JEE Main (9 April Shift 1)

Mathematics

Question 1. Let the line L intersect the lines:

$$x-2=-y=z-1, \quad 2(x+1)=2(y-1)=z+1$$

and be parallel to the line:

$$\frac{x-2}{3}=\frac{y-1}{1}=\frac{z-2}{2}.$$

Then which of the following points lies on L?

1. $\left(\frac{1}{3},-1,1\right)$
2. $\left(-\frac{1}{3},1,-1\right)$
3. $\left(-\frac{1}{3},-1,-1\right)$
4. $\left(-\frac{1}{3},-1,1\right)$

Q.2 The parabola $y^2=4x$ divides the area of the circle $x^2+y^2=5$ in two parts. The area of the smaller part is equal to:

1. $\frac{2}{3}+5\sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$
2. $\frac{1}{3}+5\sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$
3. $\frac{1}{3}+\sqrt{5}\sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$
4. $\frac{2}{3}+\sqrt{5}\sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$

Q.3 The solution curve of the differential equation

$$2y\frac{dy}{dx}+3=5\frac{dy}{dx},$$

passing through the point $(0,1)$, is a conic whose vertex lies on the line:

1. $2x+3y=9$
2. $2x+3y=-9$
3. $2x+3y=-6$
4. $2x+3y=6$

Question 4.
A ray of light coming from the point $P(1,2)$ gets reflected from the point Q on the x-axis and then passes through the point $R(4,3)$. If the point $S(h,k)$ is such that $PQRS$ is a parallelogram, then hk^2 is equal to:

1. 80
2. 90
3. 60
4. 70

Question 5.
Let $\lambda, \mu \in R$. If the system of equations

$$3x + 5y + \lambda z = 3,$$

$$7x + 11y - 9z = 2,$$

$$97x + 155y - 189z = \mu$$

has infinitely many solutions, then $\mu + 2\lambda$ is equal to:

1. 25
2. 24
3. 27
4. 22

Q.6 The coefficient of x^{70} in

$$x^2(1+x)^{98} + x^3(1+x)^{97} + x^4(1+x)^{96} + \cdots + x^{54}(1+x)^{46}$$

is ${}^{99}C_p -^{46} C_q$.
Then a possible value to $p+q$ is:

(1) 55 (2) 61 (3) 68 (4) 83

Question 7.
Let

$$\int \frac{2-\tan x}{3+\tan x}\, dx = \frac{1}{2}\left(\alpha x + \log_e |\beta \sin x + \gamma \cos x|\right) + C,$$

where C is the constant of integration. Then $\alpha + \frac{\gamma}{\beta}$ is equal to:

1. 3
2. 1

3. 4
4. 7

Question 8.
A variable line L passes through the point $(3, 5)$ and intersects the positive coordinate axes at the points A and B. The minimum area of the triangle OAB, where O is the origin, is:

1. 30
2. 25
3. 40
4. 35

Question 9.
Let

$$|\cos\theta\cos(60-\theta)\cos(60-\theta)| \le \frac{1}{8}, \quad \theta \in [0, 2\pi].$$

Then, the sum of all $\theta \in [0, 2\pi]$, where $\cos 3\theta$ attains its maximum value, is:

1. 9π
2. 18π
3. 6π
4. 15π

Question 10.
Let

$$\overrightarrow{OA} = 2\vec{a}, \quad \overrightarrow{OB} = 6\vec{a} + 5\vec{b}, \quad \text{and} \quad \overrightarrow{OC} = 3\vec{b},$$

where O is the origin. If the area of the parallelogram with adjacent sides $\overrightarrow{OA}$ and $\overrightarrow{OC}$ is 15 sq. units, then the area (in sq. units) of the quadrilateral $OABC$ is equal to:

1. 38
2. 40
3. 32
4. 35

Question 11.
If the domain of the function

$$f(x) = \sin^{-1}\left(\frac{x-1}{2x+3}\right)$$

is $R - (\alpha, \beta)$, then $12\alpha\beta$ is equal to:

1. 36
2. 24
3. 40
4. 32

Question 12.
If the sum of the series

$$\frac{1}{1\cdot(1+d)}+\frac{1}{(1+d)(1+2d)}+\frac{1}{(1+2d)(1+3d)}+\cdots+\frac{1}{(1+9d)(1+10d)}$$

is equal to 5, then $50d$ is equal to:

1. 20
2. 5
3. 15
4. 10

Question 13.
Let

$$f(x)=ax^3+bx^2+cx+41$$

be such that $f(1)=40$, $f'(1)=2$, and $f''(1)=4$. Then $a^2+b^2+c^2$ is equal to:

1. 62
2. 73
3. 54
4. 51

Question 14.
Let a circle passing through $(2,0)$ have its center at the point (h,k). Let (x_c,y_c) be the point of intersection of the lines $3x+5y=1$ and $(2+c)x+5c^2y=1$. If $h=\lim_{c\to 1}x_c$ and $k=\lim_{c\to 1}y_c$, then the equation of the circle is:

1. $25x^2+25y^2-20x+2y-60=0$
2. $5x^2+5y^2-4x-2y-12=0$
3. $25x^2+25y^2-2x+2y-60=0$
4. $5x^2+5y^2-4x+2y-12=0$

Question 15.
The shortest distance between the lines:

$$\frac{x-3}{4}=\frac{y+7}{-11}=\frac{z-1}{5} \quad \text{and} \quad \frac{x-5}{3}=\frac{y-9}{-6}=\frac{z+2}{1}$$

is:

1. $\frac{187}{\sqrt{563}}$
2. $\frac{178}{\sqrt{563}}$
3. $\frac{185}{\sqrt{563}}$
4. $\frac{179}{\sqrt{563}}$

