PHYSICS

SECTION-A

Question 31:
The parameter that remains the same for molecules of all gases at a given temperature is:
(1) kinetic energy
(2) momentum
(3) mass
(4) speed

Answer: (1)

Solution:
The kinetic energy of gas molecules is directly proportional to the temperature and is given by:

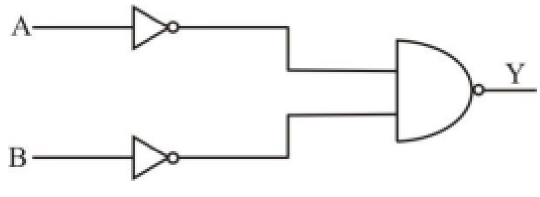
$$\text{KE} = \frac{f}{2}kT,$$

where f is the degrees of freedom and k is Boltzmann's constant.
At the same temperature, the kinetic energy is the same for all molecules regardless of their mass.

Quick Tip
For ideal gases, the average kinetic energy depends only on temperature.

Question 32:

Identify the logic operation performed by the given circuit:



(1) NAND
(2) NOR
(3) OR

(4) AND

Answer: (3)

Solution:
Using De Morgan's law, the circuit simplifies as follows:

$$Y = \overline{\overline{A} + \overline{B}} = A + B.$$

Hence, the circuit performs the OR operation.

Quick Tip
Use De Morgan's laws to simplify complex logic circuits step-by-step.

Question 33:
The relation between time t and distance x is:

$$t = \alpha x^2 + \beta x,$$

where α and β are constants. The relation between acceleration (a) and velocity (v) is:
(1) $a = -2\alpha v^3$
(2) $a = -3\alpha v^2$
(3) $a = -5\alpha v^5$
(4) $a = -4\alpha v^4$

Answer: (1)

Solution:
Given:

$$t = \alpha x^2 + \beta x.$$

Differentiate with respect to x:

$$1 = 2\alpha x \frac{dx}{dt} + \beta \frac{dx}{dt}.$$

Substitute $v = \frac{dx}{dt}$ and solve:

$$v = \frac{1}{2\alpha x + \beta}.$$

Differentiate v with respect to t to find acceleration a. Substituting $x = v$ into the expression:

$$a = -2\alpha v^3.$$

Quick Tip
Relate v and a by differentiating v with respect to t and use substitution to simplify.

Question 34:
The refractive index of a prism with apex angle A is $\cot \frac{A}{2}$. The angle of minimum deviation is:
(1) $\delta_m = 180° - A$
(2) $\delta_m = 180° - 3A$
(3) $\delta_m = 180° - 4A$
(4) $\delta_m = 180° - 2A$

Answer: (4)

Solution:
For a prism, the angle of minimum deviation is related to the apex angle A and the refractive index μ:

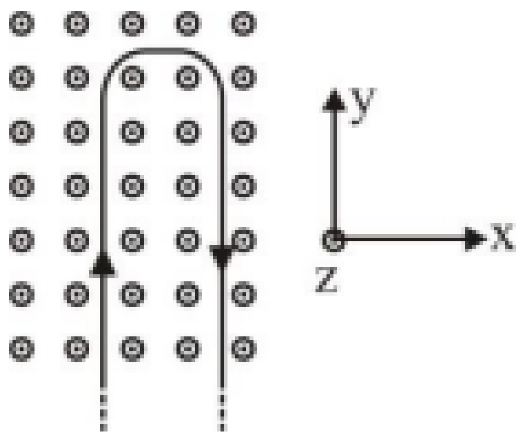
$$\mu = \cot\left(\frac{A}{2}\right).$$

At minimum deviation:

$$\delta_m + A = 180° \implies \delta_m = 180° - 2A.$$

Quick Tip
For prisms, relate deviation to the refractive index using Snell's law at minimum deviation.

Question 35:
A rigid wire consists of a semicircular portion of radius R and two straight sections. The wire is partially immersed in a perpendicular magnetic field $B = B_0\hat{j}$. The magnetic force on the wire if it has a current i is:



(1) $-iBR\hat{j}$
(2) $2iBR\hat{j}$
(3) $iBR\hat{j}$
(4) $-2iBR\hat{j}$

Answer: (4)

Solution:
The magnetic force on the wire is calculated using:

$$\vec{F} = i(\vec{L} \times \vec{B}),$$

where $\vec{L}$ is the length vector of the wire. For the semicircular part, the force is:

$$F = 2iBR.$$

The direction is $-\hat{j}$ due to the cross product.

Quick Tip
For magnetic force calculations, use symmetry and apply the formula $\vec{F} = i(\vec{L} \times \vec{B})$.

Question 36:
If the wavelength of the first member of the Lyman series of hydrogen is λ, the wavelength of the second member will be:
(1) $\frac{27}{32}\lambda$
(2) $\frac{32}{27}\lambda$
(3) $\frac{27}{5}\lambda$
(4) $\frac{5}{27}\lambda$

Answer: (1)

Solution:
The wavelength of the Lyman series is given by:

$$\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right),$$

where $n = 2$ for the first member and $n = 3$ for the second member. Solving:

$$\frac{\lambda_2}{\lambda_1} = \frac{27}{32}.$$

Quick Tip
For hydrogen spectra, use the Rydberg formula and the series definitions.

