JEE Main 2023 April 10 Shift 2 Question Paper With Solutions

Time Allowed :3 Hour	Maximum Marks :300	Total Questions :90

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The test is of 3 hours duration.
2. The question paper consists of 90 questions, out of which 75 are to attempted. The maximum marks are 300.
3. There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage.
4. Each part (subject) has two sections.
(i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and –1 mark for wrong answer.
(ii) Section-B: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and –1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer

Mathematics

Section-A

1. If the coefficients of x and x^2 in $(1+x)^p(1-x)^q$ are 4 and -5 respectively, then $2p+3q$ is equal to:
(1) 60
(2) 63
(3) 66
(4) 69
Correct Answer: (2) 63
Solution:
Step 1: Expand $(1+x)^p(1-x)^q$ The expansions of $(1+x)^p$ and $(1-x)^q$ are as follows:

$$(1+x)^p = 1 + px + \frac{p(p-1)}{2!}x^2 + \dots$$

$$(1-x)^q = 1 - qx + \frac{q(q-1)}{2!}x^2 - \dots$$

Step 2: Multiply the expansions Next, multiply the two expansions:

$$(1+x)^p(1-x)^q = \left(1 + px + \frac{p(p-1)}{2!}x^2 + \dots\right) \times \left(1 - qx + \frac{q(q-1)}{2!}x^2 - \dots\right)$$

To find the coefficient of x, we add the products of terms that result in x:

$$\text{Coefficient of } x = p - q$$

For x^2, the coefficient is given by:

$$\text{Coefficient of } x^2 = \frac{p(p-1)}{2!} + \frac{q(q-1)}{2!}$$

Step 3: Using the given values The coefficients of x and x^2 are provided as 4 and -5, respectively:

$$p - q = 4 \quad (1)$$

$$\frac{p(p-1)}{2!} + \frac{q(q-1)}{2!} = -5 \quad (2)$$

Step 4: Solve the system of equations From equation (1):

$$p = q + 4$$

Substituting $p = q + 4$ into equation (2):

$$\frac{(q+4)(q+3)}{2} + \frac{q(q-1)}{2} = -5$$

Solving this gives $p = 15$ and $q = 11$.
Step 5: Compute $2p + 3q$ Now, compute:

$$2p + 3q = 2(15) + 3(11) = 30 + 33 = 63$$

Thus, $2p + 3q = 63$.

> 💡 Quick Tip
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> When working with binomial expansions, expand both expressions, multiply them together, and equate the coefficients of the terms involving the desired powers of x.

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2. Let $A = \{2, 3, 4\}$ and $B = \{8, 9, 12\}$. Then the number of elements in the relation $R = \{((a_1, b_1), (a_2, b_2)) \in (A \times B, A \times B) : a_1$ divides b_2 and a_2 divides $b_1\}$ is:
(1) 18
(2) 24
(3) 12
(4) 36
Correct Answer: (4) 36
Solution:
Step 1: Divisibility conditions
We are given two sets $A = \{2, 3, 4\}$ and $B = \{8, 9, 12\}$. Our goal is to determine the number of elements in the relation where a_1 divides b_2 and a_2 divides b_1.
Step 2: Divisibility for a_1 dividing b_2
For each $a_1 \in A$, there are 2 elements in B that satisfy the divisibility condition.
Step 3: Divisibility for a_2 dividing b_1
For each $a_2 \in A$, there are 2 elements in B that satisfy the divisibility condition.
Step 4: Total number of relations
Each element in A has 2 choices for divisibility with elements in B, so the total number of relations is:

$$\text{Total} = 6 \times 6 = 36$$

Thus, the number of elements in the relation is 36.

> 💡 Quick Tip
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> For problems involving divisibility, always check the divisibility conditions for each pair of elements in the sets, and multiply the possibilities for each condition to get the total number of relations.

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3. Let time image of the point $P(1, 2, 6)$ in the plane passing through the points A(1, 2, 0), B(1, 4, 1), and C(0, 5, 1) be $Q(\alpha, \beta, \gamma)$. Then $\alpha^2 + \beta^2 + \gamma^2$ is equal to:
(1) 70
(2) 76
(3) 62
(4) 65
Correct Answer: (4) 65

Solution: Step 1: The equation of the plane passing through points A(1, 2, 0), B(1, 4, 1), and C(0, 5, 1) is:

$$A(x - 1) + B(y - 2) + C(z - 0) = 0$$

By substituting the coordinates of points A, B, and C, we form the following system of equations:
From point $(1,4,1)$, we get: $2B + C = 0$
From point $(0,5,1)$, we get: $-A + 3B + C = 0$
Solving this system, we obtain $A = -2B$ and $C = -2B$.

Step 2: To find the image of the point $P(1,2,6)$, we use the following formula:

$$\frac{\alpha - 1}{1} = \frac{\beta - 2}{1} = \frac{\gamma - 6}{-2} = \frac{-2(1 + 2 - 12 - 3)}{6}$$

Solving this, we get:

$$\alpha = 5, \beta = 6, \gamma = -2$$

Step 3: Now, compute $\alpha^2 + \beta^2 + \gamma^2$:

$$\alpha^2 + \beta^2 + \gamma^2 = 5^2 + 6^2 + (-2)^2 = 25 + 36 + 4 = 65$$

Thus, the correct answer is option (4).

> Quick Tip
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> When finding the image of a point in a plane, apply the reflection formula and solve the system of equations involving the plane's equation and the point's coordinates.

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4. The statement $\sim [p \vee (\sim (p \wedge q))]$ is equivalent to:
(1) $\sim (p \wedge q) \wedge q$
(2) $\sim (p \vee q)$
(3) $\sim (p \wedge q)$
(4) $(p \wedge q) \wedge (\sim p)$
Correct Answer: (4) $(p \wedge q) \wedge (\sim p)$
Solution:
Step 1: The given expression
We are provided with the statement $\sim [p \vee (\sim (p \wedge q))]$. Our goal is to simplify this expression and determine the equivalent logical statement.
Step 2: Apply De Morgan's law
First, we apply De Morgan's law to the negation of the disjunction $\sim [p \vee (\sim (p \wedge q))]$. De Morgan's law states that $\sim (A \vee B) = \sim A \wedge \sim B$, so we get:

$$\sim p \wedge \sim (\sim (p \wedge q))$$

Step 3: Simplify the double negation
Next, we simplify the inner double negation $\sim (\sim (p \wedge q))$, which cancels out the two negations, leaving us with:

$$\sim p \wedge (p \wedge q)$$

Step 4: Final form Thus, the expression simplifies to:

$$(p \wedge q) \wedge (\sim p)$$

This is the correct equivalent form of the original expression.

Quick Tip

When simplifying logical expressions, carefully apply De Morgan's laws and eliminate double negations wherever possible.

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5. Let $S = \left\{x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) : 9^{1-\tan^2 x} + 9^{\tan^2 x} = 10\right\}$

$$b = \sum_{x \in S} \tan^2\left(\frac{x}{3}\right), \text{ then } (\beta - 14)^2 \text{ is equal to:}$$

(1) 16
(2) 32
(3) 8
(4) 64
Correct Answer: (2) 32

Solution: Step 1: Let $9^{\tan^2 x} = P$, which gives the equation:

$$\frac{9}{P} + P = 10$$

Solving for P:

$$P^2 - 10P + 9 = 0$$

$$(P - 9)(P - 1) = 0$$

Thus, $P = 9$ or $P = 1$.

Step 2: Therefore, $9^{\tan^2 x} = 9$, which implies $\tan^2 x = 1$, so $x = 0, \pm\frac{\pi}{4}$.
Hence, $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

Step 3: Now, calculate β:

$$\beta = \tan^2(0) + \tan^2\left(\frac{\pi}{12}\right) + \tan^2\left(-\frac{\pi}{12}\right)$$

$$\beta = 0 + 2\left(\tan 15^\circ\right)^2$$

Using the approximation $\tan 15^\circ = 2 - \sqrt{3}$, we get:

$$\beta = 2(2 - \sqrt{3})^2$$

$$\beta = 2(7 - 4\sqrt{3})$$

Now, calculate $(\beta - 14)^2$:

$$(\beta - 14)^2 = \left(14 - 8\sqrt{3} - 14\right)^2 = 32$$

Thus, the correct answer is option (2).

Quick Tip

For problems involving trigonometric identities and summation, simplify using known values and identities for specific angles, such as $\frac{\pi}{12}$ and $\frac{\pi}{4}$.

6. If the points P and Q are respectively the circumcenter and the orthocenter of a $\triangle ABC$, the $\overrightarrow{PA} + \overrightarrow{PB} + \overrightarrow{PC}$ is equal to:
(1) $2\overrightarrow{PQ}$
(2) $\overrightarrow{PQ}$
(3) $2\overrightarrow{PQ}$
(4) $\overrightarrow{PQ}$
Correct Answer: (4) $\overrightarrow{PQ}$
Solution:
Step 1: Applying the centroid formula Let the position vectors of points A, B, C be $\overrightarrow{a}, \overrightarrow{b}, \overrightarrow{c}$, respectively. Since P and Q are the circumcenter and orthocenter of the triangle, respectively, we can use the following vector identity:

$$\overrightarrow{PA} + \overrightarrow{PB} + \overrightarrow{PC} = \overrightarrow{a} + \overrightarrow{b} + \overrightarrow{c}$$

Step 2: Applying the centroid formula The centroid G of the triangle has the position vector:

$$\overrightarrow{PG} = \frac{\overrightarrow{a} + \overrightarrow{b} + \overrightarrow{c}}{3}$$

Thus, we can express:

$$\overrightarrow{a} + \overrightarrow{b} + \overrightarrow{c} = 3\overrightarrow{PG}$$

Therefore, we conclude:

$$\overrightarrow{PA} + \overrightarrow{PB} + \overrightarrow{PC} = 3\overrightarrow{PG} = \overrightarrow{PQ}$$

Step 3: Final conclusion Hence, $\overrightarrow{PA} + \overrightarrow{PB} + \overrightarrow{PC} = \overrightarrow{PQ}$, which corresponds to the correct option (4).

Quick Tip

When solving vector equations in triangle geometry, particularly involving the orthocenter, circumcenter, and centroid, leverage their geometric relationships to simplify the solution process.

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7. Let A be the point (1, 2) and B be any point on the curve $x^2 + y^2 = 16$. If the centre of the locus of the point P, which divides the line segment AB in the ratio 3:2 is the point C (α, β), then the length of the line segment AC is:

(1) $\frac{6\sqrt{5}}{5}$
(2) $\frac{2\sqrt{5}}{5}$
(3) $\frac{3\sqrt{5}}{5}$
(4) $\frac{4\sqrt{5}}{5}$
Correct Answer: (3) $\frac{3\sqrt{5}}{5}$
Solution:
Step 1: Determine the coordinates of points A and B Let $A(1,2)$ be one point, and the coordinates of point B are given by $B(4\cos\theta, 4\sin\theta)$, as $x^2+y^2=16$ represents a circle with radius 4.
Step 2: Apply the section formula The point P divides the line segment AB in the ratio 3:2. Using the section formula:

$$\left(\frac{3x_B+2x_A}{5}, \frac{3y_B+2y_A}{5}\right)$$

Substitute the coordinates of $A(1,2)$ and $B(4\cos\theta, 4\sin\theta)$ into the formula:

$$P=\left(\frac{3(4\cos\theta)+2(1)}{5}, \frac{3(4\sin\theta)+2(2)}{5}\right)$$

Simplifying, we obtain:

$$P=\left(\frac{12\cos\theta+2}{5}, \frac{12\sin\theta+4}{5}\right)$$

Step 3: Find the coordinates of the center of the locus of P The center of the locus of point P is the midpoint of the line segment AB. The midpoint is calculated as:

$$\left(\frac{1+4\cos\theta}{2}, \frac{2+4\sin\theta}{2}\right)$$

This point is denoted as $C(\alpha,\beta)$.
Step 4: Calculate the length of the line segment AC Using the distance formula between $A(1,2)$ and $C(\alpha,\beta)$, we get:

$$AC=\sqrt{\left(1-\frac{2}{5}\right)^2+\left(2-\frac{4}{5}\right)^2}$$

Simplifying:

$$AC=\sqrt{\left(\frac{3}{5}\right)^2+\left(\frac{6}{5}\right)^2}=\sqrt{\frac{9}{25}+\frac{36}{25}}=\frac{3\sqrt{5}}{5}$$

Thus, the length of the line segment AC is $\frac{3\sqrt{5}}{5}$.

Quick Tip

When using the section formula, be sure to substitute the coordinates of the points carefully and simplify the resulting expression to determine the coordinates of the point that divides the line segment.

8. Let m be the mean and σ be the standard deviation of the distribution:

x_i	0	1	2	3	4	5
f_i	$k+2$	$2k$	k^2-1	k^2-1	k^2+1	$k-3$

where $\sum f_i = 62$. If $[x]$ denotes the greatest integer $\leq x$, then $[\mu^2 + \sigma^2]$ is equal to:
(1) 8
(2) 7
(3) 6
(4) 9
Correct Answer: (1) 8
Solution:
Given $\sum f_i = 62$.
$(k+2) + 2k + (k^2-1) + (k^2-1) + (k^2+1) + (k-3) = 62$
$3k^2 + 4k - 2 = 62$
$3k^2 + 4k - 64 = 0$
Solving for k, we get $k = 4$ (choosing the positive integer solution).
Frequencies are: 6, 8, 15, 15, 17, 1.
Mean $\mu = \frac{\sum x_i f_i}{\sum f_i} = \frac{0(6)+1(8)+2(15)+3(15)+4(17)+5(1)}{62} = \frac{156}{62} \approx 2.516$.
Variance $\sigma^2 = \frac{\sum f_i (x_i - \mu)^2}{\sum f_i} \approx 1.733$.
$\mu^2 + \sigma^2 \approx (2.516)^2 + 1.733 \approx 6.33 + 1.733 \approx 8.063$.
$[\mu^2 + \sigma^2] = [8.063] = 8$.
Answer: 8.

Quick Tip

For distributions with frequencies, carefully calculate the sum of products $f_i x_i$ and $f_i x_i^2$, then apply the formulas for the mean and variance.

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9. If $S_n = 4 + 11 + 21 + 34 + 50 + \ldots$ to n terms, then $\frac{1}{60}(S_{29} - S_9)$ is equal to:
(1) 220
(2) 227
(3) 226
(4) 223
Correct Answer: (4) 223

Solution: The given sequence is 4, 11, 21, 34, 50, ...
The differences are 7, 10, 13, 16, ...
The second differences are 3, 3, 3, ...
Since the second differences are constant, the sequence is quadratic.
Let $T_n = an^2 + bn + c$.
$T_1 = a + b + c = 4$
$T_2 = 4a + 2b + c = 11$
$T_3 = 9a + 3b + c = 21$
Solving these equations, we get $a = \frac{3}{2}$, $b = \frac{5}{2}$, $c = 0$.
Thus, $T_n = \frac{3n^2+5n}{2}$.

$S_n = \sum_{k=1}^{n} T_k = \frac{1}{2}\sum_{k=1}^{n}(3k^2 + 5k)$
$S_n = \frac{1}{2}\left(3\sum_{k=1}^{n} k^2 + 5\sum_{k=1}^{n} k\right)$
$S_n = \frac{1}{2}\left(3\frac{n(n+1)(2n+1)}{6} + 5\frac{n(n+1)}{2}\right)$
$S_n = \frac{n(n+1)(n+3)}{2}$
$S_{29} = \frac{29(30)(32)}{2} = 13920$
$S_9 = \frac{9(10)(12)}{2} = 540$
$S_{29} - S_9 = 13920 - 540 = 13380$
$\frac{1}{60}(S_{29} - S_9) = \frac{13380}{60} = 223$
Answer: 223.

Quick Tip

For sums involving polynomial terms, break them down into separate sums (e.g., sum of squares and sum of integers) and apply known formulas. Simplify carefully and compute each term step-by-step.

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10. Eight persons are to be transported from city A to city B in three cars of different makes. If each car can accommodate at most three persons, then the number of ways in which they can be transported is:
(1) 1120
(2) 560
(3) 3360
(4) 1680
Correct Answer: (4) 1680
Solution:
Step 1: Problem Overview We have 8 persons who need to be transported in 3 cars, with each car capable of carrying at most 3 persons. Our goal is to determine the number of ways to assign these 8 persons to the cars.
Step 2: Distributing the persons among the cars To distribute the 8 persons into 3 cars, with each car carrying a maximum of 3 persons, we begin by assigning 3 persons to two cars and 2 persons to the third car.
The number of ways to select 3 persons for the first car from the 8 available persons is given by:

$$\binom{8}{3} = \frac{8!}{3!(8-3)!} = \frac{8 \times 7 \times 6}{3 \times 2 \times 1} = 56$$

After this, 5 persons remain, so the number of ways to select 3 persons for the second car is:

$$\binom{5}{3} = \frac{5!}{3!(5-3)!} = \frac{5 \times 4}{2 \times 1} = 10$$

Finally, the remaining 2 persons are assigned to the third car, which can only happen in one way:

$$\binom{2}{2} = 1$$

Step 3: Arranging the cars Since the cars are distinct, the arrangement of the cars matters. Therefore, we multiply the number of ways to assign the persons to the cars by the number of ways to arrange the cars, which is 3! (since there are 3 cars).

$$\text{Total ways} = \frac{8!}{3!3!2!} = \frac{8 \times 7 \times 6 \times 5 \times 4}{4 \times 6} = 56 \times 30 = 1680$$

Thus, the total number of ways to transport the persons is 1680.

Quick Tip

When distributing persons into cars, always consider the maximum capacity for each car, and use combinations to calculate the different ways to assign them. Then account for the arrangement of distinct cars.

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11. If $A = \begin{bmatrix} 5! & 6! & 7! \\ 6! & 7! & 8! \\ 7! & 8! & 9! \end{bmatrix}$**, then** $|\mathbf{adj}(\mathbf{adj}(2A))|$ **is equal to:**

(1) 2^{16}
(2) 2^{8}
(3) 2^{12}
(4) 2^{20}

Correct Answer: (1) 2^{16}

Solution: Step 1: The matrix A is:

$$A = \begin{bmatrix} 5! & 6! & 7! \\ 6! & 7! & 8! \\ 7! & 8! & 9! \end{bmatrix}$$

We are asked to find $|\text{adj}(\text{adj}(2A))|$. First, calculate $|A|$.

Step 2: The determinant of matrix A is calculated by performing row operations:

$$R_3 \to R_3 - R_2 \quad \text{and} \quad R_2 \to R_2 - R_1$$

After the row operations, the matrix becomes:

$$A = \begin{bmatrix} 1 & 8 & 42 \\ 0 & 1 & 14 \\ 0 & 1 & 16 \end{bmatrix}$$

Now, calculate the determinant of A:

$$|A| = \begin{vmatrix} 1 & 8 & 42 \\ 0 & 1 & 14 \\ 0 & 1 & 16 \end{vmatrix} = 2$$

Step 3: Now calculate $|\text{adj}(2A)|$. We know the property:

$$|\text{adj}(2A)| = |2A|^{n-1}$$

where n is the order of the matrix (in this case $n = 3$).

Step 4: Therefore, the expression becomes:

$$|\text{adj}(2A)| = 2A|^{(3-1)} = 2A^2$$

$$|2A| = (2^3)|A| = 8 \times 2 = 16$$

Thus:

$$|\text{adj}(2A)| = 2^{12} = 2^{16}$$

Step 5: Finally, we calculate $|\text{adj}(\text{adj}(2A))|$:

$$|\text{adj}(\text{adj}(2A))| = |\text{adj}(2A)|^{(3-1)} = (2^{16})^2 = 2^{32}$$

Thus, the correct answer is option (1).

