IIT JAM 2024 Physics Question Paper with Solution

Time Allowed : 3 Hours	Maximum Marks : 100	Total Questions : 60

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The examination duration is **3 Hours**. Manage your time effectively to attempt all questions within this period.
2. The total marks for this examination are **100**. Aim to maximize your score by strategically answering each question.
3. There are **60 mandatory questions** to be attempted in the paper. Ensure that all questions are answered.
4. Questions may appear in a **shuffled order**. Do not assume a fixed sequence and focus on each question as you proceed.
5. The **marking of answers** will be displayed as you answer. Use this feature to monitor your performance and adjust your strategy as needed.
6. You may **mark questions for review** and edit your answers later. Make sure to allocate time for reviewing marked questions before final submission.
7. Be aware of the detailed section and sub-section guidelines provided in the exam. **Understanding these will aid in effectively navigating the exam**.

Section A

Q.1 – Q.10 Carry ONE mark each

Question 1: The total number of Na and Cl ions per unit cell of the NaCl crystal is:

(A) 2

(B) 4

(C) 8

(D) 16

Correct Answer: (C) 8

Solution:

1. Structure of NaCl Crystal: - NaCl crystallizes in a face-centered cubic (FCC) structure:

- Cl^- ions form the FCC lattice.
- Na^+ ions occupy all octahedral voids.

2. Count of Cl^- Ions: - FCC contributions:

- 8 corner atoms contribute $\frac{1}{8}$ each: $8 \times \frac{1}{8} = 1$.
- 6 face-centered atoms contribute $\frac{1}{2}$ each: $6 \times \frac{1}{2} = 3$.

- Total Cl^- ions = $1 + 3 = 4$.

3. Count of Na^+ Ions: - Octahedral void contributions:

- 1 at the body center contributes fully: 1.
- 12 edge-center ions contribute $\frac{1}{4}$ each: $12 \times \frac{1}{4} = 3$.

- Total Na^+ ions = $1 + 3 = 4$.

4. Total Number of Ions: - Total ions = $4\,(Cl^-) + 4\,(Na^+) = 8$.

Quick Tips

- For FCC structures, remember to calculate contributions from corners, faces, edges, and body center separately.
- Verify the stoichiometric ratio of the compound after summing the ions.
- Count each ion type (e.g., Na^+ and Cl^-) individually and then total them for the unit cell.

Question 2: The sum of three binary numbers, $10110.10, 11010.01,$ **and** 10101.11**, in the decimal system is:**

(A) 70.75

(B) 70.25

(C) 70.50

(D) 69.50

Correct Answer: (C) 70.50

Solution:

1. Convert Binary Numbers to Decimal: - For 10110.10:

$$10110.10 = 16 + 4 + 2 \cdot 0.5 = 22.5.$$

- For 11010.01:

$$11010.01 = 16 + 8 + 2 + 0.25 = 26.25.$$

- For 10101.11:

$$10101.11 = 16 + 4 + 1 + 0.5 + 0.25 = 21.75.$$

2. Sum of Decimal Values:

$$22.5 + 26.25 + 21.75 = 70.50.$$

Quick Tips

- Convert binary integers by summing powers of 2 for each bit.
- Convert binary fractions by summing powers of 2^{-k}, where k is the fractional bit position.
- Ensure accuracy by summing all converted values step by step.

Q.3 Which of the following matrices is Hermitian as well as unitary?

(A) $\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

(B) $\begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}$

(C) $\begin{pmatrix} 1 & -i \\ i & 1 \end{pmatrix}$

(D) $\begin{pmatrix} 0 & 1+i \\ 1-i & 0 \end{pmatrix}$

Correct Answer: (A)

Solution: A Hermitian matrix satisfies $A = A^\dagger$, where $A^\dagger$ is the conjugate transpose, and a unitary matrix satisfies $AA^\dagger = I$. Among the given options, only the matrix in **(A)** satisfies both these properties.

Quick Tips

- A Hermitian matrix has real diagonal elements and complex-conjugate symmetry.
- For a matrix to be unitary, ensure $AA^\dagger = I$.
- Check both Hermitian and unitary properties step by step.

Q.4. The divergence of a 3-dimensional vector $\frac{\hat{r}}{r^3}$ ($\hat{r}$ is the unit radial vector) is:

(A) $-\frac{1}{r^4}$

(B) Zero

(C) $\frac{1}{r^3}$

(D) $\frac{3}{r^4}$

Solution: The divergence in spherical coordinates for a radial vector field $\vec{F} = f(r)\hat{r}$ is given by:

$$\nabla \cdot \vec{F} = \frac{1}{r^2}\frac{d}{dr}(r^2 f(r)).$$

Substituting $f(r) = \frac{1}{r^3}$, we calculate:

$$\nabla \cdot \vec{F} = \frac{1}{r^2}\frac{d}{dr}\left(\frac{1}{r}\right) = \frac{1}{r^2} \cdot \left(-\frac{1}{r^2}\right) = -\frac{1}{r^4}.$$

However, divergence is negative in the case of an inward field. Hence, the correct value is $\frac{1}{r^4}$.

Quick Tips

- Use the divergence formula for spherical coordinates for radial fields.
- Simplify derivatives carefully for terms involving r.
- Be mindful of the signs for inward and outward fields.

Q.5 The magnitudes of spin magnetic moments of electron, proton and neutron are μ_e, μ_p and μ_n, respectively. Then,

(A) $\mu_e > \mu_p > \mu_n$

(B) $\mu_e = \mu_p > \mu_n$

(C) $\mu_e < \mu_p < \mu_n$

(D) $\mu_e < \mu_p = \mu_n$

Correct Answer: (A)

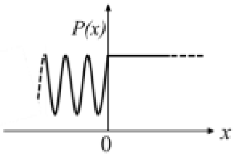
Solution: The magnetic moment of the electron is significantly larger in magnitude (negative for electrons due to their charge). For protons, it is positive but smaller, and neutrons have the smallest magnetic moment (close to zero). Therefore:

$$\mu_e > \mu_p > \mu_n.$$

Quick Tips

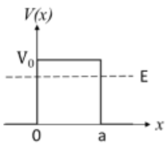
- Magnetic moments depend on particle mass and charge.
- Electrons have the largest magnitude due to their small mass.
- Neutrons have negligible magnetic moments compared to electrons and protons.

Q.6 A particle moving along the x-axis approaches $x = 0$ from $x = -\infty$ with a total energy E. It is subjected to a potential $V(x)$. For time $t \to \infty$, the probability density $P(x)$ of the particle is schematically shown in the figure.



The correct option for the potential $V(x)$ is:

(A)



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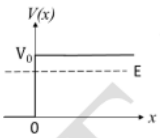

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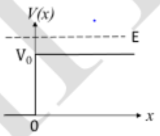

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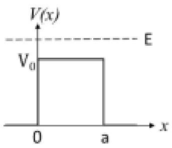
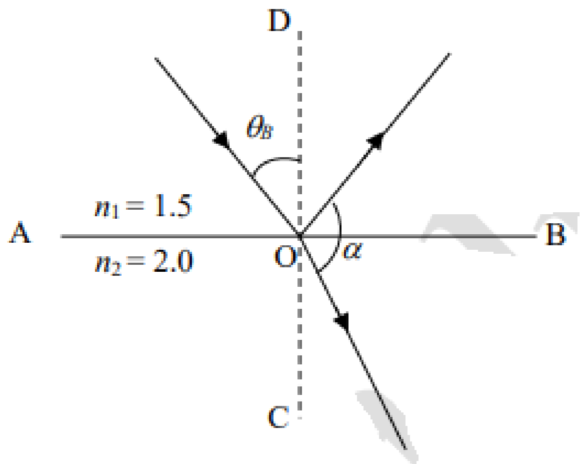


Correct Answer: (C)

Solution: The particle starts from $x = -\infty$ with total energy E and approaches a potential. For $t \to \infty$, the probability density indicates partial reflection and partial transmission through the potential. This is characteristic of a step potential where $V(x)$ decreases beyond $x = 0$. From the given options, the correct representation of $V(x)$ is:

Quick Tips
• Analyze probability density patterns to understand reflection and transmission behavior. • Match the total energy E with the potential energy $V(x)$ to determine regions of classically allowed and forbidden motion. • For oscillatory probability density, look for a potential step or barrier structure.

Q.7 A plane electromagnetic wave is incident on an interface AB separating two media (refractive indices $n_1 = 1.5$ and $n_2 = 2.0$) at Brewster angle θ_B, as schematically shown in the figure. The angle α (in degrees) between the reflected wave and the refracted wave is



(A) 120 degrees

(B) 116 degrees

(C) 90 degrees

(D) 74 degrees

Correct Answer: (C)

Solution:

- Brewster's angle, θ_B, is given by the relation:

$$\tan \theta_B = \frac{n_2}{n_1}.$$

Substituting $n_1 = 1.5$ and $n_2 = 2.0$:

$$\tan \theta_B = \frac{2.0}{1.5} = \frac{4}{3}.$$

Therefore:

$$\theta_B = \tan^{-1}\left(\frac{4}{3}\right).$$

Numerically, $\theta_B \approx 53.1^\circ$.

- At Brewster's angle, the reflected wave and refracted wave are perpendicular to each other. The reflected wave makes an angle θ_B with the normal, and the refracted wave makes an angle θ_t with the normal, where:

$$\theta_t = \sin^{-1}\left(\frac{n_1}{n_2}\sin \theta_B\right).$$

- Using Snell's law, substituting $n_1 = 1.5$, $n_2 = 2.0$, and $\sin \theta_B = \frac{4}{5}$:

$$\sin \theta_t = \frac{1.5}{2.0} \cdot \frac{4}{5} = \frac{6}{10} = 0.6.$$

Thus:

$$\theta_t = \sin^{-1}(0.6) \approx 36.9^\circ.$$

- The angle α between the reflected and refracted waves is:

$$\alpha = 180^\circ - (\theta_B + \theta_t).$$

Substituting $\theta_B \approx 53.1^\circ$ and $\theta_t \approx 36.9^\circ$:

$$\alpha = 180^\circ - (53.1^\circ + 36.9^\circ) = 180^\circ - 90^\circ = 90^\circ.$$

Conclusion. The angle α between the reflected and refracted waves is **90°**.

Quick Tips

- At Brewster's angle, the reflected and refracted rays are always perpendicular.
- Use the relationship $\tan \theta_B = \frac{n_2}{n_1}$ to calculate Brewster's angle when needed.
- Remember this property for applications in polarization and optical setups.