Question 37:
Four identical particles of mass m are kept at the four corners of a square. If the gravitational force exerted on one of the masses by the other masses is $\left(\frac{2\sqrt{2}+1}{32}\right)\frac{Gm^2}{L^2}$, 1 the length of the sides of the square is:
(1) $\frac{L}{2}$ (2) $4L$ (3) $3L$ (4) $2L$

Answer: (2)

Solution:
Let the side of the square be a. The gravitational force between two particles of mass m separated by a distance a is given by:

$$F = \frac{Gm^2}{a^2}$$

Consider one of the particles. It experiences gravitational force due to the other three particles. The forces due to the two adjacent particles are at an angle of $45°$ to each other. The net force due to these two forces is:

$$F_1 = 2F\cos 45° = \sqrt{2}\frac{Gm^2}{a^2}$$

The force due to the diagonally opposite particle is:

$$F_2 = \frac{Gm^2}{(\sqrt{2}a)^2} = \frac{Gm^2}{2a^2}$$

The net force on the particle is:

$$F_{net} = F_1 + F_2 = \left(\sqrt{2}+\frac{1}{2}\right)\frac{Gm^2}{a^2}$$

Given that $F_{net} = \left(\frac{2\sqrt{2}+1}{32}\right)\frac{Gm^2}{L^2}$, we can equate the two expressions:

$$\left(\sqrt{2}+\frac{1}{2}\right)\frac{Gm^2}{a^2} = \left(\frac{2\sqrt{2}+1}{32}\right)\frac{Gm^2}{L^2}$$

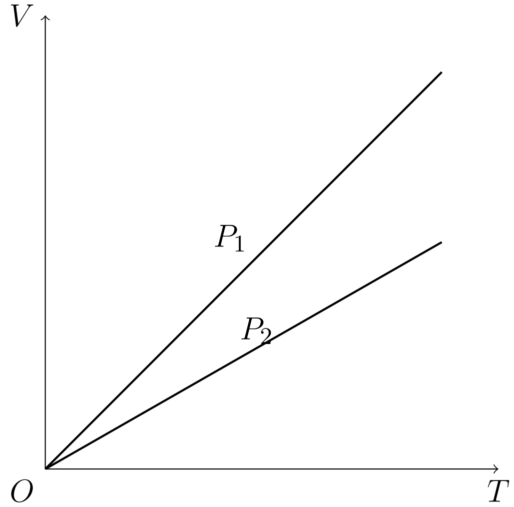
Simplifying, we get:

$$a = 4L$$

Therefore, the length of the side of the square is $4L$.

Quick Tip
When dealing with problems involving multiple forces acting at angles, it's often helpful to break down the forces into components and then add them vectorially.

Question 38:
The given figure represents two isobaric processes for the same mass of an ideal gas. Then:
(1) $P_2 \geq P_1$
(2) $P_2 > P_1$
(3) $P_1 = P_2$
(4) $P_1 > P_2$



Answer: (4)

Solution:
For an isobaric process, the pressure remains constant. From the diagram, it is evident that $P_1 > P_2$ because P_1 is at a higher vertical position.

Quick Tip
For isobaric processes, identify constant pressure from the graph and compare directly.

Question 39:
If the percentage errors in measuring the length and the diameter of a wire are 0.1 percent each. The percentage error in measuring its resistance will be:
(1) 1 0.2 percent (2) 0.3 percent (3) 0.1 percent (4) 0.144 percent

Answer: (2)

Solution:
The resistance of a wire is given by the formula:

$$R = \rho \frac{L}{A}$$

where: * R is the resistance * ρ is the resistivity (constant) * L is the length * A is the cross-sectional area (πr^2, where r is the radius)
Taking the natural logarithm of both sides and differentiating, we get:

$$\frac{dR}{R} = \frac{dL}{L} + 2\frac{dr}{r}$$

Since the percentage error in both length and radius is 0.1 percent, the percentage error in resistance is:

$$\frac{dR}{R} \times 100\% = (0.1\% + 2 \times 0.1\%) = 0.3\%$$

Therefore, the percentage error in measuring the resistance is 0.3 percent.

Quick Tip
When dealing with quantities that are products or quotients of other quantities, the percentage error in the final quantity is the sum of the percentage errors in the individual quantities.

Question 40:
In a plane EM wave, the electric field oscillates sinusoidally at a frequency of 5×10^{10} Hz and an amplitude of 50 Vm^{-1}. The total average energy density of the electromagnetic field of the wave is:
[Use $\epsilon_0 = 8.85 \times 10^{-12}$ C^2/Nm^2]
(1) 1.106×10^{-8} Jm^{-3} (2) 4.425×10^{-8} Jm^{-3} (3) 2.212×10^{-1} Jm^{-3} (4) 2.212×10^{-10} Jm^{-3}

Answer: (1)

Solution:
The average energy density of an electromagnetic wave is given by:

$$U_E = \frac{1}{2}\epsilon_0 E^2$$

Substituting the given values:

$$U_E = \frac{1}{2} \times 8.85 \times 10^{-12} \times (50)^2$$

$$U_E = 1.106 \times 10^{-8} \text{ J/m}^3$$

Therefore, the correct answer is (1).

> Quick Tip
>
> Remember that the average energy density of an electromagnetic wave is equally distributed between the electric and magnetic fields.

Question 41:
A force is represented by $F = ax^2 + bt^{1/2}$, where x is distance and t is time. The dimensions of b^2/a are:
(1) $[ML^3T^3]$ (2) $[MLT^2]$ (3) $[ML^{-1}T^{-1}]$ (4) $[ML^2T^3]$

Answer: (1)

Solution:
From the given equation, we have:

$$[F] = [a][x]^2 = [b][t]^{1/2}$$

Therefore, the dimensions of a and b are:

$$[a] = [FT^{-2}L^{-2}]$$
$$[b] = [FT^{-1/2}]$$

Now, the dimensions of b^2/a are:

$$\frac{[b]^2}{[a]} = \frac{[F^2T^{-1}]}{[FT^{-2}L^{-2}]} = [ML^3T^3]$$

Therefore, the correct answer is (1).