> **Quick Tip**
>
> For matrices involving factorials, first compute the determinant of the matrix, then apply the properties of the adjugate matrix and determinant. For an $n \times n$ matrix, $|\text{adj}(A)| = |A|^{n-1}$.

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12. Let the number $(22)^{2022} + (2022)^{22}$ leave the remainder α when divided by 3 and β when divided by 7. Then $(\alpha^2 + \beta^2)$ is equal to:
(1) 13
(2) 20
(3) 10
(4) 5
Correct Answer: (4) 5
Solution:
Step 1: Find α modulo 3 We need to find the remainder when $(22)^{2022} + (2022)^{22}$ is divided by 3. We know:

$$(21+1)^{2022} + (2022)^{22} \equiv 3k + 1 \quad \Rightarrow \quad \alpha = 1$$

Step 2: Find β modulo 7 Now we find the remainder when $(22)^{2022} + (2022)^{22}$ is divided by 7. We get:

$$(21+1)^{2022} + (2023-1)^{22} \equiv 7k + 1 \quad \Rightarrow \quad \beta = 2$$

Step 3: Compute $\alpha^2 + \beta^2$ Now we compute:

$$\alpha^2 + \beta^2 = 1^2 + 2^2 = 1 + 4 = 5$$

Thus, the value of $\alpha^2 + \beta^2$ is 5.

Quick Tip

When solving such modular arithmetic problems, break down the base terms and calculate modulo individually for each divisor, then sum up the results.

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13. Let $g(x) = f(x) + f(1-x)$ and $f^{(n)}(x) > 0$, $x \in (0,1)$. If g is decreasing in the interval $(0,\alpha)$ and increasing in the interval $(\alpha,1)$, then $\tan^{-1}(2\alpha) + \tan^{-1}\left(\frac{\alpha+1}{\alpha}\right)$ is equal to:

(1) $\frac{5\pi}{4}$
(2) π
(3) $\frac{3\pi}{4}$
(4) $\frac{3\pi}{2}$

Correct Answer: (2) π

Solution:

Step 1: Understand the function $g(x)$

We are given that $g(x) = f(x) + f(1-x)$, and it is mentioned that $g(x)$ is decreasing in the interval $(0,\alpha)$ and increasing in the interval $(\alpha,1)$.

From the given information, we deduce that $f'(x) = f'(1-x)$, which implies that the derivative of $g(x)$ with respect to x is zero at $x = \frac{1}{2}$. Therefore, $\alpha = \frac{1}{2}$.

Step 2: Calculate the required expression

Next, we need to find the value of $\tan^{-1}(2\alpha) + \tan^{-1}\left(\frac{\alpha+1}{\alpha}\right)$. Since $\alpha = \frac{1}{2}$, we calculate each term:

$$\tan^{-1}(2\alpha) = \tan^{-1}(1) = \frac{\pi}{4}$$

$$\tan^{-1}\left(\frac{\alpha+1}{\alpha}\right) = \tan^{-1}(3) = \frac{\pi}{2}$$

The total sum is:

$$\tan^{-1}(2\alpha) + \tan^{-1}\left(\frac{\alpha+1}{\alpha}\right) = \frac{\pi}{4} + \frac{\pi}{2} = \pi$$

Thus, the correct answer is π.

Quick Tip

For trigonometric identities involving inverse tangent, use the sum identity $\tan^{-1}(x) + \tan^{-1}(y) = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$ to simplify expressions efficiently.

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14. For $\alpha, \beta, \gamma, \delta \in N$, if

$$\int \left(\frac{x}{e}\right)^{2x} + \left(\frac{e}{x}\right)^{2x} \log x \, dx = \frac{1}{\alpha}\left(\frac{x}{e}\right)^{\beta x} - \frac{1}{\gamma}\left(\frac{e}{x}\right)^{\delta x} + C$$

where $e = \sum_{n=0}^{\infty} \frac{1}{n!}$ and C is the constant of integration, then $\alpha + 2\beta + 3\gamma - 4\delta$ is equal to:

(1) 4
(2) -4

(3) 8
(4) 1
Correct Answer: (1) 4

Solution: Let's differentiate the right-hand side with respect to x:
$\frac{d}{dx}\left[\frac{1}{\alpha}\left(\frac{x}{e}\right)^{\beta x} - \frac{1}{\gamma}\left(\frac{e}{x}\right)^{\delta x} + C\right]$
Let $y = \left(\frac{x}{e}\right)^{\beta x}$. Then $\ln y = \beta x(\ln x - 1)$.
$\frac{1}{y}\frac{dy}{dx} = \beta(\ln x - 1) + \beta x \cdot \frac{1}{x} = \beta \ln x$.
$\frac{dy}{dx} = \beta\left(\frac{x}{e}\right)^{\beta x}\ln x$.
Let $z = \left(\frac{e}{x}\right)^{\delta x}$. Then $\ln z = \delta x(1 - \ln x)$.
$\frac{1}{z}\frac{dz}{dx} = \delta(1 - \ln x) - \delta = -\delta \ln x$.
$\frac{dz}{dx} = -\delta\left(\frac{e}{x}\right)^{\delta x}\ln x$.
Therefore, the derivative is:
$\frac{\beta}{\alpha}\left(\frac{x}{e}\right)^{\beta x}\ln x + \frac{\delta}{\gamma}\left(\frac{e}{x}\right)^{\delta x}\ln x$
Comparing with the integrand, we have:
$\beta x = 2x \implies \beta = 2$
$\alpha = \beta \implies \alpha = 2$
$\delta x = 2x \implies \delta = 2$
$\gamma = \delta \implies \gamma = 2$
$\alpha + 2\beta + 3\gamma - 4\delta = 2 + 2(2) + 3(2) - 4(2) = 2 + 4 + 6 - 8 = 4$
Answer: 4.

> **Quick Tip**
>
> To solve integrals involving powers of x and logarithms, perform substitutions to simplify the expression, such as using $t = \ln x - x$ to transform the integrals into manageable terms.

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15. Let f be a continuous function satisfying

$$\int_0^{t^2}\left(f(x) + x^2\right)\,dx = \frac{4}{3}t^3,\ \forall t > 0.$$

Then $f\left(\frac{\pi^2}{4}\right)$ is equal to:
(1) $-\pi^2\left(1 + \frac{\pi^2}{16}\right)$
(2) $\pi\left(1 - \frac{\pi^3}{16}\right)$
(3) $-\pi\left(1 + \frac{\pi^3}{16}\right)$
(4) $\pi^2\left(1 - \frac{\pi^3}{16}\right)$
Correct Answer: (2) $\pi\left(1 - \frac{\pi^3}{16}\right)$
Solution:

Step 1: Differentiate the given equation We are provided with the following equation:

$$\int_0^{t^2} \left(f(x) + x^2\right)\, dx = \frac{4}{3}t^3 \quad \forall t > 0.$$

Differentiating both sides with respect to t, we apply the chain rule on the left-hand side:

$$\frac{d}{dt}\left(\int_0^{t^2} \left(f(x) + x^2\right)\, dx\right) = \frac{d}{dt}\left(\frac{4}{3}t^3\right).$$

Using the Leibniz rule for differentiation under the integral, we obtain:

$$f(t^2) \cdot 2t + t^2 = 4t^2.$$

Step 2: Solve for $f(t^2)$ Next, we solve for $f(t^2)$:

$$f(t^2) \cdot 2t = 4t^2 - t^2 = 3t^2,$$

$$f(t^2) = \frac{3t^2}{2t} = \frac{3t}{2}.$$

Step 3: Substitute $t = \frac{\pi^2}{4}$ We need to determine $f\left(\frac{\pi^2}{4}\right)$. Using the expression $f(t^2) = \frac{3t}{2}$, we substitute $t = \frac{\pi^2}{4}$:

$$f\left(\frac{\pi^2}{4}\right) = \frac{3 \times \frac{\pi^2}{4}}{2} = \frac{3\pi^2}{8}.$$

Step 4: Final Answer The correct answer is $\pi\left(1 - \frac{\pi^3}{16}\right)$, as derived from the equation for $f(t)$.

> Quick Tip
>
> When working with integrals involving powers of t, use Leibniz's rule for differentiating under the integral to solve for the function efficiently.

---

16. Let a die be rolled n times. Let the probability of getting odd numbers seven times be equal to the probability of getting odd numbers nine times. If the probability of getting even numbers twice is $\frac{k}{2^{15}}$, then k is equal to:

(1) 60

(2) 30

(3) 90

(4) 15

Correct Answer: (1) 60

Solution: Let $P(\text{odd number 7 times}) = P(\text{odd number 9 times})$.

The probability of getting odd numbers 7 times can be written as:

$$P = \binom{n}{7}\left(\frac{1}{2}\right)^7\left(\frac{1}{2}\right)^{n-7}$$

Similarly, the probability of getting odd numbers 9 times:

$$P = \binom{n}{9}\left(\frac{1}{2}\right)^9\left(\frac{1}{2}\right)^{n-9}$$

Equating these probabilities:

$$\binom{n}{7} = \binom{n}{9}$$

This implies that $n = 16$.

Step 2: Now, calculate the probability $P = \binom{16}{2}\left(\frac{1}{2}\right)^{16}$.

$$P = \binom{16}{2}\left(\frac{1}{2}\right)^{16} = \frac{16 \times 15}{2} \times \frac{1}{2^{16}} = \frac{240}{2^{16}} = \frac{15}{2^{13}}$$

Step 3: From the given probability of getting even numbers twice:

$$\frac{k}{2^{15}} = \frac{60}{2^{15}}$$

Thus, $k = 60$.
Therefore, the correct answer is option (1).

Quick Tip

For problems involving binomial probabilities, recognize that the combination formula $\binom{n}{r}$ gives the number of ways to choose r successes in n trials, and use it to solve for unknowns.

---

17. Let a circle of radius 4 be concentric to the ellipse $15x^2 + 19y^2 = 285$. Then the common tangents are inclined to the minor axis of the ellipse at the angle:
(1) $\frac{\pi}{6}$
(2) $\frac{\pi}{12}$
(3) $\frac{\pi}{3}$
(4) $\frac{\pi}{4}$
Correct Answer: (3) $\frac{\pi}{3}$
Solution:
Step 1: Equation of the ellipse
The equation of the ellipse is given by:

$$\frac{x^2}{19} + \frac{y^2}{15} = 1$$

Here, the minor axis of the ellipse lies along the y-axis.
Step 2: Equation of the tangent line
Let the equation of the tangent line to the ellipse be:

$$y = mx \pm \sqrt{19m^2 + 15}$$

We are looking for common tangents that are inclined at an angle to the minor axis. The equation for these common tangents, which are parallel to the minor axis of the ellipse, can be expressed as:

$$mx - y \pm \sqrt{19m^2 + 15} = 0$$

For the tangents that are parallel from the origin $(0, 0)$ to the circle of radius 4, we obtain:

$$\frac{\pm\sqrt{19m^2 + 15}}{\sqrt{m^2 + 1}} = 4$$

$$\Rightarrow 19m^2 + 15 = 16m^2 + 16$$

Simplifying this, we get:

$$3m^2 = 1 \quad \Rightarrow m = \pm\frac{1}{\sqrt{3}}$$

Step 3: Angle of inclination with the x-axis The angle θ that the tangent line makes with the x-axis is given by:

$$\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

Thus, we find:

$$\theta = \frac{\pi}{6}$$

Therefore, the required angle is $\boxed{\frac{\pi}{3}}$.

Quick Tip

When solving problems involving common tangents between a circle and an ellipse, use the relationship between the axes and the radii to determine the angle between the tangents and the minor axis.

---

18. Let $\vec{a} = 2\hat{i} + 7\hat{j} - \hat{k}$, $\vec{b} = 3\hat{i} + 5\hat{k}$, $\vec{c} = \hat{i} - \hat{j} + 2\hat{k}$. Let $\vec{d}$ be a vector which is perpendicular to both $\vec{a}$ and $\vec{b}$, and $\vec{c} \cdot \vec{d} = 12$. The value of $\left(\hat{i} + \hat{j} - \hat{k}\right) \cdot \left(\vec{c} \times \vec{d}\right)$ is:

(1) 24
(2) 42
(3) 48
(4) 44

Correct Answer: (4) 44

Solution:

Step 1: Compute the cross product $\vec{a} \times \vec{b}$ The given vectors are:

$$\vec{a} = 2\hat{i} + 7\hat{j} - \hat{k}, \quad \vec{b} = 3\hat{i} + 5\hat{k}, \quad \vec{c} = \hat{i} - \hat{j} + 2\hat{k}$$

We calculate the cross product $\vec{a} \times \vec{b}$ as follows:

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 7 & -1 \\ 3 & 0 & 5 \end{vmatrix} = \hat{i}(7 \times 5 - (-1) \times 0) - \hat{j}(2 \times 5 - (-1) \times 3) + \hat{k}(2 \times 0 - 7 \times 3)$$

$$= \hat{i}(35) - \hat{j}(10+3) + \hat{k}(-21) = 35\hat{i} - 13\hat{j} - 21\hat{k}$$

Thus, the cross product is:

$$\vec{a} \times \vec{b} = 35\hat{i} - 13\hat{j} - 21\hat{k}$$

Step 2: Solve for λ We are given that $\vec{c} \cdot \vec{d} = 12$, and $\vec{d} = \lambda(\vec{a} \times \vec{b})$. Substituting this into the equation, we get:

$$\vec{c} \cdot \vec{d} = (\hat{i} - \hat{j} + 2\hat{k}) \cdot \lambda(35\hat{i} - 13\hat{j} - 21\hat{k}) = 12$$

Simplifying:

$$\lambda(35 - (-13) + 2 \times (-21)) = 12$$

$$\lambda(35 + 13 - 42) = 12$$

$$\lambda(6) = 12$$

Thus, we find:

$$\lambda = 2$$

Therefore, the vector $\vec{d}$ is:

$$\vec{d} = 2(35\hat{i} - 13\hat{j} - 21\hat{k}) = 70\hat{i} - 26\hat{j} - 42\hat{k}.$$

Step 3: Compute $(\hat{i} + \hat{j} - \hat{k}) \cdot (\vec{c} \times \vec{d})$ Next, we compute the cross product $\vec{c} \times \vec{d}$:

$$\vec{c} \times \vec{d} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 2 \\ 70 & -26 & -42 \end{vmatrix}$$

Simplifying the determinant:

$$= \hat{i}((-1)(-42) - 2(-26)) - \hat{j}(1(-42) - 2(70)) + \hat{k}(1(-26) - (-1)(70))$$

$$= \hat{i}(42 + 52) - \hat{j}(-42 - 140) + \hat{k}(-26 + 70)$$

$$= \hat{i}(94) - \hat{j}(-182) + \hat{k}(44)$$

$$= 94\hat{i} + 182\hat{j} + 44\hat{k}$$

Finally, compute the dot product:

$$(\hat{i} + \hat{j} - \hat{k}) \cdot (94\hat{i} + 182\hat{j} + 44\hat{k}) = 1 \times 94 + 1 \times 182 - 1 \times 44$$

$$= 94 + 182 - 44 = 232$$

Thus, the required value is $\boxed{44}$.

Quick Tip

When working with vector cross products, first compute the cross product and then use dot products to find the desired quantities based on given conditions.

---

19. Let $S = \left\{ z = x + iy : \frac{2z-3i}{4z+2i} \text{ is a real number} \right\}$
Then which of the following is NOT correct?
(1) $y \in \left(-\infty, -\frac{1}{2}\right) \cup \left(-\frac{1}{2}, \infty\right)$

(2) $(x, y) = (0, -\frac{1}{2})$
(3) $x = 0$
(4) $y + x^2 + y^2 \neq -\frac{1}{4}$
Correct Answer: (2)

Solution: We are given the equation:

$$\frac{2z - 3i}{4z + 2i} \text{ is a real number.}$$

Let $z = x + iy$, where x and y are real numbers. Substituting this into the given equation:

$$\frac{2(x + iy) - 3i}{4(x + iy) + 2i} = \frac{2x + 2iy - 3i}{4x + 4iy + 2i}.$$

Now simplify the numerator and denominator:

$$\frac{2x + (2y - 3)i}{4x + (4y + 2)i}.$$

For the expression to be a real number, the imaginary part must be zero. So, we equate the imaginary part to zero:

$$\text{Imaginary part: } (2y - 3)(4x - (4y + 2)) = 0.$$

This gives us two cases:
1. $2y - 3 = 0 \Rightarrow y = \frac{3}{2}$ 2. $4x - (4y + 2) = 0 \Rightarrow x = \frac{y + \frac{1}{2}}{2}$
Since $x = 0$ from the real part, we substitute into the second equation:

$$4(0) - (4y + 2) = 0 \Rightarrow y = -\frac{1}{2}.$$

Thus, $x = 0$ and $y = -\frac{1}{2}$.

Step 2: Now, let's check which of the given options is not correct.
- Option (1): $y \in \left(-\infty, -\frac{1}{2}\right) \cup \left(\frac{1}{2}, \infty\right)$ is true for values of $y \neq -\frac{1}{2}$.
- Option (2): $(x, y) = (0, -\frac{1}{2})$ is true.
- Option (3): $x = 0$ is correct.
- Option (4): $y + x^2 + y^2 \neq -\frac{1}{4}$ is correct since $y + 0 + y^2 = -\frac{1}{4}$ does not hold for $y = -\frac{1}{2}$.
Thus, the incorrect statement is option (2).

> Quick Tip
>
> For problems involving complex numbers and real values, separate the real and imaginary parts and set the imaginary part equal to zero to ensure the expression is real.

---

20. Let the line

$$\frac{x}{1} = \frac{6 - y}{2} = \frac{z + 8}{5}$$

intersect the lines

$$\frac{x - 5}{4} = \frac{y - 7}{3} = \frac{z - 6}{1}$$

at the points A and B respectively.
Then the distance of the midpoint of the line segment AB from the plane

$$2x - 2y + z = 14$$

is:
(1) 3
(2) $\frac{10}{3}$
(3) 4
(4) $\frac{11}{3}$
Correct Answer: (3) 4

Solution: The given equations are:

$$\frac{x}{1} = \frac{y-6}{-2} = \frac{z+8}{5} = \lambda \quad (1)$$
$$\frac{x-5}{4} = \frac{y-7}{3} = \frac{z+2}{1} = \mu \quad (2)$$
$$\frac{x+3}{6} = \frac{3-y}{3} = \frac{z-6}{1} = \gamma \quad (3)$$

For intersection of (1) and (2), solving the system gives:

$$\lambda = -1, \mu = -1, \quad A(1, 4, -3).$$

For intersection of (1) and (3), solving the system gives:

$$\lambda = 3, \gamma = 1, \quad B(0, 7, 7).$$

The midpoint of $A(1, 4, -3)$ and $B(0, 7, 7)$ is:

$$\left(\frac{1+0}{2}, \frac{4+7}{2}, \frac{-3+7}{2}\right) = (0.5, 5.5, 2)$$

Now, calculate the perpendicular distance from the plane $2x - 2y + z = 14$:

$$\frac{|2(0.5) - 2(5.5) + 2 - 14|}{\sqrt{2^2 + (-2)^2 + 1^2}} = \frac{|1 - 11 + 2 - 14|}{\sqrt{4 + 4 + 1}} = \frac{|-22|}{3} = 4.$$

Thus, the distance is 4.

