April 6 2023, Shift 1

JEE Main 2023 April 6 Shift 1 Mathematics Question Paper

Total Time Allowed : 180 minutes	**Maximum Marks :** 300	**Total Questions :** 90	**Questions to be answered :** 90

General Instructions

Read the following instructions very carefully and strictly follow them:
(1) The test is of 3 hours duration.
(2) The question paper consists of 90 questions. The maximum marks are 300.
(3) There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage.
(4) Each part (subject) has two sections.
(i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and –1 mark for wrong answer.
(ii) Section-B: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and –1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

Mathematics

Section A

Question 1: The straight lines l_1 and l_2 pass through the origin and trisect the line segment of the line $L : 9x + 5y = 45$ between the axes. If m_1 and m_2 are the slopes of the lines l_1 and l_2, then the point of intersection of the line $y = (m_1 + m_2)x$ with L lies on:

(1) $6x + y = 10$
(2) $6x - y = 15$
(3) $y - 2x = 5$
(4) $y - x = 5$

Question 2: Let the position vectors of the points A, B, C and D be $5\hat{\mathbf{i}} + 5\hat{\mathbf{j}} + 2\lambda\hat{\mathbf{k}}$, $\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$, $-2\hat{\mathbf{i}} + \lambda\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$ and $-\hat{\mathbf{i}} + 5\hat{\mathbf{j}} + 6\hat{\mathbf{k}}$. Let the set $S = \{\lambda \in R$: the points A, B, C and D are coplanar. Then $\sum_{\lambda\in S}(\lambda + 2)^2$ is equal to:
(1) $\frac{37}{2}$
(2) 13
(3) 25

(4) 41

Question 3: Let $I(x) = \int \frac{x^2(x\sec^2 x+\tan x)}{(x\tan x+1)^2}dx$. If $I(0) = 0$, then $I(\frac{\pi}{4})$ is equal to:
(1) $\log_e \frac{(\pi+4)^2}{16} + \frac{\pi^2}{4(\pi+4)}$
(2) $\log_e \frac{(\pi+4)^2}{32} - \frac{\pi^2}{4(\pi+4)}$
(3) $\log_e \frac{(\pi+4)^2}{16} - \frac{\pi^2}{4(\pi+4)}$
(4) $\log_e \frac{(\pi+4)^2}{32} + \frac{\pi^2}{4(\pi+4)}$

Question 4: The sum of the first 20 terms of the series $5+11+19+29+41+\dots$ **is:**
(1) 3450
(2) 3420
(3) 3520
(4) 3250

Question 5: A pair of dice is thrown 5 times. For each throw, a total of 5 is considered a success. If probability of at least 4 successes is $\frac{k}{3^{11}}$**, then k is equal to :**
(1) 164
(2) 123
(3) 82
(4) 75

Question 6: Let $A = [a_{ij}]_{2\times 2}$**, where** $a_{ij} \neq 0$ **for all i, j, and** $A^2 = I$**. Let a be the sum of all diagonal elements of A and** $b = |A|$**. Then** $3a^2 + 4b^2$ **is equal to :**
(1) 14
(2) 4
(3) 3
(4) 7

Question 7: Let $a_1, a_2, a_3, \dots, a_n$ be n positive consecutive terms of an arithmetic progression. If $d > 0$ is its common difference, then:

$$\lim_{n\to\infty} \frac{d}{\sqrt{n}} \left(\frac{1}{\sqrt{a_1}+\sqrt{a_2}} + \frac{1}{\sqrt{a_2}+\sqrt{a_3}} + \cdots + \frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}} \right) \text{ is}$$

(1) $\frac{1}{\sqrt{d}}$ (2) 1 (3) $\sqrt{d}$ (4) 0

Question 8: If ${}^{2n}C_3 :{}^{n} C_3 : 10 : 1$**, then the ratio** $(n^2+3n) : (n^2-3n+4)$ **is:**