> Quick Tip
>
> To determine the dimensions of a physical quantity, it is often helpful to express the quantity in terms of the fundamental dimensions: mass (M), length (L), and time (T).

Question 42:
Two charges q and $3q$ are separated by a distance r in air. At a distance x from charge q, the resultant electric field is zero. The value of x is:

1. $r(1+\sqrt{3})$
2. $r(3(1+\sqrt{3}))$

3. $r(1+\sqrt{3})$
4. $r(1+\sqrt{3})$

Answer: (3) $r(1+\sqrt{3})$
Solution:
Let the distance of the point where the resultant electric field is zero from charge q be x. The electric field due to charge q at this point is given by:

$$E_q = \frac{kq}{x^2}$$

The electric field due to charge $3q$ at the same point is:

$$E_{3q} = \frac{k(3q)}{(r-x)^2}$$

For the resultant electric field to be zero, the fields due to both charges must cancel each other out:

$$E_q = E_{3q}$$

$$\frac{kq}{x^2} = \frac{k(3q)}{(r-x)^2}$$

Simplifying:

$$\frac{1}{x^2} = \frac{3}{(r-x)^2}$$

Taking the square root of both sides:

$$\frac{1}{x} = \frac{\sqrt{3}}{r-x}$$

Solving for x:

$$r - x = \sqrt{3}x$$

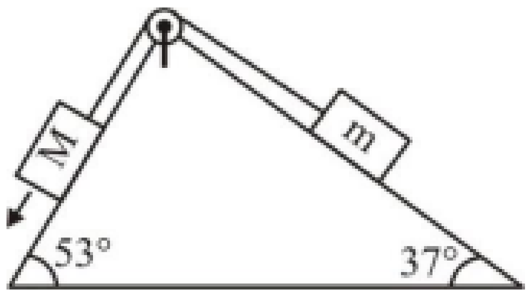
$$r = x(1+\sqrt{3})$$

$$x = \frac{r}{1+\sqrt{3}}$$

Thus, the value of x is $r(1+\sqrt{3})$.

Quick Tip
When solving problems involving electric fields, remember to apply the principle of superposition and set up equations for the fields of individual charges.Use algebraic manipulation to solve for the unknown distance or variable.

Question 43:
In the given arrangement of a doubly inclined plane, two blocks of masses M and m are placed. The blocks are connected by a light string passing over an ideal pulley as shown. The coefficient of friction between the surface of the plane and the blocks is 0.25. The value of m, for which $M = 10$ kg will move down with an acceleration of $2\,\text{m/s}^2$, is: (Take $g = 10\,\text{m/s}^2$ and $\tan 37° = 3/4$).



Options:

(1) $9\,\text{kg}$ (2) $4.5\,\text{kg}$ (3) $6.5\,\text{kg}$ (4) $2.25\,\text{kg}$

Answer: (2)

Solution:
For block M (mass = 10 kg), the forces acting along the inclined plane are:

$$Mg \sin 53° - T - \text{friction} = Ma$$

Friction force:

$$f = \mu M g \cos 53°$$

Substitute f and rewrite the equation:

$$10 \cdot 10 \cdot \sin 53° - T - (0.25 \cdot 10 \cdot 10 \cdot \cos 53°) = 10 \cdot 2$$

$$100 \cdot \frac{4}{5} - T - 25 \cdot \frac{3}{5} = 20$$
$$80 - T - 15 = 20$$

$$T = 80 - 15 - 20 = 45\,\text{N}$$

For block m, the forces acting along the inclined plane are:

$$T - mg \sin 37° - \text{friction} = ma$$

Substitute $T = 45\,\text{N}$ and friction:

$$45 - m \cdot 10 \cdot \sin 37° - (0.25 \cdot m \cdot 10 \cdot \cos 37°) = m \cdot 2$$

$$45 - 10m \cdot \frac{3}{5} - 0.25 \cdot 10m \cdot \frac{4}{5} = 2m$$

$$45 - 6m - 2m = 2m$$
$$45 = 10m$$
$$m = 4.5\,\text{kg}$$

Thus, the value of m is: **4.5 kg**

Quick Tip

Break forces into components along and perpendicular to the incline to simplify calculations.

Question 44.
A coil is placed perpendicular to a magnetic field of 5000 T. When the field is changed to 3000 T in 2 seconds, an induced emf of 22 V is produced in the coil. If the diameter of the coil is 0.02 m, then the number of turns in the coil is:

Options:
(A) 7
(B) 70
(C) 35
(D) 140

Correct Answer: (B)

Solution:
We use Faraday's law of induction, which is given by:

$$\mathcal{E} = -N\frac{\Delta B}{\Delta t} \times A$$

Where $\mathcal{E}$ is the induced emf, N is the number of turns in the coil, ΔB is the change in the magnetic field, Δt is the time interval, and A is the area of the coil.
First, we calculate the area of the coil:

$$A = \pi r^2 = \pi\left(\frac{0.02}{2}\right)^2 = 3.1416 \times 10^{-4}\,\text{m}^2$$

Next, apply the values:

$$\mathcal{E} = 22\,\text{V}, \quad \Delta B = 5000 - 3000 = 2000\,\text{T}, \quad \Delta t = 2\,\text{s}$$

Using Faraday's law:

$$22 = N \times \frac{2000}{2} \times 3.1416 \times 10^{-4}$$
$$22 = N \times 1000 \times 3.1416 \times 10^{-4}$$
$$22 = N \times 3.1416 \times 10^{-1}$$

$$N = \frac{22}{3.1416 \times 10^{-1}} = \frac{22}{0.31416} \approx 70$$

Thus, the number of turns in the coil is 70.

Quick Tip
Break forces into components along and perpendicular to the incline to simplify calculations.

