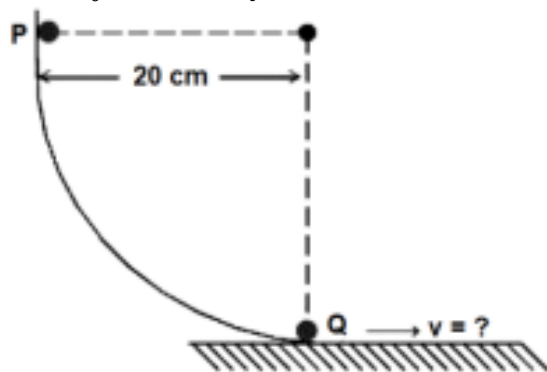
- (1) $3T^2$
 - (2) $\frac{3}{T^2}$
 - (3) $\frac{3}{T^3}$
 - (4) $\frac{3}{T}$
-

Question 3: A person has been using spectacles of power -1.0 diopter for distant vision and a separate reading glass of power 2.0 diopters. What is the least distance of distinct vision for this person?

Options:

- (1) 10 cm
 - (2) 40 cm
 - (3) 30 cm
 - (4) 50 cm
-

Question 4: As per the given figure, a small ball P slides down the quadrant of a circle and hits the other ball Q of equal mass which is initially at rest. Neglecting the effect of friction and assume the collision to be elastic, the velocity of ball Q after collision will be: ($g = 10 \text{ m/s}^2$)



Options:

- (1) 0
 - (2) 0.25 m/s
 - (3) 2 m/s
 - (4) 4 m/s
-

Question 5: Choose the correct relationship between Poisson ratio (σ), bulk modulus (K) and modulus of rigidity (η) of a given solid object:

Options:

- (1) $\sigma = \frac{3K-2\eta}{6K+2\eta}$
 - (2) $\sigma = \frac{6K+2\eta}{3K-2\eta}$
 - (3) $\sigma = \frac{3K+2\eta}{6K+2\eta}$
 - (4) $\sigma = \frac{6K-2\eta}{3K+2\eta}$
-

Question 6: The magnetic moments associated with two closely wound circular coils A and B of radius $r_A = 10$ cm and $r_B = 20$ cm respectively are equal if: (Where N_A, I_A and N_B, I_B are number of turns and current of A and B respectively)

Options:

- (1) $2N_AI_A = N_BI_B$
 - (2) $N_A = 2N_B$
 - (3) $N_AI_A = 4N_BI_B$
 - (4) $4N_AI_A = N_BI_B$
-

Question 7: A small object at rest absorbs a light pulse of power 20 mW and duration 300 ns. Assuming speed of light as 3×10^8 m/s, the momentum of the object becomes equal to:

Options:

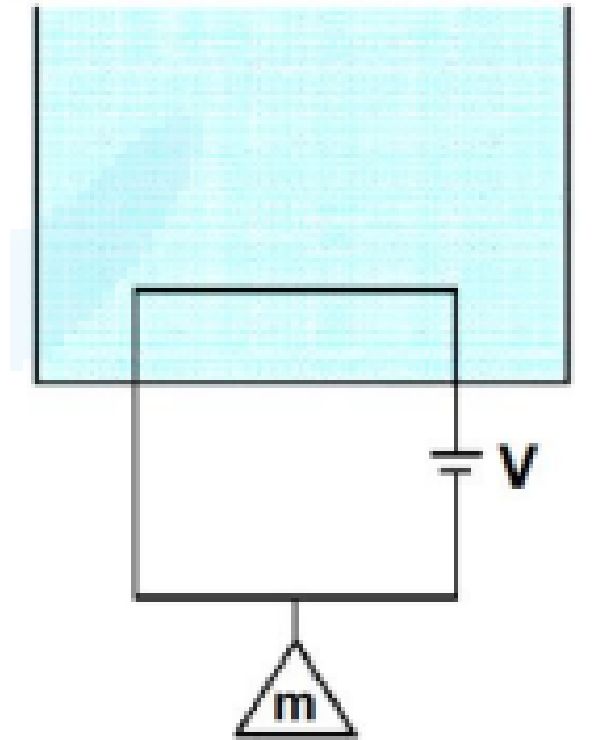
- (1) 0.5×10^{-17} kg m/s
 - (2) 2×10^{-17} kg m/s
 - (3) 3×10^{-17} kg m/s
 - (4) 1×10^{-17} kg m/s
-

Question 8: Speed of an electron in Bohr's 7th orbit for Hydrogen atom is 3.6×10^6 m/s. The corresponding speed of the electron in the 3rd orbit, in m/s, is:

Options:

- (1) 1.8×10^6 m/s
 - (2) 7.5×10^6 m/s
 - (3) 3.6×10^6 m/s
 - (4) 8.4×10^6 m/s
-

Question 9: A massless square loop, of wire resistance $10\ \Omega$, supporting a mass of 1 g , hangs vertically with one of its sides in a uniform magnetic field of 10^3 G , directed outwards in the shaded region. A dc voltage V is applied to the loop. For what value of V will the magnetic force exactly balance the weight of the supporting mass of 1 g ? (If sides of the loop = 10 cm , $g = 10\text{ m/s}^2$)



Options:

- (1) $\frac{1}{10}\text{ V}$
- (2) 100 V
- (3) 1 V
- (4) 10 V

Question 10: Two isolated metallic solid spheres of radii R and $2R$ are charged such that both have the same charge density σ . The spheres are then connected by a thin conducting wire. If the new charge density of the bigger sphere is σ' , the ratio $\frac{\sigma'}{\sigma}$ is:

Options:

- (1) $\frac{9}{4}$
- (2) $\frac{4}{3}$
- (3) $\frac{5}{3}$
- (4) $\frac{5}{6}$

Question 11: Heat is given to an ideal gas in an isothermal process.

