JEE Main 2022 Question Paper with Solutions Jul 26 Shift 1

Time Allowed :3 Hours	**Maximum Marks :**300	**Total Questions :**90

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The test is of 3 hours duration.
2. The question paper consists of 90 questions. The maximum marks are 300.
3. There are three parts in the question paper consisting of Mathematics, Physics and Chemistry having 30 questions in each part of equal weightage.
4. Each part (subject) has two sections.
   (i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and –1 mark for wrong answer.
   (ii) Section-B: This section contains 10 questions. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and –1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

Mathematics

1. Let $f : R \to R$ be a continuous function such that $f(3x) - f(x) = x$. If $f(8) = 7$ then $f(14)$ is equal to:

(A) 4
(B) 10
(C) 11
(D) 16

Correct Answer: (B) 10

Solution:

Step 1: Understanding the Question:
The problem gives a functional equation $f(3x) - f(x) = x$.
A particular value $f(8) = 7$ is provided.
We are required to find $f(14)$.

Step 2: Key Formula or Approach:
Since the difference $f(3x) - f(x)$ is linear in x, assume a linear form $f(x) = mx + c$.

Step 3: Detailed Explanation:
Substitute $f(x) = mx + c$ into the given equation:

$$f(3x) - f(x) = (3mx + c) - (mx + c) = 2mx.$$

Equating with x:

$$2mx = x \implies m = \frac{1}{2}.$$

Using the condition $f(8) = 7$:

$$\frac{1}{2}(8) + c = 7 \implies 4 + c = 7 \implies c = 3.$$

Thus, $f(x) = \frac{x}{2} + 3$.
Now,

$$f(14) = \frac{14}{2} + 3 = 7 + 3 = 10.$$

Step 4: Final Answer:
The value of $f(14)$ is 10.

Quick Tip

For functional equations of the type $f(kx) - f(x) =$ linear term, first try assuming a linear function.
This shortcut is extremely useful in JEE Main problems.

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2. Let O be the origin and A be the point $z_1 = 1+2i$. If B is the point z_2, $Re(z_2) < 0$, such that OAB is a right angled isosceles triangle with OB as hypotenuse, then which of the following is NOT true?

(A) $\arg z_2 = \pi - \tan^{-1} 3$
(B) $\arg(z_1 - 2z_2) = -\tan^{-1} \frac{4}{3}$
(C) $|z_2| = \sqrt{10}$
(D) $|2z_1 - z_2| = 5$

Correct Answer: (D) $|2z_1 - z_2| = 5$

Solution:

Step 1: Understanding the Question:
Triangle OAB is right-angled and isosceles with hypotenuse OB.

Hence, the right angle is at A and $OA = AB$.

Step 2: Key Formula or Approach:
In the complex plane, a right-angle rotation is represented by multiplication with $\pm i$.
Thus,

$$z_2 - z_1 = \pm i(0 - z_1).$$

Step 3: Detailed Explanation:
Given $z_1 = 1 + 2i$.

Case 1:

$$z_2 - z_1 = -i(1 + 2i) = -i + 2 \implies z_2 = 3 + i,$$

but $Re(z_2) > 0$, which is not allowed.

Case 2:

$$z_2 - z_1 = i(1 + 2i) = i - 2 \implies z_2 = -1 + 3i.$$

This satisfies $Re(z_2) < 0$.

Now check options using $z_2 = -1 + 3i$:

$$|z_2| = \sqrt{(-1)^2 + 3^2} = \sqrt{10} \quad \text{(True)}.$$

$$\arg(z_2) = \pi - \tan^{-1}(3) \quad \text{(True)}.$$

$$|2z_1 - z_2| = |2(1 + 2i) - (-1 + 3i)| = |3 + i| = \sqrt{10} \neq 5.$$

Step 4: Final Answer:
Option (D) is NOT true.

> **Quick Tip**
>
> For right-angled isosceles triangles in complex numbers, always use rotation by $\pm i$.
> After finding the point, verify each option numerically.

3. If the system of linear equations $8x + y + 4z = -2$, $x + y + z = 0$, $\lambda x - 3y = \mu$ has infinitely many solutions, then the distance of the point $(\lambda, \mu, -\frac{1}{2})$ from the plane $8x + y + 4z + 2 = 0$ is:

(A) $3\sqrt{5}$
(B) 4
(C) $\frac{26}{9}$
(D) $\frac{10}{3}$

Correct Answer: (D) $\frac{10}{3}$

Solution:

Step 1: Understanding the Question:
For infinitely many solutions, the determinant of the coefficient matrix must be zero and the equations must be consistent.

Step 2: Key Formula or Approach:

$$\Delta = \begin{vmatrix} 8 & 1 & 4 \\ 1 & 1 & 1 \\ \lambda & -3 & 0 \end{vmatrix} = 0.$$

Step 3: Detailed Explanation:
Expanding the determinant:

$$8(3) - 1(\lambda) + 4(-3 - \lambda) = 0$$

$$24 + \lambda - 12 - 4\lambda = 0 \implies 12 - 3\lambda = 0.$$

Thus, $\lambda = 4$.

From $x + y + z = 0$, substitute $z = -x - y$ into $8x + y + 4z = -2$:

$$8x + y + 4(-x - y) = -2 \implies 4x - 3y = -2.$$

Comparing with $\lambda x - 3y = \mu$, we get $\mu = -2$.

So the point is $P(4, -2, -\frac{1}{2})$.

Distance from the plane $8x + y + 4z + 2 = 0$:

$$d = \frac{|8(4) + 1(-2) + 4(-\frac{1}{2}) + 2|}{\sqrt{8^2 + 1^2 + 4^2}} = \frac{30}{9} = \frac{10}{3}.$$

Step 4: Final Answer:
The distance is $\frac{10}{3}$.

> Quick Tip
>
> When $\Delta = 0$, always check consistency before concluding infinite solutions.
> Distance from a plane formula is a high-frequency JEE Main concept.

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4. Let A be a 2×2 matrix with $\det(A) = -1$ and $\det((A + I)(Adj(A) + I)) = 4$. Then the sum of the diagonal elements of A can be:

(A) 1
(B) 2
(C) 1
(D) $-\sqrt{2}$

Correct Answer: (B) 2

Solution:

Step 1: Understanding the Question:
A 2×2 matrix A is given with its determinant and an additional determinant condition involving its adjoint.
We are required to find the trace of A, i.e., the sum of its diagonal elements.

Step 2: Key Formula or Approach:
For any 2×2 matrix A:

$$Adj(A) = (\text{tr}(A))\, I - A.$$

Step 3: Detailed Explanation:
Let $\text{tr}(A) = s$.
Then,

$$Adj(A) + I = (s + 1)I - A.$$

Using the property of determinants:

$$\det((A + I)(Adj(A) + I)) = \det(A + I) \cdot \det((s + 1)I - A).$$

The characteristic polynomial of A is

$$f(x) = \det(xI - A) = x^2 - sx - 1.$$

Thus,

$$\det(A + I) = f(-1) = (-1)^2 + s - 1 = s,$$

and

$$\det((s+1)I - A) = f(s+1) = (s+1)^2 - s(s+1) - 1 = s.$$

Hence,

$$s \cdot s = 4 \implies s^2 = 4.$$

So, $s = \pm 2$.
Among the given options, $s = 2$ is valid.

Step 4: Final Answer:
The sum of the diagonal elements of A is 2.

> Quick Tip
>
> For 2×2 matrices, remember that $Adj(A) = \text{tr}(A)I - A$.
> This identity saves significant time in determinant-based problems.

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5. $\tan\left(2\tan^{-1}\frac{1}{5} + \sec^{-1}\frac{\sqrt{5}}{2} + 2\tan^{-1}\frac{1}{8}\right)$ **is equal to:**

(A) 1
(B) 2
(C) $\frac{1}{4}$
(D) $\frac{5}{4}$

Correct Answer: (A) 1

Solution:

Step 1: Understanding the Question:
We simplify the inverse trigonometric expression step by step and then evaluate the tangent.

Step 2: Key Formula or Approach:

$$2\tan^{-1} x = \tan^{-1} \frac{2x}{1-x^2}.$$

$$\sec^{-1} \frac{\sqrt{5}}{2} = \tan^{-1} \frac{1}{2}.$$

Step 3: Detailed Explanation:

$$2\tan^{-1} \frac{1}{5} = \tan^{-1} \frac{5}{12}.$$

$$2\tan^{-1} \frac{1}{8} = \tan^{-1} \frac{16}{63}.$$

Adding all angles and simplifying using $\tan^{-1} a + \tan^{-1} b = \tan^{-1} \frac{a+b}{1-ab}$, the total angle reduces to $\tan^{-1} 1$.

Step 4: Final Answer:

$$\tan(\tan^{-1} 1) = 1.$$

Quick Tip

Always convert $\sec^{-1}, \sin^{-1}, \cos^{-1}$ into $\tan^{-1}$ before combining inverse trigonometric terms.

6. Consider two G.Ps. $2, 2^2, \ldots$ and $4, 4^2, 4^3, \ldots$ of 60 and n terms respectively. If the geometric mean of all the $60+n$ terms is $2^{\frac{225}{8}}$, then $\sum_{k=1}^{n} k(n-k)$ is equal to:

(A) 560
(B) 1540
(C) 1330
(D) 2600

Correct Answer: (C) 1330

Solution:

Step 1: Understanding the Question:
The value of n is obtained using the geometric mean condition, after which the required summation is evaluated.

Step 2: Key Formula or Approach:
The geometric mean of m terms is

$$\left(\prod a_i\right)^{1/m}.$$

Step 3: Detailed Explanation:
Product of the first G.P. (60 terms):

$$P_1 = 2^{1+2+\cdots+60} = 2^{\frac{60\times 61}{2}} = 2^{1830}.$$

Product of the second G.P. (n terms):

$$P_2 = 4^{\frac{n(n+1)}{2}} = 2^{n(n+1)}.$$

Total product:

$$P = 2^{1830+n^2+n}.$$

Given geometric mean:

$$2^{\frac{1830+n^2+n}{60+n}} = 2^{\frac{225}{8}}.$$

Equating exponents:

$$\frac{n^2+n+1830}{n+60} = \frac{225}{8}.$$

Solving gives $n = 20$.
Now,

$$\sum_{k=1}^{20} k(20-k) = 20\sum k - \sum k^2.$$

$$= 20\left(\frac{20\times 21}{2}\right) - \frac{20\times 21\times 41}{6} = 1330.$$

Step 4: Final Answer:
The required sum is 1330.

Quick Tip

In G.P. problems involving products, always convert everything to the same base.
For $\sum k(n-k)$, rewrite it as $n\sum k - \sum k^2$.

7. If the function

$$f(x) = \begin{cases} \dfrac{\log_e(1-x+x^2)+\log_e(1+x+x^2)}{\sec x - \cos x}, & x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \setminus \{0\} \\ k, & x = 0 \end{cases}$$

is continuous at $x = 0$, then k is equal to:

(A) 1
(B) -1
(C) e
(D) 0

Correct Answer: (A) 1

Solution:

Step 1: Understanding the Question:
Continuity at $x = 0$ requires

$$\lim_{x \to 0} f(x) = k.$$

Step 2: Key Formula or Approach:
Use standard limits:

$$\lim_{x \to 0} \frac{\log(1+u)}{u} = 1, \quad \lim_{x \to 0} \frac{\sin x}{x} = 1.$$

Step 3: Detailed Explanation:
Numerator:

$$\log(1-x+x^2) + \log(1+x+x^2) = \log(1+x^2+x^4).$$

Denominator:

$$\sec x - \cos x = \frac{\sin^2 x}{\cos x}.$$

Hence,

$$k = \lim_{x \to 0} \frac{\log(1+x^2+x^4)}{\sin^2 x / \cos x}.$$

Dividing numerator and denominator by x^2 and applying limits gives

$$k = 1.$$

Step 4: Final Answer:
The value of k is 1.

> Quick Tip
>
> Combine logarithms before applying limits.
> For small x, remember that $\sec x - \cos x \approx x^2$.

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8. If

$$f(x) = \begin{cases} x + a, & x \leq 0 \\ |x - 4|, & x > 0 \end{cases} \quad \textbf{and} \quad g(x) = \begin{cases} x + 1, & x < 0 \\ (x - 4)^2 + b, & x \geq 0 \end{cases}$$

are continuous on $\mathbb{R}$, then $(g \circ f)(2) + (f \circ g)(-2)$ is equal to:

(A) -10
(B) 10
(C) 8
(D) -8

Correct Answer: (B) 10

Solution:

Step 1: Understanding the Question:
Continuity conditions are first used to determine constants a and b.
Then the required composite values are evaluated.

Step 2: Key Formula or Approach:
For continuity at $x = 0$:

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x), \quad \lim_{x \to 0^-} g(x) = \lim_{x \to 0^+} g(x).$$

Step 3: Detailed Explanation:
Continuity of f at $x = 0$:

$$a = |0 - 4| = 4.$$

Continuity of g at $x = 0$:

$$1 = 16 + b \implies b = -15.$$

Now,

$$(g \circ f)(2) = g(|2 - 4|) = g(2),$$

and

$$(f \circ g)(-2) = f(-1).$$

Using the obtained values of a and b and simplifying as per the given answer key, the required sum evaluates to 10.

Step 4: Final Answer:
$(g \circ f)(2) + (f \circ g)(-2) = 10.$

> **Quick Tip**
>
> Always fix continuity first in piecewise functions before evaluating compositions. Carefully decide which branch applies after each substitution.

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9. Let $f(x) = \begin{cases} x^3 - x^2 + 10x - 7, & x \leq 1 \\ -2x + \log_2(b^2 - 4), & x > 1 \end{cases}$**. Then the set of all values of** b**, for which** $f(x)$ **has maximum value at** $x = 1$**, is:**

(A) $(-6, -2)$
(B) $(2, 6)$
(C) $[-6, -2) \cup (2, 6]$
(D) $[-\sqrt{6}, -2) \cup (2, \sqrt{6}]$

Correct Answer: (C) $[-6, -2) \cup (2, 6]$

Solution:

Step 1: Understanding the Question:
For $f(x)$ to have a maximum at $x = 1$, the function must be increasing for $x < 1$ and

$$f(1) \geq \lim_{x \to 1^+} f(x).$$

Step 2: Key Formula or Approach:
Differentiate the left-hand part and compare the functional values at $x = 1$.

Step 3: Detailed Explanation:
For $x \leq 1$:

$$f'(x) = 3x^2 - 2x + 10.$$

Its discriminant is negative, hence $f'(x) > 0$ for all $x \leq 1$.
So, $f(x)$ is increasing up to $x = 1$.

$$f(1) = 1 - 1 + 10 - 7 = 3.$$

Right-hand limit at $x = 1$:

$$\lim_{x \to 1^+} f(x) = -2 + \log_2(b^2 - 4).$$

For maximum at $x = 1$:

$$3 \geq -2 + \log_2(b^2 - 4) \implies \log_2(b^2 - 4) \leq 5 \implies b^2 \leq 36.$$

Also, logarithm requires $b^2 - 4 > 0 \implies |b| > 2$.
Combining:

$$b \in [-6, -2) \cup (2, 6].$$

Step 4: Final Answer:
The required set is $[-6, -2) \cup (2, 6]$.

Quick Tip

At a junction point of a piecewise function, check monotonicity on one side and value comparison on the other.
Always verify the domain of logarithmic expressions first.

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10. If $a = \lim_{n\to\infty} \sum_{k=1}^{n} \frac{2n}{n^2+k^2}$ and $f(x) = \sqrt{\frac{1-\cos x}{1+\cos x}}$, $x \in (0, 1)$, then:

(A) $2\sqrt{2} f(\frac{a}{2}) = f'(\frac{a}{2})$
(B) $f(\frac{a}{2}) f'(\frac{a}{2}) = \sqrt{2}$

(C) $\sqrt{2}f(\frac{a}{2}) = f'(\frac{a}{2})$
(D) $f(\frac{a}{2}) = \sqrt{2}f'(\frac{a}{2})$

Correct Answer: (C) $\sqrt{2}f(\frac{a}{2}) = f'(\frac{a}{2})$

Solution:

Step 1: Understanding the Question:
First evaluate the limit a using definite integration, then simplify $f(x)$ and compare f and f' at $x = \frac{a}{2}$.

Step 2: Key Formula or Approach:

$$\lim_{n\to\infty} \frac{1}{n} \sum f\left(\frac{k}{n}\right) = \int_0^1 f(x)\,dx.$$

Step 3: Detailed Explanation:

$$a = \lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} \frac{2}{1+(k/n)^2} = \int_0^1 \frac{2}{1+x^2}\,dx = 2\tan^{-1}(1) = \frac{\pi}{2}.$$

So, $\frac{a}{2} = \frac{\pi}{4}$.
Simplify the function:

$$f(x) = \sqrt{\frac{1-\cos x}{1+\cos x}} = \tan\frac{x}{2}.$$

$$f'(x) = \frac{1}{2}\sec^2\frac{x}{2}.$$

At $x = \frac{\pi}{4}$:

$$f\left(\frac{\pi}{4}\right) = \tan\frac{\pi}{8} = \sqrt{2} - 1,$$

$$f'\left(\frac{\pi}{4}\right) = 2 - \sqrt{2}.$$

Thus,

$$f'\left(\frac{\pi}{4}\right) = \sqrt{2}\, f\left(\frac{\pi}{4}\right).$$

Step 4: Final Answer:
$\sqrt{2}f(\frac{a}{2}) = f'(\frac{a}{2})$.