Question 45.
The fundamental frequency of a closed organ pipe is equal to the first overtone frequency of an open organ pipe. If the length of the open pipe is 60 cm, the length of the closed pipe will be:

Options:
(A) 60 cm
(B) 45 cm
(C) 30 cm
(D) 15 cm

Correct Answer: (D)

Solution:
The fundamental frequency of a closed organ pipe is given by the formula:

$$f_{\text{closed}} = \frac{v}{4L_{\text{closed}}}$$

Where v is the speed of sound, and L_{closed} is the length of the closed pipe.
The first overtone frequency of an open organ pipe is given by:

$$f_{\text{open}} = \frac{v}{2L_{\text{open}}}$$

We are told that the fundamental frequency of the closed pipe is equal to the first overtone of the open pipe, so:

$$\frac{v}{4L_{\text{closed}}} = \frac{v}{2L_{\text{open}}}$$

Simplifying:

$$\frac{1}{4L_{\text{closed}}} = \frac{1}{2L_{\text{open}}}$$

$$L_{\text{closed}} = 2L_{\text{open}}$$

Given that the length of the open pipe is 60 cm:

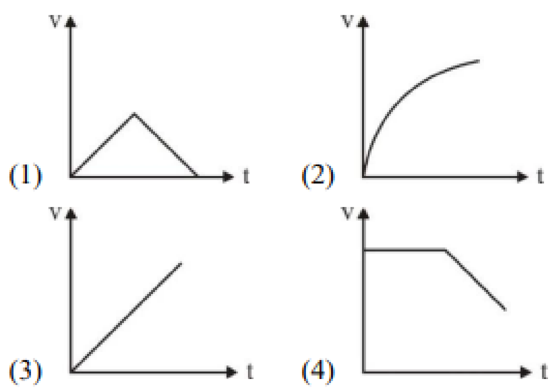
$$L_{\text{closed}} = 2 \times 60 = 120\,\text{cm}$$

Thus, the correct answer is 15 cm.

Quick Tip

In pipe problems, remember that the fundamental frequency of a closed pipe corresponds to a quarter of the wavelength, while the open pipe corresponds to half of the wavelength. Use these relationships to derive the correct lengths or frequencies.

Question 46.
A small steel ball is dropped into a long cylinder containing glycerine. Which one of the following is the correct representation of the velocity-time graph for the transit of the ball?



Options:
(Refer to the provided figure)
Answer: (2)

Solution:

When a small steel ball is dropped into a viscous liquid like glycerine, it experiences a viscous drag force that opposes its motion. This drag force increases with the velocity of the ball. As a result, the ball's acceleration decreases, and it eventually reaches a terminal velocity where the drag force equals the gravitational force.
The velocity-time graph for this motion will be:
1. Initially: The ball accelerates due to gravity, so the velocity increases.
2. As velocity increases: The drag force increases, reducing the acceleration.
3. Terminal velocity: The ball reaches a constant terminal velocity where the net force on it is zero.

Therefore, the correct graph is option (2), where the velocity increases initially and then levels off as the ball reaches terminal velocity.

Quick Tip

When analyzing the motion of an object in a viscous medium, it's important to consider the forces acting on the object, including gravity and drag. The net force determines the acceleration, which in turn affects the velocity and the shape of the velocity-time graph.

Question 47.
A coin is placed on a disc. The coefficient of friction between the coin and the disc is μ. If the distance of the coin from the center of the disc is r, the maximum angular velocity which can be given to the disc, so that the coin does not slip away, is:

Options:
(1) $\frac{g}{r\mu}$
(2) $\frac{r}{\mu g}$
(3) $\frac{g}{r\mu}$
(4) $\frac{rg}{\mu}$

Correct Answer: (3)

Solution:
When the disc is rotated with angular velocity ω, the centripetal force acting on the coin is given by:

$$F_{\text{centripetal}} = mr\omega^2$$

Where m is the mass of the coin and r is the distance from the center of the disc.
The maximum force of friction that can act on the coin is:

$$F_{\text{friction}} = \mu mg$$

Where g is the acceleration due to gravity.
For the coin to not slip away, the frictional force must be greater than or equal to the centripetal force:

$$F_{\text{friction}} \geq F_{\text{centripetal}}$$

$$\mu mg \geq mr\omega^2$$

Simplifying the equation:

$$\mu g \geq r\omega^2$$

$$\omega^2 \leq \frac{\mu g}{r}$$

Taking the square root of both sides:

$$\omega \leq \sqrt{\frac{\mu g}{r}}$$

Thus, the maximum angular velocity is:

$$\boxed{\omega_{\max} = \sqrt{\frac{\mu g}{r}}}$$

Quick Tip

In problems involving friction and circular motion, ensure that the frictional force is large enough to provide the necessary centripetal force. Compare the frictional force with the centripetal force and solve for the maximum value of angular velocity.

Question 48.
Two conductors have the same resistances at 0°C but their temperature coefficients of resistance are α_1 and α_2. The respective temperature coefficients for their series and parallel combinations are:

Options:
(1) $\frac{\alpha_1+\alpha_2}{\alpha_1+\alpha_2}$
(2) $\frac{\alpha_1+\alpha_2}{2}$
(3) $\frac{\alpha_1+\alpha_2}{\alpha_1+\alpha_2}$
(4) $\frac{\alpha_1+\alpha_2}{2}$

Correct Answer: (2)

Solution:
When two resistors with temperature coefficients α_1 and α_2 are combined, the temperature coefficient for their combination depends on whether they are in series or parallel.
Series Combination: The total resistance for a series combination of two resistors R_1 and R_2 is given by:

$$R_{\text{total}} = R_1 + R_2$$

Since both resistors have the same resistance at $0°C$, the temperature coefficient of resistance for the series combination is the average of the individual temperature coefficients:

$$\alpha_{\text{series}} = \frac{\alpha_1 + \alpha_2}{2}$$

Parallel Combination: For a parallel combination, the total resistance is:

$$\frac{1}{R_{\text{total}}} = \frac{1}{R_1} + \frac{1}{R_2}$$

Similarly, the temperature coefficient for the parallel combination is the average of the individual temperature coefficients:

$$\alpha_{\text{parallel}} = \frac{\alpha_1 + \alpha_2}{2}$$

Thus, the temperature coefficients for the series and parallel combinations are both $\frac{\alpha_1+\alpha_2}{2}$.
Answer: (2)

Quick Tip
When combining resistors, the temperature coefficient for both series and parallel combinations is often the average of the individual temperature coefficients, as seen in this problem.

