JEE Main 2023 April 12 Shift 1 Mathematics Question Paper

Time Allowed : 1 Hour	**Maximum Marks :** 100	**Total Questions :** 30

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The test is of 3 hours duration.
2. The question paper consists of 90 questions. The maximum marks are 300.
3. There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage.
4. Each part (subject) has two sections.
(i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and –1 mark for wrong answer.
(ii) Section-B: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and –1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

1. Let D be the domain of the function $f(x) = \sin^{-1}\left(\log_{3x}\left(\frac{6+2\log_3 x}{-5x}\right)\right)$. If the range of the function $g : D \to \mathbb{R}$ defined by $g(x) = x - [x]$, ($[x]$ is the greatest integer function), is (α, β), then $\alpha^2 + \frac{5}{\beta}$ is equal to
(A) 45
(B) 46
(C) 135
(D) 136

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2. Let α, β be the roots of the quadratic equation $x^2 + \sqrt{6}x + 3 = 0$. Then $\frac{\alpha^{23}+\beta^{23}+\alpha^{14}+\beta^{14}}{\alpha^{15}+\beta^{15}+\alpha^{10}+\beta^{10}}$ is equal to
(A) 9
(B) 72
(C) 81
(D) 729

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3. Let $A = \begin{bmatrix} 1 & \frac{1}{51} \\ 0 & 1 \end{bmatrix}$. If $B = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix} A \begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix}$, then the sum of all the elements of the matrix $\sum_{n=1}^{50} B^n$ is equal to
(A) 50

(B) 75
(C) 100
(D) 125

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4. The number of five digit numbers, greater than 40000 and divisible by 5, which can be formed using the digits 0, 1, 3, 5, 7 and 9 without repetition, is equal to
(A) 72
(B) 96
(C) 120
(D) 132

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5. The sum, of the coefficients of the first 50 terms in the binomial expansion of $(1-x)^{100}$, is equal to
(A) ${}^{99}C_{49}$
(B) $-{}^{99}C_{49}$
(C) ${}^{101}C_{50}$
(D) $-{}^{101}C_{50}$

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6. If $\frac{1}{n+1}{}^nC_n + \frac{1}{n}{}^nC_{n-1} + \cdots + \frac{1}{2}{}^nC_1 + {}^nC_0 = \frac{1023}{10}$, then n is equal to
(A) 6
(B) 7
(C) 8
(D) 9

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7. Let C be the circle in the complex plane with centre $z_0 = \frac{1}{2}(1+3i)$ and radius $r=1$. Let $z_1 = 1+i$ and the complex number z_2 be outside the circle C such that $|z_1 - z_0||z_2 - z_0| = 1$. If z_0, z_1 and z_2 are collinear, then the smaller value of $|z_2|^2$ is equal to
(A) $\frac{3}{2}$
(B) $\frac{5}{2}$
(C) $\frac{7}{2}$
(D) $\frac{13}{2}$

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8. Let $< a_n >$ be a sequence such that $a_1 + a_2 + \cdots + a_n = \frac{n^2+3n}{(n+1)(n+2)}$. If $28\sum_{k=1}^{10} \frac{1}{a_k} = p_1p_2p_3\ldots p_m$ where $p_1, p_2, \ldots, p_m$ are the first m prime numbers, then m is equal to
(A) 5
(B) 6
(C) 7

(D) 8

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9. If the local maximum value of the function $f(x) = \left(\frac{\sqrt{3e}}{2\sin x}\right)^{\sin^2 x}$, $x \in (0, \pi/2)$ **is** $\frac{k}{e}$**, then** $\left(\frac{k}{e}\right)^8 + \frac{k^8}{e^5} + k^8$ **is equal to**
(A) $e^5 + e^6 + e^{11}$
(B) $e^3 + e^6 + e^{10}$
(C) $e^3 + e^5 + e^{11}$
(D) $e^3 + e^6 + e^{11}$

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10. The area of the region enclosed by the curve $y = x^3$ **and its tangent at the point** $(-1, -1)$ **is**
(A) $\frac{19}{4}$
(B) $\frac{23}{4}$
(C) $\frac{27}{4}$
(D) $\frac{31}{4}$

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11. Let $y = y(x), y > 0$**, be a solution curve of the differential equation** $(1 + x^2)dy = y(x - y)dx$**. If** $y(0) = 1$ **and** $y(2\sqrt{2}) = \beta$**, then**
(A) $e^{\beta^{-1}} = e^{-2}(3 + 2\sqrt{2})$
(B) $e^{3\beta^{-1}} = e(5 + \sqrt{2})$
(C) $e^{\beta^{-1}} = e^{-2}(5 + \sqrt{2})$
(D) $e^{3\beta^{-1}} = e(3 + 2\sqrt{2})$

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12. Let $p\left(\frac{2\sqrt{3}}{\sqrt{7}}, \frac{6}{\sqrt{7}}\right)$**,** Q, R **and** S **be four points on the ellipse** $9x^2 + 4y^2 = 36$**. Let** PQ **and** RS **be mutually perpendicular and pass through the origin. If** $\frac{1}{(PQ)^2} + \frac{1}{(RS)^2} = \frac{p}{q}$**, where** p **and** q **are coprime, then** $p + q$ **is equal to**
(A) 137
(B) 143
(C) 147
(D) 157