- A. Internal energy of the gas will decrease.
- B. Internal energy of the gas will increase.
- C. Internal energy of the gas will not change.
- D. The gas will do positive work.
- E. The gas will do negative work.

Choose the correct answer from the options given below:

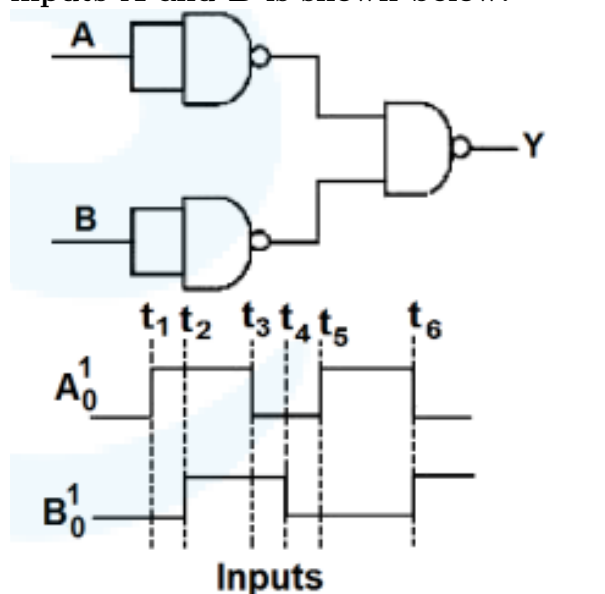
- (1) A and E only
- (2) B and D only
- (3) C and E only
- (4) C and D only

Question 12: Electric field in a certain region is given by $\mathbf{E} = \left(\frac{A}{x^2} \hat{i} + \frac{B}{y^3} \hat{j} \right)$. The SI unit of A and B are:

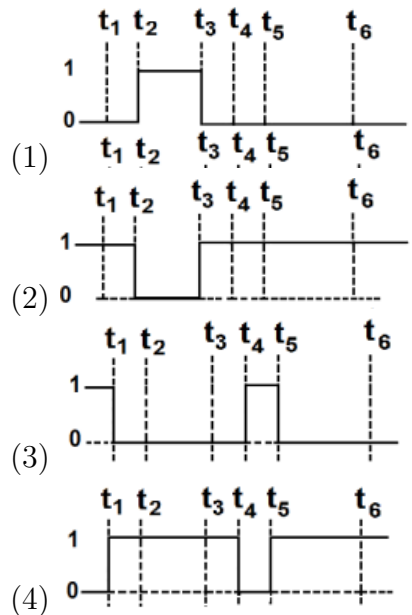
Options:

- (1) Nm^2C^{-1} , Nm^2C^{-1}
- (2) Nm^2C^{-1} , Nm^3C^{-1}
- (3) Nm^3C , Nm^3C
- (4) Nm^2C^{-1} , Nm^3C

Question 13: The output waveform of the given logical circuit for the following inputs A and B is shown below:



Options:



Question 14: The height of the liquid column raised in a capillary tube of certain radius when dipped in liquid A vertically is 5 cm. If the tube is dipped in a similar manner in another liquid B of surface tension and density double the values of liquid A, the height of the liquid column raised in liquid B would be:

Options:

- (1) 0.20
- (2) 0.5
- (3) 0.10
- (4) 0.05

Question 15: A sinusoidal carrier voltage is amplitude modulated. The resultant amplitude modulated wave has maximum and minimum amplitude of 120 V and 80 V respectively. The amplitude of each sideband is:

Options:

- (1) 15 V
- (2) 10 V
- (3) 20 V
- (4) 5 V

Question 16: In a series LR circuit with $X_L = R$, the power factor is P_1 . If a capacitor of capacitance C with $X_C = X_L$ is added to the circuit, the power factor becomes P_2 . The ratio of P_1 to P_2 will be:

Options:

- (1) 3:1
 - (2) $1:\sqrt{2}$
 - (3) 1:1
 - (4) 1:2
-

Question 17: If the gravitational field in the space is given as $-\frac{K}{r^2}$, taking the reference point to be at $r = 2$ cm with gravitational potential $V = 10$ J/kg, find the gravitational potential at $r = 3$ cm in SI units. (Given that $K = 6$ J cm/kg)

Options:

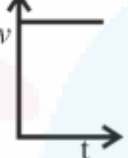
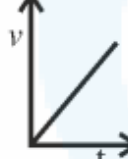
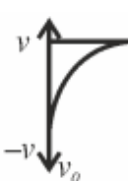
- (1) 9
 - (2) 11
 - (3) 12
 - (4) 10
-

Question 18: A ball of mass 200 g rests on a vertical post of height 20 m. A bullet of mass 10 g, travelling in horizontal direction, hits the centre of the ball. After collision both travel independently. The ball hits the ground at a distance of 30 m and the bullet at a distance of 120 m from the foot of the post. The value of initial velocity of the bullet will be (if $g = 10$ m/s²):