(1) 27 : 11
(2) 35 : 16
(3) 2 : 1
(4) 65 : 37

Question 9: Let $A = \{x \in R : |x+3| + |x+4| \leq 3\}$,

$$B = \left\{ x \in R : 3 \cdot \sum_{r=1}^{\infty} \frac{3^{x-3}}{10^r} < 3^{-3x} \right\},$$

where $[x]$ denotes the greatest integer function. Then,
(1) $A \subset B, A \neq B$
(2) $A \cap B = \phi$
(3) $A = B$
(4) $B \subset C, A \neq B$

Question 10: One vertex of a rectangular parallelepiped is at the origin O and the lengths of its edges along x, y and z axes are 3, 4 and 5 units respectively. Let P be the vertex (3, 4, 5). Then the shortest distance between the diagonal OP and an edge parallel to z axis, not passing through O or P is :
(1) $\frac{12}{5\sqrt{5}}$
(2) $12\sqrt{5}$
(3) $\frac{12}{5}$
(4) $\frac{12}{\sqrt{5}}$

Question 11: If the equation of the plane passing through the line of intersection of the planes $2x - y + z = 3$ **and** $4x - 3y + 5z + 9 = 0$ **and parallel to the line** $\frac{x+1}{-2} = \frac{y+3}{4} = \frac{z-2}{5}$ **is** $ax + by + cz + 6 = 0$**, then** $a + b + c$ **is equal to :**
(1) 15
(2) 14
(3) 13
(4) 12

Question 12: If the ratio of the fifth term from the beginning to the fifth term from the end in the expansion of $\left(\sqrt[4]{2} + \frac{1}{\sqrt[4]{3}}\right)^n$ **is** $\sqrt{6} : 1$**, then the third term from the beginning is :**
(1) $30\sqrt{2}$
(2) $60\sqrt{2}$
(3) $30\sqrt{3}$

(4) $60\sqrt{3}$

Question 13: The sum of all the roots of the equation $|x^2 - 8x + 15| - 2x + 7 = 0$ is:
(1) $11 - \sqrt{3}$
(2) $9 - \sqrt{3}$
(3) $9 + \sqrt{3}$
(4) $11 + \sqrt{3}$

Question 14: From the top A of a vertical wall AB of height 30 m, the angles of depression of the top P and bottom Q of a vertical tower PQ are 15° and 60° respectively. B and Q are on the same horizontal level. If C is a point on AB such that CB = 15 m, then the area (in m^2) of the quadrilateral BCPQ is equal to :
(1) $300(\sqrt{3} - \sqrt{5})$
(2) $300(\sqrt{3} + 1)$
(3) $300(\sqrt{3} - 1)$
(4) $600(\sqrt{3} - 1)$

Question 15: Let $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$, $\vec{b} = \hat{i} - 2\hat{j} - 2\hat{k}$ and $\vec{c} = -\hat{i} + 4\hat{j} + 3\hat{k}$. If $\vec{d}$ is a vector perpendicular to both $\vec{b}$ and $\vec{c}$, and $\vec{a} \cdot \vec{d} = 18$, then $|\vec{a} \times \vec{d}|^2$ is equal to :
(1) 760
(2) 640
(3) 720
(4) 680

Question 16: If $2x^y + 3y^x = 20$, then $\frac{dy}{dx}$ at (2, 2) is equal to :
(1) $-\frac{3+\log_e 8}{2+\log_e 4}$
(2) $-\frac{2+\log_e 8}{3+\log_e 4}$
(3) $-\frac{3+\log_e 4}{2+\log_e 8}$
(4) $-\frac{3+\log_e 16}{4+\log_e 8}$

Question 17: If the system of equations
x + y + az = b
2x + 5y + 2z = 6
x + 2y + 3z = 3
has infinitely many solutions, then 2a + 3b is equal to :

