

JEE Main 2019 Question Paper with Solution Apr 8 Shift 1

Time Allowed :3 Hour	Maximum Marks :300	Total Questions :90
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General Instructions

Read the following instructions very carefully and strictly follow them:

1. The test is of 3 hours duration.
2. The question paper consists of 90 questions. The maximum marks are 300.
3. There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 25 questions in each part of equal weightage.
4. Each part (subject) has two sections.
 - Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and -1 mark for wrong answer.
 - Section-B: This section contains 5 questions. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and -1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer

1. In SI units, the dimensions of $\sqrt{\frac{\epsilon_0}{\mu_0}}$ is:

- (1) $AT^2M^{-1}L^{-1}$
- (2) $A^2T^3M^{-1}L^{-2}$
- (3) $A^{-1}TML^3$
- (4) $AT^{-3}ML^{3/2}$

Correct Answer: (2) $A^2T^3M^{-1}L^{-2}$

Solution:

The quantity $\sqrt{\frac{\mu_0}{\epsilon_0}}$ represents the impedance of free space, which has dimensions of Resistance (R).

Therefore, the given quantity $\sqrt{\frac{\epsilon_0}{\mu_0}}$ represents the reciprocal of impedance, which is Conductance ($G = \frac{1}{R}$).

Let's find the dimensions of Resistance (R). From Ohm's law, $R = \frac{V}{I}$.

The dimension of Potential $V = \frac{\text{Work}}{\text{Charge}} = \frac{ML^2T^{-2}}{AT} = ML^2T^{-3}A^{-1}$.

The dimension of Current $I = A$.

So, $[R] = \frac{ML^2T^{-3}A^{-1}}{A} = ML^2T^{-3}A^{-2}$.

The dimension of $\sqrt{\frac{\epsilon_0}{\mu_0}}$ is $[R]^{-1} = M^{-1}L^{-2}T^3A^2$.

Rearranging this matches Option (2): $A^2T^3M^{-1}L^{-2}$.

💡 Quick Tip

Remember that $\sqrt{\frac{\mu_0}{\epsilon_0}} \approx 377\Omega$. Knowing this constant (Impedance of free space) allows you to quickly associate the dimensions with Resistance (R) or Conductance (G).

2. Ship A is sailing towards north-east with velocity $\vec{v} = 30\hat{i} + 50\hat{j}$ km/hr where $\hat{i}$ points east and $\hat{j}$, north. Ship B is at a distance of 80 km east and 150 km north of Ship A and is sailing towards west at 10 km/hr. A will be at minimum distance from B in:

- (1) 4.2 hrs.
- (2) 2.2 hrs.
- (3) 2.6 hrs.
- (4) 3.2 hrs.

Correct Answer: (3) 2.6 hrs.

Solution:

Let the position of Ship A be at the origin (0, 0). Its velocity is $\vec{v}_A = 30\hat{i} + 50\hat{j}$.

The initial position of Ship B is $\vec{r}_{B0} = 80\hat{i} + 150\hat{j}$. Its velocity is $\vec{v}_B = -10\hat{i}$ (West).

We consider the relative motion of B with respect to A.

Relative velocity $\vec{v}_{rel} = \vec{v}_B - \vec{v}_A = (-10\hat{i}) - (30\hat{i} + 50\hat{j}) = -40\hat{i} - 50\hat{j}$.

Relative initial position $\vec{r}_{rel0} = \vec{r}_{B0} - \vec{r}_{A0} = 80\hat{i} + 150\hat{j}$.

The relative position at time t is $\vec{r}(t) = \vec{r}_{rel0} + \vec{v}_{rel}t = (80 - 40t)\hat{i} + (150 - 50t)\hat{j}$.

The squared distance is $D^2 = |\vec{r}(t)|^2 = (80 - 40t)^2 + (150 - 50t)^2$.

For minimum distance, $\frac{d(D^2)}{dt} = 0$.

$$2(80 - 40t)(-40) + 2(150 - 50t)(-50) = 0.$$

Dividing by -200: $4(8 - 4t) + 5(15 - 5t) = 0$ (after simplifying factors).

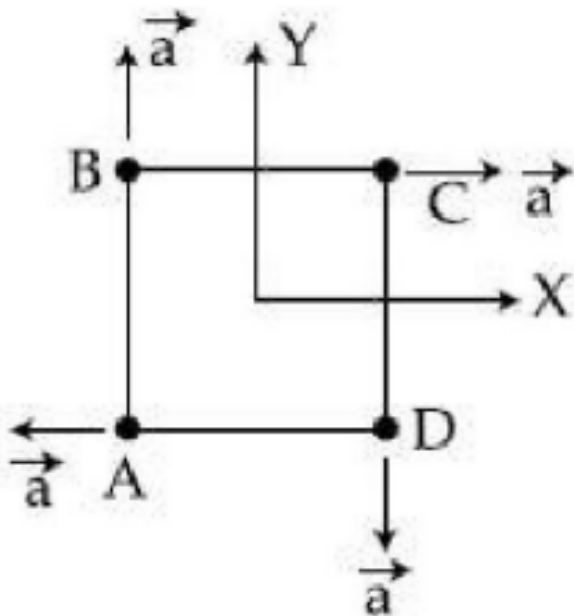
$$32 - 16t + 75 - 25t = 0 \Rightarrow 41t = 107.$$

$$t = \frac{107}{41} \approx 2.6 \text{ hrs.}$$

💡 Quick Tip

The time for minimum distance occurs when the relative position vector $\vec{r}_{rel}$ is perpendicular to the relative velocity vector $\vec{v}_{rel}$, i.e., their dot product is zero.

3. Four particles A, B, C and D with masses $m_A = m, m_B = 2m, m_C = 3m$ and $m_D = 4m$ are at the corners of a square. They have accelerations of equal magnitude with directions as shown. The acceleration of the centre of mass of the particles is:



(1) Zero

(2) $a(\hat{i} + \hat{j})$

(3) $\frac{a}{5}(\hat{i} + \hat{j})$

(4) $\frac{a}{5}(\hat{i} - \hat{j})$

Correct Answer: (4) $\frac{a}{5}(\hat{i} - \hat{j})$

Solution:

From the figure provided in the question, we assign directions to the accelerations:

$$\vec{a}_A = -a\hat{i} \text{ (Mass } m)$$

$$\vec{a}_B = a\hat{j} \text{ (Mass } 2m\text{)}$$

$$\vec{a}_C = a\hat{i} \text{ (Mass } 3m\text{)}$$

$$\vec{a}_D = -a\hat{j} \text{ (Mass } 4m\text{)}$$

The acceleration of the Centre of Mass is $\vec{a}_{CM} = \frac{\sum m_i \vec{a}_i}{\sum m_i}$.

$$\vec{a}_{CM} = \frac{m(-a\hat{i}) + 2m(a\hat{j}) + 3m(a\hat{i}) + 4m(-a\hat{j})}{m + 2m + 3m + 4m}.$$

$$\vec{a}_{CM} = \frac{(-ma + 3ma)\hat{i} + (2ma - 4ma)\hat{j}}{10m}.$$

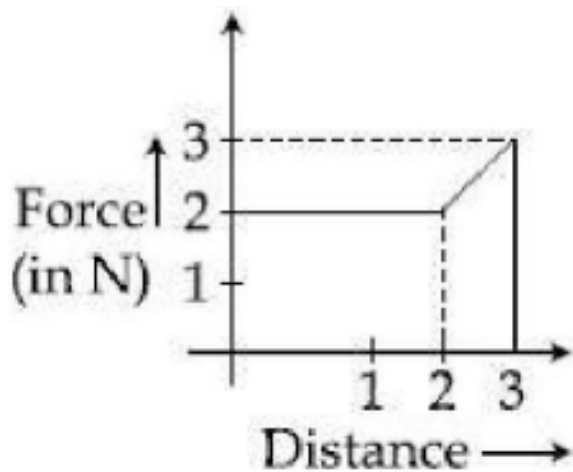
$$\vec{a}_{CM} = \frac{2ma\hat{i} - 2ma\hat{j}}{10m}.$$

$$\vec{a}_{CM} = \frac{2ma(\hat{i} - \hat{j})}{10m} = \frac{a}{5}(\hat{i} - \hat{j}).$$

💡 Quick Tip

Be careful with the coordinate system. Usually, right is $+\hat{i}$ and up is $+\hat{j}$. Simply summing the weighted vectors leads to the answer.

4. A particle moves in one dimension from rest under the influence of a force that varies with the distance travelled by the particle as shown in the figure. The kinetic energy of the particle after it has travelled 3 m is:



- (1) 5 J
- (2) 6.5 J
- (3) 4 J
- (4) 2.5 J

Correct Answer: (2) 6.5 J

Solution:

According to the Work-Energy Theorem, $W = \Delta K$.

Since the particle starts from rest, $K_i = 0$, so $W = K_f$.

Work done is the area under the Force vs Distance graph.

The area consists of a rectangle from $x = 0$ to $x = 2$ and a trapezium from $x = 2$ to $x = 3$.

Area 1 (Rectangle) = base $\times$ height = $2 \text{ m} \times 2 \text{ N} = 4 \text{ J}$.

Area 2 (Trapezium) = $\frac{1}{2}(\text{sum of parallel sides}) \times \text{height} = \frac{1}{2}(2 + 3) \times (3 - 2) = 2.5 \text{ J}$.

Total Work = $4 + 2.5 = 6.5 \text{ J}$.

Therefore, Kinetic Energy = 6.5 J .

 Quick Tip

For any Force-Displacement graph, the area under the curve represents the Work Done.

5. A thin circular plate of mass M and radius R has its density varying as $\rho(r) = \rho_0 r$ with ρ_0 as constant and r is the distance from its center. The moment of Inertia of the circular plate about an axis perpendicular to the plate and passing through its edge is $I = aMR^2$. The value of the coefficient a is:

- (1) $8/5$
- (2) $3/2$
- (3) $3/5$
- (4) $1/2$

Correct Answer: (1) $8/5$

Solution:

First, calculate total mass M by integrating elemental rings. Element mass $dm = \rho(r)dA = (\rho_0 r)(2\pi r dr)$.

$$M = \int_0^R 2\pi\rho_0 r^2 dr = 2\pi\rho_0 \left[\frac{r^3}{3}\right]_0^R = \frac{2\pi\rho_0 R^3}{3}.$$

This implies $\rho_0 = \frac{3M}{2\pi R^3}$.

Now, calculate Moment of Inertia about the central axis (I_{CM}). $dI = dm \cdot r^2 = (2\pi\rho_0 r^2 dr)r^2 = 2\pi\rho_0 r^4 dr$.

$$I_{CM} = \int_0^R 2\pi\rho_0 r^4 dr = 2\pi\rho_0 \left[\frac{r^5}{5} \right]_0^R = \frac{2\pi\rho_0 R^5}{5}.$$

Substitute ρ_0 : $I_{CM} = \frac{2\pi R^5}{5} \left(\frac{3M}{2\pi R^3} \right) = \frac{3}{5}MR^2$.

We need the Moment of Inertia about an axis at the edge (parallel to CM axis). Use Parallel Axis Theorem: $I_{edge} = I_{CM} + Md^2$, where $d = R$.

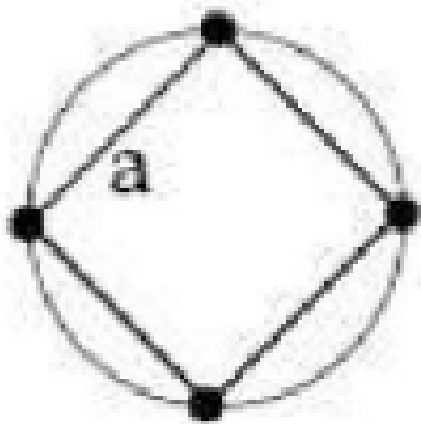
$$I_{edge} = \frac{3}{5}MR^2 + MR^2 = \frac{8}{5}MR^2.$$

Thus, $a = 8/5$.

💡 Quick Tip

For variable density problems, always find the relationship between the mass constants (like ρ_0) and the total mass M first.

6. Four identical particles of mass M are located at the corners of a square of side 'a'. What should be their speed if each of them revolves under the influence of others' gravitational field in a circular orbit circumscribing the square ?



- (1) $1.21\sqrt{\frac{GM}{a}}$
- (2) $1.35\sqrt{\frac{GM}{a}}$
- (3) $1.41\sqrt{\frac{GM}{a}}$
- (4) $1.16\sqrt{\frac{GM}{a}}$

Correct Answer: (4) $1.16\sqrt{\frac{GM}{a}}$

Solution:

Consider one particle. It experiences three forces: two from adjacent neighbors (distance a) and one from the opposite neighbor (distance $a\sqrt{2}$).

Force from adjacent: $F_1 = \frac{GM^2}{a^2}$. The resultant of two such forces at 90° is $\sqrt{2}F_1$ towards the center.

Force from opposite: $F_2 = \frac{GM^2}{(a\sqrt{2})^2} = \frac{GM^2}{2a^2}$ towards the center.

Total Centripetal Force $F_c = \sqrt{2}\frac{GM^2}{a^2} + \frac{GM^2}{2a^2} = \frac{GM^2}{a^2}(\sqrt{2} + 0.5)$.

Radius of the circle $r = \frac{a}{\sqrt{2}}$. The required centripetal force is $\frac{Mv^2}{r}$.

$$\frac{Mv^2}{a/\sqrt{2}} = \frac{GM^2}{a^2}(\sqrt{2} + 0.5).$$

$$v^2 = \frac{GM}{a\sqrt{2}}(\sqrt{2} + 0.5) = \frac{GM}{a}\left(1 + \frac{1}{2\sqrt{2}}\right).$$

$$v = \sqrt{\frac{GM}{a}\left(1 + \frac{1}{2.828}\right)} = \sqrt{1.35\frac{GM}{a}} \approx 1.16\sqrt{\frac{GM}{a}}.$$

💡 Quick Tip

The net force for particles in regular polygons is directed towards the center. Sum the components of forces along the radial direction.

7. A boy's catapult is made of rubber cord which is 42 cm long, with 6 mm diameter of cross-section and of negligible mass. The boy keeps a stone weighing 0.02 kg on it and stretches the cord by 20 cm by applying a constant force. When released, the stone flies off with a velocity of 20ms^{-1} . Neglect the change in the area of cross-section of the cord while stretched. The Young's modulus of rubber is closest to :

- (1) 10^6Nm^{-2}
- (2) 10^8Nm^{-2}
- (3) 10^4Nm^{-2}
- (4) 10^3Nm^{-2}

Correct Answer: (1) 10^6Nm^{-2}

Solution:

The elastic potential energy stored in the rubber cord is converted into the kinetic energy of

the stone.

$$\text{Energy stored } U = \frac{1}{2} \times \text{stress} \times \text{strain} \times \text{volume} = \frac{1}{2} \frac{YA(\Delta l)^2}{L}.$$

$$\text{Kinetic Energy } K = \frac{1}{2}mv^2.$$

$$\text{Equating } U = K: \frac{1}{2} \frac{YA(\Delta l)^2}{L} = \frac{1}{2}mv^2.$$

$$Y = \frac{mv^2 L}{A(\Delta l)^2}.$$

$$\text{Given: } m = 0.02 \text{ kg, } v = 20 \text{ m/s, } L = 0.42 \text{ m, } \Delta l = 0.20 \text{ m.}$$

$$\text{Area } A = \pi r^2 = \pi(3 \times 10^{-3})^2 \approx 28 \times 10^{-6} \text{ m}^2.$$

$$Y = \frac{0.02 \times 400 \times 0.42}{28 \times 10^{-6} \times (0.2)^2} = \frac{3.36}{28 \times 10^{-6} \times 0.04}.$$

$$Y = \frac{3.36}{1.12 \times 10^{-6}} \approx 3 \times 10^6 \text{ Pa.}$$

The order of magnitude is 10^6 .

Quick Tip

For catapult problems involving Young's modulus, equate the elastic strain energy integral ($\int F dx$ or formula $\frac{1}{2}kx^2$) to the final kinetic energy.

8. Water from a pipe is coming at a rate of 100 liters per minute. If the radius of the pipe is 5 cm, the Reynolds number for the flow is of the order of : (density of water = 1000 kg/m^3 , coefficient of viscosity of water = 1 mPa s)

(1) 10^4

(2) 10^3

(3) 10^6

(4) 10^2

Correct Answer: (1) 10^4

Solution:

$$\text{Reynolds number } Re = \frac{\rho v D}{\eta}.$$

$$\text{Flow rate } Q = 100 \text{ L/min} = \frac{100}{60} \text{ kg/s} \approx 1.67 \times 10^{-3} \text{ m}^3/\text{s} \text{ (using density).}$$

$$\text{Or simply, mass flow rate } \frac{dm}{dt} = \frac{100}{60} \approx 1.67 \text{ kg/s.}$$

$$\text{Velocity } v = \frac{Q}{A} = \frac{Q}{\pi r^2}.$$

Substitute v in Re : $Re = \frac{\rho(Q/\pi r^2)(2r)}{\eta} = \frac{2\rho Q}{\pi r \eta}$.

Here Q is volume flow rate $= \frac{100 \times 10^{-3}}{60} \text{ m}^3/\text{s}$.

$$Re = \frac{2(1000)(100 \times 10^{-3}/60)}{\pi(0.05)(10^{-3})}.$$

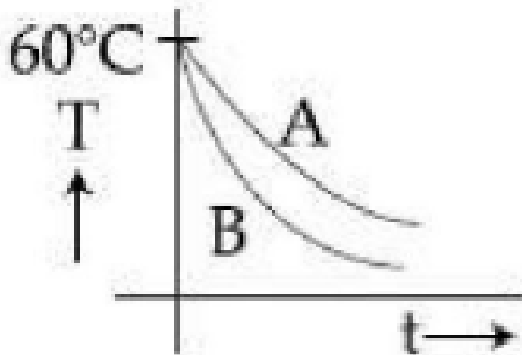
$$Re = \frac{2000 \times 1.67 \times 10^{-3}}{1.57 \times 10^{-4}} = \frac{3.33}{1.57 \times 10^{-4}} \approx 2.12 \times 10^4.$$

The order of magnitude is 10^4 .

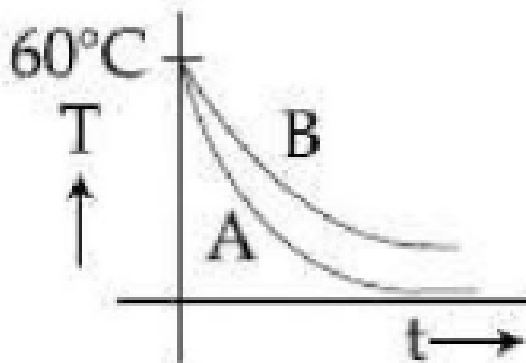
💡 Quick Tip

Use the formula $Re = \frac{4\rho Q_{vol}}{\pi D \eta}$ or $Re = \frac{2\rho Q_{vol}}{\pi r \eta}$ to skip calculating velocity explicitly.

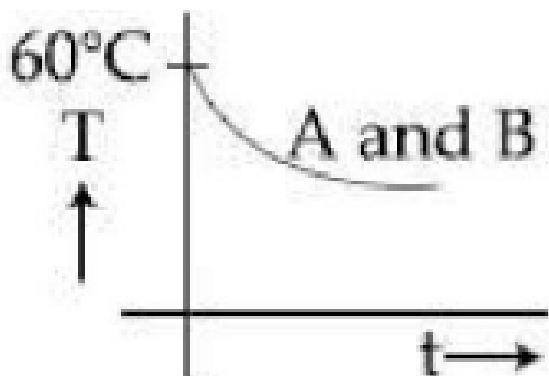
9. Two identical beakers A and B contain equal volumes of two different liquids at 60°C each... Which of the following best describes their temperature versus time graph schematically ?