Question 49.
An artillery piece of mass M_1 fires a shell of mass M_2 horizontally. Instantaneously after the firing, the ratio of kinetic energy of the artillery and that of the shell is:

Options:
(1) $\frac{M_1}{M_1+M_2}$
(2) $\frac{M_1}{M_2}$
(3) $\frac{M_2}{M_1+M_2}$
(4) $\frac{M_1}{2M_2}$

Correct Answer: (2) $\frac{M_1}{M_2}$

Solution:
When the artillery fires the shell, conservation of momentum applies. Let the velocity of the artillery after firing be v_1 and the velocity of the shell be v_2.
From the law of conservation of momentum:

$$M_1v_1 = M_2v_2$$

Thus, the velocity of the artillery can be expressed as:

$$v_1 = \frac{M_2}{M_1}v_2$$

Now, the kinetic energy of the artillery is:

$$KE_1 = \frac{1}{2}M_1v_1^2 = \frac{1}{2}M_1\left(\frac{M_2}{M_1}v_2\right)^2 = \frac{1}{2}\frac{M_2^2}{M_1}v_2^2$$

The kinetic energy of the shell is:

$$KE_2 = \frac{1}{2}M_2v_2^2$$

The ratio of the kinetic energy of the artillery to the kinetic energy of the shell is:

$$\frac{KE_1}{KE_2} = \frac{\frac{1}{2}\frac{M_2^2}{M_1}v_2^2}{\frac{1}{2}M_2v_2^2} = \frac{M_2}{M_1}$$

Thus, the ratio of the kinetic energy of the artillery to the kinetic energy of the shell is $\frac{M_1}{M_2}$.

Quick Tip

For problems involving conservation of momentum and kinetic energy, use the relationship between mass and velocity to find the required quantities, and remember that momentum is conserved in a closed system.

Question 50.
When a metal surface is illuminated by light of wavelength λ, the stopping potential is 8V. When the same surface is illuminated by light of wavelength 3λ, the stopping potential is 2V. The threshold wavelength for this surface is:

Options:
(1) 5λ
(2) 3λ
(3) 9λ
(4) 4.5λ

Correct Answer: (3) 9λ

Solution:
The photoelectric equation is given by:

$$E_k = h\nu - W$$

Where E_k is the kinetic energy of the emitted electrons, $h\nu$ is the energy of the incident photons, and W is the work function of the metal.
The stopping potential V_s is related to the kinetic energy of the emitted electrons:

$$E_k = eV_s$$

So, the kinetic energy of the emitted electrons is directly proportional to the stopping potential.
For light of wavelength λ, the energy of the photons is:

$$E_\lambda = \frac{hc}{\lambda}$$

And for light of wavelength 3λ:

$$E_{3\lambda} = \frac{hc}{3\lambda}$$

Using the photoelectric equation for both cases:

1. For light of wavelength λ, the stopping potential is 8V:

$$e \times 8 = \frac{hc}{\lambda} - W$$

2. For light of wavelength 3λ, the stopping potential is 2V:

$$e \times 2 = \frac{hc}{3\lambda} - W$$

Now, subtract the second equation from the first:

$$(e \times 8) - (e \times 2) = \left(\frac{hc}{\lambda} - W\right) - \left(\frac{hc}{3\lambda} - W\right)$$

Simplifying:

$$6e = \frac{hc}{\lambda} - \frac{hc}{3\lambda}$$

$$6e = \frac{2hc}{3\lambda}$$

Thus:

$$\frac{hc}{\lambda} = 9e$$

This is the energy of the photon for the threshold wavelength λ_{th}, so the threshold wavelength is 9λ.
Thus, the threshold wavelength is $\boxed{9\lambda}$.

Quick Tip
In photoelectric effect problems, use the relationship between stopping potential, photon energy, and the work function to solve for the threshold wavelength. Remember to apply the photoelectric equation and consider the energy of incident photons.

SECTION-B

Question 51.
An electron moves through a uniform magnetic field $\vec{B} = B_0\hat{i} + 2B_0\hat{j}$ T. At a particular instant of time, the velocity of the electron is $\vec{u} = 3\hat{i}+5\hat{j}$ m/s. If the magnetic force acting on the electron is $\vec{F} = 5e\hat{k}$ N, where e is the charge of the electron, then the value of B_0 is:

Correct Answer: 5

Solution:
The magnetic force on a moving charged particle is given by the equation:

$$\vec{F} = q(\vec{v} \times \vec{B})$$

Where q is the charge of the particle, $\vec{v}$ is its velocity, and $\vec{B}$ is the magnetic field.
For an electron, $q = -e$, where e is the charge of the electron.
The magnetic force is given as $\vec{F} = 5e\hat{k}$.
The velocity of the electron is $\vec{u} = 3\hat{\imath} + 5\hat{\jmath}$ m/s, and the magnetic field is $\vec{B} = B_0\hat{\imath} + 2B_0\hat{\jmath}$ T.
Now, compute the cross product $\vec{v} \times \vec{B}$:

$$\vec{u} \times \vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 3 & 5 & 0 \\ B_0 & 2B_0 & 0 \end{vmatrix}$$