(1) 28
(2) 20
(3) 25
(4) 23

Question 18: Statement (P $\Rightarrow$ Q) $\wedge$ (R $\Rightarrow$ Q) is logically equivalent to :
(1) (P $\vee$ R) $\Rightarrow$ Q
(2) (P $\Rightarrow$ R) $\vee$ (Q $\Rightarrow$ R)
(3) (P $\Rightarrow$ R) $\wedge$ (Q $\Rightarrow$ R)
(4) (P $\wedge$ R) $\Rightarrow$ Q

Question 19: Let $5f(x) + 4f\left(\frac{1}{x}\right) = \frac{1}{x} + 3,\ x > 0$**. Then** $18\int_1^2 f(x)\,dx$ **is equal to :**
(1) $10\log_e 2 - 6$
(2) $10\log_e 2 + 6$
(3) $5\log_e 2 - 3$
(4) $5\log_e 2 + 3$

Question 20: The mean and variance of a set of 15 numbers are 12 and 14 respectively. The mean and variance of another set of 15 numbers are 14 and σ^2 **respectively. If the variance of all the 30 numbers in the two sets is 13, then** σ^2 **is equal to :**
(1) 12
(2) 10
(3) 11
(4) 9

Section B

Question 21: Let the tangents to the curve $x^2 + 2x - 4y + 9 = 0$ **at the point P(1, 3) on it meet the y-axis at A. Let the line passing through P and parallel to the line** $x - 3y = 6$ **meet the parabola** $y^2 = 4x$ **at B. If B lies on the line** $2x - 3y = 8$**, then** $(AB)^2$ **is equal to __________.**

Question 22: Let the point (p, p + 1) lie inside the region E = {(x,y): 3 - x $\leq$ y $\leq$ $\sqrt{9 - x^2}$, 0 $\leq$ x $\leq$ 3}. If the set of all values of p is the interval (a, b), then $b^2 + b - a^2$ **is equal to __________.**

Question 23: Let $y = y(x)$ be a solution of the differential $(x \cos x)dy + (xy \sin x + y \cos x - 1)dx = 0$, $0 < x < \frac{\pi}{2}$. If $\frac{\pi}{3} y\left(\frac{\pi}{3}\right) = \sqrt{3}$, then $\left|\frac{\pi}{6} y''\left(\frac{\pi}{6}\right) + 2y'\left(\frac{\pi}{6}\right)\right|$ is equal to ________.

Question 24: Let $a \in Z$ and $[t]$ be the greatest integer $\leq t$. Then the number of points, where the function f(x) = $[a + 13 \sin x]$, $x \in (0, \pi)$ is not differentiable, is ________.

Question 25: If the area of the region S = {(x, y) : 2y - y^2 $\leq$ x $\leq$ 2y, x $\geq$ y} is equal to $\frac{n}{n+1} - \frac{\pi}{n-1}$, then the natural number n is equal to ________.

Question 26: The number of ways of giving 20 distinct oranges to 3 children such that each child gets at least one orange is ____________.

Question 27: Let the image of the point P (1, 2, 3) in the plane $2x - y + z = 9$ be Q. If the coordinates of the point R are (6, 10, 7), then the square of the area of the triangle PQR is ________.

Question 28: A circle passing through the point P(α, β) in the first quadrant touches the two coordinate axes at the points A and B. The point P is above the line AB. The point Q on the line segment AB is the foot of perpendicular from P on AB. If PQ is equal to 11 units, then value of $\alpha\beta$ is ________.

Question 29: The coefficient of x^{18} in the expansion of $\left(x^4 - \frac{1}{x^3}\right)^{15}$ is ________.

Question 30: Let A = $\{1, 2, 3, 4, \ldots, 10\}$ and B = $\{0, 1, 2, 3, 4\}$. The number of elements in the relation R = $\{(a, b) \in A \times A : 2(a-b)^2 + 3(a-b) \in B\}$ is ________.