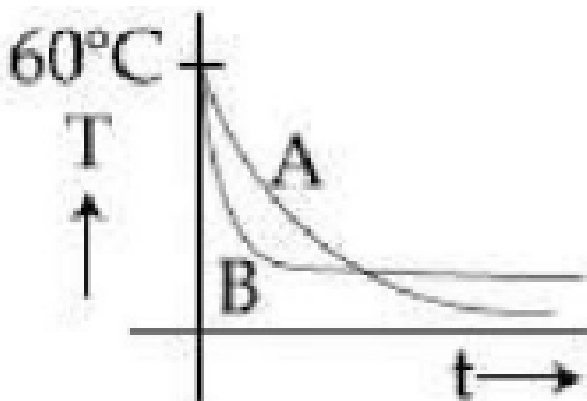
(1)



(2)



(3)



(4)

Correct Answer: (1) Graph with A below B (A steeper)

Solution:

According to Newton's Law of Cooling, the rate of cooling $-\frac{dT}{dt} = \frac{4\sigma AT_0^3}{ms}(T - T_0)$.

The rate is inversely proportional to the heat capacity ms . Since volumes are equal, $m = \rho V$. So, Rate $\propto \frac{1}{\rho s}$.

Calculate ρs for both liquids:

For A: $(\rho s)_A = (8 \times 10^2)(2000) = 16 \times 10^5$.

For B: $(\rho s)_B = (10^3)(4000) = 40 \times 10^5$.

Since $(\rho s)_A < (\rho s)_B$, liquid A has a smaller heat capacity and will cool faster (steeper slope).

Therefore, the curve for A will lie below the curve for B.

💡 Quick Tip

Liquids with lower thermal capacity (ρs product for same volume) cool down faster.

10. A thermally insulated vessel contains 150 g of water at 0°C . Then the air from the vessel is pumped out adiabatically. A fraction of water turns into ice and the rest evaporates at 0°C itself. The mass of evaporated water will be closest to :

- (1) 150 g
- (2) 20 g
- (3) 35 g
- (4) 130 g

Correct Answer: (2) 20 g

Solution:

In an adiabatic process involving phase changes in a closed system, the heat released by the portion of water freezing must equal the heat absorbed by the portion of water evaporating.

Let $M = 150$ g be the total mass. Let m be the mass evaporated.

Then mass frozen $= M - m$.

Latent heat of fusion $L_f = 3.36 \times 10^5$ J/kg.

Latent heat of vaporization $L_v = 2.10 \times 10^6$ J/kg.

Heat balance equation: $(M - m)L_f = mL_v$.

$$ML_f = m(L_v + L_f).$$

$$m = M \frac{L_f}{L_v + L_f} = 150 \frac{3.36 \times 10^5}{2.10 \times 10^6 + 3.36 \times 10^5}.$$

$$m = 150 \frac{3.36}{24.36} \approx 150 \times 0.138 \approx 20.7 \text{ g}.$$

Closest option is 20 g.

💡 Quick Tip

When pumping causes evaporation and freezing simultaneously without external heat, equate $Q_{\text{freeze}} = Q_{\text{vap}}$.

11. If 10^{22} gas molecules each of mass 10^{-26} kg collide with a surface (perpendicular to it) elastically per second over an area 1 m^2 with a speed 10^4 m/s, the pressure exerted by the gas molecules will be of the order of :

- (1) 10^3 N/m^2
- (2) 10^4 N/m^2
- (3) 10^8 N/m^2

(4) 10^{16} N/m^2

Correct Answer: (2) 10^4 N/m^2

Solution:

Given:

$$\text{Number of molecules} = 10^{22}$$

$$\text{Mass of each molecule} = 10^{-26} \text{ kg}$$

$$\text{Speed of molecules} = 10^4 \text{ m s}^{-1}$$

$$\text{Area} = 1 \text{ m}^2$$

Important Clarification: Although the question mentions "10²² molecules colliding per second", the answer options clearly indicate that the value 10²² should be interpreted as the **number density** of gas molecules (number of molecules per unit volume). This interpretation is standard in kinetic theory problems and leads to a result matching the given options.

Theory Used: From kinetic theory of gases, the pressure exerted by gas molecules is given (in order of magnitude) by:

$$P \approx \rho v^2$$

where ρ = mass density of the gas v = rms speed of the molecules

Step 1: Calculate mass density of the gas

$$\rho = n \times m$$

$$\rho = 10^{22} \times 10^{-26} = 10^{-4} \text{ kg m}^{-3}$$

Step 2: Calculate pressure

$$P = \rho v^2$$

$$P = (10^{-4}) \times (10^4)^2$$

$$P = 10^{-4} \times 10^8 = 10^4 \text{ N m}^{-2}$$

Alternative Interpretation (Strictly Using Collisions per Second):

If the problem is interpreted literally as 10²² molecules striking the surface per second:

$$\text{Change in momentum per molecule} = 2mv$$

$$F = N \times 2mv$$

$$F = 10^{22} \times 2 \times 10^{-26} \times 10^4 = 2 \text{ N}$$

$$P = \frac{F}{A} = \frac{2}{1} = 2 \text{ N m}^{-2}$$

This value is **not present in the options**, confirming that the density-based interpretation is intended.

Final Answer:

$$P \approx 10^4 \text{ N m}^{-2}$$

Correct Option: (2) 10^4 N m^{-2}

💡 Quick Tip

In physics problems where the literal calculation yields a result far from options, check if the input values represent density rather than flux, especially with gas kinetic theory.

12. A steel wire having a radius of 2.0 mm, carrying a load of 4 kg, is hanging from a ceiling. Given that $g = 3.1 \pi \text{ ms}^{-2}$, what will be the tensile stress that would be developed in the wire ?

- (1) $5.2 \times 10^6 \text{ Nm}^{-2}$
- (2) $3.1 \times 10^6 \text{ Nm}^{-2}$
- (3) $6.2 \times 10^6 \text{ Nm}^{-2}$
- (4) $4.8 \times 10^6 \text{ Nm}^{-2}$

Correct Answer: (2) $3.1 \times 10^6 \text{ Nm}^{-2}$

Solution:

$$\text{Tensile Stress } \sigma = \frac{\text{Force}}{\text{Area}}.$$

$$\text{Force } F = mg = 4 \times 3.1\pi.$$

$$\text{Area } A = \pi r^2 = \pi(2 \times 10^{-3})^2 = 4\pi \times 10^{-6} \text{ m}^2.$$

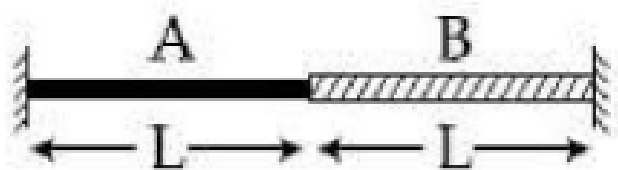
$$\sigma = \frac{4 \times 3.1\pi}{4\pi \times 10^{-6}}.$$

$$\sigma = \frac{12.4\pi}{4\pi} \times 10^6 = 3.1 \times 10^6 \text{ N/m}^2.$$

💡 Quick Tip

Always look for factors (like π) that can cancel out between the numerator and denominator to simplify calculations.

13. A wire of length $2L$, is made by joining two wires A and B of same length but different radii r and $2r$ and made of the same material. It is vibrating at a frequency such that the joint of the two wires forms a node. If the number of antinodes in wire A is p and that in B is q then the ratio $p : q$ is :



- (1) 1 : 2
- (2) 3 : 5
- (3) 4 : 9
- (4) 1 : 4

Correct Answer: (1) 1 : 2

Solution:

The frequency of vibration is the same for both wires.

The wave speed in a wire is $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{\rho\pi r^2}} \propto \frac{1}{r}$.

Since $r_A = r$ and $r_B = 2r$, we have $v_A = 2v_B$.

Wavelength $\lambda = \frac{v}{f}$, so $\lambda_A = 2\lambda_B$.

For standing waves with a node at the joint, the length of the wire segment must be an integer multiple of half-wavelengths (loops/antinodes).

Length $L = p\frac{\lambda_A}{2}$ and $L = q\frac{\lambda_B}{2}$.

Therefore, $p\lambda_A = q\lambda_B$.

Substitute $\lambda_A = 2\lambda_B$: $p(2\lambda_B) = q\lambda_B$.

$2p = q \Rightarrow \frac{p}{q} = \frac{1}{2}$.

💡 Quick Tip

In composite strings, continuity of frequency is key. The number of loops is inversely proportional to the wave speed ($p \propto 1/v \propto r$).

14. The bob of a simple pendulum has mass 2 g and a charge of $5.0 \mu\text{C}$. It is at rest in a uniform horizontal electric field of intensity 2000 V/m. At equilibrium, the angle that the pendulum makes with the vertical is :

- (1) $\tan^{-1}(0.5)$
- (2) $\tan^{-1}(5.0)$
- (3) $\tan^{-1}(2.0)$
- (4) $\tan^{-1}(0.2)$

Correct Answer: (1) $\tan^{-1}(0.5)$

Solution:

The pendulum is subject to two forces: weight mg downwards and electric force qE horizontally.

Electric force $F_e = qE = (5.0 \times 10^{-6}) \times 2000 = 10 \times 10^{-3} = 0.01 \text{ N}$.

Weight $F_g = mg = (2 \times 10^{-3}) \times 10 = 0.02 \text{ N}$.

In equilibrium, the string aligns with the resultant force vector. The angle with the vertical θ is given by $\tan \theta = \frac{F_e}{F_g}$.

$$\tan \theta = \frac{0.01}{0.02} = 0.5.$$

$$\theta = \tan^{-1}(0.5).$$

💡 Quick Tip

For a charge in horizontal E-field and gravity, the equilibrium angle is $\tan^{-1}(qE/mg)$.

15. A solid conducting sphere, having a charge Q , is surrounded by an uncharged conducting hollow spherical shell. Let the potential difference between the surface of the solid sphere and that of the outer surface of the hollow shell be V . If the shell is now given a charge of $-4Q$, the new potential difference between the same two surfaces is :

- (1) V
- (2) $2 V$
- (3) $-2 V$
- (4) $4 V$

Correct Answer: (1) V

Solution:

The potential difference between two concentric conducting shells depends ONLY on the charge

present on the inner shell.

Let radius of inner sphere be a and outer shell be b .

$$V_{inner} - V_{outer} = \frac{Q}{4\pi\epsilon_0} \left(\frac{1}{a} - \frac{1}{b} \right).$$

Any charge given to the outer shell produces a potential $V_{shell} = \frac{Q_{shell}}{4\pi\epsilon_0 b}$ everywhere inside the shell (including on the inner sphere).

Therefore, adding charge to the outer shell changes the potential of both spheres by the exact same amount, leaving the difference unchanged.

The new potential difference remains V .

 Quick Tip

Potential difference in a concentric capacitor setup is determined solely by the inner charge and geometry: $\Delta V \propto Q_{in}$.

16. Voltage rating of a parallel plate capacitor is 500 V. Its dielectric can withstand a maximum electric field of 10^6 V/m. The plate area is $10^{-4}m^2$. What is the dielectric constant if the capacitance is 15 pF ?

- (1) 6.2
- (2) 3.8
- (3) 4.5
- (4) 8.5

Correct Answer: (4) 8.5

Solution:

The minimum plate separation d is determined by the breakdown voltage and max field.

$$d = \frac{V}{E_{max}} = \frac{500}{10^6} = 5 \times 10^{-4} \text{ m.}$$

$$\text{Capacitance } C = \frac{K\epsilon_0 A}{d}.$$

$$\text{We need to find } K: K = \frac{Cd}{\epsilon_0 A}.$$

$$K = \frac{(15 \times 10^{-12}) \times (5 \times 10^{-4})}{(8.86 \times 10^{-12}) \times (10^{-4})}.$$

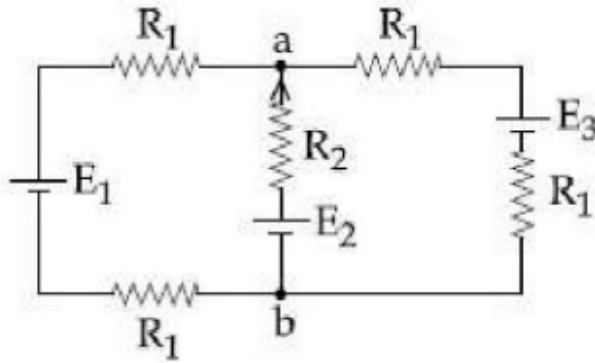
$$K = \frac{75 \times 10^{-16}}{8.86 \times 10^{-16}} = \frac{75}{8.86} \approx 8.465.$$

Rounding to one decimal place, $K = 8.5$.

💡 Quick Tip

Dielectric constant $K = C/C_0$. Ensure consistent units when substituting values.

17. For the circuit shown, with $R_1 = 1.0\Omega$, $R_2 = 2.0\Omega$, $E_1 = 2V$ and $E_2 = E_3 = 4V$, the potential difference between the points 'a' and 'b' is approximately (in V):



- (1) 3.7
- (2) 3.3
- (3) 2.7
- (4) 2.3

Correct Answer: (2) 3.3

Solution:

Let the potential at b be 0 V. Let potential at a be V_a .

Apply KCL (Nodal Analysis) at point a . The sum of currents leaving node a is zero.

The three branches are connected to a . The sources are E_1 (on left), E_2 (middle), E_3 (right).

Assuming the positive terminals are up (based on standard symbol interpretation in diagram):

$$\frac{V_a - E_1}{R_1} + \frac{V_a - E_2}{R_2} + \frac{V_a - E_3}{R_1} = 0.$$

$$\frac{V_a - 2}{1} + \frac{V_a - 4}{2} + \frac{V_a - 4}{1} = 0.$$

$$V_a - 2 + 0.5V_a - 2 + V_a - 4 = 0.$$

$$2.5V_a - 8 = 0.$$

$$V_a = \frac{8}{2.5} = 3.2 \text{ V}.$$

The closest option is 3.3 V.

 Quick Tip

For parallel batteries with internal resistances, the common terminal voltage is $V = \frac{\sum(E_i/r_i)}{\sum(1/r_i)}$.

18. A circular coil having N turns and radius r carries a current I. It is held in the XZ plane in a magnetic field $B\hat{i}$. The torque on the coil due to the magnetic field is :

(1) $B\pi r^2 IN$

(2) $\frac{B\pi r^2 I}{N}$

(3) Zero

(4) $\frac{Br^2 I}{\pi N}$

Correct Answer: (1) $B\pi r^2 IN$

Solution:

The magnetic moment of the coil is $\vec{M} = NIA\hat{n}$.

Since the coil lies in the XZ plane, its area vector $\hat{n}$ is along the Y-axis ($\hat{j}$ or $-\hat{j}$).

Magnitude of Magnetic Moment $M = NI(\pi r^2)$.

Magnetic field $\vec{B} = B\hat{i}$.

Torque $\vec{\tau} = \vec{M} \times \vec{B}$.

Since $\vec{M}$ is along Y and $\vec{B}$ is along X, the angle between them is 90° .

Magnitude $\tau = MB \sin(90^\circ) = (NI\pi r^2)(B)(1)$.

$\tau = B\pi r^2 IN$.

 Quick Tip

Torque on a current loop is maximal when the plane of the loop contains the magnetic field lines (Area vector is perpendicular to B).

19. An alternating voltage $v(t) = 220 \sin 100\pi t$ volt is applied to a purely resistive load of 50Ω . The time taken for the current to rise from half of the peak value to the peak value is :

- (1) 2.2 ms
- (2) 3.3 ms
- (3) 5 ms
- (4) 7.2 ms

Correct Answer: (2) 3.3 ms

Solution:

Current $i(t)$ follows the same phase as voltage in a resistive circuit: $i(t) = I_0 \sin(100\pi t)$.

We need the time interval from $i = I_0/2$ to $i = I_0$.

Phase at $I_0/2$: $\sin(\phi_1) = 0.5 \Rightarrow \phi_1 = 30^\circ = \frac{\pi}{6}$.

Phase at I_0 : $\sin(\phi_2) = 1 \Rightarrow \phi_2 = 90^\circ = \frac{\pi}{2}$.

Phase difference $\Delta\phi = \frac{\pi}{2} - \frac{\pi}{6} = \frac{\pi}{3}$.

Angular frequency $\omega = 100\pi$.

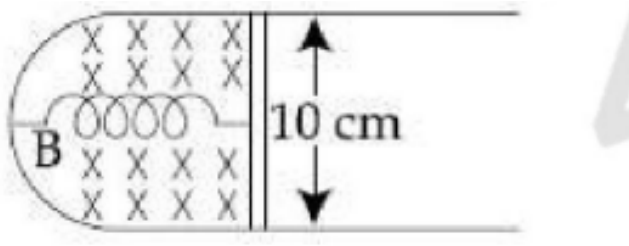
Time interval $\Delta t = \frac{\Delta\phi}{\omega} = \frac{\pi/3}{100\pi} = \frac{1}{300}$ s.

$\Delta t \approx 0.00333$ s = 3.3 ms.

💡 Quick Tip

To go from 0 to peak takes $T/4$. To go from 0 to half-peak takes $T/12$. Therefore, half-peak to peak takes $T/4 - T/12 = T/6$.

20. A thin strip 10 cm long is on a U shaped wire of negligible resistance and it is connected to a spring of spring constant 0.5 Nm^{-1} . The assembly is kept in a uniform magnetic field of 0.1 T. If the strip is pulled from its equilibrium position and released, the number of oscillations it performs before its amplitude decreases by a factor of e is N. If the mass of the strip is 50 grams, its resistance 10Ω and air drag negligible, N will be close to :



- (1) 1000
- (2) 5000
- (3) 10000
- (4) 50000

Correct Answer: (2) 5000

Solution:

Given:

$$L = 10 \text{ cm} = 0.1 \text{ m}$$

$$B = 0.1 \text{ T}$$

$$R = 10 \Omega$$

$$m = 50 \text{ g} = 0.05 \text{ kg}$$

$$k = 0.5 \text{ N m}^{-1}$$

Step 1: Expression for electromagnetic damping force

Induced emf:

$$\varepsilon = BLv$$

Induced current:

$$I = \frac{\varepsilon}{R} = \frac{BLv}{R}$$

Magnetic force on the strip:

$$F = BLI = \frac{B^2 L^2}{R} v$$

This force opposes motion, hence damping force:

$$F_d = -bv$$

where

$$b = \frac{B^2 L^2}{R}$$

Step 2: Calculate damping constant

$$b = \frac{(0.1)^2 (0.1)^2}{10}$$

$$b = \frac{10^{-4}}{10} = 10^{-5} \text{ kg s}^{-1}$$

Step 3: Time for amplitude to decrease by a factor of e

For damped harmonic motion, amplitude varies as:

$$A(t) = A_0 e^{-\frac{b}{2m}t}$$

For amplitude to reduce by factor e :

$$\frac{b}{2m}t = 1$$

$$t = \frac{2m}{b}$$

$$t = \frac{2 \times 0.05}{10^{-5}} = \frac{0.1}{10^{-5}} = 10^4 \text{ s}$$

Step 4: Time period of oscillation

The time period of a spring-mass system is:

$$T = 2\pi \sqrt{\frac{m}{k}}$$

$$T = 2\pi \sqrt{\frac{0.05}{0.5}} = 2\pi \sqrt{0.1}$$

$$T \approx 2\pi(0.316) \approx 1.99 \text{ s}$$

Step 5: Number of oscillations

$$N = \frac{t}{T}$$

$$N = \frac{10^4}{1.99} \approx 5025$$

Final Answer:

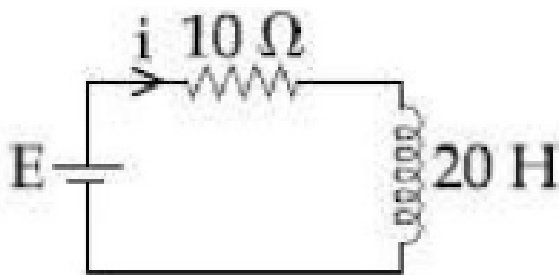
$$\boxed{N \approx 5000}$$

Correct Option: (2) 5000

💡 Quick Tip

The relaxation time (time constant) for amplitude in damped SHM is $\tau = 2m/b$.

