JEE Main 2023 Jan 31 Shift 2 Mathematics Question Paper

Total Time Allowed : 180 minutes	**Maximum Marks :** 300	**Total Questions :** 90	**Questions to be answered :** 90

General Instructions

Read the following instructions very carefully and strictly follow them:
(1) The test is of 3 hours duration.
(2) The question paper consists of 90 questions. The maximum marks are 300.
(3) There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage.
(4) Each part (subject) has two sections.
(i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and –1 mark for wrong answer.
(ii) Section-B: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and –1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

MATHEMATICS

Section-A

1. If $\phi(x) = \frac{1}{\sqrt{x}} \int_{\frac{x}{4}}^{x} \left(4\sqrt{2}\sin t - 3\phi(t)\right) \, dt$, $x > 0$,
then $\phi\left(\frac{\pi}{4}\right)$ **is equal to:**
(1) $\frac{8}{\sqrt{\pi}}$
(2) $\frac{6}{6+\sqrt{\pi}}$
(3) $\frac{8}{6+\sqrt{\pi}}$
(4) $\frac{4}{6-\sqrt{\pi}}$

---

2. If a point $P(\alpha, \beta, \gamma)$ **satisfying the equation**

$$\begin{pmatrix} 2 & 10 & 8 \\ 9 & 3 & 8 \\ 8 & 4 & 8 \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

lies on the plane $2x + 4y + 3z = 5$**, then** $6\alpha + 9\beta + 7\gamma$ **is equal to:**

(1) 1
(2) $\frac{11}{5}$
(3) $\frac{5}{4}$
(4) 11

---

3. Let $a_1, a_2, a_3, \ldots$ be an A.P. If $a_4 = 3$, the product $a_1 a_4$ is minimum and the sum of its first n terms is zero, then $n! - 4a_n(a_{n+2})$ is equal to:
(1) 24
(2) $\frac{33}{4}$
(3) $\frac{381}{4}$
(4) 9

---

4. Let $(a, b) \subset (0, 2\pi)$ be the largest interval for which

$$\sin^{-1}(\sin\theta) - \cos^{-1}(\sin\theta) > 0, \quad \theta \in (0, 2\pi)$$

holds. If

$$\alpha x^2 + \beta x + \sin^{-1}\left((x^2 - 6x + 10)\right) + \cos^{-1}\left((x^2 - 3)^2 + 1\right) = 0$$

and $\alpha - \beta = b - a$, then α is equal to:
(1) $\frac{\pi}{48}$
(2) $\frac{\pi}{16}$
(3) $\frac{\pi}{12}$
(4) $\frac{\pi}{8}$

---

5. Let $y = y(x)$ be the solution of the differential equation

$$(3y^2 - 5x^2)y\,dx + 2x(x^2 - y^2)\,dy = 0,$$

such that $y(1) = 1$. Then

$$\left(y(2)\right)^3 - 12y(2) \text{ is equal to:}$$

(1) $32\sqrt{2}$
(2) 64
(3) $16\sqrt{2}$
(4) 32

---

6. The set of all values of a^2 for which the line $x + y = 0$ bisects two distinct chords drawn from a point $P\left(\frac{1+a}{2}, \frac{1-a}{2}\right)$ on the circle

$$2x^2 + 2y^2 - (1+a)x - (1-a)y = 0$$

is equal to:
(1) $(8, \infty)$
(2) $(4, \infty)$
(3) $(0, 4)$
(4) $(2, 12)$

---

7. Among the relations

$$S = \left\{(a, b) : a, b \in R \setminus \{0\}, a^2 + b^2 > 0\right\}$$

And

$$T = \left\{(a, b) : a, b \in R, a^2 - b^2 \in Z\right\}$$

which of the following is true?
(1) S is transitive but T is not.
(2) T is symmetric but S is not.
(3) Neither S nor T is transitive.
(4) Both S and T are symmetric.

