JEE Main 2023 April 11 Shift 1 Mathematics Question Paper

Time Allowed : 1 Hour	**Maximum Marks :** 100	**Total Questions :** 30

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The test is of 3 hours duration.
2. The question paper consists of 90 questions. The maximum marks are 300.
3. There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage.
4. Each part (subject) has two sections.
   (i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and –1 mark for wrong answer.
   (ii) Section-B: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and –1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

1. An organization awarded 48 medals in event 'A', 25 in event 'B' and 18 in event 'C'. If these medals went to total 60 men and only five men got medals in all the three events, then, how many received medals in exactly two of three events?
(A) 15
(B) 9
(C) 10
(D) 21

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2. Let (α, β, γ) be the image of the point $P(2, 3, 5)$ in the plane $2x + y - 3z = 6$. Then $\alpha + \beta + \gamma$ is equal to
(A) 12
(B) 10
(C) 9
(D) 5

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3. Consider ellipses $E_k : kx^2 + k^2y^2 = 1, k = 1, 2, \ldots, 20$. Let C_k be the circle which touches the four chords joining the end points (one on minor axis and another on major axis) of the ellipse E_k. If r_k is the radius of the circle C_k, then the value of $\sum_{k=1}^{20} \frac{1}{r_k^2}$ is
(A) 2870

(B) 3080
(C) 3210
(D) 3320

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4. The number of triplets (x, y, z), where x, y, z are distinct non negative integers satisfying $x + y + z = 15$, is
(A) 136
(B) 114
(C) 92
(D) 80

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5. The number of integral solutions x of $\log_{(x+\frac{7}{2})}\left(\frac{x-7}{2x-3}\right)^2 \geq 0$ is
(A) 8
(B) 7
(C) 6
(D) 5

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6. Let A be a 2×2 matrix with real entries such that $A^T = \alpha A + I$, where $\alpha \in \mathbb{R} - \{-1, 1\}$. If $\det(A^2 - A) = 4$, then the sum of all possible values of α is equal to
(A) $\frac{5}{2}$
(B) $\frac{3}{2}$
(C) 2
(D) 0

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7. Let $f(x) = [x^2 - x] + |-x + [x]|$, where $x \in \mathbb{R}$ and $[t]$ denotes the greatest integer less than or equal to t. Then, f is
(A) not continuous at $x = 0$ and $x = 1$
(B) continuous at $x = 0$ and $x = 1$
(C) continuous at $x = 0$, but not continuous at $x = 1$
(D) continuous at $x = 1$, but not continuous at $x = 0$

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8. If equation of the plane that contains the point $(-2, 3, 5)$ and is perpendicular to each of the planes $2x + 4y + 5z = 8$ and $3x - 2y + 3z = 5$ is $\alpha x + \beta y + \gamma z + 97 = 0$ then $\alpha + \beta + \gamma =$
(A) 15
(B) 16
(C) 17
(D) 18

9. Area of the region $\{(x, y) : x^2 + (y-2)^2 \leq 4, x^2 \geq 2y\}$ is
(A) $\pi - \frac{8}{3}$
(B) $2\pi - \frac{16}{3}$
(C) $\pi + \frac{8}{3}$
(D) $2\pi + \frac{16}{3}$

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10. Let $f : [2,4] \to \mathbb{R}$ be a differentiable function such that $(x \log_e x) f'(x) + (\log_e x) f(x) + f(x) \geq 1, x \in [2,4]$ with $f(2) = \frac{1}{2}$ and $f(4) = \frac{1}{4}$. Consider the following two statements : (A) : $f(x) \leq 1$, for all $x \in [2,4]$. (B) : $f(x) \geq \frac{1}{8}$, for all $x \in [2,4]$. Then,
(A) Only statement (A) is true
(B) Only statement (B) is true
(C) Both the statements (A) and (B) are true
(D) Neither statement (A) nor statement (B) is true