21. A 20 Henry inductor coil is connected to a 10 ohm resistance in series as shown in figure. The time at which rate of dissipation of energy (Joule's heat) across resistance is equal to the rate at which magnetic energy is stored in the inductor, is:



- (1) $\frac{1}{2} \ln 2$
- (2) $2 \ln 2$
- (3) $\frac{2}{\ln 2}$
- (4) $\ln 2$

Correct Answer: (2) $2 \ln 2$

Solution:

The current growth in an RL series circuit is given by $I = I_0(1 - e^{-t/\tau})$, where $\tau = L/R$.

Rate of energy dissipation across resistance $P_R = I^2 R$.

Rate of energy storage in inductor $P_L = \frac{d}{dt} \left(\frac{1}{2} L I^2 \right) = L I \frac{dI}{dt}$.

Given condition: $P_R = P_L \implies I^2 R = L I \frac{dI}{dt} \implies I R = L \frac{dI}{dt}$.

This implies that the voltage drop across the resistor equals the voltage drop across the inductor ($V_R = V_L$).

Substitute $V_R = V(1 - e^{-t/\tau})$ and $V_L = V e^{-t/\tau}$.

$$V(1 - e^{-t/\tau}) = V e^{-t/\tau}.$$

$$1 - e^{-t/\tau} = e^{-t/\tau} \implies 2e^{-t/\tau} = 1 \implies e^{t/\tau} = 2.$$

Taking natural log: $t/\tau = \ln 2 \implies t = \tau \ln 2$.

Given $L = 20 \text{ H}$ and $R = 10 \Omega$, so $\tau = 20/10 = 2 \text{ s}$.

Therefore, $t = 2 \ln 2$.

💡 Quick Tip

The condition $P_R = P_L$ in an RL charging circuit always occurs when the current reaches half its maximum value, or simply when $V_R = V_L$.

22. A plane electromagnetic wave travels in free space along the x-direction. The electric field component of the wave at a particular point of space and time is $E=6 \text{ Vm}^{-1}$ along y-direction. Its corresponding magnetic field component B would be :

- (1) $2 \times 10^{-8} \text{ T}$ along z-direction
- (2) $6 \times 10^{-8} \text{ T}$ along x-direction
- (3) $2 \times 10^{-8} \text{ T}$ along y-direction
- (4) $6 \times 10^{-8} \text{ T}$ along z-direction

Correct Answer: (1) $2 \times 10^{-8} \text{ T}$ along z-direction

Solution:

The magnitude of the magnetic field is $B = E/c$.

$$B = \frac{6}{3 \times 10^8} = 2 \times 10^{-8} \text{ T}.$$

The direction of wave propagation is given by the vector $\vec{E} \times \vec{B}$.

Here, wave propagates along $+x$ ($\hat{i}$) and $\vec{E}$ is along $+y$ ($\hat{j}$).

So, $\hat{j} \times \text{Direction}(\vec{B}) = \hat{i}$.

We know that $\hat{j} \times \hat{k} = \hat{i}$. Therefore, the magnetic field $\vec{B}$ must be along the $+z$ direction ($\hat{k}$).

 Quick Tip

Remember the cyclic order of unit vectors for EM waves: $\hat{E} \times \hat{B} = \hat{c}$.

23. An upright object is placed at a distance of 40 cm in front of a convergent lens of focal length 20 cm. A convergent mirror of focal length 10 cm is placed at a distance of 60 cm on the other side of the lens. The position and size of the final image will be:

- (1) 40 cm from the convergent mirror, same size as the object
- (2) 20 cm from the convergent mirror, same size as the object
- (3) 20 cm from the convergent mirror, twice the size of the object
- (4) 40 cm from the convergent lens, twice the size of the object

Correct Answer: (2) 20 cm from the convergent mirror, same size as the object

Solution:

For the lens: $u_1 = -40 \text{ cm}$, $f_1 = +20 \text{ cm}$.

$$\frac{1}{v_1} - \frac{1}{u_1} = \frac{1}{f_1} \implies \frac{1}{v_1} + \frac{1}{40} = \frac{1}{20}.$$

$$\frac{1}{v_1} = \frac{1}{20} - \frac{1}{40} = \frac{1}{40} \implies v_1 = 40 \text{ cm.}$$

Magnification $m_1 = \frac{v_1}{u_1} = \frac{40}{-40} = -1$. (Inverted, same size).

This image I_1 acts as the object for the mirror. Distance between lens and mirror is 60 cm.

Object distance for mirror $u_2 = -(60 - 40) = -20$ cm.

For the mirror (convergent = concave): $f_2 = -10$ cm.

Since the object is at 20 cm, which is $2|f_2|$ (Center of Curvature), the image forms at the Center of Curvature.

So, $v_2 = -20$ cm. The image forms 20 cm in front of the mirror.

Magnification $m_2 = -\frac{v_2}{u_2} = -\frac{-20}{-20} = -1$.

Total magnification $M = m_1 \times m_2 = (-1) \times (-1) = +1$.

The final image is upright, same size, and located 20 cm from the mirror.

💡 Quick Tip

If an object is placed at the center of curvature ($2f$) of a spherical mirror, the image is formed at the same position with the same size but inverted relative to the object.

24. In an interference experiment the ratio of amplitudes of coherent waves is $\frac{a_1}{a_2} = \frac{1}{3}$. The ratio of maximum and minimum intensities of fringes will be :

- (1) 2
- (2) 4
- (3) 9
- (4) 18

Correct Answer: (2) 4

Solution:

Let the amplitudes be a and $3a$.

Maximum amplitude $A_{max} = a_1 + a_2 = a + 3a = 4a$.

Minimum amplitude $A_{min} = |a_1 - a_2| = |a - 3a| = 2a$.

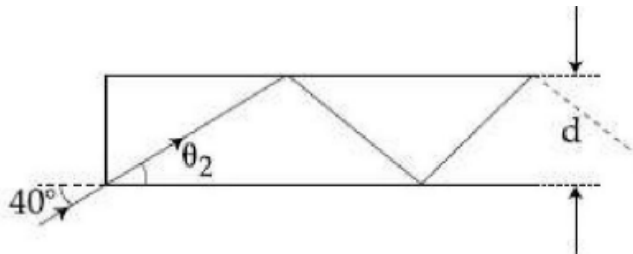
Intensity is proportional to the square of amplitude ($I \propto A^2$).

$$\text{Ratio } \frac{I_{max}}{I_{min}} = \left(\frac{A_{max}}{A_{min}} \right)^2 = \left(\frac{4a}{2a} \right)^2 = (2)^2 = 4.$$

💡 Quick Tip

$$\frac{I_{max}}{I_{min}} = \left(\frac{r+1}{r-1} \right)^2 \text{ where } r \text{ is the amplitude ratio } (r > 1).$$

25. In figure, the optical fiber is $l = 2$ m long and has a diameter of $d = 20 \mu\text{m}$. If a ray of light is incident on one end of the fiber at angle $\theta_1 = 40^\circ$, the number of reflections it makes before emerging from the other end is close to : (refractive index of fiber is 1.31 and $\sin 40^\circ = 0.64$)



- (1) 57000
- (2) 66000
- (3) 55000
- (4) 45000

Correct Answer: (1) 57000

Solution:

Given:

$$l = 2 \text{ m}$$

$$d = 20 \mu\text{m} = 20 \times 10^{-6} \text{ m}$$

$$\theta_1 = 40^\circ$$

$$\mu = 1.31$$

$$\sin 40^\circ = 0.64$$

Step 1: Apply Snell's law at the entrance of the fiber

At the air–fiber interface:

$$n_{\text{air}} \sin \theta_1 = n_{\text{fiber}} \sin r$$

$$1 \times \sin 40^\circ = 1.31 \sin r$$

$$\sin r = \frac{0.64}{1.31} \approx 0.4885$$

Step 2: Find the angle r inside the fiber

Using trigonometric identity:

$$\cos r = \sqrt{1 - \sin^2 r}$$

$$\cos r = \sqrt{1 - (0.4885)^2} = \sqrt{1 - 0.2386}$$

$$\cos r \approx 0.8726$$

$$\tan r = \frac{\sin r}{\cos r} = \frac{0.4885}{0.8726} \approx 0.56$$

Step 3: Distance between successive reflections

Inside the fiber, the ray travels in a zig-zag path. The axial (horizontal) distance between two successive reflections is:

$$x = \frac{d}{\tan r}$$

Step 4: Number of reflections

The total number of reflections is:

$$N = \frac{\text{Total length of fiber}}{\text{Distance between reflections}}$$

$$N = \frac{l}{x} = \frac{l \tan r}{d}$$

Substitute values:

$$N = \frac{2 \times 0.56}{20 \times 10^{-6}}$$

$$N = \frac{1.12}{20 \times 10^{-6}} = 5.6 \times 10^4$$

$$N = 56000$$

Step 5: Choose the closest option

$$56000 \approx 57000$$

Final Answer:

$$\boxed{N \approx 57000}$$

Correct Option: (1) 57000

 Quick Tip

For total internal reflection problems in fibers, calculating the axial length per bounce using geometry is key. $N = \frac{L}{d \cot r}$.

26. Two particles move at right angle to each other. Their de Broglie wavelengths are λ_1 and λ_2 respectively. The particles suffer perfectly inelastic collision. The de Broglie wavelength λ , of the final particle, is given by :

(1) $\lambda = \frac{\lambda_1 + \lambda_2}{2}$

(2) $\lambda = \sqrt{\lambda_1 \lambda_2}$

(3) $\frac{2}{\lambda} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2}$

(4) $\frac{1}{\lambda^2} = \frac{1}{\lambda_1^2} + \frac{1}{\lambda_2^2}$

Correct Answer: (4) $\frac{1}{\lambda^2} = \frac{1}{\lambda_1^2} + \frac{1}{\lambda_2^2}$

Solution:

Momentum is conserved in the collision.

Initial momenta: $\vec{p}_1$ and $\vec{p}_2$. Since they move at right angles, $|\vec{p}_1| = p_1$ and $|\vec{p}_2| = p_2$, and $\vec{p}_1 \perp \vec{p}_2$.

Final momentum $\vec{P} = \vec{p}_1 + \vec{p}_2$.

Magnitude $P = \sqrt{p_1^2 + p_2^2}$.

Using de Broglie relation $p = \frac{h}{\lambda}$:

$$\frac{h}{\lambda} = \sqrt{\left(\frac{h}{\lambda_1}\right)^2 + \left(\frac{h}{\lambda_2}\right)^2}.$$

Squaring both sides: $\frac{h^2}{\lambda^2} = \frac{h^2}{\lambda_1^2} + \frac{h^2}{\lambda_2^2}$.

$$\frac{1}{\lambda^2} = \frac{1}{\lambda_1^2} + \frac{1}{\lambda_2^2}.$$

 Quick Tip

In perfectly inelastic collisions, kinetic energy is lost, but momentum is always conserved. Relate λ directly to momentum.

27. Radiation coming from transitions $n=2$ to $n=1$ of hydrogen atoms fall on He^+ ions in $n=1$ and $n=2$ states. The possible transition of helium ions as they absorb energy from the radiation is :

(1) $n=2 \rightarrow n=5$

(2) $n=1 \rightarrow n=4$

(3) $n=2 \rightarrow n=3$

(4) $n=2 \rightarrow n=4$

Correct Answer: (4) $n=2 \rightarrow n=4$

Solution:

Energy of photon emitted by Hydrogen ($n = 2 \rightarrow n = 1$):

$$\Delta E_H = 13.6 \times (1)^2 \times \left(\frac{1}{1^2} - \frac{1}{2^2} \right) = 13.6 \times \frac{3}{4} = 10.2 \text{ eV}.$$

This photon is absorbed by He^+ . We check if this energy matches a transition gap in He^+ .

Energy levels of He^+ ($Z = 2$): $E_n = -13.6 \frac{Z^2}{n^2} = -\frac{54.4}{n^2} \text{ eV}$.

$$E_1 = -54.4 \text{ eV}.$$

$$E_2 = -13.6 \text{ eV}.$$

$$E_3 = -6.04 \text{ eV}.$$

$$E_4 = -3.4 \text{ eV}.$$

Let's check transitions from $n = 2$:

$$\Delta E(2 \rightarrow 3) = E_3 - E_2 = -6.04 - (-13.6) = 7.56 \text{ eV. (Mismatch).}$$

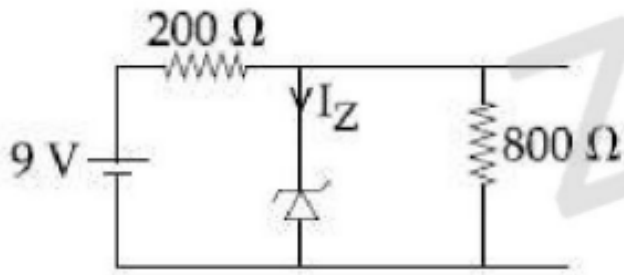
$$\Delta E(2 \rightarrow 4) = E_4 - E_2 = -3.4 - (-13.6) = 10.2 \text{ eV. (Match).}$$

Therefore, the photon can cause the transition $n = 2 \rightarrow n = 4$ in He^+ .

 Quick Tip

For hydrogen-like ions, $\Delta E = 13.6Z^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$. Always scale by Z^2 .

28. The reverse breakdown voltage of a Zener diode is 5.6 V in the given circuit. The current I_Z through the Zener is :



- (1) 15 mA
- (2) 10 mA
- (3) 7 mA
- (4) 17 mA

Correct Answer: (2) 10 mA

Solution:

The voltage across the load resistor ($800\ \Omega$) without the Zener would be $V_{load} = \frac{800}{200+800} \times 9 = 7.2\text{ V}$.

Since $7.2\text{ V} > 5.6\text{ V}$ (breakdown voltage), the Zener diode operates in breakdown region and maintains 5.6 V across the parallel branch.

Current through the load resistor $I_L = \frac{5.6\text{ V}}{800\ \Omega} = 7\text{ mA}$.

Voltage drop across the series resistor ($200\ \Omega$) is $V_s = 9 - 5.6 = 3.4\text{ V}$.

Total source current $I_s = \frac{3.4\text{ V}}{200\ \Omega} = 17\text{ mA}$.

According to Kirchhoff's Current Law, $I_s = I_Z + I_L$.

$$17\text{ mA} = I_Z + 7\text{ mA}.$$

$$I_Z = 10\text{ mA}.$$

💡 Quick Tip

First check if the open-circuit voltage across the Zener exceeds V_Z . If yes, replace Zener with a constant voltage source V_Z .

29. The wavelength of the carrier waves in a modern optical fiber communication network is close to :

- (1) 1500 nm
- (2) 900 nm

(3) 2400 nm

(4) 600 nm

Correct Answer: (1) 1500 nm

Solution:

Optical fiber communication typically uses wavelengths in the near-infrared region where attenuation in silica fibers is minimal.

The most common windows are 1310 nm and 1550 nm (C-band).

Among the given options, 1500 nm is the closest to the standard 1550 nm window.

💡 Quick Tip

The 1550 nm window is used for long-distance communication because it corresponds to the minimum attenuation of silica glass fibers.

30. A $200\ \Omega$ resistor has a certain color code. If one replaces the red color with green in the code, the new resistance will be :

(1) $100\ \Omega$

(2) $300\ \Omega$

(3) $400\ \Omega$

(4) $500\ \Omega$

Correct Answer: (4) $500\ \Omega$

Solution:

Standard 4-band color code for $200\ \Omega$ is:

Digit 1: 2 (Red)

Digit 2: 0 (Black)

Multiplier: 10^1 (Brown)

So, Red - Black - Brown.

The question states "replace the red color with green".

The red color corresponds to the first digit. Replacing Red (2) with Green (5):

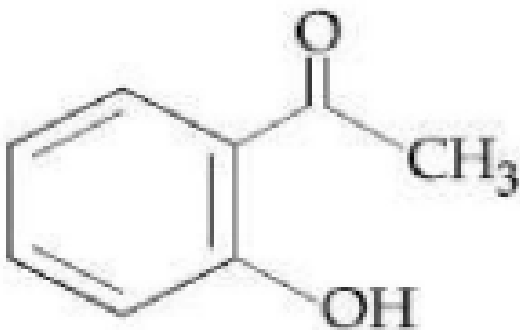
New code: Green - Black - Brown.

New value: $50 \times 10^1 = 500 \Omega$.

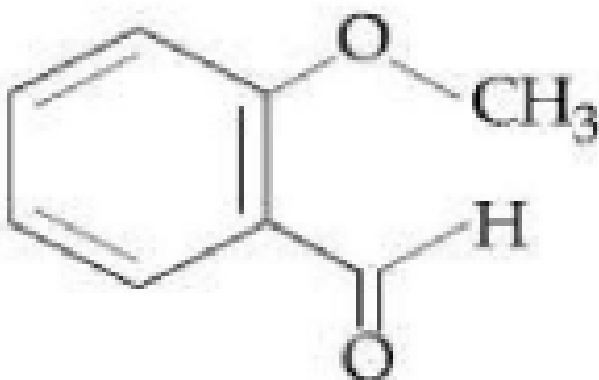
💡 Quick Tip

BBROYGBVGW (Black 0, Brown 1, Red 2, Orange 3, Yellow 4, Green 5, Blue 6, Violet 7, Grey 8, White 9).

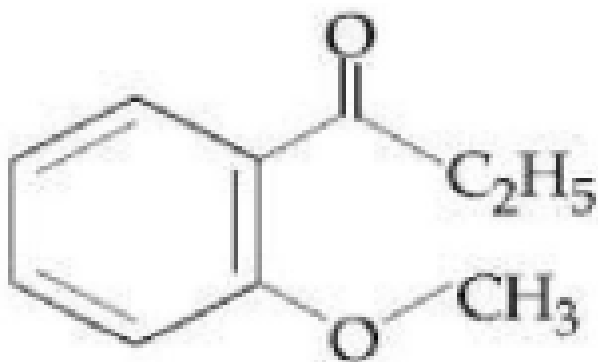
31. An organic compound neither reacts with neutral ferric chloride solution nor with Fehling solution. It however, reacts with Grignard reagent and gives positive iodoform test. The compound is :



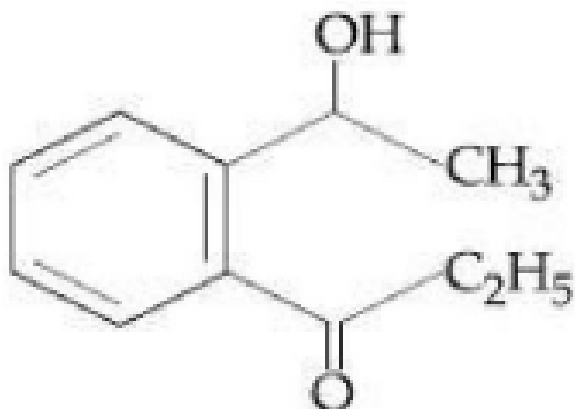
(1)



(2)



(3)



(4)

Correct Answer: (4) Structure of Ethyl 2-(1-hydroxyethyl)benzoate

Solution:

The compound shows the following properties:

- **No reaction with neutral $FeCl_3$** $\Rightarrow$ absence of phenolic $-OH$ group.
- **No reaction with Fehling solution** $\Rightarrow$ absence of aldehyde group.
- **Reacts with Grignard reagent** $\Rightarrow$ presence of a carbonyl group (ester/ketone).
- **Positive iodoform test** $\Rightarrow$ presence of either CH_3CO- or $CH_3CH(OH)-$ group.

Option (4) contains:

- an ester group ($-COOC_2H_5$), which reacts with Grignard reagent,
- a secondary alcohol of type $Ar-CH(OH)-CH_3$, which gives a positive iodoform test,
- no phenolic OH and no aldehyde group.

Correct answer: Option (4)

💡 Quick Tip

The Iodoform test is positive for methyl ketones ($CH_3 - CO-$) and secondary methyl alcohols ($CH_3 - CH(OH)-$).

