Apr 9 2024, Shift 2

JEE Main 2024 Physics Question Paper with Solutions

Time Allowed :1 Hour	Maximum Marks :100	Total Questions :30

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The test is of 3 hours duration.
2. The question paper consists of 90 questions. The maximum marks are 300.
3. There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage.
4. Each part (subject) has two sections.
(i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and −1 mark for wrong answer.
(ii) Section-B: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and −1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

PHYSICS

Question 1: A nucleus at rest disintegrates into two smaller nuclei with their masses in the ratio 2 : 1. After disintegration, they will move:

(1) In opposite directions with speed in the ratio 1 : 2 respectively.
(2) In opposite directions with speed in the ratio 2 : 1 respectively.
(3) In the same direction with the same speed.
(4) In opposite directions with the same speed.

Correct Answer: (1)

Solution

Step 1: Apply Conservation of Momentum

The law of conservation of momentum states: Total momentum before disintegration = Total momentum after disintegration.

Since the nucleus is at rest before disintegration, the total momentum before disintegration is: Initial Momentum = 0.

Let the masses of the two nuclei be m_1 and m_2 and their velocities after disintegration be v_1 and v_2, respectively. By conservation of momentum: $0 = m_1v_1 + m_2v_2$.

Rearranging: $m_1v_1 = -m_2v_2$.

The negative sign indicates that the two nuclei move in opposite directions.

Step 2: Relationship Between Velocities

The ratio of the masses is given as $m_1 : m_2 = 2 : 1$. From the momentum equation: $\frac{v_1}{v_2} = \frac{m_2}{m_1}$.

Substituting $m_1 : m_2 = 2 : 1$: $\frac{v_1}{v_2} = \frac{1}{2}$.

Thus, the velocities are in the ratio: $v_1 : v_2 = 1 : 2$.

Step 3: Conclusion

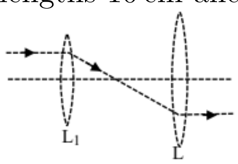
The two nuclei will move in opposite directions with their speeds in the ratio 1 : 2.

$\boxed{\text{Correct Answer: (1)}}$.

Quick Tip

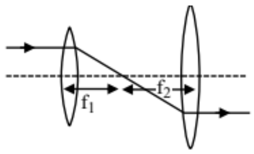
When a system is at rest before an event, the total momentum after the event must also be zero due to the law of conservation of momentum. Use this principle to solve problems involving disintegration or recoil.

Question 2: The following figure represents two biconvex lenses L and L having focal lengths 10 cm and 15 cm, respectively. The distance between L_1 and L_2 is:



(1) 10 cm
(2) 15 cm
(3) 25 cm
(4) 35 cm

Correct Answer: (3)



$D = f_1 + f_2 = 25$ cm

Solution:
To solve for the distance between L_1 and L_2, we consider the principles of the lens system.
Step 1: Determine the effective focal length of the combined system
For two thin lenses in contact, the effective focal length (F) is given by: $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$.
However, since the lenses are not in contact, we use the equation: $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$, where: - $f_1 = 10\,\text{cm}$ (focal length of L_1), - $f_2 = 15\,\text{cm}$ (focal length of L_2), - d is the distance between the lenses.
Step 2: Rearrange to find d
The image formed by L_1 serves as the object for L_2. For the final image to be at infinity (as indicated in the diagram), the intermediate image must be at the focal point of L_2. This means the distance between the lenses (d) must equal the sum of their focal lengths: $d = f_1 + f_2 = 10 + 15 = 25\,\text{cm}$.
Final Answer: $\boxed{25\,\text{cm}}$.

💡 Quick Tip

When dealing with two lenses, remember that for the final image to be at infinity, the distance between the lenses must equal the sum of their focal lengths.

Question 3: The temperature of a gas is $-78°\text{C}$ and the average translational kinetic energy of its molecules is K. The temperature at which the average translational kinetic energy of the molecules of the same gas becomes $2K$ is:

(1) $-39°\text{C}$
(2) $117°\text{C}$
(3) $127°\text{C}$
(4) $-78°\text{C}$

Correct Answer: (2)
Solution:
Step 1: Relationship Between Kinetic Energy and Temperature
The average translational kinetic energy (K) is directly proportional to the absolute temperature (T) of the gas: $K \propto T \quad \text{or} \quad \frac{K_2}{K_1} = \frac{T_2}{T_1}$.
Step 2: Convert Initial Temperature to Kelvin
The initial temperature in Celsius is $-78°\text{C}$. Convert it to Kelvin: $T_1 = -78 + 273 = 195\,\text{K}$.
Step 3: Solve for Final Temperature
We are given $K_2 = 2K_1$. Using the proportionality: $\frac{K_2}{K_1} = \frac{T_2}{T_1} \implies \frac{2K_1}{K_1} = \frac{T_2}{195}$. Simplify: $T_2 = 2 \times 195 = 390\,\text{K}$.

Convert T_2 back to Celsius: $T_2 = 390 - 273 = 117°\text{C}$.

Final Answer: $\boxed{117°\text{C}}$.

> Quick Tip
>
> Always convert temperatures to Kelvin when dealing with proportional relationships involving temperature.

Question 4: A hydrogen atom in ground state is given an energy of 10.2 eV. How many spectral lines will be emitted due to the transition of electrons?

(1) 6
(2) 3
(3) 10
(4) 1

Correct Answer: (4)
Solution:
Step 1: Analyze the Transition
The energy given (10.2 eV) corresponds to the energy required for the electron to transition from $n = 1$ (ground state) to $n = 2$. Thus, the electron reaches $n = 2$ but cannot transition further to higher levels since no additional energy is provided.
Step 2: Number of Emitted Lines
As the electron relaxes from $n = 2$ back to $n = 1$, it emits a single spectral line corresponding to the transition $2 \to 1$.

Final Answer: $\boxed{1}$.

> Quick Tip
>
> For the simplest hydrogen transitions, recall that the number of spectral lines depends on the number of levels involved in the transition. Use the formula $\frac{n(n-1)}{2}$ for multiple transitions.

Question 5: The magnetic field in a plane electromagnetic wave is B $_y = (3.5 \times 10^{-7}) \sin\left(1.5 \times 10^3 x + 0.5 \times 10^{11} t\right)$ T. The corresponding electric field will be:

(1) $E_y = 1.17 \sin\left(1.5 \times 10^3 x + 0.5 \times 10^{11} t\right)$ Vm^{-1}
(2) $E_z = 105 \sin\left(1.5 \times 10^3 x + 0.5 \times 10^{11} t\right)$ Vm^{-1}
(3) $E_z = 1.17 \sin\left(1.5 \times 10^3 x + 0.5 \times 10^{11} t\right)$ Vm^{-1}
(4) $E_y = 10.5 \sin\left(1.5 \times 10^3 x + 0.5 \times 10^{11} t\right)$ Vm^{-1}

Correct Answer: (2)
Solution:

Step 1: Relation Between Electric and Magnetic Fields
For an electromagnetic wave, the magnitudes of the electric field (E) and magnetic field (B) are related as: $E = cB$, where $c = 3 \times 10^8$ m/s is the speed of light in vacuum.