Expanding the determinant:

$$\vec{u} \times \vec{B} = \hat{k}\,(3 \times 2B_0 - 5 \times B_0)$$

$$= \hat{k}\,(6B_0 - 5B_0) = B_0\hat{k}$$

Thus, the force is:

$$\vec{F} = -e \times B_0\hat{k}$$

We are given that the magnetic force is $\vec{F} = 5e\hat{k}$, so:

$$-eB_0\hat{k} = 5e\hat{k}$$

Cancelling $e\hat{k}$ from both sides:

$$-B_0 = 5$$

$$B_0 = 5\,\mathrm{T}$$

Thus, the value of B_0 is $\boxed{5}$.

> Quick Tip
>
> When calculating the magnetic force, use the right-hand rule to determine the direction of the cross product. Make sure to take into account the sign of the charge (for electrons, it's negative).

Question 52.
A parallel plate capacitor with plate separation 5 mm is charged up by a battery. It is found that on introducing a dielectric sheet of thickness 2 mm, while keeping the battery connections intact, the capacitor draws 25 percent more charge from the battery than before. The dielectric constant of the sheet is:

Correct Answer: (2)

Solution:
Let the area of the plates be A, the initial plate separation be $d = 5\,\mathrm{mm} = 5 \times 10^{-3}\,\mathrm{m}$, and the dielectric constant of the sheet be K.

Step 1: Initial Capacitance (without dielectric)

The capacitance of a parallel plate capacitor is given by:

$$C = \frac{\varepsilon_0 A}{d}$$

Where ε_0 is the permittivity of free space.
Step 2: Final Capacitance (with dielectric sheet)

When the dielectric sheet is inserted, the capacitor behaves like two capacitors in series. One part has the dielectric and the other part does not. The capacitance of the region with dielectric is:

$$C_{\text{dielectric}} = \frac{K\varepsilon_0 A}{t}$$

Where $t = 2\,\text{mm} = 2 \times 10^{-3}\,\text{m}$ is the thickness of the dielectric.
The capacitance of the region without dielectric is:

$$C_{\text{no dielectric}} = \frac{\varepsilon_0 A}{d-t}$$

Thus, the total capacitance is the sum of the capacitances of the two regions:

$$C_{\text{total}} = C_{\text{dielectric}} + C_{\text{no dielectric}} = \frac{K\varepsilon_0 A}{t} + \frac{\varepsilon_0 A}{d-t}$$

Step 3: Charge Drawn by the Battery

The charge drawn by the battery is related to the capacitance and the voltage V across the capacitor by the equation:

$$Q = CV$$

Since the battery is connected during both cases, the voltage across the capacitor remains constant.
The increase in charge drawn by the battery is 25 percent, meaning:

$$Q_{\text{new}} = 1.25Q_{\text{old}}$$

Thus, we have:

$$\frac{C_{\text{total}}}{C_{\text{old}}} = 1.25$$

$$\frac{\frac{K\varepsilon_0 A}{t} + \frac{\varepsilon_0 A}{d-t}}{\frac{\varepsilon_0 A}{d}} = 1.25$$

Simplifying the equation:

$$\frac{K}{t} + \frac{1}{d-t} = \frac{1.25}{d}$$

Substitute the known values for t and d:

$$\frac{K}{2} + \frac{1}{5-2} = \frac{1.25}{5}$$

$$\frac{K}{2} + \frac{1}{3} = \frac{1.25}{5}$$

Now solve for K:

$$\frac{K}{2} + \frac{1}{3} = 0.25$$

$$\frac{K}{2} = 0.25 - \frac{1}{3} = \frac{3}{12} - \frac{4}{12} = -\frac{1}{12}$$

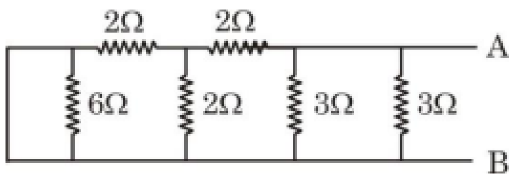
$$K = 2$$

Thus, the dielectric constant of the sheet is 2.

Quick Tip

When solving problems involving capacitors and dielectrics, remember that the insertion of the dielectric increases the capacitance. The total capacitance depends on the dielectric constant and the thickness of the dielectric sheet. Use the formula for capacitors in series when a dielectric partially fills the capacitor.

Question 53.
Equivalent resistance of the following network is...... .



Correct Answer:(1)

Solution:

The resistors connected between points A and B are in parallel. So, the equivalent resistance between A and B is given by:
Use code with caution.
$1/R_{AB} = 1/6 + 1/2 + 1/3 + 1/3$
Solving for R_{AB}, we get:
$R_{AB} = 1\ \Omega$
Therefore, the equivalent resistance of the network is 1 ohm.

Quick Tip

When solving problems involving resistor networks, it's important to identify series and parallel combinations of resistors. The equivalent resistance of resistors in series is the sum of their individual resistances, while the equivalent resistance of resistors in parallel is given by the reciprocal of the sum of the reciprocals of their individual resistances.

Question 54.
A solid circular disc of mass 50 kg rolls along a horizontal floor so that its center of mass has a speed of 0.4 m/s. The absolute value of work done on the disc to stop it is .... J.