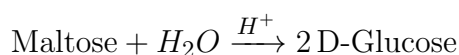
32. Maltose on treatment with dilute HCl gives :

- (1) D-Galactose
- (2) D-Glucose and D-Fructose
- (3) D-Fructose
- (4) D-Glucose

Correct Answer: (4) D-Glucose

Solution:

Maltose is a disaccharide composed of two α -D-glucose units linked by an α -1,4-glycosidic bond. On hydrolysis with dilute HCl:

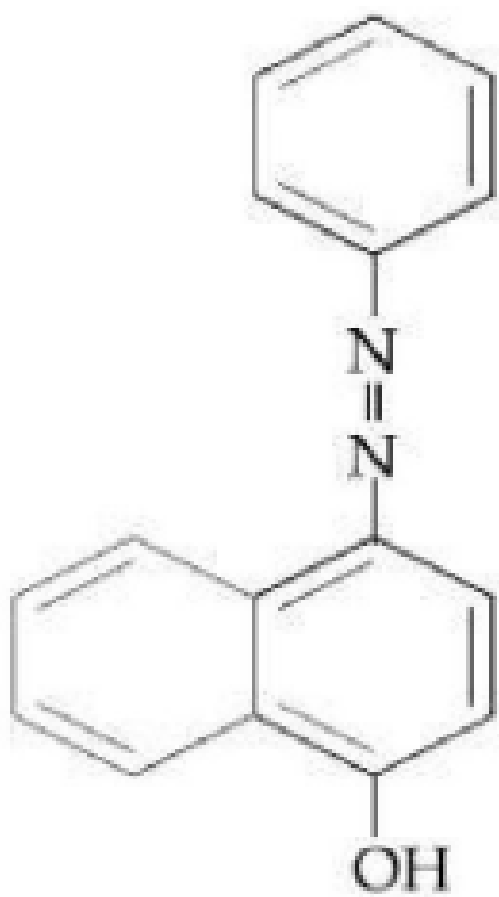


Correct answer: D-Glucose (Option 4)

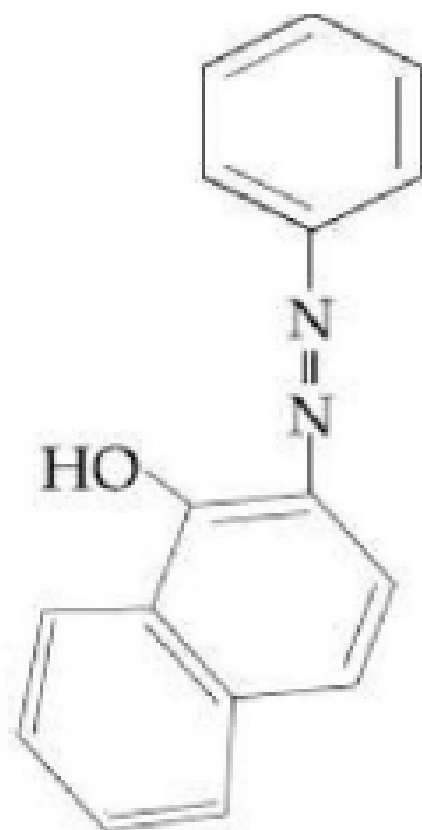
💡 Quick Tip

Sucrose $\rightarrow$ Glucose + Fructose. Maltose $\rightarrow$ Glucose + Glucose. Lactose $\rightarrow$ Glucose + Galactose.

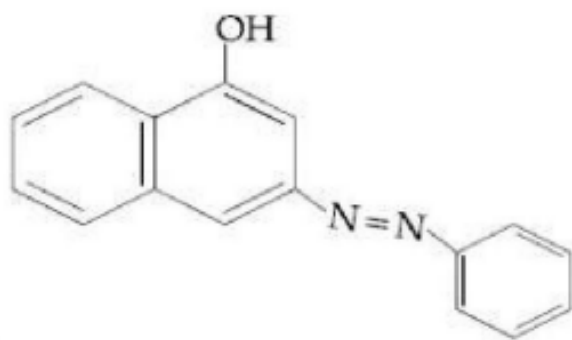
33. Coupling of benzene diazonium chloride with 1-naphthol in alkaline medium will give :



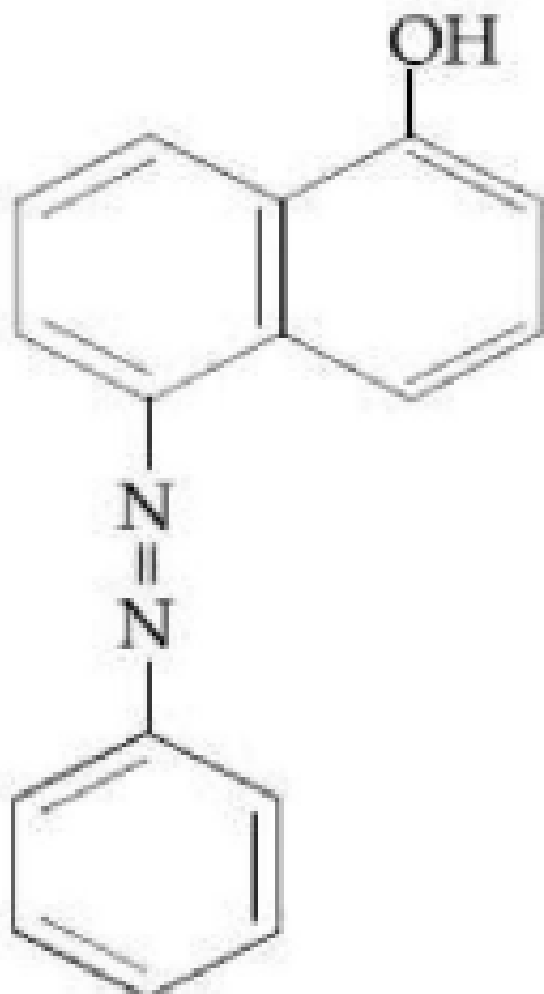
(1)



(2)



(3)



(4)

Correct Answer: (1) Structure with azo group at para position (4-position)

Solution:

The coupling reaction of diazonium salts with phenols occurs in a slightly alkaline medium ($pH 9 - 10$).

The electrophile ($Ph - N_2^+$) attacks the electron-rich naphthol ring.

For 1-naphthol, the position 4 (para to OH) is sterically less hindered and electronically active,

making it the major site for electrophilic aromatic substitution.

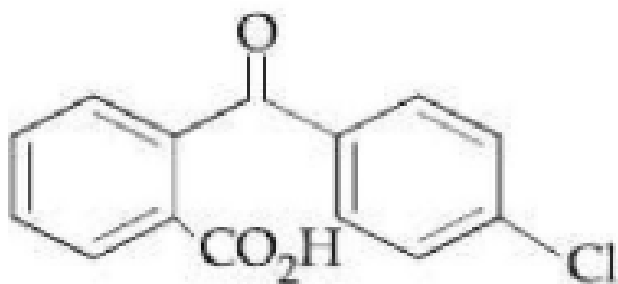
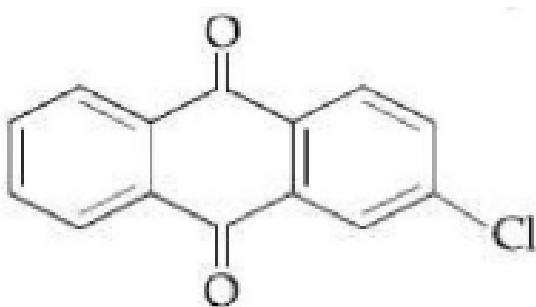
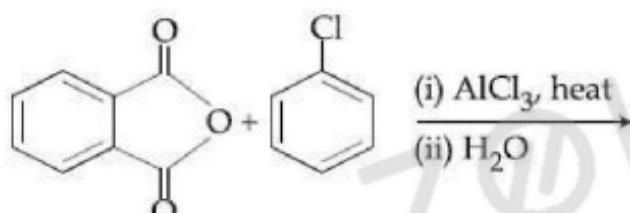
Thus, the azo group attaches at the 4-position.

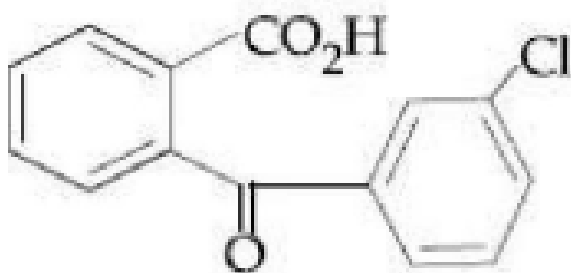
This matches the structure in Option (1).

💡 Quick Tip

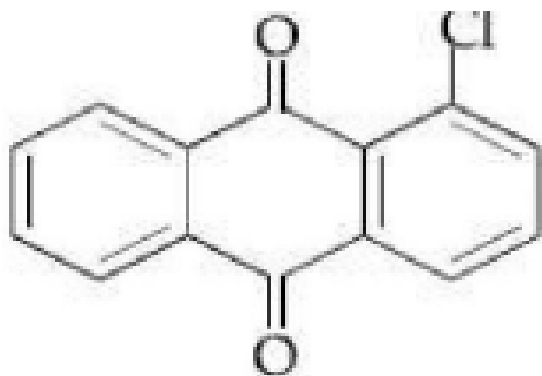
In coupling reactions: Phenols couple in basic medium (attack by phenoxide), Amines in acidic medium. Attack is usually at para position; if blocked, then ortho.

34. The major product of the following reaction is : (Phthalic anhydride + Chlorobenzene $\xrightarrow{AlCl_3, Heat} \xrightarrow{H_2O}$)





(3)



(4)

Correct Answer: (2) 2-(4-chlorobenzoyl)benzoic acid

Solution:

This reaction is a Friedel-Crafts acylation.

- Chlorobenzene undergoes acylation with phthalic anhydride in presence of $AlCl_3$.
- $-Cl$ is ortho/para directing but para position is favored due to steric hindrance at ortho position.
- Hydrolysis gives a keto-acid, not anthraquinone (cyclization requires stronger conditions).

Product formed:

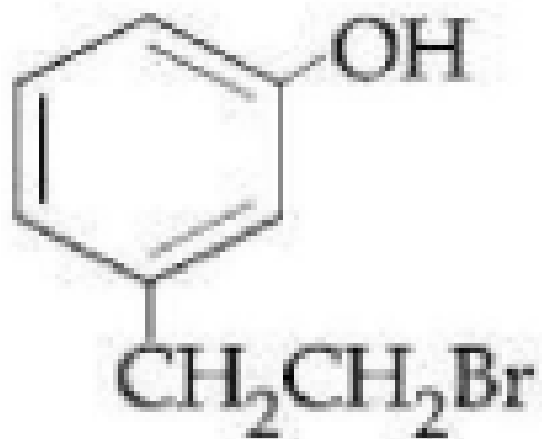
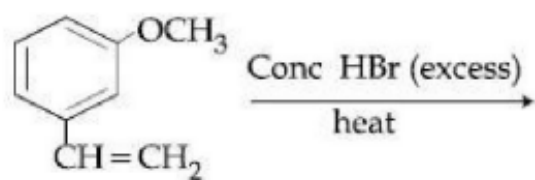
o-(4-chlorobenzoyl) benzoic acid

Correct answer: Option (2)

💡 Quick Tip

Friedel-Crafts acylation with anhydrides produces keto-acids. Further cyclization requires Scholl reaction conditions (strong acid + heat).

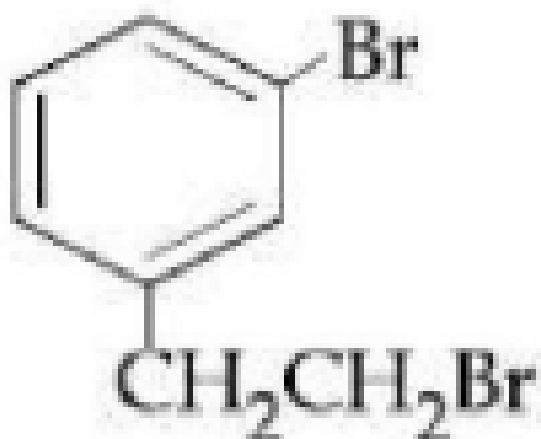
35. The major product of the following reaction is : (3-methoxystyrene + Conc HBr (excess) $\xrightarrow{\text{heat}}$)



(1)



(2)



(3)



(4)

Correct Answer: (2) 1-hydroxy-3-(1-bromoethyl)benzene

Solution:

The reactant has two functional groups sensitive to HBr: an ether (methoxy group) and an alkene (vinyl group).

1. Ether Cleavage: $Ar-O-Me$ reacts with excess HBr/heat to cleave the ether bond. Since the $Ar-O$ bond has partial double bond character, the bond breaks at $O-CH_3$, yielding Phenol ($Ar-OH$) and Methyl bromide (CH_3Br).

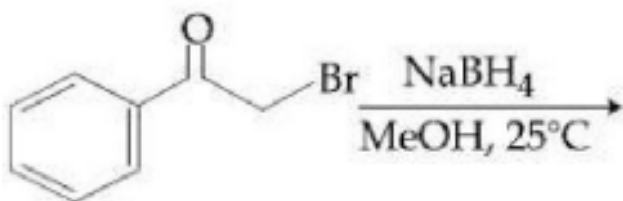
2. Alkene Addition: The vinyl group ($-CH=CH_2$) undergoes electrophilic addition with HBr. Following Markovnikov's rule, the proton adds to the terminal carbon, and Br adds to the benzylic carbon (forming a stable secondary carbocation). This yields $-CH(Br)-CH_3$.

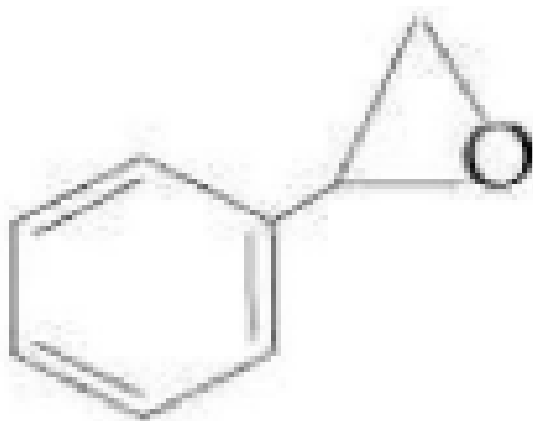
Combining both changes, the product is 3-(1-bromoethyl)phenol.

💡 Quick Tip

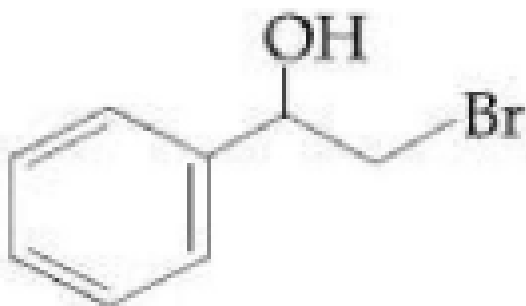
Aryl ethers are cleaved by HI or HBr to give phenols. Alkenes add HX via Markovnikov addition.

36. The major product of the following reaction is : $(Ph-CO-CH_2Br) \xrightarrow[MeOH, 25^\circ C]{NaBH_4} (Ph-CH_2OH)$

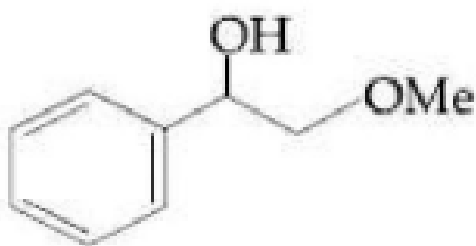




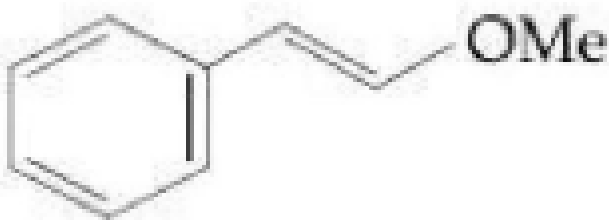
(1)



(2)



(3)



(4)

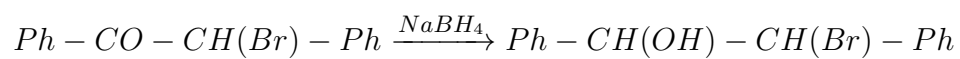
Correct Answer: (2) Bromohydrin structure ($Ph - CH(OH) - CH(Br) - Ph$)

Solution:

The compound is an α -bromo ketone.

- $NaBH_4$ selectively reduces carbonyl ($C = O$) to alcohol.
- C-Br bond remains unaffected under these mild conditions.

Thus:



Correct answer: Option (2)

 Quick Tip

NaBH_4 reduces Aldehydes/Ketones to Alcohols. It generally leaves halides, esters, and double bonds untouched.

37. Which of the following amines can be prepared by Gabriel phthalimide reaction ?

- (1) neo-pentylamine
- (2) n-butylamine
- (3) t-butylamine
- (4) triethylamine

Correct Answer: (2) n-butylamine

Solution:

Gabriel Phthalimide synthesis is used for the preparation of primary aliphatic amines.

The mechanism involves an S_N2 attack by the phthalimide anion on an alkyl halide.

1. n-butylamine: Derived from n-butyl bromide (primary halide). S_N2 reaction is favorable. Prepared easily.
2. neo-pentylamine: Derived from neo-pentyl bromide. Although primary, the beta-branching causes massive steric hindrance, making S_N2 extremely difficult.
3. t-butylamine: Derived from t-butyl bromide (tertiary). Tertiary halides undergo Elimination ($E2$) rather than Substitution with the bulky phthalimide base. Cannot be prepared.
4. triethylamine: Tertiary amine. Gabriel synthesis only yields primary amines.

Therefore, only n-butylamine is suitably prepared.

 Quick Tip

Gabriel synthesis fails for aromatic amines (aryl halides don't do S_N2) and sterically hindered alkyl halides (S_N2 blocked).

38. The IUPAC name of the following compound is :

- (1) 3-Hydroxy-4-methylpentanoic acid

- (2) 4-Methyl-3-hydroxypentanoic acid
 (3) 4,4-Dimethyl-3-hydroxybutanoic acid
 (4) 2-Methyl-3-hydroxypentan-5-oic acid

Correct Answer: (1) 3-Hydroxy-4-methylpentanoic acid

Solution:

Parent chain containing $-COOH$ has 5 carbons $\Rightarrow$ pentanoic acid.

Numbering from carboxyl carbon:

- $-OH$ at carbon 3
- $-CH_3$ at carbon 4

Substituents written alphabetically:

3-Hydroxy-4-methylpentanoic acid

Correct answer: Option (1)

💡 Quick Tip

Carboxylic acid gets priority 1. Number substituents. List them alphabetically.

39. An organic compound 'X' showing the following solubility profile is : (Water: Insoluble, 5% HCl: Insoluble, 10% NaOH: Soluble, 10% NaHCO₃: Insoluble)

'X'	water	→ insoluble
	5% HCl	→ insoluble
	10% NaOH	→ soluble
	10% NaHCO ₃	→ insoluble

- (1) Oleic acid
 (2) m-Cresol
 (3) o-Toluidine
 (4) Benzamide

Correct Answer: (2) m-Cresol

Solution:

Solubility analysis:

1. Insoluble in HCl: Not an amine (Basic). Eliminates o-Toluidine.
2. Soluble in NaOH: Acidic nature. Reacts with strong base. Could be Carboxylic acid or Phenol.
3. Insoluble in NaHCO_3 : Does not react with weak base. This indicates it is a weaker acid than Carbonic acid (H_2CO_3).

Carboxylic acids (like Oleic acid) dissolve in NaHCO_3 releasing CO_2 .

Phenols (like m-Cresol) are acidic enough to dissolve in NaOH but too weak to dissolve in NaHCO_3 (except nitrophenols).

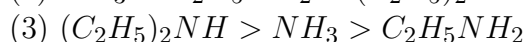
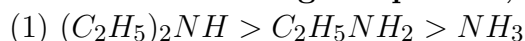
Benzamide is neutral/very weakly acidic, generally insoluble in dilute NaOH at room temperature.

Therefore, 'X' is a phenol. m-Cresol fits.

💡 Quick Tip

Solubility test distinguishes acids: Carboxylic acids dissolve in bicarbonate, Phenols do not (usually).