Options:

- (1) 120 m/s
 - (2) 60 m/s
 - (3) 400 m/s
 - (4) 360 m/s
-

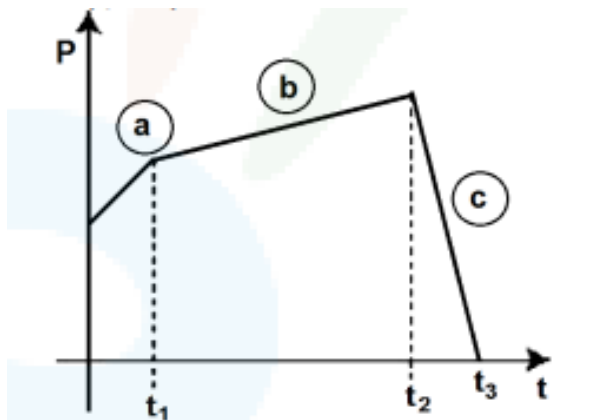
Question 19: Match Column-I with Column-II:

Column-I (x-t graphs)		Column-II (v-t graphs)	
A.		I.	
B.		II.	
C.		III.	
D.		IV.	

Options:

- (1) A-II, B-IV, C-III, D-I
- (2) A-I, B-II, C-III, D-IV
- (3) A-II, B-III, C-IV, D-II
- (4) A-I, B-III, C-IV, D-I

Question 20: The figure represents the momentum time ($p - t$) curve for a particle moving along an axis under the influence of the force. Identify the regions on the graph where the magnitude of the force is maximum and minimum respectively? If $t_3 - t_2 < t_1$:



Options:

- (1) c and a
- (2) b and c
- (3) c and b
- (4) a and b

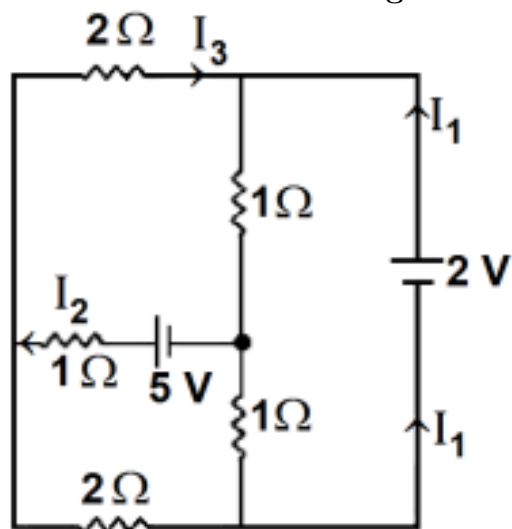
Section - B

Question 21: The general displacement of a simple harmonic oscillator is $x = A \sin(\omega t)$. Let T be its time period. The slope of its potential energy (U) - time (t) curve will be maximum when $t = \frac{T}{\beta}$. The value of β is:

Question 22: A capacitor of capacitance $900 \mu\text{F}$ is charged by a 100 V battery. The capacitor is disconnected from the battery and connected to another uncharged identical capacitor such that one plate of the uncharged capacitor is connected to the positive plate and another plate of the uncharged capacitor is connected to the negative plate of the charged capacitor. The loss of energy in this process is measured as $x \times 10^{-2} \text{ J}$. The value of x is:

Question 23: In Young's double slit experiment, two slits S_1 and S_2 are 'd' distance apart and the separation from slits to screen is D (as shown in figure). Now if two transparent slabs of equal thickness 0.1 mm but refractive index 1.51 and 1.55 are introduced in the path of beam ($\lambda = 4000 \text{ \AA}$) from S_1 and S_2 respectively. The central bright fringe spot will shift by ____ number of fringes.

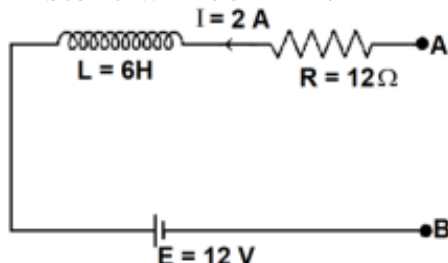
Question 24: In the following circuit, the magnitude of current I_1 is ____ A.



Question 25: A horse rider covers half the distance with 5 m/s speed. The remaining part of the distance was travelled with speed 10 m/s for half the time and with speed 15 m/s for other half of the time. The mean speed of the rider averaged over the whole time of motion is $\frac{x}{7}$ m/s. The value of x is:

Question 26: A point source of light is placed at the centre of curvature of a hemispherical surface. The source emits a power of 24 W. The radius of curvature of the hemisphere is 10 cm and the inner surface is completely reflecting. The force on the hemisphere due to the light falling on it is ____ $\times 10^{-8}$ N.

Question 27: As per the given figure, if $\frac{dI}{dt} = -1$ A/s, then the value of V_{AB} at this instant will be ____ V.



Question 28: In a screw gauge, there are 100 divisions on the circular scale and the main scale moves by 0.5 mm on a complete rotation of the circular scale. The zero of circular scale lies 6 divisions below the line of graduation

when two studs are brought in contact with each other. When a wire is placed between the studs, 4 linear scale divisions are clearly visible while the 46th division the circular scale coincide with the reference line. The diameter of the wire is $\text{-----} \times 10^{-2}$ mm.