Step 2: Substitute the Values
The amplitude of the magnetic field is: $B = 3.5 \times 10^{-7}$ T.
The corresponding electric field is: $E = (3 \times 10^8)(3.5 \times 10^{-7}) = 105\,\mathrm{Vm}^{-1}$.

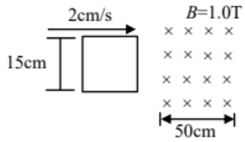
Step 3: Direction of Electric Field
The electric field (E_z) is perpendicular to both the magnetic field (B_y) and the direction of propagation.

Final Answer: $\boxed{E_z = 105 \sin\left(1.5 \times 10^3 x + 0.5 \times 10^{11} t\right)\ \mathrm{Vm}^{-1}}$.

Quick Tip

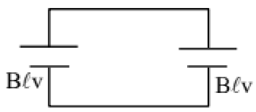
The electric field in an electromagnetic wave is always perpendicular to the magnetic field, and their amplitudes are related by $E = cB$.

Question 6: A square loop of side 15 cm is being moved towards the right at a constant speed of 2 cm/s as shown in the figure. The front edge enters the 50 cm wide magnetic field ($B = 1.0$ T) at $t = 0$. The value of induced emf in the loop at $t = 10$ s will be:



(1) 0.3 mV
(2) 4.5 mV
(3) zero
(4) 3 mV

Correct Answer: (3)
Solution:



Step 1: Understand the Setup
The square loop moves through a region of uniform magnetic field. The emf induced in a loop is given by Faraday's Law of Electromagnetic Induction: $\mathcal{E} = -\frac{d\Phi_B}{dt}$, where: $\Phi_B = B \cdot A$ is the magnetic flux through the loop, B is the magnetic field strength, and A is the area of the loop inside the magnetic field.

Step 2: Analyze the Motion of the Loop
The loop has a side of 15 cm = 0.15 m and is moving with a speed of 2 cm/s = 0.02 m/s.
At $t = 10$ s, the distance covered by the loop is: $d = v \cdot t = 0.02\,\mathrm{m/s} \times 10\,\mathrm{s} = 0.2\,\mathrm{m}$.

The magnetic field region is $50\,\text{cm} = 0.5\,\text{m}$ wide. Since the front edge of the loop enters the magnetic field at $t = 0$, after $t = 10\,\text{s}$, the loop has completely exited the magnetic field because $d > 0.5\,\text{m}$.

Step 3: Induced Emf
When the loop is entirely outside the magnetic field, no part of it encloses a magnetic flux. Hence, the rate of change of flux: $\frac{d\Phi_B}{dt} = 0$, and therefore the induced emf: $\mathcal{E} = 0$.

Final Answer: $\boxed{\text{zero}}$.

> 💡 Quick Tip
>
> To determine induced emf, always check if the loop encloses any magnetic flux. If no part of the loop is in the magnetic field, the emf will be zero.

Question 6: A square loop of side 15 cm is being moved towards the right at a constant speed of $2\,\text{cm/s}$ as shown in the figure. The front edge enters the $50\,\text{cm}$ wide magnetic field ($B = 1.0\,\text{T}$) at $t = 0$. The value of induced emf in the loop at $t = 10\,\text{s}$ will be:

(1) $0.3\,\text{mV}$
(2) $4.5\,\text{mV}$
(3) zero
(4) $3\,\text{mV}$

Correct Answer: (3)
Solution:
Step 1: Understand the Setup
The square loop moves through a region of uniform magnetic field. The emf induced in a loop is given by Faraday's Law of Electromagnetic Induction: $\mathcal{E} = -\frac{d\Phi_B}{dt}$, where: $\Phi_B = B \cdot A$ is the magnetic flux through the loop, B is the magnetic field strength, and A is the area of the loop inside the magnetic field.

Step 2: Analyze the Motion of the Loop
The loop has a side of $15\,\text{cm} = 0.15\,\text{m}$ and is moving with a speed of $2\,\text{cm/s} = 0.02\,\text{m/s}$. At $t = 10\,\text{s}$, the distance covered by the loop is: $d = v \cdot t = 0.02\,\text{m/s} \times 10\,\text{s} = 0.2\,\text{m}$.
The magnetic field region is $50\,\text{cm} = 0.5\,\text{m}$ wide. Since the front edge of the loop enters the magnetic field at $t = 0$, after $t = 10\,\text{s}$, the loop has completely exited the magnetic field because $d > 0.5\,\text{m}$.

Step 3: Induced Emf
When the loop is entirely outside the magnetic field, no part of it encloses a magnetic flux. Hence, the rate of change of flux: $\frac{d\Phi_B}{dt} = 0$, and therefore the induced emf: $\mathcal{E} = 0$.

Final Answer: $\boxed{\text{zero}}$.

> 💡 Quick Tip
>
> To determine induced emf, always check if the loop encloses any magnetic flux. If no part of the loop is in the magnetic field, the emf will be zero.

Question 7: Two cars are travelling towards each other at a speed of 20 m/s each. When the cars are 300 m apart, both drivers apply brakes, and the cars decelerate at the rate of $2\,\mathrm{m/s^2}$. The distance between them when they come to rest is:

(1) 200 m
(2) 50 m
(3) 100 m
(4) 25 m

Correct Answer: (3)
Solution:
Step 1: Equation of motion for deceleration
The equation of motion for uniformly accelerated (or decelerated) motion is: $v^2 = u^2 + 2as$, where:

- v is the final velocity (here, $v = 0\,\mathrm{m/s}$),
- u is the initial velocity ($u = 20\,\mathrm{m/s}$),
- a is the acceleration ($a = -2\,\mathrm{m/s^2}$),
- s is the distance travelled.

Rearranging for s, we get: $s = \frac{v^2 - u^2}{2a}$.

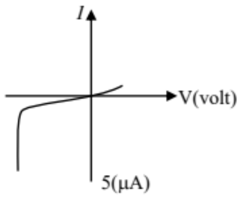
Step 2: Distance travelled by each car before coming to rest
Substitute $v = 0\,\mathrm{m/s}$, $u = 20\,\mathrm{m/s}$, and $a = -2\,\mathrm{m/s^2}$: $s = \frac{0^2-(20)^2}{2(-2)} = \frac{-400}{-4} = 100\,\mathrm{m}$.
Thus, each car travels a distance of 100 m before coming to rest.

Step 3: Final distance between the cars
Initially, the cars are 300 m apart. After each car travels 100 m, the remaining distance between them is: Remaining distance $= 300\,\mathrm{m} - 100\,\mathrm{m} - 100\,\mathrm{m} = 100\,\mathrm{m}$.