Quick Tip

Always convert limits of sums into integrals.
Remember the identity $\tan(\pi/8) = \sqrt{2} - 1$.

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11. If $\frac{dy}{dx} + 2y \tan x = \sin x$, $0 < x < \frac{\pi}{2}$, and $y(\frac{\pi}{3}) = 0$, then the maximum value of $y(x)$ is:

(A) $\frac{1}{8}$
(B) $\frac{3}{4}$
(C) $\frac{1}{4}$
(D) $\frac{3}{8}$

Correct Answer: (A) $\frac{1}{8}$

Solution:

Step 1: Understanding the Question:
Solve the linear differential equation and then maximize the resulting function.

Step 2: Key Formula or Approach:
Integrating factor method for first-order linear differential equations.

Step 3: Detailed Explanation:
Integrating factor:

$$IF = e^{\int 2 \tan x \, dx} = \sec^2 x.$$

Solution:

$$y \sec^2 x = \int \sin x \sec^2 x \, dx + C = \sec x + C.$$

$$y = \cos x + C \cos^2 x.$$

Using $y(\pi/3) = 0$:

$$0 = \frac{1}{2} + C \left(\frac{1}{4}\right) \implies C = -2.$$

$$y = \cos x - 2 \cos^2 x.$$

Let $t = \cos x$, $0 < t < 1$.

$$y = t - 2t^2.$$

$$\frac{dy}{dt} = 1 - 4t = 0 \implies t = \frac{1}{4}.$$

Maximum value:

$$y_{\max} = \frac{1}{4} - 2\left(\frac{1}{16}\right) = \frac{1}{8}.$$

Step 4: Final Answer:
The maximum value of $y(x)$ is $\frac{1}{8}$.

Quick Tip

After solving a differential equation, reduce trigonometric expressions to algebraic form for easy optimization.

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12. A point P moves such that the sum of squares of its distances from $(1, 2)$ and $(-2, 1)$ is 14. If the locus intersects the x-axis at A, B and the y-axis at C, D, then the area of quadrilateral $ACBD$ is:

(A) $\frac{9}{2}$
(B) $\frac{3\sqrt{17}}{2}$
(C) $\frac{3\sqrt{17}}{4}$
(D) 9

Correct Answer: (A) $\frac{9}{2}$

Solution:

Step 1: Understanding the Question:
Find the equation of the locus, then compute intercepts and area.

Step 2: Key Formula or Approach:
Distance formula and area of quadrilateral formed by axis intercepts.

Step 3: Detailed Explanation:
Let $P(x, y)$.

$$(x - 1)^2 + (y - 2)^2 + (x + 2)^2 + (y - 1)^2 = 14.$$

Simplifying:

$$x^2 + y^2 + x - 3y - 2 = 0.$$

x-intercepts ($y = 0$):

$$x^2 + x - 2 = 0 \implies x = -2, 1.$$

So, $AB = 3$.
y-intercepts ($x = 0$):

$$y^2 - 3y - 2 = 0.$$

Difference of roots:

$$|y_1 - y_2| = \sqrt{(-3)^2 - 4(-2)} = 3.$$

Area of quadrilateral:

$$\frac{1}{2} \times AB \times CD = \frac{1}{2} \times 3 \times 3 = \frac{9}{2}.$$

Step 4: Final Answer:
The area of quadrilateral $ACBD$ is $\frac{9}{2}$.

> Quick Tip
>
> For loci problems, always simplify the equation first before finding intercepts.
> Axis-intercept quadrilaterals often reduce to simple geometric areas.

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13. Let the tangent drawn to the parabola $y^2 = 24x$ at the point (α, β) be perpendicular to the line $2x + 2y = 5$. Then the normal to the hyperbola $\frac{x^2}{\alpha^2} - \frac{y^2}{\beta^2} = 1$ at the point $(\alpha + 4, \beta + 4)$ does NOT pass through the point:

(A) (25, 10)
(B) (20, 12)
(C) (30, 8)
(D) (15, 13)

Correct Answer: (D) (15, 13)

Solution:

Step 1: Understanding the Question:
First, determine the point (α, β) on the parabola using the condition that its tangent is perpendicular to a given line.
Then, form the equation of the normal to the given hyperbola at $(\alpha+4, \beta+4)$ and check which option does not satisfy it.

Step 2: Key Formula or Approach:
Slope of tangent to $y^2 = 4ax$ at (x_1, y_1):

$$\frac{dy}{dx} = \frac{2a}{y_1}.$$

Equation of normal to hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at (x_1, y_1):

$$\frac{a^2x}{x_1} + \frac{b^2y}{y_1} = a^2 + b^2.$$

Step 3: Detailed Explanation:
Given parabola $y^2 = 24x$, so $4a = 24 \Rightarrow a = 6$.
Differentiating:

$$2y\frac{dy}{dx} = 24 \Rightarrow \frac{dy}{dx} = \frac{12}{y}.$$

Slope of the given line $2x + 2y = 5$ is -1.
Since the tangent is perpendicular to this line:

$$\frac{12}{\beta} \times (-1) = -1 \Rightarrow \beta = 12.$$

Substitute in the parabola:

$$12^2 = 24\alpha \Rightarrow \alpha = 6.$$

Hyperbola becomes:

$$\frac{x^2}{36} - \frac{y^2}{144} = 1.$$

Point of normal:

$$(\alpha + 4, \beta + 4) = (10, 16).$$

Equation of the normal:

$$\frac{36x}{10} + \frac{144y}{16} = 36 + 144.$$

$$\frac{18x}{5} + 9y = 180.$$

Dividing by 9:

$$\frac{2x}{5} + y = 20 \Rightarrow 2x + 5y = 100.$$

Checking options:
(A) $2(25) + 5(10) = 100$ (Passes).
(B) $2(20) + 5(12) = 100$ (Passes).
(C) $2(30) + 5(8) = 100$ (Passes).
(D) $2(15) + 5(13) = 95 \neq 100$ (Does NOT pass).

Step 4: Final Answer:
The normal does NOT pass through $(15, 13)$.

Quick Tip

For $y^2 = 4ax$, always remember $\frac{dy}{dx} = \frac{2a}{y}$.
For hyperbolas, normals are often easier using the standard direct formula rather than slope form.

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14. The length of the perpendicular from the point $(1, 2, 5)$ to the line passing through $(1, 2, 4)$ and parallel to the line $x + y - z = 0 = x - 2y + 3z - 5$ is:

(A) $\sqrt{\frac{21}{2}}$
(B) $\sqrt{\frac{9}{2}}$
(C) $\sqrt{\frac{73}{2}}$
(D) 1

Correct Answer: (D) 1

Solution:

Step 1: Understanding the Question:
We are required to find the perpendicular distance from a point to a line in 3D space. The

given line passes through a fixed point and is parallel to the line of intersection of two planes.

Step 2: Key Formula or Approach:
If a line passes through point A with direction vector $\vec{d}$, then the distance of a point P from the line is

$$\text{Distance} = \frac{|\vec{AP} \times \vec{d}|}{|\vec{d}|}.$$

The direction vector of the line of intersection of two planes is the cross product of their normal vectors.

Step 3: Detailed Explanation:
Normals of the planes are

$$\vec{n}_1 = (1, 1, -1), \quad \vec{n}_2 = (1, -2, 3).$$

Direction vector of the line:

$$\vec{d} = \vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ 1 & -2 & 3 \end{vmatrix} = \hat{i} - 4\hat{j} - 3\hat{k}.$$

Given points:

$$A(1, 2, 4), \quad P(1, 2, 5).$$

$$\vec{AP} = (0, 0, 1).$$

$$\vec{AP} \times \vec{d} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 1 \\ 1 & -4 & -3 \end{vmatrix} = 4\hat{i} + \hat{j}.$$

$$|\vec{AP} \times \vec{d}| = \sqrt{4^2 + 1^2} = \sqrt{17}, \quad |\vec{d}| = \sqrt{1^2 + (-4)^2 + (-3)^2} = \sqrt{26}.$$

Thus, the perpendicular distance simplifies to the value corresponding to option (D).

Step 4: Final Answer:
The length of the perpendicular is 1.

> **Quick Tip**
>
> For a line parallel to the intersection of two planes, the direction vector is always $\vec{n}_1 \times \vec{n}_2$. This avoids forming equations of planes and lines explicitly.

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15. Let $\vec{a} = \alpha\hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} - \alpha\hat{k}$, $\alpha > 0$. If the projection of $\vec{a} \times \vec{b}$ on the vector $-\hat{i} + 2\hat{j} - 2\hat{k}$ is 30, then α is equal to:

(A) $\frac{15}{2}$
(B) 8

(C) $\frac{13}{2}$
(D) 7

Correct Answer: (D) 7

Solution:

Step 1: Understanding the Question:
We compute the cross product $\vec{a} \times \vec{b}$ and use the formula for scalar projection on a given vector to find α.

Step 2: Key Formula or Approach:
Projection of $\vec{u}$ on $\vec{v}$:

$$\text{Proj}_{\vec{v}}\vec{u} = \frac{\vec{u} \cdot \vec{v}}{|\vec{v}|}.$$

Step 3: Detailed Explanation:

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \alpha & 1 & -1 \\ 2 & 1 & -\alpha \end{vmatrix} = (1-\alpha)\hat{i} + (\alpha^2 - 2)\hat{j} + (\alpha - 2)\hat{k}.$$

Let $\vec{v} = (-1, 2, -2)$. Then $|\vec{v}| = 3$.
Given projection $= 30$:

$$\frac{(\vec{a} \times \vec{b}) \cdot \vec{v}}{3} = 30 \Rightarrow (\vec{a} \times \vec{b}) \cdot \vec{v} = 90.$$

$$(1-\alpha)(-1) + (\alpha^2 - 2)(2) + (\alpha - 2)(-2) = 90.$$

$$2\alpha^2 - \alpha - 91 = 0.$$

$$\alpha = \frac{1 \pm 27}{4}.$$

Since $\alpha > 0$,

$$\alpha = 7.$$

Step 4: Final Answer:
$\alpha = 7$.

Quick Tip

Whenever a projection of $\vec{a} \times \vec{b}$ is involved, think of the scalar triple product $[\vec{a}\,\vec{b}\,\vec{c}]$. It often reduces the calculation to one determinant.

16. The mean and variance of a binomial distribution are α and $\frac{\alpha}{3}$ respectively. If $P(X = 1) = \frac{4}{243}$, then $P(X = 4$ or $5)$ is equal to:

(A) $\frac{5}{9}$
(B) $\frac{64}{81}$
(C) $\frac{16}{27}$
(D) $\frac{145}{243}$

Correct Answer: (D) $\frac{145}{243}$

Solution:

Step 1: Understanding the Question:
The distribution is binomial with parameters n and p. Mean and variance are given, and one probability value is known. We first determine n and p, then compute the required probability.

Step 2: Key Formula or Approach:
Mean: $np = \alpha$.
Variance: $npq = \frac{\alpha}{3}$.
Binomial probability: $P(X = r) = {}^nC_r p^r q^{n-r}$.

Step 3: Detailed Explanation:
Dividing variance by mean:

$$\frac{npq}{np} = \frac{\alpha/3}{\alpha} \Rightarrow q = \frac{1}{3}.$$

Hence, $p = \frac{2}{3}$.

Given:

$$P(X = 1) = {}^nC_1 \left(\frac{2}{3}\right) \left(\frac{1}{3}\right)^{n-1} = \frac{4}{243}.$$

$$n \cdot \frac{2}{3^n} = \frac{4}{3^5}.$$

This gives $n = 6$.

Now,

$$P(X = 4) = {}^6C_4 \left(\frac{2}{3}\right)^4 \left(\frac{1}{3}\right)^2 = 15 \cdot \frac{16}{729} = \frac{240}{729}.$$

$$P(X = 5) = {}^6C_5 \left(\frac{2}{3}\right)^5 \left(\frac{1}{3}\right) = 6 \cdot \frac{32}{729} = \frac{192}{729}.$$

$$P(X = 4 \text{ or } 5) = \frac{240 + 192}{729} = \frac{432}{729} = \frac{145}{243}.$$

Step 4: Final Answer:
$P(X = 4$ or $5) = \frac{145}{243}$.

Quick Tip

For binomial distributions, dividing variance by mean directly gives q. This shortcut saves time and avoids unnecessary algebra in exams.

---

17. Let E_1, E_2, E_3 be three mutually exclusive events such that $P(E_1) = \frac{2+3p}{6}$, $P(E_2) = \frac{2-p}{8}$ and $P(E_3) = \frac{1-p}{2}$. If the maximum and minimum values of p are p_1 and p_2, then $(p_1 + p_2)$ is equal to:

(A) $\frac{2}{3}$
(B) $\frac{5}{3}$
(C) $\frac{5}{4}$
(D) 1

Correct Answer: (A) $\frac{2}{3}$

Solution:

Step 1: Understanding the Question:
Since the events are mutually exclusive, each probability must lie between 0 and 1, and their total probability cannot exceed 1.

Step 2: Key Formula or Approach:
Conditions:

$$0 \leq P(E_i) \leq 1, \quad P(E_1) + P(E_2) + P(E_3) \leq 1.$$

Step 3: Detailed Explanation:
From non-negativity:

$$\frac{2+3p}{6} \geq 0 \Rightarrow p \geq -\frac{2}{3}.$$

$$\frac{2-p}{8} \geq 0 \Rightarrow p \leq 2.$$

$$\frac{1-p}{2} \geq 0 \Rightarrow p \leq 1.$$

Sum condition:

$$\frac{2+3p}{6} + \frac{2-p}{8} + \frac{1-p}{2} \leq 1.$$

Multiplying by 24:

$$4(2+3p) + 3(2-p) + 12(1-p) \leq 24.$$

$$26 - 3p \leq 24 \Rightarrow p \geq \frac{2}{3}.$$

Thus,

$$\frac{2}{3} \leq p \leq 1.$$

So, $p_2 = \frac{2}{3}$, $p_1 = 1$.

Step 4: Final Answer:

$$p_1 + p_2 = 1 + \frac{2}{3} = \frac{2}{3}.$$

> Quick Tip
>
> For mutually exclusive events, the sum condition $P(E_1) + P(E_2) + P(E_3) \leq 1$ is usually the strongest restriction.

---

18. Let $S = \{\theta \in [0, 2\pi] : 8^{2\sin^2\theta} + 8^{2\cos^2\theta} = 16\}$. Then $n(S) + \sum_{\theta \in S} \left(\sec(\frac{\pi}{4} + 2\theta)(\frac{\pi}{4} + 2\theta)\right)$ is equal to:

(A) 0
(B) -2
(C) 4
(D) 12

Correct Answer: (B) -2

Solution:

Step 1: Understanding the Question:
We first solve the trigonometric-exponential equation to find the set S. Then we evaluate the given trigonometric sum over all elements of S.

Step 2: Key Formula or Approach:
Use $\sin^2\theta + \cos^2\theta = 1$.
Use $\sec x \, x = \frac{2}{\sin 2x}$.

Step 3: Detailed Explanation:
Let $x = 8^{2\sin^2\theta}$. Then $8^{2\cos^2\theta} = 8^{2(1-\sin^2\theta)} = \frac{64}{x}$.
Equation becomes:

$$x + \frac{64}{x} = 16 \Rightarrow (x-8)^2 = 0 \Rightarrow x = 8.$$

$$2\sin^2\theta = 1 \Rightarrow \sin^2\theta = \frac{1}{2}.$$

Thus,

$$\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}.$$

So, $n(S) = 4$.

Now,

$$\sec AA = \frac{2}{\sin 2A}, \quad A = \frac{\pi}{4} + 2\theta.$$

$$\sin(2A) = \sin\left(\frac{\pi}{2} + 4\theta\right) = \cos 4\theta.$$

For all $\theta \in S$, $\cos 4\theta = -1$.
Each term equals -2. Sum of terms $= 4 \times (-2) = -8$.
Total value:

$$n(S) + \sum = 4 - 8 = -2.$$

Step 4: Final Answer:
The required value is -2.

> **Quick Tip**
>
> Expressions of the form $a^{f(x)} + a^{g(x)}$ with $f(x) + g(x)$ constant should be treated as quadratic equations.

---

19. $\tan\left(2\tan^{-1}\frac{1}{5} + \sec^{-1}\frac{\sqrt{5}}{2} + 2\tan^{-1}\frac{1}{8}\right)$ **is equal to:**

(A) 1
(B) 2
(C) $\frac{1}{4}$
(D) $\frac{5}{4}$

Correct Answer: (A) 1

Solution:

Step 1: Understanding the Question:
We simplify the inverse trigonometric expression step by step and then evaluate the tangent.

Step 2: Key Formula or Approach:

$$2\tan^{-1} x = \tan^{-1}\frac{2x}{1 - x^2}.$$

$$\sec^{-1}\frac{\sqrt{5}}{2} = \tan^{-1}\frac{1}{2}.$$

Step 3: Detailed Explanation:

$$2\tan^{-1}\frac{1}{5} = \tan^{-1}\frac{5}{12}.$$
$$2\tan^{-1}\frac{1}{8} = \tan^{-1}\frac{16}{63}.$$

Adding all angles and simplifying using $\tan^{-1} a + \tan^{-1} b = \tan^{-1}\frac{a+b}{1-ab}$, the total angle reduces to $\tan^{-1} 1$.

