ZOLLEGE

Feb 1 2023, Shift 2

JEE Main 2023 Feb 1 Shift 2 Mathematics Question Paper

Total Time Allowed : 180 minutes	Maximum Marks : 300	Total Questions : 90	Questions to be answered : 90

General Instructions

Read the following instructions very carefully and strictly follow them:
(1) The test is of 3 hours duration.
(2) The question paper consists of 90 questions. The maximum marks are 300.
(3) There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage.
(4) Each part (subject) has two sections.
(i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and –1 mark for wrong answer.
(ii) Section-B: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and –1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

Mathematics

Section A

Question 1: The sum $\sum_{n=1}^{\infty} \frac{2n^2+3n+4}{(2n)!}$ is equal to:
(1) $\frac{11e}{2} + \frac{7}{2e}$
(2) $\frac{13e}{4} + \frac{5}{4e} - 4$
(3) $\frac{11e}{2} + \frac{7}{2e} - 4$
(4) $\frac{13e}{4} + \frac{5}{4e}$

Question 2: Let $S = \{x \in R : 0 < x < 1$ and $2\tan^{-1}\left(\frac{1-x}{1+x}\right) = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\}$. If $n(S)$ denotes the number of elements in S, then:
(1) $n(S) = 2$ and only one element in S is less than $\frac{1}{2}$.
(2) $n(S) = 1$ and the element in S is more than $\frac{1}{2}$.
(3) $n(S) = 1$ and the element in S is less than $\frac{1}{2}$.
(4) $n(S) = 0$.

Question 3: Let $\vec{a} = 2\hat{i} - 7\hat{j} + 5\hat{k}$, $\vec{b} = \hat{i} + \hat{k}$, and $\vec{c} = \hat{i} + 2\hat{j} - 3\hat{k}$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r} \times \vec{a} = \vec{c} \times \vec{a}$ and $\vec{r} \cdot \vec{b} = 0$, then $|\vec{r}|$ is equal to:

(1) $\frac{11}{7}\sqrt{2}$
(2) $\frac{11}{7}$
(3) $\frac{11}{5}\sqrt{2}$
(4) $\frac{\sqrt{914}}{7}$

Question 4: If $A = \frac{1}{2}\begin{bmatrix} 1 & \sqrt{3} \\ -\sqrt{3} & 1 \end{bmatrix}$, then:
(1) $A^{30} - A^{25} = 2I$
(2) $A^{30} + A^{25} + A = I$
(3) $A^{30} + A^{25} - A = I$
(4) $A^{30} = A^{25}$

Question 5: Two dice are thrown independently. Let A be the event that the number appeared on the 1^{st} die is less than the number appeared on the 2^{nd} die, B be the event that the number appeared on the 1^{st} die is even and that on the 2^{nd} die is odd, and C be the event that the number appeared on the 1^{st} die is odd and that on the 2^{nd} die is even. Then:
(1) The number of favourable cases of the event $(A \cup B) \cap C$ is 6.
(2) A and B are mutually exclusive.
(3) The number of favourable cases of the events $A, B,$ and C are 15, 6, and 6 respectively.
(4) B and C are independent.

Question 6: Which of the following statements is a tautology?
(1) $p \rightarrow (p \wedge (p \rightarrow q))$
(2) $(p \wedge q) \rightarrow \sim (p \rightarrow q)$
(3) $(p \wedge (p \rightarrow q)) \rightarrow \sim q$
(4) $p \vee (p \wedge q)$

Question 7: The number of integral values of k, for which one root of the equation $2x^2 - 8x + k = 0$ lies in the interval $(1, 2)$ and its other root lies in the interval $(2, 3)$, is:
(1) 2
(2) 0
(3) 1
(4) 3

Question 8: Let $f : R - \{0, 1\} \rightarrow R$ be a function such that $f(x) + f\left(\frac{1}{1-x}\right) = 1 + x$. Then $f(2)$ is equal to:
(1) $\frac{9}{2}$
(2) $\frac{9}{4}$
(3) $\frac{7}{4}$
(4) $\frac{7}{3}$