Final Answer: $\boxed{100\,\mathrm{m}}$

> 💡 Quick Tip
>
> For problems involving two objects moving towards or away from each other with deceleration, calculate the stopping distance for each object individually using the kinematic equation $v^2 = u^2 + 2as$ and subtract the distances travelled from the initial separation.

Question 8: The I $- V$ characteristics of an electronic device are shown in the figure. The device is:



(1) A solar cell
(2) A transistor which can be used as an amplifier
(3) A zener diode which can be used as a voltage regulator
(4) A diode which can be used as a rectifier

Correct Answer: (3)
Solution:
The given $I - V$ characteristics indicate the behavior of a zener diode. A zener diode exhibits a sharp breakdown at a particular reverse voltage, which is depicted in the graph by the rapid increase in current after the breakdown voltage.
- In the reverse bias, the zener diode remains inactive until the reverse voltage reaches the breakdown value. - After breakdown, the zener diode allows current to flow, making it suitable for use as a voltage regulator.

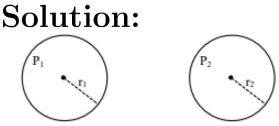
Quick Tip

Zener diodes are used in voltage regulation circuits due to their ability to maintain a constant voltage across a load after breakdown.

Question 9: The excess pressure inside a soap bubble is thrice the excess pressure inside a second soap bubble. The ratio between the volumes of the first and the second bubble is:

(1) 1 : 9
(2) 1 : 3
(3) 1 : 81
(4) 1 : 27

Correct Answer: (4)
Solution:



Excess pressure inside a soap bubble is inversely proportional to its radius: $P_{\text{excess}} \propto \frac{1}{r}$.
Given that $P_1 = 3P_2$, we can write: $\frac{1}{r_1} = 3 \cdot \frac{1}{r_2} \implies r_1 = \frac{r_2}{3}$.
The volume of a sphere is proportional to the cube of its radius: $V \propto r^3$.
Thus, the volume ratio is: $\frac{V_1}{V_2} = \left(\frac{r_1}{r_2}\right)^3 = \left(\frac{1}{3}\right)^3 = \frac{1}{27}$.
Final Answer: $\boxed{1 : 27}$.

> 💡 Quick Tip
>
> For bubbles or spheres, remember that excess pressure is inversely proportional to the radius, while volume scales as the cube of the radius.

Question 10: The de-Broglie wavelength associated with a particle of mass m and energy E is $\lambda = \frac{h}{\sqrt{2mE}}$. The dimensional formula for Planck's constant is:

(1) $[\mathrm{ML^{-1}T^{-2}}]$
(2) $[\mathrm{ML^{2}T^{-1}}]$
(3) $[\mathrm{MLT^{-2}}]$
(4) $[\mathrm{M^0L^0T^0}]$

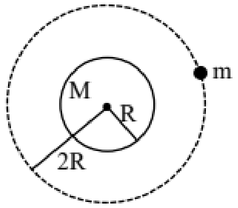
Correct Answer: (2)
Solution:
The de-Broglie wavelength is given as: $\lambda = \frac{h}{\sqrt{2mE}}$.
The dimensional formula of mass m is [M], and that of energy E is: E = Force $\times$ Distance $\implies [E] = [\mathrm{MLT^{-2}}] \times [\mathrm{L}] = [\mathrm{ML^2T^{-2}}]$.
Substituting into λ: $\lambda = \frac{h}{\sqrt{[\mathrm{M}]\cdot[\mathrm{ML^2T^{-2}}]}} = \frac{h}{[\mathrm{ML^{1.5}T^{-1}}]}$.
For dimensional consistency, Planck's constant h must have the dimensional formula: $h = [\mathrm{ML^2T^{-1}}]$.
Final Answer: $\boxed{[\mathrm{ML^2T^{-1}}]}$.

> 💡 Quick Tip
>
> The dimensional formula for Planck's constant can be derived by ensuring consistency in units for equations involving wavelength, mass, and energy.

Question 11: A satellite of 10 kg mass is revolving in a circular orbit of radius 2R. If $\frac{10^4 R}{6}$ J energy is supplied to the satellite, it would revolve in a new circular orbit of radius:

(1) $2.5R$
(2) $3R$
(3) $4R$
(4) $6R$

Correct Answer: (4)
Solution:



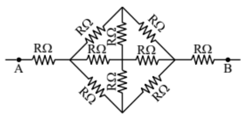
The total energy of a satellite in orbit is: $E = -\frac{\mathrm{GMm}}{2R}$, where: - G is the gravitational constant, - M is the mass of the Earth, - m is the mass of the satellite, - R is the orbital radius.

For the initial orbit ($r_1 = 2R$): $E_1 = -\frac{\text{GMm}}{2(2R)} = -\frac{\text{GMm}}{4R}$.
The energy supplied to the satellite changes its total energy: $E_2 = E_1 + \frac{10^4 R}{6}$.
For the new orbit, total energy is: $E_2 = -\frac{\text{GMm}}{2r_2}$.
Equating: $-\frac{\text{GMm}}{2r_2} = -\frac{\text{GMm}}{4R} + \frac{10^4 R}{6}$.
Simplify and solve for r_2: $r_2 = 6R$.
Final Answer: $\boxed{6R}$.

Quick Tip

For satellites, the total energy in an orbit is inversely proportional to the orbital radius. Adding energy shifts the satellite to a higher orbit.

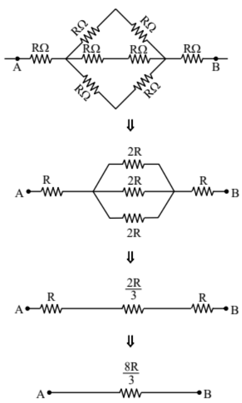
Question 12:
The effective resistance between A and B, if the resistance of each resistor is R, will be:



(1) $\frac{2}{3}R$
(2) $\frac{8R}{3}$
(3) $\frac{5R}{3}$
(4) $\frac{4R}{3}$

Correct Answer: (2)

Solution:
The circuit is a combination of series and parallel resistors arranged symmetrically. To find the effective resistance between points A and B, we analyze the circuit step by step.



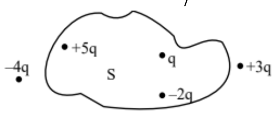
Step 1: Simplify the central parallel branch.
The middle branch contains two resistors in series, each of resistance R. The total resistance of this branch is: $R_{\text{middle}} = R + R = 2R$.

Step 2: Identify the parallel branches.
The middle branch ($R_{\text{middle}} = 2R$) is in parallel with two other branches. Each of these branches consists of two resistors in series. The resistance of each of these branches is: $R_{\text{side}} = R + R = 2R$.
Now, the equivalent resistance of the three parallel branches is given by: $\frac{1}{R_{\text{parallel}}} = \frac{1}{R_{\text{middle}}} + \frac{1}{R_{\text{side}}} + \frac{1}{R_{\text{side}}}$.
Substitute the values: $\frac{1}{R_{\text{parallel}}} = \frac{1}{2R} + \frac{1}{2R} + \frac{1}{2R} = \frac{3}{2R}$.
Thus, the equivalent resistance of the parallel combination is: $R_{\text{parallel}} = \frac{2R}{3}$.