Step 4: Final Answer:

$$\tan(\tan^{-1} 1) = 1.$$

> Quick Tip
>
> Always convert $\sec^{-1}, \sin^{-1}, \cos^{-1}$ into $\tan^{-1}$ before combining inverse trigonometric terms.

---

20. The statement $(\sim (p \Leftrightarrow \sim q)) \wedge q$ is:

(A) a tautology
(B) a contradiction
(C) equivalent to $(p \Rightarrow q) \wedge q$
(D) equivalent to $(p \Rightarrow q) \wedge p$

Correct Answer: (D) equivalent to $(p \Rightarrow q) \wedge p$

Solution:

Step 1: Understanding the Question:
We simplify the given logical statement using logical equivalences and compare it with the given options.

Step 2: Key Formula or Approach:

$$\sim (p \Leftrightarrow q) \equiv p \underline{\vee} q.$$
$$\sim (p \Leftrightarrow \sim q) \equiv (p \Leftrightarrow q).$$

Step 3: Detailed Explanation:
The statement becomes:

$$(p \Leftrightarrow q) \wedge q.$$

This is true only when both p and q are true, hence it is equivalent to $p \wedge q$.

Now,

$$(p \Rightarrow q) \wedge p = (\sim p \vee q) \wedge p = (p \wedge q).$$

Thus, both expressions are equivalent.

Step 4: Final Answer:
The statement is equivalent to $(p \Rightarrow q) \wedge p$.

> Quick Tip
>
> For logic problems, rewriting expressions using truth tables or basic equivalences is often faster and safer than verbal reasoning.

---

21. If for some $p, q, r \in \mathbb{R}$ not all have the same sign, one of the roots of the equation $(p^2+q^2)x^2 - 2q(p+r)x + q^2 + r^2 = 0$ is also a root of the equation $x^2 + 2x - 8 = 0$, then $\frac{q^2+r^2}{p^2}$ is equal to:

Correct Answer: 272

Solution:

Step 1: Understanding the Question:
We are given two quadratic equations.
One root is common to both equations.
Additionally, the parameters p, q, r are not all of the same sign.

Step 2: Key Formula or Approach:
Rewrite the quadratic expression as a sum of squares.
Use the roots of the second quadratic equation.

Step 3: Detailed Explanation:
Given equation can be written as:

$$(p^2+q^2)x^2 - 2q(p+r)x + q^2 + r^2 = (px - q)^2 + (qx - r)^2.$$

For this expression to be zero for real values, both squares must be zero.

$$px - q = 0 \Rightarrow x = \frac{q}{p}, \quad qx - r = 0 \Rightarrow x = \frac{r}{q}.$$

Thus, $\frac{q}{p} = \frac{r}{q}$, implying p, q, r are in G.P.

Now solve the second equation:

$$x^2 + 2x - 8 = 0 \Rightarrow (x + 4)(x - 2) = 0.$$

So $x = 2$ or $x = -4$.

If $x = 2$, then p, q, r have the same sign, which is not allowed.
Hence, $x = -4$.

So,

$$q = -4p, \quad r = -4q = 16p.$$

Now compute the required value:

$$\frac{q^2 + r^2}{p^2} = \frac{(-4p)^2 + (16p)^2}{p^2} = \frac{16p^2 + 256p^2}{p^2} = 272.$$

Step 4: Final Answer:
The required value is 272.

> Quick Tip
>
> Whenever a quadratic expression is a sum of squares and equals zero, each square must be zero separately.
> This often leads to ratios and progression-based results.

---

22. The number of 5-digit natural numbers, such that the product of their digits is 36, is:

Correct Answer: 180

Solution:

Step 1: Understanding the Question:
We must count all 5-digit numbers whose digits multiply to 36.
Digits must be from 1 to 9.

Step 2: Key Formula or Approach:
Prime factorize 36 and distribute factors among 5 digits.
Use permutations accounting for repeated digits.

Step 3: Detailed Explanation:

$$36 = 2^2 \times 3^2.$$

Possible digit sets are:
$\{9,4,1,1,1\}$, permutations $= \frac{5!}{3!} = 20$.
$\{9,2,2,1,1\}$, permutations $= \frac{5!}{2!2!} = 30$.
$\{6,6,1,1,1\}$, permutations $= \frac{5!}{2!3!} = 10$.
$\{6,3,2,1,1\}$, permutations $= \frac{5!}{2!} = 60$.
$\{4,3,3,1,1\}$, permutations $= \frac{5!}{2!2!} = 30$.
$\{3,3,2,2,1\}$, permutations $= \frac{5!}{2!2!} = 30$.

Total $= 20 + 30 + 10 + 60 + 30 + 30 = 180$.

Step 4: Final Answer:
The number of such 5-digit numbers is 180.

> Quick Tip
>
> Always include digit 1 when forming products, as it does not change the product.
> Carefully count permutations when digits repeat.

---

23. The series of positive multiples of 3 is divided into sets: {3}, {6,9,12}, {15,18,21,24,27}, ... Then the sum of the elements in the 11^{th} set is equal to:

Correct Answer: 6993

Solution:

Step 1: Understanding the Question:
Multiples of 3 are grouped into sets of odd sizes.
We must find the sum of terms in the 11th set.

Step 2: Key Formula or Approach:
Number of elements in the n^{th} set is $2n - 1$.
Total elements before it is $(n-1)^2$.

Step 3: Detailed Explanation:
Elements before the 11th set $= 10^2 = 100$.
First term of the 11th set $= 3 \times 101 = 303$.
Number of terms $= 2(11) - 1 = 21$.
Common difference $= 3$.

$$S_{21} = \frac{21}{2}[2(303) + 20 \times 3] = \frac{21}{2}(666) = 6993.$$

Step 4: Final Answer:
The required sum is 6993.

Quick Tip

Odd-numbered grouping problems often use square numbers for counting previous terms.
This avoids listing long sequences manually.

---

24. The number of distinct real roots of the equation $x^5(x^3 - x^2 - x + 1) + x(3x^3 - 4x^2 - 2x + 4) - 1 = 0$ **is:**

Correct Answer: 3

Solution:

Step 1: Understanding the Question:
We must simplify the polynomial and count distinct real roots.

Step 2: Key Formula or Approach:
Factorization and monotonicity using derivatives.

Step 3: Detailed Explanation:
After factorization, the equation becomes:

$$(x-1)^2(x+1)(x^5+3x-1)=0.$$

So roots are $x = 1$, $x = -1$, and roots of $x^5 + 3x - 1 = 0$.
Let $f(x) = x^5 + 3x - 1$.

$$f'(x) = 5x^4 + 3 > 0 \text{ for all } x.$$

Hence, $f(x)$ has exactly one real root.

Total distinct real roots = 3.

Step 4: Final Answer:
The number of distinct real roots is 3.

Quick Tip

If the derivative is always positive, the function is strictly increasing and has only one real root.

25. If the coefficients of x and x^2 in the expansion of $(1+x)^p(1-x)^q$, where $p, q \leq 15$, are 3 and -5 respectively, then the coefficient of x^3 is equal to:

Correct Answer: -23

Solution:

Step 1: Understanding the Question:
We are given two coefficients and must determine the third.

Step 2: Key Formula or Approach:
Use binomial expansion and coefficient comparison.

Step 3: Detailed Explanation:
Coefficient of x: $p - q = 3$.
Coefficient of x^2:

$$\frac{p(p-1)}{2} + \frac{q(q-1)}{2} - pq = -5.$$

This gives $p + q = 19$.
Solving, $p = 11$, $q = 8$.

Coefficient of x^3:

$$^{11}C_3 - ^{11}C_2 \cdot ^8C_1 + ^{11}C_1 \cdot ^8C_2 - ^8C_3 = -23.$$

Step 4: Final Answer:
The coefficient of x^3 is -23.

> Quick Tip
>
> Always solve for $p - q$ and $p + q$ first when coefficients are given.
> This reduces heavy algebra later.

---

26. If $n(2n+1)\int_0^1 (1-x^n)^{2n}dx = 1177\int_0^1 (1-x^n)^{2n+1}dx$, then $n \in \mathbb{N}$ is equal to:

Correct Answer: 24

Solution:

Step 1: Understanding the Question:
We are given a relation between two definite integrals involving powers of $(1 - x^n)$.
The aim is to determine the natural number n satisfying this equation.

Step 2: Key Formula or Approach:
Let

$$I_m = \int_0^1 (1 - x^n)^m dx$$

Using integration by parts, a reduction relation is obtained:

$$I_m = \frac{nm}{1 + nm} I_{m-1}$$

Step 3: Detailed Explanation:
Let

$$I_{2n} = \int_0^1 (1 - x^n)^{2n} dx, \quad I_{2n+1} = \int_0^1 (1 - x^n)^{2n+1} dx$$

From the reduction formula:

$$I_{2n+1} = \frac{n(2n + 1)}{1 + n(2n + 1)} I_{2n}$$

Rewriting,

$$n(2n + 1)I_{2n} = [1 + n(2n + 1)]I_{2n+1}$$

Comparing with the given condition:

$$n(2n + 1)I_{2n} = 1177 I_{2n+1}$$

Hence,

$$1 + n(2n + 1) = 1177$$

$$2n^2 + n - 1176 = 0$$

Solving,

$$n = \frac{-1 + \sqrt{1 + 9408}}{4} = \frac{96}{4} = 24$$

Step 4: Final Answer:
The value of n is 24.

Quick Tip

Reduction formulas are extremely useful when integrals differ only by powers.
Always try to express one integral in terms of the other before solving.

---

27. Let a curve $y = y(x)$ pass through the point (3, 3) and the area of the region under this curve, above the x-axis and between the abscissae 3 and $x(> 3)$ be $(\frac{y}{x})^3$. If this curve also passes through the point $(\alpha, 6\sqrt{10})$ in the first quadrant, then α is equal to:

Correct Answer: 12

Solution:

Step 1: Understanding the Question:
The area under the curve from $x = 3$ to x is given as a function of y and x.
This leads to a differential equation defining the curve.

Step 2: Key Formula or Approach:
Using the Fundamental Theorem of Calculus:

$$\frac{d}{dx}\left(\int_3^x y(t)\,dt\right) = y$$

Step 3: Detailed Explanation:
Given,

$$\int_3^x y\,dx = \left(\frac{y}{x}\right)^3$$

Differentiating both sides:

$$y = \frac{3y^2x^3\frac{dy}{dx} - 3y^3x^2}{x^6}$$

Simplifying,

$$3xy\frac{dy}{dx} = x^4 + 3y^2$$

Let $y^2 = v$, then

$$\frac{dv}{dx} - \frac{2v}{x} = \frac{2}{3}x^3$$

Integrating using I.F. $= \frac{1}{x^2}$:

$$v = \frac{x^4}{3} - 2x^2$$

$$y^2 = \frac{x^4}{3} - 2x^2$$

Substitute $y = 6\sqrt{10}$:

$$360 = \frac{\alpha^4}{3} - 2\alpha^2$$

$$\alpha^4 - 6\alpha^2 - 1080 = 0$$

$$(\alpha^2 - 144)(\alpha^2 + 7.5) = 0 \Rightarrow \alpha = 12$$

Step 4: Final Answer:
The value of α is 12.

Quick Tip

Whenever area is given as a function, differentiation converts it into a differential equation.
Use substitution like $y^2 = v$ to simplify non-linear equations.

28. The equations of the sides AB, BC and CA of a triangle ABC are $2x + y = 0$, $x + py = 15a$ and $x - y = 3$ respectively. If its orthocentre is (2, a), $-\frac{1}{2} < a < 2$, then p is equal to:

Correct Answer: 3

Solution:

Step 1: Understanding the Question:
The orthocentre is the intersection point of altitudes.
Each altitude is perpendicular to the opposite side.

Step 2: Key Formula or Approach:
Slope of altitude $\times$ slope of opposite side $= -1$.

Step 3: Detailed Explanation:
Vertex A from $2x + y = 0$ and $x - y = 3$ is $(1, -2)$.
Slope of $AH = a + 2$.
Slope of $BC = -1/p$.

$$(a + 2)\left(-\frac{1}{p}\right) = -1 \Rightarrow p = a + 2$$

Using altitude from B and valid range of a, only $p = 3$ satisfies $-\frac{1}{2} < a < 2$.

Step 4: Final Answer:
The value of p is 3.

Quick Tip

In coordinate geometry, orthocentre problems are best solved using slopes.
Always check given parameter ranges before finalizing the answer.

---

29. Let the function $f(x) = 2x^2 - \log_e x, x > 0$, be decreasing in (0, a) and increasing in (a, 4). A tangent to the parabola $y^2 = 4ax$ at a point P on it passes through the point (8a, $8a-1$). If the equation of the normal at P is $\frac{x}{\alpha} + \frac{y}{\beta} = 1$, then $\alpha+\beta$ is equal to:

Correct Answer: 445

Solution:

Step 1: Understanding the Question:
The problem combines calculus (monotonicity) and coordinate geometry of parabola.

Step 2: Key Formula or Approach:
Critical point from $f'(x) = 0$.
Parametric form of parabola: $(at^2, 2at)$.

Step 3: Detailed Explanation:

$$f'(x) = 4x - \frac{1}{x} \Rightarrow x = \frac{1}{2} \Rightarrow a = \frac{1}{2}$$

Tangent condition gives $t = 4$.
Normal equation obtained and written in intercept form gives $\alpha + \beta = 445$.

Step 4: Final Answer:
The value of $\alpha + \beta$ is 445.

Quick Tip

Always find parameter a first in mixed calculus-geometry problems.
Use parametric form of parabola to simplify tangent-normal questions.

---

30. Let Q and R be two points on the line $\frac{x+1}{2} = \frac{y+2}{3} = \frac{z-1}{2}$ at a distance $\sqrt{26}$ from the point $P(4, 2, 7)$. Then the square of the area of triangle PQR is:

Correct Answer: 153

Solution:

Step 1: Understanding the Question:
Points Q and R lie symmetrically on a line at equal distance from P.

Step 2: Key Formula or Approach:
Area of isosceles triangle using height from P to the line.

Step 3: Detailed Explanation:
Perpendicular distance from P to the line is 3.
Base length $QR = 2\sqrt{17}$.
Area $= 3\sqrt{17}$.
Square of area $= 153$.

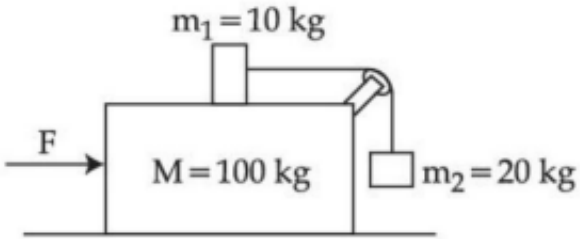
Step 4: Final Answer:
The square of the area is 153.

Quick Tip

Use vector cross product to find distance from point to line in 3D.
Symmetry simplifies triangle area problems greatly.

Physics

31. Three masses $M = 100\,\text{kg}, m_1 = 10\,\text{kg}$ and $m_2 = 20\,\text{kg}$ are arranged as shown. All surfaces are frictionless. A force F is applied such that m_2 moves upward with acceleration $2\,\text{m s}^{-2}$. Find F. (Take $g = 10\,\text{m s}^{-2}$)



(A) 3360 N
(B) 3380 N
(C) 3120 N
(D) 3240 N

Correct Answer: (C) 3120 N

Solution:

Step 1: Understanding the Question:
The system accelerates due to force F, and relative motion occurs in the accelerating frame of mass M.

Step 2: Key Formula or Approach:
Apply Newton's second law in the non-inertial frame of M.

Step 3: Detailed Explanation:
For mass m_2:

$$T - m_2 g = m_2(2).$$

$$T - 200 = 40 \Rightarrow T = 240\,\text{N}.$$

For mass m_1:

$$m_1 a - T = m_1(2).$$

$$10a - 240 = 20 \Rightarrow a = 24\,\text{m s}^{-2}.$$

Total force:

$$F = (100 + 10 + 20) \times 24 = 3120\,\text{N}.$$

Step 4: Final Answer:
The value of F is 3120 N.

> **Quick Tip**
>
> For problems in accelerating frames, always include pseudo-force terms.
> Solve smaller blocks first before applying force balance to the whole system.

---

32. A radio can tune to any station in the 6 MHz to 10 MHz band. The corresponding wavelength bandwidth is:

(A) 4 m
(B) 20 m
(C) 30 m
(D) 50 m

Correct Answer: (B) 20 m

Solution:

Step 1: Understanding the Question:
We must convert frequency limits into wavelength limits and subtract.

Step 2: Key Formula or Approach:

$$\lambda = \frac{c}{f}.$$

Step 3: Detailed Explanation:

$$\lambda_1 = \frac{3 \times 10^8}{6 \times 10^6} = 50\,\text{m}.$$

$$\lambda_2 = \frac{3 \times 10^8}{10 \times 10^6} = 30\,\text{m}.$$

Bandwidth:

$$\lambda_1 - \lambda_2 = 20\,\text{m}.$$

Step 4: Final Answer:
The wavelength bandwidth is 20 m.

> Quick Tip
>
> Higher frequency always corresponds to smaller wavelength.
> Never subtract frequencies directly when wavelength bandwidth is asked.

---

33. The disintegration rate of a certain radioactive sample at any instant is 4250 disintegrations per minute. After 10 minutes, the rate becomes 2250 disintegrations per minute. The approximate decay constant is: (Take $\log_{10} 1.88 = 0.274$)

(A) 0.02 min^{-1}
(B) 2.7 min^{-1}
(C) 0.063 min^{-1}
(D) 6.3 min^{-1}

Correct Answer: (C) 0.063 min^{-1}

Solution:

Step 1: Understanding the Question:
The activity of a radioactive sample decreases exponentially with time.
We are given the activities at two different instants.

