JEE Main 2023 April 13 Shift 1 Mathematics Question Paper

Time Allowed :1 Hour	**Maximum Marks :**100	**Total Questions :**30

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The test is of 3 hours duration.
2. The question paper consists of 90 questions. The maximum marks are 300.
3. There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage.
4. Each part (subject) has two sections.
(i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and –1 mark for wrong answer.
(ii) Section-B: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and –1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

1. For the differentiable function $f : \mathbb{R} - \{0\} \to \mathbb{R}$, let $3f(x) + 2f\left(\frac{1}{x}\right) = \frac{1}{x} - 10$, then $\left|f(3) + f'\left(\frac{1}{4}\right)\right|$ is equal to
(A) 7
(B) $\frac{29}{5}$
(C) $\frac{33}{5}$
(D) 13

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2. The set of all $a \in \mathbb{R}$ for which the equation $x|x-1| + |x+2| + a = 0$ has exactly one real root, is
(A) $(-\infty, -3)$
(B) $(-6, \infty)$
(C) $(-\infty, \infty)$
(D) $(-6, -3)$

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3. Let $B = \begin{bmatrix} 1 & 3 & \alpha \\ 1 & 2 & 3 \\ \alpha & \alpha & 4 \end{bmatrix}$, $\alpha > 2$ be the adjoint of a matrix A and $|A| = 2$. Then
$\begin{bmatrix} \alpha & -2\alpha & \alpha \end{bmatrix} B \begin{bmatrix} \alpha \\ -2\alpha \\ \alpha \end{bmatrix}$ **is equal to**

(A) 0
(B) -16
(C) 16
(D) 32

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4. For the system of linear equations:
$2x + 4y + 2az = b$
$x + 2y + 3z = 4$
$2x - 5y + 2z = 8$
which of the following is NOT correct?
(A) It has unique solution if $a = b = 6$
(B) It has unique solution if $a = b = 8$
(C) It has infinitely many solutions if $a = 3, b = 8$
(D) It has infinitely many solutions if $a = 3, b = 6$

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5. The number of symmetric matrices of order 3, with all the entries from the set $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$**, is**
(A) 10^9
(B) 9^{10}
(C) 10^6
(D) 6^{10}

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6. Let $s_1, s_2, s_3, \ldots, s_{10}$ **respectively be the sum to 12 terms of 10 A.P.s whose first terms are 1, 2, 3, . . . , 10 and the common differences are 1, 3, 5, . . . , 19 respectively. Then** $\sum_{i=1}^{10} s_i$ **is equal to**
(A) 7260
(B) 7220
(C) 7360
(D) 7380

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7. Among
(S1) : $\lim_{n\to\infty} \frac{1}{n^2}(2 + 4 + 6 + \cdots + 2n) = 1$
(S2) : $\lim_{n\to\infty} \frac{1}{n^{16}}(1^{15} + 2^{15} + 3^{15} + \cdots + n^{15}) = \frac{1}{16}$
(A) Only (S1) is true
(B) Only (S2) is true
(C) Both (S1) and (S2) are true
(D) Both (S1) and (S2) are false

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8. Let the equation of plane passing through the line of intersection of the planes $x + 2y + az = 2$ **and** $x - y + z = 3$ **be** $5x - 11y + bz = 6a - 1$**. For** $c \in \mathbb{Z}$**, if the distance**

of this plane from the point $(a, -c, c)$ is $\frac{2}{\sqrt{a}}$, then $\frac{a+b}{c}$ is equal to
(A) -4
(B) -2
(C) 2
(D) 4

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9. Let $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$, $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$. If a vector $\vec{d}$ satisfies $\vec{d} \times \vec{b} = \vec{c} \times \vec{b}$ and $\vec{d} \cdot \vec{a} = 24$, then $|\vec{d}|^2$ is equal to
(A) 323
(B) 313
(C) 423
(D) 413

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10. The maximum value of $\left\{x - 2\sin x \cos x + \frac{1}{3}\sin 3x\right\}$ for $0 \leq x \leq \pi$ is
(A) $\frac{\pi+2-3\sqrt{3}}{6}$
(B) $\frac{5\pi+2+3\sqrt{3}}{6}$
(C) π
(D) 0

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11. The value of the definite integral $\int_0^\infty \frac{6}{e^{3x}+6e^{2x}+11e^x+6} dx$ is equal to
(A) $\log_e\left(\frac{512}{81}\right)$
(B) $\log_e\left(\frac{64}{27}\right)$
(C) $\log_e\left(\frac{256}{81}\right)$
(D) $\log_e\left(\frac{32}{27}\right)$

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12. The area of the region enclosed by the curve $f(x) = \max\{\sin x, \cos x\}$, $-\pi \leq x \leq \pi$ and the x-axis is
(A) $2\sqrt{2}(\sqrt{2}+1)$
(B) 4
(C) $2(\sqrt{2}+1)$
(D) $4\sqrt{2}$

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13. Fractional part of the number $\frac{4^{2022}}{15}$ is equal to
(A) $\frac{1}{15}$
(B) $\frac{4}{15}$
(C) $\frac{8}{15}$
(D) $\frac{14}{15}$

14. Let $y = y_1(x)$ and $y = y_2(x)$ be the solution curves of the differential equation $\frac{dy}{dx} = y + 7$ with initial conditions $y_1(0) = 0$ and $y_2(0) = 1$ respectively. Then the curves $y = y_1(x)$ and $y = y_2(x)$ intersect at

(A) no point
(B) one point
(C) two points
(D) infinite number of points

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15. Let the tangent and normal at the point $(3\sqrt{3}, 1)$ on the ellipse $\frac{x^2}{36} + \frac{y^2}{4} = 1$ meet the y-axis at the points A and B respectively. Let the circle C be drawn taking AB as a diameter and the line $x = 2\sqrt{5}$ intersect C at the points P and Q. If the tangents at the points P and Q on the circle intersect at the point (α, β), then $\alpha^2 - \beta^2$ is equal to

(A) 60
(B) 61
(C) $\frac{304}{5}$
(D) $\frac{314}{5}$

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16. Let PQ be a focal chord of the parabola $y^2 = 36x$ of length 100, making an acute angle with the positive x-axis. Let the ordinate of P be positive and M be the point on the line segment PQ such that $PM : MQ = 3 : 1$. Then which of the following points does NOT lie on the line passing through M and perpendicular to the line PQ?