---

8. The equation

$$e^x + 8e^{2x} + 13e^x - 8e^x + 1 = 0, \quad x \in R$$

has:
(1) two solutions and both are negative
(2) no solution
(3) four solutions, two of which are negative
(4) two solutions and only one of them is negative

---

9. The number of values of $r \in \{p, q, \neg p, \neg q\}$ for which

$$((p \wedge q) \Leftrightarrow (r \vee q)) \wedge ((p \wedge r) \Leftrightarrow q)$$

is a tautology, is:
(1) 3
(2) 2
(3) 1
(4) 4

---

10. Let $f : R \setminus \{2, 6\} \to R$ be the real-valued function defined as

$$f(x) = \frac{x^2 + 2x + 1}{x^2 - 8x + 12}.$$

Then the range of f is:
(1) $\left(-\infty, \frac{-21}{4}\right] \cup [0, \infty)$
(2) $\left(-\infty, \frac{-21}{4}\right] \cup (0, \infty)$
(3) $\left(-\infty, \frac{-21}{4}\right] \cup \left[\frac{21}{4}, \infty\right)$
(4) $\left[\frac{-21}{4}, \infty\right) \cup [0, \infty)$

---

11. Evaluate the limit:

$$\lim_{x \to 1} \frac{\left(\sqrt{3x+1} + \sqrt{3x-1}\right)^6}{(x + \sqrt{x^2 - 1})^3 + \left(\sqrt{3x+1} - \sqrt{3x-1}\right)^6}$$

(1) is equal to 9
(2) is equal to 27
(3) does not exist
(4) is equal to $\frac{27}{2}$

---

12. Let P be the plane, passing through the point $(1, -1, -5)$ and perpendicular to the line joining the points $(4, 1, -3)$ and $(2, 4, 3)$. Then the distance of P from the point $(3, -2, 2)$ is:
(1) 6
(2) 4
(3) 5
(4) 7

---

13. The absolute minimum value of the function

$f(x) = |x^2 - x + 1| + \lfloor x^2 - x + 1 \rfloor$, where $[t]$ denotes the greatest integer function, in the interval $[-1, 2$

(1) $\frac{3}{4}$
(2) $\frac{3}{2}$
(3) $\frac{1}{4}$
(4) $\frac{5}{4}$

---

14. Let the plane $P : 8x + \alpha y + \alpha z + 12 = 0$ be parallel to the line

$$L : \frac{x+2}{2} = \frac{y-3}{3} = \frac{z+4}{5}.$$

If the intercept of P on the y-axis is 1, then the distance between P and L is:
(1) $\sqrt{14}$
(2) $\frac{6}{\sqrt{14}}$
(3) $\frac{\sqrt{2}}{7}$
(4) $\frac{\sqrt{7}}{2}$

---

15. The foot of perpendicular from the origin O to a plane P which meets the coordinate axes at the points A, B, C is $(2,4,4)$. If the volume of the tetrahedron $OABC$ is 144 unit^3, then which of the following points is NOT on P?
(1) $(2,2,4)$
(2) $(0,4,4)$
(3) $(3,0,4)$
(4) $(0,6,6)$

---

16. Let the mean and standard deviation of marks of class A of 100 students be respectively 40 and $\alpha > 0$, and the mean and standard deviation of marks of class B of n students be respectively 55 and $30-\alpha$. If the mean and variance of the marks of the combined class of $100+n$ students are respectively 50 and 350, then the sum of variances of classes A and B is:
(1) 500
(2) 650
(3) 450
(4) 900

---

17. Let

$$\mathbf{a} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}, \quad \mathbf{b} = \hat{\imath} - \hat{\jmath} + 2\hat{k}, \quad \mathbf{c} = 5\hat{\imath} - 3\hat{\jmath} + 3\hat{k}$$

be three vectors. If r is a vector such that $\mathbf{r} \times \mathbf{b} = \mathbf{c} \times \mathbf{b}$ and $\mathbf{r} \cdot \mathbf{a} = 0$, then $25|\mathbf{r}|^2$ is equal to:
(1) 449
(2) 336
(3) 339
(4) 560

