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Jan 30 2023, Shift 1

JEE Main 2023 Jan 30 Shift 1 Mathematics Question Paper

Total Time Allowed : 180 minutes	**Maximum Marks :** 300	**Total Questions :** 90	**Questions to be answered :** 90

General Instructions

Read the following instructions very carefully and strictly follow them:
(1) The test is of 3 hours duration.
(2) The question paper consists of 90 questions. The maximum marks are 300.
(3) There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage.
(4) Each part (subject) has two sections.
(i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and −1 mark for wrong answer.
(ii) Section-B: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and −1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

Mathematics

Section A

Question 1: Let

$$A = \begin{bmatrix} m & n \\ p & q \end{bmatrix}, \ d = |A| \neq 0, \textbf{ and } |A - d(\textbf{Adj}A)| = 0.$$

Then:

(1) $(1+d)^2 = (m+q)^2$
(2) $1+d^2 = (m+q)^2$
(3) $(1+d)^2 = m^2+q^2$
(4) $1+d^2 = m^2+q^2$

Question 2: The line ℓ_1 passes through the point $(2, 6, 2)$ and is perpendicular to the plane $2x + y - 2z = 10$. Then the shortest distance between the line ℓ_1 and the line

$$\frac{x+1}{2} = \frac{y+4}{-3} = \frac{z}{2}$$

is:
(1) 7
(2) $\frac{19}{3}$
(3) $\frac{19}{2}$
(4) 9

Question 3: If an unbiased die, marked with $-2, -1, 0, 1, 2, 3$ on its faces, is thrown five times, then the probability that the product of the outcomes is positive, is:
(1) $\frac{881}{2592}$
(2) $\frac{521}{2592}$
(3) $\frac{440}{2592}$
(4) $\frac{27}{288}$

Question 4: Let the system of linear equations

$$x + y + kz = 22x + 3y - z = 13x + 4y + 2z = k$$

have infinitely many solutions. Then the system

$$(k+1)x + (2k-1)y = 7(2k+1)x + (k+5)y = 10$$

has:
(1) infinitely many solutions
(2) unique solution satisfying $x - y = 1$
(3) no solution
(4) unique solution satisfying $x + y = 1$

Question 5: If

$$\tan 15^\circ + \frac{1}{\tan 75^\circ} + \tan 105^\circ + \tan 195^\circ = 2a,$$

then the value of $a + \frac{1}{a}$ is:
(1) 4
(2) $4 - 2\sqrt{3}$
(3) 2
(4) $5 - 3\sqrt{3}$

Question 6: Suppose $f : R \to (0, \infty)$ be a differentiable function such that $5f(x+y) = f(x) \cdot f(y)$, $\forall x, y \in R$. If $f(3) = 320$, then $\sum_{n=0}^{5} f(n)$ is equal to:

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(1) 6875
(2) 6575
(3) 6825
(4) 6528

Question 7: If

$$a_n = \frac{-2}{4n^2 - 16n + 15}, \quad \text{then} \quad a_1 + a_2 + \cdots + a_5 \text{ is equal to:}$$

(1) $\frac{51}{144}$
(2) $\frac{49}{138}$
(3) $\frac{50}{141}$
(4) $\frac{52}{147}$

Question 8: If the coefficient of x^{15} in the expansion of

$$\left(ax^3 + \frac{1}{bx^3}\right)^{15}$$

is equal to the coefficient of x^{-15} in the expansion of

$$\left(\frac{a}{x^3} - \frac{1}{bx^3}\right)^{15},$$

where a and b are positive real numbers, then for each such ordered pair (a, b):
(1) $a = b$
(2) $ab = 1$
(3) $a = 3b$
(4) $ab = 3$