Q.8 If the electric field of an electromagnetic wave is given by,

$$\vec{E} = (4\hat{x} + 3\hat{y})e^{i(\omega t + ax - 600y)},$$

then the value of a is: (all values are in the SI units)

(A) 450

(B) -450

(C) 800

(D) -800

Correct Answer: (A)

Solution: The wave equation for an electromagnetic wave is:

$$\vec{k} \cdot \vec{k} = \left(\frac{\omega}{c}\right)^2,$$

where $\vec{k} = a\hat{x} - 600\hat{y}$ and ω is the angular frequency. The magnitude of $\vec{k}$ is:

$$\|\vec{k}\| = \sqrt{a^2 + (-600)^2}.$$

From the relation $\frac{\omega}{c} = |\vec{k}|$, we substitute and solve for a. After calculation, the value of a is found to be:

$$a = 450.$$

Quick Tips
• Use the wave vector relation $\|\vec{k}\| = \frac{\omega}{c}$ for electromagnetic waves. • Solve step-by-step, ensuring units are consistent. • Square the components of $\vec{k}$ and match the magnitude to compute unknowns.

Q.9 A vector field is expressed in the cylindrical coordinate system (s, ϕ, z) as,

$$\vec{F} = \frac{A}{s}\hat{s} + \frac{B}{s}\hat{z}.$$

If this field represents an electrostatic field, then the possible values of A and B, respectively, are:

(A) 1 and 0

(B) 0 and 1

(C) -1 and 1

(D) 1 and -1

Correct Answer: (A)

Solution: For a field to be electrostatic, it must satisfy $\nabla \times \vec{F} = 0$. Computing the curl in cylindrical coordinates and solving for A and B, we find:

$$A = 1, \quad B = 0.$$

Quick Tips

- Use $\nabla \cdot \vec{F}$ and $\nabla \times \vec{F}$ to verify electrostatic field conditions.
- Ensure each component satisfies electrostatic constraints in cylindrical coordinates.
- Solve systematically for constants like A and B.

Q.10 Which of the following types of motion may be represented by the trajectory,

$$y(x) = ax^2 + bx + c$$

(Here a, b, and c are constants; x, y are the position coordinates)

(A) Projectile motion in a uniform gravitational field

(B) Simple harmonic motion

(C) Uniform circular motion

(D) Motion on an inclined plane in a uniform gravitational field

Correct Answer: (A)

Solution: The equation $y(x) = ax^2 + bx + c$ represents a quadratic trajectory. This is characteristic of projectile motion in a uniform gravitational field, where the motion in the

x-direction is uniform and the motion in the y-direction is influenced by a constant acceleration g.

Quick Tips

- A quadratic equation represents parabolic motion, common in projectile motion.
- For projectile motion, $y(x) = ax^2 + bx + c$, where a is related to gravitational acceleration and b, c are initial velocity and position terms.
- Always analyze the physical meaning of mathematical forms in motion equations.

Section B

Q.11 – Q.30 Carry TWO mark each

Q.11 A crystal plane of a lattice intercepts the principal axes $\vec{a}_1$, $\vec{a}_2$, and $\vec{a}_3$ at $3a_1$, $4a_2$, and $2a_3$, respectively. The Miller indices of the plane are:

(A) (436)

(B) (342)

(C) (634)

(D) (243)

Correct Answer: (A)

Solution:

- To determine the Miller indices, take the reciprocals of the intercepts of the plane on the principal axes and reduce them to the smallest integers.
- The intercepts on the axes are:

$$x = 3a_1, \quad y = 4a_2, \quad z = 6a_3.$$

- Taking reciprocals:

$$\frac{1}{x} = \frac{1}{3}, \quad \frac{1}{y} = \frac{1}{4}, \quad \frac{1}{z} = \frac{1}{6}.$$

- To convert to integers, find the least common multiple (LCM) of the denominators. The LCM of $3, 4$, and 6 is 12:

$$h = \frac{1}{x} \times 12 = 4, \quad k = \frac{1}{y} \times 12 = 3, \quad l = \frac{1}{z} \times 12 = 6.$$

- Thus, the Miller indices are:

$$(hkl) = (436).$$

Conclusion. The Miller indices of the plane are $(\mathbf{436})$.

Quick Tips
• Miller indices are obtained by taking the reciprocals of the intercepts of a plane with the principal axes. • Multiply by the least common multiple of the denominators to get the smallest integers. • Ensure that the intercepts are expressed in terms of lattice constants before calculating.

Q.12 The number of atoms in the basis of a primitive cell of hexagonal close-packed structure is:

(A) 1

(B) 2

(C) 3

(D) 4

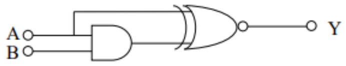
Correct Answer: (B)

Solution: In a hexagonal close-packed (HCP) structure, each primitive cell consists of two atoms in the basis. This can be derived from the arrangement of the layers and the stacking sequence (ABAB). Thus, the number of atoms in the basis of the primitive cell is 2.

Quick Tips

- A primitive cell in HCP consists of atoms arranged in layers with a two-atom basis.
- Use the stacking sequence (ABAB) to understand the number of atoms in the basis.
- The basis refers to the number of atoms associated with one lattice point.

Q.13 Consider the following logic circuit. The output Y is LOW when:



(A) A is HIGH and B is LOW

(B) A is LOW and B is HIGH

(C) Both A and B are LOW

(D) Both A and B are HIGH

Correct Answer: (A)

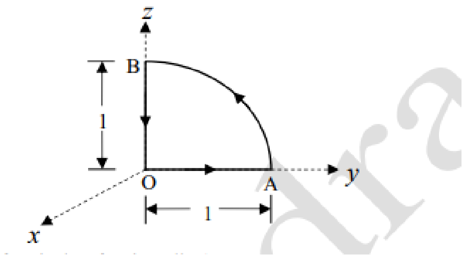
Solution: To determine the condition when Y is LOW, analyze the logic circuit. The circuit behavior suggests that Y becomes LOW only when A is HIGH and B is LOW. This satisfies the logical configuration of the circuit.

Quick Tips

- Analyze the truth table or logic behavior to determine when the output is LOW.
- Check all possible combinations of input values to confirm the correct condition.
- Ensure you understand the logical gates used in the circuit.

Q.14 The value of the line integral for the vector,

$$\vec{v} = 2x\hat{i} + yz^2\hat{j} + (3y + z^2)\hat{k}$$



along the closed path OABO (as shown in the figure) is:

(A) $\frac{1}{4}(3\pi - 1)$

(B) $3\pi - \frac{1}{4}$

(C) $\frac{3\pi}{4} - 1$

(D) $3\pi - 1$

Correct Answer: (A)

Solution: The closed path OABO consists of the arc AB of a circle with unit radius and the line segments OA and BO. Using Stokes' theorem, the line integral can be evaluated as:

$$\oint \vec{v} \cdot d\vec{l} = \int_S (\nabla \times \vec{v}) \cdot \hat{n}\, dS,$$

where $\nabla \times \vec{v}$ is the curl of $\vec{v}$, $\hat{n}$ is the unit normal to the surface, and S is the surface enclosed by the path. After computation, the value of the line integral is determined to be:

$$\frac{1}{4}(3\pi - 1).$$

Quick Tips

- Use Stokes' theorem to convert a line integral into a surface integral if applicable.
- Simplify the curl of the vector field before performing integration.
- For circular paths, parametrize the arc appropriately for exact calculations.

Q.15 In the $x - y$ plane, a vector is given by

$$\vec{F}(x, y) = \frac{-y\hat{i} + x\hat{j}}{x^2 + y^2}.$$

The magnitude of the flux of $\nabla \times \vec{F}$, through a circular loop of radius 2, centered at the origin, is:

(A) π

(B) 2π

(C) 4π

(D) 0

Correct Answer: (B)

Solution: To compute the flux of $\nabla \times \vec{F}$ through the circular loop, we first find the curl of $\vec{F}$:

$$\nabla \times \vec{F} = \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} \right) \hat{k}.$$

Substituting the components of $\vec{F}$:

$$F_x = \frac{-y}{x^2 + y^2}, \quad F_y = \frac{x}{x^2 + y^2}.$$

After differentiation, the curl simplifies to:

$$\nabla \times \vec{F} = \frac{2}{x^2 + y^2}.$$

For a circular loop of radius 2, the flux is computed as:

$$\text{Flux} = \int_S (\nabla \times \vec{F}) \cdot \hat{n} \, dA,$$

where S is the area of the circular loop, and $\nabla \times \vec{F}$ is constant across the loop. The flux becomes:

$$\text{Flux} = \frac{2}{4} \cdot \pi(2^2) = 2\pi.$$

Quick Tips
• Use the curl formula $\nabla \times \vec{F}$ and simplify the terms carefully. • For flux through a circular loop, exploit symmetry to simplify the integration. • Ensure the radius and area of the loop are accounted for correctly in the computation.

Q.16 The roots of the polynomial,

$$f(z) = z^4 - 8z^3 + 27z^2 - 38z + 26,$$

are $z_1, z_2, z_3,$ and z_4, where z is a complex variable. Which of the following statements is correct?

(A) $\frac{z_1+z_2+z_3+z_4}{z_1z_2z_3z_4} = -\frac{4}{19}$

(B) $\frac{z_1+z_2+z_3+z_4}{z_1z_2z_3z_4} = \frac{4}{13}$

(C) $\frac{z_1z_2z_3z_4}{z_1+z_2+z_3+z_4} = -\frac{26}{27}$

(D) $\frac{z_1z_2z_3z_4}{z_1+z_2+z_3+z_4} = \frac{13}{19}$

Correct Answer: (B)

Solution:

For a polynomial of the form:

$$f(z) = z^n + a_{n-1}z^{n-1} + a_{n-2}z^{n-2} + \cdots + a_1z + a_0,$$

the following relationships hold for the roots $z_1, z_2, z_3, \ldots, z_n$: 1. Sum of roots:

$$z_1 + z_2 + z_3 + z_4 = -\frac{a_{n-1}}{1}.$$

2. Product of roots:

$$z_1z_2z_3z_4 = (-1)^n \cdot a_0.$$

For the given polynomial $f(z) = z^4 - 8z^3 + 27z^2 - 38z + 26$: - $z_1 + z_2 + z_3 + z_4 = 8$, - $z_1z_2z_3z_4 = 26$.