Question 29: In an experiment for estimating the value of focal length of a converging mirror, image of an object placed at 40 cm from the pole of the mirror is formed at a distance 120 cm from the pole of the mirror. These distances are measured with a modified scale in which there are 20 small divisions in 1 cm. The value of error in measurement of focal length of the mirror is $1/K$ cm. The value of K is ----- .

Question 30: A thin uniform rod of length 2m, cross-sectional area 'A' and density 'd' is rotated about an axis passing through the center and perpendicular to its length with angular velocity ω . If value of ω in terms of its rotational kinetic energy E is:

$$E = \frac{\alpha E}{Ad}$$

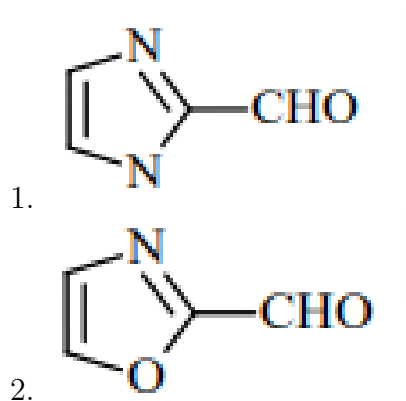
Then the value of α is ----- .

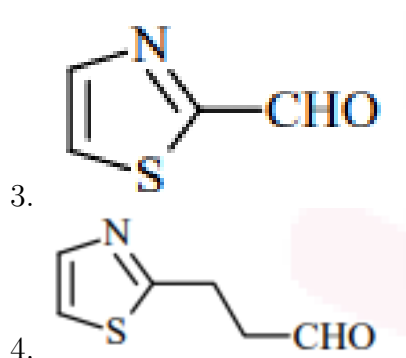
Chemistry

Section A

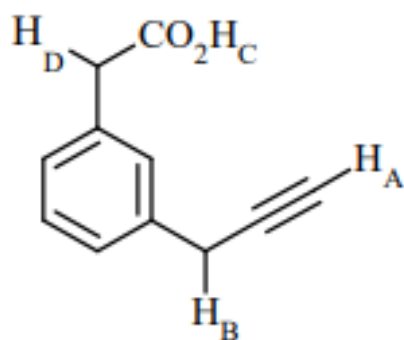
Question 31: Which of the following compounds would give the following set of qualitative analysis?

- (i) Fehling's Test: Positive
- (ii) Na fusion extract upon treatment with sodium nitroprusside gives a blood red colour but not





Question 32: What is the correct order of acidity of the protons marked A-D in the given compounds?



- (1) $H_C > H_D > H_B > H_A$
- (2) $H_C > H_D > H_A > H_B$
- (3) $H_D > H_C > H_B > H_A$
- (4) $H_C > H_A > H_D > H_B$

Question 33: Given below are two statements: one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A): Ketoses give Seliwanoff's test faster than Aldoses.

Reason (R): Ketoses undergo β -elimination followed by formation of furfural.

In light of the above statements, choose the correct answer from the options given below:

- (1) (A) is false but (R) is true
- (2) Both (A) and (R) are true and (R) is the correct explanation of (A)
- (3) (A) is true but (R) is false
- (4) Both (A) and (R) are true but (R) is not the correct explanation of (A)

Question 34: In the extraction of copper, its sulphide ore is heated in a reverberatory furnace after mixing with silica to:

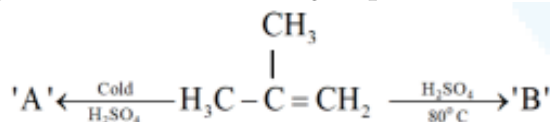
- (1) separate CuO as $CuSiO_3$

- (2) remove calcium as $CaSiO_3$
- (3) decrease the temperature needed for roasting of Cu_2S
- (4) remove FeO as $FeSiO_3$

Question 35: Amongst the following compounds, which one is an antacid?

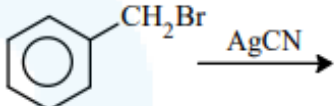
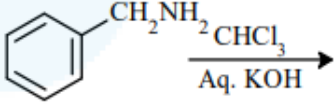
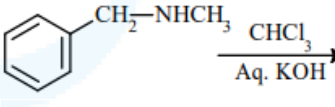
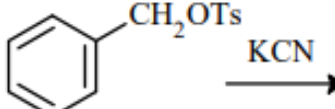
- (1) Ranitidine
- (2) Meprobamate
- (3) Terfenadine
- (4) Brompheniramine

Question 36: The major products 'A' and 'B', respectively, are:



- (1) $\text{CH}_3\text{C}_6\text{H}_4$ and $\text{CH}_3\text{C}_6\text{H}_4\text{OSO}_3$
- (2) $\text{CH}_3\text{C}_6\text{H}_5$ and $\text{CH}_3\text{C}_6\text{H}_4\text{OSO}_3$
- (3) $\text{CH}_3\text{C}_6\text{H}_5$ and $\text{CH}_3\text{C}_6\text{H}_4\text{OH}$
- (4) $\text{CH}_3\text{C}_6\text{H}_5$ and $\text{CH}_3\text{C}_6\text{H}_3\text{OH}$

Question 37: Benzyl isocyanide can be obtained by:

- A.  $\text{C}_6\text{H}_5\text{CH}_2\text{Br} \xrightarrow{\text{AgCN}}$
- B.  $\text{C}_6\text{H}_5\text{CH}_2\text{NH}_2 \xrightarrow[\text{Aq. KOH}]{\text{CHCl}_3}$
- C.  $\text{C}_6\text{H}_5\text{CH}_2\text{NHCH}_3 \xrightarrow[\text{Aq. KOH}]{\text{CHCl}_3}$
- D.  $\text{C}_6\text{H}_5\text{CH}_2\text{OTs} \xrightarrow{\text{KCN}}$

Choose the correct answer from the options given below:

- (1) A and D
- (2) Only B
- (3) A and B

(4) B and C

Question 38: Given below are two statements: one is labelled as Assertion (A) and the other is labelled as Reason (R).