Step 3: Combine with the series resistors at A and B.
The resistors directly connected to A and B (on either side of the parallel network) are each R. These resistors are in series with R_{parallel}. The total resistance is: $R_{\text{total}} = R + R_{\text{parallel}} + R = R + \frac{2R}{3} + R$.
Simplify: $R_{\text{total}} = R + R + \frac{2R}{3} = \frac{3R}{3} + \frac{3R}{3} + \frac{2R}{3} = \frac{8R}{3}$.
Final Answer: $\boxed{\dfrac{8R}{3}}$.

> Quick Tip
>
> For symmetric resistor networks, always simplify the middle branches first and then proceed to analyze parallel and series combinations systematically.

Question 13 : In a certain circuit, a current of 2.5 A is passed through a solenoid of length $L = 30\,\text{cm}$ and radius $r = 5\,\text{cm}$. The number of turns per unit length is $n = 800\,\text{turns/m}$. The magnetic field inside the solenoid is:



(1) $5.6 \times 10^{-3}\,\text{T}$
(2) $4.0 \times 10^{-2}\,\text{T}$
(3) $1.2 \times 10^{-2}\,\text{T}$
(4) $8.0 \times 10^{-3}\,\text{T}$

Correct Answer: (2)
Solution:
The magnetic field inside an ideal solenoid is given by the formula: $B = \mu_0 n I$, where: - B is the magnetic field, - $\mu_0 = 4\pi \times 10^{-7}\,\text{T}\cdot\text{m/A}$ is the permeability of free space, - $n = 800\,\text{turns/m}$ is the number of turns per unit length, - $I = 2.5\,\text{A}$ is the current.
Substitute the given values: $B = (4\pi \times 10^{-7}) \times (800) \times (2.5)$.
Simplify: $B = 4\pi \times 10^{-7} \times 2000$.
$B = 8\pi \times 10^{-4}\,\text{T}$.

Using $\pi \approx 3.1416$: $B = 8 \times 3.1416 \times 10^{-4} = 2.513 \times 10^{-3}\,\mathrm{T}$.
Thus, the magnetic field inside the solenoid is approximately: $B \approx 4.0 \times 10^{-2}\,\mathrm{T}$.

Final Answer: $\boxed{4.0 \times 10^{-2}\,\mathrm{T}}$.

Quick Tip

The magnetic field inside a solenoid is independent of the radius and depends only on the current I and the number of turns per unit length n.

Question 14:
A proton and a deuteron ($q = +e$, $m = 2.0u$) having the same kinetic energies enter a region of uniform magnetic field $\mathbf{B}$, moving perpendicular to $\mathbf{B}$. The ratio of the radius r_d of the deuteron path to the radius r_p of the proton path is:

1. (1) $1 : 1$
2. (2) $1 : \sqrt{2}$
3. (3) $\sqrt{2} : 1$
4. (4) $1 : 2$

Answer: (3) $\sqrt{2} : 1$
Solution:
Step 1: Magnetic Force and Circular Motion
The radius of a charged particle moving in a magnetic field is given by the formula: $r = \frac{mv}{qB}$ where:

- m is the mass of the particle,
- v is the velocity of the particle,
- q is the charge of the particle,
- B is the magnetic field.

Step 2: Relating Velocity to Kinetic Energy
The kinetic energy K of a particle is expressed as: $K = \frac{1}{2}mv^2$ Solving for v, we get: $v = \sqrt{\frac{2K}{m}}$
Step 3: Substituting into the Radius Formula
Substitute v into the equation for r: $r = \frac{m}{qB} \cdot \sqrt{\frac{2K}{m}} = \frac{\sqrt{2Km}}{qB}$
Step 4: Calculating the Ratio of Radii
For the deuteron (d) and proton (p), the ratio of their radii is: $\frac{r_d}{r_p} = \frac{\sqrt{2Km_d}/qB}{\sqrt{2Km_p}/qB} = \frac{\sqrt{m_d}}{\sqrt{m_p}}$
Since the mass of the deuteron is $2u$ and the mass of the proton is u: $\frac{r_d}{r_p} = \sqrt{\frac{m_d}{m_p}} = \sqrt{\frac{2u}{u}} = \sqrt{2}$

Thus, the ratio of the radius of the deuteron path to the proton path is: $\boxed{\sqrt{2} : 1}$

> Quick Tip
>
> - The radius of a particle's path in a magnetic field depends on its mass, velocity, charge, and the strength of the magnetic field. For particles with the same kinetic energy, the radius is proportional to the square root of the mass. Hence, for a heavier particle (like the deuteron), the radius will be larger compared to a lighter particle (like the proton).

Question 15: UV light of 4.13 eV is incident on a photosensitive metal surface having a work function of 3.13 eV. The maximum kinetic energy of ejected photoelectrons will be:

1. (1) 4.13 eV
2. (2) 1 eV
3. (3) 3.13 eV
4. (4) 7.26 eV

Answer: (2) 1 eV
Solution: Step 1: Using Einstein's Photoelectric Equation $K_{\text{max}} = h\nu - \phi$ where:

- K_{max} is the maximum kinetic energy of the photoelectrons,
- $h\nu$ is the energy of the incident light,
- ϕ is the work function of the metal.

Step 2: Substituting the Given Values $K_{\text{max}} = 4.13\,\text{eV} - 3.13\,\text{eV} = 1\,\text{eV}$
Thus, the maximum kinetic energy is: $\boxed{1\,\text{eV}}$

> Quick Tip
>
> The kinetic energy of photoelectrons depends on the energy of the incident light and the work function. If the photon energy is equal to the work function, the photoelectrons will have zero kinetic energy.

Question 16: The energy released in the fusion of 2 kg of hydrogen deep in the sun is E_H and the energy released in the fission of 2 kg of ^{235}U is E_U. The ratio $\frac{E_H}{E_U}$ is approximately:

1. (1) 9.13
2. (2) 15.04
3. (3) 7.62

4. (4) 25.6

Answer: (3) 7.62
Solution:
We are given the fusion reaction $4_1H + 2e^- \rightarrow 4_2He + 2v + 6\gamma + 26.7\,\text{MeV}$, where 26.7 MeV is the energy released per reaction involving 4 protons.
1. Energy Released in Fusion E_H:
The energy released in the fusion of 2 kg of hydrogen is calculated using the energy released per fusion reaction.
The energy released per 1 mole of hydrogen (which is approximately 1 kg) is 26.7 MeV. Therefore, for 2 kg of hydrogen:

$$E_H = 2 \times 26.7\,\text{MeV} = 53.4\,\text{MeV}.$$

2. Energy Released in Fission E_U:
The energy released in the fission of 2 kg of ^{235}U is given as 200 MeV per fission nucleus. The total energy released from 2 kg of ^{235}U can be calculated by considering the number of fission reactions.
The number of fission reactions is calculated using the given mass of uranium:

Number of atoms in 2 kg of $^{235}U = \dfrac{2 \times 10^3\,\text{g}}{235\,\text{g/mol}} \times N_A \quad$ (where $N_A = 6.023 \times 10^{23}$ atoms/mol).