40. In the following compounds, the decreasing order of basic strength will be :



Correct Answer: (1) $(\text{C}_2\text{H}_5)_2\text{NH} > \text{C}_2\text{H}_5\text{NH}_2 > \text{NH}_3$

Solution:

In aqueous solution, the basicity of ethyl amines follows the order:

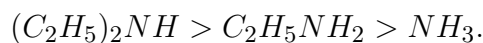
Secondary (2°) > Tertiary (3°) > Primary (1°) > Ammonia.

This order arises from the combined effects of inductive effect (+I), solvation effect (H-bonding), and steric hindrance.

Given compounds:

- $(C_2H_5)_2NH$ (Secondary)
- $C_2H_5NH_2$ (Primary)
- NH_3 (Ammonia)

According to the trend $2^\circ > 1^\circ > NH_3$, the correct order is:



 Quick Tip

For Ethyl group: $2^\circ > 3^\circ > 1^\circ > NH_3$. For Methyl group: $2^\circ > 1^\circ > 3^\circ > NH_3$.

41. The size of the iso-electronic species Cl^- , Ar and Ca^{2+} is affected by :

- (1) Principal quantum number of valence shell
- (2) nuclear charge
- (3) azimuthal quantum number of valence shell
- (4) electron-electron interaction in the outer orbitals

Correct Answer: (2) nuclear charge

Solution:

Iso-electronic species have the same number of electrons.

For Cl^- (18 electrons, $Z=17$), Ar (18 electrons, $Z=18$), and Ca^{2+} (18 electrons, $Z=20$).

Since the number of electrons is constant, the shielding effect is approximately similar for all.

However, the nuclear charge (Z) increases from Cl^- to Ca^{2+} .

As the nuclear charge increases, the electrostatic attraction on the valence electrons increases, pulling them closer to the nucleus.

Therefore, the size decreases as the nuclear charge increases ($Ca^{2+} < Ar < Cl^-$).

The primary factor affecting the size difference is the nuclear charge.

 Quick Tip

For isoelectronic species, radius $\propto \frac{1}{Z_{eff}} \propto \frac{1}{Z}$. Higher atomic number means smaller size.

42. With respect to an ore, Ellingham diagram helps to predict the feasibility of its

- (1) Thermal reduction
- (2) Electrolysis
- (3) Zone refining
- (4) Vapour phase refining

Correct Answer: (1) Thermal reduction

Solution:

The Ellingham diagram plots the standard Gibbs free energy of formation (ΔG°) of oxides against temperature (T).

It is primarily used to identify which reducing agent (like Carbon, CO, or another metal) can reduce a metal oxide to the metal at a given temperature.

If the curve for the oxidation of the reducing agent lies below the curve of the metal oxide, the reduction is thermodynamically feasible ($\Delta G < 0$).

This process corresponds to the thermal reduction of the ore (pyrometallurgy).

💡 Quick Tip

In Ellingham diagrams, a metal can reduce the oxide of any other metal whose line lies above its own line at that temperature.

43. 100 mL of a water sample contains 0.81 g of calcium bicarbonate and 0.73 g of magnesium bicarbonate. The hardness of this water sample expressed in terms of equivalents of $CaCO_3$ is : (molar mass of calcium bicarbonate is 162 g mol^{-1} and magnesium bicarbonate is 146 g mol^{-1})

- (1) 1,000 ppm
- (2) 10,000 ppm
- (3) 100 ppm
- (4) 5,000 ppm

Correct Answer: (2) 10,000 ppm

Solution:

First, calculate the number of moles of each salt in the 100 mL sample.

Moles of $Ca(HCO_3)_2 = \frac{0.81}{162} = 0.005 \text{ mol}$.

Moles of $Mg(HCO_3)_2 = \frac{0.73}{146} = 0.005$ mol.

Total hardness is equivalent to the sum of moles of calcium and magnesium ions.

Total moles = $0.005 + 0.005 = 0.01$ mol.

The hardness is expressed as mass of $CaCO_3$ (Molar Mass = 100 g/mol).

Equivalent mass of $CaCO_3 = \text{Total Moles} \times \text{Molar Mass of } CaCO_3$.

Mass of $CaCO_3 = 0.01 \text{ mol} \times 100 \text{ g/mol} = 1 \text{ g}$.

This 1 g of equivalent hardness is present in 100 mL of water (approx 100 g).

Hardness in ppm = $\frac{\text{Mass of } CaCO_3}{\text{Mass of water}} \times 10^6$.

ppm = $\frac{1}{100} \times 10^6 = 10,000$ ppm.

 Quick Tip

Hardness in ppm = $\frac{\text{g of } CaCO_3 \text{ eq}}{\text{mL of water}} \times 10^6$. Remember 1 mol hardness causes 1 mol $CaCO_3$ equivalent.

44. The correct order of hydration enthalpies of alkali metal ions is :

(1) $Li^+ > Na^+ > K^+ > Rb^+ > Cs^+$

(2) $Li^+ > Na^+ > K^+ > Cs^+ > Rb^+$

(3) $Na^+ > Li^+ > K^+ > Rb^+ > Cs^+$

(4) $Na^+ > Li^+ > K^+ > Cs^+ > Rb^+$

Correct Answer: (1) $Li^+ > Na^+ > K^+ > Rb^+ > Cs^+$

Solution:

Hydration enthalpy depends on the charge density of the ion (Charge / Size).

For alkali metals, the charge is +1 for all.

The ionic size increases down the group: $Li^+ < Na^+ < K^+ < Rb^+ < Cs^+$.

Therefore, the charge density decreases down the group.

Higher charge density attracts more water molecules, releasing more energy.

Thus, hydration enthalpy follows the order: $Li^+ > Na^+ > K^+ > Rb^+ > Cs^+$.

💡 Quick Tip

Smaller ions have higher charge density and thus higher magnitude of hydration enthalpy ($\Delta H_{hyd} \propto \frac{1}{r}$).

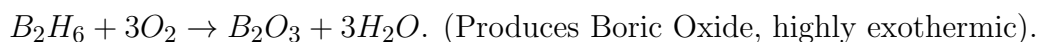
45. Diborane (B_2H_6) reacts independently with O_2 and H_2O to produce, respectively :

- (1) H_3BO_3 and B_2O_3
- (2) HBO_2 and H_3BO_3
- (3) B_2O_3 and H_3BO_3
- (4) B_2O_3 and $[BH_4]^-$

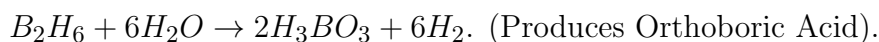
Correct Answer: (3) B_2O_3 and H_3BO_3

Solution:

Reaction with Oxygen (Combustion):



Reaction with Water (Hydrolysis):



Therefore, the products are B_2O_3 and H_3BO_3 respectively.

💡 Quick Tip

Boranes are electron-deficient and hydrolyze rapidly to give boric acid. Combustion yields the thermodynamic stable oxide B_2O_3 .

46. The lanthanide ion that would show colour is :

- (1) Gd^{3+}
- (2) Sm^{3+}
- (3) La^{3+}
- (4) Lu^{3+}

Correct Answer: (2) Sm^{3+}

Solution:

Colour in lanthanide ions arises due to f-f transitions, which require partially filled f-orbitals.

Electronic configurations of the ions:

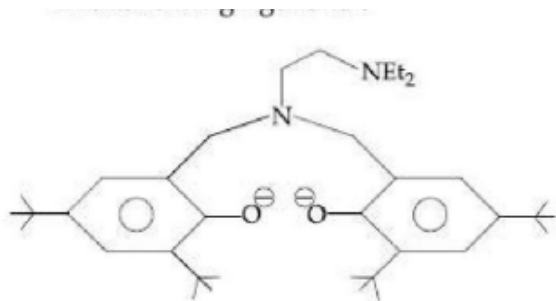
- (1) Gd^{3+} ($Z = 64$): $[Xe]4f^7$. Half-filled stable configuration. f-f transitions are spin-forbidden and weak. Often colourless (or very faint UV absorptions).
- (2) Sm^{3+} ($Z = 62$): $[Xe]4f^5$. Partially filled. Shows colour (typically yellow).
- (3) La^{3+} ($Z = 57$): $[Xe]4f^0$. Empty f-shell. Colourless.
- (4) Lu^{3+} ($Z = 71$): $[Xe]4f^{14}$. Fully filled f-shell. Colourless.

Therefore, Sm^{3+} is the one that shows colour.

💡 Quick Tip

Lanthanide ions with f^0, f^7, f^{14} configurations are generally colourless. Others are coloured.

47. The following ligand is :



- (1) bidentate
- (2) tridentate
- (3) tetradentate
- (4) hexadentate

Correct Answer: (3) tetradentate

Solution:

Identify the donor atoms in the structure provided in the image.

The structure consists of a central Nitrogen atom bonded to three groups:

1. A chain ending in a diethyl amine group ($-CH_2-CH_2-NEt_2$). The Nitrogen in the amine is a donor.
2. A methyl-phenol group. The phenolate Oxygen (O^-) is a donor.
3. Another methyl-phenol group. The phenolate Oxygen (O^-) is a donor.

The central tertiary Nitrogen atom is also a donor.

Total number of donor atoms = 1 (Central N) + 1 (Amine N) + 2 (Phenolate O) = 4.

Since it can coordinate through 4 donor atoms, it is a tetradentate ligand.

💡 Quick Tip

Count the atoms with lone pairs capable of binding to the metal center simultaneously (Nitrogen amines, Oxygen anions).

48. The correct order of the spin-only magnetic moment of metal ions in the following low-spin complexes, $[V(CN)_6]^{4-}$, $[Fe(CN)_6]^{4-}$, $[Ru(NH_3)_6]^{3+}$, and $[Cr(NH_3)_6]^{2+}$, is :

- (1) $V^{2+} > Cr^{2+} > Ru^{3+} > Fe^{2+}$
- (2) $V^{2+} > Ru^{3+} > Cr^{2+} > Fe^{2+}$
- (3) $Cr^{2+} > Ru^{3+} > Fe^{2+} > V^{2+}$
- (4) $Cr^{2+} > V^{2+} > Ru^{3+} > Fe^{2+}$

Correct Answer: (1) $V^{2+} > Cr^{2+} > Ru^{3+} > Fe^{2+}$

Solution:

48. Solution:

The complexes are stated to be **low-spin**. For low-spin octahedral complexes, electrons preferentially pair in the t_{2g} orbitals due to large crystal field splitting (Δ_o).

The spin-only magnetic moment is given by:

$$\mu = \sqrt{n(n+2)} \text{ BM}$$

where n = number of unpaired electrons.

Step 1: Determine oxidation state and electronic configuration

1. $[V(CN)_6]^{4-}$

Oxidation state of V:

$$x + 6(-1) = -4 \Rightarrow x = +2$$

V^{2+} configuration: $3d^3$

Low-spin octahedral:

$$t_{2g}^3 e_g^0$$

Number of unpaired electrons:

$$n = 3$$

Magnetic moment:

$$\mu = \sqrt{3(3+2)} = \sqrt{15} \approx 3.87 \text{ BM}$$

2. $[Cr(NH_3)_6]^{2+}$

Oxidation state of Cr:

$$x = +2$$

Cr^{2+} configuration: $3d^4$

Low-spin octahedral:

$$t_{2g}^4 e_g^0$$

Number of unpaired electrons:

$$n = 2$$

Magnetic moment:

$$\mu = \sqrt{2(2+2)} = \sqrt{8} \approx 2.83 \text{ BM}$$

3. $[Ru(NH_3)_6]^{3+}$

Oxidation state of Ru:

$$x = +3$$

Ru^{3+} configuration: $4d^5$

For 4d metals, Δ_o is large, hence low-spin:

$$t_{2g}^5 e_g^0$$

Number of unpaired electrons:

$$n = 1$$

Magnetic moment:

$$\mu = \sqrt{1(1+2)} = \sqrt{3} \approx 1.73 \text{ BM}$$

4. $[Fe(CN)_6]^{4-}$

Oxidation state of Fe:

$$x + 6(-1) = -4 \Rightarrow x = +2$$

Fe^{2+} configuration: $3d^6$

CN^- is a strong field ligand $\Rightarrow$ low-spin:

$$t_{2g}^6 e_g^0$$

Number of unpaired electrons:

$$n = 0$$

Magnetic moment:

$$\mu = 0 \text{ BM}$$

Step 2: Arrange in decreasing order of magnetic moment

$$V^{2+}(n=3) > Cr^{2+}(n=2) > Ru^{3+}(n=1) > Fe^{2+}(n=0)$$

Final Answer:

$$V^{2+} > Cr^{2+} > Ru^{3+} > Fe^{2+}$$

Correct Option: (1)

💡 Quick Tip

Spin-only magnetic moment $\mu = \sqrt{n(n+2)}$. Higher number of unpaired electrons (n) implies higher magnetic moment.

49. Which is wrong with respect to our responsibility as a human being to protect our environment ?

- (1) Setting up compost tin in gardens.
- (2) Using plastic bags.
- (3) Restricting the use of vehicles
- (4) Avoiding the use of floodlighted facilities.

Correct Answer: (2) Using plastic bags.

Solution:

We need to identify the action that is detrimental (wrong) for environmental protection.

- 1. Setting up compost tin: Good. Reduces waste, creates manure.
- 2. Using plastic bags: Bad. Plastic is non-biodegradable and causes pollution. This is "wrong" behaviour for protection.
- 3. Restricting use of vehicles: Good. Reduces air pollution.
- 4. Avoiding floodlighted facilities: Good. Reduces light pollution and energy consumption.

Therefore, "Using plastic bags" is the incorrect practice.

💡 Quick Tip

The 3 Rs: Reduce, Reuse, Recycle. Avoiding single-use plastics is a primary environmental responsibility.

50. Assertion : Ozone is destroyed by CFCs in the upper stratosphere. Reason : Ozone holes increase the amount of UV radiation reaching the earth.

- (1) Assertion and reason are correct, but the reason is not the explanation for the assertion.
- (2) Assertion is false, but the reason is correct.
- (3) Assertion and reason are both correct, and the reason is the correct explanation for the assertion.
- (4) Assertion and reason are incorrect.

Correct Answer: (1) Assertion and reason are correct, but the reason is not the explanation for the assertion.

Solution:

Assertion: CFCs release chlorine radicals in the upper stratosphere which catalyze the breakdown of ozone (O_3) into oxygen (O_2). This statement is Correct.

Reason: The depletion of the ozone layer (ozone holes) allows more harmful UV-B radiation to penetrate the atmosphere and reach the Earth's surface. This statement is Correct.

Connection: The Reason describes the consequence of the Assertion. It does not explain why or how ozone is destroyed by CFCs.

Therefore, both are true, but the reason is not the correct explanation for the assertion.

💡 Quick Tip

Check if the Reason answers "Why?" for the Assertion. Here, "Why is ozone destroyed?" is not answered by "Because UV increases".

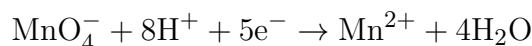
51. In order to oxidise a mixture of one mole of each of FeC_2O_4 , $Fe_2(C_2O_4)_3$, $FeSO_4$ and $Fe_2(SO_4)_3$ in acidic medium, the number of moles of $KMnO_4$ required is :

- (1) 1
- (2) 1.5
- (3) 2
- (4) 3

Correct Answer: (3) 2

Solution:

In acidic medium, potassium permanganate acts as a strong oxidizing agent and is reduced as:



Thus, **1 mole of $KMnO_4$ accepts 5 moles of electrons.**

We calculate the total number of electrons released by oxidation of each compound (1 mole each).

(i) FeC_2O_4

- $Fe^{2+} \rightarrow Fe^{3+} + e^-$ (1 electron)
- $C_2O_4^{2-} \rightarrow 2CO_2 + 2e^-$

Total electrons released:

$$1 + 2 = 3$$

(ii) $Fe_2(C_2O_4)_3$

- Iron is already in +3 oxidation state $\Rightarrow$ no oxidation.
- Each oxalate ion releases 2 electrons.
- Number of oxalate ions = 3

Total electrons released:

$$3 \times 2 = 6$$

(iii) $FeSO_4$

- $Fe^{2+} \rightarrow Fe^{3+} + e^-$ (1 electron)
- Sulphate ion does not undergo oxidation.

Total electrons released:

$$1$$

(iv) $Fe_2(SO_4)_3$

Both iron and sulphate ions are already in their highest oxidation states.

$$\text{Electrons released} = 0$$

Step 2: Total electrons released

$$3 + 6 + 1 + 0 = 10 \text{ electrons}$$

Step 3: Calculate moles of $KMnO_4$ required

$$\text{Moles of } KMnO_4 = \frac{\text{Total electrons released}}{5} = \frac{10}{5} = 2$$

Final Answer:

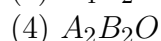
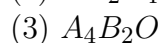
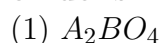
$$\boxed{2}$$

Correct Option: (3)

 Quick Tip

Always sum the n-factors for all oxidizable parts of a molecule (e.g., both Cation and Anion in Ferrous Oxalate).

52. Element 'B' forms ccp structure and 'A' occupies half of the octahedral voids, while oxygen atoms occupy all the tetrahedral voids. The structure of bimetallic oxide is :



Correct Answer: (2) AB_2O_4

Solution:

Let the number of atoms of element B in the ccp (face-centered cubic) packing be N .

So, Number of B = N .

In ccp packing:

Number of Octahedral Voids (OV) = N .

Number of Tetrahedral Voids (TV) = $2N$.

Element A occupies half of the octahedral voids:

Number of A = $\frac{1}{2} \times N = \frac{N}{2}$.

Oxygen atoms occupy all the tetrahedral voids:

Number of O = $2N$.

Ratio of atoms A : B : O = $\frac{N}{2} : N : 2N$.

Multiply by 2 to get whole numbers: 1 : 2 : 4.

Empirical Formula: AB_2O_4 .

 Quick Tip

In a close-packed lattice of N atoms, there are N Octahedral voids and $2N$ Tetrahedral voids.

53. The quantum number of four electrons are given below :

I. $n=4$, $l=2$, $m_l=-2$, $m_s=-1/2$

II. $n=3$, $l=2$, $m_l=1$, $m_s=+1/2$

III. $n=4, l=1, m_l=0, m_s=+1/2$

IV. $n=3, l=1, m_l=1, m_s=-1/2$

The correct order of their increasing energies will be :

(1) $I < II < III < IV$

(2) $I < III < II < IV$

(3) $IV < III < II < I$

(4) $IV < II < III < I$

Correct Answer: (4) $IV < II < III < I$

Solution:

The energy of an electron in a multi-electron atom depends on the $(n + l)$ value.

Higher $(n + l)$ means higher energy. If $(n + l)$ is the same, higher n means higher energy.

Calculate $(n + l)$ for each:

I. $n = 4, l = 2 \Rightarrow n + l = 6$. (4d orbital)

II. $n = 3, l = 2 \Rightarrow n + l = 5$. (3d orbital)

III. $n = 4, l = 1 \Rightarrow n + l = 5$. (4p orbital)

IV. $n = 3, l = 1 \Rightarrow n + l = 4$. (3p orbital)

Arranging by value:

Lowest: IV (4).

Next: Both II and III have 5. Comparing n : II ($n = 3$) ; III ($n = 4$).

Highest: I (6).

Correct Order: $IV < II < III < I$.

💡 Quick Tip

Aufbau Principle rule: Energy order is based on increasing $(n + l)$. For tie-breaking, use n .

54. For silver, $C_p(JK^{-1}mol^{-1}) = 23 + 0.01T$. If the temperature (T) of 3 moles of silver is raised from 300 K to 1000 K at 1 atm pressure, the value of ΔH will be close to :

(1) 13 kJ

(2) 21 kJ

(3) 16 kJ

(4) 62 kJ

Correct Answer: (4) 62 kJ

Solution:

For a process at constant pressure, $\Delta H = n \int_{T_1}^{T_2} C_p dT$.