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13. If the point $\left(\alpha, \frac{7\sqrt{3}}{3}\right)$ **lies on the curve traced by the mid-points of the line segments of the lines** $x\cos\theta + y\sin\theta = 7$**,** $\theta \in \left(0, \frac{\pi}{2}\right)$ **between the co-ordinates axes, then** α **is equal to**
(A) -7
(B) -73

(C) 7
(D) 73

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14. Let the plane $P : 4x - y + z = 10$ be rotated by an angle $\frac{\pi}{2}$ about its line of intersection with the plane $x + y - z = 4$. If α is the distance of the point $(2, 3, -4)$ from the new position of the plane P, then 35α is equal to
(A) 85
(B) 90
(C) 105
(D) 126

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15. Let the lines $l_1 : \frac{x+5}{3} = \frac{y+4}{1} = \frac{z-\alpha}{-2}$ and $l_2 : 3x + 2y + z - 2 = 0 = x - 3y + 2z - 13$ be coplanar. If the point $P(a, b, c)$ on l_1 is nearest to the point $Q(-4, -3, 2)$, then $|a| + |b| + |c|$ is equal to
(A) 8
(B) 10
(C) 12
(D) 14

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16. Let a, b, c be three distinct real numbers, none equal to one. If the vectors $a\hat{i} + \hat{j} + \hat{k}$, $\hat{i} + b\hat{j} + \hat{k}$ and $\hat{i} + \hat{j} + c\hat{k}$ are coplanar, then $\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c}$ is equal to
(A) -1
(B) 1
(C) -2
(D) 2

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17. Let $\lambda \in \mathbb{Z}$, $\vec{a} = \lambda\hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = 3\hat{i} - \hat{j} + 2\hat{k}$. Let $\vec{c}$ be a vector such that $(\vec{a} + \vec{b} + \vec{c}) \times \vec{c} = \vec{0}$, $\vec{a} \cdot \vec{c} = -17$ and $\vec{b} \cdot \vec{c} = -20$. Then $|\vec{c} \times (\lambda\hat{i} + \hat{j} + \hat{k})|^2$ is equal to
(A) 46
(B) 49
(C) 53
(D) 62

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18. Two dice A and B are rolled. Let the numbers obtained on A and B be α and β respectively. If the variance of $\alpha - \beta$ is $\frac{p}{q}$, where p and q are co-prime, then the sum of the positive divisors of p is equal to
(A) 31
(B) 36
(C) 48

(D) 72

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19. In a triangle ABC, if $\cos A + 2\cos B + \cos C = 2$ and the lengths of the sides opposite to the angles A and C are 3 and 7 respectively, then $\cos A - \cos C$ is equal to

(A) 3/7
(B) 10/7
(C) 5/7
(D) 9/7

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20. Among the two statements
(S1) : $(p \Rightarrow q) \wedge (p \wedge (\sim q))$ **is a contradiction and**
(S2) : $(p \wedge q) \vee ((\sim p) \wedge q) \vee (p \wedge (\sim q)) \vee ((\sim p) \wedge (\sim q))$ **is a tautology**
(A) both are true.
(B) both are false.
(C) only (S1) is true.
(D) only (S2) is true.

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21. The number of relations, on the set {1, 2, 3} containing (1, 2) and (2, 3), which are reflexive and transitive but not symmetric, is ________.

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22. Let $D_k = \begin{vmatrix} 1 & 2k & 2k-1 \\ n & n^2+n+2 & n^2 \\ n & n^2+n & n^2+n+2 \end{vmatrix}$. **If** $\sum_{k=1}^{n} D_k = 96$, **then** n **is equal to ________.**

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23. Let the digits a, b, c be in A. P. Nine-digit numbers are to be formed using each of these three digits thrice such that three consecutive digits are in A.P. at least once. How many such numbers can be formed?

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24. Let the positive numbers a_1, a_2, a_3, a_4 **and** a_5 **be in a G.P. Let their mean and variance be** $\frac{31}{10}$ **and** $\frac{m}{n}$ **respectively, where m and n are co-prime. If the mean of their reciprocals is** $\frac{31}{40}$ **and** $a_3 + a_4 + a_5 = 14$, **then** $m + n$ **is equal to ________.**

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25. Let [x] be the greatest integer $\leq x$. **Then the number of points in the interval** $(-2, 1)$, **where the function** $f(x) = [x] + \sqrt{x - [x]}$ **is discontinuous, is ________.**

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26. If $\int_{-0.15}^{0.15} |100x^2 - 1| dx = \frac{k}{3000}$, then k is equal to ________.

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27. Let $I(x) = \int \sqrt{\frac{x+7}{x}} dx$ and $I(9) = 12 + 7\log_e 7$. If $I(1) = \alpha + 7\log_e(1 + 2\sqrt{2})$, then α^4 is equal to ________.

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28. Two circles in the first quadrant of radii r_1 and r_2 touch the coordinate axes. Each of them cuts off an intercept of 2 units with the line $x + y = 2$. Then $r_1^2 + r_2^2 - r_1 r_2$ is equal to ________.

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29. Let the plane $x + 3y - 2z + 6 = 0$ meet the co-ordinate axes at the points A, B, C. If the orthocenter of the triangle ABC is $(\alpha, \beta, 6/7)$, then $98(\alpha + \beta)^2$ is equal to ________.

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30. A fair n (n $>$ 1) faces die is rolled repeatedly until a number less than n appears. If the mean of the number of tosses required is $\frac{n}{9}$, then n is equal to ________.