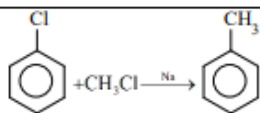
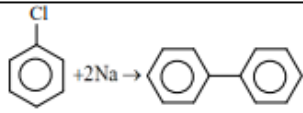
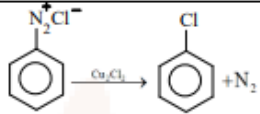
Assertion (A): In expensive scientific instruments, silica gel is kept in watch-glasses or in semipermeable membrane bags.

Reason (R): Silica gel adsorbs moisture from air via adsorption, thus protects the instrument from water corrosion (rusting) and/or prevents malfunctioning.

In the light of the above statements, choose the correct answer from the options given below:

- (1) A is false but (R) is true
- (2) A is true but (R) is false
- (3) Both (A) and (R) are true and (R) is the correct explanation of (A)
- (4) Both (A) and (R) are true but (R) is not the correct explanation of (A)

Question 39: Match List I with List II:

List I		List II	
A		I	Fitting reaction
B		II	Wurtz Fitting reaction
C		III	Finkelstein reaction
D	$\text{C}_2\text{H}_5\text{Cl} + \text{NaI} \rightarrow \text{C}_2\text{H}_5\text{I} + \text{NaCl}$	IV	Sandmeyer reaction

- (1) A – IV, B – I, C – III, D – II
- (2) A – I, B – II, C – IV, D – III
- (3) A – I, B – III, C – II, D – I
- (4) A – II, B – I, C – IV, D – III

Question 40: Caprolactam when heated at high temperature in presence of water gives:

- (1) Teflon
- (2) Dacron
- (3) Nylon 6, 6
- (4) Nylon 6

Question 41: The alkaline earth metal sulphate(s) which are readily soluble in water is/are:

- (A) BeSO_4
- (B) MgSO_4
- (C) CaSO_4
- (D) SrSO_4
- (E) BaSO_4

Choose the **correct answer** from the options given below:

- 1) A only
- 2) B only
- 3) A and B
- 4) B and C

Question 42: Which of the following is the correct order of ligand field strength?

- (1) $\text{CO} < \text{en} < \text{NH}_3 < \text{C}_2\text{O}_4^{2-} < \text{S}^{2-}$
- (2) $\text{S}^{2-} < \text{C}_2\text{O}_4^{2-} < \text{NH}_3 < \text{en} < \text{CO}$
- (3) $\text{NH}_3 < \text{en} < \text{CO} < \text{S}^{2-} < \text{C}_2\text{O}_4^{2-}$
- (4) $\text{S}^{2-} < \text{NH}_3 < \text{en} < \text{CO} < \text{C}_2\text{O}_4^{2-}$

Question 43: Formation of photochemical smog involves the following reaction in which A, B, and C are respectively.

- (i) $\text{NO}_2 \xrightarrow{h\nu} \text{A} + \text{B}$
- (ii) $\text{B} + \text{O}_2 \rightarrow \text{C}$
- (iii) $\text{A} + \text{C} \rightarrow \text{NO}_2 + \text{O}_2$

Choose the correct answer from the options given below:

- (1) O, NO, and NO_3^-
- (2) N_2O and NO
- (3) N, O_2 , and O_3
- (4) NO, O, and O_3

Question 44: During the qualitative analysis of SO_3^{2-} using dilute H_2SO_4 , SO_2 gas is evolved which turns $\text{K}_2\text{Cr}_2\text{O}_7$ solution (acidified with dilute H_2SO_4):

- (1) Black
- (2) Red
- (3) Green
- (4) Blue

Question 45: To inhibit the growth of tumours, identify the compounds used from the following:

- (A) EDTA
- (B) Coordination Compounds of Pt
- (C) D – Penicillamine
- (D) Cis – Platin

Choose the correct answer from the options given below:

- (1) B and D Only
- (2) C and D Only
- (3) A and B Only
- (4) A and C Only

Question 46: In the wet tests for identification of various cations by precipitation, which transition element cation doesn't belong to group IV in qualitative inorganic analysis?

- (1) Fe^{3+}
- (2) Zn^{2+}
- (3) Co^{2+}
- (4) Ni^{2+}

Question 47: Match List I with List II

LIST-I (molecules/ions)	LIST-II (No. of lone pairs of e^- on central atom)
(A) IF_7	I. Three
(B) ICl_4^-	II. One
(C) XeF_6	III. Two
(D) XeF_2	IV. Zero

Choose the **correct answer** from the options given below:

- (1) A – II, B – III, C – IV, D – I
- (2) A – IV, B – III, C – II, D – I

(3) A – II, B – I, C – IV, D – III

(4) A – IV, B – I, C – II, D – III

Question 48: For OF_2 molecule consider the following:

(A) Number of lone pairs on oxygen is 2.