---

18. Let H be the hyperbola, whose foci are $(1 \pm \sqrt{2}, 0)$ and eccentricity is $\sqrt{2}$. Then the length of its latus rectum is:
(1) 2

(2) 3
(3) $\frac{5}{2}$
(4) $\frac{3}{2}$

---

19. Let $\alpha > 0$. If

$$\int_{\alpha}^{x} \frac{x}{\sqrt{x+\alpha} - \sqrt{x}}\, dx = \frac{16+20\sqrt{2}}{15},$$

then α is equal to:
(1) 2
(2) 4
(3) $\sqrt{2}$
(4) $2\sqrt{2}$

---

20. The complex number

$$z = \frac{i-1}{\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}}$$

is equal to:
(1) $\sqrt{2}\left(\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}\right)$
(2) $\cos\frac{\pi}{12} - i\sin\frac{\pi}{12}$
(3) $\sqrt{2}\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)$
(4) $\sqrt{2}\left(\cos\frac{5\pi}{12} - i\sin\frac{5\pi}{12}\right)$

Section - B

21. The coefficient of x^{-6}, in the expansion of

$$\left(\frac{4x}{5} + \frac{5}{2x^2}\right)^9, \text{ is:}$$

---

22. Let the area of the region

$$\left\{(x,y) : |2x-1| \leq y \leq x^2 - x, 0 \leq x \leq 1\right\} \quad \text{be } A.$$

Then $(6A+11)^2$ is equal to:

---

23. If

$$\frac{(2n+1)P_{n-1}}{2nP_n} = \frac{11}{21}, \quad \text{then} \quad n^2+n+15 \,\text{is equal to:}$$

---

24. If the constant term in the binomial expansion of

$$\left(\frac{x^{5/2}}{2} - \frac{4}{x}\right)^9 \text{ is } -84 \text{ and the coefficient of } x^{-3} \text{ is } 2\alpha\beta,$$

where $\beta < 0$ is an odd number, then $|\alpha - \beta|$ is equal to:

---

25. Let $\vec{a}, \vec{b}, \vec{c}$ be three vectors such that

$$|\vec{a}| = \sqrt{31}, \quad |\vec{b}| = 4, \quad |\vec{c}| = 2, \quad 2(\vec{a} \times \vec{b}) = 3(\vec{c} \times \vec{a}).$$

If the angle between $\vec{b}$ and $\vec{c}$ is $\frac{2\pi}{3}$, then $\left(\frac{\vec{a}\times\vec{c}}{\vec{a}\cdot\vec{b}}\right)^2$ is equal to:

---

26. Let S be the set of all $a \in N$ such that the area of the triangle formed by the tangent at the point $P(b,c), b, c \in N$ on the parabola

$$y^2 = 2ax \quad \text{and the lines} \quad x = b,\ y = 0 \quad \text{is}\, 16\, \text{unit}^2,\ \text{then} \quad \sum_{a \in S} a\, \text{is equal to:}$$

---

27. The sum

$$1^2 - 2 \cdot 3^2 + 3.5^2 - 4.7^2 + 5.9^2 - \cdots + 15.29^2 \text{ is:}$$

---

28. Let A be the event that the absolute difference between two randomly chosen real numbers in the sample space

$$[0, 60] \quad \text{is less than or equal to}\, a. \,\text{If}\, P(A) = \frac{11}{36},\, \text{then}\, a\, \text{is equal to:}$$

---

29. Let $A = [a_{ij}]$, where $a_{ij} \in Z \cap [0,4], 1 \leq i, j \leq 2$. The number of matrices A such that the sum of all entries is a prime number $p \in \{2, 13\}$ is:

---

30. Let A be an $n \times n$ matrix such that $|A| = 2$. If the determinant of the matrix

$$\text{Adj}(2 \cdot \text{Adj}(2A^{-1})) \text{ is } 2^{84},\ \text{then } n \text{ is equal to:}$$