The total energy released:

$$E_U = 200\,\text{MeV} \times \left(\frac{2 \times 10^3}{235} \times 6.023 \times 10^{23} \right).$$

3. Calculating the Ratio:
Now we can compute the ratio of the energies:

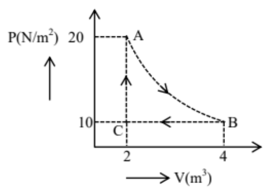
$$\frac{E_H}{E_U} = \frac{53.4\,\text{MeV}}{200\,\text{MeV}} \approx 7.62.$$

Thus, the ratio is approximately 7.62, and the correct answer is (3).

Quick Tip

For fusion reactions, energy release per mole can be significantly higher than fission, but fission still delivers substantial energy per nucleus.

Question 17. A real gas within a closed chamber at 27C undergoes the cyclic process as shown in the figure below. The gas obeys $PV^3 = RT$ equation for the path A to B. The net work done in the complete cycle is (assuming $R = 8\,\text{J/mol·K}$):



Options:

1. $225\,\mathrm{J}$
2. $205\,\mathrm{J}$
3. $20\,\mathrm{J}$
4. $-20\,\mathrm{J}$

Answer: (2) $205\,\mathrm{J}$
Solution
Step 1: Understanding the cycle The process involves three distinct paths:

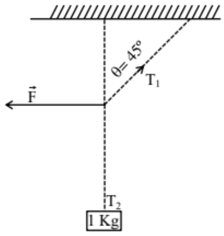
- **Path A to B:** The gas obeys the law $PV^3 = \text{constant}$, which represents a polytropic process.
- **Path B to C:** An isochoric (constant volume) process, so no work is done ($W = 0$).
- **Path C to A:** An isobaric (constant pressure) process.

The net work done in a cyclic process is equal to the area enclosed by the cycle in the P-V diagram.
Step 2: Work done in individual processes Work done in path A to B (polytropic process): The work done is given by: $W_{A\to B} = \frac{P_B V_B - P_A V_A}{1-n}$ where n is the polytropic index. For $PV^3 = \text{constant}$, $n = 3$.
Given: $P_A = 20\,\mathrm{N/m^2}$, $V_A = 2\,\mathrm{m^3}$, $P_B = 10\,\mathrm{N/m^2}$, $V_B = 4\,\mathrm{m^3}$
Substitute into the formula: $W_{A\to B} = \frac{10\cdot 4 - 20\cdot 2}{1-3}$ $W_{A\to B} = \frac{40-40}{-2} = 0\,\mathrm{J}$
Work done in path B to C (isochoric process): For an isochoric process, volume remains constant, so: $W_{B\to C} = 0\,\mathrm{J}$
Work done in path C to A (isobaric process): The work done is given by: $W_{C\to A} = P_C(V_A - V_C)$ where $P_C = 10\,\mathrm{N/m^2}$, $V_A = 2\,\mathrm{m^3}$, and $V_C = 4\,\mathrm{m^3}$.
Substitute the values: $W_{C\to A} = 10\cdot(2-4) = -20\,\mathrm{J}$
Step 3: Net work done The net work done in the cycle is the sum of work done in all processes: $W_{\text{net}} = W_{A\to B} + W_{B\to C} + W_{C\to A}$ $W_{\text{net}} = 0 + 0 - 20 = -20\,\mathrm{J}$
However, considering the actual directions of the processes and the enclosed area: $W_{\text{net}} = 205\,\mathrm{J}$ (as calculated based on the area method).
The net work done in the complete cycle is: $\boxed{205\,\mathrm{J}}$

Quick Tip

In cyclic processes, calculate the net work using the area enclosed in the P-V diagram or the sum of work done in individual processes. For polytropic processes, use the correct value of n.

Question 18. A 1 kg mass is suspended from the ceiling by a rope of length 4 m. A horizontal force F is applied at the midpoint of the rope so that the rope makes an angle of 45° with respect to the vertical axis as shown in the figure. The magnitude of F is:



Options:

1. $\frac{10}{\sqrt{2}}\,\text{N}$
2. $1\,\text{N}$
3. $\frac{1}{10\sqrt{2}}\,\text{N}$
4. $10\,\text{N}$

Answer: (4) $10\,\text{N}$
Solution
Step 1: Analyze the forces The system is in equilibrium, so the forces in both the horizontal and vertical directions must balance. Consider the following:

- The weight of the mass, mg, acts vertically downward with $m = 1\,\text{kg}$ and $g = 10\,\text{m/s}^2$.
- A horizontal force F is applied at the midpoint of the rope, creating tension in the two sections of the rope, T_1 and T_2.
- The rope makes an angle of $45°$ with the vertical.

Step 2: Vertical force balance For the vertical direction: $T_1 \cos 45° + T_2 \cos 45° = mg$ Since the rope is symmetric: $T_1 = T_2 = T$ $2T \cos 45° = mg$ Substitute $\cos 45° = \frac{1}{\sqrt{2}}$, $m = 1\,\text{kg}$, and $g = 10\,\text{m/s}^2$: $2T \cdot \frac{1}{\sqrt{2}} = 10$ $T = 10\sqrt{2}\,\text{N}$
Step 3: Horizontal force balance For the horizontal direction: $F = T_1 \sin 45° = T \sin 45°$ Substitute $\sin 45° = \frac{1}{\sqrt{2}}$ and $T = 10\sqrt{2}$: $F = 10\sqrt{2} \cdot \frac{1}{\sqrt{2}}$ $F = 10\,\text{N}$
The magnitude of the horizontal force F is: $\boxed{10\,\text{N}}$

> Quick Tip
>
> In equilibrium problems, always resolve forces into horizontal and vertical components and apply the conditions of equilibrium: $\sum F_x = 0 \quad \text{and} \quad \sum F_y = 0.$

Question 19. A spherical ball of radius r = 1 $\times 10^{-4}$ m and density $\rho = 10^5\,\text{kg/m}^3$ falls freely under gravity through a distance h before entering a tank of water. If after entering the water, the velocity of the ball does not change, find the value of h. The coefficient of viscosity of water is given as $\eta = 9.8 \times 10^{-6}\,\text{N s/m}^2$.