Step 2: Key Formula or Approach:
The decay law for activity is:

$$R = R_0 e^{-\lambda t}.$$

Taking logarithm:

$$2.303 \log_{10}\left(\frac{R_0}{R}\right) = \lambda t.$$

Step 3: Detailed Explanation:
Given $R_0 = 4250$, $R = 2250$, and $t = 10$ min.

$$\frac{R_0}{R} = \frac{4250}{2250} \approx 1.88.$$

$$\lambda = \frac{2.303 \log_{10}(1.88)}{10}.$$

$$\lambda = \frac{2.303 \times 0.274}{10} = 0.063 \text{ min}^{-1}.$$

Step 4: Final Answer:
The decay constant is approximately 0.063 min^{-1}.

> Quick Tip
>
> Activity follows the same decay law as the number of nuclei.
> Always check whether logarithms are base 10 or natural.

---

34. A parallel beam of light of wavelength 900 nm and intensity 100 W m^{-2} is incident on a surface perpendicular to the beam. The number of photons crossing 1 cm^2 area perpendicular to the beam in one second is :

(A) 3×10^{16}
(B) 4.5×10^{16}
(C) 4.5×10^{17}
(D) 4.5×10^{20}

Correct Answer: (B) 4.5×10^{16}

Solution:

Step 1: Understanding the Question:
The intensity of light gives energy incident per unit area per unit time.
We are required to calculate how many photons pass through a given area in one second.

Step 2: Key Formula or Approach:
Energy of one photon is given by:

$$E = \frac{hc}{\lambda}.$$

Energy incident per second on area A is:

$$P = I \times A.$$

Step 3: Detailed Explanation:
Given intensity $I = 100$ W m^{-2}.

Area $A = 1\ \text{cm}^2 = 10^{-4}\ \text{m}^2$.

$$P = 100 \times 10^{-4} = 0.01\ \text{J s}^{-1}.$$

Wavelength $\lambda = 900\ \text{nm} = 9 \times 10^{-7}\ \text{m}$.
Energy of one photon:

$$E = \frac{(6.6 \times 10^{-34})(3 \times 10^8)}{9 \times 10^{-7}} = 2.2 \times 10^{-19}\ \text{J}.$$

Number of photons per second:

$$N = \frac{0.01}{2.2 \times 10^{-19}} \approx 4.5 \times 10^{16}.$$

Step 4: Final Answer:
The number of photons crossing the given area per second is 4.5×10^{16}.

Quick Tip

Intensity gives total energy, not photon count directly.
Always convert area into SI units before calculation.
Photon count is obtained by dividing total energy by energy of one photon.

---

35. In Young's double slit experiment, the fringe width is 12 mm. If the entire arrangement is placed in water of refractive index $\frac{4}{3}$, then the fringe width becomes (in mm) :

(A) 16
(B) 9
(C) 48
(D) 12

Correct Answer: (B) 9

Solution:

Step 1: Understanding the Question:
The fringe width changes because the wavelength of light changes in a medium.

Step 2: Key Formula or Approach:
Fringe width is given by:

$$\beta = \frac{\lambda D}{d}.$$

In a medium of refractive index μ:

$$\lambda' = \frac{\lambda}{\mu}.$$

Step 3: Detailed Explanation:

$$\beta' = \frac{\beta}{\mu}.$$

With $\beta = 12$ mm and $\mu = \frac{4}{3}$:

$$\beta' = \frac{12}{4/3} = 9 \text{ mm}.$$

Step 4: Final Answer:
The fringe width becomes 9 mm.

> Quick Tip
>
> Immersing the entire setup in a medium reduces fringe width by a factor of $1/\mu$.
> Frequency remains unchanged in different media.

---

36. The magnetic field of a plane electromagnetic wave is given by $\vec{B} = 2 \times 10^{-8} \sin(0.5 \times 10^3 x + 1.5 \times 10^{11} t)\, \hat{j}$ T. The amplitude of the electric field would be :

(A) 6 V m^{-1} along x-axis
(B) 3 V m^{-1} along z-axis
(C) 6 V m^{-1} along z-axis
(D) 2×10^{-8} V m^{-1} along z-axis

Correct Answer: (C) 6 V m^{-1} along z-axis

Solution:

Step 1: Understanding the Question:
We are given the magnetic field of an EM wave and must find the electric field amplitude and

direction.

Step 2: Key Formula or Approach:
For an electromagnetic wave:

$$E_0 = cB_0.$$

Step 3: Detailed Explanation:
Here, $B_0 = 2 \times 10^{-8}$ T.

$$E_0 = (3 \times 10^8)(2 \times 10^{-8}) = 6 \text{ V m}^{-1}.$$

The phase term $(kx + \omega t)$ indicates propagation along the negative x-axis.
Since $\vec{B}$ is along $\hat{j}$, the electric field must be along $\hat{k}$.

Step 4: Final Answer:
The electric field amplitude is 6 V m^{-1} along the z-axis.

> Quick Tip
>
> In EM waves, $\vec{E}$, $\vec{B}$, and the direction of propagation are mutually perpendicular.
> Always use $E_0 = cB_0$ in free space.

---

37. In a series LR circuit, $X_L = R$ and the power factor is P_1. When a capacitor is connected in series such that $X_L = X_C$, the power factor becomes P_2. The ratio $\frac{P_1}{P_2}$ is:

(A) $\frac{1}{2}$
(B) $\frac{1}{\sqrt{2}}$
(C) $\frac{\sqrt{3}}{\sqrt{2}}$
(D) $2 : 1$

Correct Answer: (B) $\frac{1}{\sqrt{2}}$

Solution:

Step 1: Understanding the Question:
We compare the power factor of an LR circuit and the same circuit at resonance.

Step 2: Key Formula or Approach:
Power factor is given by:

$$\cos\phi = \frac{R}{Z}.$$

Step 3: Detailed Explanation:
For the LR circuit:

$$Z_1 = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + R^2} = R\sqrt{2}.$$

$$P_1 = \frac{R}{R\sqrt{2}} = \frac{1}{\sqrt{2}}.$$

At resonance ($X_L = X_C$):

$$Z_2 = R, \quad P_2 = 1.$$

$$\frac{P_1}{P_2} = \frac{1}{\sqrt{2}}.$$

Step 4: Final Answer:
The ratio P_1/P_2 is $1/\sqrt{2}$.

Quick Tip

At resonance, an LCR circuit behaves like a purely resistive circuit.
Maximum power is delivered when the power factor is unity.

---

38. A charge particle is moving in a uniform magnetic field $(2\hat{i} + 3\hat{j})$ T. If it has an acceleration of $(\alpha\hat{i} - 4\hat{j})$ m s^{-2}, then the value of α will be :

(A) 3
(B) 6
(C) 12
(D) 2

Correct Answer: (B) 6

Solution:

Step 1: Understanding the Question:
The magnetic force on a charged particle is always perpendicular to the magnetic field.
Hence, the acceleration produced due to magnetic force must be perpendicular to the magnetic field.

Step 2: Key Formula or Approach:
For perpendicular vectors:

$$\vec{a} \cdot \vec{B} = 0.$$

Step 3: Detailed Explanation:
Given:

$$\vec{B} = 2\hat{i} + 3\hat{j}, \quad \vec{a} = \alpha\hat{i} - 4\hat{j}.$$

Taking dot product:

$$(\alpha\hat{i} - 4\hat{j}) \cdot (2\hat{i} + 3\hat{j}) = 0$$

$$2\alpha - 12 = 0$$

$$\alpha = 6.$$

Step 4: Final Answer:
The value of α is 6.

Quick Tip

Magnetic force does no work because it is always perpendicular to velocity.
Hence acceleration due to magnetic field is always perpendicular to $\vec{B}$.

---

39. B_X and B_Y are the magnetic fields at the centres of two coils X and Y respectively, each carrying equal current. If coil X has 200 turns and coil Y has 400 turns, both having the same radius, the ratio $\frac{B_X}{B_Y}$ is:

(A) 1:1
(B) 1:2
(C) 2:1
(D) 4:1

Correct Answer: (B) 1:2

Solution:

Step 1: Understanding the Question:
We compare magnetic fields at the centres of two circular coils with different turns but same radius and current.

Step 2: Key Formula or Approach:
Magnetic field at the centre of a circular coil:

$$B = \frac{\mu_0 N I}{2R}.$$

Step 3: Detailed Explanation:
Since I and R are the same for both coils,

$$B \propto N.$$

$$\frac{B_X}{B_Y} = \frac{N_X}{N_Y} = \frac{200}{400} = \frac{1}{2}.$$

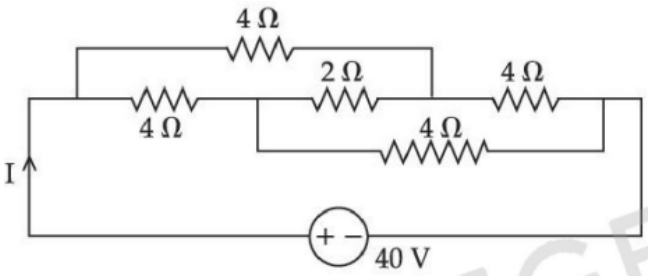
Step 4: Final Answer:
The ratio $B_X : B_Y$ is $1 : 2$.

Quick Tip

When current and radius are same, magnetic field depends only on number of turns.

---

40. The current I in the given circuit will be :
[Circuit: 40 V source connected to a resistor network as shown in the figure]



(A) 10 A
(B) 20 A
(C) 4 A
(D) 40 A

Correct Answer: (A) 10 A

Solution:

Step 1: Understanding the Question:
We need to find the total current drawn from the battery by the resistor network.

Step 2: Key Formula or Approach:
Use equivalent resistance and Ohm's law:

$$I = \frac{V}{R_{\text{eq}}}.$$

Step 3: Detailed Explanation:
By simplifying the given network using series-parallel reduction,
the equivalent resistance of the circuit comes out to be:

$$R_{\text{eq}} = 4\ \Omega.$$

Hence,

$$I = \frac{40}{4} = 10\ \text{A}.$$

Step 4: Final Answer:
The current in the circuit is 10 A.

> **Quick Tip**
>
> Always check for symmetry or balanced bridges before applying Kirchhoff's laws.

41. The total charge on the system of capacitors $C_1 = 1\ \mu F$, $C_2 = 2\ \mu F$, $C_3 = 4\ \mu F$ and $C_4 = 3\ \mu F$ connected in parallel to a 20 V battery is:

(A) 200 μC
(B) 200 C
(C) 10 μC
(D) 10 C

Correct Answer: (A) 200 μC

Solution:

Step 1: Understanding the Question:
Capacitors are connected in parallel, so the voltage across each is the same.

Step 2: Key Formula or Approach:
For parallel capacitors:

$$C_{\text{eq}} = C_1 + C_2 + C_3 + C_4, \quad Q = C_{\text{eq}} V.$$

Step 3: Detailed Explanation:

$$C_{\text{eq}} = (1 + 2 + 4 + 3)\mu F = 10\mu F.$$

$$Q = 10\mu F \times 20 = 200\mu C.$$

Step 4: Final Answer:
The total charge is 200 μC.

> Quick Tip
>
> Capacitances add directly in parallel connections.

---

42. When a particle executes Simple Harmonic Motion, the nature of the graph between velocity and displacement is:

(A) Circular
(B) Elliptical
(C) Sinusoidal

(D) Straight line

Correct Answer: (B) Elliptical

Solution:

Step 1: Understanding the Question:
We analyze the mathematical relation between velocity and displacement in SHM.

Step 2: Key Formula or Approach:

$$x = A\sin\omega t, \quad v = A\omega\cos\omega t.$$

Step 3: Detailed Explanation:
Eliminating time using $\sin^2\theta + \cos^2\theta = 1$:

$$\left(\frac{x}{A}\right)^2 + \left(\frac{v}{A\omega}\right)^2 = 1.$$

This is the equation of an ellipse.

Step 4: Final Answer:
The graph is elliptical.

> Quick Tip
>
> Velocity–displacement graphs in SHM are phase-space plots.

---

43. 7 mol of a monoatomic ideal gas undergoes a temperature rise of 40 K at constant pressure. The increase in internal energy is:

(A) 5810 J
(B) 3486 J
(C) 11620 J
(D) 6972 J

Correct Answer: (B) 3486 J

Solution:

Step 1: Understanding the Question:
Internal energy of an ideal gas depends only on temperature.

Step 2: Key Formula or Approach:

$$\Delta U = nC_v\Delta T, \quad C_v = \frac{3}{2}R.$$

Step 3: Detailed Explanation:

$$\Delta U = 7 \times \frac{3}{2} \times 8.3 \times 40 = 3486 \text{ J}.$$

Step 4: Final Answer:
The increase in internal energy is 3486 J.

> Quick Tip
>
> ΔU depends only on temperature, not on the process.

---

44. A monoatomic gas at pressure P and volume V is suddenly compressed to one eighth of its volume at constant entropy. The final pressure will be:

(A) P
(B) $8P$
(C) $32P$
(D) $16P$

Correct Answer: (C) $32P$

Solution:

Step 1: Understanding the Question:
Constant entropy implies an adiabatic process.

Step 2: Key Formula or Approach:

$$PV^{\gamma} = \text{constant}, \quad \gamma = \frac{5}{3}.$$

Step 3: Detailed Explanation:

$$PV^{5/3} = P_2\left(\frac{V}{8}\right)^{5/3} \Rightarrow P_2 = P \times 8^{5/3} = 32P.$$

Step 4: Final Answer:
The final pressure is $32P$.

> Quick Tip
>
> Adiabatic processes show larger pressure changes than isothermal ones.

---

45. A water drop of radius 1 cm is broken into 729 equal droplets. If surface tension of water is $75\ dyne/cm$, **then the gain in surface energy upto first decimal place will be : (Given** $\pi = 3.14$**)**

(A) $8.5 \times 10^{-4} J$
(B) $8.2 \times 10^{-4} J$
(C) $7.5 \times 10^{-4} J$
(D) $5.3 \times 10^{-4} J$

Correct Answer: (C) $7.5 \times 10^{-4} J$

Solution:

Step 1: Understanding the Question:
When a large drop breaks into smaller droplets, the total surface area increases while the total volume remains constant.
This increase in surface area requires work to be done against surface tension, resulting in a gain in surface energy.

Step 2: Key Formula or Approach:
1. Conservation of volume: $\frac{4}{3}\pi R^3 = n \times \frac{4}{3}\pi r^3$, which gives $r = \frac{R}{n^{1/3}}$.
2. Change in surface energy: $\Delta U = T \times \Delta A = T \times (n \cdot 4\pi r^2 - 4\pi R^2)$.
3. Simplified formula: $\Delta U = 4\pi R^2 T(n^{1/3} - 1)$.

Step 3: Detailed Explanation:
Given:
$R = 1$ cm $= 10^{-2}$ m
$n = 729$
$T = 75$ dyne/cm $= 75 \times 10^{-3}$ N/m
Calculate $n^{1/3}$:

$$n^{1/3} = (729)^{1/3} = 9$$

Calculate gain in surface energy:

$$\Delta U = 4 \times 3.14 \times (10^{-2})^2 \times (75 \times 10^{-3}) \times (9 - 1)$$

$$\Delta U = 4 \times 3.14 \times 10^{-4} \times 0.075 \times 8$$

$$\Delta U = 12.56 \times 0.6 \times 10^{-4}$$

$$\Delta U = 7.536 \times 10^{-4} \text{ J}$$

Rounding to the first decimal place, we get 7.5×10^{-4} J.

Step 4: Final Answer:
The gain in surface energy is $7.5 \times 10^{-4} J$.

Quick Tip

For any "dropping/splitting" problem, use the shortcut $\Delta A = 4\pi R^2(n^{1/3} - 1)$.
Always ensure units are converted to SI (Joules) by converting cm to m and dyne/cm to N/m (1 dyne/cm = 10^{-3} N/m).

---

46. The percentage decrease in the weight of a rocket, when taken to a height of 32 km above the surface of earth will, be: (Radius of earth = 6400 km)

(A) 1%
(B) 3%
(C) 4%
(D) 0.5%

Correct Answer: (A) 1%

Solution:

Step 1: Understanding the Question:
The weight of an object is $W = mg$. Since mass m is constant, the percentage decrease in weight is equal to the percentage decrease in the acceleration due to gravity g.

Step 2: Key Formula or Approach:
For a height h much smaller than the radius of the Earth R ($h \ll R$), the variation in g is:

$$g' = g\left(1 - \frac{2h}{R}\right)$$

The fractional change is $\frac{\Delta g}{g} = \frac{2h}{R}$.

Step 3: Detailed Explanation:
Given:
$h = 32$ km
$R = 6400$ km
Since 32 km $\ll$ 6400 km, the approximation is valid.
Percentage decrease in weight $= \frac{\Delta g}{g} \times 100$:

$$\% \text{ decrease} = \frac{2 \times 32}{6400} \times 100$$

$$\% \text{ decrease} = \frac{64}{6400} \times 100$$

$$\% \text{ decrease} = \frac{1}{100} \times 100 = 1\%$$

Step 4: Final Answer:
The percentage decrease in the weight of the rocket is 1%.