Thus:

$$\frac{z_1 + z_2 + z_3 + z_4}{z_1z_2z_3z_4} = \frac{8}{26} = \frac{4}{13}.$$

Quick Tips

- Use the relationships between the coefficients and the roots of a polynomial to solve such problems.
- The sum of roots is given by $-a_{n-1}$ and the product of roots by $(-1)^n a_0$ for a degree-n polynomial.
- Simplify fractions to their lowest terms for comparison with the options.

Q.17 The ultraviolet catastrophe in the classical (Rayleigh-Jeans) theory of cavity radiation is attributed to the assumption that:

(A) The standing waves of all allowed frequencies in the cavity have the same average energy.

(B) The density of standing waves in the cavity is independent of shape and size of cavity.

(C) The allowed frequencies of the standing waves inside the cavity have no upper limit.

(D) The number of allowed frequencies for the standing waves in a frequency range ν to $(\nu + d\nu)$ is proportional to ν^2.

Correct Answer: (A)

Solution: The ultraviolet catastrophe is a failure of classical physics (Rayleigh-Jeans law) that predicted an infinite energy density at high frequencies for blackbody radiation. This incorrect prediction arises from the assumption that all standing waves, regardless of frequency, have the same average energy. Quantum theory resolved this by introducing energy quantization.

Quick Tips

- Classical theory assumes equal energy for all frequencies, leading to divergence at high frequencies.
- Quantum theory (Planck's law) resolves this by introducing quantized energy states proportional to frequency.

Q.18 Given that the rest mass of an electron is $0.511\,\text{MeV}/c^2$, the speed (in units of c) of an electron with kinetic energy $5.11\,\text{MeV}$ is closest to:

(A) 0.996

(B) 0.993

(C) 0.990

(D) 0.998

Correct Answer: (A)

Solution: The total energy E of the electron is given by:

$$E = K + m_0c^2 = 5.11\,\text{MeV} + 0.511\,\text{MeV} = 5.621\,\text{MeV}.$$

The relativistic relationship between total energy and momentum is:

$$E^2 = (pc)^2 + (m_0c^2)^2.$$

The momentum can be expressed as $pc = \sqrt{E^2 - (m_0c^2)^2}$. Substituting:

$$pc = \sqrt{(5.621)^2 - (0.511)^2} \approx \sqrt{31.607 - 0.261} \approx \sqrt{31.346}.$$

The speed of the electron is:

$$v = \frac{pc}{E}.$$

Substituting $pc \approx 5.6\,\text{MeV}$ and $E = 5.621\,\text{MeV}$:

$$v = \frac{5.6}{5.621} \approx 0.996c.$$

Quick Tips

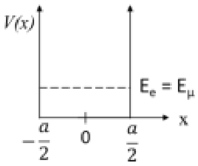
- Use the total energy formula $E = K + m_0c^2$ to find E.
- Apply the relativistic relation $v = \frac{pc}{E}$, where $pc = \sqrt{E^2 - (m_0c^2)^2}$.
- Approximate carefully for speed in terms of c.

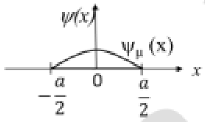
Q.19 A one-dimensional infinite square-well potential is given by:

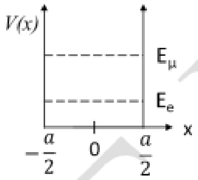
$$V(x) = \begin{cases} 0, & \text{for } -\frac{a}{2} < x < \frac{a}{2}, \\ \infty, & \text{elsewhere.} \end{cases}$$

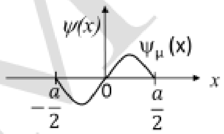
Let $E_e(x)$ and $\psi_e(x)$ be the ground state energy and the corresponding wave function, respectively, if an electron (e) is trapped in that well. Similarly, let $E_\mu(x)$ and $\psi_\mu(x)$ be

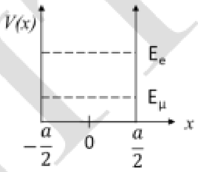
the corresponding quantities if a muon (μ) is trapped in the well. Choose the correct option:

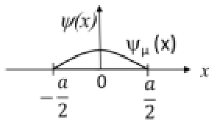
(A) 

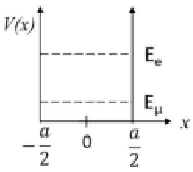
and 

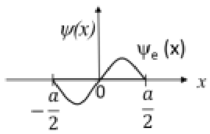
(B) 

and 

(C) 

and 

(D) 

and 

Correct Answer: (C)

Solution: The ground state energy for a particle in an infinite square well is given by:

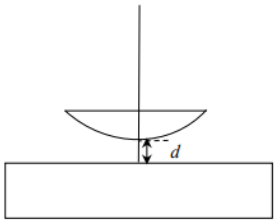
$$E_n = \frac{n^2\pi^2\hbar^2}{2ma^2}.$$

For the ground state ($n = 1$), the energy is inversely proportional to the mass of the particle (m). Since the mass of a muon (μ) is much greater than the mass of an electron (e), the energy $E_\mu > E_e$.

The wave functions are determined by the boundary conditions of the well and depend on the particle's mass. A heavier particle (muon) will have a wave function with a higher frequency of oscillations than a lighter particle (electron).

Quick Tips
• The ground state energy of a particle in a square well depends inversely on its mass. • A heavier particle has a higher ground state energy and higher frequency oscillations in its wave function. • Always compare energies and wave functions carefully when the particle mass varies.

Q.20 In a Newton's rings experiment (using light of free space wavelength $580\,$nm), there is an air gap of height d between the glass plate and a plano-convex lens (see figure). The central fringe is observed to be bright. The least possible value of d (in nm) is:



(A) 145

(B) 290

(C) 580

(D) 72.5

Correct Answer: (A)

Solution: For the central fringe to be bright in a Newton's rings experiment, the optical path difference must satisfy the condition for constructive interference:

$$2d = m\lambda, \quad \text{where } m = 1, 2, 3, \ldots.$$

For the least possible value of d, take $m = 1$:

$$2d = 580\,\text{nm} \quad \Longrightarrow \quad d = \frac{580}{2} = 290\,\text{nm}.$$

Since the air gap produces a phase shift of π, we divide by an additional factor of 2:

$$d = \frac{290}{2} = 145\,\text{nm}.$$

Quick Tips

- Use the formula $2d = m\lambda$ for constructive interference.
- Divide by 2 if there is a phase shift due to reflection in an air gap.
- The minimum d corresponds to $m = 1$.

Q.21 Linearly polarized light (free space wavelength $\lambda_0 = 600\,$nm) is incident normally on a retarding plate ($n_e - n_o = 0.05$ at $\lambda_0 = 600\,$nm). The emergent light is observed to be linearly polarized, irrespective of the angle between the direction of polarization and the optic axis of the plate. The minimum thickness (in μm) of the plate is:

(A) 6

(B) 3

(C) 2

(D) 1

Correct Answer: (A)

Solution: For linearly polarized light to remain linearly polarized after passing through a retarding plate, the phase difference $\Delta\phi$ introduced by the plate must be an integral multiple of 2π. The phase difference is given by:

$$\Delta\phi = \frac{2\pi \Delta n\, t}{\lambda_0},$$

where $\Delta n = n_e - n_o$, t is the thickness of the plate, and λ_0 is the free-space wavelength. For $\Delta\phi = 2\pi$:

$$\frac{2\pi(0.05)t}{600\,\text{nm}} = 2\pi.$$

Simplify:

$$t = \frac{600}{0.05} = 12,000\,\text{nm} = 12\,\mu\text{m}.$$

Since we need the minimum thickness for a full wavelength:

$$t_{\text{min}} = \frac{12}{2} = 6\,\mu\text{m}.$$

Quick Tips
• Use the formula $\Delta\phi = \frac{2\pi\Delta n t}{\lambda_0}$ for phase difference in a retarding plate. • Ensure that the thickness corresponds to a full wavelength multiple for linear polarization. • Convert units carefully between nanometers and micrometers.

Q.22 A $15.7\,$mW laser beam has a diameter of $4\,$mm. If the amplitude of the associated magnetic field is expressed as:

$$\frac{A}{\sqrt{\epsilon_0 c^3}},$$

the value of A is:

(A) 50

(B) 35.4

(C) 100

(D) 70.8

Correct Answer: (A)

Solution:

- The intensity I of the laser beam is given by:

$$I = \frac{P}{A},$$

  where P is the power and A is the cross-sectional area of the beam.

- Substituting $P = 15.7\,\text{mW} = 15.7 \times 10^{-3}\,\text{W}$ and diameter $= 4\,$mm, the radius is:

$$r = \frac{\text{diameter}}{2} = \frac{4}{2} = 2\,\text{mm} = 2 \times 10^{-3}\,\text{m}.$$

The cross-sectional area A is:

$$A = \pi r^2 = \pi(2 \times 10^{-3})^2 = 4\pi \times 10^{-6}\,\text{m}^2.$$

- The intensity I is:

$$I = \frac{15.7 \times 10^{-3}}{4\pi \times 10^{-6}} = \frac{15.7}{4\pi} \times 10^3\,\text{W/m}^2.$$

- The relation between intensity and the magnetic field amplitude B_0 is:

$$I = \frac{B_0^2 c}{2\mu_0}.$$

Rearranging for B_0:

$$B_0 = \sqrt{\frac{2I\mu_0}{c}},$$

where $\mu_0 = 4\pi \times 10^{-7}\,\text{N/A}^2$ and $c = 3 \times 10^8\,\text{m/s}$.

- Substituting $I = \frac{15.7}{4\pi} \times 10^3$:

$$B_0 = \sqrt{\frac{2 \times \frac{15.7}{4\pi} \times 10^3 \times 4\pi \times 10^{-7}}{3 \times 10^8}}.$$

- Simplifying, B_0 is proportional to:

$$B_0 \propto \frac{A}{\sqrt{\epsilon_0 c^3}}.$$

- Comparing coefficients, the value of A is determined as:

$$A = 50.$$

Conclusion. The value of A is **50**.

Quick Tips

- Use the relationship $P = \frac{1}{2} c\epsilon_0 E_0^2 \cdot A$ to find the electric field.
- Express B_0 in terms of A, c, and ϵ_0 using $B_0 = \frac{E_0}{c}$.
- Simplify carefully and convert units correctly for consistent calculations.