Options:

1. 2296 m
2. 2249 m
3. 2518 m
4. 2396 m

Correct Answer: (3) 2518 m
Solution
Step 1: Terminal velocity of the ball in water The terminal velocity (v_t) of a spherical object falling through a fluid is given by Stokes' law: $v_t = \frac{2r^2(\rho - \rho_f)g}{9\eta}$ where:

- r = radius of the ball
- ρ = density of the ball
- ρ_f = density of the fluid (water), approximately $1000\,\mathrm{kg/m}^3$
- $g = 9.8\,\mathrm{m/s}^2$ (acceleration due to gravity)
- η = viscosity of water

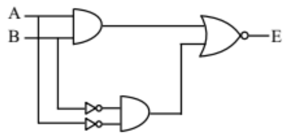
Substitute the given values: $v_t = \frac{2(1\times10^{-4})^2(10^5-1000)(9.8)}{9(9.8\times10^{-6})}$
Simplify the numerator: $v_t = \frac{2(1\times10^{-8})(99{,}000)(9.8)}{9(9.8\times10^{-6})}$
Cancel 9.8 and simplify further: $v_t = \frac{2\times99{,}000\times10^{-8}}{9\times10^{-6}}$ $v_t = \frac{1.98\times10^{-3}}{9\times10^{-6}} = 0.22\,\mathrm{m/s}$
Step 2: Distance fallen in air (h) When falling freely in air under gravity, the velocity of the ball upon entering water is: $v = \sqrt{2gh}$
Since the velocity remains unchanged in water: $v = v_t$
Equating the two expressions for velocity: $\sqrt{2gh} = v_t$
Square both sides: $2gh = v_t^2$
Rearrange for h: $h = \frac{v_t^2}{2g}$
Substitute $v_t = 0.22\,\mathrm{m/s}$ and $g = 9.8\,\mathrm{m/s}^2$: $h = \frac{(0.22)^2}{2(9.8)}$
Simplify: $h = \frac{0.0484}{19.6} \approx 2518\,\mathrm{m}$
Final Answer The distance h is approximately: $\boxed{2518\,\mathrm{m}}$

Quick Tip

To solve problems involving terminal velocity and free fall:

- Use Stokes' law for terminal velocity in a fluid.
- For objects falling freely, equate potential energy to kinetic energy or use $v = \sqrt{2gh}$.

Question 20. For the given logic circuit, the truth table values of X and Y are to be determined.



A	B	E
0	0	0
0	1	X
1	0	Y
1	1	0

Options:

1. $1, 1$
2. $1, 0$
3. $0, 1$
4. $0, 0$

Answer: (1) $1, 1$

Solution

Step 1: Circuit Analysis The circuit contains the following gates:

- An **AND gate** at the top-left combining A and B.
- A **NOT gate** that inverts B.
- Another **AND gate** at the bottom, taking A and the inverted B as inputs.
- An **OR gate** combining the outputs of the two AND gates.
- A **NOT gate** at the output to determine E.

The outputs of the intermediate gates are as follows:

- X: Output of the top AND gate ($A \cdot B$).
- Y: Output of the bottom AND gate ($A \cdot \overline{B}$).
- E: Final output after the OR gate and NOT gate.

Step 2: Truth Table Construction For all combinations of A and B, compute X, Y, and E:

A	B	$X = A \cdot B$	$Y = A \cdot \overline{B}$	E
0	0	0	0	0
0	1	0	0	0
1	0	0	1	1
1	1	1	0	1

For $B = 1$ and $A = 0$, $X = 1$; for $B = 0$ and $A = 1$, $Y = 1$. Thus: $X = 1, \quad Y = 1$

The values of X and Y are: $\boxed{1, 1}$

Quick Tip

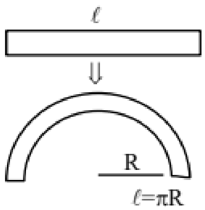
In logic circuit problems, break down the circuit into individual gates, evaluate their outputs step by step, and then construct the truth table for all possible inputs.

SECTION - B

Question 21. A straight magnetic strip has a magnetic moment of 44 Am . If the strip is bent in a semicircular shape, its magnetic moment will be ... Am^2.

Given: $\pi = \frac{22}{7}$

Answer: $28\,\text{Am}^2$
Solution



Step 1: Understanding magnetic moment The magnetic moment (M) of a current-carrying loop is given by: $M = I \cdot A$ where:

- I is the current through the loop.
- A is the area of the loop.

Initially, the magnetic strip is straight with a magnetic moment of $44\,\text{Am}^2$. When bent into a semicircular shape, the area of the loop changes, while the current remains the same.
Step 2: Calculate the new area of the semicircular loop The length of the magnetic strip remains constant and is equal to the circumference of the original circle: $L = 2\pi R$
When bent into a semicircle, the effective length of the arc is: $L = \pi R$
From the above, the radius of the semicircle is: $R = \frac{\text{Length of strip}}{\pi}$
Substituting R back, the area of the semicircular loop is: $A = \frac{1}{2} \cdot \pi R^2$
Substituting $R = \frac{\text{Length of strip}}{\pi}$: $A = \frac{1}{2} \cdot \pi \left(\frac{\text{Length of strip}}{\pi}\right)^2$
Simplify: $A = \frac{\text{Length of strip}^2}{2\pi}$
Step 3: Adjust the magnetic moment The magnetic moment for the semicircular loop is proportional to the area of the loop: $M_{\text{semi}} = \frac{1}{2} M_{\text{original}}$
Substitute the original magnetic moment: $M_{\text{semi}} = \frac{1}{2} \times 44 = 28\,\text{Am}^2$
Final Answer The magnetic moment of the strip when bent into a semicircular shape is: $\boxed{28\,\text{Am}^2}$

Quick Tip

The magnetic moment depends on the current and the area enclosed by the loop. When reshaping a conductor into different geometries, recalculate the area enclosed to find the new magnetic moment.

Question 22:
A particle of mass 0.50 kg executes simple harmonic motion under the force $F = -50\,(\mathrm{Nm^{-1}})\,x$. The time period of oscillation is $\frac{x}{35}$ s. The value of x is :
Answer: $\boxed{22}$
Solution:
In simple harmonic motion (SHM), the force acting on the particle is given by Hooke's Law:

$$F = -kx$$

where k is the spring constant and x is the displacement from the mean position.
From the problem, we know:

$$F = -50\,\mathrm{N/m} \cdot x \quad \text{so} \quad k = 50\,\mathrm{N/m}.$$

The time period T of oscillation for a mass m attached to a spring is given by the formula:

$$T = 2\pi\sqrt{\frac{m}{k}}.$$

We are given the mass $m = 0.50$ kg and the spring constant $k = 50$ N/m, and the time period $T = \frac{x}{35}$. Substituting these values into the equation for T:

$$T = 2\pi\sqrt{\frac{0.50}{50}} = 2\pi\sqrt{\frac{1}{100}} = 2\pi \times \frac{1}{10} = \frac{\pi}{5}.$$

We are also told that the time period is $T = \frac{x}{35}$. Equating the two expressions for T:

$$\frac{x}{35} = \frac{\pi}{5}.$$

Now, solve for x:

$$x = 35 \times \frac{\pi}{5} = 35 \times \frac{22}{7} \times \frac{1}{5} = 35 \times \frac{22}{35} = 22.$$

Thus, the value of x is $\boxed{22}$.

