April 10 2023, Shift 1

JEE Main 2023 April 10 Shift 1 Mathematics Question Paper

Total Time Allowed : 180 minutes	**Maximum Marks : 300**	**Total Questions : 90**	**Questions to be answered : 90**

General Instructions

Read the following instructions very carefully and strictly follow them:
(1) The test is of 3 hours duration.
(2) The question paper consists of 90 questions. The maximum marks are 300.
(3) There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage.
(4) Each part (subject) has two sections.
(i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and –1 mark for wrong answer.
(ii) Section-B: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and –1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

Mathematics

Section A

Question 1: An arc PQ of a circle subtends a right angle at its centre O. The midpoint of the arc PQ is R. If $\vec{OP} = \vec{u}$, $\vec{OR} = \vec{v}$ and $\vec{OQ} = \alpha\vec{u} + \beta\vec{v}$, then α, β^2 are the roots of the equation:

(1) $3x^2 - 2x - 1 = 0$
(2) $3x^2 + 2x - 1 = 0$
(3) $x^2 - x - 2 = 0$
(4) $x^2 + x - 2 = 0$

Question 2: A square piece of tin of side 30 cm is to be made into a box without top by cutting a square from each corner and folding up the flaps to form a box. If the volume of the box is maximum, then its surface area (in cm^2) is equal to :

(1) 800
(2) 1025

(3) 900
(4) 675

Question 3: Let O be the origin and the position vector of the point P be $-\hat{i}-2\hat{j}+3\hat{k}$. If the position vectors of the A, B and C are $-2\hat{i}+\hat{j}-3\hat{k}$, $2\hat{i}+4\hat{j}-2\hat{k}$ and $-4\hat{i}+2\hat{j}-\hat{k}$ respectively, then the projection of the vector $\vec{OP}$ on a vector perpendicular to the vectors $\vec{AB}$ and $\vec{AC}$ is:

(1) $\frac{10}{3}$
(2) $\frac{8}{3}$
(3) $\frac{7}{3}$
(4) 3

Question 4: If A is a 3×3 matrix and $|A|=2$, then $|3\text{adj}(|3A|A^2)|$ is equal to :

(1) $3^{12}\cdot 6^{10}$
(2) $3^{11}\cdot 6^{10}$
(3) $3^{12}\cdot 6^{11}$
(4) $3^{10}\cdot 6^{11}$

Question 5: Let two vertices of a triangle ABC be $(2,4,6)$ and $(0,-2,-5)$, and its centroid be $(2,1,-1)$. If the image of the third vertex in the plane $x+2y+4z=11$ is (α,β,γ), then $\alpha\beta+\beta\gamma+\gamma\alpha$ is equal to:

(1) 76
(2) 74
(3) 70
(4) 72

Question 6: The negation of the statement $(p\vee q)\wedge(q\vee(\sim r))$ is

(1) $((\sim p)\vee r)\wedge(\sim q)$
(2) $((\sim p)\vee(\sim q))\wedge(\sim r)$
(3) $((\sim p)\vee(\sim q))\vee(\sim r)$
(4) $(p\vee r)\wedge(\sim q)$

Question 7: The shortest distance between the lines $\frac{x+2}{1}=\frac{y}{-2}=\frac{z-5}{2}$ and $\frac{x-4}{1}=\frac{y-1}{2}=\frac{z+3}{0}$ is:

(1) 8
(2) 7
(3) 6
(4) 9

Question 8: If the coefficient of x^7 in $\left(ax - \frac{1}{bx^2}\right)^{13}$ and the coefficient of x^{-5} in $\left(ax + \frac{1}{bx^2}\right)^{13}$ are equal, then a^4b^4 is equal to:

(1) 22
(2) 44
(3) 11
(4) 33

Question 9: A line segment AB of length λ moves such that the points A and B remain on the periphery of a circle of radius γ. Then the locus of the point, that divides the line segment AB in the ratio 2 : 3, is a circle of radius :

(1) $\frac{2}{3}\lambda$
(2) $\frac{\sqrt{19}}{7}\lambda$
(3) $\frac{3}{5}\lambda$
(4) $\frac{\sqrt{19}}{5}\lambda$

Question 10: For the system of linear equations

$$2x - y + 3z = 5$$

$$3x + 2y - z = 7$$

$$4x + 5y + \alpha z = \beta,$$

which of the following is <u>NOT</u> correct ?

(1) The system in inconsistent for $\alpha = -5$ and $\beta = 8$
(2) The system has infinitely many solutions for $\alpha = -6$ and $\beta = 9$
(3) The system has a unique solution for $\alpha \neq -5$ and $\beta = 8$
(4) The system has infinitely many solutions for $\alpha = -5$ and $\beta = 9$

Question 11: Let the first term a and the common ratio r of a geometric progression be positive integers. If the sum of squares of its first three is 33033,

then the sum of these terms is equal to:

(1) 210
(2) 220
(3) 231
(4) 241

Question 12: Let P be the point of intersection of the line $\frac{x+3}{3} = \frac{y+2}{1} = \frac{1-z}{2}$ **and the plane** $x + y + z = 2$**. If the distance of the point P from the plane** $3x - 4y + 12z = 32$ **is q, then q and 2q are the roots of the equation:**