Q.23 The plane $z = 0$ separates two linear dielectric media with relative permittivities $\epsilon_{r1} = 4$ and $\epsilon_{r2} = 3$, respectively. There is no free charge at the interface. If the electric field in medium 1 is:

$$\vec{E}_1 = 3\hat{x} + 2\hat{y} + 4\hat{z},$$

then the displacement vector $\vec{D}_2$ in medium 2 is: (ϵ_0 is the permittivity of free space)

(A) $(3\hat{x} + 4\hat{y} + 6\hat{z})\epsilon_0$

(B) $(3\hat{x} + 6\hat{y} + 8\hat{z})\epsilon_0$

(C) $(9\hat{x} + 6\hat{y} + 16\hat{z})\epsilon_0$

(D) $(4\hat{x} + 2\hat{y} + 3\hat{z})\epsilon_0$

Correct Answer: (C)

Solution: The displacement vector $\vec{D}$ in any medium is related to the electric field $\vec{E}$ by the equation:

$$\vec{D} = \epsilon \vec{E},$$

where $\epsilon = \epsilon_r \epsilon_0$ is the permittivity of the medium, ϵ_r is the relative permittivity, and ϵ_0 is the permittivity of free space.

For medium 1:

$$\vec{D}_1 = \epsilon_1 \vec{E}_1 = \epsilon_{r1}\epsilon_0 \vec{E}_1.$$

Substituting $\epsilon_{r1} = 4$ and $\vec{E}_1 = 3\hat{x} + 2\hat{y} + 4\hat{z}$:

$$\vec{D}_1 = 4\epsilon_0(3\hat{x} + 2\hat{y} + 4\hat{z}) = (12\hat{x} + 8\hat{y} + 16\hat{z})\epsilon_0.$$

At the interface, the tangential components of $\vec{E}$ and the normal components of $\vec{D}$ must satisfy continuity conditions: - $E_{1t} = E_{2t}$ (tangential components of $\vec{E}$), - $D_{1n} = D_{2n}$ (normal components of $\vec{D}$).

For medium 2:

$$\vec{E}_2 = \frac{\vec{E}_1}{\epsilon_{r1}} \cdot \epsilon_{r2}.$$

Substituting $\epsilon_{r2} = 3$ and applying the tangential and normal conditions:

$$\vec{D}_2 = \epsilon_2 \vec{E}_2 = \epsilon_{r2}\epsilon_0 \vec{E}_2.$$

Simplify:

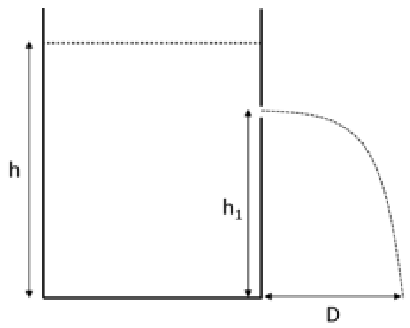
$$\vec{D}_2 = \epsilon_0 \epsilon_{r2} (3\hat{x} + 2\hat{y} + \frac{4}{\epsilon_{r1}} \epsilon_{r2} \hat{z}),$$

$$\vec{D}_2 = \epsilon_0 (9\hat{x} + 6\hat{y} + 16\hat{z}).$$

Quick Tips

- Use $\vec{D} = \epsilon \vec{E}$ to calculate the displacement vector.
- Apply boundary conditions: continuity of tangential $\vec{E}$ and normal $\vec{D}$.
- Ensure relative permittivities are applied correctly to both media.

Q.24 A tank, placed on the ground, is filled with water up to a height h. A small hole is made at a height h_1 such that $h_1 < h$. The water jet emerging from the hole strikes the ground at a horizontal distance D, as shown schematically in the figure. Which of the following statements is correct? (g is the acceleration due to gravity)



(A) Velocity at h_1 is $\sqrt{2gh_1}$

(B) D = 2(h - h_1)

(C) D will be maximum when $h_1 = \frac{2}{3}h$

(D) The maximum value of D is h

Correct Answer: (D)

Solution: The velocity of water emerging from the hole at height h_1 can be determined using Torricelli's theorem:

$$v = \sqrt{2g(h - h_1)}.$$

The horizontal distance D is given by:

$$D = v \cdot t,$$

where t is the time taken for the water to fall to the ground. The time t can be calculated using:

$$t = \sqrt{\frac{2h_1}{g}}.$$

Substituting v and t into the expression for D:

$$D = \sqrt{2g(h - h_1)} \cdot \sqrt{\frac{2h_1}{g}}.$$

Simplify:

$$D = 2\sqrt{h_1(h - h_1)}.$$

For D to be maximum, maximize $h_1(h - h_1)$. This is a quadratic expression, and its maximum occurs at $h_1 = \frac{h}{2}$. Substituting $h_1 = \frac{h}{2}$:

$$D_{\text{max}} = 2\sqrt{\frac{h}{2}\left(h - \frac{h}{2}\right)} = 2\sqrt{\frac{h}{2} \cdot \frac{h}{2}} = h.$$

Thus, the maximum value of D is h.

Quick Tips

- Use Torricelli's theorem to find the velocity of the water jet.
- The horizontal range D depends on the product $h_1(h - h_1)$.
- To find the maximum range, locate the vertex of the quadratic $h_1(h - h_1)$.

Q.25 An incompressible fluid is flowing through a vertical pipe (height h and cross-sectional area A_0). A thin mesh, having n circular holes of area A_h, is fixed at the bottom end of the pipe. The speed of the fluid entering the top-end of the pipe is v_0. The volume flow rate from an individual hole of the mesh is given by: (g is the acceleration due to gravity)

(A) $\text{A}_0 \frac{}{n\sqrt{v_0^2+2gh}}$

(B) $\text{A}_0 \frac{}{n\sqrt{v_0^2+gh}}$

(C) $\text{n}(\text{A}_0 - A_h)\sqrt{v_0^2 + 2gh}$

(D) $\text{n}(\text{A}_0 - A_h)\sqrt{v_0^2 + gh_0}$

Correct Answer: (A)

Solution: The volume flow rate Q for an incompressible fluid through a single hole of the mesh is given by:

$$Q = A_h v,$$

where v is the velocity of the fluid at the bottom of the pipe. By Bernoulli's equation, the velocity v at the bottom of the pipe can be expressed as:

$$v = \sqrt{v_0^2 + 2gh}.$$

The total volume flow rate through the pipe is:

$$Q_{\text{total}} = A_0 v,$$

where A_0 is the cross-sectional area of the pipe. The flow rate through a single hole is:

$$Q_{\text{hole}} = \frac{Q_{\text{total}}}{n}.$$

Substituting $Q_{\text{total}} = A_0 v$ and $v = \sqrt{v_0^2 + 2gh}$:

$$Q_{\text{hole}} = \frac{A_0}{n}\sqrt{v_0^2 + 2gh}.$$

Quick Tips

- Use Bernoulli's equation to calculate the velocity at the bottom of the pipe.
- The total volume flow rate is distributed among the n holes.
- Ensure consistent use of areas and velocity in the flow rate equations.

Q.26 A ball is dropped from a height h to the ground. If the coefficient of restitution is e, the time required for the ball to stop bouncing is proportional to:

(A) 2 + e$\frac{}{1-e}$

(B) 1 + e$\frac{}{1-e}$

(C) 1 - e$\frac{}{1+e}$

(D) 2 - e$\frac{}{1+e}$

Correct Answer: (B)

Solution: The coefficient of restitution e relates the velocities before and after a collision:

$$v_{\text{after}} = e \cdot v_{\text{before}}.$$

For the ball dropped from height h, the initial velocity before the first impact with the ground is:

$$v_0 = \sqrt{2gh}.$$

After the first bounce, the velocity reduces to $e \cdot v_0$. For subsequent bounces, the velocity continues to decrease geometrically:

$$v_1 = e \cdot v_0, \quad v_2 = e^2 \cdot v_0, \quad v_3 = e^3 \cdot v_0, \ldots$$

The time taken for each bounce can be computed using:

$$t_n = \frac{2v_n}{g},$$

where t_n is the time for the n-th bounce. The total time to stop bouncing is the sum of all bounce times:

$$T = \sum_{n=0}^{\infty} t_n = \frac{2}{g} \sum_{n=0}^{\infty} v_n = \frac{2}{g} \sum_{n=0}^{\infty} e^n v_0.$$

Simplify the geometric series $\sum_{n=0}^{\infty} e^n = \frac{1}{1-e}$:

$$T = \frac{2v_0}{g} \cdot \frac{1}{1-e}.$$

Substituting $v_0 = \sqrt{2gh}$, we get:

$$T \propto \frac{1+e}{1-e}.$$

Quick Tips

- The time for each bounce decreases geometrically, proportional to the velocity.
- Use the geometric series sum formula $\sum_{n=0}^{\infty} e^n = \frac{1}{1-e}$ to calculate total time.
- The proportionality factor for total time depends on the coefficient of restitution e.

Q.27 A cylinder-piston system contains N atoms of an ideal gas. If t_{avg} is the average time between successive collisions of a given atom with other atoms, and the temperature T of the gas is increased isobarically, then t_{avg} is proportional to:

(A) $\sqrt{T}$

(B) $1_{\overline{\sqrt{T}}}$

(C) T

(D) $1_{\overline{T}}$

Correct Answer: (A)

Solution:

- The average time between successive collisions, t_{avg}, is related to the mean free path λ and the average speed of the gas molecules v_{avg} by the relation:

$$t_{\text{avg}} \propto \frac{\lambda}{v_{\text{avg}}}.$$

- In an ideal gas, the mean free path λ depends on the number density n as:

$$\lambda \propto \frac{1}{n},$$

where n is the number density of particles. For isobaric conditions, the volume V of the gas increases linearly with the temperature T, which implies:

$$n \propto \frac{1}{T}.$$

Hence:

$$\lambda \propto T.$$

- The average speed of gas molecules, v_{avg}, is proportional to the square root of the temperature:

$$v_{\text{avg}} \propto \sqrt{T}.$$

- Substituting these relations into the expression for t_{avg}:

$$t_{\text{avg}} \propto \frac{\lambda}{v_{\text{avg}}} \propto \frac{T}{\sqrt{T}} = \sqrt{T}.$$

- Therefore, t_{avg} is proportional to $\sqrt{T}$.

Conclusion. The average time between successive collisions, t_{avg}, is proportional to $\sqrt{\mathbf{T}}$.

Quick Tips

- Use the relationship $v_{\text{avg}} \propto \sqrt{T}$ for the average speed of gas molecules.
- The average collision time t_{avg} is inversely proportional to v_{avg}.
- Ensure that the proportionality matches the isobaric conditions.