Given: $n = 3$ moles, $T_1 = 300$ K, $T_2 = 1000$ K.

$$C_p = 23 + 0.01T.$$

$$\Delta H = 3 \int_{300}^{1000} (23 + 0.01T) dT.$$

$$\Delta H = 3 \left[23T + \frac{0.01T^2}{2} \right]_{300}^{1000}.$$

$$\Delta H = 3 [23(1000 - 300) + 0.005(1000^2 - 300^2)].$$

$$\Delta H = 3 [23(700) + 0.005(1000000 - 90000)].$$

$$\Delta H = 3 [16100 + 0.005(910000)].$$

$$\Delta H = 3 [16100 + 4550] = 3[20650] = 61950 \text{ J}.$$

Converting to kJ: $\Delta H \approx 62$ kJ.

 Quick Tip

When C_p is temperature dependent, you must integrate. Don't just use average temperature.

55. Which one of the following equations does not correctly represent the first law of thermodynamics for the given processes involving an ideal gas ? (Assume non-expansion work is zero)

(1) Isothermal process : $q = -w$

(2) Cyclic process : $q = -w$

(3) Isochoric process : $\Delta U = q$

(4) Adiabatic process : $\Delta U = -w$

Correct Answer: (4) Adiabatic process : $\Delta U = -w$

Solution:

According to the IUPAC (chemistry) sign convention, the First Law of Thermodynamics is:

$$\Delta U = q + w$$

where q = heat absorbed by the system, w = work done **on** the system.

(Non-expansion work is zero as stated.)

Check each process:

1. **Isothermal process** ($T = \text{constant}$)

For an ideal gas, internal energy depends only on temperature.

$$\Delta U = 0$$

From the first law:

$$0 = q + w \Rightarrow q = -w$$

Hence, option (1) is **correct**.

2. **Cyclic process**

In a cyclic process, the system returns to its initial state.

$$\Delta U = 0$$

Thus:

$$0 = q + w \Rightarrow q = -w$$

Hence, option (2) is **correct**.

3. **Isochoric process** ($V = \text{constant}$)

Work done:

$$w = -P\Delta V = 0$$

Therefore:

$$\Delta U = q + 0 = q$$

Hence, option (3) is **correct**.

4. **Adiabatic process**

By definition:

$$q = 0$$

From the first law:

$$\Delta U = 0 + w = w$$

However, option (4) states:

$$\Delta U = -w$$

This is **incorrect** under the IUPAC convention used in chemistry.

Final Answer:

Option (4) does not correctly represent the first law of thermodynamics

 Quick Tip

Pay attention to sign conventions. Chemistry usually uses $\Delta U = q + w$ (w = work on system). Physics uses $\Delta Q = \Delta U + \Delta W$ (W = work by system).

56. The vapour pressures of pure liquids A and B are 400 and 600 mmHg, respectively at 298 K. On mixing the two liquids, the sum of their initial volumes is equal to the volume of the final mixture. The mole fraction of liquid B is 0.5 in the mixture. The vapour pressure of the final solution, the mole fractions of components A and B in vapour phase, respectively are :

(1) 500 mmHg, 0.4, 0.6

(2) 450 mmHg, 0.5, 0.5

(3) 500 mmHg, 0.5, 0.5

(4) 450 mmHg, 0.4, 0.6

Correct Answer: (1) 500 mmHg, 0.4, 0.6

Solution:

56. Solution:

The statement “*sum of initial volumes equals the final volume of the mixture*” implies:

$$\Delta V_{\text{mix}} = 0$$

Hence, the solution behaves as an **ideal solution** and obeys **Raoult’s law**.

Given:

$$P_A^\circ = 400 \text{ mmHg}, \quad P_B^\circ = 600 \text{ mmHg}$$

Mole fractions in liquid phase:

$$x_B = 0.5, \quad x_A = 1 - x_B = 0.5$$

Step 1: Calculate partial vapour pressures

According to Raoult’s law:

$$P_A = x_A P_A^\circ = 0.5 \times 400 = 200 \text{ mmHg}$$

$$P_B = x_B P_B^\circ = 0.5 \times 600 = 300 \text{ mmHg}$$

Step 2: Calculate total vapour pressure

$$P_{\text{total}} = P_A + P_B$$

$$P_{\text{total}} = 200 + 300 = 500 \text{ mmHg}$$

Step 3: Calculate vapour phase mole fractions

Using Dalton’s law:

$$y_A = \frac{P_A}{P_{\text{total}}} = \frac{200}{500} = 0.4$$

$$y_B = \frac{P_B}{P_{\text{total}}} = \frac{300}{500} = 0.6$$

Final Answer:

$$P_{\text{total}} = 500 \text{ mmHg}, \quad y_A = 0.4, \quad y_B = 0.6$$

Correct Option: (1)

💡 Quick Tip

Raoult's Law for ideal solutions: $P_{\text{total}} = \sum P_i^\circ x_i$. Composition in vapour phase is richer in the more volatile component ($y_B > x_B$ since $P_B^\circ > P_A^\circ$).

57. If solubility product of $Zr_3(PO_4)_4$ is denoted by K_{sp} and its molar solubility is denoted by S , then which of the following relation between S and K_{sp} is correct ?

(1) $S = \left(\frac{K_{sp}}{144}\right)^{1/6}$

(2) $S = \left(\frac{K_{sp}}{929}\right)^{1/9}$

(3) $S = \left(\frac{K_{sp}}{6912}\right)^{1/7}$

(4) $S = \left(\frac{K_{sp}}{216}\right)^{1/7}$

Correct Answer: (3) $S = \left(\frac{K_{sp}}{6912}\right)^{1/7}$

Solution:

Dissociation equilibrium: $Zr_3(PO_4)_4(s) \rightleftharpoons 3Zr^{4+}(aq) + 4PO_4^{3-}(aq)$.

If solubility is S , then:

$$[Zr^{4+}] = 3S$$

$$[PO_4^{3-}] = 4S$$

Solubility Product $K_{sp} = [Zr^{4+}]^3 [PO_4^{3-}]^4$.

$$K_{sp} = (3S)^3 (4S)^4.$$

$$K_{sp} = (27S^3)(256S^4).$$

$$K_{sp} = (27 \times 256)S^{3+4}.$$

$$K_{sp} = 6912S^7.$$

Therefore, $S = \left(\frac{K_{sp}}{6912}\right)^{1/7}$.

💡 Quick Tip

For salt A_xB_y , $K_{sp} = x^x y^y S^{x+y}$. Here $x = 3, y = 4$, so factor is $3^3 4^4 = 27 \times 256 = 6912$.

58. Given that $E_{O_2/H_2O}^{\ominus} = +1.23 \text{ V}$; $E_{S_2O_8^{2-}/SO_4^{2-}}^{\ominus} = 2.05 \text{ V}$; $E_{Br_2/Br^-}^{\ominus} = +1.09 \text{ V}$; $E_{Au^{3+}/Au}^{\ominus} = +1.4 \text{ V}$. The strongest oxidizing agent is :

- (1) $S_2O_8^{2-}$
- (2) Au^{3+}
- (3) Br_2
- (4) O_2

Correct Answer: (1) $S_2O_8^{2-}$

Solution:

Standard Reduction Potential ($E^{\ominus}$) measures the tendency of a species to get reduced.

A higher positive value of $E^{\ominus}$ indicates a stronger tendency to gain electrons and undergo reduction.

The species that gets reduced acts as an Oxidizing Agent.

Values given:

1. $S_2O_8^{2-}$: +2.05 V (Highest)
2. Au^{3+} : +1.4 V
3. O_2 : +1.23 V
4. Br_2 : +1.09 V

Since $S_2O_8^{2-}$ has the highest reduction potential, it is the strongest oxidizing agent.

💡 Quick Tip

Highest $E_{red}^{\ominus} \implies$ Strongest Oxidizing Agent. Lowest $E_{red}^{\ominus} \implies$ Strongest Reducing Agent.

59. For the reaction $2A + B \rightarrow C$, the values of initial rate at different reactant concentrations are given in the table below. The rate law for the reaction is :

[A] (mol L^{-1})	[B] (mol L^{-1})	Initial Rate ($\text{mol L}^{-1}\text{s}^{-1}$)
0.05	0.05	0.045
0.10	0.05	0.090
0.20	0.10	0.72

- (1) Rate = $k[A][B]$
- (2) Rate = $k[A]^2[B]$
- (3) Rate = $k[A][B]^2$
- (4) Rate = $k[A]^2[B]^2$

Correct Answer: (3) Rate = $k[A][B]^2$

Solution:

Let the rate law for the reaction be:

$$\text{Rate} = k[A]^x[B]^y$$

where x and y are the orders with respect to reactants A and B , respectively.

Experimental Data (from the table):

Experiment	$[A]$	$[B]$	Rate
1	0.05	0.05	0.045
2	0.10	0.05	0.090
3	0.20	0.10	0.72

Step 1: Determine the order with respect to A

Compare Experiment 1 and Experiment 2, where concentration of B is constant.

$$\frac{\text{Rate}_2}{\text{Rate}_1} = \left(\frac{[A]_2}{[A]_1} \right)^x$$

$$\frac{0.090}{0.045} = \left(\frac{0.10}{0.05} \right)^x$$

$$2 = 2^x \Rightarrow x = 1$$

Thus, the reaction is **first order in A** .

Step 2: Determine the order with respect to B

Compare Experiment 2 and Experiment 3.

$$\frac{\text{Rate}_3}{\text{Rate}_2} = \left(\frac{[A]_3}{[A]_2} \right)^x \left(\frac{[B]_3}{[B]_2} \right)^y$$

Substitute known values and $x = 1$:

$$\frac{0.72}{0.090} = \left(\frac{0.20}{0.10} \right)^1 \left(\frac{0.10}{0.05} \right)^y$$

$$8 = 2 \times 2^y$$

$$4 = 2^y \Rightarrow y = 2$$

Thus, the reaction is **second order in B**.

Step 3: Write the rate law

$$\text{Rate} = k[A][B]^2$$

Final Answer:

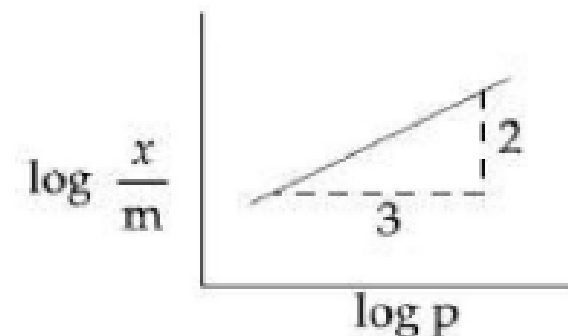
$$\text{Rate} = k[A][B]^2$$

Correct Option: (3)

💡 Quick Tip

Use the ratio method. Hold one concentration constant to find the order of the other. If no pair is constant, divide the rate equations of any two runs.

60. Adsorption of a gas follows Freundlich adsorption isotherm. x is the mass of the gas adsorbed on mass m of the adsorbent. The plot of $\log \frac{x}{m}$ versus $\log p$ is shown in the given graph. $\frac{x}{m}$ is proportional to :



- (1) p^2
- (2) $p^{3/2}$
- (3) $p^{2/3}$
- (4) p^3

Correct Answer: (3) $p^{2/3}$

Solution:

The Freundlich adsorption isotherm is given by $\frac{x}{m} = kp^{1/n}$.

Taking logarithm on both sides:

$$\log \left(\frac{x}{m} \right) = \log k + \frac{1}{n} \log p.$$

This equation represents a straight line ($y = c + mx$) when plotting $\log(x/m)$ vs $\log p$.
Slope of the line = $1/n$.

From the provided graph geometry (Triangle indicated with vertical side 2 and horizontal side 3):

$$\text{Slope} = \frac{\Delta y}{\Delta x} = \frac{2}{3}.$$

$$\text{Therefore, } \frac{1}{n} = \frac{2}{3}.$$

Substituting back into the isotherm equation:

$$\frac{x}{m} \propto p^{2/3}.$$

💡 Quick Tip

In log-log plots for power laws ($y = ax^b$), the slope of the line equals the exponent b .

61. If $f(x) = \log_e \left(\frac{1-x}{1+x} \right)$, $|x| < 1$, then $f \left(\frac{2x}{1+x^2} \right)$ is equal to :

(1) $2f(x^2)$

(2) $-2f(x)$

(3) $2f(x)$

(4) $(f(x))^2$

Correct Answer: (3) $2f(x)$

Solution:

$$\text{Given } f(x) = \ln \left(\frac{1-x}{1+x} \right).$$

Let's evaluate $f \left(\frac{2x}{1+x^2} \right)$:

$$f \left(\frac{2x}{1+x^2} \right) = \ln \left(\frac{1 - \frac{2x}{1+x^2}}{1 + \frac{2x}{1+x^2}} \right).$$

Simplify the argument of the log:

$$\frac{\frac{1+x^2-2x}{1+x^2}}{\frac{1+x^2+2x}{1+x^2}} = \frac{1+x^2-2x}{1+x^2+2x} = \frac{(1-x)^2}{(1+x)^2} = \left(\frac{1-x}{1+x} \right)^2.$$

$$\text{So, } f \left(\frac{2x}{1+x^2} \right) = \ln \left(\left(\frac{1-x}{1+x} \right)^2 \right).$$

Using the property $\ln(a^b) = b \ln a$, we get:

$$2 \ln \left(\frac{1-x}{1+x} \right) = 2f(x).$$

 Quick Tip

Recognize the standard substitution form: if $x = \tan \theta$, then $\frac{2x}{1+x^2} = \sin 2\theta$? No, simpler algebra works here. Note logarithmic identity $\ln(A^k) = k \ln A$.

62. If α and β be the roots of the equation $x^2 - 2x + 2 = 0$, then the least value of n for which $\left(\frac{\alpha}{\beta}\right)^n = 1$ is :

- (1) 5
- (2) 4
- (3) 3
- (4) 2

Correct Answer: (2) 4

Solution:

Solve $x^2 - 2x + 2 = 0$ using the quadratic formula:

$$x = \frac{2 \pm \sqrt{4-8}}{2} = \frac{2 \pm 2i}{2} = 1 \pm i.$$

Let $\alpha = 1 + i$ and $\beta = 1 - i$.

Calculate the ratio $\frac{\alpha}{\beta}$:

$$\frac{\alpha}{\beta} = \frac{1+i}{1-i} = \frac{(1+i)(1+i)}{(1-i)(1+i)} = \frac{1+2i+i^2}{1-i^2} = \frac{2i}{2} = i.$$

We need to find the least n such that $(i)^n = 1$.

We know that $i^1 = i$, $i^2 = -1$, $i^3 = -i$, $i^4 = 1$.

The least positive integer n is 4.

 Quick Tip

Remember that $\frac{1+i}{1-i} = i$ and $\frac{1-i}{1+i} = -i$.

63. The sum of the solutions of the equation $|\sqrt{x} - 2| + \sqrt{x}(\sqrt{x} - 4) + 2 = 0$, ($x > 0$) is equal to :

- (1) 4
- (2) 9
- (3) 10

(4) 12

Correct Answer: (3) 10

Solution:

Let $t = \sqrt{x}$. The equation becomes $|t - 2| + t(t - 4) + 2 = 0$.

$$|t - 2| + t^2 - 4t + 2 = 0.$$

Case 1: $t \geq 2$.

$$(t - 2) + t^2 - 4t + 2 = 0 \implies t^2 - 3t = 0 \implies t(t - 3) = 0.$$

Possible values $t = 0, 3$. Since $t \geq 2$, we accept $t = 3$.

$$t = 3 \implies \sqrt{x} = 3 \implies x = 9.$$

Case 2: $t < 2$.

$$-(t - 2) + t^2 - 4t + 2 = 0 \implies -t + 2 + t^2 - 4t + 2 = 0 \implies t^2 - 5t + 4 = 0.$$

$(t - 4)(t - 1) = 0$. Possible values $t = 1, 4$. Since $t < 2$, we accept $t = 1$.

$$t = 1 \implies \sqrt{x} = 1 \implies x = 1.$$

The solutions are $x = 9$ and $x = 1$.

Sum of solutions = $9 + 1 = 10$.

 Quick Tip

When dealing with modulus equations involving a variable, always split the problem into cases based on the sign of the expression inside the modulus.

64. Let $A = \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix}$, $(\alpha \in R)$ **such that** $A^{32} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$. **Then a value of** α **is**

:

- (1) 0
- (2) $\frac{\pi}{64}$
- (3) $\frac{\pi}{32}$
- (4) $\frac{\pi}{16}$

Correct Answer: (2) $\frac{\pi}{64}$

Solution:

The given matrix

$$A = \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix}$$

is a **rotation matrix** representing rotation through an angle α in the plane.

Step 1: Use the property of rotation matrices

For a rotation matrix,

$$A^n = \begin{pmatrix} \cos(n\alpha) & -\sin(n\alpha) \\ \sin(n\alpha) & \cos(n\alpha) \end{pmatrix}$$

Step 2: Apply the given condition

Given:

$$A^{32} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

Comparing with the standard form:

$$\begin{pmatrix} \cos(32\alpha) & -\sin(32\alpha) \\ \sin(32\alpha) & \cos(32\alpha) \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

Thus,

$$\cos(32\alpha) = 0 \quad \text{and} \quad \sin(32\alpha) = 1$$

Step 3: Solve for α

The angle for which $\sin \theta = 1$ and $\cos \theta = 0$ is:

$$\theta = \frac{\pi}{2} + 2k\pi, \quad k \in \mathbb{Z}$$

Hence,

$$32\alpha = \frac{\pi}{2} + 2k\pi$$

Taking the principal value ($k = 0$):

$$32\alpha = \frac{\pi}{2}$$

$$\alpha = \frac{\pi}{64}$$

Final Answer:

$$\boxed{\alpha = \frac{\pi}{64}}$$

Correct Option: (2)

💡 Quick Tip

Rotation matrices sum angles when multiplied: $R(\alpha) \cdot R(\beta) = R(\alpha + \beta)$. Thus $R(\alpha)^n = R(n\alpha)$.

65. The greatest value of $c \in R$ for which the system of linear equations $x - cy - cz = 0$, $cx - y + cz = 0$, $cx + cy - z = 0$ has a non-trivial solution, is :

(1) -1

(2) 0

(3) $\frac{1}{2}$

(4) 2

Correct Answer: (3) $\frac{1}{2}$

Solution:

For a homogeneous system to have a non-trivial solution, the determinant of the coefficient matrix must be zero.

$$\Delta = \begin{vmatrix} 1 & -c & -c \\ c & -1 & c \\ c & c & -1 \end{vmatrix} = 0.$$

Expanding along row 1:

$$1((-1)(-1) - c(c)) - (-c)(c(-1) - c(c)) + (-c)(c(c) - c(-1)) = 0.$$

$$1(1 - c^2) + c(-c - c^2) - c(c^2 + c) = 0.$$

$$1 - c^2 - c^2 - c^3 - c^3 - c^2 = 0.$$

$$-2c^3 - 3c^2 + 1 = 0 \implies 2c^3 + 3c^2 - 1 = 0.$$

Check roots: If $c = -1$, $2(-1) + 3(1) - 1 = 0$. So $(c + 1)$ is a factor.

$$\text{Divide by } (c + 1): (c + 1)(2c^2 + c - 1) = 0.$$

$$(c + 1)(2c - 1)(c + 1) = 0.$$

Roots are $c = -1$ and $c = \frac{1}{2}$.

The greatest value is $\frac{1}{2}$.

 Quick Tip

For homogeneous systems $AX = 0$, non-trivial solutions exist if and only if $\det(A) = 0$.

66. All possible numbers are formed using the digits 1, 1, 2, 2, 2, 2, 3, 4, 4 taken all at a time. The number of such numbers in which the odd digits occupy even

places is :

- (1) 160
- (2) 162
- (3) 175
- (4) 180

Correct Answer: (4) 180

Solution:

Total digits = 9.

Digits: 1, 1 (Two 1s), 2, 2, 2, 2 (Four 2s), 3 (One 3), 4, 4 (Two 4s).

Odd digits: 1, 1, 3 (Total 3 digits).

Even digits: 2, 2, 2, 2, 4, 4 (Total 6 digits).

Positions in a 9-digit number: 1, 2, 3, 4, 5, 6, 7, 8, 9.

Even places are: 2, 4, 6, 8 (Total 4 places).

Condition: Odd digits must occupy even places.

Step 1: Select 3 positions from the 4 available even places.