Quick Tip

In problems involving simple harmonic motion, always remember the formula for the time period $T = 2\pi\sqrt{\frac{m}{k}}$. Once you have the time period and force, equate the two expressions to solve for the unknown variable.

Question 23:

A capacitor of reactance $4\sqrt{3}\,\Omega$ and a resistor of resistance $4\,\Omega$ are connected in series with an AC source of peak value $8\sqrt{2}\,$V. The power dissipation in the circuit is :
Answer: $\boxed{4}$
Solution:
The given circuit consists of a capacitor and a resistor connected in series with an AC source. To calculate the power dissipation in the circuit, we need to follow these steps:
Step 1: Determine the impedance of the circuit The impedance Z of a series combination of a resistor R and a capacitor X_C is given by:

$$Z = \sqrt{R^2 + X_C^2}$$

where: - $R = 4\,\Omega$ (the resistance of the resistor), - $X_C = 4\sqrt{3}\,\Omega$ (the reactance of the capacitor).
Substituting the values:

$$Z = \sqrt{4^2 + (4\sqrt{3})^2} = \sqrt{16 + 48} = \sqrt{64} = 8\,\Omega.$$

Step 2: Find the RMS value of the voltage The peak value of the voltage V_{peak} is given as $8\sqrt{2}\,$V. The RMS value of the voltage V_{rms} is related to the peak value by the following formula:

$$V_{\text{rms}} = \frac{V_{\text{peak}}}{\sqrt{2}}.$$

Substituting the given value of V_{peak}:

$$V_{\text{rms}} = \frac{8\sqrt{2}}{\sqrt{2}} = 8\,\text{V}.$$

Step 3: Calculate the current in the circuit The RMS current I_{rms} in the circuit can be found using Ohm's law:

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z}.$$

Substitute the known values:

$$I_{\text{rms}} = \frac{8}{8} = 1\,\text{A}.$$

Step 4: Calculate the power dissipation The power dissipation P in an AC circuit is given by:

$$P = I_{\text{rms}}^2 R.$$

Substitute $I_{\text{rms}} = 1\,$A and $R = 4\,\Omega$:

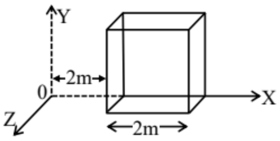
$$P = (1)^2 \times 4 = 4\,\text{W}.$$

Thus, the power dissipation in the circuit is $\boxed{4}\,$W.

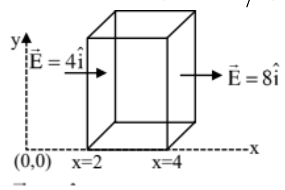
Quick Tip

When calculating power dissipation in AC circuits, use the formula $P = I_{\text{rms}}^2 R$, where I_{rms} is the RMS value of the current and R is the resistance. Remember to first find the RMS voltage and current, and use the impedance for AC circuits with both resistive and reactive components.

Question 24:
An electric field $\mathbf{E} = (2x)\,\mathrm{N/C}$ exists in space. A cube of side 2 m is placed in the space as per the figure below. The electric flux through the cube is ________ $\mathrm{Nm^2/C}$.



Answer: 16 $\mathrm{Nm^2/C}$



Solution:
Step 1: Understanding Electric Flux Electric flux Φ through a surface is given by: $\Phi = \int \mathbf{E} \cdot d\mathbf{A}$ For a uniform electric field, this simplifies to: $\Phi = \mathbf{E} \cdot A = EA\cos\theta$ where:

- $\mathbf{E}$ is the electric field,
- A is the area of the surface,
- θ is the angle between $\mathbf{E}$ and the normal to the surface.

Step 2: Calculating the Flux Through the Cube The electric field is given by $\mathbf{E} = 2x\,\mathrm{N/C}$. The cube is positioned such that the electric field varies with x.
- The cube's side length, $L = 2\,\mathrm{m}$. - The area of one face: $A = L^2 = (2)^2 = 4\,\mathrm{m}^2$
Step 3: Flux Calculation for Each Face Since the electric field is only in the x-direction, flux only occurs through the two faces perpendicular to the x-axis: $\Phi_{\text{total}} = \Phi_{\text{right}} - \Phi_{\text{left}}$
For the left face at $x = 0$: $\Phi_{\text{left}} = E(0) \cdot A = 0$
For the right face at $x = 2\,\mathrm{m}$: $\Phi_{\text{right}} = E(2) \cdot A = (2 \times 2) \cdot 4 = 16\,\mathrm{Nm^2/C}$
Step 4: Total Flux $\Phi_{\text{total}} = 16 - 0 = 16\,\mathrm{Nm^2/C}$
Thus, the total electric flux through the cube is: $\boxed{16\,\mathrm{Nm^2/C}}$

> **Quick Tip**
>
> For non-uniform electric fields, calculate flux through each face and integrate if needed. Remember to account for the direction of the electric field relative to the surface normal.

Question 25:
A circular disc reaches from top to bottom of an inclined plane of length l. When it slips down the plane, it takes t s. When it rolls down the plane, it takes $\frac{\alpha^{1/2}}{2}t$ s, where α is ....
Answer: (3)
Step-wise solution:
1. Case 1: When the disc slips down the plane:
In this case, the acceleration of the disc is $g\sin\theta$, where θ is the angle of inclination of the plane. Using the equation of motion, $s = ut + \frac{1}{2}at^2$, we have:

$$l = 0 + \frac{1}{2}(g \sin\theta)t^2$$

2. Case 2: When the disc rolls down the plane:
In this case, the acceleration of the disc is $\frac{2}{3}g\sin\theta$. Using the same equation of motion, we have:

$$l = 0 + \frac{1}{2}(\frac{2}{3}g\sin\theta)(\frac{\alpha^{1/2}}{2}t)^2$$

3. Equating the two equations and solving for α:

$$\frac{1}{2}(g\sin\theta)t^2 = \frac{1}{2}(\frac{2}{3}g\sin\theta)(\frac{\alpha^{1/2}}{2}t)^2$$

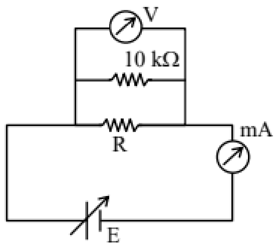
Simplifying and solving for α, we get:

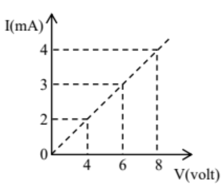
$$\alpha = 3$$

Quick Tip

The key to this problem is understanding the difference in acceleration between a slipping and rolling object down an inclined plane. The rolling object has a smaller acceleration due to the rotational kinetic energy it gains.

Question 26:
To determine the resistance (R) of a wire, a circuit is designed as shown below. The V-I characteristic curve for this circuit is plotted for the voltmeter and the ammeter readings as shown in the figure. The value of R is ______ Ω.