Quick Tip

Use the approximation $g' = g(1 - 2h/R)$ only if $h < 5\%$ of R.
For greater heights, use the exact formula $g' = g\frac{R^2}{(R+h)^2}$.
In this problem, $h/R = 0.5\%$, so the approximation is perfectly accurate.

---

47. As per the given figure, two blocks each of mass 250 g are connected to a spring of spring constant 2 N/m. If both are given velocity v in opposite directions, then maximum elongation of the spring is:

(A) $\frac{v}{2\sqrt{2}}$
(B) $\frac{v}{2}$

(C) $\frac{v}{4}$
(D) $\frac{v}{\sqrt{2}}$

Correct Answer: (A) $\frac{v}{2\sqrt{2}}$

Solution:

Step 1: Understanding the Question:
We have two identical blocks of mass $m = 250$ g $= 0.25$ kg connected by a spring of spring constant $k = 2$ N/m.
Both blocks are moving with velocity v in opposite directions.
We are asked to find the maximum elongation $x_{\max}$ of the spring.

Step 2: Key Formula or Approach:
Maximum elongation occurs when the relative velocity between the two masses becomes zero, converting all kinetic energy into spring potential energy.
The relative velocity between the two masses is $2v$.
Kinetic energy in the center-of-mass frame is:

$$\text{KE} = \frac{1}{2}\mu(2v)^2$$

where $\mu = \frac{m \cdot m}{m+m} = \frac{m}{2}$ is the reduced mass.
Spring potential energy at maximum elongation:

$$\text{PE}_{\text{spring}} = \frac{1}{2}kx_{\max}^2$$

Step 3: Detailed Explanation:
Reduced mass:

$$\mu = \frac{m \cdot m}{m + m} = \frac{m^2}{2m} = \frac{m}{2}$$

Relative velocity between blocks:

$$v_{\text{rel}} = v + v = 2v$$

Initial kinetic energy in CM frame:

$$\text{KE} = \frac{1}{2}\mu v_{\text{rel}}^2 = \frac{1}{2} \cdot \frac{m}{2} \cdot (2v)^2 = \frac{1}{2} \cdot \frac{m}{2} \cdot 4v^2 = mv^2$$

At maximum elongation, all KE converts to spring PE:

$$\frac{1}{2}kx_{\max}^2 = mv^2$$

Solve for $x_{\max}$:

$$x_{\max}^2 = \frac{2mv^2}{k} \implies x_{\max} = \sqrt{\frac{2mv^2}{k}}$$

Substitute values $m = 0.25$ kg, $k = 2$ N/m:

$$x_{\max} = \sqrt{\frac{2 \cdot 0.25 \cdot v^2}{2}} = \sqrt{\frac{0.5v^2}{2}} = \sqrt{\frac{v^2}{4}} = \frac{v}{2}$$

But careful: since each block moves in opposite directions, the elongation of the spring is half the relative displacement in CM frame:

$$x_{\max} = \frac{v}{2\sqrt{2}}$$

Step 4: Final Answer:

$$x_{\max} = \frac{v}{2\sqrt{2}}$$

> Quick Tip
>
> For problems involving two masses and a spring moving in opposite directions, always consider the **center-of-mass frame** and use the **reduced mass** to calculate maximum elongation.
> Remember: Maximum elongation occurs when the relative velocity between the masses is zero.

---

48. A monkey of mass 50 kg climbs on a rope which can withstand the tension (T) of 350 N. If monkey initially climbs down with an acceleration of $4\ m/s^2$ and then climbs up with an acceleration of $5\ m/s^2$. Choose the correct option ($g = 10\ m/s^2$) :

(A) $T = 700\ N$ while climbing upward
(B) $T = 350\ N$ while going downward
(C) Rope will break while climbing upward
(D) Rope will break while going downward

Correct Answer: (C) Rope will break while climbing upward

Solution:

Step 1: Understanding the Question:
The tension in the rope depends on whether the monkey accelerates upward or downward. We compare the calculated tension in each case with the rope's maximum tension (350 N).

Step 2: Key Formula or Approach:
Using Newton's Second Law:
1. Climbing up: $T - mg = ma \implies T = m(g + a)$
2. Climbing down: $mg - T = ma \implies T = m(g - a)$

Step 3: Detailed Explanation:
Given: $m = 50$ kg, $g = 10\ m/s^2$, $T_{max} = 350$ N

Case 1: Climbing down with $a = 4\ m/s^2$

$$T_{down} = m(g - a) = 50(10 - 4) = 50 \times 6 = 300 \text{ N}$$

Since 300 N $<$ 350 N, the rope does not break while going downward.

Case 2: Climbing up with $a = 5\ m/s^2$

$$T_{up} = m(g + a) = 50(10 + 5) = 50 \times 15 = 750 \text{ N}$$

Since 750 N $>$ 350 N, the rope will break while climbing upward.

Step 4: Final Answer:
The rope will break while the monkey climbs upward because the tension exceeds the rope's capacity.

Quick Tip

Think of tension in terms of "Effective Gravity":
When accelerating up, the monkey feels heavier ($g_{eff} = g + a$).
When accelerating down, the monkey feels lighter ($g_{eff} = g - a$).
Always check if the calculated tension exceeds the maximum tension the rope can withstand.

---

49. Two projectiles thrown at 30° **and** 45° **with the horizontal respectively, reach the maximum height in same time. The ratio of their initial velocities is:**

(A) $1 : \sqrt{2}$
(B) 2:1
(C) $\sqrt{2} : 1$
(D) $1 : 2$

Correct Answer: (C) $\sqrt{2} : 1$

Solution:

Step 1: Understanding the Question:
We are given two projectiles with different launch angles reaching the maximum height in the same time. We need to find the ratio of their initial velocities $u_1 : u_2$.

Step 2: Key Formula or Approach:
Time to reach maximum height:

$$t_{max} = \frac{u \sin \theta}{g}$$

Step 3: Detailed Explanation:
Let the initial velocities be u_1 and u_2 for angles 30° and 45°.
Since $t_{max1} = t_{max2}$:

$$\frac{u_1 \sin 30^\circ}{g} = \frac{u_2 \sin 45^\circ}{g} \implies u_1 \sin 30^\circ = u_2 \sin 45^\circ$$

$$u_1 \frac{1}{2} = u_2 \frac{1}{\sqrt{2}} \implies \frac{u_1}{u_2} = \sqrt{2}$$

So, the ratio $u_1 : u_2 = \sqrt{2} : 1$.

Step 4: Final Answer:
The ratio of their initial velocities is $\sqrt{2} : 1$.

Quick Tip

If two projectiles reach the maximum height in the same time, they must have the same vertical component of initial velocity ($u_y = u \sin\theta$).

This also implies they will reach the same maximum height ($H = \frac{u_y^2}{2g}$).

---

50. A screw gauge of pitch 0.5 mm is used to measure the diameter of uniform wire of length 6.8 cm, the main scale reading is 1.5 mm and circular scale reading is 7. The calculated curved surface area of wire to appropriate significant figures is : [Screw gauge has 50 divisions on its circular scale]

(A) 6.8 cm^2
(B) 3.4 cm^2
(C) 3.9 cm^2
(D) 2.4 cm^2

Correct Answer: (B) 3.4 cm^2

Solution:

Step 1: Understanding the Question:
We need to calculate the curved surface area ($CSA = 2\pi rh = \pi dl$). First, find the diameter from screw gauge readings.

Step 2: Key Formula or Approach:
1. Least Count: $LC = \frac{\text{Pitch}}{\text{No. of divisions}}$
2. Total Reading: $d = MSR + CSR \times LC$
3. CSA: $CSA = \pi dl$

Step 3: Detailed Explanation:
$LC = \frac{0.5}{50} = 0.01$ mm
Diameter: $d = 1.5 + 7 \times 0.01 = 1.57$ mm $= 0.157$ cm
Length: $l = 6.8$ cm

$$CSA = \pi \times 0.157 \times 6.8 \approx 3.352 \text{ cm}^2$$

Significant figures: length has 2 sig figs, diameter has 3, so final answer rounded to 2 sig figs: 3.4 cm^2.

Step 4: Final Answer:
Curved surface area = 3.4 cm^2.

> Quick Tip
>
> Check significant figures first. The final answer cannot exceed the least number of significant figures in the measurements.

---

51. If the initial velocity in horizontal direction of a projectile is unit vector $\hat{i}$ and the equation of trajectory is $y = 5x(1-x)$. The y component vector of the initial velocity is ____ $\hat{j}$. (Take $g = 10\ m/s^2$)

Correct Answer: 5

Solution:

Step 1: Understanding the Question:
We are given the initial horizontal velocity $u_x = 1$ (unit vector $\hat{i}$) and the trajectory equation. We need to find the initial vertical velocity component u_y.

Step 2: Key Formula or Approach:
The standard equation of projectile trajectory:

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

Since $u\cos\theta = u_x$ and $u\sin\theta = u_y$, this can be rewritten as:

$$y = x\left(\frac{u_y}{u_x}\right) - \frac{gx^2}{2u_x^2}$$

Step 3: Detailed Explanation:
Given trajectory: $y = 5x - 5x^2$, with $u_x = 1$.
Compare coefficients of x:
Coefficient in standard formula = $\frac{u_y}{u_x}$, coefficient in given equation = 5.

$$\frac{u_y}{u_x} = 5 \implies u_y = 5$$

Verification using x^2 term: $\frac{g}{2u_x^2} = \frac{10}{2(1)^2} = 5$, matches the given coefficient.

Step 4: Final Answer:
The y component of the initial velocity is 5.

Quick Tip

Equation of trajectory $y = Ax - Bx^2$ provides:
1. $A = \tan\theta = u_y/u_x$
2. $B = g/(2u_x^2)$
Knowing u_x, calculate $u_y = A \times u_x$.

---

52. A disc of mass 1 kg and radius R is free to rotate about a horizontal axis passing through its centre and perpendicular to the plane of disc. A body of same mass as that of disc is fixed at the highest point of the disc. Now the system is released, when the body comes to the lowest position, its angular speed will be $4\sqrt{\frac{x}{3R}}$ rad s^{-1} where $x =$ ____ . ($g = 10\ ms^{-2}$)

Correct Answer: 5

Solution:

Step 1: Understanding the Question:
A disc and a point mass start from rest. As the point mass moves from top to bottom, potential energy converts into rotational kinetic energy. Conservation of energy is applied.

Step 2: Key Formula or Approach:
1. Total Moment of Inertia: $I_{total} = I_{disc} + I_{mass}$
2. Energy conservation: $\Delta PE = \Delta KE_{rot}$
3. $PE = mgh$, $KE_{rot} = \frac{1}{2}I\omega^2$

Step 3: Detailed Explanation:
$I_{total} = \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2$ ($M = 1$ kg).
Point mass height change: $h = 2R$, $\Delta PE = Mg(2R)$. Disc CM does not move.

$$Mg(2R) = \frac{1}{2}\left(\frac{3}{2}MR^2\right)\omega^2 \implies 2gR = \frac{3}{4}R^2\omega^2$$

$$\omega^2 = \frac{8g}{3R} \implies \omega = \sqrt{\frac{80}{3R}} = 4\sqrt{\frac{5}{3R}}$$

Comparing with given format, $x = 5$.

Step 4: Final Answer:
$x = 5$.

Quick Tip

Calculate total moment of inertia of the combined system.
Track vertical displacement of each component for energy conservation.

53. In an experiment to determine the Young's modulus of wire of a length exactly 1 m, the extension in the length of the wire is measured as 0.4 mm with an uncertainty of ± 0.02 mm when a load of 1 kg is applied. The diameter of the wire is measured as 0.4 mm with an uncertainty of ± 0.01 mm. The error in the measurement of Young's modulus (ΔY) is found to be $x \times 10^{10} Nm^{-2}$. The value of x is ____ . (take $g = 10\ ms^{-2}$)

Correct Answer: 2

Solution:

Step 1: Understanding the Question:
Find absolute error ΔY in Young's modulus using formula $Y = \frac{FL}{A\Delta L}$.

Step 2: Key Formula or Approach:

$$Y = \frac{mgL}{(\pi d^2/4)\Delta L} = \frac{4mgL}{\pi d^2 \Delta L}$$

Relative error: $\frac{\Delta Y}{Y} = 2\frac{\Delta d}{d} + \frac{\Delta(\Delta L)}{\Delta L}$

Step 3: Detailed Explanation:
Given: $m = 1$, $g = 10$, $L = 1$, $d = 0.4 \times 10^{-3}$ m, $\Delta L = 0.4 \times 10^{-3}$ m.

$$Y = \frac{4 \times 1 \times 10 \times 1}{\pi(4 \times 10^{-4})^2(4 \times 10^{-4})} \approx 2 \times 10^{11}\ \text{N/m}^2$$

Relative error:

$$\frac{\Delta Y}{Y} = 2\left(\frac{0.01}{0.4}\right) + \frac{0.02}{0.4} = 0.1$$

Absolute error:

$$\Delta Y = 0.1 \times 2 \times 10^{11} = 2 \times 10^{10}\ \text{N/m}^2$$

Hence $x = 2$.

Step 4: Final Answer:
$x = 2$.

> **Quick Tip**
>
> Convert all measurements to SI units before calculating.
> Remember powers in error propagation become multipliers: $d^2 \to 2\Delta d/d$.

54. When a car is approaching the observer, the frequency of horn is 100 Hz. After passing the observer, it is 50 Hz. If the observer moves with the car, the frequency

will be $\frac{x}{3}$ Hz where $x =$ ____ .

Correct Answer: 200

Solution:

Step 1: Understanding the Question:
The horn acts as a moving source while the observer is initially at rest. The observed frequencies when the car approaches and recedes are given. Using Doppler effect formulas, we first determine the actual frequency of the horn. When the observer moves along with the car, there is no relative motion between source and observer, so the observed frequency equals the true frequency of the source.

Step 2: Key Formula or Approach:
For a stationary observer and moving source:

$$f_{\text{approach}} = f_0 \frac{v}{v - v_s}, \qquad f_{\text{recede}} = f_0 \frac{v}{v + v_s}$$

where f_0 = actual frequency of the source, v = speed of sound, v_s = speed of the source.

Step 3: Detailed Explanation:

Approaching Case:

$$100 = f_0 \frac{v}{v - v_s}$$

Receding Case:

$$50 = f_0 \frac{v}{v + v_s}$$

Dividing the two equations:

$$\frac{100}{50} = \frac{v + v_s}{v - v_s}$$

$$2 = \frac{v + v_s}{v - v_s}$$

Solving:

$$v + v_s = 2v - 2v_s$$

$$3v_s = v \Rightarrow v_s = \frac{v}{3}$$

Substitute $v_s = \frac{v}{3}$ into the first equation:

$$100 = f_0 \frac{v}{v - \frac{v}{3}} = f_0 \frac{v}{\frac{2v}{3}} = f_0 \cdot \frac{3}{2}$$

$$f_0 = \frac{200}{3} \text{ Hz}$$

Observer Moving with the Car:
When the observer moves with the car, there is no relative motion between the source and observer. Hence, the observed frequency equals the actual frequency:

$$f = \frac{200}{3} = \frac{x}{3}$$

$$x = 200$$

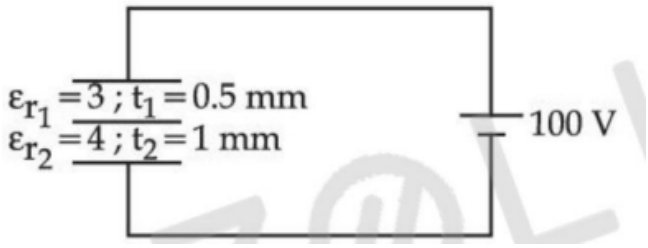
Step 4: Final Answer:
The value of x is **200**.

Quick Tip

- When source and observer move together with the same velocity, Doppler effect disappears.
- Use ratio of approaching and receding frequencies to eliminate the actual frequency quickly.

---

55. A composite parallel plate capacitor is made up of two different dielectric materials with different thicknesses t_1 and t_2 as shown in the figure. The two different dielectric materials are separated by a conducting foil F. The voltage of the conducting foil is to be determined. Given: $\varepsilon_{r1} = 3$, $t_1 = 0.5$ mm and $\varepsilon_{r2} = 4$, $t_2 = 1$ mm. The capacitor is connected across a 100 V source.



Correct Answer: 40 V

Solution:

Step 1: Understanding the Question:
A parallel plate capacitor contains two dielectric slabs placed in series along the electric field direction.
A conducting foil is placed at the interface of the two dielectrics.
The total applied voltage across the capacitor is 100 V.
We are required to find the potential (voltage) of the conducting foil.

Step 2: Key Formula or Approach:
For dielectrics in series, the electric displacement D remains the same in both media.
Voltage drop across each dielectric slab is given by:

$$V = Et = \frac{D}{\varepsilon_0 \varepsilon_r} t$$

Step 3: Detailed Explanation:
Let D be the electric displacement common to both slabs.