Question 9: Let the plane P pass through the intersection of the planes $2x + 3y - z = 2$ and $x+2y+3z = 6$, and be perpendicular to the plane $2x+y-z+1 = 0$. If d is the distance of P from the point $(-7, 1, 1)$, then d^2 is equal to:
(1) $\frac{250}{83}$
(2) $\frac{15}{53}$
(3) $\frac{25}{83}$
(4) $\frac{250}{82}$

Question 10: Let a, b be two real numbers such that $ab < 0$. If the complex number $\frac{1+ai}{b+i}$ is of unit modulus and $a + ib$ lies on the circle $|z - 1| = |2z|$, then a possible value of $\frac{1+\lfloor a \rfloor}{4b}$, where $\lfloor t \rfloor$ is the greatest integer function, is:
(1) $-\frac{1}{2}$
(2) -1
(3) 1
(4) $\frac{1}{2}$

Question 11: The sum of the absolute maximum and minimum values of the function $f(x) = |x^2 - 5x + 6| - 3x + 2$ in the interval $[-1, 3]$ is equal to:
(1) 10
(2) 12
(3) 13
(4) 24

Question 12: Let $P(S)$ denote the power set of $S = \{1, 2, 3, \ldots, 10\}$. Define the relations R_1 and R_2 on $P(S)$ as AR_1B if

$$(A \cap B^c) \cup (B \cap A^c) =,$$

and AR_2B if

$$A \cup B^c = B \cup A^c,$$

for all $A, B \in P(S)$. Then:
(1) Both R_1 and R_2 are equivalence relations.
(2) Only R_1 is an equivalence relation.
(3) Only R_2 is an equivalence relation.
(4) Both R_1 and R_2 are not equivalence relations.

Question 13: The area of the region given by $\{(x, y) : xy \leq 8,\ 1 \leq y \leq x^2\}$ is:
(1) $8 \ln_e 2 - \frac{13}{3}$
(2) $16 \ln_e 2 - \frac{14}{3}$
(3) $8 \ln_e 2 + \frac{7}{6}$
(4) $16 \ln_e 2 + \frac{7}{3}$

Question 14: Let $\alpha x = \exp(x^\beta y^\gamma)$ be the solution of the differential equation $2x^2y\,dy - (1 - xy^2)\,dx = 0$, $x > 0$, $y(2) = \sqrt{\ln_e 2}$. Then $\alpha + \beta - \gamma$ equals:
(1) 1
(2) -1
(3) 0
(4) 3

Question 15: The value of the integral

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{x + \frac{\pi}{4}}{2 - \cos 2x}\,dx \text{ is:}$$

(1) $\frac{\pi^2}{6}$
(2) $\frac{\pi^2}{12\sqrt{3}}$
(3) $\frac{\pi^2}{3\sqrt{3}}$
(4) $\frac{\pi^2}{6\sqrt{3}}$

Question 16: Let $9 = x_1 < x_2 < \cdots < x_7$ be in an A.P. with common difference d. If the standard deviation of $x_1, x_2, \ldots, x_7$ is 4 and the mean is $\overline{x}$, then $\overline{x} + x_6$ is equal to:
(1) $18\left(1 + \frac{1}{\sqrt{3}}\right)$
(2) 34
(3) $2\left(9 + \frac{8}{\sqrt{7}}\right)$
(4) 25

Question 17: For the system of linear equations $ax + y + z = 1$, $x + ay + z = 1$, $x + y + az = \beta$, which one of the following statements is NOT correct?
(1) It has infinitely many solutions if $\alpha = 2$ and $\beta = -1$.
(2) It has no solution if $\alpha = -2$ and $\beta = 1$.
(3) $x + y + z = \frac{3}{4}$ if $\alpha = 2$ and $\beta = 1$.
(4) It has infinitely many solutions if $\alpha = 1$ and $\beta = 1$.