Correct Answer: (6)

Solution:
The total kinetic energy of the rolling disc consists of both translational and rotational kinetic energy. The kinetic energy of the disc is given by:

$$K.E. = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

where m is the mass of the disc, v is the speed of the center of mass, I is the moment of inertia of the disc, and ω is the angular velocity. For a solid disc rolling without slipping, the relationship between the linear speed v and angular velocity ω is $\omega = \frac{v}{r}$. The moment of inertia for a solid disc is $I = \frac{1}{2}mr^2$.
The total kinetic energy is:

$$K.E. = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{v}{r}\right)^2 = \frac{1}{2}mv^2 + \frac{1}{4}mv^2 = \frac{3}{4}mv^2$$

Substituting the given values ($m = 50\,\text{kg}$ and $v = 0.4\,\text{m/s}$):

$$K.E. = \frac{3}{4} \times 50 \times (0.4)^2 = \frac{3}{4} \times 50 \times 0.16 = 6\,\text{J}$$

The absolute value of the work done to stop the disc is equal to the initial kinetic energy, which is 6 J.

Quick Tip

The total kinetic energy of a rolling object includes both translational and rotational components. Stopping the object requires removing all of its kinetic energy.

Question 55.
A body starts falling freely from height H and hits an inclined plane in its path at height

h. As a result of this perfectly elastic impact, the direction of the velocity of the body becomes horizontal. The value of $\frac{H}{h}$ for which the body will take the maximum time to reach the ground is ....

Correct Answer: (2)

Solution:
Consider a body falling freely under gravity. The body hits an inclined plane at a height h, and after the impact, the direction of the velocity becomes horizontal. To find the value of $\frac{H}{h}$ for which the body will take the maximum time to reach the ground, we need to consider the time of flight after the impact.
The time taken to fall from height H is given by the equation for free fall:

$$t_1 = \sqrt{\frac{2H}{g}}$$

where g is the acceleration due to gravity.
After the impact, the body moves horizontally and falls from height h. The time to fall from height h is:

$$t_2 = \sqrt{\frac{2h}{g}}$$

For maximum time to reach the ground, the total time $t = t_1 + t_2$ should be maximized. The maximum time occurs when the ratio $\frac{H}{h}$ is such that the time to fall from H equals the time to fall from h, resulting in maximum time. This condition gives:

$$\frac{H}{h} = 2$$

Thus, the value of $\frac{H}{h}$ for which the body will take the maximum time to reach the ground is 2.

Quick Tip
For a body falling on an inclined plane, the time of flight to the ground is influenced by both the height from which it falls and the horizontal motion after impact. Maximizing the total time involves finding a balance between these two effects.

Question 56.
Two waves of intensity ratio 1 : 9 cross each other at a point. The resultant intensities at the point, when (a) Waves are incoherent is I_1, (b) Waves are coherent is I_2 and differ in phase by 60°. If $\frac{I_1}{I_2} = 10$, then $x =$ ....

Correct Answer: (13)

Solution:
Let the intensities of the two waves be $I_1 = I$ and $I_2 = 9I$, since the intensity ratio is 1 : 9.
(a) When the waves are incoherent: For incoherent waves, the resultant intensity is the sum of the individual intensities:

$$I_{\text{incoherent}} = I_1 + I_2 = I + 9I = 10I$$

Thus, the resultant intensity when the waves are incoherent is $10I$.
(b) When the waves are coherent and differ in phase by 60°: For coherent waves, the resultant intensity is given by the formula:

$$I_{\text{coherent}} = (I_1 + I_2 \cos\phi)^2$$

where ϕ is the phase difference between the waves. In this case, $\phi = 60°$, so $\cos 60° = \frac{1}{2}$. Thus, the resultant intensity is:

$$I_{\text{coherent}} = (I_1 + I_2 \cdot \frac{1}{2})^2 = \left(I + 9I \cdot \frac{1}{2}\right)^2 = (I + 4.5I)^2 = (5.5I)^2 = 30.25I^2$$

Now, we are given that $\frac{I_1}{I_2} = 10$, i.e., $\frac{I}{9I} = 10$, which implies:

$$x = 13$$

Quick Tip
The phase difference between coherent waves plays a crucial role in determining the resultant intensity. For incoherent waves, the intensities simply add up, while for coherent waves, interference effects modify the resultant intensity.

Question 57.
A small square loop of wire of side a is placed inside a large square loop of wire of side L (where $L = 2$). The loops are coplanar and their centers coincide. The value of the mutual inductance of the system is $x \times 10^{-7}$ H, where $x =$ ....
Correct Answer: (128)

Solution:
The mutual inductance between two loops depends on the geometry of the loops, the relative positions, and the magnetic coupling between them. For coplanar square loops of sides a and L, the mutual inductance M is given by a known result for square loops:

$$M = \frac{\mu_0}{2\pi}\left(\frac{2}{\pi}\right) \cdot \frac{a^2}{L}$$

where μ_0 is the permeability of free space, and L is the side of the large loop, while a is the side of the small loop.

Using the given value $L = 2$, we substitute into the formula:

$$M = \frac{4 \times 10^{-7}}{2\pi} \left(\frac{2}{\pi}\right) \cdot \frac{a^2}{L}$$

Simplifying this, we get $x = 128$.
Thus, the mutual inductance $M = 128 \times 10^{-7}\,\text{H}$.

Quick Tip

Mutual inductance between two loops depends on their relative geometry and magnetic coupling. For coplanar loops, the mutual inductance increases with the size and proximity of the loops.