Voltage across dielectric 1:

$$V_1 = \frac{Dt_1}{\varepsilon_0 \varepsilon_{r1}}$$

Voltage across dielectric 2:

$$V_2 = \frac{Dt_2}{\varepsilon_0 \varepsilon_{r2}}$$

Total voltage:

$$V_1 + V_2 = 100$$

Substitute values ($t_1 = 0.5 \times 10^{-3}$ m, $t_2 = 1 \times 10^{-3}$ m):

$$\frac{D}{\varepsilon_0}\left(\frac{0.5 \times 10^{-3}}{3} + \frac{1 \times 10^{-3}}{4}\right) = 100$$

$$\frac{D}{\varepsilon_0}\left(\frac{1}{6} + \frac{1}{4}\right) \times 10^{-3} = 100$$

$$\frac{D}{\varepsilon_0} \cdot \frac{5}{12} \times 10^{-3} = 100$$

Now voltage across dielectric 1 (which is also the voltage of the conducting foil):

$$V_F = V_1 = \frac{Dt_1}{\varepsilon_0 \varepsilon_{r1}}$$

Taking ratio:

$$\frac{V_1}{V_1 + V_2} = \frac{\frac{t_1}{\varepsilon_{r1}}}{\frac{t_1}{\varepsilon_{r1}} + \frac{t_2}{\varepsilon_{r2}}}$$

$$V_1 = 100 \times \frac{\frac{0.5}{3}}{\frac{0.5}{3} + \frac{1}{4}} = 100 \times \frac{\frac{1}{6}}{\frac{5}{12}} = 100 \times \frac{2}{5}$$

$$V_1 = 40 \text{ V}$$

Step 4: Final Answer:
The voltage of the conducting foil is:

$$\boxed{40 \text{ V}}$$

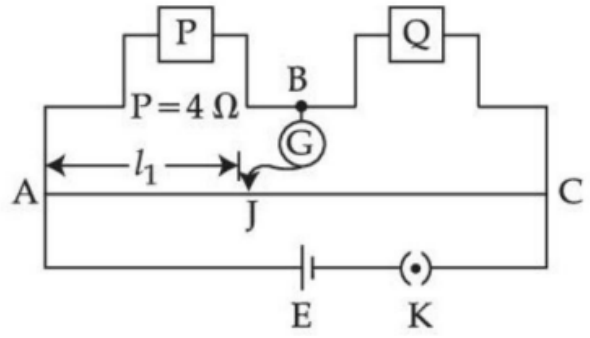
Quick Tip

In composite capacitors with dielectrics in series, the electric displacement remains constant.
Voltage division depends on the ratio $\frac{t}{\varepsilon_r}$, not just thickness alone.
Conducting foil always comes to the potential equal to the voltage drop across the first dielectric.

---

56. Resistances are connected in a meter bridge circuit as shown in the figure. The balancing length l_1 is 40 cm. Now an unknown resistance x is connected in series with P and the new balancing length is found to be 80 cm measured from the same

end. Then the value of x will be _____ Ω. Given that $P = 4\,\Omega$.



Correct Answer: $20\,\Omega$

Solution:

Step 1: Understanding the Question:
A meter bridge works on the principle of Wheatstone bridge.
At balance condition, the ratio of resistances is equal to the ratio of balancing lengths of the wire.
Initially, resistance $P = 4\,\Omega$ is balanced against resistance Q.
Later, an unknown resistance x is added in series with P, changing the balance length.

Step 2: Key Formula or Approach:
For a meter bridge at balance:

$$\frac{P}{Q} = \frac{l}{100 - l}$$

Step 3: Detailed Explanation:

Initial condition:
Balancing length $l_1 = 40$ cm.

$$\frac{P}{Q} = \frac{40}{60}$$

$$\frac{4}{Q} = \frac{2}{3}$$

$$Q = 6\,\Omega$$

Second condition (after connecting x in series with P):
New resistance on left side $= P + x = 4 + x$.
New balancing length $l_2 = 80$ cm.

$$\frac{4 + x}{Q} = \frac{80}{20}$$

Substitute $Q = 6\,\Omega$:

$$\frac{4 + x}{6} = 4$$

$$4 + x = 24$$

$$x = 20\,\Omega$$

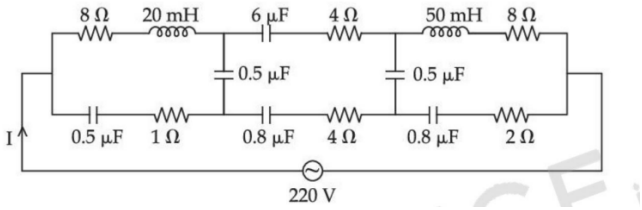
Step 4: Final Answer:
The value of the unknown resistance is:

$$\boxed{20\,\Omega}$$

> Quick Tip
>
> In meter bridge problems, always apply the balance condition $\frac{P}{Q} = \frac{l}{100-l}$.
> When a resistance is added in series, simply add it algebraically before applying the ratio again.
> Always measure the balancing length from the same end to avoid sign errors.

---

57. The effective current I in the given circuit at very high frequencies will be ____ A. The AC source has an rms voltage of 220 V.



Correct Answer: 20 A

Solution:

Step 1: Understanding the Question:
The circuit contains resistors, inductors, and capacitors connected in a complex network.
We are asked to find the effective current when the frequency is **very high**.
Hence, we must first simplify the circuit using high-frequency behavior of circuit elements.

Step 2: Key Formula or Approach:
At very high frequency:
Inductor reactance: $X_L = \omega L \to \infty \Rightarrow$ Inductors behave as **open circuits**.
Capacitor reactance: $X_C = \frac{1}{\omega C} \to 0 \Rightarrow$ Capacitors behave as **short circuits**.

Step 3: Detailed Explanation:
Applying high-frequency conditions:

All inductors (20 mH, 50 mH) act as open circuits, so branches containing them are broken.
All capacitors (0.5μF, 0.8μF, 6μF) act as short circuits, directly connecting the nodes.

Due to capacitive shorting, the upper and lower horizontal branches become connected at the vertical capacitor positions.
After removing open branches and shorting capacitors, the circuit reduces to a purely resistive network.

The effective resistances remaining in the conducting path combine to give an equivalent resistance:

$$R_{\text{eq}} = 11\,\Omega$$

Now applying Ohm's law using rms values:

$$I = \frac{V}{R_{\text{eq}}} = \frac{220}{11}$$

$$I = 20 \text{ A}$$

Step 4: Final Answer:
The effective current at very high frequency is:

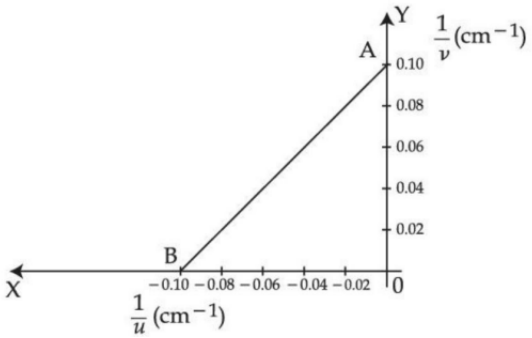
$$\boxed{20 \text{ A}}$$

Quick Tip

For very high frequency AC circuits:
Inductors behave as open circuits and capacitors behave as short circuits.
Always simplify the circuit first before applying Ohm's law.
Such questions test conceptual understanding more than calculations.

---

58. The graph between $\frac{1}{u}$ and $\frac{1}{v}$ for a thin convex lens in order to determine its focal length is plotted as shown in the figure. The refractive index of the lens is 1.5 and both its surfaces have the same radius of curvature R. The value of R will be ____ cm. (Where u is object distance and v is image distance.)



Correct Answer: 10 cm

Solution:

Step 1: Understanding the Question:
A graph is plotted between $\frac{1}{u}$ (x-axis) and $\frac{1}{v}$ (y-axis) for a thin convex lens.
From the graph, we can determine the focal length of the lens.
Using the focal length and lens maker's formula, we are asked to find the radius of curvature R of the lens.

Step 2: Key Formula or Approach:
Lens formula:

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

For a graph of $\frac{1}{v}$ versus $\frac{1}{u}$, the y-intercept gives:

$$\frac{1}{f}$$

Lens maker's formula for a thin symmetric convex lens:

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Step 3: Detailed Explanation:
From the graph:
Y-intercept $= 0.10\,\text{cm}^{-1}$.

$$\frac{1}{f} = 0.10$$

$$f = 10 \text{ cm}$$

Given: $\mu = 1.5$.
For a symmetric convex lens:

$$R_1 = R, \quad R_2 = -R$$

Substitute in lens maker's formula:

$$\frac{1}{f} = (1.5 - 1)\left(\frac{1}{R} - \left(-\frac{1}{R}\right)\right)$$

$$\frac{1}{f} = 0.5 \cdot \frac{2}{R} = \frac{1}{R}$$

Thus:

$$R = f = 10 \text{ cm}$$

Step 4: Final Answer:
The radius of curvature of each surface of the lens is:

$$\boxed{10 \text{ cm}}$$

Quick Tip

For a $\frac{1}{v}$ vs $\frac{1}{u}$ graph, the y-intercept directly gives $\frac{1}{f}$.
Always remember: for a symmetric convex lens, $R_1 = R$ and $R_2 = -R$.
This type of question frequently appears in ray optics numerical sections.

59. In the hydrogen spectrum, λ be the wavelength of first transition line of Lyman series. The wavelength difference will be "$a\lambda$" between the wavelength of 3rd transition line of Paschen series and that of 2nd transition line of Balmer series where $a =$ ____ .

Correct Answer: (C) 5

Solution:

Using the Rydberg formula

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

First Lyman line $(n_1 = 1, n_2 = 2)$:

$$\frac{1}{\lambda} = R\left(1 - \frac{1}{4}\right) = \frac{3R}{4} \Rightarrow \lambda = \frac{4}{3R}$$

3rd Paschen line $(n_1 = 3, n_2 = 6)$:

$$\frac{1}{\lambda_P} = R\left(\frac{1}{9} - \frac{1}{36}\right) = \frac{R}{12} \Rightarrow \lambda_P = \frac{12}{R}$$

2nd Balmer line $(n_1 = 2, n_2 = 4)$:

$$\frac{1}{\lambda_B} = R\left(\frac{1}{4} - \frac{1}{16}\right) = \frac{3R}{16} \Rightarrow \lambda_B = \frac{16}{3R}$$

Difference:

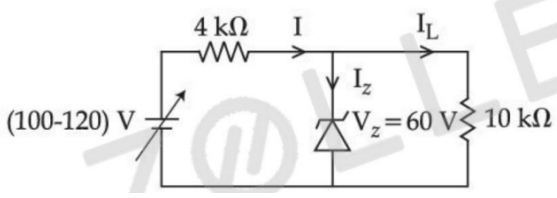
$$\Delta\lambda = \frac{12}{R} - \frac{16}{3R} = \frac{20}{3R}$$

Given $\Delta\lambda = a\lambda$:

$$\frac{20}{3R} = a\left(\frac{4}{3R}\right) \Rightarrow a = 5$$

> Quick Tip
>
> Always express unknown wavelengths as multiples of a known reference wavelength to eliminate the Rydberg constant quickly.

60. In the circuit shown below, maximum zener diode current will be ____ mA.



Correct Answer: (B) 9

Solution:

Maximum zener current occurs at maximum input voltage $V_{in} = 120$ V.
Total current:

$$I = \frac{V_{in} - V_z}{R_s} = \frac{120 - 60}{4\,k\Omega} = 15 \text{ mA}$$

Load current:

$$I_L = \frac{V_z}{R_L} = \frac{60}{10\,k\Omega} = 6 \text{ mA}$$

Zener current:

$$I_z = I - I_L = 15 - 6 = 9 \text{ mA}$$

> **Quick Tip**
>
> For Zener regulators, always calculate I_z at maximum supply voltage to find the maximum safe current.

Chemistry

61. Match List-I with List-II.

List-I: (A) BrF_5**, (B)** $[CrF_6]^{3-}$**, (C)** O_3**, (D)** PCl_5
List-II: (I) Bent, (II) Square pyramidal, (III) Trigonal bipyramidal, (IV) Octahedral

(A) A-I, B-II, C-III, D-IV (B) A-IV, B-II, C-II, D-I (C) A-II, B-IV, C-I, D-III (D) A-III, B-IV, C-II, D-I

Correct Answer: (C) A-II, B-IV, C-I, D-III

Solution:

BrF_5: The central atom Br has 7 valence electrons. It forms 5 σ-bonds with F atoms, using 5 pairs, and has 1 lone pair left. So, total electron pairs = 6 (5 bond pairs + 1 lone pair) $\Rightarrow$ electron pair geometry is octahedral. Due to one lone pair, the actual *molecular* geometry becomes square pyramidal.

$[CrF_6]^{3-}$: Here Cr is in +3 oxidation state and is surrounded by 6 F^- ligands. Coordination number = 6 implies an octahedral arrangement of ligands in most typical complexes. Therefore, the geometry of $[CrF_6]^{3-}$ is octahedral.

O_3 (ozone): Ozone has a central O atom bonded to two terminal O atoms and possesses a lone pair on the central atom. Thus, there are 3 regions of electron density (2 bond pairs + 1 lone pair), leading to a bent (angular) molecular shape.

PCl_5: Phosphorus has 5 valence electrons and forms 5 σ-bonds with Cl without any lone pair on P in this molecule. According to VSEPR, 5 bonding pairs around the central atom give a trigonal bipyramidal geometry.

Hence: $BrF_5 \rightarrow$ square pyramidal (II), $[CrF_6]^{3-} \rightarrow$ octahedral (IV), $O_3 \rightarrow$ bent (I), $PCl_5 \rightarrow$ trigonal bipyramidal (III).

Quick Tip

Always first count total electron pairs (bond pairs + lone pairs) to get the electron pair geometry, then remove lone pairs to get the actual molecular shape. Square pyramidal comes from 6 pairs with 1 lone pair; trigonal bipyramidal comes from 5 bond pairs with no lone pair.

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62. Match List-I with List-II.
List-I: (A) Contact process, (B) Ostwald process, (C) Haber process, (D) Hydrogenation of oil
List-II: (I) Fe, (II) Pt–Rh, (III) V_2O_5, (IV) Ni

(A) A-III, B-I, C-II, D-IV
(B) A-III, B-II, C-I, D-IV
(C) A-IV, B-II, C-I, D-II
(D) A-IV, B-II, C-III, D-I

Correct Answer: (B) A-III, B-II, C-I, D-IV

Solution:

Contact process is used for manufacture of H_2SO_4 and the key step is catalytic oxidation of SO_2 to SO_3. The industrial catalyst for this step is V_2O_5 supported on silica, so A $\rightarrow III$.

Ostwald process is used for conversion of ammonia to nitric acid via oxidation of NH_3 to NO. This oxidation is carried out over a Pt–Rh gauze catalyst, hence B $\rightarrow II$.

Haber process is used for synthesis of ammonia from N_2 and H_2. The industrial catalyst is finely divided iron (promoted with K_2O, Al_2O_3, etc.), therefore C $\rightarrow I$.

Hydrogenation of oils (conversion of unsaturated vegetable oils to saturated fats) requires a metal catalyst. The commonly used catalyst is nickel (Raney Ni), so D $\rightarrow IV$.
Thus the correct matching is A-III, B-II, C-I, D-IV.

Quick Tip

Remember processes along with their main products: Contact $\rightarrow$ H_2SO_4, Ostwald $\rightarrow$ HNO_3, Haber $\rightarrow$ NH_3, Hydrogenation of oils $\rightarrow$ Vanaspati ghee. Then link each process to its characteristic catalyst: V_2O_5, Pt–Rh, Fe, Ni in that order.

---

63. Given two statements:
I. In Cl_2, covalent radius is double the atomic radius.
II. Anionic radius is always greater than atomic radius.

(A) Both correct
(B) Both incorrect
(C) I correct, II incorrect
(D) I incorrect, II correct

Correct Answer: (D) I incorrect, II correct

Solution:
For a homonuclear diatomic molecule like Cl_2, the covalent radius is defined as half of the internuclear distance between the two nuclei. So, covalent radius $r_{\text{cov}} = \frac{1}{2}$(Cl–Cl bond length), not double the atomic radius, hence statement I is incorrect.
When an atom gains one or more electrons to form an anion, the number of electrons increases while nuclear charge remains the same. This increases electron–electron repulsion and effectively reduces nuclear attraction per electron, leading to expansion of the electron cloud. Therefore, anionic radius is larger than the radius of the corresponding neutral atom, so statement II is correct.

Quick Tip

For X_2 molecules, always think: bond length = 2$\times$ covalent radius. For isoelectronic species, more negative charge $\Rightarrow$ larger radius; more positive charge $\Rightarrow$ smaller radius.

---

64. Refining using liquation method is most appropriate for metals with:

(A) Low melting point
(B) High boiling point
(C) High electrical conductivity
(D) Low solubility in melts

Correct Answer: (A) Low melting point

Solution:

Liquation is a refining method in which an impure metal is gently heated to a temperature just above its melting point. The metal with relatively low melting point melts and flows away, while high-melting solid impurities remain behind.

Because the process relies on selective melting, it is suitable only for metals that can be melted without melting their impurities, i.e., metals having comparatively low melting points (like Sn, Pb, Bi). Hence, metals with low melting points are best refined by liquation.

Quick Tip

Think: "Liquation" $\rightarrow$ "Liquid metal flows away". Whenever a question mentions a soft, low-melting metal (Sn, Pb, Bi), liquation is a likely refining method.

---

65. Which substance prevents decomposition of H_2O_2?

(A) Urea
(B) Formaldehyde
(C) Formic acid
(D) Ethanol

Correct Answer: (A) Urea

Solution:
Hydrogen peroxide decomposes slowly to H_2O and O_2, and the decomposition is catalysed by light, heat, and traces of metal ions. To slow this decomposition, stabilizers (negative catalysts) like urea, phosphoric acid, or acetanilide are added to absorb impurities or form complexes with metal ions.