(A) $(3, 33)$
(B) $(-6, 45)$
(C) $(-3, 43)$
(D) $(6, 29)$

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17. The distance of the point $(-1, 2, 3)$ from the plane $\vec{r} \cdot (\hat{i} - 2\hat{j} + 3\hat{k}) = 10$ measured parallel to the line of the shortest distance between the lines $\vec{r} = (\hat{i} - \hat{j}) + \lambda(2\hat{i} + \hat{k})$ and $\vec{r} = (2\hat{i} - \hat{j}) + \mu(\hat{i} - \hat{j} + \hat{k})$ is

(A) $3\sqrt{5}$
(B) $2\sqrt{6}$
(C) $3\sqrt{6}$
(D) $2\sqrt{5}$

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18. A coin is biased so that the head is 3 times as likely to occur as tail. This coin is tossed until a head or three tails occur. If X denotes the number of tosses of the coin, then the mean of X is

(A) $\frac{21}{16}$
(B) $\frac{15}{16}$
(C) $\frac{81}{64}$
(D) $\frac{37}{16}$

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19. For $x \in \mathbb{R}$, two real valued functions $f(x)$ and $g(x)$ are such that $g(x) = \sqrt{x} + 1$ and $f(g(x)) = x + 3 - \sqrt{x}$. Then $f(0)$ is equal to
(A) -3
(B) 0
(C) 1
(D) 5

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20. The negation of the statement $((A \wedge (B \vee C)) \implies (A \vee B)) \implies A$ is
(A) a fallacy
(B) equivalent to $B \vee \sim C$
(C) equivalent to $\sim A$
(D) equivalent to $\sim C$

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21. Let $w = z\bar{z} + k_1 z + k_2 iz + \lambda(1+i)$. $k_1, k_2 \in \mathbb{R}$. Let $\mathbf{Re}(w) = 0$ be the circle C of radius 1 in the first quadrant touching the line $y = 1$ and the y-axis. If the curve $\mathbf{Im}(w) = 0$ intersects C at A and B, then $30(AB)^2$ is equal to ______.

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22. The number of seven digit positive integers formed using the digits 1, 2, 3 and 4 only and sum of the digits equal to 12 is ______.

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23. Let α be the constant term in the binomial expansion of $\left(\sqrt{x} - \frac{6}{x^{3/2}}\right)^n$, $n \leq 15$. If the sum of the coefficients of the remaining terms in the expansion is 649 and the coefficient of x^{-n} is $\lambda\alpha$, then λ is equal to ______.

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24. The sum to 20 terms of the series $2 \cdot 2^2 - 3^2 + 2 \cdot 4^2 - 5^2 + 2 \cdot 6^2 - \ldots$ is equal to ______.

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25. Let for $x \in \mathbb{R}$, $S_0(x) = x$, $S_k(x) = C_k x + k \int_0^x S_{k-1}(t)dt$, where
$C_0 = 1, C_k = 1 - \int_0^1 S_{k-1}(x)dx$, $k = 1, 2, 3, \ldots$. **Then $S_2(3) + 6C_3$ is equal to ______.**

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26. **Let m_1 and m_2 be the slopes of the tangents drawn from the point $P(4,1)$ to the hyperbola $H : \frac{y^2}{25} - \frac{x^2}{16} = 1$. If Q is the point from which the tangents drawn to H have slopes $|m_1|$ and $|m_2|$ and they make positive intercepts α and β on the x-axis, then $\frac{(PQ)^2}{\alpha\beta}$ is equal to ______.**

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27. **Let the image of the point $\left(\frac{5}{3}, \frac{5}{3}, \frac{8}{3}\right)$ in the plane $x - 2y + z - 2 = 0$ be P. If the distance of the point $Q(6, -2, \alpha)$, $\alpha > 0$, from P is 13, then α is equal to ______.**

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28. **Let $\vec{a} = 3\hat{i} + \hat{j} - \hat{k}$ and $\vec{c} = 2\hat{i} - 3\hat{j} + 3\hat{k}$. If $\vec{b}$ is a vector such that $\vec{a} = \vec{b} \times \vec{c}$ and $|\vec{b}|^2 = 50$, then $|72 - |\vec{b} + \vec{c}|^2|$ is equal to ______.**

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29. **Let the mean of the data be 5. If m and σ^2 are respectively the mean deviation about the mean and the variance of the data, then $\frac{3\alpha}{m+\sigma^2}$ is equal to ______.**

x	1	3	5	7	9
Frequency (f)	4	24	28	α	8

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30. **If $S = \left\{x \in \mathbb{R} : \sin^{-1}\left(\frac{x+1}{\sqrt{x^2+2x+2}}\right) - \sin^{-1}\left(\frac{x}{\sqrt{x^2+1}}\right) = \frac{\pi}{4}\right\}$, then $\sum_{x \in S}\left(\sin\left((x^2+x+5)\frac{\pi}{2}\right) - \cos\left((x^2+x+5)\pi\right)\right)$ is equal to ______.**