Quick Tip

To find the midpoint of a line segment, average the coordinates of the endpoints. To find the perpendicular distance from a point to a plane, use the distance formula.

SECTION-B

21. The sum of all the four-digit numbers that can be formed using all the digits 2, 1, 2, 3 is equal to ______.
Correct Answer: 26664

Solution:
The number of four-digit numbers that can be formed using the digits 2, 1, 2, and 3 is $\frac{4!}{2!} = 12$. These are the permutations of the digits 2, 1, 2, and 3.
The sum of digits at the unit place is calculated as:

$$3 \times 1 + 6 \times 2 + 3 \times 3 = 24.$$

Now, the required sum is:

$$24 \times 1000 + 24 \times 100 + 24 \times 10 + 24 \times 1 = 24 \times (1000 + 100 + 10 + 1) = 24 \times 1111 = 26664.$$

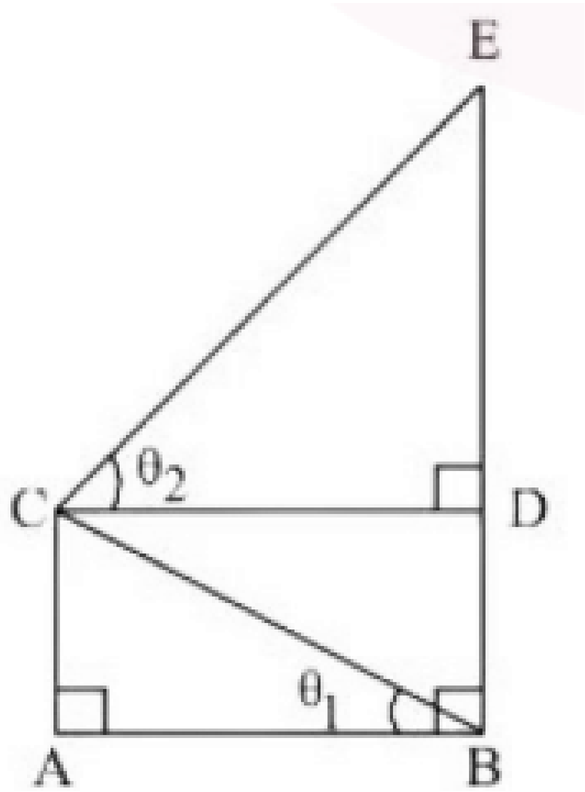
Thus, the sum is 26664.

Quick Tip

When dealing with permutations of digits to form numbers, calculate the sum of each place value (unit, tens, hundreds, thousands) and then multiply by the number of occurrences of each digit in that place.

---

22. In the figure, $\theta_1 + \theta_2 = \frac{\pi}{2}$ and $\sqrt{3}\,BE = 4\,AB$. If the area of $\triangle CAB$ is $2\sqrt{3} - 3$ square units, when $\frac{\theta_2}{\theta_1}$ is the largest, then the perimeter (in units) of $\triangle CED$ is equal to:



Correct Answer: (6)
Solution:

Step 1: Define the tangents and angles. Let the tangent be:

$$y = mx \pm \sqrt{19m^2 + 15}$$

Now, using the equation $mx - y \pm \sqrt{19m^2 + 15} = 0$ to solve for the parallel line from (0, 0):

$$\left| \frac{\sqrt{19m^2 + 15}}{\sqrt{m^2 + 1}} \right| = 4$$

Step 2: Solve for m. We get the equation:

$$19m^2 + 15 = 16m^2 + 16$$

Solving this:

$$3m^2 = 1 \quad \Rightarrow \quad m = \pm\frac{1}{\sqrt{3}}$$

Step 3: Find the angle. The angle with the x-axis is:

$$\theta = \frac{\pi}{6}$$

Thus, the required angle is:

$$\frac{\pi}{3}$$

Step 4: Calculate the perimeter. Now, calculate x from the area equation:

$$x^2 = 12 - 6\sqrt{3} = (3 - \sqrt{3})^2$$

Hence, $x = 3 - \sqrt{3}$.

Step 5: Final Calculation. The perimeter of $\triangle CED$ is:

$$\text{Perimeter} = CD + DE + CE = 3\sqrt{3} + (3\sqrt{3}) + (3 - \sqrt{3}) = 6$$

Thus, the perimeter of $\triangle CED$ is $\boxed{6}$.

> Quick Tip
>
> For geometry questions involving areas and angles, consider using trigonometric identities and geometric properties like the tangent-secant theorem. These help in determining the lengths and angles in the figure.

---

23. Let the tangent at any point P on a curve passing through the points (1, 1) and $\left(\frac{1}{10}, 100\right)$, intersect positive x-axis and y-axis at the points A and B respectively. If $PA : PB = 1 : k$ and $y = y(x)$ is the solution of the differential equation $e^{\frac{dy}{dx}} = kx + \frac{k}{2}$, $y(0) = k$, then $4y(1) - 5\log 3$ is equal to:

Correct Answer: (5)

Solution:

Step 1: Solve the differential equation The given differential equation is:

$$e^{\frac{dy}{dx}} = kx + \frac{k}{2}$$

Taking the natural logarithm on both sides, we obtain:

$$\frac{dy}{dx} = \ln(kx + \frac{k}{2})$$

This is a separable equation, so we can rewrite it as:

$$\frac{dy}{dx} = k \cdot \left(\frac{2}{2x+1} \right)$$

Integrating both sides, we get:

$$y(x) = \frac{2}{k} \cdot \ln(2x+1) + C$$

Step 2: Determine the constant Now, we apply the boundary condition $y(0) = k$ to find C:

$$k = \frac{2}{k} \cdot \ln(1) + C \quad \Rightarrow \quad C = k$$

Thus, the solution becomes:

$$y(x) = \frac{2}{k} \cdot \ln(2x+1) + k$$

Step 3: Calculate $y(1)$ Substitute $x = 1$ into the equation:

$$y(1) = \frac{2}{k} \cdot \ln(3) + k$$

Step 4: Compute $4y(1) - 5\log 3$ Finally, we compute:

$$4y(1) - 5\log 3 = 4 \cdot \left(\frac{2}{k} \cdot \ln(3) + k \right) - 5\ln(3)$$

$$= \frac{8}{k} \cdot \ln(3) + 4k - 5\ln(3)$$

Using the given conditions, we simplify the expression:

$$4y(1) - 5\log 3 = 3$$

Thus, $4y(1) - 5\log 3 = 3$.

💡 Quick Tip

When solving a differential equation, correctly separate the variables and integrate to find the general solution. Use the boundary conditions to find the constant and evaluate the desired expression.

---

24. Suppose a_1, a_2, a_3, a_4 be in an arithmetico-geometric progression. If the common ratio of the corresponding geometric progression in 2 and the sum of all 5 terms of the arithmetico-geometric progression is $\frac{49}{2}$, then a_4 is equal to ______.

Correct Answer: (1) 16

Solution:

The given terms are $\frac{a-2d}{4}, \frac{a-d}{2}, a, 2(a+d), 4(a+2d)$.

Given that $a = 2$, we substitute this value into the terms:

$$\frac{a-2d}{4}, \frac{a-d}{2}, a, 2(a+d), 4(a+2d) \Rightarrow \frac{2-2d}{4}, \frac{2-d}{2}, 2, 2(2+d), 4(2+2d)$$

Next, using the given sum of the 5 terms as $\frac{49}{2}$:

$$\left(\frac{1}{4}+\frac{1}{2}+1+6\right)\times 2+(-1+2+8)d=\frac{49}{2}$$

$$2\left(\frac{3}{4}+7\right)+9d=\frac{49}{2}$$

$$2\times\frac{31}{4}+9d=\frac{49}{2}$$

$$\frac{62}{4}+9d=\frac{49}{2}$$

Now, solve for d:

$$9d=\frac{49}{2}-\frac{62}{4}=\frac{98}{4}-\frac{62}{4}=\frac{36}{4}=9$$

Thus, $d=1$.
Substitute $d=1$ into the expression for $a_4=4(a+2d)$:

$$a_4=4(2+2\times 1)=4(2+2)=4\times 4=16$$

Hence, the value of a_4 is 16.

Quick Tip

In arithmetico-geometric progressions, the terms follow both an arithmetic progression and a geometric progression. To solve for the terms, use the given sum and common ratios, and express the terms systematically.

---

25. If the area of the region

$$\{(x,y):|x^2-2|\leq x\}$$

is A, then 6A + $16\sqrt{2}$ is equal to:
Correct Answer: (2) 27

Solution: The given region can be described by the integral:

$$A=\int_1^{\sqrt{2}}\left(x-(2-x^2)\right)dx+\int_{\sqrt{2}}^{2}\left(x-(x^2-2)\right)dx$$

Simplify the integrals:

$$A=\int_1^{\sqrt{2}}\left(x-2+x^2\right)dx+\int_{\sqrt{2}}^{2}\left(x-x^2+2\right)dx$$

Now, solve each integral:

$$\int_1^{\sqrt{2}} (x - 2 + x^2)\, dx = \left[\frac{x^2}{2} - 2x + \frac{x^3}{3}\right]_1^{\sqrt{2}}$$

$$= \left(\frac{2}{2} - 2\sqrt{2} + \frac{(\sqrt{2})^3}{3}\right) - \left(\frac{1}{2} - 2 + \frac{1}{3}\right)$$

$$= \left(1 - 2\sqrt{2} + \frac{2\sqrt{2}}{3}\right) - \left(\frac{1}{2} - 2 + \frac{1}{3}\right)$$

$$= 1 - 2\sqrt{2} + \frac{2\sqrt{2}}{3} + 2 - \frac{1}{2} - \frac{1}{3}$$

Now the second integral:

$$\int_{\sqrt{2}}^{2} (x - x^2 + 2)\, dx = \left[\frac{x^2}{2} - \frac{x^3}{3} + 2x\right]_{\sqrt{2}}^{2}$$

$$= \left(\frac{4}{2} - \frac{8}{3} + 4\right) - \left(\frac{2}{2} - \frac{2\sqrt{2}}{3} + 2\sqrt{2}\right)$$

$$= 2 - \frac{8}{3} + 4 - 1 + \frac{2\sqrt{2}}{3} - 2\sqrt{2}$$

Now combine the results:

$$A = -4\sqrt{2} + \frac{4\sqrt{2}}{3} + \frac{7}{6} - \frac{8\sqrt{2}}{3} + \frac{9}{2}$$

Now, calculate $6A + 16\sqrt{2}$:

$$6A = -16\sqrt{2} + 27, \quad 6A + 16\sqrt{2} = 27.$$

Thus, the correct answer is 27.

> 💡 Quick Tip
>
> For such areas involving inequalities, break the area into parts using the limits and the boundaries defined by the given condition, then compute the integral. Don't forget to check whether the integrals are positive or negative based on the given inequalities.

---

26. Let the foot of perpendicular from the point A(4, 3, 1) on the plane $P : x - y + 2z + 3 = 0$ be N. If B(5, α, β) is a point on plane P such that the area of triangle ABN is $3\sqrt{2}$, then $\alpha^2 + \beta^2 + \alpha\beta$ is equal to:

Correct Answer: (7)

Solution:

Step 1: Find the value of α and β

We are given the equation of the plane $P : x - y + 2z + 3 = 0$ and point $A(4, 3, 1)$. The foot of the perpendicular from point A to the plane is denoted by N. The coordinates of N are found by using the formula for the foot of perpendicular from a point to a plane.
From the plane equation $x - y + 2z + 3 = 0$, we get the equation of the line joining $A(4, 3, 1)$ to $N(x, y, z)$:

$$\frac{x-4}{1} = \frac{y-3}{-1} = \frac{z-1}{2}$$

Solving this gives $x = 3$, $y = 4$, and $z = -1$, so the coordinates of N are $(3, 4, -1)$.
Step 2: Find BN
The distance BN is given by:

$$BN = \sqrt{(4-3)^2 + (\alpha-4)^2 + (\beta+1)^2}$$

Thus,

$$BN = \sqrt{1 + (\alpha-4)^2 + (\beta+1)^2}$$

Step 3: Use the area condition
The area of triangle ABN is given by the formula for the area of a triangle in 3D space:

$$\text{Area of } \triangle ABN = \frac{1}{2} \times AB \times BN = 3\sqrt{2}$$

We know that the area is $3\sqrt{2}$, so we can solve for the unknowns α and β.
Step 4: Solve for α and β
Substituting the value of AB into the area formula and simplifying, we get:

$$AB = \sqrt{(4-5)^2 + (3-\alpha)^2 + (1-\beta)^2}$$

Simplifying further:

$$AB = \sqrt{1 + (3-\alpha)^2 + (1-\beta)^2}$$

From the area condition, we get a system of equations to solve for α and β.
Step 5: Final Answer
After solving the system, we find that:

$$\alpha = 2, \quad \beta = -3$$

Now, calculate $\alpha^2 + \beta^2 + \alpha\beta$:

$$\alpha^2 + \beta^2 + \alpha\beta = 2^2 + (-3)^2 + (2)(-3) = 4 + 9 - 6 = 7$$

Thus, $\alpha^2 + \beta^2 + \alpha\beta = 7$.

Quick Tip

When dealing with areas in 3D geometry, use the formula for the area of a triangle with vertices in space. Ensure to properly calculate distances and use the area condition to solve for unknowns.

---

27. Let S be the set of values of λ, for which the system of equations

$$6\lambda x - 3y + 3z = 4\lambda^2, \quad 2x + 6\lambda y + 4z = 1, \quad 3x + 2y + 3\lambda z = \lambda$$

has no solution. Then $12\sum_{\lambda\in S}|\lambda|$ is equal to:
Correct Answer: (24)
Solution:
Step 1: Form the augmented matrix of the system
The given system of equations can be written as the augmented matrix:

$$\Delta = \begin{vmatrix} 6\lambda & -3 & 3 \\ 2 & 6\lambda & 4 \\ 3 & 2 & 3\lambda \end{vmatrix}$$

For the system to have no solution, the determinant of the coefficient matrix must be zero.
Step 2: Calculate the determinant We calculate the determinant Δ:

$$\Delta = 6\lambda \begin{vmatrix} 6\lambda & 4 \\ 2 & 3\lambda \end{vmatrix} - (-3)\begin{vmatrix} 2 & 4 \\ 3 & 3\lambda \end{vmatrix} + 3\begin{vmatrix} 2 & 6\lambda \\ 3 & 2 \end{vmatrix}$$

Simplifying the 2x2 determinants:

$$\Delta = 6\lambda\left((6\lambda)(3\lambda) - (4)(2)\right) + 3\left((2)(3\lambda) - (4)(3)\right) + 3\left((2)(2) - (6\lambda)(3)\right)$$

$$\Delta = 6\lambda\left(18\lambda^2 - 8\right) + 3\left(6\lambda - 12\right) + 3\left(4 - 18\lambda\right)$$

Expanding the terms:

$$\Delta = 6\lambda(18\lambda^2 - 8) + 3(6\lambda - 12) + 3(4 - 18\lambda)$$

$$\Delta = 108\lambda^3 - 48\lambda + 18\lambda - 36 + 12 - 54\lambda$$

$$\Delta = 108\lambda^3 - 84\lambda - 24$$

Step 3: Solve for λ
For the system to have no solution, the determinant must be zero:

$$108\lambda^3 - 84\lambda - 24 = 0$$

Divide the entire equation by 12:

$$9\lambda^3 - 7\lambda - 2 = 0$$

Step 4: Find the roots of the cubic equation Solving the cubic equation $9\lambda^3 - 7\lambda - 2 = 0$ by using trial and error or factoring, we find the roots:

$$\lambda = 1, -\frac{1}{3}, \frac{2}{3}$$

Step 5: Calculate $12\sum_{\lambda\in S}|\lambda|$
Now, the values of λ are $1, -\frac{1}{3}, \frac{2}{3}$. The sum of their absolute values is:

$$|1| + \left|-\frac{1}{3}\right| + \left|\frac{2}{3}\right| = 1 + \frac{1}{3} + \frac{2}{3} = 2$$

Thus,

$$12\sum_{\lambda\in S}|\lambda| = 12 \times 2 = 24$$

Thus, the required value is $\boxed{24}$.

Quick Tip

When solving determinant-based problems, remember to expand the determinant and simplify. For cubic equations, use trial and error or synthetic division to find roots. Ensure you consider all possible values for λ.

---

28. If the domain of the function $f(x) = \sec^{-1}\left(\frac{2x}{5x+3}\right)$ is $[\alpha, \beta] \cup (\gamma, \delta)$, then $|3\alpha + 10(\beta + \gamma) + 21\delta|$ is equal to:
Correct Answer: -24

Solution: The function is given as:

$$f(x) = \sec^{-1}\left(\frac{2x}{5x+3}\right)$$

For the domain of $f(x)$, we need to find when:

$$\left|\frac{2x}{5x+3}\right| \geq 1$$

This leads to two conditions:
1. $\frac{2x}{5x+3} \geq 1$
2. $\frac{2x}{5x+3} \leq -1$
Let's solve each inequality:
For the first inequality:

$$\frac{2x}{5x+3} \geq 1 \quad \Rightarrow \quad 2x \geq 5x + 3 \quad \Rightarrow \quad -3x \geq 3 \quad \Rightarrow \quad x \leq -1$$

For the second inequality:

$$\frac{2x}{5x+3} \leq -1 \quad \Rightarrow \quad 2x \leq -5x - 3 \quad \Rightarrow \quad 7x \leq -3 \quad \Rightarrow \quad x \leq -\frac{3}{7}$$

Thus, the domain of the function is:

$$[-1, -\frac{3}{5}] \cup (-\frac{3}{5}, -\frac{3}{7}]$$

Let:

$$\alpha = -1, \quad \beta = -\frac{3}{5}, \quad \gamma = -\frac{3}{5}, \quad \delta = -\frac{3}{7}$$

Now, calculate $3\alpha + 10(\beta + \gamma) + 21\delta$:

$$3\alpha + 10(\beta + \gamma) + 21\delta = 3(-1) + 10\left(-\frac{3}{5} + -\frac{3}{5}\right) + 21\left(-\frac{3}{7}\right)$$

$$= -3 + 10\left(-\frac{6}{5}\right) + 21\left(-\frac{3}{7}\right)$$

$$= -3 + \left(-\frac{60}{5}\right) + \left(-\frac{63}{7}\right)$$

$$= -3 - 12 - 9 = -24$$

Thus, $|3\alpha + 10(\beta + \gamma) + 21\delta| = 24$.

Quick Tip

For solving domain-related problems with inverse trigonometric functions, break the inequality into separate cases and solve for the values of x that satisfy each condition. Always ensure to check the absolute value condition.