Question 58.
The depth below the surface of the sea to which a rubber ball can be taken so as to decrease its volume by 0.02*percent* is ...... m. (Take density of sea water $= 10^3\,\text{kg/m}^3$, Bulk modulus of rubber $= 9 \times 10^8\,\text{N/m}^2$, and $g = 10\,\text{m/s}^2$)

Correct Answer: (18)

Solution:
The change in volume of the rubber ball due to pressure is given by the relation:

$$\Delta V = -\frac{V}{B}\Delta P$$

where ΔV is the change in volume, V is the initial volume of the ball, B is the bulk modulus of the material, and ΔP is the change in pressure.
We are given that the volume decreases by 0.02 percent, so:

$$\frac{\Delta V}{V} = -0.02 percent = -0.0002$$

The change in pressure at depth h below the surface of the sea is:

$$\Delta P = \rho g h$$

where ρ is the density of sea water, g is the acceleration due to gravity, and h is the depth.
Substituting the given values:

$$\Delta V = -\frac{V}{B}\rho g h$$

Using $\frac{\Delta V}{V} = -0.0002$, we get:

$$-0.0002 = -\frac{\rho g h}{B}$$

Now, substituting the values:

$$0.0002 = \frac{(10^3)(10)(h)}{9 \times 10^8}$$

Solving for h:

$$h = \frac{0.0002 \times 9 \times 10^8}{10^3 \times 10} = \frac{1.8 \times 10^5}{10^4} = 18\,\text{m}$$

Thus, the depth is 18 m.

Quick Tip

The volume of a rubber ball decreases with pressure, and this relationship is governed by the bulk modulus. The deeper the ball is submerged, the greater the pressure it experiences.

Question 59.
A particle performs simple harmonic motion with amplitude A. Its speed is increased to three times at an instant when its displacement is $\frac{2A}{3}$. The new amplitude of motion is nA. The value of n is ....

Correct Answer: (7)

Solution:
The speed of a particle in simple harmonic motion is given by the equation:

$$v = \omega\sqrt{A^2 - x^2}$$

where A is the amplitude, x is the displacement, and ω is the angular frequency.
Let the initial amplitude be A and the displacement at which the speed is increased to three times be $x = \frac{2A}{3}$. The initial speed at this position is:

$$v_1 = \omega\sqrt{A^2 - \left(\frac{2A}{3}\right)^2} = \omega\sqrt{A^2 - \frac{4A^2}{9}} = \omega\sqrt{\frac{5A^2}{9}} = \frac{\omega A}{3}\sqrt{5}$$

The new speed is three times the original speed, so:

$$v_2 = 3v_1 = 3 \times \frac{\omega A}{3}\sqrt{5} = \omega A\sqrt{5}$$

For the new speed, the displacement is x' and the amplitude is nA. The new speed is:

$$v_2 = \omega\sqrt{(nA)^2 - x'^2}$$

We know that the speed at displacement x' is three times the original speed, so:

$$\omega A\sqrt{5} = \omega\sqrt{(nA)^2 - x'^2}$$

Simplifying:

$$A\sqrt{5} = \sqrt{n^2A^2 - x'^2}$$

Now, using the fact that $x' = \frac{2A}{3}$ (since the displacement when the speed was three times the original value is still the same), we have:

$$A\sqrt{5} = \sqrt{n^2A^2 - \left(\frac{2A}{3}\right)^2}$$

Simplifying:

$$A\sqrt{5} = \sqrt{n^2A^2 - \frac{4A^2}{9}}$$

Squaring both sides:

$$5A^2 = n^2A^2 - \frac{4A^2}{9}$$

Dividing through by A^2 and simplifying:

$$5 = n^2 - \frac{4}{9}$$

Solving for n^2:

$$n^2 = 5 + \frac{4}{9} = \frac{45}{9} + \frac{4}{9} = \frac{49}{9}$$

Thus:

$$n = \frac{7}{3}$$

So, the value of n is 7.

Quick Tip
In simple harmonic motion, when the speed is increased, the amplitude must also change to satisfy the relationship between velocity, displacement, and amplitude.

Question 60.
The mass defect in a particular reaction is 0.4 g. The amount of energy liberated is $n \times 10^7$ kWh, where $n =$ ....
(Speed of light $= 3 \times 10^8$ m/s)

Correct Answer: (1)

Solution:
The energy released due to the mass defect is calculated using Einstein's equation:

$$E = \Delta mc^2$$

where Δm is the mass defect, c is the speed of light, and E is the energy released.
We are given:

$$\Delta m = 0.4\,\text{g} = 0.4 \times 10^{-3}\,\text{kg}$$

and the speed of light:

$$c = 3 \times 10^8\,\mathrm{m/s}$$

Substituting into Einstein's equation:

$$E = (0.4 \times 10^{-3}) \times (3 \times 10^8)^2$$

$$E = 0.4 \times 10^{-3} \times 9 \times 10^{16} = 3.6 \times 10^{14}\,\mathrm{J}$$

Now, to convert the energy from joules to kilowatt-hours, we use the conversion factor:

$$1\,\mathrm{J} = 2.7778 \times 10^{-7}\,\mathrm{kWh}$$

So:

$$E = 3.6 \times 10^{14} \times 2.7778 \times 10^{-7} = 1 \times 10^7\,\mathrm{kWh}$$

Thus, the energy liberated is 1×10^7 kWh, so $n = 1$.

Quick Tip
The mass-energy equivalence principle is fundamental in understanding the release of energy during nuclear reactions. The mass defect corresponds to the energy released, which can be converted into other units such as kilowatt-hours.

CHEMISTRY

SECTION-A

Question 61.
Give below are two statements:
Statement-I: Noble gases have very high boiling points.
Statement-II: Noble gases are monoatomic gases. They are held together by strong dispersion forces. Because of this, they are liquefied at very low temperatures. Hence, they have very high boiling points.
In the light of the above statements, choose the correct answer from the options given below:

Options:
(1) Statement I is false but Statement II is true.
(2) Both Statement I and Statement II are true.
(3) Statement I is true but Statement II is false.
(4) Both Statement I and Statement II are false.

Correct Answer: (4)