(B) F–O–F angle is less than 104.5° .

(C) Oxidation state of O is –2.

(D) Molecule is bent ‘V’ shaped.

(E) Molecular geometry is linear.

Correct options are:

(1) C, D, E only

(2) B, E, A only

(3) A, C, D only

(4) A, B, D only

Question 49: Lithium aluminium hydride can be prepared from the reaction of:

(1) LiCl and Al_2H_6

(2) LiH and Al_2Cl_6

(3) LiCl, Al and H_2

(4) LiH and $\text{Al}(\text{OH})_3$

Question 50: Match List – I with List – II

LIST–I (Atomic number)	LIST–II (Block of periodic table)
(A) 37 (K)	I. p-block
(B) 78 (Pt)	II. d-block
(C) 52 (Te)	III. f-block
(D) 65 (Tb)	IV. s-block

Choose the **correct answer** from the options given below:

(1) A – II, B – IV, C – I, D – III

(2) A – I, B – III, C – IV, D – II

(3) A – IV, B – III, C – II, D – I

(4) A – IV, B – II, C – I, D – III

Section B

Question 51: Consider the cell



When the potential of the cell is 0.712 V at 298 K, the ratio $\frac{[\text{Fe}^{2+}]}{[\text{Fe}^{3+}]}$ is _____.
(Nearest integer)

Given:



$$\frac{2.303RT}{F} = 0.06 \text{ V}.$$

Question 52: A 300 mL bottle of soft drink has 0.2 M CO_2 dissolved in it. Assuming CO_2 behaves as an ideal gas, the volume of the dissolved CO_2 at STP is _____ mL. (Nearest integer)

Given: At STP, molar volume of an ideal gas is 22.7 L mol^{-1} .

Question 53: A solution containing 2 g of a non-volatile solute in 20 g of water boils at 373.52 K. The molecular mass of the solute is _____ g mol^{-1} . (Nearest integer)

Given: Water boils at 373 K, K_b for water = $0.52 \text{ K kg mol}^{-1}$.

Question 54: If compound A reacts with B following first-order kinetics with rate constant $2.011 \times 10^{-3} \text{ s}^{-1}$, the time taken by A (in seconds) to reduce from 7 g to 2 g will be _____. (Nearest Integer)

Given:

$$\log 5 = 0.698, \quad \log 7 = 0.845, \quad \log 2 = 0.301.$$

Question 55: The energy of one mole of photons of radiation of frequency $2 \times 10^{12} \text{ Hz}$ in J mol^{-1} is _____. (Nearest integer)

Given: $h = 6.626 \times 10^{-34} \text{ Js}$, $N_A = 6.022 \times 10^{23} \text{ mol}^{-1}$.

Question 56: The number of electrons involved in the reduction of permanganate to manganese dioxide in acidic medium is _____.

Question 57: When 2 liters of ideal gas expands isothermally into a vacuum to a total volume of 6 liters, the change in internal energy is _____ J. (Nearest integer)

Question 58: 600 mL of 0.01 M HCl is mixed with 400 mL of 0.01 M H₂SO₄. The pH of the mixture is $\text{-----} \times 10^{-2}$. (Nearest integer)

Given:

$$\log 2 = 0.30, \quad \log 3 = 0.48, \quad \log 5 = 0.69, \quad \log 7 = 0.84, \quad \log 11 = 1.04.$$

Question 59: A trisubstituted compound 'A', C₁₀H₁₂O₂, gives neutral FeCl₃ test positive. Treatment of compound 'A' with NaOH and CH₃Br gives C₁₁H₁₄O₂, with hydroiodic acid gives methyl iodide and with hot conc. NaOH gives a compound 'B', C₁₀H₁₂O₂. Compound 'A' also decolourises alkaline KMnO₄. The number of π bond/s present in the compound 'A' is ----- .

Question 60: Some amount of dichloromethane (CH₂Cl₂) is added to 671.141 mL of chloroform (CHCl₃) to prepare a 2.6×10^{-3} M solution of CH₂Cl₂ (DCM). The concentration of DCM is ----- ppm (by mass).

Given: Atomic mass C = 12, H = 1, Cl = 35.5, density of CHCl₃ = 1.49 g cm⁻³.

Mathematics

Section A

Question 61: Let

$$A = \begin{bmatrix} m & n \\ p & q \end{bmatrix}, \quad d = |A| \neq 0, \quad \text{and} \quad |A - d(\text{Adj}A)| = 0.$$

Then:

- (1) $(1 + d)^2 = (m + q)^2$
 - (2) $1 + d^2 = (m + q)^2$
 - (3) $(1 + d)^2 = m^2 + q^2$
 - (4) $1 + d^2 = m^2 + q^2$
-

Question 62: The line ℓ_1 passes through the point (2, 6, 2) and is perpendicular to the plane $2x + y - 2z = 10$. Then the shortest distance between the line ℓ_1 and the line

$$\frac{x+1}{2} = \frac{y+4}{-3} = \frac{z}{2}$$

is:

- (1) 7
- (2) $\frac{19}{3}$

- (3) $\frac{19}{2}$
 (4) 9
-

Question 63: If an unbiased die, marked with $-2, -1, 0, 1, 2, 3$ on its faces, is thrown five times, then the probability that the product of the outcomes is positive, is:

- (1) $\frac{881}{2592}$
 (2) $\frac{521}{2592}$
 (3) $\frac{440}{2592}$
 (4) $\frac{27}{288}$
-

Question 64: Let the system of linear equations

$$x + y + kz = 22x + 3y - z = 13x + 4y + 2z = k$$

have infinitely many solutions. Then the system

$$(k + 1)x + (2k - 1)y = 7(2k + 1)x + (k + 5)y = 10$$

has:

- (1) infinitely many solutions
 (2) unique solution satisfying $x - y = 1$
 (3) no solution
 (4) unique solution satisfying $x + y = 1$
-

Question 65: If

$$\tan 15^\circ + \frac{1}{\tan 75^\circ} + \tan 105^\circ + \tan 195^\circ = 2a,$$

then the value of $a + \frac{1}{a}$ is:

- (1) 4
 (2) $4 - 2\sqrt{3}$
 (3) 2
 (4) $5 - 3\sqrt{3}$
-

Question 66: Suppose $f : R \rightarrow (0, \infty)$ be a differentiable function such that $5f(x + y) = f(x) \cdot f(y)$, $\forall x, y \in R$. If $f(3) = 320$, then $\sum_{n=0}^5 f(n)$ is equal to:

- (1) 6875
 (2) 6575

(3) 6825

(4) 6528

Question 67: If

$$a_n = \frac{-2}{4n^2 - 16n + 15}, \quad \text{then } a_1 + a_2 + \cdots + a_5 \text{ is equal to:}$$

(1) $\frac{51}{144}$

(2) $\frac{49}{138}$

(3) $\frac{50}{141}$

(4) $\frac{52}{147}$

Question 68: If the coefficient of x^{15} in the expansion of

$$\left(ax^3 + \frac{1}{bx^3}\right)^{15}$$

is equal to the coefficient of x^{-15} in the expansion of

$$\left(\frac{a}{x^3} - \frac{1}{bx^3}\right)^{15},$$

where a and b are positive real numbers, then for each such ordered pair (a, b) :

(1) $a = b$

(2) $ab = 1$

(3) $a = 3b$

(4) $ab = 3$

Question 69: If $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are three non-zero vectors and $\hat{n}$ is a unit vector perpendicular to $\mathbf{c}$ such that

$$\mathbf{a} = \alpha \mathbf{b} - \hat{n}, \quad (\alpha \neq 0)$$

and

$$\vec{b} \cdot \vec{c} = 12, \quad \text{then } \left| \vec{c} \times (\vec{a} \times \vec{b}) \right|$$

is equal to:

(1) 15

(2) 9

(3) 12

(4) 6

Question 70: The number of points on the curve

$$y = 54x^5 - 135x^4 - 70x^3 + 180x^2 + 210x$$

at which the normal lines are parallel to

$$x + 90y + 2 = 0$$

is:

- (1) 2
 - (2) 3
 - (3) 4
 - (4) 0
-

Question 71: Let

$$y = x + 2, \quad 4y = 3x + 6, \quad \text{and} \quad 3y = 4x + 1$$

be three tangent lines to the circle

$$(x - h)^2 + (y - k)^2 = r^2.$$

Then $h + k$ is equal to:

- (1) 5
 - (2) $5(1 + \sqrt{2})$
 - (3) 6
 - (4) $5\sqrt{2}$
-

Question 72: Let the solution curve $y = y(x)$ of the differential equation

$$\frac{dy}{dx} - \frac{3x^5 \tan^{-1}(x^3)}{(1 + x^6)^{3/2}} y = 2x$$
$$\exp \frac{x^3 - \tan^{-1} x^3}{\sqrt{(1 + x)^6}}$$

pass through the origin. Then $y(1)$ is equal to:

- (1) $\exp \left(\frac{4 - \pi}{4\sqrt{2}} \right)$
- (2) $\exp \left(\frac{\pi - 4}{4\sqrt{2}} \right)$
- (3) $\exp \left(\frac{1 - \pi}{4\sqrt{2}} \right)$

(4) $\exp\left(\frac{4+\pi}{4\sqrt{2}}\right)$

Question 73: Let a unit vector $\overrightarrow{OP}$ make angles α, β, γ with the positive directions of the coordinate axes OX, OY, OZ respectively, where $\beta \in (0, \frac{\pi}{2})$, and $\overrightarrow{OP}$ is perpendicular to the plane through points $(1, 2, 3)$, $(2, 3, 4)$, and $(1, 5, 7)$. Then which one of the following is true?

(1) $\alpha \in (\frac{\pi}{2}, \pi)$ and $\gamma \in (\frac{\pi}{2}, \pi)$

(2) $\alpha \in (0, \frac{\pi}{2})$ and $\gamma \in (0, \frac{\pi}{2})$

(3) $\alpha \in (\frac{\pi}{2}, \pi)$ and $\gamma \in (0, \frac{\pi}{2})$

(4) $\alpha \in (0, \frac{\pi}{2})$ and $\gamma \in (\frac{\pi}{2}, \pi)$

Question 74: If $[t]$ denotes the greatest integer $\leq t$, then the value of

$$\frac{3(e-1)^2}{e} \int_1^2 x^2 e^{[x]+[x^3]} dx$$

is:

(1) $e^9 - e$

(2) $e^8 - e$

(3) $e^7 - 1$

(4) $e^8 - 1$

Question 75: If $P(h, k)$ be a point on the parabola $x = 4y^2$, which is nearest to the point $Q(0, 33)$, then the distance of P from the directrix of the parabola $y^2 = 4(x + y)$ is equal to:

(1) 2

(2) 4

(3) 8

(4) 6

Question 76: A straight line cuts off the intercepts $OA = a$ and $OB = b$ on the positive directions of the x -axis and y -axis, respectively. If the perpendicular from the origin O to this line makes an angle of $\frac{\pi}{6}$ with the positive direction of the y -axis and the area of $\triangle OAB$ is $\frac{98}{3}\sqrt{3}$, then $a^2 - b^2$ is equal to:

(1) $\frac{392}{3}$

- (2) 196
 - (3) $\frac{196}{3}$
 - (4) 98
-

Question 77: The coefficient of x^{301} in

$$(1+x)^{500} + x(1+x)^{499} + x^2(1+x)^{498} + \dots + x^{500}$$

is:

- (1) ${}^{501}C_{302}$
 - (2) ${}^{500}C_{301}$
 - (3) ${}^{500}C_{300}$
 - (4) ${}^{501}C_{200}$
-

Question 78: Among the statements:

- (S1) $((p \vee q) \Rightarrow r) \Leftrightarrow (p \Rightarrow r)$
- (S2) $((p \vee q) \Rightarrow r) \Leftrightarrow ((p \Rightarrow r) \vee (q \Rightarrow r))$

Which of the following is true?

- (1) Only (S1) is a tautology
 - (2) Neither (S1) nor (S2) is a tautology
 - (3) Only (S2) is a tautology
 - (4) Both (S1) and (S2) are tautologies
-

Question 79: The minimum number of elements that must be added to the relation

$$R = \{(a, b), (b, c)\}$$

on the set

$$\{a, b, c\}$$

so that it becomes symmetric and transitive is:

- (1) 4
 - (2) 7
 - (3) 5
 - (4) 3
-

Question 80: If the solution of the equation

$$\log_{\cos x} \cot x + 4 \log_{\sin x} \tan x = 1, x \in \left(0, \frac{\pi}{2}\right),$$

is

$$\sin^{-1} \left(\frac{\alpha + \sqrt{\beta}}{2} \right),$$

where α, β are integers, then $\alpha + \beta$ is equal to:

- (1) 3
- (2) 5
- (3) 6
- (4) 4

Section B

Question 81: Let $S = \{1, 2, 3, 4, 5, 6\}$. Then the number of one-one functions $f : S \rightarrow P(S)$, where $P(S)$ denotes the power set of S , such that $f(n) \subset f(m)$ where $n < m$, is ----.

Question 82: Let α be the area of the larger region bounded by the curve

$$y^2 = 8x$$

and the lines

$$y = x \quad \text{and} \quad x = 2,$$

which lies in the first quadrant. Then the value of 3α is equal to:

Question 83: $\lambda_1 < \lambda_2$ are two values of λ such that the angle between the planes

$$P_1 : \vec{r} \cdot (3\hat{i} - 5\hat{j} + \hat{k}) = 7$$

and

$$P_2 : \vec{r} \cdot (\lambda\hat{i} + \hat{j} - 3\hat{k}) = 9$$

is $\sin^{-1} \left(\frac{2\sqrt{6}}{5} \right)$, then the square of the length of the perpendicular from the point $(38\lambda, 10\lambda, 2)$ to the plane P_1 is -----.

Question 84: Let $z = 1 + i$ and $z_1 = \frac{1+i\bar{z}}{\bar{z}(1-z) + \frac{1}{z}}$. Then $\frac{12}{\pi} \arg(z_1)$ is equal to ----.

Question 85:

$$\lim_{x \rightarrow 0} \frac{48}{x^4} \int_0^x \frac{t^3}{t^6 + 1} dt \text{ is equal to } \text{-----}.$$

Question 86: The mean and variance of 7 observations are 8 and 16, respectively. If one observation 14 is omitted and a and b are respectively the mean and variance of the remaining 6 observations, then $a + 3b - 5$ is equal to:

Question 87: If the equation of the plane passing through the point $(1, 1, 2)$ and perpendicular to the line

$$x - 3y + 2z - 1 = 0, \quad 4x - y + z = 0 \quad \text{is} \quad Ax + By + Cz = 1,$$

then $140(C - B + A)$ is equal to:

Question 88: Let

$$\sum_{n=0}^{\infty} \frac{n^3((2n)!) + (2n-1)(n!)}{(n!)(2n)!} = ae + \frac{b}{e} + c,$$

where $a, b, c \in \mathbb{Z}$ and $e = \sum_{n=0}^{\infty} \frac{1}{n!}$. Then $a^2 - b + c$ is equal to -----.

Question 89: Number of 4-digit numbers (the repetition of digits is allowed) which are made using the digits 1, 2, 3, and 5 and are divisible by 15 is equal to:

Question 90: Let

$$f^1(x) = \frac{3x+2}{2x+3}, \quad x \in \mathbb{R}, \quad \mathbb{R} - \left(-\frac{3}{2}\right).$$

For $n \geq 2$, define $f^n(x) = f^1 \circ f^{n-1}(x)$ and if

$$f^5(x) = \frac{ax+b}{bx+a}, \quad \gcd(a, b) = 1, \quad \text{then } a+b \text{ is equal to:}$$