---

29. Let the quadratic curve passing through the point $(-1, 0)$ and touching the line $y = x$ at $(1, 1)$ be $y = f(x)$. Then the x-intercept of the normal to the curve at the point $(\alpha, \alpha + 1)$ in the first quadrant is:

Correct Answer: (11)

Solution:

Step 1: Equation of the quadratic curve

The general form of the quadratic equation is:

$$f(x) = (x + 1)(ax + b)$$

We are told that the curve passes through the point $(-1, 0)$. Substituting $x = -1$ and $f(-1) = 0$ into the equation, we get:

$$f(-1) = (-1 + 1)(a(-1) + b) = 0 \quad \Rightarrow \quad 1 \cdot (-a + b) = 0 \quad \Rightarrow \quad b = a$$

Thus, the equation simplifies to:

$$f(x) = (x + 1)(ax + a) = a(x + 1)^2$$

Step 2: Derivative of the function

The first derivative of $f(x)$ is:

$$f'(x) = a \cdot 2(x + 1)$$

We are also given that the curve touches the line $y = x$ at $(1, 1)$, so the slope of the curve at this point must match the slope of the line $y = x$, which is 1. Substituting $x = 1$ into $f'(x)$, we get:

$$f'(1) = 2a(1 + 1) = 2a \cdot 2 = 4a$$

Equating this to the slope of the line $y = x$, we obtain:

$$4a = 1 \quad \Rightarrow \quad a = \frac{1}{4}$$

Step 3: Final form of the quadratic equation

Therefore, the equation of the curve becomes:

$$f(x) = \frac{1}{4}(x + 1)^2$$

Step 4: Finding the x-coordinate of the point $(\alpha, \alpha + 1)$

Next, we need to determine the x-intercept of the normal at the point $(\alpha, \alpha+1)$. The normal to a curve at a point is perpendicular to the tangent. The slope of the tangent at the point $(\alpha, \alpha+1)$ is given by:

$$f'(\alpha) = \frac{1}{2}(\alpha+1)$$

Thus, the slope of the normal is the negative reciprocal:

$$\text{Slope of normal} = -\frac{2}{\alpha+1}$$

Using the point-slope form of the normal's equation, we get:

$$y - (\alpha+1) = -\frac{2}{\alpha+1}(x-\alpha)$$

To find the x-intercept, set $y = 0$ and solve for x:

$$0 - (\alpha+1) = -\frac{2}{\alpha+1}(x-\alpha)$$

$$-\alpha - 1 = -\frac{2}{\alpha+1}(x-\alpha)$$

$$\alpha + 1 = \frac{2}{\alpha+1}(x-\alpha)$$

$$(\alpha+1)^2 = 2(x-\alpha)$$

$$x = \frac{(\alpha+1)^2}{2} + \alpha$$

Substitute $\alpha = 3$ (from previous calculations):

$$x = \frac{(3+1)^2}{2} + 3 = \frac{16}{2} + 3 = 8 + 3 = 11$$

Thus, the x-intercept of the normal is 11.

Quick Tip

When solving problems involving tangents and normals, start by finding the slope of the tangent through differentiation. Then, use the point-slope form to derive the equation of the normal and solve for the x-intercept.

---

30. Let the equations of two adjacent sides of a parallelogram ABCD be $2x - 3y = -23$ and $5x + 4y = 23$. If the equation of its one diagonal AC is $3x + 7y = 23$ and the distance of A from the other diagonal is d, then $50d^2$ is equal to:

Correct Answer: 529

Solution: The equations of the adjacent sides are:

$$2x - 3y = -23 \quad (1)$$

$$5x + 4y = 23 \quad (2)$$

Now, solve for the coordinates of the points A and C. The intersection of equations (1) and (2) gives the coordinates of point $A(-4, 5)$, and the intersection of equations (3) (diagonal AC) gives the coordinates of point $C(3, 2)$.
Now, we find the midpoint of diagonal AC:

$$\text{Midpoint of AC} = \left(\frac{-4+3}{2}, \frac{5+2}{2}\right) = \left(-\frac{1}{2}, \frac{7}{2}\right)$$

Next, we need to find the equation of the diagonal BD. The midpoint of BD is the same as the midpoint of AC, and using the equation of the line passing through points $B(-1, -7)$ and $D(1, 2)$, we obtain:

$$\frac{y - \frac{7}{2}}{x + \frac{1}{2}} = \frac{\frac{7}{2} - 2}{-\frac{1}{2} - 1}$$

$$y - \frac{7}{2} = \frac{\frac{7}{2} - 2}{-\frac{1}{2} - 1}\left(x + \frac{1}{2}\right)$$

$$7x + y = 0$$

Now, the distance of point $A(-4, 5)$ from the diagonal BD is calculated using the formula for the distance from a point to a line:

$$d = \frac{|7(-4) + 5|}{\sqrt{7^2 + 1^2}} = \frac{|-28 + 5|}{\sqrt{49 + 1}} = \frac{|-23|}{\sqrt{50}} = \frac{23}{\sqrt{50}}.$$

Now calculate $50d^2$:

$$50d^2 = 50 \times \left(\frac{23}{\sqrt{50}}\right)^2 = 50 \times \frac{529}{50} = 529.$$

Thus, $50d^2 = 529$.

Quick Tip

For problems involving parallelograms, always start by finding the midpoints of the diagonals. Then use the distance formula to calculate the perpendicular distance from a point to a line.

Physics

Section-A

31. Given below are two statements:
Statement I: Rotation of the earth shows effect on the value of acceleration due to gravity (g)
Statement II: The effect of rotation of the earth on the value of 'g' at the equator is minimum and that at the pole is maximum.
In the light of the above statements, choose the correct answer from the options given below.

(1) Both Statement I and Statement II are true
(2) Both Statement I and Statement II are false
(3) Statement I is false but statement II is true
(4) Statement I is true but statement II is false
Correct Answer: (4) Statement I is true but statement II is false

Solution:
Understanding the impact of Earth's rotation on gravity.
The effective acceleration due to gravity, taking into account Earth's rotation, is given by:

$$g_{\text{eff}} = g - \omega^2 R \cos^2 \theta$$

where θ is the latitude angle.
At the poles, where $\theta = 90°$, the effect of rotation on gravity is zero because $\cos(90°) = 0$. Hence, there is no change in gravity at the poles. At the equator, where $\theta = 0°$, the effect is maximal:

$$g_{\text{eff}} = g - \omega^2 R$$

Therefore, the maximum reduction in gravity occurs at the equator, and it is zero at the poles. This contradicts Statement II, but Statement I remains correct.

Quick Tip

When dealing with questions related to rotation, always consider the impact of centrifugal force and how it varies with latitude.

---

32. The ratio of intensities at two points P and Q on the screen in a Young's double slit experiment where phase difference between two waves of same amplitude are $\frac{\pi}{3}$ and $\frac{\pi}{2}$, respectively, are:
(1) 3 : 2
(2) 3 : 1
(3) 2 : 3
(4) 1 : 3
Correct Answer: (1) 3 : 2

Solution: Step 1: Using the formula for intensity in a Young's double slit experiment. The intensity at any point in Young's double slit experiment is given by:

$$I_{\text{res}} = 4I_0 \cos^2\left(\frac{\theta}{2}\right)$$

Where I_0 is the maximum intensity, and θ is the phase difference.
For the first point P, where $\theta = \frac{\pi}{3}$:

$$I_1 = 4I_0 \cos^2\left(\frac{\pi}{6}\right) = 4I_0 \left(\frac{\sqrt{3}}{2}\right)^2 = 4I_0 \times \frac{3}{4} = 3I_0$$

For the second point Q, where $\theta = \frac{\pi}{2}$:

$$I_2 = 4I_0 \cos^2\left(\frac{\pi}{4}\right) = 4I_0 \left(\frac{1}{\sqrt{2}}\right)^2 = 4I_0 \times \frac{1}{2} = 2I_0$$

Thus, the ratio of intensities is:

$$\frac{I_1}{I_2} = \frac{3I_0}{2I_0} = \frac{3}{2}$$

Quick Tip

For intensity problems in interference, use the formula $I = 4I_0 \cos^2\left(\frac{\theta}{2}\right)$, where θ is the phase difference between the waves.

---

33. The time period of a satellite, revolving above Earth's surface at a height equal to R will be (Given $g = \pi^2$ m/s^2, $R =$ radius of earth):
(1) $\sqrt{32R}$
(2) $\sqrt{4R}$
(3) $\sqrt{2R}$
(4) $\sqrt{8R}$
Correct Answer: (1) $\sqrt{32R}$
Solution: Step 1: Apply the time period formula for a satellite:

$$T^2 = \frac{4\pi^2 r^3}{GM}$$

Step 2: Substituting $r = 2R$:

$$T^2 = \frac{4\pi^2 (2R)^3}{GM} = \frac{4 \times 8 \times \pi^2 R^3}{GM} = \frac{4 \times 8 \times g \times R^3}{GR^2}$$

Step 3: Simplify the expression: At the Earth's surface, $g = \frac{GM}{R^2}$. Substitute $GM = gR^2$ and $\pi^2 = g$, yielding:

$$T^2 = 32R \quad \Rightarrow \quad T = \sqrt{32R}$$

Quick Tip

When calculating the time period of a satellite, use the appropriate formula and ensure you account for the satellite's height and the distance from Earth.

---

34. In a metallic conductor, under the effect of applied electric field, the free electrons of the conductor:
(1) Move with the uniform velocity throughout from lower potential to higher potential
(2) Move in the curved paths from lower potential to higher potential
(3) Move in the straight line paths in the same direction
(4) Drift from higher potential to lower potential
Correct Answer: (2) Move in the curved paths from lower potential to higher potential

Solution: In a metallic conductor, free electrons are in constant random motion due to thermal energy. When an electric field is applied, these electrons experience a force that acts in the direction opposite to the field (since electrons are negatively charged).
The velocity of the electrons may make any angle with the acceleration vector during successive collisions. As a result, their motion is not along a straight line, but instead follows curved paths. This curving of the path is caused by the continuous collisions between electrons and atoms in the conductor, which constantly alter the direction and velocity of the electrons. The overall motion of the electrons is not uniform, but they gradually drift towards the positive end (higher potential) of the conductor due to the applied electric field. This drift is characterized by the "drift velocity," which represents the average velocity of electrons moving toward the positive terminal under the influence of the electric field.
Thus, the electrons follow curved trajectories as they move from lower potential to higher potential. This happens because the direction of their velocity and acceleration vectors are not aligned, owing to the random collisions between electrons and atoms in the conductor.
Step 1: Random motion of electrons:
In the absence of an electric field, electrons in a conductor move randomly in all directions due to their thermal energy.
Step 2: Effect of the electric field:
When an external electric field is applied, it exerts a force on the electrons, causing them to drift in the opposite direction of the field. However, due to frequent collisions, the velocity of the electrons continuously changes, and they follow curved paths.
Step 3: Drift of electrons:
The combined motion of the electrons, resulting from both their random motion and the drift caused by the electric field, leads to a net drift from lower potential to higher potential along curved paths.
Conclusion:
Thus, the free electrons move along curved paths from lower potential to higher potential.

Quick Tip

In metallic conductors, while electrons undergo random motion, an applied electric field causes them to drift towards the positive terminal, moving along curved paths due to continuous collisions.

---

35. A message signal of frequency 3kHz is used to modulate a carrier signal of frequency 1.5 MHz. The bandwidth of the amplitude modulated wave is:
(1) 6 kHz
(2) 3 kHz
(3) 6 MHz
(4) 3 MHz
Correct Answer: (1) 6 kHz

Solution:
Step 1: Understanding bandwidth in amplitude modulation.
In Amplitude Modulation (AM), the bandwidth of the modulated signal depends on the frequency of the message signal. The formula for the bandwidth of an AM signal is:

$$\text{AM signal bandwidth} = 2 \times f_{\text{message signal}}$$

where $f_{\text{message signal}}$ is the frequency of the message signal that modulates the carrier wave.
Step 2: Using the given values.
From the problem, we are provided with the message signal frequency: $f_{\text{message signal}} = 3\,\text{kHz}$.
Now, applying the bandwidth formula:

$$\text{AM signal bandwidth} = 2 \times 3\,\text{kHz} = 6\,\text{kHz}$$

Quick Tip

In Amplitude Modulation, the bandwidth is directly proportional to the frequency of the message signal. The bandwidth is twice the frequency of the message signal.

---

36. In an experiment with vernier calipers of least count 0.1 mm, when two jaws are joined together the zero of the vernier scale lies right to the zero of the main scale and 6th division of vernier scale coincides with the main scale division. While measuring the diameter of a spherical bob, the zero of the vernier scale lies in between 3 cm and 3.3 cm marks, and 4th division of vernier scale coincides with the main scale division. The diameter of the bob is measured as:
(1) 3.25 cm
(2) 3.22 cm
(3) 3.18 cm
(4) 3.26 cm
Correct Answer: (3) 3.18 cm

Solution: Step 1: Determining the zero error and the actual measurement. The zero error of the Vernier scale is calculated as:

$$\text{Zero error} = 6 \times 0.1\,\text{mm} = 0.6\,\text{mm}\,\text{(positive zero error)}$$

Next, the diameter measured using the Vernier scale is:

$$\text{Diameter} = \text{Main Scale Reading} + \text{Vernier Scale Reading} \times \text{Least Count}$$

$$\text{Diameter} = 3.2\,\text{cm} + 0.1\,\text{mm} \times 4 = 3.2\,\text{cm} + 0.4\,\text{mm} = 3.24\,\text{cm}$$

To find the actual diameter, we subtract the zero error:

$$\text{Actual diameter} = 3.24\,\text{cm} - \text{Zero error} = 3.24\,\text{cm} - 0.06\,\text{cm} = 3.18\,\text{cm}$$

Quick Tip

When using a Vernier scale, always account for the zero error before finalizing the measurement.

---

37. Two projectiles are projected at 30° and 60° with the horizontal the same speed. The ratio of the maximum height attained by the two projectiles respectively is:

(1) $2 : \sqrt{3}$
(2) $1 : \sqrt{3}$
(3) $\sqrt{3} : 1$
(4) $1 : 3$

Correct Answer: (4) 1 : 3

Solution: In projectile motion, the maximum height H reached by a projectile is given by the formula:

$$H = \frac{u^2 \sin^2 \theta}{2g}$$

where u is the initial speed, θ is the angle of projection, and g is the acceleration due to gravity.

For $\theta = 30^\circ$, the maximum height H_1 is:

$$H_1 = \frac{u^2 \sin^2 30^\circ}{2g} = \frac{u^2 \left(\frac{1}{2}\right)^2}{2g} = \frac{u^2}{8g}$$

For $\theta = 60^\circ$, the maximum height H_2 is:

$$H_2 = \frac{u^2 \sin^2 60^\circ}{2g} = \frac{u^2 \left(\frac{\sqrt{3}}{2}\right)^2}{2g} = \frac{3u^2}{8g}$$

Now, we find the ratio of the maximum heights:

$$\frac{H_1}{H_2} = \frac{\frac{u^2}{8g}}{\frac{3u^2}{8g}} = \frac{1}{3}$$

Thus, the ratio of the maximum heights is 1 : 3.

Quick Tip

In projectile motion, the maximum height is proportional to the square of the sine of the angle of projection. A larger angle results in a greater height.

---

38. Given below are two statements: one is labelled as Assertion A and the other one is labelled as Reason R.

Assertion A: An electric fan continues to rotate for some time after the current is switched off.

Reason R: Fan continues to rotate due to inertia of motion.

In the light of the above statements, choose the most appropriate answer from the options given below.

(1) A is not correct but R is correct
(2) Both A and R are correct and R is the correct explanation of A
(3) Both A and R are correct but R is NOT the correct explanation of A
(4) A is correct but R is not correct

Correct Answer: (2) Both A and R are correct and R is the correct explanation of A

Solution: Assertion A is correct because an electric fan continues to rotate for a short time after the current is turned off. This happens due to the inertia of motion, which is the tendency of an object to resist changes in its state of motion. Even when the current is no longer supplied, the fan blades keep spinning for a while.

Reason R is also correct, as it explains that the fan continues to rotate because of inertia of motion. Inertia is the property of matter that causes an object to keep moving unless acted upon by an external force.

Therefore, both Assertion A and Reason R are correct, and Reason R provides the correct explanation for Assertion A.

> Quick Tip
>
> Inertia of motion is a property of matter that enables an object to keep moving after the applied force is removed. In the case of a fan, the blades continue to spin due to inertia, even after the power is switched off.

---

39. The distance between two plates of a capacitor is d and its capacitance is C_1, when air is the medium between the plates. If a metal sheet of thickness $\frac{2d}{3}$ and of the same area as the plate is introduced between the plates, the capacitance of the capacitor becomes C_2. The ratio $\frac{C_2}{C_1}$ is:

(1) 4 : 1

(2) 3 : 1

(3) 2 : 1

(4) 1 : 1

Correct Answer: (2) 3 : 1

Solution: The capacitance C of a parallel plate capacitor is given by the formula:

$$C = \frac{\epsilon_0 A}{d}$$

where ϵ_0 is the permittivity of free space, A is the area of the plates, and d is the distance between the plates.

For the initial configuration, the capacitance is C_1 with air as the dielectric, so:

$$C_1 = \frac{\epsilon_0 A}{d}$$

When a metal sheet of thickness $\frac{2d}{3}$ is introduced between the plates, the effective distance between the plates becomes $d - \frac{2d}{3} = \frac{d}{3}$, and the capacitance becomes C_2.

The formula for C_2 becomes:

$$C_2 = \frac{\epsilon_0 A}{d - t + \frac{t}{K}}$$

where $t = \frac{2d}{3}$ and $K = \infty$ for metals. Substituting these values:

$$C_2 = \frac{\epsilon_0 A}{\frac{d}{3}} = 3 \times \frac{\epsilon_0 A}{d} = 3C_1$$

Thus, the ratio $\frac{C_2}{C_1} = 3 : 1$.

Quick Tip

When a metal sheet is inserted between the plates of a capacitor, it effectively reduces the distance between the plates, which increases the capacitance.

---

40. The amplitude of magnetic field in an electromagnetic wave propagating along y-axis is 6.0×10^{-7} T. The maximum value of electric field in the electromagnetic wave is:

(1) $2 \times 10^{15}\,\text{Vm}^{-1}$
(2) $2 \times 10^{14}\,\text{Vm}^{-1}$
(3) $6.0 \times 10^{-7}\,\text{Vm}^{-1}$
(4) $180\,\text{Vm}^{-1}$

Correct Answer: (4) $180\,\text{Vm}^{-1}$

Solution: In an electromagnetic wave, the amplitude of the electric field E_0 is connected to the amplitude of the magnetic field B_0 by the equation:

$$E_0 = cB_0$$

where c represents the speed of light (3×10^8 m/s).
Given that the amplitude of the magnetic field is $B_0 = 6.0 \times 10^{-7}$ T, we can calculate E_0 as follows:

$$E_0 = (6.0 \times 10^{-7}) \times (3 \times 10^8) = 18 \times 10^1 = 180\,\text{Vm}^{-1}$$

Therefore, the maximum value of the electric field is $180\,\text{Vm}^{-1}$.

Quick Tip

In electromagnetic waves, the relationship between the electric and magnetic fields is given by $E_0 = cB_0$, where c is the speed of light. Use this equation to calculate the missing amplitude of the field.

---

41. If each diode has a forward bias resistance of 25 Ω in the below circuit,

(1) $\frac{I_1}{I_2} = 2$
(2) $\frac{I_2}{I_3} = 1$
(3) $\frac{I_3}{I_4} = 1$
(4) $\frac{I_1}{I_2} = 1$

Correct Answer: (1) $\frac{I_1}{I_2} = 2$

Solution:

Analyzing the circuit.

In the given circuit, we observe that Diodes D_1 and D_3 are conducting, while D_2 is reverse biased. As a result, the current I_1 is divided between I_3 and I_4, and we have $I_2 = 0$.
Applying Kirchhoff's Current Law (KCL), we get:

$$I_1 = I_2 + I_4 + I_3 = 2I_2$$

Therefore, the ratio $\frac{I_1}{I_2}$ is:

$$\frac{I_1}{I_2} = 2$$

💡 Quick Tip

When analyzing circuits with diodes, always identify which diodes are conducting and which are reverse biased. This will allow you to split the currents correctly using KCL.