Question 16.
The frequency distribution of the age of students in a class of 40 students is given below:

Age	15	16	17	18	19	20
No. of Students	5	8	5	12	x	y

If the mean deviation about the median is 1.25, then $4x+5y$ is equal to:

1. 43
2. 44
3. 47
4. 46

Question 17.
The solution of the differential equation:

$$(x^2+y^2)dx-5xy\,dy=0, \quad y(1)=0,$$

is:

1. $|x^2-4y^2|^5=x^2$
2. $|x^2-2y^2|^6=x$
3. $|x^2-4y^2|^6=x$
4. $|x^2-2y^2|^5=x^2$

Question 18.
Let three vectors

$$\vec{a}=\alpha\hat{i}+4\hat{j}+2\hat{k}, \quad \vec{b}=5\hat{i}+3\hat{j}+4\hat{k}, \quad \vec{c}=x\hat{i}+y\hat{j}+z\hat{k},$$

form a triangle such that $\vec{c}=\vec{a}-\vec{b}$ and the area of the triangle is $5\sqrt{6}$. If α is a positive real number, then $|\vec{c}|^2$ is:

1. 16
2. 14
3. 12
4. 10

Question 19.
Let α, β be the roots of the equation:

$$x^2 + 2\sqrt{2}x - 1 = 0.$$

The quadratic equation whose roots are $\alpha^4 + \beta^4$ and $\frac{1}{10}(\alpha^6 + \beta^6)$ is:

1. $x^2 - 190x + 9466 = 0$
2. $x^2 - 195x + 9466 = 0$
3. $x^2 - 195x + 9506 = 0$
4. $x^2 - 180x + 9506 = 0$

Question 20.
Let $f(x) = x^2 + 9$, $g(x) = \frac{x}{x-9}$, and:

$$a = f(g(10)), \quad b = g(f(3)).$$

If e and ℓ denote the eccentricity and the length of the latus rectum of the ellipse:

$$\frac{x^2}{a} + \frac{y^2}{b} = 1,$$

then $8e^2 + \ell^2$ is equal to:

1. 16
2. 8
3. 6
4. 12

Question 21.
Let a, b, c denote the outcomes of three independent rolls of a fair tetrahedral die, whose four faces are marked $1, 2, 3, 4$. If the probability that:

$$ax^2 + bx + c = 0$$

has all real roots is $\frac{m}{n}$, where gcd$(m, n) = 1$, then $m + n$ is equal to:

Question 22.
The sum of the square of the modulus of the elements in the set:

$$\{z = a + ib : a, b \in Z, z \in C, |z - 1| \leq 1, |z - 5| \leq |z - 5i|\}$$

is _______.

Question 23.
Let the set of all positive values of λ, for which the point of local minimum of the function:

$$f(x) = \left(1 + x(\lambda^2 - x^2)\right) \quad \text{satisfies} \quad \frac{x^2 + x + 2}{x^2 + 5x + 6} < 0,$$

be (α, β). Then $\alpha^2 + \beta^2$ is equal to ______.

Question 24.
Let

$$\lim_{n \to \infty} \left(\frac{n}{\sqrt{n^4 + 1}} - \frac{2n}{(n^2 + 1)\sqrt{n^4 + 1}} + \frac{n}{\sqrt{n^4 + 16}} - \frac{8n}{(n^2 + 4)\sqrt{n^4 + 16}} + \cdots + \frac{n}{\sqrt{n^4 + n^4}} - \frac{2n \cdot n^2}{(n^2 + n^2)\sqrt{n^4 + n^4}} \right)$$

be equal to:

$$\frac{\pi}{k}$$

using only the principal values of the inverse trigonometric functions. Then, k^2 is equal to ....

Question 25.
The remainder when 428^{2024} is divided by 21 is ______.

Question 26.
Let $f : (0, \pi) \to R$ be a function given by:

$$f(x) = \begin{cases} \left(\frac{8}{7}\right)^{\tan 8x / \tan 7x}, & 0 < x < \frac{\pi}{2} \\ a - 8, & x = \frac{\pi}{2} \\ (1 + |\cot x|)^{\frac{b}{a} \tan |x|}, & \frac{\pi}{2} < x < \pi \end{cases}$$

where $a, b \in Z$. If f is continuous at $x = \frac{\pi}{2}$, find $a^2 + b^2$.

Question 27.
Let A be a non-singular matrix of order 3. If:

$$\det\left(3\text{adj}(2\text{adj}((\det A)A))\right) = 3^{-13} \cdot 2^{-10},$$

and:

$$\det(3\text{adj}(2A)) = 2^m \cdot 3^n,$$

then $|3m + 2n|$ is equal to ______.

Question 28.
Let the centre of a circle, passing through the points $(0, 0)$, $(1, 0)$, and touching the circle $x^2 + y^2 = 9$, be (h, k). Then for all possible values of the coordinates of the centre (h, k), $4(h^2 + k^2)$ is equal to ______.

Question 29.
If a function f satisfies $f(m + n) = f(m) + f(n)$ for all $m, n \in N$, and $f(1) = 1$, then the largest natural number λ such that:

$$\sum_{k=1}^{2022} f(\lambda + k) \leq (2022)^2,$$

is equal to ______.

Question 30.
Let $A = \{2, 3, 6, 7\}$ and $B = \{4, 5, 6, 8\}$. Let R be a relation defined on $A \times B$ by:

$$(a_1, b_1)\, R\, (a_2, b_2) \iff a_1 + a_2 = b_1 + b_2.$$

Then the number of elements in R is ______.