Question 9: If $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are three non-zero vectors and $\hat{n}$ is a unit vector perpendicular to $\mathbf{c}$ such that

$$\mathbf{a} = \alpha\mathbf{b} - \hat{n}, \quad (\alpha \neq 0)$$

and

$$\overrightarrow{b} \cdot \overrightarrow{c} = 12, \quad \text{then} \quad \left|\overrightarrow{c} \times (\overrightarrow{a} \times \overrightarrow{b})\right|$$

is equal to:
(1) 15
(2) 9

(3) 12
(4) 6

Question 10: The number of points on the curve

$$y = 54x^5 - 135x^4 - 70x^3 + 180x^2 + 210x$$

at which the normal lines are parallel to

$$x + 90y + 2 = 0$$

is:
(1) 2
(2) 3
(3) 4
(4) 0

Question 11: Let

$$y = x + 2, \quad 4y = 3x + 6, \quad \text{and} \quad 3y = 4x + 1$$

be three tangent lines to the circle

$$(x - h)^2 + (y - k)^2 = r^2.$$

Then $h + k$ is equal to:
(1) 5
(2) $5(1 + \sqrt{2})$
(3) 6
(4) $5\sqrt{2}$

Question 12: Let the solution curve $y = y(x)$ of the differential equation

$$\frac{dy}{dx} - \frac{3x^5 \tan^{-1}(x^3)}{(1 + x^6)^{3/2}} y = 2x$$

$$exp\frac{x^3 - \tan^{-1} x^3}{\sqrt{(1 + x)^6}}$$

pass through the origin. Then $y(1)$ is equal to:
(1) $\exp\left(\frac{4-\pi}{4\sqrt{2}}\right)$
(2) $\exp\left(\frac{\pi-4}{4\sqrt{2}}\right)$

(3) $\exp\left(\frac{1-\pi}{4\sqrt{2}}\right)$
(4) $\exp\left(\frac{4+\pi}{4\sqrt{2}}\right)$

Question 13: Let a unit vector $\overrightarrow{OP}$ make angles α, β, γ with the positive directions of the coordinate axes OX, OY, OZ respectively, where $\beta \in \left(0, \frac{\pi}{2}\right)$, and $\overrightarrow{OP}$ is perpendicular to the plane through points $(1, 2, 3)$, $(2, 3, 4)$, and $(1, 5, 7)$. Then which one of the following is true?
(1) $\alpha \in \left(\frac{\pi}{2}, \pi\right)$ and $\gamma \in \left(\frac{\pi}{2}, \pi\right)$

(2) $\alpha \in \left(0, \frac{\pi}{2}\right)$ and $\gamma \in \left(0, \frac{\pi}{2}\right)$

(3) $\alpha \in \left(\frac{\pi}{2}, \pi\right)$ and $\gamma \in \left(0, \frac{\pi}{2}\right)$

(4) $\alpha \in \left(0, \frac{\pi}{2}\right)$ and $\gamma \in \left(\frac{\pi}{2}, \pi\right)$

Question 14: If $[t]$ denotes the greatest integer ≤ 1, then the value of

$$\frac{3(e-1)^2}{e} \int_1^2 x^2 e^{[x]+[x^3]} dx$$

is:
(1) $e^9 - e$
(2) $e^8 - e$
(3) $e^7 - 1$
(4) $e^8 - 1$

Question 15: If $P(h, k)$ be a point on the parabola $x = 4y^2$, which is nearest to the point $Q(0, 33)$, then the distance of P from the directrix of the parabola $y^2 = 4(x + y)$ is equal to:
(1) 2
(2) 4
(3) 8
(4) 6

Question 16: A straight line cuts off the intercepts $OA = a$ and $OB = b$ on the positive directions of the x-axis and y-axis, respectively. If the perpendicular from the origin O to this line makes an angle of $\frac{\pi}{6}$ with the positive direction of the y-axis and the area of $\triangle OAB$ is $\frac{98}{3}\sqrt{3}$, then $a^2 - b^2$ is equal to:

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(1) $\frac{392}{3}$
(2) 196
(3) $\frac{196}{3}$
(4) 98

Question 17: The coefficient of x^{301} in

$$(1+x)^{500} + x(1+x)^{499} + x^2(1+x)^{498} + \cdots + x^{500}$$

is:
(1) ${}^{501}C_{302}$
(2) ${}^{500}C_{301}$
(3) ${}^{500}C_{300}$
(4) ${}^{501}C_{200}$