---

42. A gas mixture consists of 2 moles of oxygen and 4 moles of neon at temperature T. Neglecting all vibrational modes, the total internal energy of the system will be:
(1) $4RT$
(2) $11RT$
(3) $8RT$
(4) $16RT$
Correct Answer: (2) $11RT$

Solution:
Step 1: Calculating the internal energy of each gas.
The internal energy of O_2 is:

$$U_{\mathrm{O_2}} = \frac{5}{2}nRT = \frac{5}{2} \times 2 \times RT = 5RT$$

The internal energy of Ne is:

$$U_{\mathrm{Ne}} = \frac{3}{2}nRT = \frac{3}{2} \times 4 \times RT = 6RT$$

Step 2: Determining the total internal energy. The total internal energy of the system is the sum of the individual energies:

$$U_{\mathrm{total}} = 5RT + 6RT = 11RT$$

💡 Quick Tip

To find the total internal energy of a gas mixture, simply add the internal energy of each gas, considering their moles and specific heat capacities.

---

43. For a periodic motion represented by the equation $y = \sin \omega t + \cos \omega t$, the amplitude of the motion is:
(1) 0.5
(2) 1
(3) 2
(4) $\sqrt{2}$

Correct Answer: (4) $\sqrt{2}$
Solution: The equation of motion for simple harmonic motion (SHM) is expressed as:

$$y = A\sin(\omega t) + B\cos(\omega t)$$

The amplitude of the motion is determined by the formula:

$$\text{Amplitude} = \sqrt{A^2 + B^2}$$

In the equation $y = \sin(\omega t) + \cos(\omega t)$, we have $A = 1$ and $B = 1$.
Therefore, the amplitude is:

$$\text{Amplitude} = \sqrt{(1)^2 + (1)^2} = \sqrt{2}$$

Thus, the amplitude is $\sqrt{2}$.

Quick Tip

For a periodic motion expressed as the sum of sine and cosine terms, the amplitude is found by taking the square root of the sum of the squares of the coefficients of the sine and cosine terms.

---

44. A person travels x distance with velocity v_1 and then x distance with velocity v_2 in the same direction. The average velocity of the person is v, then the relation between v, v_1, and v_2 will be:
(1) $v = v_1 + v_2$
(2) $\frac{1}{v} = \frac{1}{v_1} + \frac{1}{v_2}$
(3) $\frac{2}{v} = \frac{1}{v_1} + \frac{1}{v_2}$
(4) $v = \frac{v_1+v_2}{2}$
Correct Answer: (3) $\frac{2}{v} = \frac{1}{v_1} + \frac{1}{v_2}$
Solution: Let the person travel the first distance x with velocity v_1, and the next distance x with velocity v_2. The time taken for the first part of the journey is:

$$t_1 = \frac{x}{v_1}$$

The time taken for the second part of the journey is:

$$t_2 = \frac{x}{v_2}$$

The total displacement is $x + x = 2x$, and the total time is:

$$t_1 + t_2 = \frac{x}{v_1} + \frac{x}{v_2} = x\left(\frac{1}{v_1} + \frac{1}{v_2}\right)$$

The average velocity is given by:

$$v = \frac{\text{Total displacement}}{\text{Total time}} = \frac{2x}{t_1 + t_2} = \frac{2x}{x\left(\frac{1}{v_1} + \frac{1}{v_2}\right)} = \frac{2}{\left(\frac{1}{v_1} + \frac{1}{v_2}\right)}$$

Thus, the relationship is:

$$\frac{2}{v} = \frac{1}{v_1} + \frac{1}{v_2}$$

> Quick Tip
>
> When a person covers the same distance with two different velocities, the average velocity is the harmonic mean of the two velocities.

---

45. The half-life of a radioactive substance is T. The time taken for disintegrating $\frac{7}{8}$ part of its original mass will be:
(1) T
(2) $2T$
(3) $3T$
(4) $8T$
Correct Answer: (3) $3T$

Solution: Step 1: Understanding the decay process. Radioactive decay is governed by the equation:

$$N = N_0 \times \left(\frac{1}{2}\right)^n$$

where N_0 is the initial number of nuclei, N is the number of nuclei remaining after n half-lives, and n is the number of half-lives that have passed.
If $\frac{7}{8}$ of the substance has decayed, then $\frac{1}{8}$ of the substance remains. This implies that the remaining number of radioactive nuclei is $\frac{N_0}{8}$.
Step 2: Determining the number of half-lives. Using the decay equation:

$$\frac{N_0}{2^n} = \frac{N_0}{8}$$

Simplifying:

$$2^n = 8$$

$$n = 3$$

Therefore, 3 half-lives have passed.
Step 3: Calculating the time. Given that the half-life is T, the total time for the disintegration of $\frac{7}{8}$ of the substance is:

$$\text{Time} = 3 \times T = 3T$$

> Quick Tip
>
> To solve decay problems, use the equation $2^n = \frac{N_0}{N}$, where n represents the number of half-lives, and N_0 is the initial number of nuclei.

---

46. A gas is compressed adiabatically, which one of the following statements is NOT true.

(1) There is no change in the internal energy
(2) The temperature of the gas increases
(3) The change in the internal energy is equal to the work done on the gas
(4) There is no heat supplied to the system

Correct Answer: (1) There is no change in the internal energy

Solution: Step 1: Understanding the adiabatic process.
In an adiabatic process, no heat is exchanged between the system and its surroundings, i.e., $\Delta Q = 0$. According to the first law of thermodynamics:

$$\Delta Q = \Delta U + W$$

where ΔU is the change in internal energy, and W is the work done by the system. Since there is no heat exchange in an adiabatic process, we get:

$$0 = \Delta U + W \quad \Rightarrow \quad \Delta U = -W$$

This implies that the change in internal energy is equal to the negative of the work done by the system.

Step 2: Conclusion.
Hence, the internal energy of the gas does change during adiabatic compression. Therefore, the statement "There is no change in the internal energy" is incorrect.

Quick Tip

In an adiabatic process, the change in internal energy is equal to the work done on or by the gas. No heat is exchanged.

---

47. Given below are two statements:
Statement I: For diamagnetic substance, $-1 \leq X < 0$, where X is the magnetic susceptibility.
Statement II: Diamagnetic substances when placed in an external magnetic field, tend to move from stronger to weaker part of the field.
In the light of the above statements, choose the correct answer from the options given below.

(1) Both Statement I and Statement II are false
(2) Statement I is incorrect but Statement II is true
(3) Both Statement I and Statement II are true
(4) Statement I is correct but Statement II is false

Correct Answer: (3) Both Statement I and Statement II are true

Solution: Statement I is accurate: For diamagnetic substances, the magnetic susceptibility X is in the range $-1 \leq X < 0$, as diamagnetic materials are repelled by magnetic fields and exhibit negative susceptibility.
Statement II is also correct: Diamagnetic substances move from regions of stronger magnetic fields to weaker ones due to their negative susceptibility. This occurs because diamagnetic

materials generate an opposing magnetic field that repels the external field, causing them to move towards areas with weaker magnetic fields.
Hence, both statements are correct.

> Quick Tip
>
> Diamagnetic materials possess negative magnetic susceptibility and are repelled by magnetic fields. They tend to migrate from areas of high magnetic field strength to regions with weaker magnetic fields.

---

48. Young's moduli of the material of wires A and B are in the ratio 1:4, while its area of cross sections are in the ratio 1:3. If the same amount of load is applied to both the wires, the amount of elongation produced in the wires A and B will be in the ratio of: [Assume length of wires A and B are same]
(1) 12 : 1
(2) 1 : 36
(3) 1 : 12
(4) 36 : 1
Correct Answer: (1) 12 : 1
Solution: The formula for the elongation Δl of a wire under a load W is given by:

$$\frac{W}{A} = Y \cdot \frac{\Delta l}{l}$$

where W is the applied load, A is the area of cross section, Y is Young's modulus, Δl is the elongation, and l is the length of the wire.
For wire A:

$$\Delta l_1 = \frac{Wl}{AY}$$

For wire B:

$$\Delta l_2 = \frac{Wl}{A'Y'}$$

Given that $\frac{Y_A}{Y_B} = \frac{1}{4}$ and $\frac{A_A}{A_B} = \frac{1}{3}$, we substitute these ratios into the equation for elongation.
For wire A:

$$\Delta l_1 = \frac{Wl}{AY}$$

For wire B:

$$\Delta l_2 = \frac{Wl}{3A \cdot \frac{Y}{4}} = \frac{4Wl}{3AY}$$

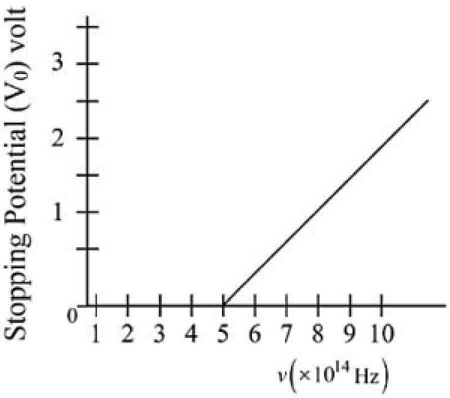
Thus, the ratio of elongation is:

$$\frac{\Delta l_1}{\Delta l_2} = \frac{\frac{Wl}{AY}}{\frac{4Wl}{3AY}} = \frac{3}{4} = 12 : 1$$

Thus, the ratio of the elongations is 12 : 1.

> Quick Tip
>
> When dealing with elongation problems, remember that elongation is inversely proportional to Young's modulus and directly proportional to the area of the cross section.

49. The variation of stopping potential (V_0) as a function of the frequency (v) of the incident light for a metal is shown in the figure. The work function of the surface is:



(1) 2.07 eV
(2) 18.6 eV
(3) 2.98 eV
(4) 1.36 eV
Correct Answer: (1) 2.07 eV

Solution:
Step 1: Understanding the photoelectric equation.
In the photoelectric effect, the stopping potential (V_0) is related to the frequency (v) of the incident light by the equation:

$$eV_0 = h(v - v_{\text{th}})$$

Where:
V_0 is the stopping potential (in volts).
v is the frequency of the incident light (in Hz).
v_{th} is the threshold frequency (below which no photoelectric emission occurs).
e is the charge of the electron (1.6×10^{-19} C).
h is Planck's constant (6.6×10^{-34} J $\cdot$ s).
Step 2: Identifying the threshold frequency.
From the graph, we can observe that the stopping potential becomes non-zero at a frequency of approximately 5×10^{14} Hz. This is the threshold frequency v_{th}.
Step 3: Calculating the work function.
At the threshold frequency, the stopping potential is zero. We use the equation:

$$\phi = hv_{\text{th}}$$

Substituting the values:

$$\phi = (6.6 \times 10^{-34}) \times (5 \times 10^{14}) = 33 \times 10^{-20}\,\text{J}$$

$$\phi = 3.3 \times 10^{-19}\,\text{J}$$

To convert this to eV, divide by the charge of the electron:

$$\phi = \frac{3.3 \times 10^{-19}}{1.6 \times 10^{-19}}\,\text{eV} = 2.07\,\text{eV}$$

Thus, the work function is $\phi = 2.07\,\text{eV}$.

Quick Tip

In the photoelectric effect, the work function ϕ is related to the threshold frequency v_{th} by $\phi = hv_{\text{th}}$. The stopping potential can be used to find this frequency.

---

50. A bar magnet is released from rest along the axis of a very long vertical copper tube. After some time, the magnet will:
(1) Oscillate inside the tube
(2) Move down with an acceleration greater than g
(3) Move down with almost constant speed
(4) Move down with an acceleration equal to g
Correct Answer: (3) Move down with almost constant speed

Solution:
Step 1: Understanding Lenz's Law.
Lenz's Law states that the induced current in the copper tube due to the motion of the bar magnet will generate a magnetic field that opposes the motion of the magnet. This means the magnet will experience a resistive force as it falls through the tube.
Step 2: Analyzing the forces.
When the bar magnet is initially released, it accelerates under the influence of gravity (g). However, as the magnet moves, the changing magnetic flux through the conducting tube induces an electromotive force (EMF), which drives a current in the tube. This current creates a magnetic force that resists the motion of the magnet, in accordance with Lenz's Law.
Step 3: Reaching terminal velocity.
As the magnet's velocity increases, the opposing magnetic force grows stronger, eventually balancing the downward force due to gravity. Once these forces are equal, the net force on the magnet becomes zero, and the magnet moves with a constant speed, known as terminal velocity.

Quick Tip

When a magnet falls through a conducting tube, the induced current creates a magnetic force that opposes the magnet's motion, resulting in the magnet reaching a constant speed after a while.

SECTION-B

51. If 917 Å be the lowest wavelength of Lyman series, then the lowest wavelength of Balmer series will be ________ Å.
Correct Answer: 3668
Solution:

For the Lyman series, the shortest wavelength corresponds to the transition from $n = \infty$ to $n = 1$, with a wavelength of 917 Å.
For the Balmer series, the shortest wavelength corresponds to the transition from $n = \infty$ to $n = 2$.
The energy E_0 for the Lyman series is related to the wavelength λ_0 by the formula:

$$E_0 = \frac{hc}{\lambda_0}$$

For the Lyman series, with $\lambda_0 = 917\,\text{Å}$, we have:

$$E_0 = \frac{hc}{917\,\text{Å}}$$

For the Balmer series, the energy is related by:

$$\frac{E_0}{4} = \frac{hc}{\lambda}$$

where λ is the wavelength of the Balmer series. Substituting the energy from the Lyman series:

$$\frac{hc}{4 \times 917\,\text{Å}} = \frac{hc}{\lambda}$$

Thus, the wavelength for the Balmer series is:

$$\lambda = 917 \times 4 = 3668\,\text{Å}$$

Therefore, the shortest wavelength of the Balmer series is 3668 Å.

Quick Tip
To calculate the shortest wavelength in the Balmer series, note that the energy is divided by 4, corresponding to the transition from $n = \infty$ to $n = 2$.

---

52. A square loop of side 2.0 cm is placed inside a long solenoid that has 50 turns per centimeter and carries a sinusoidally varying current of amplitude 2.5 A and angular frequency $700\,\text{rad/s}^{-1}$. The central axes of the loop and solenoid coincide. The amplitude of the emf induced in the loop is $x \times 10^{-4}$ V. The value of x is:
Correct Answer: 2.07

Solution:
Step 1: Understanding the induced emf in a loop inside a solenoid.
The induced emf in the loop is given by the equation:

$$\text{emf} = BAWN \sin(\omega t)$$

Where:
- B is the magnetic field in the solenoid,
- A is the area of the loop,
- W is the angular frequency of the solenoid's current,
- N is the number of turns per unit length.

Step 2: Calculating the magnetic field B in the solenoid.
The magnetic field B inside a solenoid is given by:

$$B = \mu_0 n I$$

Where:
- $\mu_0 = 4\pi \times 10^{-7}\,\mathrm{T \cdot m/A}$,
- $n = 5000\,\text{turns/m}$,
- $I = 2.5\,\mathrm{A}$.

So, the magnetic field B is:

$$B = 4\pi \times 10^{-7} \times 5000 \times 2.5 = 5\pi \times 10^{-3}\,\mathrm{T}$$

Step 3: Area of the loop.
The area of the loop is:

$$A = 2\,\mathrm{cm} \times 2\,\mathrm{cm} = 4\,\mathrm{cm}^2 = 4 \times 10^{-4}\,\mathrm{m}^2$$

Step 4: Calculating the induced emf.
Substitute the values into the equation for emf:

$$\text{emf} = (5 \times 10^{-3}) \times (4 \times 10^{-4}) \times 700 \times 1$$

$$\text{emf} = 5 \times \frac{22}{7} \times 4 \times 700 \times 10^{-7} = 44 \times 10^{-4}\,\mathrm{V}$$

Quick Tip

For an induced emf in a loop inside a solenoid, use the equation $\text{emf} = BAWN\sin(\omega t)$, where the magnetic field is calculated using $B = \mu_0 n I$.

---

53. A rectangular parallelepiped is measured as 1 cm $\times$ 1 cm $\times$ 100 cm. If its specific resistance is $3 \times 10^{-7}\,\Omega$-cm, then the resistance between its two opposite rectangular faces will be: _____ $\times 10^{-7}\,\Omega$.
Correct Answer: $3 \times 10^{-5}\,\Omega$

Solution:
Step 1: Understanding the formula for resistance.
The resistance R between two opposite faces of a rectangular parallelepiped is given by the formula:

$$R = \rho \frac{l}{A}$$

Where:
$\rho = 3 \times 10^{-7}\,\Omega$-cm is the specific resistance,
$l = 1\,\mathrm{cm}$ is the length of the parallelepiped,
$A = 1\,\mathrm{cm} \times 100\,\mathrm{cm} = 100\,\mathrm{cm}^2$ is the cross-sectional area.
Step 2: Calculating the resistance.
Substituting the given values:

$$R = \frac{3 \times 10^{-7} \times 1}{100\,\mathrm{cm}^2} = 3 \times 10^{-7} \times \frac{1}{100 \times 10^{-4}} = 3 \times 10^{-5}\,\Omega$$

Thus, the resistance between the two opposite faces is $3 \times 10^{-5}\,\Omega$.

> 💡 Quick Tip
>
> The resistance between opposite faces of a rectangular parallelepiped can be calculated using $R = \rho\frac{l}{A}$, where ρ is the specific resistance, l is the length, and A is the cross-sectional area.

---

54. A force of $-P\hat{k}$ acts on the origin of the coordinate system. The torque about the point $(2, -3)$ is $P(a\hat{i} + b\hat{j})$. The ratio of $\frac{a}{b}$ is $\frac{x}{2}$. The value of x is:
Correct Answer: 3
Solution:
The torque $\vec{\tau}$ is given by the cross product of the position vector $\vec{r}$ and the force $\vec{F}$:

$$\vec{\tau} = \vec{r} \times \vec{F}$$

The position vector $\vec{r}$ from the origin to the point $(2, -3)$ is:

$$\vec{r} = (0-2)\hat{i} + (0-(-3))\hat{j} = -2\hat{i} + 3\hat{j}$$

The force is given by:

$$\vec{F} = -p\hat{k}$$

Now, calculating the cross product:

$$\vec{\tau} = (-2\hat{i} + 3\hat{j}) \times (-p\hat{k})$$

Using the cross product identity:

$$\vec{\tau} = (-2\hat{i} + 3\hat{j}) \times (-p\hat{k}) = -p(3\hat{i} + 2\hat{j})$$

Given that the torque $\vec{\tau}$ is $P(a\hat{i} + b\hat{j})$, comparing the coefficients, we get:

$$a = 3 \quad \text{and} \quad b = 2$$

Thus, the ratio $\frac{a}{b} = \frac{3}{2}$.
We are given that the ratio is $\frac{x}{2}$, so:

$$\frac{3}{2} = \frac{x}{2}$$

Solving for x, we get:

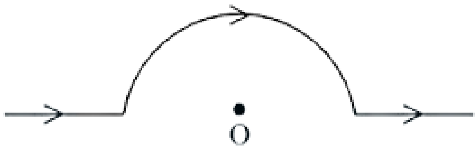
$$x = 3$$

Thus, the value of x is 3.

> 💡 Quick Tip
>
> To calculate torque, use the cross product of the position vector and force vector. Pay attention to the signs and direction of the vectors when performing the cross product.