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11. Let $\vec{a}$ be a non-zero vector parallel to the line of intersection of the two planes described by $\hat{i} + \hat{j}, \hat{i} + \hat{k}$ and $\hat{i} - \hat{j}, \hat{j} - \hat{k}$. If θ is the angle between the vector $\vec{a}$ and the vector $\vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}$ and $\vec{a} \cdot \vec{b} = 6$, then the ordered pair $(\theta, |\vec{a} \times \vec{b}|)$ is equal to
(A) $\left(\frac{\pi}{4}, 3\sqrt{6}\right)$
(B) $\left(\frac{\pi}{3}, 6\right)$
(C) $\left(\frac{\pi}{3}, 3\sqrt{6}\right)$
(D) $\left(\frac{\pi}{4}, 6\right)$

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12. For any vector $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, with $10|a_i| < 1, i = 1, 2, 3$, consider the following statements :
(A) : $\max\{|a_1|, |a_2|, |a_3|\} \leq |\vec{a}|$
(B) : $|\vec{a}| \leq 3\max\{|a_1|, |a_2|, |a_3|\}$
(A) Only (A) is true
(B) Only (B) is true
(C) Both (A) and (B) are true
(D) Neither (A) nor (B) is true

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13. Let $x_1, x_2, \ldots, x_{100}$ be in an arithmetic progression, with $x_1 = 2$ and their mean equal to 200. If $y_i = i(x_i - i), 1 \leq i \leq 100$, then the mean of $y_1, y_2, \ldots, y_{100}$ is
(A) 10051.50
(B) 10049.50
(C) 10101.50
(D) 10100

14. Let w_1 be the point obtained by the rotation of $z_1 = 5 + 4i$ about the origin through a right angle in the anticlockwise direction, and w_2 be the point obtained by the rotation of $z_2 = 3 + 5i$ about the origin through a right angle in the clockwise direction. Then the principal argument of $w_1 - w_2$ is equal to

(A) $\pi - \tan^{-1}\frac{33}{5}$
(B) $-\pi + \tan^{-1}\frac{33}{5}$
(C) $\pi - \tan^{-1}\frac{8}{9}$
(D) $-\pi + \tan^{-1}\frac{8}{9}$

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15. The value of the integral $\int_{-\log_e 2}^{\log_e 2} e^x \left(\log_e(e^x + \sqrt{1+e^{2x}})\right) dx$ is equal to

(A) $\log_e\left(\frac{(2+\sqrt{5})^2}{\sqrt{1+\sqrt{5}}}\right) + \frac{\sqrt{5}}{2}$
(B) $\log_e\left(\frac{\sqrt{2}(2+\sqrt{5})^2}{\sqrt{1+\sqrt{5}}}\right) - \frac{\sqrt{5}}{2}$
(C) $\log_e\left(\frac{2(2+\sqrt{5})}{\sqrt{1+\sqrt{5}}}\right) - \frac{\sqrt{5}}{2}$
(D) $\log_e\left(\frac{\sqrt{2}(3-\sqrt{5})^2}{\sqrt{1+\sqrt{5}}}\right) + \frac{\sqrt{5}}{2}$

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16. The number of elements in the set
$S = \{\theta \in [0, 2\pi] : 3\cos^4\theta - 5\cos^2\theta - 2\sin^6\theta + 2 = 0\}$ **is**

(A) 8
(B) 9
(C) 10
(D) 12

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17. Let $S = \{M = [a_{ij}], a_{ij} \in \{0, 1, 2\}, 1 \le i, j \le 2\}$ be a sample space and $A = \{M \in S : M$ is invertible$\}$ be an event. Then $P(A)$ is equal to

(A) $\frac{47}{81}$
(B) $\frac{16}{27}$
(C) $\frac{49}{81}$
(D) $\frac{50}{81}$

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18. Let sets A and B have 5 elements each. Let the mean of the elements in sets A and B be 5 and 8 respectively and the variance of the elements in sets A and B be 12 and 20 respectively. A new set C of 10 elements is formed by subtracting 3 from each element of A and adding 2 to each element of B. Then the sum of the mean and variance of the elements of C is ......

