JEE Main 2023 April 13 Shift 2 Question Paper

Time Allowed : 3 Hour	**Maximum Marks :** 300	**Total Questions :** 90

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The test is of 3 hours duration.
2. The question paper consists of 90 questions, out of which 75 are to attempted. The maximum marks are 300.
3. There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage.
4. Each part (subject) has two sections.
   (i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and –1 mark for wrong answer.
   (ii) Section-B: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and –1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer

SECTION-A

1. The area of the region $\left\{(x, y) : x^2 \leq y \leq |x^2 - 4|, y \geq 1\right\}$ **is:**
(1) $\frac{3}{4}\left(4\sqrt{2}+1\right)$
(2) $\frac{4}{3}\left(4\sqrt{2}-1\right)$
(3) $\frac{3}{4}\left(4\sqrt{2}-1\right)$
(4) $\frac{4}{3}\left(4\sqrt{2}+1\right)$

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2. If $\lim_{x\to 0} \frac{e^{ax}-\cos(bx)-\frac{cxe^{-cx}}{2}}{1-\cos(2x)} = 17$**, then** $5a^2 + b^2$ **is equal to:**
(1) 76
(2) 72
(3) 64
(4) 68

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3. The line that is coplanar to the line $\frac{x+3}{-3} = \frac{y-1}{1} = \frac{z-5}{5}$ **is:**
(1) $\frac{x+1}{-1} = \frac{y-2}{2} = \frac{z-5}{5}$
(2) $\frac{x+1}{1} = \frac{y-2}{2} = \frac{z-5}{5}$
(3) $\frac{x+1}{-1} = \frac{y-2}{2} = \frac{z-5}{4}$
(4) $\frac{x+1}{-1} = \frac{y-2}{2} = \frac{z-5}{5}$

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4. The plane, passing through the points $(0, -1, 2)$ **and** $(-1, 2, 1)$**, and parallel to the line passing through** $(5, 1, -7)$ **and** $(1, -1, -1)$**, also passes through the point:**
(1) $(0, 5, -2)$
(2) $(-2, 5, 0)$
(3) $(2, 0, 1)$
(4) $(1, -2, 1)$

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5. Let for a triangle ABC,

$$\overrightarrow{AB} = -2\hat{i} + \hat{j} + 3\hat{k}, \quad \overrightarrow{CB} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}, \quad \overrightarrow{CA} = 4\hat{i} + 3\hat{j} + \delta\hat{k}$$

If $\delta > 0$ and the area of the triangle ABC is $5\sqrt{6}$, then $\overrightarrow{CB} \cdot \overrightarrow{CA}$ is equal to:
(1) 108
(2) 60
(3) 54
(4) 120

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6. Let for $A = \begin{bmatrix} 1 & 2 & 3 \\ \alpha & 3 & 1 \\ 1 & 1 & 2 \end{bmatrix}$, $|A| = 2$. If $|2\, adj(2\, adj(2A))| = 32^n$, then $3n + \alpha$ is equal to:
(1) 10
(2) 9
(3) 12
(4) 11

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7. The range of $f(x) = 4\sin^{-1}\left(\frac{x^2}{x^2+1}\right)$ is:
(1) $[0, \pi]$
(2) $[0, \pi]$
(3) $[0, 2\pi]$
(4) $[0, 2\pi]$

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8. Let $a_1, a_2, a_3, \ldots$ be a G.P. of increasing positive numbers. Let the sum of its 6th and 8th terms be 2 and the product of its 3rd and 5th terms be $\frac{1}{9}$. Then $6(a_2 + a_4)(a_4 + a_6)$ is equal to:
(1) 2
(2) 3
(3) $3\sqrt{3}$
(4) $2\sqrt{2}$

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9. If the system of equations

$$2x + y - z = 52x - 5y + \lambda z = \mu x + 2y - 5z = 7$$

has infinitely many solutions, then $(\lambda + \mu)^2 + (\lambda - \mu)^2$ is equal to:
(1) 904
(2) 916

(3) 912
(4) 920

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10. The statement $(p \wedge \sim q) \vee ((\sim p) \wedge q) \vee ((\sim p) \wedge (\sim q))$ **is equivalent to:**
(1) $(\sim p) \vee (\sim q)$
(2) $(\sim p) \wedge (\sim q)$
(3) $p \vee q$
(4) $p \vee \sim q$

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11. Let $S = \left\{ z \in C : \overline{z} = i\left(z^2 + \mathbf{Re}(\overline{z})\right)\right\}$**. Then** $\sum_{z \in S} |z|^2$ **is equal to:**
(1) 4
(2) $\frac{7}{2}$
(3) 3
(4) $\frac{5}{2}$

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12. Let α, β **be the roots of the equation** $x^2 - \sqrt{2}x + 2 = 0$**, then** $\alpha^{14} + \beta^{14}$ **is equal to:**
(1) $-128\sqrt{2}$
(2) $-64\sqrt{2}$
(3) -128
(4) -64

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13. Let $|\mathbf{a}| = 2$, $|\mathbf{b}| = 3$**, and the angle between the vectors a and b be** $\frac{\pi}{4}$**. Then** $|(\mathbf{a} + 2\mathbf{b}) \times (2\mathbf{a} - 3\mathbf{b})|^2$ **is equal to:**
(1) 482
(2) 841
(3) 882
(4) 441