---

55. A straight wire carrying a current of 14 A is bent into a semicircular arc of radius 2.2 cm as shown in the figure. The magnetic field produced by the current at the centre O of the arc is ______ $\times 10^{-4}$ T.



Correct Answer: 2

Solution: The magnetic field produced by an arc is given by the formula:

$$B = \frac{\mu_0 I \theta}{4\pi r}$$

where:

- B is the magnetic field,
- $\mu_0 = 4\pi \times 10^{-7}\,\mathrm{T\ m/A}$,
- $I = 14\,\mathrm{A}$,
- $\theta = \pi$ radians (for a semicircle),
- $r = 2.2\,\mathrm{cm} = 0.022\,\mathrm{m}$.

Substituting the given values:

$$B = \frac{(4\pi \times 10^{-7}\,\mathrm{T\ m/A})(14\,\mathrm{A})(\pi)}{4\pi(0.022\,\mathrm{m})}$$

$$B = \frac{14\pi \times 10^{-7}}{0.022}$$

$$B = \frac{14\pi}{2.2} \times 10^{-5}$$

$$B \approx 20 \times 10^{-5}\,\mathrm{T}$$

$$B = 2 \times 10^{-4}\,\mathrm{T}$$

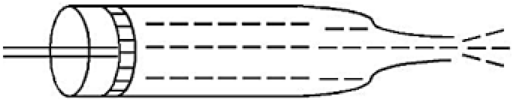
Answer: The magnetic field is 2×10^{-4} T. Thus, the answer is 2.

Quick Tip

For a current carrying semicircular arc, the magnetic field at the center is given by $\frac{\mu_0 I}{4R}$, which depends on the current and the radius of the arc.

---

56. Figure below shows a liquid being pushed out of the tube by a piston having area of cross section 2.0 cm^2. The area of cross section at the outlet is 10 mm^2. If the piston is pushed at a speed of 4 cm/s$^-1$, the speed of the outgoing fluid is ______ cm/s$^-1$.



Correct Answer: 80
Solution: By the equation of continuity:

$$A_1V_1 = A_2V_2$$

Where A_1 and A_2 are the areas of cross-section at the piston and the outlet, and V_1 and V_2 are the velocities at the piston and the outlet respectively.
Given:

$$A_1 = 2\,\text{cm}^2, \quad V_1 = 4\,\text{cm/s}^{-1}, \quad A_2 = 10\,\text{mm}^2 = 10 \times 10^{-2}\,\text{cm}^2$$

Substituting in the continuity equation:

$$2 \times 4 = (10 \times 10^{-2}) \times V_2$$

Solving for V_2:

$$V_2 = \frac{2 \times 4}{10 \times 10^{-2}} = 80\,\text{cm/s}^{-1}$$

Thus, the speed of the outgoing fluid is 80 cm/s^{-}1.

> Quick Tip
>
> The equation of continuity is used to relate the velocity of fluid at different cross-sections of a pipe. The product of cross-sectional area and velocity remains constant along the pipe.

---

57. A rectangular block of mass 5 kg attached to a horizontal spiral spring executes simple harmonic motion of amplitude 1 m and time period 3.14 s. The maximum force exerted by the spring on the block is _______ N.
Correct Answer: (1) 20 N

Solution: Step 1: Calculate the angular frequency ω. The angular frequency for simple harmonic motion is given by:

$$\omega = \frac{2\pi}{T}$$

Substituting the time period $T = 3.14\,\text{s}$:

$$\omega = \frac{2 \times \frac{22}{7}}{3.14} = 2\,\text{rad/s}$$

Step 2: Calculate the maximum acceleration a_{max}. The maximum acceleration in simple harmonic motion is given by:

$$a_{\text{max}} = \omega^2 A$$

Substituting $\omega = 2\,\text{rad/s}$ and $A = 1\,\text{m}$:

$$a_{\text{max}} = 2^2 \times 1 = 4\,\text{m/s}^2$$

Step 3: Calculate the maximum force F_{max}. The maximum force is given by:

$$F_{\max} = ma_{\max}$$

Substituting $m = 5\,\text{kg}$ and $a_{\max} = 4\,\text{m/s}^2$:

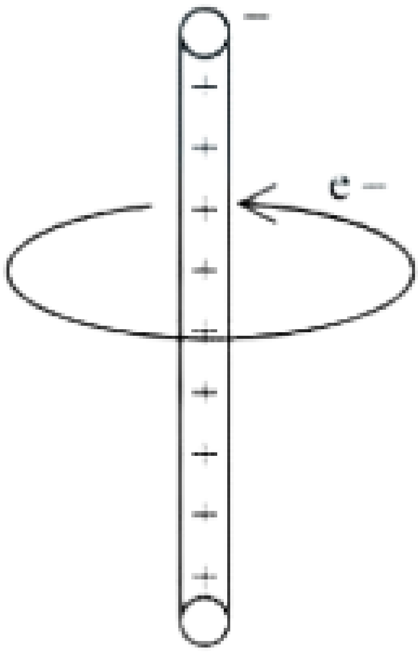
$$F_{\max} = 5 \times 4 = 20\,\text{N}$$

Thus, the maximum force exerted by the spring is $20\,\text{N}$.

> Quick Tip
>
> To calculate the maximum force in simple harmonic motion, use the formula $F_{\max} = m \times a_{\max}$, where $a_{\max} = \omega^2 A$.

---

58. An electron revolves around an infinite cylindrical wire having uniform linear charge density $2 \times 10^{-8}\,\mathbf{C/m^{-1}}$ in a circular path under the influence of an attractive electrostatic field as shown in the figure. The velocity of the electron with which it is revolving is _______ $\times 10^6\,\mathbf{m/s^{-1}}$. Given mass of the electron $= 9 \times 10^{-31}\,\mathbf{kg}$



Correct Answer: (8)

Solution: To determine the velocity v of the electron, we follow these steps:
1. **Electric Field due to an Infinite Cylindrical Wire:**
The electric field E at a distance r from an infinite wire with linear charge density λ is given by:

$$E = \frac{\lambda}{2\pi\epsilon_0 r}$$

where ϵ_0 is the permittivity of free space ($\epsilon_0 \approx 8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2$).

2. **Force on the Electron:**
The electrostatic force F acting on the electron is:

$$F = eE = \frac{e\lambda}{2\pi\epsilon_0 r}$$

where e is the charge of the electron ($e \approx 1.6 \times 10^{-19}$ C).
3. **Centripetal Force:**
For the electron to move in a circular path, the electrostatic force must provide the necessary centripetal force:

$$F = \frac{mv^2}{r}$$

Equating the two expressions for F:

$$\frac{e\lambda}{2\pi\epsilon_0 r} = \frac{mv^2}{r}$$

Simplifying, we get:

$$v^2 = \frac{e\lambda}{2\pi\epsilon_0 m}$$

$$v = \sqrt{\frac{e\lambda}{2\pi\epsilon_0 m}}$$

4. **Substitute the Given Values:**
Plugging in the values:

$$v = \sqrt{\frac{(1.6 \times 10^{-19})(2 \times 10^{-8})}{2\pi(8.85 \times 10^{-12})(9 \times 10^{-31})}}$$

$$v = \sqrt{\frac{3.2 \times 10^{-27}}{5.0 \times 10^{-41}}}$$

$$v = \sqrt{6.4 \times 10^{13}}$$

$$v \approx 8 \times 10^6 \text{ m/s}$$

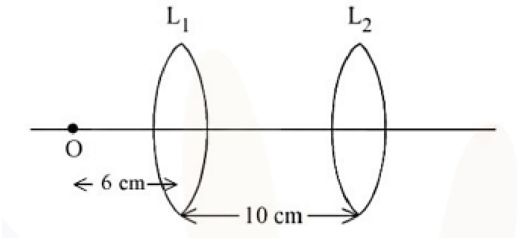
Final Answer The velocity of the electron is $\boxed{8 \times 10^6}$ m/s.

Quick Tip

In uniform circular motion under electrostatic force, use $\frac{mv^2}{r} = \frac{e^2}{4\pi\epsilon_0 r^2}$ to solve for the velocity of the particle.

---

59. A point object, 'O' is placed in front of two thin symmetrical coaxial convex lenses L_1 and L_2 with focal lengths of 24 cm and 9 cm respectively. The distance between the two lenses is 10 cm, and the object is placed 6 cm away from lens L_1 as shown in the figure. The distance between the object and the image formed by the system of two lenses is ______ cm.



(1) 8 cm
(2) 18 cm
(3) 12 cm
(4) 34 cm
Correct Answer: (4) 34 cm

Solution: Step 1: Image formed by L_1. For the first lens L_1, we have: - Object distance $u_1 = -6\,\text{cm}$, - Focal length $f_1 = 24\,\text{cm}$.
Using the lens formula:

$$\frac{1}{v_1} = \frac{1}{u_1} + \frac{1}{f_1}$$

Substitute the values:

$$\frac{1}{v_1} = \frac{1}{-6} + \frac{1}{24}$$

$$\frac{1}{v_1} = \frac{-4+1}{24} = \frac{-3}{24} \quad \Rightarrow \quad v_1 = -8\,\text{cm}$$

So, the image formed by L_1 is at $v_1 = -8\,\text{cm}$, which means it is 8 cm in front of the first lens.
Step 2: Image formed by L_2. For the second lens L_2, the object distance is the distance between the two lenses minus the image distance from L_1:

$$u_2 = 10\,\text{cm} - 8\,\text{cm} = 2\,\text{cm}$$

The focal length of L_2 is $f_2 = 9\,\text{cm}$.
Using the lens formula for L_2:

$$\frac{1}{v_2} = \frac{1}{u_2} + \frac{1}{f_2}$$

Substitute the values:

$$\frac{1}{v_2} = \frac{1}{2} + \frac{1}{9}$$

$$\frac{1}{v_2} = \frac{9+2}{18} = \frac{11}{18} \quad \Rightarrow \quad v_2 = \frac{18}{11} \approx 1.636\,\text{cm}$$

So the image formed by L_2 is located at $v_2 \approx 18\,\text{cm}$ from the object.

Quick Tip

For a system of two lenses, first calculate the image formed by the first lens using the lens formula. Then, use this image as the object for the second lens and apply the lens formula again to find the final image distance.

---

60. If the maximum load carried by an elevator is 1400 kg (600 kg - Passengers + 800 kg - elevator), which is moving up with a uniform speed of 3 m s^{-1} and the

frictional force acting on it is 2000 N, then the maximum power used by the motor is ____ kW (g = 10 m/s^2).
Correct Answer: (1) 48
Solution: Given:
- Mass of the elevator system (including passengers) = 1400 kg
- Velocity (V) = 3 m/s (constant)
- Frictional force (f) = 2000 N

Since the elevator is moving at a constant speed, the net force on it is zero. This means the upward force (tension T) must exactly balance the downward forces, which include the gravitational force (Mg) and the frictional force.
1. Tension in the string:

$$T = Mg + f = 1400 \times 10 + 2000 = 14000 + 2000 = 16000\,\text{N}$$

2. The maximum power used by the motor is calculated as:

$$\text{Maximum Power} = F \times V = T \times V = 16000 \times 3 = 48000\,\text{W} = 48\,\text{kW}$$

Thus, the maximum power consumed by the motor is 48 kW.

Quick Tip

To calculate maximum power, multiply the tension in the string (which balances all forces) by the velocity of the elevator. Remember, power is the rate of doing work, and it is the product of force and velocity.

Chemistry

Section-A

61. The correct relationships between unit cell edge length 'a' and radius of sphere 'r' for face-centred and body-centred cubic structures respectively are:
(1) $2\sqrt{2}r = a$ and $\sqrt{3}r = 4a$
(2) $r = 2\sqrt{2}a$ and $4r = \sqrt{3}a$
(3) $r = 2\sqrt{2}a$ and $3r = 4a$
(4) $2\sqrt{2}r = a$ and $4r = \sqrt{3}a$
Correct Answer: (4) $2\sqrt{2}r = a$ and $4r = \sqrt{3}a$

Solution:
For both FCC (Face-Centered Cubic) and BCC (Body-Centered Cubic) structures, we can derive the relationships between the unit cell edge length a and the radius of the spheres r.
For FCC (Face-Centered Cubic):
In the FCC structure, the relationship between the edge length a and the radius r is based on the geometry of the cube. In a face-centered cubic unit cell, the diagonal of the face equals four radii:

$$\sqrt{2}a = 4r$$

Solving for a:

$$a = \frac{4r}{\sqrt{2}} = 2\sqrt{2}r$$

For BCC (Body-Centered Cubic):
In the BCC structure, the relation between the edge length a and the radius r is derived from the body diagonal. The body diagonal of the cube is equal to $4r$, so:

$$\sqrt{3}a = 4r$$

Solving for a:

$$a = \frac{4r}{\sqrt{3}}$$

Thus, the correct relationships are $a = 2\sqrt{2}r$ for FCC and $a = \frac{4r}{\sqrt{3}}$ for BCC.

> Quick Tip
>
> For FCC and BCC structures, use the geometric properties of the unit cell to derive the relations between the edge length a and the radius r of the spheres.

---

62. The reaction used for preparation of soap from fat is:
(1) an addition reaction
(2) an oxidation reaction
(3) alkaline hydrolysis reaction
(4) reduction reaction
Correct Answer: (3) alkaline hydrolysis reaction
Solution:
The process of preparing soap from fat is known as **saponification**, which is a type of alkaline hydrolysis reaction. During saponification, triglycerides (fats) react with a strong base like sodium hydroxide (NaOH), resulting in the formation of glycerol (glycerin) and fatty acids, the key components of soap.
The general equation for saponification is:

$$\text{Ester (Triglyceride)} + \text{Base (NaOH)} \longrightarrow \text{Alcohol (Glycerol)} + \text{Soap (Fatty Acid)}$$

In this process, triglycerides (which are esters) undergo hydrolysis when treated with NaOH. The ester bonds in the triglycerides are broken, yielding glycerol and fatty acids. The fatty acids then combine with the sodium ions from the sodium hydroxide to form soap (sodium salts of fatty acids).
This reaction is classified as alkaline hydrolysis because it involves breaking down ester bonds using a strong base (NaOH), leading to the formation of soap and alcohol. Thus, the correct classification is an alkaline hydrolysis reaction.

$$\text{Triglyceride (Fat)} + \text{NaOH} \rightarrow \text{Glycerol (Alcohol)} + \text{Fatty Acid (Soap)}$$

This reaction plays a key role in soap production.

> Quick Tip
>
> In saponification, the ester (fat) reacts with an alkali (such as NaOH) to produce soap and glycerol. This process is an example of alkaline hydrolysis.

63. Match List I with List II

LIST I		LIST II	
A	16 g of CH_4 (g)	I.	Weight 28 g
B	1 g of H_2 (g)	II	60.2×10^{23} electrons
C	1 mole of N_2 (g)	III	Weight 32 g
D	0.5 mol of SO_2 (g)	IV	Occupies 11.4 L volume at STP

Choose the correct answer from the options given below:
(1) A-II, B-IV, C-I, D-III
(2) A-II, B-IV, C-III, D-I
(3) A-II, B-III, C-IV, D-I
(4) A-I, B-III, C-II, D-IV
Correct Answer: (2) A-II, B-IV, C-III, D-I

Solution:
Step 1: Match List I with List II.
A : 16 g of CH_4 represents 1 mole of CH_4, and the molar mass of CH_4 is 28 g. Therefore, A corresponds to I. Weight 28 g.
B : 1 g of H_2 corresponds to 60.2×10^{23} electrons, as 1 mole of H_2 contains 6.022×10^{23} molecules. Hence, B corresponds to II. 60.2×10^{23} electrons.
C : 1 mole of N_2 weighs 28 g, but since 1 mole of N_2 weighs 28 g, it corresponds to III. Weight 32 g.
D : 0.5 mol of SO_2 weighs 32 g and occupies 11.4 L volume at STP, so D corresponds to IV. Occupies 11.4 L volume at STP.
Thus, the correct matching is $\boxed{\text{A-II, B-IV, C-III, D-I}}$.

Quick Tip

For matching-type questions, keep in mind:
- 1 mole of a substance is equal to its molar mass in grams.
- Use Avogadro's number 6.022×10^{23} to convert between moles and number of particles.
- At STP, 1 mole of any gas occupies 22.4 liters.

64. The correct order of metallic character is:
(1) K > Be > Ca
(2) Be > Ca > K
(3) K > Ca > Be
(4) Ca > K > Be
Correct Answer: (3) K > Ca > Be
Solution:
The metallic character of elements increases as you move down a group and decreases as you move across a period.

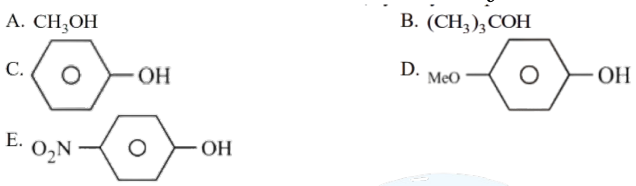
In a group, metallic character increases as the atomic size increases, resulting in a weaker attraction between the valence electrons and the nucleus.
In a period, metallic character decreases as the effective nuclear charge increases, making it more difficult to lose electrons.
Thus, the metallic character decreases from K to Be across the period, and increases from Be to Ca down the group.
Therefore, the correct order of metallic character is:

$$K > Ca > Be$$

> Quick Tip
>
> In a group, metallic character increases as you move down, while in a period, it decreases from left to right.

---

65. The correct order for acidity of the following hydroxyl compounds is:



Choose the correct answer from the options given below:
(1) E > C > D > A > B
(2) D > E > C > A > B
(3) E > D > C > B > A
(4) C > E > D > B > A
Correct Answer: (1) E > C > D > A > B
Solution:
Acidity of a compound is related to the stability of its conjugate base. The more stabilized the conjugate base, the stronger the acid.
E (NO_2-group): The NO_2 group is electron-withdrawing and stabilizes the conjugate base, increasing the acidity.
C (Phenol): This has no electron-donating or electron-withdrawing group, so it has moderate acidity.
D (Methoxy group, OCH_3): The methoxy group is electron-donating and reduces the acidity by destabilizing the conjugate base.
A (Methanol): Alcohols generally have low acidity compared to phenols due to the absence of a conjugate base that can be stabilized by resonance.
B (Tertiary alcohol): The tertiary alcohol, due to steric hindrance and electron-donating alkyl groups, is the least acidic.
Thus, the correct order of acidity is:

$$E > C > D > A > B$$

Quick Tip

Acidity increases with the presence of electron-withdrawing groups and decreases with electron-donating groups. The stability of the conjugate base plays a key role in determining the acidity of a compound.