Number of ways = $\binom{4}{3} = 4$.

Step 2: Arrange the 3 odd digits (1, 1, 3) in these selected places.

Number of ways = $\frac{3!}{2!1!} = 3$.

Step 3: Arrange the remaining 6 even digits (2, 2, 2, 2, 4, 4) in the remaining 6 places (1 even + 5 odd places).

Number of ways = $\frac{6!}{4!2!} = \frac{720}{24 \times 2} = 15$.

Total ways = $4 \times 3 \times 15 = 180$.

 Quick Tip

Break the problem into selection of places $\times$ arrangement of specific items in those places $\times$ arrangement of remaining items. Handle identical items using division by factorials.

67. The sum of the co-efficients of all even degree terms in x in the expansion of $(x + \sqrt{x^3 - 1})^6 + (x - \sqrt{x^3 - 1})^6$, ($x > 1$) is equal to :

(1) 24

(2) 26

(3) 29

(4) 32

Correct Answer: (1) 24

Solution:

Let $a = x$ and $b = \sqrt{x^3 - 1}$.

The expression is $(a + b)^6 + (a - b)^6 = 2\left(\binom{6}{0}a^6 + \binom{6}{2}a^4b^2 + \binom{6}{4}a^2b^4 + \binom{6}{6}b^6\right)$.

Substitute back:

$$2[1 \cdot x^6 + 15x^4(x^3 - 1) + 15x^2(x^3 - 1)^2 + 1(x^3 - 1)^3].$$

$$2[x^6 + 15x^7 - 15x^4 + 15x^2(x^6 - 2x^3 + 1) + (x^9 - 3x^6 + 3x^3 - 1)].$$

$$2[x^6 + 15x^7 - 15x^4 + 15x^8 - 30x^5 + 15x^2 + x^9 - 3x^6 + 3x^3 - 1].$$

Group terms by degree. We need the sum of coefficients of even degree terms (x^8, x^6, x^4, x^2, x^0).

Coefficients from inside the bracket:

$$x^8 \rightarrow 15$$

$$x^6 \rightarrow 1 - 3 = -2$$

$$x^4 \rightarrow -15$$

$$x^2 \rightarrow 15$$

$$x^0 \rightarrow -1$$

$$\text{Sum} = 15 - 2 - 15 + 15 - 1 = 12.$$

Multiply by the outer factor of 2: Total Sum = $2 \times 12 = 24$.

Quick Tip

To find sum of coefficients, usually set $x = 1$. Here, the term involves $\sqrt{x^3 - 1}$, so setting $x = 1$ makes it 0. Alternatively, identify the powers directly.

68. The sum of all natural numbers 'n' such that $100 < n < 200$ and H.C.F. (91, n) is 1 is :

(1) 3203

(2) 3221

(3) 3121

(4) 3303

Correct Answer: (3) 3121

Solution:

$$91 = 7 \times 13.$$

$HCF(91, n) > 1$ means n shares a factor with 91. Thus, n must be divisible by 7 OR 13.

We need sum of numbers in range (100, 200) divisible by 7 or 13.

$$\text{Sum} = \text{Sum}(\text{div by } 7) + \text{Sum}(\text{div by } 13) - \text{Sum}(\text{div by } 91).$$

Multiples of 7: 105, 112, ..., 196.

$$a = 105, l = 196. \quad 196 = 105 + (N - 1)7 \implies 91 = (N - 1)7 \implies N = 14.$$

$$\text{Sum}_7 = \frac{14}{2}(105 + 196) = 7(301) = 2107.$$

Multiples of 13: 104, 117, ..., 195.

$$a = 104, l = 195. \quad 195 = 104 + (N - 1)13 \implies 91 = (N - 1)13 \implies N = 8.$$

$$\text{Sum}_{13} = \frac{8}{2}(104 + 195) = 4(299) = 1196.$$

Multiples of 91: 182 is the only one in range.

$$\text{Sum}_{91} = 182.$$

$$\text{Total Sum} = 2107 + 1196 - 182 = 3303 - 182 = 3121.$$

 Quick Tip

Use the Principle of Inclusion-Exclusion for sums: $S(A \cup B) = S(A) + S(B) - S(A \cap B)$.

69. The sum of the series $2 \cdot {}^{20}C_0 + 5 \cdot {}^{20}C_1 + 8 \cdot {}^{20}C_2 + \dots + 62 \cdot {}^{20}C_{20}$ is equal to :

(1) 2^{24}

(2) 2^{25}

(3) 2^{23}

(4) 2^{26}

Correct Answer: (2) 2^{25}

Solution:

The general term is $T_r = (3r + 2) \cdot {}^{20}C_r$ for $r = 0$ to 20 .

$$\text{Sum } S = \sum_{r=0}^{20} (3r + 2) \binom{20}{r}.$$

$$S = 3 \sum_{r=0}^{20} r \binom{20}{r} + 2 \sum_{r=0}^{20} \binom{20}{r}.$$

Using identities: $\sum r \binom{n}{r} = n2^{n-1}$ and $\sum \binom{n}{r} = 2^n$.

Here $n = 20$.

$$S = 3(20 \cdot 2^{19}) + 2(2^{20}).$$

$$S = 60 \cdot 2^{19} + 2 \cdot 2 \cdot 2^{19} = 60 \cdot 2^{19} + 4 \cdot 2^{19}.$$

$$S = (60 + 4)2^{19} = 64 \cdot 2^{19}.$$

$$S = 2^6 \cdot 2^{19} = 2^{25}.$$

 Quick Tip

Separate the arithmetic part ($Ar + B$) from the binomial coefficient. Use $\sum rC_r = n2^{n-1}$.

70. $\lim_{x \rightarrow 0} \frac{\sin^2 x}{\sqrt{2} - \sqrt{1 + \cos x}}$ equals :

(1) $4\sqrt{2}$

(2) 4

(3) $\sqrt{2}$

(4) $2\sqrt{2}$

Correct Answer: (1) $4\sqrt{2}$

Solution:

Multiply numerator and denominator by the conjugate $\sqrt{2} + \sqrt{1 + \cos x}$.

$$L = \lim_{x \rightarrow 0} \frac{\sin^2 x (\sqrt{2} + \sqrt{1 + \cos x})}{(\sqrt{2})^2 - (1 + \cos x)}.$$

$$L = \lim_{x \rightarrow 0} \frac{\sin^2 x (\sqrt{2} + \sqrt{1 + \cos x})}{2 - 1 - \cos x} = \lim_{x \rightarrow 0} \frac{\sin^2 x (\sqrt{2} + \sqrt{1 + \cos x})}{1 - \cos x}.$$

$$\text{Use } \sin^2 x = 1 - \cos^2 x = (1 - \cos x)(1 + \cos x).$$

$$L = \lim_{x \rightarrow 0} \frac{(1 - \cos x)(1 + \cos x)(\sqrt{2} + \sqrt{1 + \cos x})}{1 - \cos x}.$$

$$L = \lim_{x \rightarrow 0} (1 + \cos x)(\sqrt{2} + \sqrt{1 + \cos x}).$$

Substitute $x = 0$:

$$L = (1 + 1)(\sqrt{2} + \sqrt{1 + 1}) = 2(\sqrt{2} + \sqrt{2}) = 2(2\sqrt{2}) = 4\sqrt{2}.$$

💡 Quick Tip

Rationalization is the standard technique for limits involving roots. Also $1 - \cos x = 2 \sin^2(x/2)$ is useful.

71. If $2y = \left(\cot^{-1} \left(\frac{\sqrt{3} \cos x + \sin x}{\cos x - \sqrt{3} \sin x} \right) \right)^2$, $x \in (0, \frac{\pi}{2})$, then $\frac{dy}{dx}$ is equal to :

- (1) $\frac{\pi}{6} - x$
- (2) $\frac{\pi}{3} - x$
- (3) $x - \frac{\pi}{6}$
- (4) $2x - \frac{\pi}{3}$

Correct Answer: (3) $x - \frac{\pi}{6}$

Solution:

Divide numerator and denominator inside $\cot^{-1}$ by 2 (or $\cos x$ then adjust, but dividing by 2 reveals angle formula).

$$\frac{\frac{\sqrt{3}}{2} \cos x + \frac{1}{2} \sin x}{\frac{1}{2} \cos x - \frac{\sqrt{3}}{2} \sin x} = \frac{\sin(\frac{\pi}{3}) \cos x + \cos(\frac{\pi}{3}) \sin x}{\cos(\frac{\pi}{3}) \cos x - \sin(\frac{\pi}{3}) \sin x}.$$

$$\text{This is } \frac{\sin(\frac{\pi}{3} + x)}{\cos(\frac{\pi}{3} + x)} = \tan\left(\frac{\pi}{3} + x\right).$$

$$\text{So, term is } \cot^{-1}(\tan(\frac{\pi}{3} + x)).$$

$$\text{Using } \cot^{-1}(y) = \frac{\pi}{2} - \tan^{-1}(y).$$

$$= \frac{\pi}{2} - \tan^{-1}(\tan(\frac{\pi}{3} + x)).$$

Since $x \in (0, \pi/2)$, angle is in range.

$$= \frac{\pi}{2} - \left(\frac{\pi}{3} + x\right) = \frac{\pi}{6} - x.$$

$$\text{Given } 2y = \left(\frac{\pi}{6} - x\right)^2.$$

Differentiate wrt x :

$$2\frac{dy}{dx} = 2\left(\frac{\pi}{6} - x\right) \cdot (-1).$$

$$\frac{dy}{dx} = -\left(\frac{\pi}{6} - x\right) = x - \frac{\pi}{6}.$$

💡 Quick Tip

Convert inverse trigonometric arguments into the form $\cot(\theta)$ or $\tan(\theta)$ using compound angle formulas.

72. Let $f : [0, 2] \rightarrow \mathbb{R}$ be a twice differentiable function such that $f''(x) > 0$, for all $x \in (0, 2)$. If $\phi(x) = f(x) + f(2 - x)$, then ϕ is :

- (1) increasing on $(0, 1)$ and decreasing on $(1, 2)$.
- (2) decreasing on $(0, 1)$ and increasing on $(1, 2)$.
- (3) increasing on $(0, 2)$
- (4) decreasing on $(0, 2)$

Correct Answer: (2) decreasing on $(0, 1)$ and increasing on $(1, 2)$.

Solution:

$$\text{Given } \phi(x) = f(x) + f(2 - x).$$

$$\text{Differentiate: } \phi'(x) = f'(x) + f'(2 - x) \cdot (-1) = f'(x) - f'(2 - x).$$

Given $f''(x) > 0$. This implies $f'(x)$ is a strictly increasing function.

$$\text{Case 1: } x > 2 - x \implies 2x > 2 \implies x > 1.$$

$$\text{Since } f' \text{ is increasing, } f'(x) > f'(2 - x).$$

$$\text{Therefore, } \phi'(x) > 0 \text{ for } x \in (1, 2). \text{ (}\phi \text{ is increasing).}$$

$$\text{Case 2: } x < 2 - x \implies x < 1.$$

$$\text{Since } f' \text{ is increasing, } f'(x) < f'(2 - x).$$

$$\text{Therefore, } \phi'(x) < 0 \text{ for } x \in (0, 1). \text{ (}\phi \text{ is decreasing).}$$

Conclusion: Decreasing on $(0, 1)$ and Increasing on $(1, 2)$.

 Quick Tip

If $g(x)$ is increasing, then $a > b \implies g(a) > g(b)$. Use this to determine the sign of the derivative difference.

73. If S_1 and S_2 are respectively the sets of local minimum and local maximum points of the function, $f(x) = 9x^4 + 12x^3 - 36x^2 + 25, x \in R$, then :

(1) $S_1 = \{-2, 0\}; S_2 = \{1\}$

(2) $S_1 = \{-2, 1\}; S_2 = \{0\}$

(3) $S_1 = \{-1\}; S_2 = \{0, 2\}$

(4) $S_1 = \{-2\}; S_2 = \{0, 1\}$

Correct Answer: (2) $S_1 = \{-2, 1\}; S_2 = \{0\}$

Solution:

Find $f'(x)$:

$$f'(x) = 36x^3 + 36x^2 - 72x.$$

Set $f'(x) = 0$:

$$36x(x^2 + x - 2) = 0.$$

$$36x(x + 2)(x - 1) = 0.$$

Critical points are $x = 0, -2, 1$.

Use Second Derivative Test:

$$f''(x) = 108x^2 + 72x - 72.$$

At $x = 0$: $f''(0) = -72 < 0$. Maxima. So $0 \in S_2$.

At $x = 1$: $f''(1) = 108 + 72 - 72 = 108 > 0$. Minima. So $1 \in S_1$.

At $x = -2$: $f''(-2) = 108(4) - 144 - 72 = 432 - 216 = 216 > 0$. Minima. So $-2 \in S_1$.

Thus, $S_1 = \{-2, 1\}$ and $S_2 = \{0\}$.

💡 Quick Tip

First derivative finds critical points. Second derivative sign confirms Min (+) or Max (-).

74. $\int \frac{\sin \frac{5x}{2}}{\sin \frac{x}{2}} dx$ is equal to : (where c is a constant of integration.)

- (1) $x + 2 \sin x + \sin 2x + c$
- (2) $x + 2 \sin x + 2 \sin 2x + c$
- (3) $2x + \sin x + \sin 2x + c$
- (4) $2x + \sin x + 2 \sin 2x + c$

Correct Answer: (1) $x + 2 \sin x + \sin 2x + c$

Solution:

Let $I = \int \frac{\sin \frac{5x}{2}}{\sin \frac{x}{2}} dx$.

Use the identity $\frac{\sin(2n+1)\theta}{\sin \theta} = 1 + 2 \cos 2\theta + 2 \cos 4\theta + \cdots + 2 \cos 2n\theta$.

Here $\frac{5x}{2} = (2(2) + 1)\frac{x}{2}$. So $n = 2$ and $\theta = x/2$.

Integrand $= 1 + 2 \cos(2 \cdot \frac{x}{2}) + 2 \cos(4 \cdot \frac{x}{2}) = 1 + 2 \cos x + 2 \cos 2x$.

Integrate term by term:

$$I = \int (1 + 2 \cos x + 2 \cos 2x) dx.$$

$$I = x + 2 \sin x + 2 \frac{\sin 2x}{2} + c.$$

$$I = x + 2 \sin x + \sin 2x + c.$$

💡 Quick Tip

Using geometric progression of complex exponentials or standard trig series identities simplifies such ratios.

75. If $f(x) = \frac{2-x \cos x}{2+x \cos x}$ and $g(x) = \log_e x, (x > 0)$ then the value of the integral $\int_{-\pi/4}^{\pi/4} g(f(x)) dx$ is :

- (1) $\log_e 2$
- (2) $\log_e 3$
- (3) $\log_e 1$
- (4) $\log_e e$

Correct Answer: (3) $\log_e 1$

Solution:

We need to evaluate $I = \int_{-\pi/4}^{\pi/4} \ln \left(\frac{2-x \cos x}{2+x \cos x} \right) dx$.

Let $h(x) = \ln \left(\frac{2-x \cos x}{2+x \cos x} \right)$.

Check if $h(x)$ is odd or even.

$$h(-x) = \ln \left(\frac{2-(-x) \cos(-x)}{2+(-x) \cos(-x)} \right) = \ln \left(\frac{2+x \cos x}{2-x \cos x} \right).$$

$$h(-x) = \ln \left(\left(\frac{2-x \cos x}{2+x \cos x} \right)^{-1} \right) = -\ln \left(\frac{2-x \cos x}{2+x \cos x} \right) = -h(x).$$

Since $h(x)$ is an odd function, the integral over the symmetric interval $[-\pi/4, \pi/4]$ is zero.

$$0 = \log_e 1.$$

 Quick Tip

Integral of an odd function from $-a$ to a is always zero. $\log(1/A) = -\log A$.

76. The area (in sq.units) of the region $A = \{(x, y) \in R \times R | 0 \leq x \leq 3, 0 \leq y \leq 4, y \leq x^2 + 3x\}$ is :

- (1) 8
- (2) $\frac{26}{3}$
- (3) $\frac{59}{6}$
- (4) $\frac{53}{6}$

Correct Answer: (3) $\frac{59}{6}$

Solution:

The region is bounded by $x = 0, x = 3, y = 0, y = 4$ and $y \leq x^2 + 3x$.

Intersection of curve $y = x^2 + 3x$ and line $y = 4$:

$$x^2 + 3x = 4 \implies x^2 + 3x - 4 = 0 \implies (x + 4)(x - 1) = 0.$$

For $x \geq 0$, intersection is at $x = 1$.

For $0 \leq x \leq 1$, the curve $x^2 + 3x \leq 1 + 3 = 4$. So area is under the curve.

For $1 \leq x \leq 3$, the curve $x^2 + 3x \geq 4$. Since the region is limited by $y \leq 4$, the area is bounded by $y = 4$ (flat top).

$$\text{Total Area} = \int_0^1 (x^2 + 3x) dx + \int_1^3 4 dx.$$

$$\text{Area 1} = \left[\frac{x^3}{3} + \frac{3x^2}{2} \right]_0^1 = \frac{1}{3} + \frac{3}{2} = \frac{2+9}{6} = \frac{11}{6}.$$

$$\text{Area 2} = [4x]_1^3 = 4(3 - 1) = 8.$$

$$\text{Total Area} = \frac{11}{6} + 8 = \frac{11+48}{6} = \frac{59}{6}.$$

💡 Quick Tip

Sketch the region to identify which constraint is active (y_{curve} vs y_{line}). Split the integral at the intersection point.

77. Let $y = y(x)$ be the solution of the differential equation, $(x^2 + 1)^2 \frac{dy}{dx} + 2x(x^2 + 1)y = 1$ such that $y(0) = 0$. If $\sqrt{a}y(1) = \frac{\pi}{32}$, then the value of 'a' is :

- (1) $\frac{1}{16}$
- (2) $\frac{1}{2}$
- (3) 1
- (4) $\frac{1}{4}$

Correct Answer: (1) $\frac{1}{16}$

Solution:

Divide equation by $(x^2 + 1)^2$:

$$\frac{dy}{dx} + \frac{2x}{x^2+1}y = \frac{1}{(x^2+1)^2}.$$

$$\text{Integrating Factor (IF)} = e^{\int \frac{2x}{x^2+1} dx} = e^{\ln(x^2+1)} = x^2 + 1.$$

$$\text{Solution: } y \cdot (IF) = \int Q \cdot (IF) dx.$$

$$y(x^2 + 1) = \int \frac{1}{(x^2+1)^2} (x^2 + 1) dx = \int \frac{1}{x^2+1} dx.$$

$$y(x^2 + 1) = \tan^{-1} x + C.$$

$$\text{Given } y(0) = 0 \implies 0 = 0 + C \implies C = 0.$$

$$y(x) = \frac{\tan^{-1} x}{x^2+1}.$$

Evaluate at $x = 1$:

$$y(1) = \frac{\tan^{-1}(1)}{1+1} = \frac{\pi/4}{2} = \frac{\pi}{8}.$$

$$\text{Given } \sqrt{a}y(1) = \frac{\pi}{32}.$$

$$\sqrt{a}\left(\frac{\pi}{8}\right) = \frac{\pi}{32} \implies \sqrt{a} = \frac{1}{4}.$$

$$a = \frac{1}{16}.$$

 Quick Tip

This is a Linear Differential Equation $\frac{dy}{dx} + Py = Q$. IF is $e^{\int P dx}$.