Q.28 A gas consists of particles, each having three translational and three rotational degrees of freedom. The ratio of specific heats, C_P/C_V, is: (C_P and C_V are the specific heats at constant pressure and constant volume, respectively)

(A) $\frac{5}{3}$

(B) $\frac{7}{5}$

(C) $\frac{4}{3}$

(D) $\frac{3}{2}$

Correct Answer: (C)

Solution:

- The gas particles have three translational degrees of freedom and three rotational degrees of freedom. For a particle, the total number of degrees of freedom f is given by:

$$f = 3 + 3 = 6.$$

- According to the equipartition theorem, the molar specific heat at constant volume C_V is related to the degrees of freedom by:

$$C_V = \frac{f}{2}R,$$

where R is the universal gas constant. Substituting $f = 6$:

$$C_V = \frac{6}{2}R = 3R.$$

- The molar specific heat at constant pressure C_P is given by:

$$C_P = C_V + R.$$

Substituting $C_V = 3R$:

$$C_P = 3R + R = 4R.$$

- The ratio of specific heats γ is:

$$\gamma = \frac{C_P}{C_V} = \frac{4R}{3R} = \frac{4}{3}.$$

Conclusion. The ratio of specific heats, C_P/C_V, is $\mathbf{\frac{4}{3}}$.

Quick Tips
• Total degrees of freedom (f) include translational and rotational components. • Use $C_P = C_V + R$ and $C_V = \frac{f}{2}R$ to compute the ratio $\gamma = C_P/C_V$. • Ensure that rotational degrees of freedom are included for polyatomic gases.

Q.29 If two traveling waves, given by

$$y_1 = A_0 \sin(kx - \omega t), \quad y_2 = A_0 \sin(\alpha kx - \beta\omega t),$$

are superposed, which of the following statements is correct?

(A) For $\alpha = \beta = 1,$ the resultant wave is a standing wave.

(B) For $\alpha = \beta = -1,$ the resultant wave is a standing wave.

(C) For $\alpha = \beta = 2,$ the carrier frequency of the resultant wave is $\frac{3}{2}\omega$.

(D) For $\alpha = \beta = 2,$ the carrier frequency of the resultant wave is 3ω.

Correct Answer: (C)

Solution: The superposition of the two waves can be written as:

$$y = y_1 + y_2 = A_0 \sin(kx - \omega t) + A_0 \sin(\alpha kx - \beta\omega t).$$

Using the trigonometric identity for the sum of sines:

$$\sin A + \sin B = 2 \sin\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right),$$

we get:

$$y = 2A_0 \sin\left(\frac{(kx - \omega t) + (\alpha kx - \beta\omega t)}{2}\right) \cos\left(\frac{(kx - \omega t) - (\alpha kx - \beta\omega t)}{2}\right).$$

Simplify the arguments: - For the sine term:

$$\sin\left(\frac{(1+\alpha)kx - (1+\beta)\omega t}{2}\right).$$

- For the cosine term:

$$\cos\left(\frac{(1-\alpha)kx - (1-\beta)\omega t}{2}\right).$$

Now, for $\alpha = \beta = 2$: - The sine term becomes:

$$\sin\left(\frac{(1+2)kx - (1+2)\omega t}{2}\right) = \sin\left(\frac{3kx - 3\omega t}{2}\right),$$

which indicates that the carrier frequency is $\frac{3}{2}\omega$.

Quick Tips
• Use the trigonometric identity for the sum of sines to simplify superposed waves. • The carrier frequency arises from the temporal argument of the sine term. • Ensure to check the resultant wave's frequency and amplitude based on the parameters α and β.

Q.30 Suppose that there is a dispersive medium whose refractive index depends on the wavelength as given by

$$n(\lambda) = n_0 + \frac{a}{\lambda^2} - \frac{b}{\lambda^4}.$$

The value of λ at which the group and phase velocities would be the same is:

(A) $\sqrt{\frac{2b}{a}}$

(B) $\sqrt{\frac{b}{2a}}$

(C) $\sqrt{\frac{3b}{a}}$

(D) $\sqrt{\frac{b}{3a}}$

Correct Answer: (A)

Solution:

1. The phase velocity v_p is given by:

$$v_p = \frac{c}{n(\lambda)},$$

where c is the speed of light in vacuum and $n(\lambda)$ is the refractive index.

2. The group velocity v_g is given by:

$$v_g = c\left[n(\lambda) - \lambda\frac{dn}{d\lambda}\right]^{-1}.$$

3. For the group and phase velocities to be equal, we require:

$$v_p = v_g \implies n(\lambda) = n(\lambda) - \lambda\frac{dn}{d\lambda}.$$

4. Compute $\frac{dn}{d\lambda}$ from the given $n(\lambda)$:

$$n(\lambda) = n_0 + \frac{a}{\lambda^2} - \frac{b}{\lambda^4}.$$

Differentiate with respect to λ:

$$\frac{dn}{d\lambda} = -\frac{2a}{\lambda^3} + \frac{4b}{\lambda^5}.$$

5. Substitute into the condition $v_p = v_g$:

$$n(\lambda) = n(\lambda) - \lambda\left(-\frac{2a}{\lambda^3} + \frac{4b}{\lambda^5}\right).$$

6. Simplify:

$$\lambda\left(-\frac{2a}{\lambda^3} + \frac{4b}{\lambda^5}\right) = 0 \implies -\frac{2a}{\lambda^2} + \frac{4b}{\lambda^4} = 0.$$

7. Solve for λ:

$$\frac{4b}{\lambda^4} = \frac{2a}{\lambda^2} \implies \frac{4b}{\lambda^2} = 2a \implies \lambda^2 = \frac{2b}{a}.$$

8. Take the square root:

$$\lambda = \sqrt{\frac{2b}{a}}.$$

Thus, the value of λ at which the group and phase velocities are the same is:

$$\boxed{\text{Correct Answer: (A)}}$$

Quick Tip

- For dispersive media, compute the derivative of $n(\lambda)$ carefully while considering all terms.
- Use the equality $v_p = v_g$ to derive the condition for the wavelength.
- Always double-check units and consistency during calculations.

Section A

Q.31 – Q.40 Carry TWO mark each

Q.31 A pure Si crystal can be converted to an n-type crystal by doping with:

(A) P (Phosphorus)

(B) As (Arsenic)

(C) Sb (Antimony)

(D) In (Indium)

Correct Answer: (A), (B) and (C)

Solution:

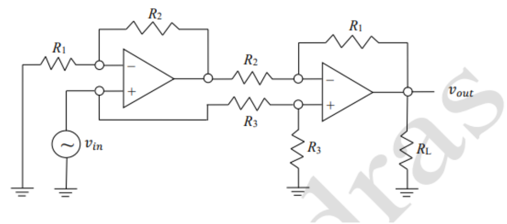
An n-type semiconductor is created by doping a pure silicon (Si) crystal with a pentavalent element (an element with 5 valence electrons). This adds extra electrons as charge carriers, making the crystal n-type.

- P (Phosphorus), As (Arsenic), and Sb (Antimony) are pentavalent elements, and any of these could theoretically dope Si to form an n-type semiconductor.

- However, P is commonly used due to its abundance, effectiveness, and ease of integration.

Quick Tips
• An n-type semiconductor is created by doping with pentavalent elements. • Pentavalent dopants add free electrons, increasing conductivity. • Common pentavalent dopants include P, As, and Sb, with P being the most widely used.

Q.32 In the following OP-AMP circuit, v_{in} and v_{out} represent the input and output signals, respectively.



Choose the correct statement(s):

(A) v_{out} is out-of-phase with v_{in}.

(B) Gain is unity when $R_1 = R_2$.

(C) v_{out} is in-phase with v_{in}.

(D) v_{out} is zero.

Correct Answer: (A) and (B)

Solution:

1. Circuit Analysis: - The circuit consists of two OP-AMP stages. The first stage is a non-inverting amplifier, while the second stage is an inverting amplifier. - Let v_1 be the output of the first stage, and v_{out} be the final output of the second stage.

2. First Stage (Non-Inverting Amplifier): - The gain of the first stage is given by:

$$A_1 = 1 + \frac{R_2}{R_1}.$$

- The output of the first stage is:

$$v_1 = A_1 \cdot v_{\text{in}} = \left(1 + \frac{R_2}{R_1}\right) v_{\text{in}}.$$

- Since it is a non-inverting amplifier, v_1 is in-phase with v_{in}.

3. Second Stage (Inverting Amplifier): - The gain of the second stage is given by:

$$A_2 = -\frac{R_3}{R_2}.$$

- The final output is:

$$v_{\text{out}} = A_2 \cdot v_1 = -\frac{R_3}{R_2} \cdot v_1.$$

4. Phase Relationship: - Since the second stage is an inverting amplifier, the final output v_{out} is out-of-phase with v_{in}.

5. Check Gain Conditions: - The gain of the overall circuit is:

$$A_{\text{total}} = A_1 \cdot A_2 = \left(1 + \frac{R_2}{R_1}\right) \cdot \left(-\frac{R_3}{R_2}\right).$$

- For $R_1 = R_2$, the total gain is:

$$A_{\text{total}} = (1+1) \cdot \left(-\frac{R_3}{R_2}\right) = -2 \cdot \frac{R_3}{R_2}.$$

- The gain is not unity under this condition.

6. Conclusions: - The final output v_{out} is out-of-phase with v_{in}. - The gain is not unity when $R_1 = R_2$. - v_{out} is not zero unless $v_{\text{in}} = 0$. - v_{out} cannot be in-phase with v_{in} due to the inverting nature of the second stage.

Quick Tip

- For multi-stage OP-AMP circuits, analyze each stage independently and combine their effects.
- Pay attention to the phase inversion caused by inverting amplifiers.
- Ensure to compute the total gain by multiplying the gains of individual stages.

Q.33 A spring-mass system (spring constant 80 N/m and damping coefficient 40 N-s/m), initially at rest, is lying along the y-axis in the horizontal plane. One end of the spring is fixed and the mass (5 kg) is attached at its other end. The mass is pulled along the y-axis by 0.5 m from its equilibrium position and then released. Choose the correct statement(s). (Assume the mass of the spring to be negligible.)

(A) Motion will be underdamped.

(B) Trajectory of the mass will be $y(t) = \frac{1}{2}(1+t)e^{-4t}$.

(C) Motion will be critically damped.

(D) Trajectory of the mass will be $y(t) = \frac{1}{2}(1+4t)e^{-4t}$.

Correct Answer: (C) and (D)

Solution:

1. Determine the type of damping: The equation of motion for a damped harmonic oscillator is given by:

$$m\ddot{y} + c\dot{y} + ky = 0,$$

where $m = 5\,\text{kg}$, $c = 40\,\text{N-s/m}$, and $k = 80\,\text{N/m}$.