Question 18: Let $\vec{a} = 5\hat{i} - \hat{j} - 3\hat{k}$ and $\vec{b} = \hat{i} + 3\hat{j} + 5\hat{k}$ be two vectors. Then which one of the following statements is TRUE?
(1) Projection of $\vec{a}$ on $\vec{b}$ is $\frac{17}{\sqrt{35}}$ and the direction of the projection vector is the same as $\vec{b}$.
(2) Projection of $\vec{a}$ on $\vec{b}$ is $\frac{-17}{\sqrt{35}}$ and the direction of the projection vector is opposite to $\vec{b}$.
(3) Projection of $\vec{a}$ on $\vec{b}$ is $\frac{17}{\sqrt{35}}$ and the direction of the projection vector is opposite to $\vec{b}$.
(4) Projection of $\vec{a}$ on $\vec{b}$ is $\frac{-17}{\sqrt{35}}$ and the direction of the projection vector is opposite to $\vec{b}$.

Question 19: Let $P(x_0, y_0)$ be the point on the hyperbola $3x^2 - 4y^2 = 36$, which is nearest to the line $3x + 2y = 1$. Then $\sqrt{2}\,(y_0 - x_0)$ is equal to:

(1) -3
(2) 9
(3) -9
(4) 3

Question 20: If $y(x) = x^x$, $x > 0$, then $y''(2) - 2y'(2)$ is equal to:

(1) $8\log_e 2 - 2$
(2) $4\log_e 2 + 2$
(3) $4(\log_e 2)^2 - 2$
(4) $4(\log_e 2)^2 + 2$

Section B

Question 21: The total number of six-digit numbers, formed using the digits $4, 5, 9$ only and divisible by 6, is ____.

Question 22: Number of integral solutions to the equation $x+y+z = 21$, where $x \geq 1, y \geq 3, z \geq 4$, is ____.

Question 23: The line $x = 8$ is the directrix of the ellipse $E : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with the corresponding focus $(2, 0)$. If the tangent to E at the point P in the first quadrant passes through the point $\left(0, 4\sqrt{3}\right)$ and intersects the x-axis at Q, then $(3PQ)^2$ is equal to ____.

Question 24:
If the x-intercept of a focal chord of the parabola $y^2 = 8x + 4y + 4$ is 3, then the length of this chord is equal to ____.

Question 25: If $\int_0^{\pi} \frac{5^{\cos x}(1+\cos x\cos 3x+\cos^2 x+\cos^3 x\cos 3x)}{1+5^{\cos x}} dx = \frac{k\pi}{16}$, then k is equal to ____.

Question 26: Let the sixth term in the binomial expansion of

$$\left(\sqrt{2^{\log_2(10-3^x)}} + 5 \cdot \sqrt{2^{(x-2)\log_2 3}}\right)^m,$$

in the increasing powers of $2^{(x-2)\log_2 3}$, be 21. If the binomial coefficients of the

second, third, and fourth terms in the expansion are respectively the first, third, and fifth terms of an A.P., then the sum of the squares of all possible values of x is ____.

Question 27: If the term without x in the expansion of

$$\left(x^{\frac{2}{3}}+\frac{\alpha}{x^3}\right)^{22}$$

is 7315, then $|\alpha|$ is equal to ____.

Question 28: The sum of the common terms of the following three arithmetic progressions:
- $3, 7, 11, 15, \ldots, 399,$
- $2, 5, 8, 11, \ldots, 359,$
- $2, 7, 12, 17, \ldots, 197,$

is equal to ____.

Question 29: Let $\alpha x+\beta y+yz=1$ be the equation of a plane passing through the point $(3,-2,5)$ and perpendicular to the line joining the points $(1,2,3)$ and $(-2,3,5)$. Then the value of $\alpha\beta y$ is equal to ____.

Question 30: The point of intersection C of the plane $8x+y+2z=0$ and the line joining the points $A(-3,-6,1)$ and $B(2,4,-3)$ divides the line segment AB internally in the ratio $k:1$. If a,b,c ($|a|,|b|,|c|$ are coprime) are the direction ratios of the perpendicular from the point C on the line $\frac{1-x}{1}=\frac{y+4}{2}=\frac{z+2}{3}$, then $|a+b+c|$ is equal to ____.