Question 18: Among the statements:
(S1) $((p \vee q) \Rightarrow r) \Leftrightarrow (p \Rightarrow r)$
(S2) $((p \vee q) \Rightarrow r) \Leftrightarrow ((p \Rightarrow r) \vee (q \Rightarrow r))$
Which of the following is true?
(1) Only (S1) is a tautology
(2) Neither (S1) nor (S2) is a tautology
(3) Only (S2) is a tautology
(4) Both (S1) and (S2) are tautologies

Question 19: The minimum number of elements that must be added to the relation

$$R = \{(a,b),(b,c)\}$$

on the set

$$\{a,b,c\}$$

so that it becomes symmetric and transitive is:
(1) 4
(2) 7
(3) 5
(4) 3

Question 20: If the solution of the equation

$$\log_{\cos x} \cot x + 4 \log_{\sin x} \tan x = 1,\ x \in \left(0, \frac{\pi}{2}\right),$$

is

$$\sin^{-1}\left(\frac{\alpha + \sqrt{\beta}}{2}\right),$$

where α, β are integers, then $\alpha + \beta$ is equal to:
(1) 3
(2) 5
(3) 6
(4) 4

Section B

Question 21: Let $S = \{1, 2, 3, 4, 5, 6\}$. Then the number of one-one functions $f : S \to P(S)$, where $P(S)$ denotes the power set of S, such that $f(n) \subset f(m)$ where $n < m$, is ____.

Question 22: Let α be the area of the larger region bounded by the curve

$$y^2 = 8x$$

and the lines

$$y = x \quad \text{and} \quad x = 2,$$

which lies in the first quadrant. Then the value of 3α is equal to:

Question 23: $\lambda_1 < \lambda_2$ are two values of λ such that the angle between the planes

$$P_1 : \vec{r} \cdot (3\hat{i} - 5\hat{j} + \hat{k}) = 7$$

and

$$P_2 : \vec{r} \cdot (\lambda\hat{i} + \hat{j} - 3\hat{k}) = 9$$

is $\sin^{-1}\left(\frac{2\sqrt{6}}{5}\right)$, then the square of the length of the perpendicular from the point $(38\lambda, 10\lambda, 2)$ to the plane P_1 is _____.

Question 24: Let $z = 1 + i$ and $z_1 = \frac{1+i\bar{z}}{\bar{z}(1-z)+\frac{1}{z}}$. Then $\frac{12}{\pi}\arg(z_1)$ is equal to ____.

Question 25:

$$\lim_{x\to 0} \frac{48}{x^4}\int_0^x \frac{t^3}{t^6+1}\,dt \textbf{ is equal to } \_\_\_\_\_.$$

Question 26: The mean and variance of 7 observations are 8 and 16, respectively. If one observation 14 is omitted and a and b are respectively the mean and variance of the remaining 6 observations, then $a + 3b - 5$ is equal to:

Question 27: If the equation of the plane passing through the point $(1, 1, 2)$ and perpendicular to the line

$$x - 3y + 2z - 1 = 0, \quad 4x - y + z = 0 \quad \text{is} \quad Ax + By + Cz = 1,$$

then $140(C - B + A)$ is equal to:

Question 28: Let

$$\sum_{n=0}^{\infty} \frac{n^3\big((2n)!\big) + (2n-1)(n!)}{(n!)(2n)!} = ae + \frac{b}{e} + c,$$

where $a, b, c \in Z$ and $e = \sum_{n=0}^{\infty} \frac{1}{n!}$. Then $a^2 - b + c$ is equal to ______.

Question 29: Number of 4-digit numbers (the repetition of digits is allowed) which are made using the digits 1, 2, 3, and 5 and are divisible by 15 is equal to:

Question 30: Let

$$f^1(x) = \frac{3x+2}{2x+3}, \quad x \in R, \quad R - \left(-\frac{3}{2}\right).$$

For $n \geq 2$, define $f^n(x) = f^1 0 f^{n-1}(x)$ and if

$$f^5(x) = \frac{ax+b}{bx+a}, \quad \gcd(a, b) = 1, \quad \text{then} \quad a + b \text{ is equal to:}$$