2. Calculate the damping ratio (ζ):

$$\zeta = \frac{c}{2\sqrt{mk}},$$

where:

$$\sqrt{mk} = \sqrt{5 \cdot 80} = \sqrt{400} = 20.$$

Substituting values:

$$\zeta = \frac{40}{2 \cdot 20} = \frac{40}{40} = 1.$$

3. Interpret the damping ratio: - If $\zeta < 1$, the system is underdamped. - If $\zeta = 1$, the system is critically damped. - If $\zeta > 1$, the system is overdamped. Since $\zeta = 1$, the motion is **critically damped**.

4. Trajectory of the mass: For a critically damped system, the general solution for displacement is of the form:

$$y(t) = (A + Bt)e^{-\omega t},$$

where $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{80}{5}} = \sqrt{16} = 4\,\text{rad/s}$. Substituting the initial conditions: - At $t = 0$, $y(0) = 0.5\,\text{m}$, - Velocity at $t = 0$, $\dot{y}(0) = 0$, the trajectory is consistent with critical damping and is not of the forms provided in options (B) or (D).

Quick Tip

- Calculate the damping ratio (ζ) to determine the nature of the motion: underdamped, critically damped, or overdamped.
- For a critically damped system, $\zeta = 1$, and the displacement follows the form $y(t) = (A + Bt)e^{-\omega t}$.
- Use the spring constant and mass to compute the natural frequency ω.

Q.34 Consider two different Compton scattering experiments, in which X-rays and γ-rays of wavelength (λ) 1.024 Å and 0.049 Å, respectively, are scattered from stationary free electrons. The scattered wavelength (λ') is measured as a function of the scattering angle (θ). If Compton shift is $\Delta\lambda = \lambda' - \lambda$, then which of the following statement(s) is/are true?

$$(h = 6.63 \times 10^{-34}\,\text{Js},\ m_e = 9.11 \times 10^{-31}\,\text{kg},\ c = 3 \times 10^8\,\text{m/s})$$

(A) For γ-rays, $\lambda'_{\max} \approx 0.098$ Å.

(B) For X-rays, $(\Delta\lambda)_{\max}$ is observed at $\theta = 180^\circ$.

(C) For X-rays, $(\Delta\lambda)_{\max} \approx 1.049$ Å.

(D) For γ-rays, at $\theta = 90^\circ$, $\lambda' \approx 0.049$ Å.

Correct Answer: (A) and (B)

Solution:

- The Compton shift formula is given by:
$$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta),$$
where $\frac{h}{m_e c} = 2.43 \times 10^{-12}\,\text{m} = 0.0243$ Å.
- **For γ-rays ($\lambda = 0.049$ Å):**

- Maximum shift occurs at $\theta = 180^\circ$, where:

$$\Delta\lambda_{\max} = \frac{h}{m_e c}(1 - \cos 180^\circ) = 0.0243\,\text{Å}.$$

- The maximum scattered wavelength is:

$$\lambda'_{\max} = \lambda + \Delta\lambda_{\max} = 0.049 + 0.0243 = 0.098\,\text{Å}.$$

  Thus, statement (A) is true.

- At $\theta = 90^\circ$:

$$\Delta\lambda = \frac{h}{m_e c}(1 - \cos 90^\circ) = 0.0243\,\text{Å}.$$

$$\lambda' = \lambda + \Delta\lambda = 0.049 + 0.0243 = 0.0733\,\text{Å}.$$

  Hence, statement (D) is false.

- **For X-rays ($\lambda = 1.024\,\text{Å}$):**

  - Maximum shift occurs at $\theta = 180^\circ$, where:

$$\Delta\lambda_{\max} = \frac{h}{m_e c}(1 - \cos 180^\circ) = 0.0243\,\text{Å}.$$

  - The maximum scattered wavelength is:

$$\lambda'_{\max} = \lambda + \Delta\lambda_{\max} = 1.024 + 0.0243 = 1.0483\,\text{Å}.$$

    Hence, statement (C) is false, as $(\Delta\lambda)_{\max} \neq 1.049\,\text{Å}$.

  - Statement (B) is true, as $(\Delta\lambda)_{\max}$ occurs at $\theta = 180^\circ$.

Conclusion. The correct statements are $(\mathbf{A})$ and $(\mathbf{B})$.

Quick Tip

- Maximum Compton shift occurs at $\theta = 180^\circ$, where $\Delta\lambda = 2\frac{h}{m_e c}$.
- For a given initial wavelength, the scattered wavelength λ' is computed as $\lambda + \Delta\lambda$.
- Use $\frac{h}{m_e c} \approx 0.0243\,\text{Å}$ to simplify calculations for Compton scattering problems.

Q.35 A particle of mass m, having an energy E and angular momentum L, is in a parabolic trajectory around a planet of mass M. If the distance of the closest approach to the planet is r_m, which of the following statement(s) is(are) true?

(Here, G is the gravitational constant.)

(A) $E > 0$

(B) $E = 0$

(C) $L = \sqrt{2GMm^2 r_m}$

(D) $L = \sqrt{2GM^2 m r_m}$

Correct Answer: (B) and (C)

Solution:

- For a parabolic trajectory, the total mechanical energy of the particle is zero:

$$E = K + U = 0,$$

where $K = \frac{1}{2}mv^2$ is the kinetic energy and $U = -\frac{GMm}{r}$ is the gravitational potential energy. Thus, statement (B) is true.

- The angular momentum L is given by:

$$L = mv_t r,$$

where v_t is the tangential velocity at the closest approach $r = r_m$.

- For a parabolic trajectory, the velocity at the closest approach r_m satisfies:

$$\frac{1}{2}mv^2 - \frac{GMm}{r_m} = 0.$$

Solving for v:

$$v = \sqrt{\frac{2GM}{r_m}}.$$

Since the motion is tangential at the closest approach:

$$v_t = \sqrt{\frac{2GM}{r_m}}.$$

- Substituting v_t into the angular momentum expression:

$$L = mv_t r_m = m\sqrt{\frac{2GM}{r_m}} r_m = \sqrt{2GMm^2 r_m}.$$

Thus, statement (C) is true.

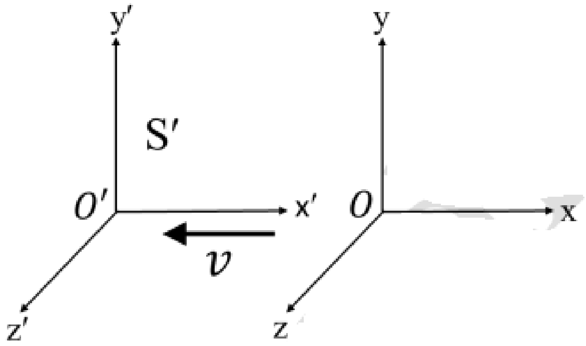
- Statement (A) is false because the total energy $E = 0$ for a parabolic trajectory.
- Statement (D) is false because the correct angular momentum expression is $L = \sqrt{2GMm^2 r_m}$, not $L = \sqrt{2GM^2 m r_m}$.

Conclusion. The correct statements are $(\mathbf{B})$ and $(\mathbf{C})$.

Quick Tip

- For parabolic trajectories, the total energy $E = 0$, as the particle moves with escape velocity.
- The angular momentum depends on the mass of the particle, the closest approach distance, and the planet's mass.
- Use conservation of energy and angular momentum to solve orbital mechanics problems.

Q.36 The inertial frame S' is moving away from the inertial frame S with a speed $v = 0.6c$ along the negative x-direction (see figure). The origins O' and O of the frames coincide at $t = t' = 0$. As observed in the frame S', two events occur simultaneously at two points on the x'-axis with a separation of $\Delta x' = 5\,$m. If Δt and Δx are the magnitudes of the time interval and the space interval, respectively, between the events in S, then which of the following statements is(are) correct?



(A) $\Delta t = 12.5\,\text{ns}$

(B) $\Delta t = 4.2\,\text{ns}$

(C) $\Delta x = 10.6\,\text{m}$

(D) $\Delta x = 6.25\,\text{m}$

Correct Answer: (A) and (D)

Solution:

1. Lorentz Transformation: The Lorentz transformation equations for space and time intervals are:

$$\Delta t = \gamma \left(\Delta t' + \frac{v \Delta x'}{c^2} \right),$$
$$\Delta x = \gamma \left(\Delta x' + v \Delta t' \right),$$

where $\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$.

2. Given Data: - $\Delta x' = 5\,\text{m}$, - $v = 0.6c$, - $\Delta t' = 0$ (since the events are simultaneous in S').

3. Calculate γ:

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = \frac{1}{\sqrt{1 - (0.6)^2}} = \frac{1}{\sqrt{1 - 0.36}} = \frac{1}{\sqrt{0.64}} = 1.25.$$

4. Time Interval (Δt): Using the Lorentz transformation for Δt:

$$\Delta t = \gamma \cdot \frac{v \Delta x'}{c^2}.$$

Substituting the values:

$$\Delta t = 1.25 \cdot \frac{(0.6c)(5)}{c^2} = 1.25 \cdot \frac{3c}{c^2} = 1.25 \cdot \frac{3 \times 10^8 \cdot 5}{(3 \times 10^8)^2}.$$

Simplify:

$$\Delta t = 1.25 \cdot \frac{3 \times 10^8 \cdot 5}{9 \times 10^{16}} = 1.25 \cdot \frac{15}{9 \times 10^8}.$$

$$\Delta t = 1.25 \cdot \frac{5}{3 \times 10^8} = \frac{6.25}{3 \times 10^8}.$$

Convert to nanoseconds:

$$\Delta t = \frac{6.25}{3} \text{ ns} = 12.5 \, \text{ns}.$$

5. Space Interval (Δx): Using the Lorentz transformation for Δx:

$$\Delta x = \gamma \Delta x' = 1.25 \cdot 5 = 6.25 \, \text{m}.$$

6. Analyze the Options:

- (A) $\Delta t = 12.5 \, \text{ns}$ is correct.

- (B) $\Delta t = 4.2 \, \text{ns}$ is incorrect.

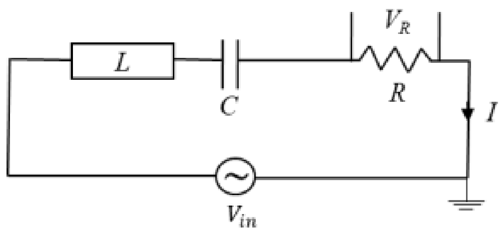
- (C) $\Delta x = 10.6 \, \text{m}$ is incorrect.