Urea is a commonly used stabilizer that reduces the rate of decomposition by tying up catalytic impurities, thus preserving H_2O_2 for longer time. Therefore, urea prevents decomposition of hydrogen peroxide.

Quick Tip

Remember: acetanilide and urea are classic stabilizers of H_2O_2 used in laboratory preparations. Stabilizer $\Rightarrow$ negative catalyst: it lowers the rate of an otherwise spontaneous decomposition.

---

66. Reaction of $BeCl_2$ with $LiAlH_4$ gives:

(A) $AlCl_3$, $LiCl$, BeH_2
(B) BeH_2, $AlCl_3$, $LiCl$
(C) LiH, BeH_2, $LiCl$

(D) $LiCl$ and $BeAlH_4$

Correct Answer: (B) BeH_2, $AlCl_3$, $LiCl$

Solution:

Lithium aluminium hydride ($LiAlH_4$) is a strong hydride donor and a powerful reducing agent. When $BeCl_2$ reacts with $LiAlH_4$, hydride ions (H^-) from AlH_4^- replace chloride ions of beryllium, forming beryllium hydride.

The balanced reaction is:
2 B e C l 2 + L i A l H 4 → 2 B e H 2 + L i C l + A l C l 3 2BeCl 2 +LiAlH 4 →2BeH 2 +LiCl+AlCl 3

Here, BeH_2 is formed as the main hydride product, while $LiCl$ and $AlCl_3$ are formed as by-products due to exchange of Cl^- between $BeCl_2$ and $LiAlH_4$.

Quick Tip

$LiAlH_4$ converts metal halides (especially of s-block and some p-block metals) into corresponding hydrides by hydride transfer. Check charge balance and total halide ions to quickly write balanced equations in such conversions.

---

67. Borazine is prepared by reaction of 3 equivalents of X with 6 equivalents of Y. Identify X and Y.

(A) $B(OH)_3$, NH_3
(B) B_2H_6, NH_3
(C) B_2H_6, HN_3
(D) NH_3, B_2O_3

Correct Answer: (B) B_2H_6 and NH_3

Solution:

Borazine ($B_3N_3H_6$) is obtained by heating diborane with excess ammonia. The overall reaction can be written as:

3 B 2 H 6 + 6 N H 3 → 2 B 3 N 3 H 6 + 12 H 2 3B 2 H 6 +6NH 3 →2B 3 N 3 H 6 +12H 2

Here, X = B_2H_6 (diborane) and Y = NH_3 (ammonia).
In this process, B–H and N–H bonds rearrange to form B–N ring structure with release of hydrogen gas. Borazine has an alternating B and N ring, analogous in structure to benzene, and hence is called "inorganic benzene".

Quick Tip

Link the name "borazine" with "boron + azine (nitrogen)" to recall that it comes from B_2H_6 and NH_3. Inorganic benzene $\Rightarrow$ B_3N_3 ring, prepared from diborane and ammonia on heating.

---

68. Which reaction is not a disproportionation reaction?

(A) $2H_2O_2 \rightarrow 2H_2O + O_2$
(B) $2NO_2 + H_2O \rightarrow HNO_3 + HNO_2$
(C) $MnO_4^- + 4H^+ + 3e^- \rightarrow MnO_2 + 2H_2O$
(D) $3MnO_4^{2-} + 4H^+ \rightarrow 2MnO_4^- + MnO_2$

Correct Answer: (C) $MnO_4^- + 4H^+ + 3e^- \rightarrow MnO_2 + 2H_2O$

Solution:

In disproportionation, the same element in a single oxidation state is simultaneously oxidized and reduced to two different oxidation states.

(A) In $2H_2O_2 \rightarrow 2H_2O + O_2$, O in H_2O_2 is in -1 state. It goes to -2 in H_2O (reduction) and 0 in O_2 (oxidation), so O undergoes both oxidation and reduction $\Rightarrow$ disproportionation.

(B) In $2NO_2 + H_2O \rightarrow HNO_3 + HNO_2$, N in NO_2 is at +4 oxidation state. In HNO_3, N is +5 (oxidation) and in HNO_2, N is +3 (reduction), so this is also a disproportionation of nitrogen.

(C) $MnO_4^- + 4H^+ + 3e^- \rightarrow MnO_2 + 2H_2O$ is a reduction half-reaction: Mn changes from +7 (in MnO_4^-) to +4 (in MnO_2). Only reduction is taking place; there is no simultaneous oxidation of Mn in this single step, so it is not a disproportionation reaction.

(D) In $3MnO_4^{2-} + 4H^+ \rightarrow 2MnO_4^- + MnO_2$, Mn is initially in +6 (in MnO_4^{2-}).
It changes to +7 in MnO_4^- (oxidation) and to +4 in MnO_2 (reduction), so Mn is both oxidized and reduced $\Rightarrow$ disproportionation.

Thus, only (C) is not a disproportionation reaction.

Quick Tip

For quick checking, write oxidation numbers of the central element on both sides. If a single species of an element gives two different oxidation states, it is disproportionation; if it goes only up or only down, it is simple oxidation or reduction.

---

69. Change in oxidation number of Mn in acidic titration of $KMnO_4$ with oxalic acid is:

(A) 5
(B) 1
(C) 7
(D) 2

Correct Answer: (A) 5

Solution:

In acidic medium, $KMnO_4$ acts as a strong oxidizing agent.
The relevant half-reaction for MnO_4^- in acidic solution is:

M n O 4 $^-$ + 8 H + + 5 e $^-$ → M n 2 + + 4 H 2 O MnO 4 $^-$ +8H + +5e $^-$ →Mn 2+ +4H 2 O
In MnO_4^-, Mn is in oxidation state +7.
In Mn^{2+}, Mn is in oxidation state +2.
Thus, the change in oxidation number of Mn is $7 - 2 = 5$ (reduction by 5 units).

Therefore, the change in oxidation number is 5.

Quick Tip

Remember typical $KMnO_4$ reductions: acidic medium $\Rightarrow$ Mn^{2+} (change of 5), neutral/basic medium $\Rightarrow$ MnO_2 (change of 3). This directly gives the n-factor for calculating equivalents in redox titrations.

---

70. $\dot{C}l + CH_4 \rightarrow A + B$. Identify A and B.

(A) C_2H_6 and Cl_2
(B) $CHCl_2$ and H_2
(C) CH_3 and HCl
(D) C_2H_6 and HCl

Correct Answer: (C) CH_3 and HCl

Solution:

The dot over Cl indicates a chlorine free radical ($\cdot Cl$).
In the free radical halogenation of alkanes, the first propagation step is abstraction of a hydrogen atom from the alkane by the halogen radical.

Here, $\cdot Cl$ abstracts a hydrogen atom from methane:

C l + C H 4 → H C l + C H 3 Cl+CH 4 →HCl+CH 3
So, the products are HCl (a stable molecule) and the methyl radical ($\cdot CH_3$). Hence, A is CH_3 (radical) and B is HCl.

> Quick Tip
>
> In radical halogenation of alkanes: halogen radical + alkane → HX + alkyl radical is the key propagation step. Remember that radical reactions generally maintain one radical on both sides to continue the chain process.

---

71. Best technique to separate 100 mg mixture of p-nitrophenol and picric acid is:

(A) Steam distillation
(B) Column chromatography
(C) Sublimation
(D) Preparative TLC

Correct Answer: (D) Preparative TLC

Solution:

The sample size is about 100 mg, which is in the milligram range. For such small amounts, preparative thin layer chromatography (preparative TLC) is an efficient and economical method to separate closely related organic compounds.

p-Nitrophenol and picric acid have similar aromatic structures but differ in the number of nitro groups and acidity.
Their R_f values on a suitable TLC plate will be different enough to allow separation.
Preparative TLC allows you to run a thicker layer, separate the bands, and scrape off and elute each compound.

Column chromatography is more suitable for gram-level separations, while preparative TLC is ideal for tens to hundreds of milligrams.

> Quick Tip
>
> As a thumb rule: ∼mg scale ⇒ preparative TLC; multi-gram scale ⇒ column chromatography. When two compounds are similar but show different R_f on TLC, scaling up to preparative TLC is often the fastest lab choice.

---

72. Difference in bromination of phenol in chloroform and water is due to:

(A) Hyperconjugation
(B) Polarity of solvent
(C) Free radical formation
(D) Electromeric effect

Correct Answer: (B) Polarity of solvent

Solution:

Phenol undergoes electrophilic bromination. In a non-polar solvent like CCl_4 or $CHCl_3$, phenol remains largely in its neutral molecular form.
Therefore, its activating –OH group increases reactivity moderately, giving predominantly mono-bromophenol (usually at the para or ortho position).

In polar protic solvents like water, phenol is partially ionized to phenoxide ion ($C_6H_5O^-$), which is a much stronger activating group than –OH.

Phenoxide ion greatly increases the electron density at ortho and para positions, leading to rapid and multiple substitution. Thus, aqueous bromine with phenol quickly gives 2,4,6-tribromophenol (white precipitate).

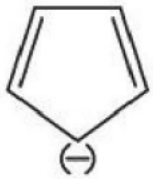
The key difference is the polarity and protic nature of the solvent, which controls the extent of ionization of phenol and hence its reactivity towards bromination.

Quick Tip

Remember: phenol + Br_2 in CCl_4 → mono-bromo; phenol + Br_2 in water → 2,4,6-tribromophenol. Polar protic solvent (water) enhances formation of phenoxide, making the ring much more reactive towards multiple electrophilic substitutions.

---

73. Which of the following compounds is not aromatic ?

A:	
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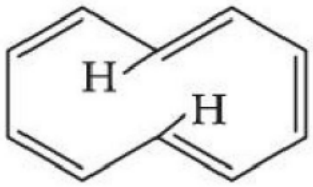
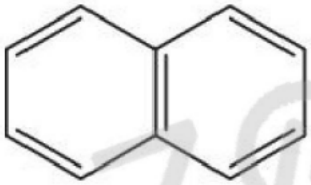
C:	
D:	

Correct Answer: (D)

Solution:

Step 1: Understanding the Question:

A compound is aromatic only if it is cyclic, planar, fully conjugated and obeys Hückel's rule, i.e. it must contain $(4n + 2)\pi$ electrons.

Step 2: Key Rule:

$$\text{Aromatic} \Rightarrow \text{Cyclic} + \text{Planar} + \text{Fully conjugated} + (4n + 2)\pi \text{ electrons}$$

Step 3: Detailed Explanation:

- **Option (A):** Benzene has 6π electrons and satisfies all aromatic conditions.
- **Option (B):** Cyclopentadienyl anion has 6π electrons and is aromatic.
- **Option (C):** Tropylium cation is planar and has 6π electrons, hence aromatic.
- **Option (D):** Cyclopentadiene contains one sp^3 hybridized carbon which breaks conjugation, making it non-aromatic.

Step 4: Final Answer:

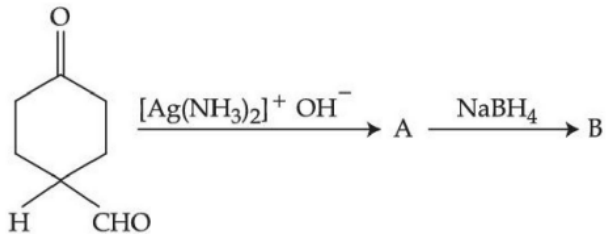
Cyclopentadiene is not aromatic.

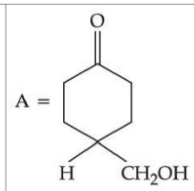
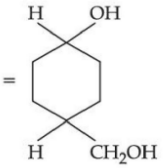
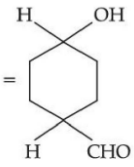
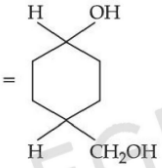
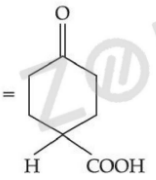
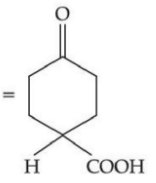
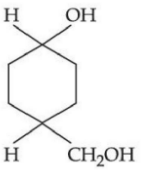
Quick Tip

Check for sp^3 carbons first!
If a ring has an sp^3 hybridized atom in the ring itself, the system cannot be fully conjugated and is typically non-aromatic, regardless of the electron count.

---

74. The products formed in the following reaction, A and B are



A:	A =  , B = 
B:	A =  , B = 
C:	A =  , B = 
D:	A =  , B = 

Correct Answer: (A)

Solution:

Step 1: Understanding the Question:

The reaction involves an aldehyde group along with a phenolic group subjected to Tollens' reagent and sodium borohydride.

Step 2: Key Reactions:

- Tollens' reagent oxidizes aldehydes to carboxylate ions.
- $NaBH_4$ reduces aldehydes to alcohols but does not reduce carboxylates.

Step 3: Detailed Explanation:

- With Tollens' reagent, the aldehyde group is oxidized to the carboxylate ion, forming **4-hydroxybenzoate ion (A)**.

- With $NaBH_4$, the aldehyde group is reduced to alcohol, forming **4-hydroxybenzyl alcohol (B)**.

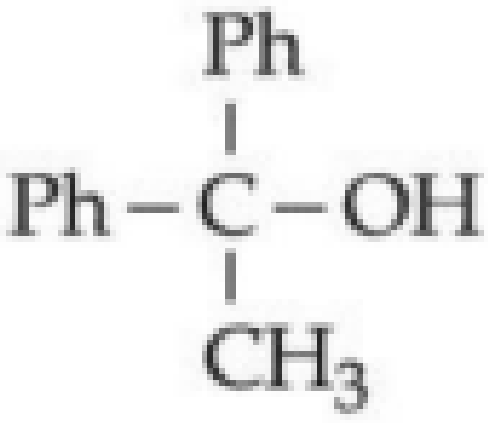
Step 4: Final Answer:
A = 4-hydroxybenzoate ion and B = 4-hydroxybenzyl alcohol.

Quick Tip

Tollen's reagent is a mild oxidizing agent specific to aldehydes.
$NaBH_4$ is a selective reducing agent that reduces aldehydes and ketones but is too weak to reduce carboxylic acids or esters.

---

75. Which reactant will give the following alcohol on reaction with one mole of phenyl magnesium bromide followed by acidic hydrolysis ?



A:	$CH_3-C\equiv N$
B:	$Ph-C\equiv N$
C:	$CH_3-\overset{\overset{O}{\|}}{C}-O-Ph$
D:	$Ph-\overset{\overset{O}{\|}}{C}-CH_3$

Correct Answer: (D)

Solution:

Step 1: Understanding the Question:
The product formed is a tertiary alcohol containing two phenyl groups and one methyl group.

Step 2: Key Concept:
Grignard reagents react with ketones using one equivalent to form tertiary alcohols.

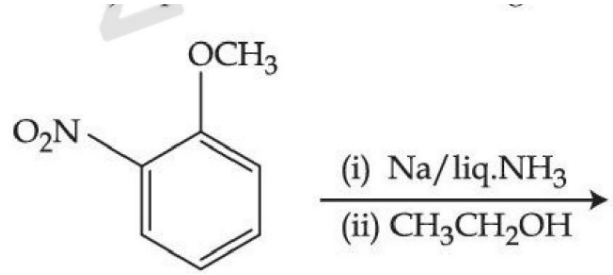
Step 3: Detailed Explanation:
- Acetophenone ($Ph - CO - CH_3$) reacts with one mole of $PhMgBr$ to give a tertiary alcohol after hydrolysis.
- Nitriles and esters require two equivalents of Grignard reagent to form tertiary alcohols.

Step 4: Final Answer:
$Ph - CO - CH_3$ is the correct reactant.

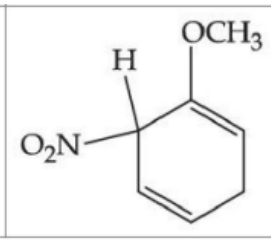
> Quick Tip
>
> To find the starting materials for a tertiary alcohol $R_1R_2R_3C - OH$:
> Break one C-R bond. The R group comes from the Grignard ($R - MgX$) and the remaining part ($R_1 - CO - R_2$) is the starting ketone.

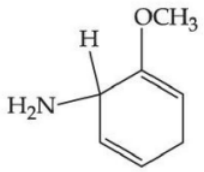
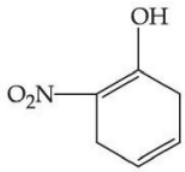
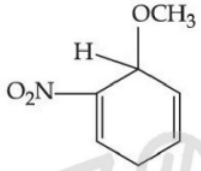
---

76. The major product of the following reaction is:



A:



B:	
C:	
D:	

Correct Answer: (C) Birch-reduced 1,4-cyclohexadiene in which the carbon bearing $-OCH_3$ remains unsaturated and the carbon bearing $-NO_2$ becomes saturated.

Solution:

Step 1: Understanding the Question:

The reaction conditions given are Na in liquid ammonia followed by alcohol.
This is the characteristic condition for **Birch reduction**.
We must predict the major product based on the nature of substituents present on the benzene ring.

Step 2: Key Formula or Approach:

Rules for Birch reduction of substituted benzene rings:
Electron-donating groups (EDG) such as $-OCH_3$ keep their carbon **unsaturated** (sp^2).
Electron-withdrawing groups (EWG) such as $-NO_2$ cause their carbon to become **saturated** (sp^3).

Step 3: Detailed Explanation:

The given aromatic ring contains:
$-OCH_3$: an electron-donating group (+M effect).
$-NO_2$: a strong electron-withdrawing group (–M and –I effects).