(1) $x^2 + 18x - 72 = 0$
(2) $x^2 + 18x + 72 = 0$
(3) $x^2 - 18x - 72 = 0$
(4) $x^2 - 18x + 72 = 0$

Question 13: Let f be a differentiable function such that $x^2 f(x) - x = 4\int_0^x t f(t) dt$**,** $f(1) = \frac{2}{3}$**. Then** $18f(3)$ **is equal to:**

(1) 180
(2) 150
(3) 210
(4) 160

Question 14: Let N denote the sum of the numbers obtained when two dice are rolled. If the probability that $2^N < N!$ **is** $\frac{m}{n}$**, where m and n are coprime, then** $4m - 3n$ **equal to:**

(1) 12
(2) 8
(3) 10
(4) 6

Question 15: If $I(x) = \int e^{\sin^2 x} \cos x \sin 2x - \sin x \, dx$ **and** $I(0) = 1$**, then** $I\left(\frac{\pi}{3}\right)$ **is equal to:**

(1) $e^{\frac{3}{4}}$
(2) $-e^{\frac{3}{4}}$
(3) $\frac{1}{2}e^{\frac{3}{4}}$

(4) $-\frac{1}{2}e^{\frac{3}{4}}$

Question 16: $96\cos\frac{\pi}{33}\cos\frac{2\pi}{33}\cos\frac{4\pi}{33}\cos\frac{8\pi}{33}\cos\frac{16\pi}{33}$ **is equal to :**

(1) 4
(2) 2
(3) 3
(4) 1

Question 17: Let the complex number $z = x + iy$ **be such that** $\frac{2z-3i}{2z+i}$ **is purely imaginary. If** $x + y^2 = 0$**, then** $y^4 + y^2 - y$ **is equal to:**

(1) $\frac{3}{2}$
(2) $\frac{2}{3}$
(3) $\frac{4}{3}$
(4) $\frac{3}{4}$

Question 18: If $f(x) = \frac{(\tan 1^\circ)x+\log_e(123)}{x\log_e(1234)-(\tan 1^\circ)}$, $x > 0$**, then the least value of** $f(f(x)) + f\left(f\left(\frac{4}{x}\right)\right)$ **is:**

(1) 2
(2) 4
(3) 8
(4) 0

Question 19: The slope of tangent at any point (x, y) on a curve $y = y(x)$ **is** $\frac{x^2+y^2}{2xy}$, $x > 0$**. If** $y(2) = 0$**, then a value of** $y(8)$ **is:**

(1) $4\sqrt{3}$
(2) $-4\sqrt{2}$
(3) $-2\sqrt{3}$
(4) $2\sqrt{3}$

Question 20: Let the ellipse E : $x^2 + 9y^2 = 9$ **intersect the positive x- and y-axes at the points A and B respectively. Let the major axis of E be a diameter of the circle C. Let the line passing through A and B meet the circle C at the point P. If the area of the triangle with vertices A, P and the origin O is** $\frac{m}{n}$,

where m and n are coprime, then m - n is equal to :

(1) 16
(2) 15
(3) 18
(4) 17

Section B

Question 21: Some couples participated in a mixed doubles badminton tournament. If the number of matches played, so that no couple in a match, is 840, then the total numbers of persons, who participated in the tournament, is ______.

Correct Answer: 16

Question 22: The number of elements in the set $\{n \in Z : |n^2 - 10n + 19| < 6\}$ **is ______.**

Question 23: The number of permutations of the digits 1, 2, 3, ..., 7 without repetition, which neither contain the string 153 nor the string 2467, is ______.

Question 24: Let $f : (-2, 2) \to R$ **be defined by**

$$f(x) = \begin{cases} x\lfloor x \rfloor, & -2 < x < 0 \\ (x-1)\lfloor x \rfloor, & 0 \leq x < 2 \end{cases}$$

where $\lfloor x \rfloor$ **denotes the greatest integer function. If m and n respectively are the number of points in (-2, 2) at which** $y = |f(x)|$ **is not continuous and not differentiable, then m + n is equal to ______.**

Question 25: Let a common tangent to the curves $y^2 = 4x$ **and** $(x-4)^2 + y^2 = 16$ **touch the curves at the points P and Q. Then** $(PQ)^2$ **is equal to ______.**

Question 26: If the mean of the frequency distribution

Class	0-10	10-20	20-30	30-40	40-50
Frequency	2	3	x	5	4

is 28, then its variance is ______.

Question 27: The coefficient of x^7 **in** $(1 - x + 2x^3)^{10}$ **is ______.**

Question 28: Let $y = p(x)$ **be the parabola passing through the points** $(-1, 0)$, $(0, 1)$ **and** $(1, 0)$. **If the area of the region** $\{(x, y) : (x+1)^2 + (y-1)^2 \leq 1, y \leq p(x)\}$ **is A, then** $12(\pi - 4A)$ **is equal to ______.**

Question 29: Let a, b, c be three distinct positive real numbers such that $(2a)^{\log_e a} = (bc)^{\log_e b}$ **and** $b^{\log_e^2 a} = a^{\log_e c}$. **Then** $6a + 5bc$ **is equal to ______.**

Question 30: The sum of all those terms, of the arithmetic progression 3, 8, 13, ..., 373, which are *not* divisible by 3, is ______.