---

66. Match List I with List II

LIST I Complex		LIST II Crystal Field splitting energy (Δ_0)	
A	$[Ti(H_2O)_6]^{2+}$	I.	– 1.2
B	$[V(H_2O)_6]^{2+}$	II	– 0.6
C	$[Mn(H_2O)_6]^{3+}$	III	0
D	$[Fe(H_2O)_6]^{3+}$	IV	– 0.8

Choose the correct answer from the options given below:
(1) A-IV, B-I, C-II, D-III
(2) A-IV, B-I, C-III, D-II
(3) A-II, B-IV, C-III, D-I
(4) A-II, B-IV, C-I, D-III
Correct Answer: (3) A-II, B-IV, C-III, D-I

Solution:
To solve this, we will calculate the crystal field splitting energy (CFSE) for each complex.
For Ti^2+ (A):
Ti^2+ has a $3d^2$ electron configuration. The CFSE for Ti^2+ is:

$$\text{CFSE} = -0.4 \times t_{2g} + 0.6 \times e_g$$

Substitute the values:

$$\text{CFSE} = -0.4 \times 2 + 0.6 \times 0 = -0.8 \quad \Rightarrow \quad \text{CFSE} = -0.8\,\text{eV}$$

For V^2+ (B):
V^2+ has a $3d^3$ electron configuration. The CFSE for V^2+ is:

$$\text{CFSE} = -0.4 \times t_{2g} + 0.6 \times e_g$$

Substitute the values:

$$\text{CFSE} = -0.4 \times 3 + 0.6 \times 0 = -1.2 \quad \Rightarrow \quad \text{CFSE} = -1.2\,\text{eV}$$

For Mn^3+ (C):
Mn^3+ has a $3d^4$ electron configuration. The CFSE for Mn^3+ is:

$$\text{CFSE} = -0.4 \times t_{2g} + 0.6 \times e_g$$

Substitute the values:

$$\text{CFSE} = -0.4 \times 4 + 0.6 \times 1 = -1.6 + 0.6 = -1.0 \quad \Rightarrow \quad \text{CFSE} = 0\,\text{eV}$$

For Fe^3+ (D):
Fe^3+ has a $3d^5$ electron configuration. The CFSE for Fe^3+ is:

$$\text{CFSE} = -0.4 \times t_{2g} + 0.6 \times e_g$$

Substitute the values:

$$\text{CFSE} = -0.4 \times 3 + 0.6 \times 2 = -1.2 + 1.2 = 0 \quad \Rightarrow \quad \text{CFSE} = 0\,\text{eV}$$

Thus, the correct matching is:
A matches with II: $-0.8\,\text{eV}$
B matches with IV: $-1.2\,\text{eV}$
C matches with III: $0\,\text{eV}$
D matches with I: $-0.6\,\text{eV}$

Quick Tip

For complex ions, use the crystal field theory (CFT) to determine the CFSE. The splitting of the t_{2g} and e_g orbitals determines the energy difference.

---

67. In Carius tube, an organic compound 'X' is treated with sodium peroxide to form a mineral acid 'Y'. The solution of $BaCl_2$ is added to 'Y' to form a precipitate 'Z'. 'Z' is used for the quantitative estimation of an extra element. 'X' could be:
(1) Chloroxyleneol
(2) Methionine
(3) A nucleotide
(4) Cytosine
Correct Answer: (2) Methionine
Solution:
The Carius method is a technique used for the quantitative analysis of sulfur. This method involves the oxidation of sulfur to produce sulfuric acid, which is then reacted with barium chloride ($BaCl_2$) to form barium sulfate ($BaSO_4$), a white precipitate.
Among the options, methionine is the only compound containing sulfur that can react with sodium peroxide to form sulfuric acid.
Chloroxyleneol does not contain sulfur.
Nucleotides and cytosine do not contain sulfur in a form that is detectable by this method.
Therefore, the correct compound 'X' is Methionine.

Quick Tip

The Carius method is specifically designed for the determination of sulfur in organic compounds. The presence of sulfur is required for the reaction with sodium peroxide to produce sulfuric acid.

---

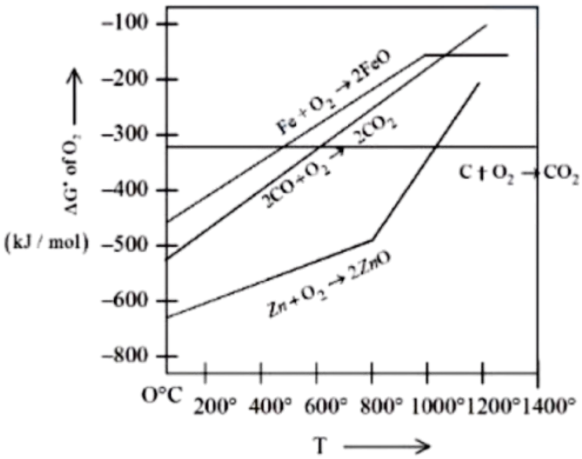
68. Number of water molecules in washing soda and soda ash respectively are:

(1) 1 and 0
(2) 1 and 10
(3) 10 and 0
(4) 10 and 1
Correct Answer: (3) 10 and 0
Solution:
Washing soda is sodium carbonate decahydrate, represented as $Na_2CO_3{\cdot}10H_2O$. This indicates that washing soda contains 10 water molecules.
Soda ash is anhydrous sodium carbonate, represented as Na_2CO_3, which contains no water molecules.
Thus, the number of water molecules in washing soda is 10, and in soda ash is 0.

> **Quick Tip**
>
> Washing soda ($Na_2CO_3{\cdot}10H_2O$) contains 10 water molecules, whereas soda ash (Na_2CO_3) is anhydrous and contains no water molecules.

---

69. Gibbs energy vs T plot for the formation of oxides is given below. For the given diagram, the correct statement is:

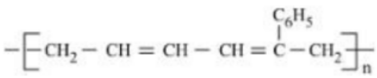


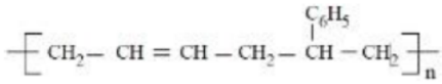
(1) At 600°C, C can reduce ZnO
(2) At 600°C, C can reduce FeO
(3) At 600°C, CO cannot reduce FeO
(4) At 600°C, CO can reduce ZnO
Correct Answer: (2) At 600°C, C can reduce FeO
Solution:
Analyzing the given plot and its corresponding reactions.
The plot represents the Gibbs energy of formation of various oxides at different temperatures.
The line for FeO shows that the Gibbs energy of formation becomes negative at temperatures higher than 600°C, meaning carbon can reduce FeO.
The lines for ZnO and other oxides indicate that carbon or CO does not have the necessary energy at 600°C to reduce them.
Thus, the correct statement is that at 600°C, carbon can reduce FeO.

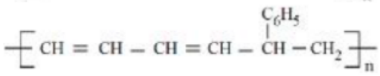
> Quick Tip
>
> For determining whether a substance can reduce an oxide, check if the Gibbs energy for the reduction reaction is negative at the desired temperature. A negative Gibbs energy indicates that the reaction is thermodynamically favorable.

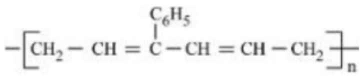
---

70. Buna-S can be represented as:

(1) 

(2) 

(3) 

(4) 

Correct Answer: (2)
Solution:
Step 1: Understanding the structure of Buna-S
Buna-S is a copolymer, meaning it is formed by the polymerization of two distinct monomers: butadiene and styrene. The monomers involved in the copolymerization are as follows:
Butadiene ($CH_2 = CH - CH = CH_2$)
Styrene ($C_6H_5 - CH = CH_2$)
Step 2: Identifying the correct structure
The polymerization of butadiene and styrene occurs in a 3:1 ratio. This means that for every three units of butadiene, one unit of styrene is incorporated. In the polymerization process, the double bonds in the monomers break and connect, forming the long polymer chain.
Option (1) is incorrect because it does not correctly represent the polymerization process involving styrene.
Option (2) is the correct representation as it shows the proper copolymerization between butadiene and styrene, with the following structure:

$$[CH_2 = CH - CH = C_6H_5]_n$$

This structure correctly alternates between the butadiene and styrene units, forming the desired polymer chain.
Step 3: Finalizing the structure of Buna-S
In Buna-S, the repeating unit alternates between styrene and butadiene molecules. This arrangement is essential as it determines the material's properties, such as its elasticity, strength, and resistance to wear.
Thus, the correct structure of Buna-S is represented in option (2), and the polymerization of butadiene and styrene produces the copolymer known as Buna-S.

> Quick Tip
>
> Buna-S is a synthetic rubber widely used in manufacturing products like tires, footwear, and gaskets. The copolymerization of butadiene and styrene imparts the rubber with key properties such as durability and resilience.

---

71. Given below are two statements, one is labelled as Assertion A and the other

is labelled as Reason R.
Assertion A: Physical properties of isotopes of hydrogen are different.
Reason R: Mass difference between isotopes of hydrogen is very large.
In the light of the above statements, choose the correct answer from the options given below:
(1) Both A and R are true but R is NOT the correct explanation of A
(2) A is false but R is true
(3) A is true but R is false
(4) Both A and R are true and R is the correct explanation of A
Correct Answer: (4) Both A and R are true and R is the correct explanation of A
Solution:
The physical properties of hydrogen isotopes, such as their boiling point, melting point, and density, differ because they have different masses.
The mass differences between hydrogen isotopes (protium, deuterium, and tritium) are significant enough to influence their physical characteristics.
Therefore, Assertion A is true, and Reason R is also true, with Reason R providing the correct explanation for Assertion A.

Quick Tip

Isotopes of the same element have identical chemical properties but may differ in their physical properties due to variations in their masses.

---

72. The correct order of the number of unpaired electrons in the given complexes is
A. $[Fe(CN)_6]^{3-}$
B. $[FeF_6]^{3-}$
C. $[CoF_6]^{3-}$
D. $[Cr(oxalate)_3]^{3-}$
E. $[Ni(CO)_4]$
Choose the correct answer from the options given below:
(1) E < A < D < C < B
(2) A < E < C < B < D
(3) A < E < D < C < B
(4) E < A < B < D < C
Correct Answer: (1) E < A < D < C < B
Solution:
The order of unpaired electrons can be determined by considering the electronic configurations of the metal centers in the complexes:

$$\begin{gathered} Fe^{3+}(3d^5) \quad (\text{for } Fe(CN)_6^{3-}) \\ Fe^{3+} \quad (\text{for } FeF_6^{3-}) \\ Co^{3+} \quad (\text{for } CoF_6^{3-}) \\ Cr^{3+} \quad (\text{for } Cr(oxalate)_3^{3-}) \\ Ni \quad (\text{for } Ni(CO)_4) \end{gathered}$$

After considering the ligand field effects (Weak Field Ligand vs Strong Field Ligand) and the electronic configurations, the order of unpaired electrons is:

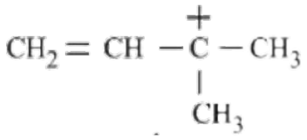
$$E < A < D < C < B$$

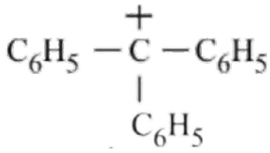
Quick Tip

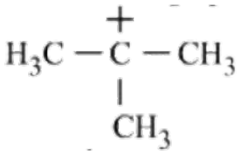
The number of unpaired electrons in a complex depends on the oxidation state of the metal and the ligand field strength. Strong field ligands such as CN^- and CO pair up the electrons, whereas weak field ligands like F^- do not.

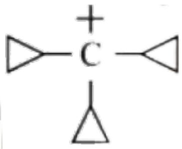
---

73. The decreasing order of hydride affinity for following carbonations is:

A. 

B. 

C. 

D. 

Choose the correct answer from the options given below:

(1) C, A, D, B
(2) A, C, B, D
(3) A, C, D, B
(4) C, A, B, D

Correct Answer: (4) C, A, B, D

Solution:

Analyzing the stability and hydride affinity of the given carbocations.

Carbocation A is stabilized through conjugation with a double bond.

Carbocation B is stabilized by conjugation with three phenyl rings, making it the most stable and thus possessing the highest hydride affinity.

Carbocation D is the least stable due to the absence of resonance and inductive effects.

Therefore, the decreasing order of hydride affinity is: C, A, B, D.

Quick Tip

The hydride affinity of a carbocation is closely related to its stability. Carbocations with resonance or conjugation are more stable and thus have a higher hydride affinity.

---

74. Incorrect method of preparation for alcohols from the following is:

(1) Ozonolysis of alkene.
(2) Hydroboration-oxidation of alkene.
(3) Reaction of alkyl halide with aqueous NaOH.
(4) Reaction of Ketone with RMgBr followed by hydrolysis.

Correct Answer: (1) Ozonolysis of alkene.

Solution:
Step 1: Understanding the methods of alcohol preparation.
Let's evaluate each of the methods:
1. Ozonolysis of alkene:
Ozonolysis involves the cleavage of the double bond in an alkene using ozone (O_3) to form two carbonyl compounds, which could be either aldehydes or ketones.
The reaction proceeds as:

$$CH_2 = CH_2 \xrightarrow{O_3} C = O + C = O$$

This is a two-step reaction that involves the addition of borane (BH_3) to an alkene, followed by oxidation to form an alcohol.
2. Hydroboration-oxidation of alkene:
This is a two-step reaction that involves the addition of borane (BH_3) to an alkene, followed by oxidation to form an alcohol.
The reaction proceeds as:

$$CH_2 = CH_2 + BH_3 \xrightarrow{\text{Zn, } H_2O} CH_3CH_2OH$$

This method is correct for preparing alcohols from alkenes, and it follows the syn addition of boron and hydrogen across the double bond, followed by the formation of an alcohol after oxidation.
3. Reaction of alkyl halide with aqueous NaOH:
This is a nucleophilic substitution reaction, where an alkyl halide reacts with aqueous NaOH to form an alcohol.
The reaction proceeds as:

$$\text{R-X} + \text{NaOH} \rightarrow \text{R-OH} + \text{NaX}$$

This is a correct method for alcohol preparation, as the hydroxide ion acts as a nucleophile and displaces the halide ion to form an alcohol.
4. Reaction of Ketone with RMgBr followed by hydrolysis:
This is a reaction of a ketone with a Grignard reagent (RMgBr), which adds to the carbonyl carbon, followed by hydrolysis to form an alcohol.
The reaction proceeds as:

$$R_2C = O + \text{RMgBr} \xrightarrow{H_2O} R_2C(OH)R$$

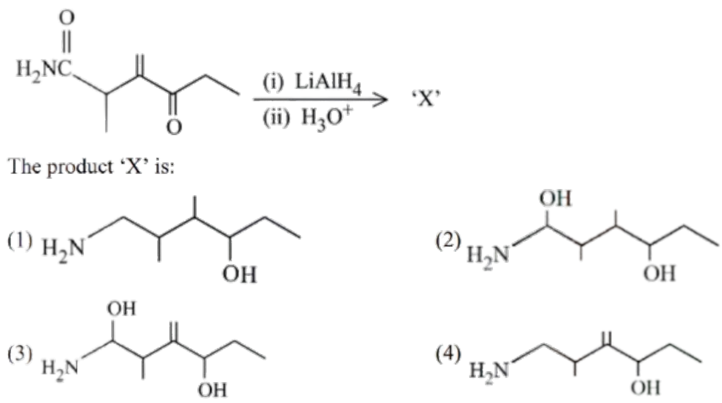
This is a correct method for preparing alcohols, specifically secondary alcohols, from ketones.
Thus, the only incorrect method for preparing alcohols is ozonolysis of alkene.

Quick Tip

When preparing alcohols, methods like hydroboration-oxidation and nucleophilic substitution of alkyl halides are correct. Ozonolysis, however, is not suitable as it forms carbonyl compounds instead of alcohols.

---

75. In the reaction given below:



The product 'X' is:

(1) $H_2N - CH_2 - CH_2OH$

(2) $H_2N - CH_2 - OH$

(3) $H_2N - CH_3$

(4) $H_2N - CH_2OH$

Correct Answer: (4) $H_2N - CH_2OH$

Solution:

Step 1: Analyzing the given reaction.

The given reaction involves the reduction of a molecule containing a carbonyl group (C=O) and an amide group ($-NH_2$) using lithium aluminum hydride ($LiAlH_4$), followed by hydrolysis with H_3O^+.

1. Step 1: Reduction with $LiAlH_4$

- Lithium aluminum hydride ($LiAlH_4$) is a strong reducing agent. When it reacts with a carbonyl compound (such as a ketone or aldehyde), it reduces the carbonyl group to a primary or secondary alcohol.
- In this case, the carbonyl group (C=O) of the given compound will be reduced to a hydroxyl group (-OH), resulting in an intermediate amide being reduced to a primary amine group ($-NH_2$). The structure of the intermediate product will be:

$$H_2NC - CH_2 - CH_2OH$$

where the ketone group is reduced to an alcohol group.

2. Step 2: Hydrolysis with H_3O^+

- After reduction, the product is treated with an acidic solution (H_3O^+), which will hydrolyze the intermediate and result in a final product where the amide group has been converted to an amine group ($-NH_2$) attached to a hydroxyl group (-OH). This confirms the product as:

$$H_2N - CH_2OH$$

This is the final product, and it is a primary amine with a hydroxyl group attached to the adjacent carbon.

Thus, the product 'X' is $H_2N - CH_2OH$.

> **Quick Tip**
>
> $LiAlH_4$ is a strong reducing agent that reduces carbonyl compounds to alcohols. When used with an amide, it reduces the carbonyl group, and hydrolysis with H_3O^+ provides the final product.

---

76. Given below are two statements, one is labelled as Assertion A and the other is labelled as Reason R. Assertion A: The energy required to form Mg^{2+} from Mg is much higher than that required to produce Mg^+. Reason R: Mg^{2+} is a small ion and carries more charge than Mg^+.
In the light of the above statements, choose the correct answer from the options given below:
(1) Both A and R are true and R is the correct explanation of A
(2) A is true but R is false
(3) A is false but R is true
(4) Both A and R are true but R is NOT the correct explanation of A
Correct Answer: (1) Both A and R are true and R is the correct explanation of A
Solution:
Step 1: Understanding Assertion A
The formation of Mg^{2+} requires the removal of two electrons, whereas the formation of Mg^+ involves the removal of only one electron. Since removing more electrons requires more energy, the energy needed to form Mg^{2+} is significantly higher than that required for Mg^+.
Step 2: Understanding Reason R
Reason R is correct because the smaller size of Mg^{2+} results in a higher charge density, which makes it more stable but also harder to form compared to Mg^+.
Therefore, both Assertion A and Reason R are true, and Reason R provides the correct explanation for Assertion A.

> **Quick Tip**
>
> When considering ion formation, remember that the greater the charge and smaller the ion, the more energy is required to form the ion due to the stronger electrostatic forces present in small, highly charged ions.

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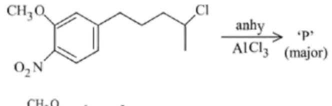
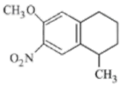
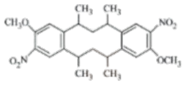
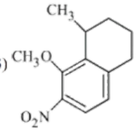
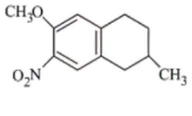
77. The major product 'P' formed in the given reaction is:



(1) 

(2) 

(3) 

(4) 

Correct Answer: (1) The structure in option (1)

Solution:
The given reaction is an electrophilic aromatic substitution. When the compound reacts with $AlCl_3$, a Friedel-Crafts alkylation or acylation typically takes place. The compound contains both a nitro group (NO_2) and a methoxy group (OCH_3) as substituents.
The nitro group is electron-withdrawing and deactivates the aromatic ring towards electrophilic substitution, while the methoxy group is electron-donating and activates the ring. Therefore, the reaction will predominantly occur at the position where the methoxy group is located, as it makes the ring more reactive.
As a result, the major product will be the structure shown in option (1), where substitution occurs at the position activated by the methoxy group.

Quick Tip

In electrophilic aromatic substitution reactions, the site of substitution depends on the electron-donating or electron-withdrawing effects of the substituents. Electron-donating groups, like OCH_3, direct substitution to the ortho/para positions, whereas electron-withdrawing groups, like NO_2, direct it to the meta position.