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14. The value of

$$\frac{e^{-\pi/4} + \int_0^{\pi/4} e^{-x}\left(\tan^{50} x\right)\, dx}{\int_0^{\pi/4} e^{-x}\left(\tan^{49} x + \tan^{51} x\right)\, dx}$$

is:
(1) 25
(2) 51
(3) 50
(4) 49

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15. The coefficient of x^5 in the expansion of $\left(2x^3 - \frac{1}{3}x^2\right)^5$ is:
(1) $\frac{80}{9}$
(2) 8
(3) 9
(4) $\frac{26}{3}$

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16. The random variable X follows binomial distribution B(n, p), for which the difference of the mean and the variance is 1. If $2P(x = 2) = 3P(x = 1)$, then $n^2P(X > 1)$ is equal to:
(1) 16
(2) 11
(3) 12
(4) 15

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17. Let the centre of a circle C be (α, β) and its radius $r < 8$. Let $3x + 4y = 24$ and $3x - 4y = 32$ be two tangents and $4x + 3y = 1$ be a normal to C. Then $(\alpha - \beta + r)$ is equal to:
(1) 5
(2) 6
(3) 7
(4) 9

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18. Let N be the foot of the perpendicular from the point $P(1, -2, 3)$ on the line passing through the points $(4, 5, 8)$ and $(1, -7, 5)$. Then the distance of N from the plane $2x - 2y + z + 5 = 0$ is:
(1) 6
(2) 7
(3) 9
(4) 8

19. All words, with or without meaning, are made using all the letters of the word MONDAY. These words are written as in a dictionary with serial numbers. The serial number of the word MONDAY is:
(1) 328
(2) 327
(3) 324
(4) 326

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20. Let (α, β) be the centroid of the triangle formed by the lines $15x - y = 82$, $6x - 5y = -4$, and $9x + 4y = 17$. Then $\alpha + 2\beta$ and $2\alpha - \beta$ are the roots of the equation:
(1) $x^2 - 13x + 42 = 0$
(2) $x^2 - 10x + 25 = 0$
(3) $x^2 - 7x + 12 = 0$
(4) $x^2 - 14x + 48 = 0$

SECTION-B

21. Let $A = \{-4, -3, -2, 0, 1, 3, 4\}$ and $R = \{(a, b) \in A \times A : b = |a|$ or $b^2 = a + 1\}$ be a relation on A. Then the minimum number of elements that must be added to the relation R so that it becomes reflexive and symmetric, is:

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22. $\mathrm{f}_n = \int_0^{\frac{\pi}{2}} \left(\sum_{k=1}^{n} \sin^{k-1} x\right) \left(\sum_{k=1}^{n} (2k-1) \sin^{k-1} x\right) \cos x \, dx, \quad n \in N.$
Then $\mathrm{f}_{21} - f_{20}$ is equal to:

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23. If $y = y(x)$ is the solution of the differential equation

$$\frac{dy}{dx} + \frac{4x}{x^2 - 1} y = \frac{x + 2}{(x^2 - 1)^{5/2}}, \quad x > 1$$

such that

$$y(2) = \frac{2}{9} \log_e \left(2 + \sqrt{3}\right) \quad \text{and} \quad y(\sqrt{2}) = \alpha \log_e \left(\sqrt{\alpha + \beta}\right) + \beta - \sqrt{\gamma}, \quad \alpha, \beta, \gamma \in N,$$

then $\alpha\beta\gamma$ is equal to __.

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24. Total numbers of 3-digit numbers that are divisible by 6 and can be formed by using the digits 1, 2, 3, 4, 5 with repetition, is:

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25. The remainder, when 7^{103} **is divided by 17, is:**

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26. Let $f(x) = \sum_{k=1}^{10} kx^k, x \in R$**. If** $2f(2) - f'(2) = 119(2)^n + 1$**, then** n **is equal to:**

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27. For $x \in (-1, 1)$**, the number of solutions of the equation** $\sin^{-1} x = 2\tan^{-1} x$ **is equal to:**
(1) 1
(2) 2
(3) 3
(4) 0

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28. The mean and standard deviation of the marks of 10 students were found to be 50 and 12 respectively. Later, it was observed that two marks 20 and 25 were wrongly read as 45 and 50 respectively. Then the correct variance is:

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29. The foci of a hyperbola are $(\pm 2, 0)$ **and its eccentricity is** $\frac{3}{2}$**. A tangent, perpendicular to the line** $2x + 3y = 6$**, is drawn at a point in the first quadrant on the hyperbola. If the intercepts made by the tangent on the x and y-axes are** a **and** b **respectively, then** $|6a| + |5b|$ **is equal to __.**

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30. Let $\lfloor \alpha \rfloor$ denote the greatest integer $\leq \alpha$. Then

$$\left\lfloor \sqrt{1} \right\rfloor + \left\lfloor \sqrt{2} \right\rfloor + \left\lfloor \sqrt{3} \right\rfloor + \cdots + \left\lfloor \sqrt{120} \right\rfloor$$

is equal to __.