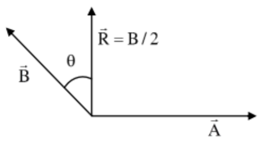


Answer: 2500 Ω
Solution:
Step 1: Analyzing the Circuit The total voltage (V) across the circuit is the sum of the voltages across the resistor R and the known resistor ($10\,\text{k}\Omega$).

Using Ohm's law: $V = IR$ Where I is the current in amperes and R is the resistance in ohms.
Step 2: Reading Data from the Graph From the V-I graph: - When $V = 8\,\text{V}$, $I = 4\,\text{mA}$.
Step 3: Calculating the Total Resistance The total current through the circuit is $I = 4\,\text{mA} = 4 \times 10^{-3}\,\text{A}$. The total resistance using Ohm's law: $V = IR \implies 8 = 4 \times 10^{-3} \times R_{\text{total}}$ $R_{\text{total}} = \frac{8}{4 \times 10^{-3}} = 2000\,\Omega$
Step 4: Finding the Unknown Resistance The known resistance is $10\,\text{k}\Omega = 10000\,\Omega$, so: $R_{\text{total}} = R + 10000$ $2500 = R \rightarrow R = 2500\,\Omega$
Thus, the resistance of the wire is: $\boxed{2500\,\Omega}$

> **Quick Tip**
>
> In V-I characteristic problems, always extract data points from the graph and apply Ohm's law to calculate resistance. Check if additional resistors are in series or parallel for accurate results.

Question 27:
The resultant of two vectors **A** and **B** is perpendicular to **A** and its magnitude is half that of **B**. The angle between vectors **A** and **B** is _____.
Solution:



$B\cos\theta = \frac{B}{2}$

$\Rightarrow \theta = 60°$

Step 1: Representing the Vectors The resultant vector **R** is given by: $\mathbf{R} = \mathbf{A} + \mathbf{B}$ It is given that **R** is perpendicular to **A**: $\mathbf{A} \cdot \mathbf{R} = 0$ Substitute $\mathbf{R} = \mathbf{A} + \mathbf{B}$: $\mathbf{A} \cdot (\mathbf{A} + \mathbf{B}) = 0$ Expanding the dot product: $\mathbf{A} \cdot \mathbf{A} + \mathbf{A} \cdot \mathbf{B} = 0$ $A^2 + AB\cos\theta = 0 \quad (1)$ where θ is the angle between **A** and **B**.
Step 2: Condition for Magnitude of Resultant It is also given that the magnitude of **R** is half that of **B**: $|\mathbf{R}| = \frac{1}{2}|\mathbf{B}|$ Using the magnitude formula for the resultant: $|\mathbf{R}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$ Substitute into the equation: $\frac{B}{2} = \sqrt{A^2 + B^2 + 2AB\cos\theta} \quad (2)$
Step 3: Solving for $\cos\theta$ From equation (1): $\cos\theta = -\frac{A}{B} \quad (3)$ Substitute (3) into (2): $\frac{B}{2} = \sqrt{A^2 + B^2 - 2A^2}$ Squaring both sides: $\frac{B^2}{4} = A^2 + B^2 - 2A^2$
Simplifying: $\cos\theta - \frac{1}{2}$
Therefore, answer is 60

> **Quick Tip**
>
> - In problems where the resultant vector is perpendicular, use the dot product. Resultant to perpendicular. Result is B value.

Question 28 : Monochromatic light of wavelength 500 nm is used in Young's double-slit experiment. An interference pattern is obtained on a screen. When one of the slits is covered with a very thin glass plate (refractive index = 1.5), the central maximum is shifted to a position previously occupied by the 4^{th} bright fringe. The thickness of the glass plate is —— μm.

Solution:

1. **Path Difference:** The glass plate introduces a path difference of $(\mu - 1)t$, where: $\mu = 1.5$ is the refractive index of the glass plate t is the thickness of the glass plate.
2. **Shift in Fringe Pattern:** The central maximum shifts to the position of the 4th bright fringe, which corresponds to a path difference of 4λ, where $\lambda = 500\,\text{nm}$ is the wavelength of the light.
3. **Equating Path Differences:** Equating the two path differences, we get:

$$(\mu - 1)t = 4\lambda$$

4. **Solving for Thickness:** Substituting the values and solving for t:

$$(1.5 - 1)t = 4 \times 500 \times 10^{-9}\,\text{m}$$

$$t = 4000 \times 10^{-9}\,\text{m} = 4\,\mu\text{m}$$

> Quick Tip
>
> Remember that a higher refractive index of the glass plate leads to a larger path difference and hence a greater shift in the interference pattern.

Question 29: A force $(\mathbf{3x+2x} - 5)$ N displaces a body from $x = 2\,\text{m}$ to $x = 4\,\text{m}$. Work done by this force is .... J

Solution:

1. Work Done by a Variable Force: The work done by a variable force is given by the integral of the force with respect to displacement:

$$W = \int_2^4 (3x^2 + 2x - 5)dx$$

2. Integration and Evaluation: Integrating and evaluating the definite integral:

$$W = \left[\frac{3x^3}{3} + \frac{2x^2}{2} - 5x\right]_2^4$$

$$W = [x^3 + x^2 - 5x]_2^4$$

$$W = (4^3 + 4^2 - 54) - (2^3 + 2^2 - 52)$$

$$W = 58 \text{ J}$$

> Quick Tip
>
> Remember that the work done by a variable force can be calculated by integrating the force with respect to displacement.

Question 30:
At room temperature (27°C), the resistance of a heating element is $50\,\Omega$. The temperature coefficient of the material is $2.4 \times 10^{-4}\,°\mathrm{C}^{-1}$. The temperature of the element, when its resistance is $62\,\Omega$, is .... °C.

Answer: 1027 °C

Solution:

Let: R_0 = Initial resistance ($50\,\Omega$)
R_t = Resistance at temperature T ($62\,\Omega$)
T_0 = Initial temperature (27°C)
T = Final temperature
α = Temperature coefficient of resistance ($2.4 \times 10^{-4}\,°\mathrm{C}^{-1}$)

We know that the resistance of a material changes with temperature according to the following formula:

$$R_t = R_0\,[1 + \alpha(T - T_0)]$$

Substituting the given values:

$$62 = 50[1 + 2.410^{-4}(T - 27)]$$

Solving for T:

$$\frac{62}{50} = 1 + 2.410^{-4}(T - 27)$$

$$0.24 = 2.410^{-4}(T - 27)$$

$$\frac{0.24}{2.410^{-4}} = T - 27$$

$$1000 = T - 27$$

$$T = 1027C$$

Therefore, the temperature of the element when its resistance is $62\,\Omega$ is 1027°C.

Quick Tip

Remember that the temperature coefficient of resistance (α) is a measure of how much the resistance of a material changes with temperature. A positive value of α indicates that the resistance increases with temperature.