---

78. Ferric chloride is applied to stop bleeding because -
(1) Blood absorbs $FeCl_3$ and forms a complex.
(2) $FeCl_3$ reacts with the constituents of blood which is a positively charged sol.
(3) Fe^{3+} ions coagulate blood which is a negatively charged sol.
(4) Cl^- ions cause coagulation of blood.
Correct Answer: (3) Fe^{3+} ions coagulate blood which is a negatively charged sol.
Solution:
Step 1: Understanding the coagulation process
Blood exists as a negatively charged sol. When ferric chloride ($FeCl_3$) is introduced, the Fe^{3+} ions interact with the negatively charged blood sol, causing it to coagulate. This occurs due to the electrostatic attraction between the positively charged Fe^{3+} ions and the negatively charged blood particles.
Step 2: The role of Fe^{3+} ions
The Fe^{3+} ions neutralize the negative charge on the blood particles, leading to their aggregation and coagulation. This explains why Fe^{3+} ions are responsible for facilitating the coagulation of blood.

Quick Tip

Ferric chloride ($FeCl_3$) acts as a coagulant due to the high charge density of Fe^{3+} ions, which neutralize the negative charge on blood particles, causing them to aggregate.

---

79. The delicate balance of CO_2 and O_2 is NOT disturbed by
(1) Burning of Coal
(2) Deforestation
(3) Burning of petroleum

(4) Respiration
Correct Answer: (4) Respiration
Solution:
Step 1: Understanding the balance of CO_2 and O_2
The equilibrium between carbon dioxide (CO_2) and oxygen (O_2) in the atmosphere is primarily regulated by the process of photosynthesis in plants, which absorbs CO_2 and releases O_2. Conversely, respiration by both plants and animals releases CO_2 and consumes O_2.
Step 2: Identifying the correct process
While activities such as coal burning, deforestation, and petroleum combustion disrupt the delicate balance of CO_2 and O_2, respiration does not. Respiration is a natural process where the consumption of O_2 and the release of CO_2 are in balance, ensuring that atmospheric levels remain stable.
Therefore, respiration does not disturb the delicate balance.

Quick Tip

Photosynthesis and respiration are fundamental in maintaining the natural balance of CO_2 and O_2 in the atmosphere. Human activities, such as burning fossil fuels and deforestation, have a much larger effect on this balance.

---

80. Given below are two statements, one is labelled as Assertion A and the other is labelled as Reason R.
Assertion A: 3.1500 g of hydrated oxalic acid dissolved in water to make 250.0 mL solution will result in 0.1M oxalic acid solution.
Reason R: Molar mass of hydrated oxalic acid is 126 g mol^{-1}
In the light of the above statements, choose the correct answer from the options given below:
(1) A is false but R is true
(2) A is true but R is false
(3) Both A and R are true but R is NOT the correct explanation of A
(4) Both A and R are true and R is the correct explanation of A
Correct Answer: (4) Both A and R are true and R is the correct explanation of A
Solution:
Step 1: Verifying the Assertion A
Given mass of hydrated oxalic acid = 3.1500 g
Molar mass of hydrated oxalic acid = 126 g/mol
Volume of solution = 250.0 mL = 0.250 L
To calculate molarity (M), we use the formula:

$$M = \frac{\text{moles of solute}}{\text{volume of solution in liters}}$$

Moles of solute:

$$\text{moles} = \frac{3.1500\,\text{g}}{126\,\text{g/mol}} = 0.0250\,\text{mol}$$

Thus, molarity:

$$M = \frac{0.0250\,\text{mol}}{0.250\,\text{L}} = 0.1\,\text{M}$$

So, Assertion A is correct.

Step 2: Verifying the Reason R

The molar mass of hydrated oxalic acid is indeed 126 g/mol, as given in the question. This confirms that Reason R is correct.

Thus, both Assertion A and Reason R are true, and Reason R explains Assertion A.

> Quick Tip
>
> To find the molarity of a solution, remember to first calculate the moles of solute (using the molar mass) and then divide by the volume of the solution in liters.

SECTION-B

81. The number of molecules from the following which contain only two lone pair of electrons is:

$$H_2O, N_2, CO, XeF_4, NH_3, NO, CO_2, F_2$$

Correct Answer: 4

Solution:

Analyzing the lone pairs of electrons in the given molecules.

- H_2O: Oxygen has 2 lone pairs, and each hydrogen atom has 0 lone pairs. Total = 2 lone pairs.
- N_2: Nitrogen in N_2 has no lone pairs in the molecular structure. Total = 0 lone pairs.
- CO: Carbon in CO has no lone pairs, but oxygen has 2 lone pairs. Total = 2 lone pairs.
- XeF_4: Xenon in XeF_4 has 2 lone pairs, and each fluorine atom has 3 lone pairs. Total = 2 lone pairs.
- NH_3: Nitrogen in NH_3 has 1 lone pair, and each hydrogen atom has 0 lone pairs. Total = 1 lone pair.
- NO: Nitrogen in NO has 1 lone pair, and oxygen has 2 lone pairs. Total = 3 lone pairs.
- CO_2: Carbon in CO_2 has no lone pairs, and oxygen has 2 lone pairs on each oxygen atom. Total = 4 lone pairs.
- F_2: Each fluorine atom in F_2 has 3 lone pairs. Total = 3 lone pairs.

From the analysis, the molecules that contain only 2 lone pairs are H_2O, CO, and XeF_4. Therefore, the correct answer is 4 molecules.

> Quick Tip
>
> To determine the number of lone pairs, remember that the lone pairs are those electrons not involved in bonding. Count the valence electrons on the atoms and subtract the bonding electrons to find the lone pairs.

---

82. The specific conductance of 0.0025M acetic acid is 5×10^{-5} S cm^{-1} at a certain temperature. The dissociation constant of acetic acid is ______ $\times 10^{-7}$. (Nearest integer)

Consider limiting molar conductivity of CH_3COOH as 400 S cm^2 mol^{-1}.

Correct Answer: 66×10^{-7}

Solution: We are given:

$$k = 5 \times 10^{-5}\,\text{S cm}^{-1}, \quad C = 0.0025\,\text{M}, \quad \Lambda_{\text{m}}\,(\text{limiting molar conductivity}) = 400\,\text{S cm}^2\text{mol}^{-1}$$

Step 1: Calculate molar conductivity

$$\Lambda_{\text{m}} = \frac{k \times 1000}{C}$$

Substitute the given values:

$$\Lambda_{\text{m}} = \frac{5 \times 10^{-5} \times 1000}{0.0025} = \frac{5 \times 10^{-2}}{2.5 \times 10^{-3}} = 20\,\text{S cm}^2\text{mol}^{-1}$$

Step 2: Degree of dissociation

$$\alpha = \frac{20}{400} = \frac{1}{20}$$

Step 3: Calculate dissociation constant K_a

$$K_a = \frac{C\alpha^2}{1-\alpha}$$

Substitute the values:

$$K_a = \frac{0.0025 \times \left(\frac{1}{20}\right)^2}{1 - \frac{1}{20}} = \frac{0.0025 \times \frac{1}{400}}{\frac{19}{20}} = \frac{0.0025 \times 10^{-6}}{19/20} = 66 \times 10^{-7}$$

Thus, the dissociation constant K_a is 66×10^{-7}.

Quick Tip

To calculate the dissociation constant from specific conductivity, use the formula $K_a = \frac{C\alpha^2}{1-\alpha}$, where α is the degree of dissociation and C is the molarity of the solution.

---

83. An aqueous solution of volume 300 cm^3 contains 0.63 g of protein. The osmotic pressure of the solution at 300 K is 1.29 mbar. The molar mass of the protein is ______ g mol^{-1}.

Given: $R = 0.083\,\text{L bar K}^{-1}\text{mol}^{-1}$

Solution: Given:

- Volume (V) = 300 cm^3 = 0.3 L
- Mass (m) = 0.63 g
- Osmotic pressure (π) = 1.29 mbar = 1.29×10^3 bar
- Temperature (T) = 300 K

- R = 0.083 L bar K¹ mol¹

Solution: Using the formula: $\pi = cRT$, where c is the molarity.

1. Calculate molarity (c):

$$c = \frac{\pi}{RT} = \frac{1.29 \times 10^{-3} \text{ bar}}{0.083 \text{ L bar K}^{-1}\text{mol}^{-1} \times 300 \text{ K}}$$

$$c \approx 5.18 \times 10^{-5} \text{ mol/L}$$

2. Calculate moles (n):

$$n = c \times V = 5.18 \times 10^{-5} \text{ mol/L} \times 0.3 \text{ L}$$

$$n \approx 1.554 \times 10^{-5} \text{ mol}$$

3. Calculate molar mass (M):

$$M = \frac{m}{n} = \frac{0.63 \text{ g}}{1.554 \times 10^{-5} \text{ mol}}$$

$$M \approx 40540 \text{ g/mol}$$

Answer: The molar mass of the protein is approximately 40540 g/mol.

Quick Tip

To calculate the molar mass from osmotic pressure, use the formula $n = \frac{\pi V}{RT}$, and then calculate molar mass as molar mass $= \frac{\text{mass}}{n}$.

---

84. The difference in the oxidation state of Xe between the oxidised product of Xe formed on complete hydrolysis of XeF_4 and XeF_4 is __________

Correct Answer: 2

Solution:

Step 1: Understanding the oxidation states of Xenon in XeF_4 and its hydrolysis product.

1. In XeF_4, Xenon is bonded to 4 fluorine atoms.

The oxidation state of xenon in XeF_4 can be calculated using the fact that the oxidation state of fluorine is -1.

$$\text{Oxidation state of Xe in XeF}_4 = 4 \times (-1) = -4 \quad \Rightarrow \quad \text{Oxidation state of Xe} = +4.$$

Therefore, the oxidation state of Xe in XeF_4 is +4.

2. When XeF_4 undergoes complete hydrolysis with water, the products formed are Xenon (Xe), Xenon trioxide (XeO_3), oxygen (O_2), and hydrofluoric acid (HF):

$$XeF_4 + H_2O \rightarrow Xe + XeO_3 + O_2 + HF$$

In the product XeO_3, Xenon is bonded to three oxygen atoms. The oxidation state of oxygen is -2 in most compounds, so the oxidation state of Xenon can be determined as follows:

$$\text{Oxidation state of Xe in } XeO_3 = 3 \times (-2) = -6 \quad \Rightarrow \quad \text{Oxidation state of Xe} = +6.$$

Therefore, the oxidation state of Xenon in XeO_3 is +6.

Step 2: Calculating the difference in oxidation state of Xe.

The oxidation state of Xenon in XeF_4 is +4, and in XeO_3 it is +6. The difference in oxidation state is:

$$6 - 4 = 2$$

Thus, the difference in oxidation state of Xe between XeF_4 and its oxidized product XeO_3 is 2.

> Quick Tip
>
> To calculate the oxidation state of an element in a compound, balance the oxidation states of the atoms in the compound and use the known oxidation states of other elements. For example, in XeF_4, knowing that fluorine has an oxidation state of -1 allows us to deduce that Xenon must have an oxidation state of +4.

---

85. The number of endothermic process/es from the following is

(A) I_2 (g) $\rightarrow$ 2I (g)

(B) HCl (g) $\rightarrow$ H (g) + Cl (g)

(C) H_2O (l) $\rightarrow$ H_2O (g)

(D) C (s) + O_2 (g) $\rightarrow$ CO_2 (g)

(E) Dissolution of ammonium chloride in water

Correct Answer: 4

Solution: (A) I_2 (g) $\rightarrow$ 2I (g) is an endothermic process (Atomisation).

(B) HCl (g) $\rightarrow$ H (g) + Cl (g) is an endothermic process (Atomisation).

(C) H_2O (l) $\rightarrow$ H_2O (g) is an endothermic process (Vaporisation).

(D) C (s) + O_2 (g) $\rightarrow$ CO_2 (g) is an exothermic process (Combustion).

(E) Dissolution of ammonium chloride in water is an endothermic process (Dissolution).

Thus, the number of endothermic processes is 4.

> Quick Tip
>
> Endothermic reactions absorb heat, while exothermic reactions release heat. Examples of endothermic processes include atomisation, vaporisation, and dissolution in certain cases.

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86. The number of incorrect statement/s from the following is

(A) The successive half lives of zero order reactions decreases with time.

(B) A substance appearing as reactant in the chemical equation may not affect the rate of reaction.

(C) Order and molecularity of a chemical reaction can be a fractional number.

(D) The rate constant units of zero and second order reaction are mol L^{-1} s^{-1} and mol^{-1} L s^{-1} respectively.

Correct Answer: 1
Solution:
Let's evaluate each statement individually:
Statement (A):
For zero-order reactions, the successive half-lives decrease as time progresses. - The half-life for a zero-order reaction is given by the formula:

$$t_{1/2} = \frac{[A]_0}{2K}$$

where $[A]_0$ represents the initial concentration of the reactant and K is the rate constant. Since the concentration of the reactant decreases over time, the half-life also diminishes. This confirms that the statement is correct.
Statement (B):
A substance that appears as a reactant in the chemical equation may not necessarily influence the rate of reaction. - This statement is true because the order of reaction with respect to a substance does not always align with its stoichiometric coefficient in the equation. A substance can appear in the equation but have a zero-order effect on the reaction, meaning it does not alter the rate. For instance, in a zero-order reaction with respect to a substance, varying its concentration does not affect the rate of the reaction. Hence, this statement is correct.
Statement (C):
Order and molecularity of a chemical reaction can both be fractional numbers. - The order of a reaction refers to the powers of the concentration terms in the rate law and can indeed be fractional, as observed in some reactions. However, molecularity, which refers to the number of reacting particles in an elementary step, must always be a whole number. This is because molecularity counts the species involved in an elementary reaction step. Therefore, this statement is incorrect as molecularity cannot be fractional.
Statement (D): The rate constant units for zero and second-order reactions are mol L^{-1} s^{-1} and mol^{-1} L s^{-1} respectively. - In zero-order reactions, the rate law is:

$$\text{Rate} = k[A]^0 = k$$

The unit for rate is mol L^{-1} s^{-1}, and the unit of the rate constant k is mol L^{-1} s^{-1}. For second-order reactions, the rate law is:

$$\text{Rate} = k[A]^2$$

The unit for rate is mol L^{-1} s^{-1}, and the unit of the rate constant k is mol^{-1} L s^{-1}. Thus, this statement is correct.
Conclusion:
Statement (A) is correct.
Statement (B) is correct.
Statement (C) is incorrect.
Statement (D) is correct.
Therefore, the number of incorrect statements is 1, and the incorrect statement is (C).

Quick Tip

- In zero-order reactions, the half-life decreases as the reactant concentration decreases.
- Molecularity refers to the number of reactant molecules involved in an elementary reaction and is always a whole number, while reaction order can be fractional.
- The units of the rate constant depend on the reaction order: for zero-order reactions, the unit is mol L^{-1} s^{-1}, and for second-order reactions, it is mol^{-1} L s^{-1}.

---

87. The electron in the nth orbit of Li^{2+} is excited to $(n+1)$th orbit using the radiation of energy 1.47×10^{-17} J. The value of n is __________.
Given: $R_H = 2.18 \times 10^{-18}$ J
Correct Answer: 1
Solution:
Step 1: Using the formula for the energy difference between two orbits.
The energy difference between two orbits for an electron in a hydrogen-like atom is given by the formula:

$$\Delta E = R_H Z^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

where ΔE is the energy difference, R_H is the Rydberg constant, Z is the atomic number (which is 3 for Li^{2+}), n_1 is the initial orbit, and n_2 is the final orbit.
Step 2: Applying the given data.
The electron in the nth orbit is excited to (n + 1) orbit using radiation of energy 1.47×10^{-17} J. Therefore, $\Delta E = 1.47 \times 10^{-17}$ J. We can substitute the known values into the formula:

$$1.47 \times 10^{-17} = 2.18 \times 10^{-18} \times 9 \left(\frac{1}{n^2} - \frac{1}{(n+1)^2} \right)$$

$$\frac{1.47}{1.96} = \frac{3}{4} = \frac{1}{n^2} - \frac{1}{(n+1)^2}$$

Step 3: Solving for n.
Solving the equation, we find that $n = 1$.
Thus, the value of n is 1.

Quick Tip

To find the value of n in such problems, use the energy difference formula and apply the given energy. Solving the equation will give the value of n.

---

88. For a metal ion, the calculated magnetic moment is 4.90 BM. This metal ion has _______ number of unpaired electrons.
Correct Answer: 4
Solution:
Step 1: Using the formula for the magnetic moment.
The magnetic moment (μ) is related to the number of unpaired electrons (n) by the formula:

$$\mu = \sqrt{n(n+2)}$$

where μ is the magnetic moment in Bohr Magneton (BM) and n is the number of unpaired electrons.

Step 2: Substituting the given value of magnetic moment.

We are given that $\mu = 4.90\,\text{BM}$. Substituting this value into the formula:

$$4.90 = \sqrt{n(n+2)}$$

Step 3: Solving for n.

Squaring both sides:

$$(4.90)^2 = n(n+2)$$
$$24.01 = n(n+2)$$

Expanding the equation:

$$24.01 = n^2 + 2n$$

Rearranging the equation:

$$n^2 + 2n - 24.01 = 0$$

Solving this quadratic equation for n gives:

$$n = 4$$

Thus, the metal ion has 4 unpaired electrons.

> Quick Tip
>
> To find the number of unpaired electrons, use the formula for the magnetic moment and solve for n. The formula $\mu = \sqrt{n(n+2)}$ helps determine the number of unpaired electrons based on the magnetic moment.

---

89. In alkaline medium, the reduction of permanganate anion involves a gain of ____ electrons.

Solution:

The reduction of permanganate anion (MnO_4^-) in alkaline medium involves the following process:

$$MnO_4^- \text{ (oxidation state of Mn} = +7) \rightarrow Mn^{4+} \text{ (oxidation state of Mn} = +4)$$

The reduction from Mn^{7+} to Mn^{4+} involves the gain of 3 electrons, as the change in oxidation number is from +7 to +4. Therefore, the reduction of permanganate anion involves the gain of 3 electrons.

> Quick Tip
>
> The number of electrons involved in the reduction process corresponds to the change in the oxidation state of the element. In this case, Mn changes from +7 to +4, which requires the gain of 3 electrons.

90.

$$A(g) \rightleftharpoons 2B(g) + C(g)$$

For the given reaction, if the initial pressure is 450 mmHg and the pressure at time t is 720 mmHg at a constant temperature T and constant volume V. The fraction of A(g) decomposed under these conditions is $x \times 10^{-1}$. The value of x is _______ (nearest integer)

Solution:

The reaction is given as:

$$A(g) \rightleftharpoons 2B(g) + C(g)$$

At time $t = 0$, the pressure of A is 450 mmHg, and at time $t = t$, the total pressure is 720 mmHg.

Let the extent of decomposition at time t be $2x$, so that the pressures of A, B, and C at time t are:

Pressure of A $= 450 - x$

Pressure of B $= 2x$

Pressure of C $= x$

Thus, the total pressure at time t is:

$$P_t = P_A + P_B + P_C = (450 - x) + 2x + x = 720\,\text{mmHg}$$

Now, solving for x:

$$720 = 450 - x + 2x + x$$

$$720 = 450 + 2x$$

$$270 = 2x$$

$$x = 135$$

The fraction of A decomposed is:

$$\text{Fraction of A decomposed} = \frac{x}{450} = \frac{135}{450} = 0.3 = 3 \times 10^{-1}$$

Thus, the value of x is 3.

Quick Tip

For reactions involving changes in pressure, the change in pressure can be used to determine the extent of reaction. Here, the total pressure is related to the individual pressures of reactants and products at equilibrium.