78. Let $O(0, 0)$ and $A(0, 1)$ be two fixed points. Then the locus of a point P such that the perimeter of $\triangle AOP$ is 4, is :

(1) $9x^2 - 8y^2 + 8y = 16$

(2) $9x^2 + 8y^2 - 8y = 16$

(3) $8x^2 - 9y^2 + 9y = 18$

(4) $8x^2 + 9y^2 - 9y = 18$

Correct Answer: (2) $9x^2 + 8y^2 - 8y = 16$

Solution:

$$\text{Perimeter } P = OP + AP + OA = 4.$$

$$OA = \sqrt{0^2 + 1^2} = 1.$$

$$\text{So } OP + AP = 4 - 1 = 3.$$

The sum of distances of P from two fixed points $(0,0)$ and $(0,1)$ is constant (3). This is an ellipse with foci at $O(0,0)$ and $A(0,1)$.

$$2a = 3 \implies a = 3/2.$$

$$\text{Distance between foci } 2ae = 1 \implies e = \frac{1}{2a} = \frac{1}{3}.$$

$$\text{Semi-minor axis } b^2 = a^2(1 - e^2) = \frac{9}{4}\left(1 - \frac{1}{9}\right) = \frac{9}{4} \cdot \frac{8}{9} = 2.$$

Center is midpoint of foci: $(0, 1/2)$. Major axis is along y-axis.

$$\text{Equation: } \frac{(x-0)^2}{b^2} + \frac{(y-1/2)^2}{a^2} = 1.$$

$$\frac{x^2}{2} + \frac{(y-0.5)^2}{2.25} = 1 \implies \frac{x^2}{2} + \frac{(2y-1)^2/4}{9/4} = 1 \implies \frac{x^2}{2} + \frac{(2y-1)^2}{9} = 1.$$

Multiply by 18: $9x^2 + 2(4y^2 - 4y + 1) = 18$.

$$9x^2 + 8y^2 - 8y + 2 = 18.$$

$$9x^2 + 8y^2 - 8y = 16.$$

💡 Quick Tip

Definition of Ellipse: $PS + PS' = 2a$. Here foci are on the y-axis, so the a^2 term goes with the y term.

79. A point on the straight line, $3x + 5y = 15$ which is equidistant from the coordinate axes will lie only in :

- (1) 1st, 2nd and 4th quadrants
- (2) 1st quadrant
- (3) 1st and 2nd quadrants
- (4) 4th quadrant

Correct Answer: (3) 1st and 2nd quadrants

Solution:

A point equidistant from coordinate axes satisfies $|x| = |y|$, i.e., $y = x$ or $y = -x$.

Case 1: $y = x$.

Substitute into line equation: $3x + 5x = 15 \implies 8x = 15 \implies x = \frac{15}{8}$.

Point $(\frac{15}{8}, \frac{15}{8})$. Since $x > 0, y > 0$, this is in the 1st quadrant.

Case 2: $y = -x$.

Substitute into line equation: $3x + 5(-x) = 15 \implies -2x = 15 \implies x = -\frac{15}{2}$.

$$y = -(-\frac{15}{2}) = \frac{15}{2}.$$

Point $(-\frac{15}{2}, \frac{15}{2})$. Since $x < 0, y > 0$, this is in the 2nd quadrant.

The points lie in 1st and 2nd quadrants only.

 Quick Tip

Points equidistant from axes lie on the angle bisectors $y = x$ and $y = -x$. Find intersection of the given line with these bisectors.

80. The sum of the squares of the lengths of the chords intercepted on the circle, $x^2 + y^2 = 16$, by the lines, $x + y = n$, $n \in N$, where N is the set of all natural numbers, is :

- (1) 105
- (2) 210
- (3) 320
- (4) 160

Correct Answer: (2) 210

Solution:

Circle center $O(0, 0)$, Radius $R = 4$.

Line $x + y - n = 0$.

Distance from center $d = \frac{|0+0-n|}{\sqrt{1^2+1^2}} = \frac{n}{\sqrt{2}}$.

For intersection, $d < R \implies \frac{n}{\sqrt{2}} < 4 \implies n < 4\sqrt{2} \approx 5.65$.

Since $n \in N$, possible values are $n = 1, 2, 3, 4, 5$.

Length of chord $L_n = 2\sqrt{R^2 - d^2} = 2\sqrt{16 - \frac{n^2}{2}}$.

Square of length $L_n^2 = 4(16 - \frac{n^2}{2}) = 64 - 2n^2$.

We need the sum $S = \sum_{n=1}^5 (64 - 2n^2)$.

$$S = \sum_{n=1}^5 64 - 2 \sum_{n=1}^5 n^2.$$

$$S = 5 \times 64 - 2 \times \frac{5(6)(11)}{6}.$$

$$S = 320 - 2 \times 55 = 320 - 110 = 210.$$

 Quick Tip

Length of chord $= 2\sqrt{R^2 - d^2}$ where d is perpendicular distance from center to chord.

“latex **81. The shortest distance between the line $y = x$ and the curve $y^2 = x - 2$ is :**

- (1) $\frac{11}{4\sqrt{2}}$
- (2) $\frac{7}{4\sqrt{2}}$
- (3) $\frac{7}{8}$
- (4) 2

Correct Answer: (2) $\frac{7}{4\sqrt{2}}$

Solution:

The shortest distance between a line and a curve occurs along the common normal. This means the tangent to the curve at the point of shortest distance is parallel to the given line.

Given Line: $y = x$ or $x - y = 0$. Slope $m = 1$.

Given Curve: $y^2 = x - 2$.

Differentiating the curve equation with respect to x :

$$2y \frac{dy}{dx} = 1 \implies \frac{dy}{dx} = \frac{1}{2y}.$$

Set the slope of the tangent equal to the slope of the line:

$$\frac{1}{2y} = 1 \implies y = \frac{1}{2}.$$

Find the corresponding x -coordinate on the curve:

$$\left(\frac{1}{2}\right)^2 = x - 2 \implies \frac{1}{4} = x - 2 \implies x = 2 + \frac{1}{4} = \frac{9}{4}.$$

So the point on the curve is $P\left(\frac{9}{4}, \frac{1}{2}\right)$.

The perpendicular distance from point $P(x_1, y_1)$ to the line $Ax + By + C = 0$ is $d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$.

Here, line is $x - y = 0$.

$$d = \frac{\left|\frac{9}{4} - \frac{1}{2}\right|}{\sqrt{1^2 + (-1)^2}} = \frac{\left|\frac{9}{4} - \frac{2}{4}\right|}{\sqrt{2}} = \frac{\frac{7}{4}}{\sqrt{2}} = \frac{7}{4\sqrt{2}}.$$

💡 Quick Tip

To find the shortest distance between a curve and a line, find a point on the curve where the tangent is parallel to the line, then calculate the perpendicular distance.

82. If the tangents on the ellipse $4x^2 + y^2 = 8$ at the points $(1, 2)$ and (a, b) are perpendicular to each other, then a^2 is equal to :

- (1) $\frac{2}{17}$
- (2) $\frac{128}{17}$
- (3) $\frac{4}{17}$
- (4) $\frac{64}{17}$

Correct Answer: (1) $\frac{2}{17}$

Solution:

Equation of ellipse: $4x^2 + y^2 = 8$.

Equation of tangent at point $(1, 2)$: $4x(1) + y(2) = 8 \implies 4x + 2y = 8 \implies y = -2x + 4$.

Slope of this tangent $m_1 = -2$.

Since the tangent at (a, b) is perpendicular to the first tangent, its slope m_2 satisfies $m_1 m_2 = -1$.

$$(-2)m_2 = -1 \implies m_2 = \frac{1}{2}.$$

Equation of tangent at (a, b) is $4ax + by = 8 \implies y = -\frac{4a}{b}x + \frac{8}{b}$.

$$\text{Slope } m_2 = -\frac{4a}{b} = \frac{1}{2} \implies b = -8a.$$

Since (a, b) lies on the ellipse, it satisfies the equation:

$$4a^2 + b^2 = 8.$$

Substitute $b = -8a$:

$$4a^2 + (-8a)^2 = 8 \implies 4a^2 + 64a^2 = 8 \implies 68a^2 = 8.$$

$$a^2 = \frac{8}{68} = \frac{2}{17}.$$

💡 Quick Tip

Condition for perpendicular tangents: product of slopes is -1. Also, the locus of the intersection of perpendicular tangents is the director circle ($x^2 + y^2 = a^2 + b^2$).

83. The length of the perpendicular from the point $(2, -1, 4)$ on the straight line,

$$\frac{x+3}{10} = \frac{y-2}{-7} = \frac{z}{1} \text{ is :}$$

- (1) less than 2
- (2) greater than 2 but less than 3
- (3) greater than 3 but less than 4
- (4) greater than 4

Correct Answer: (3) greater than 3 but less than 4

Solution:

Let point $P = (2, -1, 4)$.

The line passes through point $A(-3, 2, 0)$ and has direction vector $\vec{d} = 10\hat{i} - 7\hat{j} + \hat{k}$.

Vector $\vec{AP} = (2 - (-3))\hat{i} + (-1 - 2)\hat{j} + (4 - 0)\hat{k} = 5\hat{i} - 3\hat{j} + 4\hat{k}$.

The square of the distance is given by $D^2 = |\vec{AP}|^2 - (\text{Projection of AP on } \vec{d})^2$.

$$|\vec{AP}|^2 = 5^2 + (-3)^2 + 4^2 = 25 + 9 + 16 = 50.$$

$$\text{Projection} = \frac{\vec{AP} \cdot \vec{d}}{|\vec{d}|}.$$

$$\vec{AP} \cdot \vec{d} = 5(10) + (-3)(-7) + 4(1) = 50 + 21 + 4 = 75.$$

$$|\vec{d}| = \sqrt{10^2 + (-7)^2 + 1^2} = \sqrt{100 + 49 + 1} = \sqrt{150} = 5\sqrt{6}.$$

$$\text{Projection} = \frac{75}{5\sqrt{6}} = \frac{15}{\sqrt{6}}.$$

$$\text{Square of Projection} = \frac{225}{6} = \frac{75}{2} = 37.5.$$

$$D^2 = 50 - 37.5 = 12.5.$$

$$D = \sqrt{12.5}.$$

Since $\sqrt{9} = 3$ and $\sqrt{16} = 4$, $\sqrt{12.5}$ lies between 3 and 4 (specifically ≈ 3.53).

💡 Quick Tip

Use the vector formula for perpendicular distance $d = \frac{|\vec{AP} \times \vec{d}|}{|\vec{d}|}$ or Pythagoras theorem with projection.

84. The equation of a plane containing the line of intersection of the planes $2x - y - 4 = 0$ and $y + 2z - 4 = 0$ and passing through the point $(1, 1, 0)$ is :

(1) $x - y - z = 0$

(2) $x + 3y + z = 4$

(3) $x - 3y - 2z = -2$

(4) $2x - z = 2$

Correct Answer: (1) $x - y - z = 0$

Solution:

The equation of any plane passing through the intersection of two planes P_1 and P_2 is $P_1 + \lambda P_2 = 0$.

$$(2x - y - 4) + \lambda(y + 2z - 4) = 0.$$

This plane passes through the point $(1, 1, 0)$. Substitute these coordinates:

$$(2(1) - 1 - 4) + \lambda(1 + 2(0) - 4) = 0.$$

$$(2 - 5) + \lambda(1 - 4) = 0.$$

$$-3 - 3\lambda = 0 \implies \lambda = -1.$$

Substitute $\lambda = -1$ back into the equation:

$$(2x - y - 4) - 1(y + 2z - 4) = 0.$$

$$2x - y - 4 - y - 2z + 4 = 0.$$

$$2x - 2y - 2z = 0.$$

Dividing by 2: $x - y - z = 0$.

💡 Quick Tip
Family of planes concept ($P_1 + \lambda P_2 = 0$) is the most efficient way to solve problems involving the line of intersection of two planes.

85. The magnitude of the projection of the vector $2\hat{i} + 3\hat{j} + \hat{k}$ on the vector perpendicular to the plane containing the vectors $\hat{i} + \hat{j} + \hat{k}$ and $\hat{i} + 2\hat{j} + 3\hat{k}$, is :

(1) $3\sqrt{6}$

(2) $\frac{\sqrt{3}}{2}$

(3) $\sqrt{6}$

(4) $\sqrt{\frac{3}{2}}$

Correct Answer: (4) $\sqrt{\frac{3}{2}}$

Solution:

Let $\vec{a} = 2\hat{i} + 3\hat{j} + \hat{k}$.

Let the vectors in the plane be $\vec{b} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} + 2\hat{j} + 3\hat{k}$.

The vector perpendicular to the plane is the cross product $\vec{n} = \vec{b} \times \vec{c}$.

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & 2 & 3 \end{vmatrix} = \hat{i}(3-2) - \hat{j}(3-1) + \hat{k}(2-1) = \hat{i} - 2\hat{j} + \hat{k}.$$

The projection of $\vec{a}$ on $\vec{n}$ is $\frac{|\vec{a} \cdot \vec{n}|}{|\vec{n}|}$.

$$\vec{a} \cdot \vec{n} = (2)(1) + (3)(-2) + (1)(1) = 2 - 6 + 1 = -3.$$

$$|\vec{n}| = \sqrt{1^2 + (-2)^2 + 1^2} = \sqrt{6}.$$

$$\text{Magnitude of Projection} = \frac{|-3|}{\sqrt{6}} = \frac{3}{\sqrt{6}} = \frac{\sqrt{3} \cdot \sqrt{3}}{\sqrt{2} \cdot \sqrt{3}} = \sqrt{\frac{3}{2}}.$$

💡 Quick Tip

The vector perpendicular to a plane containing $\vec{b}$ and $\vec{c}$ is $\vec{b} \times \vec{c}$. Projection of $\vec{a}$ on $\vec{n}$ is $\frac{\vec{a} \cdot \vec{n}}{|\vec{n}|}$.

86. The mean and variance of seven observations are 8 and 16, respectively. If 5 of the observations are 2, 4, 10, 12, 14, then the product of the remaining two observations is :

- (1) 40
- (2) 49
- (3) 45
- (4) 48

Correct Answer: (4) 48

Solution:

Let the two unknown observations be x and y .

Total number of observations $n = 7$.

$$\text{Mean } \bar{x} = 8 \implies \sum x_i = 7 \times 8 = 56.$$

$$\text{Given sum} = 2 + 4 + 10 + 12 + 14 = 42.$$

$$x + y = 56 - 42 = 14.$$

$$\text{Variance } \sigma^2 = \frac{\sum x_i^2}{n} - (\bar{x})^2 = 16.$$

$$\frac{\sum x_i^2}{7} - 64 = 16 \implies \sum x_i^2 = 7(80) = 560.$$

Sum of squares of given observations = $2^2 + 4^2 + 10^2 + 12^2 + 14^2 = 4 + 16 + 100 + 144 + 196 = 460$.

$$x^2 + y^2 = 560 - 460 = 100.$$

We have system: $x + y = 14$ and $x^2 + y^2 = 100$.

Use identity $(x + y)^2 = x^2 + y^2 + 2xy$.

$$14^2 = 100 + 2xy \implies 196 = 100 + 2xy.$$

$$2xy = 96 \implies xy = 48.$$

💡 Quick Tip

Variance formula: $\sigma^2 = \frac{\sum x^2}{N} - (\bar{x})^2$. Always useful to solve for $\sum x^2$.

87. Let A and B be two non-null events such that $A \subset B$. Then, which of the following statements is always correct ?

- (1) $P(A|B) = 1$
- (2) $P(A|B) \leq P(A)$
- (3) $P(A|B) \geq P(A)$
- (4) $P(A|B) = P(B) - P(A)$

Correct Answer: (3) $P(A|B) \geq P(A)$

Solution:

Conditional Probability $P(A|B) = \frac{P(A \cap B)}{P(B)}$.

Since $A \subset B$, we have $A \cap B = A$.

Therefore, $P(A|B) = \frac{P(A)}{P(B)}$.

Since B is an event in the sample space, $P(B) \leq 1$.

Dividing $P(A)$ by a number less than or equal to 1 increases (or keeps constant) the value.

$$\frac{P(A)}{P(B)} \geq P(A) \text{ (assuming } P(A) > 0 \text{)}.$$

Thus, $P(A|B) \geq P(A)$.

 Quick Tip

If $A \subset B$, then A occurring implies B occurring, but given B has occurred, the probability of A (which is a subset) becomes relative to the size of B , which is ≤ 1 .

88. If $\cos(\alpha + \beta) = \frac{3}{5}$, $\sin(\alpha - \beta) = \frac{5}{13}$ and $0 < \alpha, \beta < \frac{\pi}{4}$, then $\tan(2\alpha)$ is equal to :

- (1) $\frac{21}{16}$
- (2) $\frac{63}{16}$
- (3) $\frac{63}{52}$
- (4) $\frac{33}{52}$

Correct Answer: (2) $\frac{63}{16}$

Solution:

Let $A = \alpha + \beta$ and $B = \alpha - \beta$.

Then $A + B = (\alpha + \beta) + (\alpha - \beta) = 2\alpha$.

Given $\cos A = \frac{3}{5}$ and A is acute (since sum of acute angles). $\tan A = \frac{4}{3}$.

Given $\sin B = \frac{5}{13}$ and B is acute. $\tan B = \frac{5}{12}$.

$$\tan(2\alpha) = \tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}.$$

$$\tan(2\alpha) = \frac{\frac{4}{3} + \frac{5}{12}}{1 - \frac{4}{3} \cdot \frac{5}{12}}.$$

$$\text{Numerator} = \frac{16+5}{12} = \frac{21}{12}.$$

$$\text{Denominator} = 1 - \frac{20}{36} = \frac{36-20}{36} = \frac{16}{36} = \frac{4}{9}.$$

$$\tan(2\alpha) = \frac{21}{12} \times \frac{9}{4} = \frac{7}{4} \times \frac{9}{4} = \frac{63}{16}.$$

 Quick Tip

Express the required angle (2α) as the sum or difference of the given compound angles ($\alpha + \beta$ and $\alpha - \beta$).

89. If $\alpha = \cos^{-1}(\frac{3}{5})$, $\beta = \tan^{-1}(\frac{1}{3})$, where $0 < \alpha, \beta < \frac{\pi}{2}$, then $\alpha - \beta$ is equal to :

- (1) $\tan^{-1}(\frac{9}{14})$
- (2) $\sin^{-1}(\frac{9}{5\sqrt{10}})$
- (3) $\cos^{-1}(\frac{9}{5\sqrt{10}})$

(4) $\tan^{-1}\left(\frac{9}{5\sqrt{10}}\right)$

Correct Answer: (2) $\sin^{-1}\left(\frac{9}{5\sqrt{10}}\right)$

Solution:

From $\alpha = \cos^{-1} \frac{3}{5}$, we get $\tan \alpha = \frac{4}{3}$.

From $\beta = \tan^{-1} \frac{1}{3}$, we get $\tan \beta = \frac{1}{3}$.

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta} = \frac{\frac{4}{3} - \frac{1}{3}}{1 + \frac{4}{3} \cdot \frac{1}{3}} = \frac{\frac{1}{3}}{1 + \frac{4}{9}} = \frac{\frac{1}{3}}{\frac{13}{9}} = \frac{9}{13}.$$

This gives $\alpha - \beta = \tan^{-1} \frac{9}{13}$. This does not match Option 1.

Let's convert to sine. Construct a triangle with opposite = 9, adjacent = 13.

$$\text{Hypotenuse } h = \sqrt{9^2 + 13^2} = \sqrt{81 + 169} = \sqrt{250} = 5\sqrt{10}.$$

$$\sin(\alpha - \beta) = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{9}{5\sqrt{10}}.$$

This matches Option (2).

 Quick Tip

Calculate $\tan(\alpha - \beta)$ first, then assume a triangle to convert the result into $\sin^{-1}$ or $\cos^{-1}$ to match the options.

90. The contrapositive of the statement "If you are born in India, then you are a citizen of India", is :

- (1) If you are born in India, then you are not a citizen of India.
- (2) If you are a citizen of India, then you are born in India.
- (3) If you are not born in India, then you are not a citizen of India.
- (4) If you are not a citizen of India, then you are not born in India.

Correct Answer: (4) If you are not a citizen of India, then you are not born in India.

Solution:

Let p be the statement "You are born in India".

Let q be the statement "You are a citizen of India".

The given statement is an implication: $p \rightarrow q$.

The contrapositive of an implication $p \rightarrow q$ is $\sim q \rightarrow \sim p$.

$\sim q$: "You are not a citizen of India".

$\sim p$: "You are not born in India".

Therefore, the contrapositive is: "If you are not a citizen of India, then you are not born in India."

 Quick Tip

The contrapositive of "If P then Q" is always "If not Q then not P". Both statements are logically equivalent.