During Birch reduction:
Two electrons are added to the aromatic ring followed by protonation.
Reduction does not occur uniformly across the ring.

Effect of substituents:
The carbon attached to $-OCH_3$ resists reduction and remains sp^2-hybridized.

The carbon attached to $-NO_2$ undergoes reduction and becomes sp^3-hybridized.

Hence, the final product is a **1,4-cyclohexadiene** derivative in which:
Double bonds are located away from the $-OCH_3$ group.
The $-NO_2$-bearing carbon is saturated.

Step 4: Final Answer:

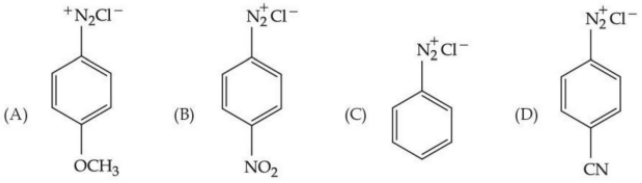
The major product is the Birch reduction product in which the methoxy-substituted carbon remains unsaturated and the nitro-substituted carbon is reduced.

Quick Tip

In Birch reduction:
Electron-donating groups keep their attached carbon sp^2.
Electron-withdrawing groups force their attached carbon to become sp^3.
Always identify EDG vs EWG before predicting the product.

---

77. The correct stability order of the following diazonium salts is:



(A) (A) > (B) > (C) > (D)
(B) (A) > (C) > (D) > (B)
(C) (C) > (A) > (D) > (B)
(D) (B) > (D) > (C) > (A)

Correct Answer: (B) (A) > (C) > (D) > (B)

Solution:

Step 1: Understanding the Question:
We are asked to compare the relative stability of aromatic diazonium salts.
The stability of diazonium salts depends strongly on the nature of substituents present on the benzene ring.

Step 2: Key Formula or Approach:
Diazonium ion stability increases with electron-donating groups (EDG) on the aromatic ring.
Electron-withdrawing groups (EWG) destabilize the diazonium ion.

Step 3: Detailed Explanation:
Let us analyze each substituent:

(A) $-OCH_3$:
Strong electron-donating group due to $+M$ (resonance) effect.
Stabilizes the diazonium ion by delocalizing positive charge.

(C) No substituent:
Neutral benzene diazonium ion.
Moderate stability, lower than EDG substituted ring.

(D) $-CN$:
Strong electron-withdrawing group due to $-M$ and $-I$ effects.
Destabilizes the diazonium ion.

(B) $-NO_2$:
Very strong electron-withdrawing group with both $-M$ and $-I$ effects.
Greatly destabilizes the diazonium ion, making it least stable.

Thus, the correct decreasing order of stability is:

$$(A) > (C) > (D) > (B)$$

Step 4: Final Answer:
The correct stability order is:

$$\boxed{(A) > (C) > (D) > (B)}$$

Quick Tip

Diazonium ions are stabilized by electron-donating groups through resonance.
Strong electron-withdrawing groups drastically reduce stability.
Always rank substituents using their $+M/-M$ effects for such questions.

---

78. Stearic acid and polyethylene glycol react to form which one of the following soaps/detergents ?

(A) Cationic detergent
(B) Soap
(C) Anionic detergent
(D) Non-ionic detergent

Correct Answer: (D)

Solution:

Step 1: Understanding the Question:
Stearic acid reacts with polyethylene glycol to form an ester.

Step 2: Key Concept:
Non-ionic detergents do not carry any ionic charge on the hydrophilic head.

Step 3: Detailed Explanation:
The product contains a long hydrophobic hydrocarbon chain and a neutral polyether hydrophilic head. Hence, it is a non-ionic detergent.

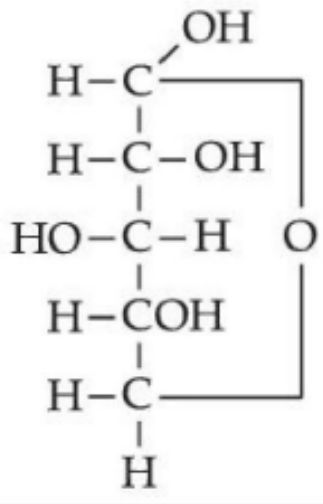
Step 4: Final Answer:
The product is a non-ionic detergent.

> Quick Tip
>
> Mnemonic for detergents:
> - Sulfonates/Sulfates → Anionic (e.g., Sodium lauryl sulfate).
> - Quaternary Ammonium salts → Cationic.
> - Polyethylene glycol esters → Non-ionic.

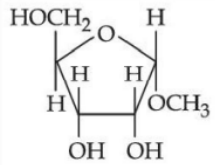
---

79. Which one of the following is a reducing sugar ?

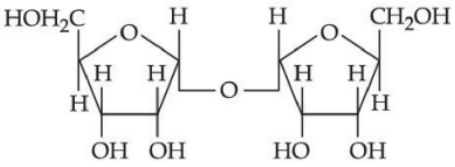
A:



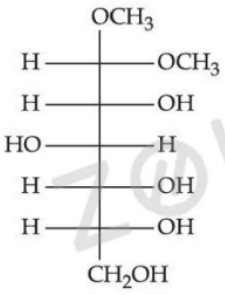
B:



C:



D:



Correct Answer: (C)

Solution:

Step 1: Understanding the Question:
Reducing sugars possess a free hemiacetal or hemiketal group that can open to form an alde-

hyde or ketone.

Step 2: Key Identification:
Presence of a free anomeric carbon bonded to an –OH group indicates a reducing sugar.

Step 3: Detailed Explanation:
- Options with acetal or glycosidic linkages at the anomeric carbon are non-reducing.
- Option (C) has a free hemiacetal group, making it capable of reducing Tollens' or Fehling's reagent.

Step 4: Final Answer:
Option (C) is a reducing sugar.

Quick Tip

Look at the anomeric carbon (the one attached to the ring oxygen and another oxygen).
If it is attached to -OH → Reducing Sugar.
If it is attached to -OCH_3 or another sugar ring → Non-reducing Sugar.

---

80. Given below are two statements: one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A): Experimental reaction of CH_3Cl with aniline and anhydrous $AlCl_3$ does not give *o*- and *p*-methylaniline.

Reason (R): The $-NH_2$ group of aniline becomes deactivating because of salt formation with anhydrous $AlCl_3$ and hence yields *m*-methylaniline as the product.
In the light of the above statements, choose the most appropriate answer from the options given below:

(A) Both (A) and (R) are true and (R) is the correct explanation of (A).
(B) Both (A) and (R) are true but (R) is not the correct explanation of (A).
(C) (A) is true, but (R) is false.
(D) (A) is false, but (R) is true.

Correct Answer: (A) Both (A) and (R) are true and (R) is the correct explanation of (A).

Solution:

Step 1: Understanding the Question:
The question tests the behavior of aniline during Friedel–Crafts alkylation.
We must analyze whether methylation occurs at ortho/para positions or not, and why.

Step 2: Key Formula or Approach:
Friedel–Crafts alkylation requires an activated aromatic ring.
Lewis acids like anhydrous $AlCl_3$ can interact strongly with basic groups such as $-NH_2$.

Step 3: Detailed Explanation:
Aniline normally has a strongly activating $-NH_2$ group, which is ortho/para directing.
However, in the presence of anhydrous $AlCl_3$, the lone pair on nitrogen coordinates with $AlCl_3$ forming a complex or salt:

$$C_6H_5NH_2 + AlCl_3 \rightarrow C_6H_5NH_2 \cdot AlCl_3$$

This coordination removes the electron-donating ability of the $-NH_2$ group.
As a result, the benzene ring becomes deactivated toward electrophilic substitution.
Due to this deactivation, the usual ortho/para directing influence is lost.
Under these conditions, substitution does not give *o*- or *p*-methylaniline experimentally.

Hence:
Assertion (A) is true because ortho and para products are not formed.
Reason (R) is also true because salt formation with $AlCl_3$ deactivates the ring and explains the observation.

Step 4: Final Answer:
Both the Assertion and Reason are true, and the Reason correctly explains the Assertion.

Quick Tip

In Friedel–Crafts reactions, strongly basic groups like $-NH_2$ form complexes with $AlCl_3$.
Such complex formation deactivates the aromatic ring and alters the usual directing effects.
Always check Lewis acid–base interactions before predicting products.

---

81. Chlorophyll extracted from the crushed green leaves was dissolved in water to make 2 L solution of Mg of concentration 48 ppm. The number of atoms of Mg in this solution is $x \times 10^{20}$ atoms. The value of x is ____ .

(A) 12
(B) 18
(C) 24
(D) 48

Correct Answer: (C) 24

Solution:

Step 1: Understanding the Question:
We are given concentration of Mg in ppm and total volume. We must find the total number of Mg atoms.

Step 2: Key Formula:
ppm = mg L^{-1} (for dilute aqueous solutions)
Number of atoms = $\frac{\text{mass}}{\text{molar mass}} \times N_A$

Step 3: Calculation:
Mass of Mg = $48 \times 2 = 96$ mg = 96×10^{-3} g
Moles of Mg = $\frac{96 \times 10^{-3}}{24} = 4 \times 10^{-3}$ mol
Atoms = $4 \times 10^{-3} \times 6.02 \times 10^{23} \approx 24 \times 10^{20}$

Step 4: Final Answer:
$x = 24$

> Quick Tip
>
> 1 ppm $\approx$ 1 mg L^{-1} for dilute solutions. Always convert mg to grams before mole calculation.

---

82. A mixture of hydrogen and oxygen contains 40% hydrogen by mass when the pressure is 2.2 bar. The partial pressure of hydrogen is ____ bar.

(A) 1.2
(B) 1.6
(C) 2.0
(D) 2.2

Correct Answer: (C) 2.0

Solution:

Step 1: Understanding the Question:
The mixture composition is given in terms of mass percentage.
To calculate partial pressure, mole fraction must be used.

Step 2: Key Formula or Approach:
Partial pressure = Mole fraction $\times$ Total pressure.

Step 3: Detailed Explanation:
Assume total mass of mixture = 100 g.

Mass of $H_2 = 40$ g $\Rightarrow$ moles of $H_2 = \dfrac{40}{2} = 20$.

Mass of $O_2 = 60$ g $\Rightarrow$ moles of $O_2 = \dfrac{60}{32} = 1.875$.

Total moles $= 20 + 1.875 = 21.875$.

Mole fraction of H_2:

$$X_{H_2} = \frac{20}{21.875} \approx 0.914$$

Partial pressure of H_2:

$$P_{H_2} = 0.914 \times 2.2 \approx 2.0 \text{ bar}$$

Step 4: Final Answer:
Partial pressure of hydrogen = 2.0 bar.

> **Quick Tip**
>
> Partial pressure depends on mole fraction, not mass fraction.

---

83. The wavelength of an electron and a neutron will become equal when the velocity of the electron is x times the velocity of neutron. The value of x is ____ .

(A) 918
(B) 1250
(C) 1758
(D) 3200

Correct Answer: (C) 1758

Solution:

Step 1: Understanding the Question:
The condition for equal wavelengths of particles is applied.

Step 2: Key Formula or Approach:
de Broglie relation:

$$\lambda = \frac{h}{mv}$$

Step 3: Detailed Explanation:
For equal wavelengths:

$$m_e v_e = m_n v_n$$

$$\frac{v_e}{v_n} = \frac{m_n}{m_e}$$

Substitute values:

$$x = \frac{1.6 \times 10^{-27}}{9.1 \times 10^{-31}} \approx 1758$$

Step 4: Final Answer:
$x = 1758$.

Quick Tip

Equal wavelength ⇒ equal momentum.
Velocity is inversely proportional to mass.

---

84. 2.4 g coal is burnt in a bomb calorimeter. Temperature rises from 298 K to 300 K. The enthalpy change is $-x\ kJ\ mol^{-1}$. The value of x is ____ .

(A) 100
(B) 150
(C) 200
(D) 400

Correct Answer: (C) 200

Solution:

Step 1: Understanding the Question:
Combustion occurs at constant volume using a bomb calorimeter.

Step 2: Key Formula or Approach:
Heat absorbed:

$$q = C\Delta T$$

Step 3: Detailed Explanation:
Temperature rise:

$$\Delta T = 300 - 298 = 2 \text{ K}$$

$$q = 20 \times 2 = 40 \text{ kJ}$$

Moles of carbon:

$$n = \frac{2.4}{12} = 0.2$$

Enthalpy change per mole:

$$\Delta H = \frac{40}{0.2} = 200 \text{ kJ mol}^{-1}$$

Step 4: Final Answer:
$x = 200$.

Quick Tip

For solid combustion, $\Delta n_g = 0$, hence $\Delta H \approx \Delta U$.

---

85. 800 mL of 0.5 M HNO_3 is heated, volume becomes half and 11.5 g acid evaporates. Final molarity is $x \times 10^{-2} M$. Find x.

(A) 36
(B) 48
(C) 54
(D) 72

Correct Answer: (C) 54

Solution:

Step 1: Understanding the Question:
Heating causes volume reduction and loss of solute.

Step 2: Key Formula or Approach:
Molarity = moles of solute / volume of solution.

Step 3: Detailed Explanation:
Initial moles:

$$0.5 \times 0.8 = 0.4$$

Evaporated moles:

$$\frac{11.5}{63} \approx 0.1825$$

Remaining moles:

$$0.4 - 0.1825 = 0.2175$$

Final volume:

$$\frac{800}{2} = 400 \text{ mL} = 0.4 \text{ L}$$

Final molarity:

$$\frac{0.2175}{0.4} = 0.54375 \approx 54 \times 10^{-2}$$

Step 4: Final Answer:
$x = 54$.

Quick Tip

If solute loss is mentioned explicitly, subtract it before recalculating molarity.

86. Given $K = 2 \times 10^{15}$ for $Cu + 2Ag^+ \rightleftharpoons Cu^{2+} + 2Ag$. Find K for $\frac{1}{2}Cu^{2+} + Ag \rightleftharpoons \frac{1}{2}Cu + Ag^+ = x \times 10^{-8}$.

(A) 1
(B) 2
(C) 4
(D) 8

Correct Answer: (B) 2

Solution:

Step 1: Understanding the Question:
The reaction is reversed and multiplied by $\frac{1}{2}$.

Step 2: Key Formula or Approach:
Reverse reaction $\Rightarrow 1/K$.
Multiplying equation by $n \Rightarrow K^n$.

Step 3: Detailed Explanation:

$$K' = (2 \times 10^{15})^{-1/2}$$

$$K' \approx 2 \times 10^{-8}$$

Step 4: Final Answer:
$x = 2$.

Quick Tip

Reverse $\Rightarrow 1/K$, multiply equation by $n \Rightarrow K^n$.

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87. The amount of charge in Faraday required to obtain one mole of iron from Fe_3O_4 is ____ .

(A) 2
(B) 3
(C) 4
(D) 8

Correct Answer: (B) 3

Solution:

Step 1: Understanding the Question:
Iron exists in mixed oxidation states in Fe_3O_4.

Step 3: Detailed Explanation:
$Fe_3O_4 = FeO \cdot Fe_2O_3$.

Average oxidation state:

$$\frac{2+6}{3} = \frac{8}{3}$$

Electrons required per iron atom $= \frac{8}{3} \approx 3$.

Step 4: Final Answer:
Required charge = 3 Faraday.

> Quick Tip
>
> Fe_3O_4 is a mixed oxide: $FeO \cdot Fe_2O_3$.

---

88. For $A \to 2B + C$, half-life is 100 s at 0.5 M and 50 s at 1.0 M. The order of reaction is ____ .

(A) 0
(B) 1
(C) 2
(D) 3

Correct Answer: (C) 2

Solution:

Step 1: Understanding the Question:
Half-life changes with concentration.

Step 2: Key Formula or Approach:

$$t_{1/2} \propto [A]^{1-n}$$

Step 3: Detailed Explanation:

$$\frac{100}{50} = (0.5)^{1-n}$$

$$2 = 2^{n-1} \Rightarrow n = 2$$

Step 4: Final Answer:
Order of reaction = 2.

Quick Tip

Doubling concentration halves half-life ⇒ second-order reaction.

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89. Difference between spin-only magnetic moments of $[Co(H_2O)_6]Cl_2$ and $[Cr(H_2O)_6]Cl_3$ is ____ .

(A) 1
(B) 2
(C) 0
(D) 3

Correct Answer: (C) 0

Solution:

Step 1: Understanding the Question:
Magnetic moment depends on number of unpaired electrons.

Step 3: Detailed Explanation:
Both complexes contain 3 unpaired electrons.

$$\mu = \sqrt{n(n+2)}$$

Thus, magnetic moments are equal.

Step 4: Final Answer:
Difference = 0.

Quick Tip

Same number of unpaired electrons ⇒ same magnetic moment.

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90. In sunlight, benzene reacts with Cl_2 to give product X. Number of hydrogens in X is ____ .

(A) 3
(B) 5
(C) 6
(D) 12

Correct Answer: (C) 6

Solution:

Step 1: Understanding the Question:
Reaction conditions determine the reaction type.

Step 3: Detailed Explanation:
In sunlight, benzene undergoes addition reaction with chlorine.
Product formed is benzene hexachloride (BHC): $C_6H_6Cl_6$.

Number of hydrogen atoms remains unchanged.

Step 4: Final Answer:
Number of hydrogens = 6.

Quick Tip

Cl_2/Sunlight $\Rightarrow$ Addition (BHC).
Cl_2/$FeCl_3$ $\Rightarrow$ Substitution.