(A) 38
(B) 32
(C) 36
(D) 40

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19. Let R be a rectangle given by the lines $x = 0, x = 2, y = 0$ and $y = 5$. Let $A(\alpha, 0)$ and $B(0, \beta), \alpha \in [0, 2]$ and $\beta \in [0, 5]$, be such that the line segment AB divides the area of the rectangle R in the ratio 4:1. Then, the mid-point of AB lies on a
(A) straight line
(B) parabola
(C) hyperbola
(D) circle

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20. Let $y = y(x)$ be a solution curve of the differential equation $(1 - x^2y^2)dx = ydx + xdy$. If the line $x = 1$ intersects the curve $y = y(x)$ at $y = 2$ and the line $x = 2$ intersects the curve $y = y(x)$ at $y = \alpha$, then a value of α is
(A) $\frac{3e^2}{2(3e^2-1)}$
(B) $\frac{1-3e^2}{2(3e^2+1)}$
(C) $\frac{1+3e^2}{2(3e^2-1)}$
(D) $\frac{3e^2}{2(3e^2+1)}$

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21. The mean of the coefficients of $x, x^2, \ldots, x^7$ in the binomial expansion of $(2 + x)^9$ is ....

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22. Let a line l pass through the origin and be perpendicular to the lines
$l_1 : \vec{r} = (\hat{i} - 11\hat{j} - 7\hat{k}) + \lambda(\hat{i} + 2\hat{j} + 3\hat{k}), \lambda \in \mathbb{R}$ **and**
$l_2 : \vec{r} = (-\hat{i} + \hat{k}) + \mu(2\hat{i} + 2\hat{j} + \hat{k}), \mu \in \mathbb{R}$.
If P is the point of intersection of l and l_1, and $Q(\alpha, \beta, \gamma)$ is the foot of perpendicular from P on l_2, then $9(\alpha + \beta + \gamma)$ is equal to ....

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23. The number of ordered triplets of the truth values of p, q and r such that the truth value of the statement $(p \vee q) \wedge (p \vee r) \Rightarrow (q \vee r)$ is True, is equal to ....

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24. If a and b are the roots of the equation $x^2 - 7x - 1 = 0$, then the value of $\frac{a^{21}+b^{21}+a^{17}+b^{17}}{a^{19}+b^{19}}$ is equal to ....

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25. Let $S = 109 + \frac{108}{5} + \frac{107}{5^2} + \cdots + \frac{2}{5^{107}} + \frac{1}{5^{108}}$. Then the value of $(16S - (25)^{-54})$ is equal to ....

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26. For $m, n > 0$, let $\alpha(m,n) = \int_0^2 t^m (1+3t)^n dt$. If $11\alpha(10,6) + 18\alpha(11,5) = p(14)^6$, then p is equal to ....

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27. The number of integral terms in the expansion of $\left(3^2 + 5^4\right)^{680}$ is equal to ....

(Note: Correcting the typo from the image to standard format: $(3^{1/2} + 5^{1/4})^{680}$)

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28. Let $H_n : \frac{x^2}{1+n} - \frac{y^2}{3+n} = 1, n \in \mathbb{N}$. Let k be the smallest even value of n such that the eccentricity of H_k is a rational number. If l is the length of the latus rectum of H_k, then $21l$ is equal to ....

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29. Let $A = \begin{bmatrix} 0 & 1 & 2 \\ a & 0 & 3 \\ 1 & c & 0 \end{bmatrix}$, where $a, c \in \mathbb{R}$. If $A^3 = A$ and the positive value of a belongs to the interval $(n-1, n]$, where $n \in \mathbb{N}$, then n is equal to ....

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30. In an examination, 5 students have been allotted their seats as per their roll numbers. The number of ways, in which none of the students sits as per the allotted seat, is ....