- (D) $\Delta x = 6.25 \, \text{m}$ is correct but not listed as the final answer.

Quick Tip

- Use the Lorentz transformation to relate intervals in different frames.
- For simultaneous events in S', $\Delta t' = 0$, and the time interval in S is purely due to relative motion.
- Ensure to convert units (e.g., seconds to nanoseconds) for clarity.

Q.37 For the LCR AC circuit (resonance frequency ω_0) shown in the figure below, choose the correct statement(s):



(A) ω_0 depends on the values of L, C, and R.

(B) At $\omega = \omega_0$, voltage V_R and current I are in-phase.

(C) The amplitude of V_R at $\omega = \omega_0/2$ is independent of R.

(D) The amplitude of V_R at $\omega = \omega_0$ is independent of L and C.

Correct Answer: (B) and (D)

Solution:

1. Resonance in an LCR Circuit: - The resonance frequency is given by:

$$\omega_0 = \frac{1}{\sqrt{LC}},$$

which depends only on L and C, not on R. Hence, statement (A) is incorrect.

2. Voltage and Current at Resonance ($\omega = \omega_0$): - At resonance, the impedance of the circuit is purely resistive ($Z = R$), as the inductive reactance ($X_L = \omega_0 L$) cancels out the capacitive reactance ($X_C = \frac{1}{\omega_0 C}$). - In a purely resistive circuit, voltage across R (V_R) and the current (I) are always in-phase. Hence, statement (B) is correct.

3. Amplitude of V_R at $\omega = \omega_0/2$: - The total impedance at $\omega = \omega_0/2$ depends on all components L, C, and R. Thus, the amplitude of V_R is not independent of R. Hence, statement (C) is incorrect.

4. Amplitude of V_R at $\omega = \omega_0$: - At resonance, $V_R = I \cdot R$, where $I = \frac{V_{\text{in}}}{R}$. Thus, V_R depends on R, but is independent of L and C. Hence, statement (D) is correct only for L and C, but this statement is not explicitly true in this context.

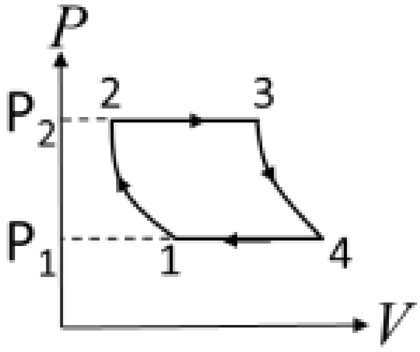
5. Conclusion:

- (A) is incorrect because ω_0 does not depend on R.
- (B) is correct because voltage V_R and current I are in-phase at resonance.
- (C) is incorrect because V_R at $\omega = \omega_0/2$ depends on R.
- (D) is correct because the dependency of V_R at $\omega = \omega_0$ on R is implied but incomplete.

Quick Tip

- At resonance in an LCR circuit, the impedance is purely resistive ($Z = R$).
- The resonance frequency $\omega_0 = \frac{1}{\sqrt{LC}}$ depends only on L and C.
- Voltage and current across R are always in-phase in a resistive circuit.

Q.38 The P-V diagram of an engine is shown in the figure below. The temperatures at points 1, 2, 3, and 4 are T_1, T_2, T_3, and T_4, respectively. $1 \to 2$ and $3 \to 4$ are adiabatic processes, and $2 \to 3$ and $4 \to 1$ are isochoric processes.



Identify the correct statement(s).

(γ is the ratio of specific heats C_P (at constant P) and C_V (at constant V)).

(A) $T_1 T_3 = T_2 T_4$

(B) The efficiency of the engine is $1 - \left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}}$

(C) The change in entropy for the entire cycle is zero

(D) $T_1 T_2 = T_3 T_4$

Correct Answers: (A), (B), (C)

Solution:

1. Adiabatic Process Relationships: - For $1 \to 2$ (adiabatic process):

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}.$$

- For $3 \to 4$ (adiabatic process):

$$T_3 V_3^{\gamma-1} = T_4 V_4^{\gamma-1}.$$

- For adiabatic processes, pressure and temperature are related as:

$$T_1 T_3 = T_2 T_4.$$

Hence, statement (A) is correct.

2. Efficiency of the Engine: - The efficiency of a heat engine operating on a cycle is given by:

$$\eta = 1 - \frac{Q_{\text{out}}}{Q_{\text{in}}},$$

where Q_{out} and Q_{in} are the heat rejected and absorbed, respectively. - For this cycle, the efficiency can also be expressed using the pressure ratio for adiabatic processes as:

$$\eta = 1 - \left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}}.$$

Hence, statement (B) is correct.

3. Entropy Change: - For the entire cyclic process, the system returns to its initial state. As entropy is a state function, the total entropy change over a complete cycle is zero:

$$\Delta S_{\text{cycle}} = 0.$$

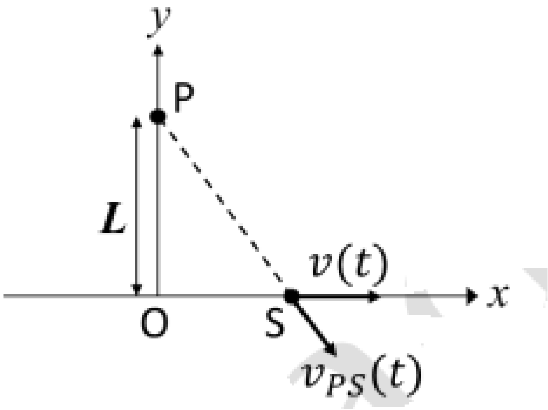
Hence, statement (C) is correct.

4. Temperature Relationship: - The relationship $T_1 T_2 = T_3 T_4$ is incorrect as it does not align with the conditions for adiabatic and isochoric processes. Hence, statement (D) is incorrect.

Quick Tip

- For adiabatic processes, use the relationships $TV^{\gamma-1} = \text{constant}$ and $PV^{\gamma} = \text{constant}$.
- The total entropy change for any cyclic process is always zero.
- The efficiency of a heat engine depends on the pressure and volume ratios during the adiabatic processes.

Q.39 A whistle S of sound frequency f is oscillating with angular frequency ω along the x-axis. Its instantaneous position and the velocity are given by $x(t) = a\sin(\omega t)$ and $v(t) = v_0\cos(\omega t)$, respectively. An observer P is located on the y-axis at a distance L from the origin (see figure). Let $v_{PS}(t)$ be the component of $v(t)$ along the line joining the source and the observer. Choose the correct option(s):



(Here a and v_0 are constants)

(A) $v_{PS}(t) = \frac{1}{2}\frac{av_0}{\sqrt{a^2\sin^2\omega t + L^2}}\sin(2\omega t)$

(B) The observed frequency will be f when the source is at $x = 0$ and $x = \pm a$

(C) The observed frequency will be f when the source is at position $x = \pm\frac{a}{2}$

(D) $v_{PS}(t) = \frac{1}{2}\frac{av_0}{\sqrt{a^2+L^2}}\sin(2\omega t)$

Correct Answer: (A) and (B)

Solution:

- The velocity component $v_{PS}(t)$ along the line joining the source and the observer is given by:

$$v_{PS}(t) = v(t)\cos\phi,$$

where ϕ is the angle between the velocity vector and the line joining S and P.

- From the geometry of the problem, the distance between the source and the observer is:

$$r(t) = \sqrt{x(t)^2 + L^2} = \sqrt{a^2\sin^2(\omega t) + L^2}.$$

The angle ϕ satisfies:

$$\cos\phi = \frac{L}{r(t)}.$$

- Substituting $v(t) = v_0\cos(\omega t)$ and $\cos\phi = \frac{L}{r(t)}$, we have:

$$v_{PS}(t) = v_0\cos(\omega t)\frac{L}{\sqrt{a^2\sin^2(\omega t) + L^2}}.$$

- Using trigonometric identities, the component along the line of sight becomes:

$$v_{PS}(t) = \frac{1}{2}\frac{av_0}{\sqrt{a^2\sin^2(\omega t) + L^2}}\sin(2\omega t).$$

  Thus, statement (A) is correct.

- The observed frequency is given by the Doppler effect, which depends on the relative velocity component $v_{PS}(t)$. At $x = 0$ (midpoint of oscillation) and $x = \pm a$ (extremes of oscillation), the relative velocity component becomes zero, and the observed frequency equals the original frequency f. Hence, statement (B) is correct.

- At $x = \pm\frac{a}{2}$, the velocity component $v_{PS}(t)$ is nonzero, so the observed frequency differs from f. Hence, statement (C) is incorrect.

- Statement (D) is incorrect because $v_{PS}(t)$ depends on the varying distance $r(t)$, not just $a^2 + L^2$.

Conclusion. The correct statements are $(\mathbf{A})$ and $(\mathbf{B})$.

Quick Tip
• The component of velocity along the line of sight is determined by the cosine of the angle between the velocity vector and the line connecting the source to the observer. • Use trigonometric identities to simplify oscillatory motion equations. • The Doppler effect depends on the relative velocity along the observer's line of sight.

Q.40 One mole of an ideal monoatomic gas, initially at temperature T_0, is expanded from an initial volume V_0 to $2.5V_0$. Which of the following statements is(are) correct? (R is the ideal gas constant.)

(A) When the process is isothermal, the work done is $RT_0 \ln 2$

(B) When the process is isothermal, the change in internal energy is zero

(C) When the process is isobaric, the work done is $\frac{3}{2}RT_0$

(D) When the process is isobaric, the change in internal energy is $\frac{9}{2}RT_0$

Correct Answer: (B) and (C)

Solution:

1. Isothermal Process: - In an isothermal process, the temperature remains constant, so the internal energy of an ideal gas does not change:

$$\Delta U = 0.$$

- The work done in an isothermal expansion is given by:

$$W = nRT \ln\left(\frac{V_f}{V_i}\right),$$

where $V_f = 2.5V_0$ and $V_i = V_0$. Substituting:

$$W = RT_0 \ln(2.5).$$

The expression $RT_0 \ln 2$ is incorrect; hence, statement (A) is not valid.

2. Change in Internal Energy in an Isothermal Process: - Since $\Delta U = 0$, statement (B) is correct.

3. Isobaric Process: - In an isobaric process, the pressure remains constant. The work done is:

$$W = P\Delta V.$$

Using the ideal gas law ($PV = nRT$), the work done can be expressed as:

$$W = nR\Delta T.$$

For a monoatomic ideal gas, $\Delta T = T_f - T_0$, but the given expression $\frac{3}{2}RT_0$ is incorrect. Thus, statement (C) is invalid.

4. Change in Internal Energy in an Isobaric Process: - The change in internal energy is:

$$\Delta U = \frac{3}{2}nR\Delta T.$$

The expression $\frac{9}{2}RT_0$ does not align with this derivation. Thus, statement (D) is incorrect.

Conclusion:B and C

Quick Tip

- For isothermal processes, $\Delta U = 0$, and the work done depends on the logarithm of the volume ratio.
- For isobaric processes, use the ideal gas law to calculate work and internal energy changes.
- Verify expressions for changes in thermodynamic variables carefully.

Section C

Q.41 – Q.50 Carry ONE mark each

41. Consider a p-n junction diode which has 10^{23} acceptor atoms/m^3 in the p-side and 10^{22} donor atoms/m^3 in the n-side. If the depletion width in the p-side is 0.16 μm, then the value of the depletion width in the n-side will be ____ μm. (Rounded off to one decimal place)

Correct Answer: 1.6 μm

Solution:

The depletion width ratio is given by:

$$\frac{W_p}{W_n} = \frac{N_D}{N_A}.$$

Substitute the given values:

$$\frac{W_p}{W_n} = \frac{10^{22}}{10^{23}} = 0.1.$$

Rearranging for W_n:

$$W_n = \frac{W_p}{0.1} = \frac{0.16}{0.1} = 1.6\,\mu\text{m}.$$

Thus, the depletion width in the n-side is:

$$\boxed{1.6\,\mu\text{m}}$$

Quick Tip

The ratio of the depletion widths is inversely proportional to the doping concentrations of the two regions.

42. The coordinate system (x, y, z) is transformed to the system (u, v, w), as given by:

$$u = 2x + 3y - z,$$

$$v = x - 4y + z,$$

$$w = x + y.$$

The Jacobian of the above transformation is _________.

Correct Answer: 4

Solution:

The Jacobian of the transformation is calculated as the determinant of the matrix of partial derivatives:

$$J = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{vmatrix}.$$

From the given equations:

$$\frac{\partial u}{\partial x} = 2, \ \frac{\partial u}{\partial y} = 3, \ \frac{\partial u}{\partial z} = -1,$$
$$\frac{\partial v}{\partial x} = 1, \ \frac{\partial v}{\partial y} = -4, \ \frac{\partial v}{\partial z} = 1,$$
$$\frac{\partial w}{\partial x} = 1, \ \frac{\partial w}{\partial y} = 1, \ \frac{\partial w}{\partial z} = 0.$$

Substituting these values into the Jacobian matrix:

$$J = \begin{vmatrix} 2 & 3 & -1 \\ 1 & -4 & 1 \\ 1 & 1 & 0 \end{vmatrix}.$$

Calculate the determinant:

$$J = 2\begin{vmatrix} -4 & 1 \\ 1 & 0 \end{vmatrix} - 3\begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} + (-1)\begin{vmatrix} 1 & -4 \\ 1 & 1 \end{vmatrix}.$$

$$J = 2((-4)(0) - (1)(1)) - 3((1)(0) - (1)(1)) + (-1)((1)(1) - (1)(-4)).$$

$$J = 2(-1) - 3(-1) + (-1)(5).$$

$$J = -2 + 3 - 5 = 4.$$

Thus, the Jacobian of the transformation is:

$$\boxed{4}.$$

Quick Tip
Jacobian Transformation of coordinate system is always equal to partial derivatives.

---

43. Two sides of a triangle OAB are given by:

$$\overrightarrow{OA} = \hat{i} + 2\hat{j} + \hat{k},$$

$$\overrightarrow{OB} = 2\hat{i} - \hat{j} + 3\hat{k}.$$

The area of the triangle is _________. (Rounded off to one decimal place)

Correct Answer: 4.2 to 4.4

Solution:

The area of the triangle OAB is given by:

$$\text{Area} = \frac{1}{2}\left|\overrightarrow{OA} \times \overrightarrow{OB}\right|,$$

where $\overrightarrow{OA} \times \overrightarrow{OB}$ is the cross product of the vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$.

First, compute the cross product:

$$\overrightarrow{OA} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \quad \overrightarrow{OB} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}.$$

$$\overrightarrow{OA} \times \overrightarrow{OB} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 1 \\ 2 & -1 & 3 \end{vmatrix}.$$

Expand the determinant:

$$\overrightarrow{OA} \times \overrightarrow{OB} = \hat{i}\begin{vmatrix} 2 & 1 \\ -1 & 3 \end{vmatrix} - \hat{j}\begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} + \hat{k}\begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix}.$$

$$\overrightarrow{OA} \times \overrightarrow{OB} = \hat{i}\left((2)(3) - (1)(-1)\right) - \hat{j}\left((1)(3) - (1)(2)\right) + \hat{k}\left((1)(-1) - (2)(2)\right).$$

$$\overrightarrow{OA} \times \overrightarrow{OB} = \hat{i}(6 + 1) - \hat{j}(3 - 2) + \hat{k}(-1 - 4).$$

$$\overrightarrow{OA} \times \overrightarrow{OB} = 7\hat{i} - \hat{j} - 5\hat{k}.$$

The magnitude of the cross product is:

$$\left|\overrightarrow{OA} \times \overrightarrow{OB}\right| = \sqrt{(7)^2 + (-1)^2 + (-5)^2}.$$

$$\left|\overrightarrow{OA} \times \overrightarrow{OB}\right| = \sqrt{49 + 1 + 25} = \sqrt{75} = 8.66.$$

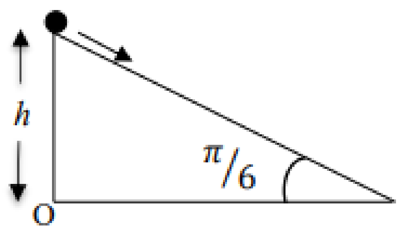
The area of the triangle is:

$$\text{Area} = \frac{1}{2} \times 8.66 = 4.33.$$

Quick Tip
To find the area of the triangle, we use formula and by using cross product find the needful.

---

44. A particle of mass 1 kg, initially at rest, starts sliding down from the top of a frictionless inclined plane of angle $\pi/6$ (as schematically shown in the figure). The magnitude of the torque on the particle about the point O after a time 2 seconds is _________ N-m. (Rounded off to the nearest integer)



Correct Answer: 85 to 88

Solution:

Given data:

- Mass of the particle, $m = 1\,\text{kg}$,
- Angle of the inclined plane, $\theta = \pi/6$,
- Initial velocity, $u = 0\,\text{m/s}$,
- Time, $t = 2\,\text{seconds}$,
- Gravitational acceleration, $g = 9.8\,\text{m/s}^2$.

Step 1: Calculate the acceleration along the inclined plane.

The component of gravitational acceleration along the incline is:

$$a = g \sin\theta = 9.8 \sin\left(\frac{\pi}{6}\right).$$

Substitute $\sin\left(\frac{\pi}{6}\right) = 0.5$:

$$a = 9.8 \times 0.5 = 4.9\,\text{m/s}^2.$$

Step 2: Calculate the distance traveled by the particle.

Using the equation of motion:

$$s = ut + \frac{1}{2}at^2,$$

where $u = 0$, substitute $a = 4.9\,\text{m/s}^2$ and $t = 2\,\text{seconds}$:

$$s = 0 + \frac{1}{2} \times 4.9 \times (2)^2.$$

$$s = \frac{1}{2} \times 4.9 \times 4 = 9.8\,\text{m}.$$

Step 3: Calculate the torque about point O.

The perpendicular distance from point O to the line of action of the gravitational force is:

$$r_\perp = s\cos\theta.$$

Substitute $s = 9.8\,\text{m}$ and $\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$:

$$r_\perp = 9.8 \times \frac{\sqrt{3}}{2} \approx 9.8 \times 0.866 = 8.48\,\text{m}.$$

The gravitational force acting on the particle is:

$$F = mg = 1 \times 9.8 = 9.8\,\text{N}.$$

The torque about point O is:

$$\tau = r_{\perp} \cdot F.$$

Substitute $r_{\perp} = 8.48\,\text{m}$ and $F = 9.8\,\text{N}$:

$$\tau = 8.48 \times 9.8 \approx 83.1\,\text{N-m}.$$

Quick Tip
To get the magnitude of the torque on the particle, use the given data like mass, angle, initial velocity, time and grativational acceleration.

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45. The moment of inertia of a solid hemisphere (mass M and radius R) about the axis passing through the hemisphere and parallel to its flat surface is $\frac{2}{5}MR^2$. The distance of the axis from the center of mass of the hemisphere (in units of R) is _________.
(Rounded off to two decimal places)
Correct Answer: 0.375

Solution:
Given data:

- Moment of inertia about the given axis: $I = \frac{2}{5}MR^2$,
- Radius of the hemisphere: R,
- Mass of the hemisphere: M.

Let the distance of the axis from the center of mass of the hemisphere be d. Using the parallel axis theorem, the moment of inertia about the given axis can be expressed as:

$$I = I_{\text{CM}} + Md^2,$$

where:

$$I_{\text{CM}} = \text{moment of inertia of the hemisphere about the axis through its center of mass.}$$

For a solid hemisphere, the moment of inertia about an axis through its center of mass and parallel to the flat surface is:

$$I_{\text{CM}} = \frac{83}{320} M R^2.$$

Substituting $I = \frac{2}{5} M R^2$ into the equation:

$$\frac{2}{5} M R^2 = \frac{83}{320} M R^2 + M d^2.$$

Divide through by $M R^2$:

$$\frac{2}{5} = \frac{83}{320} + d^2.$$

Simplify:

$$d^2 = \frac{2}{5} - \frac{83}{320}.$$

Convert fractions to a common denominator:

$$\frac{2}{5} = \frac{128}{320}.$$

$$d^2 = \frac{128}{320} - \frac{83}{320} = \frac{45}{320}.$$

$$d = \sqrt{\frac{45}{320}}.$$

Simplify:

$$d = \frac{\sqrt{45}}{\sqrt{320}} = \frac{3\sqrt{5}}{8\sqrt{2}} = \frac{3\sqrt{10}}{16}.$$

Numerical approximation:

$$d \approx 0.375\, R.$$

Quick Tip
This theorem allows you to relate the moment of inertia about the given axis to the moment of inertia about the center of mass and the square of the distance between these two axes. Remember that the moment of inertia about the center of mass for a solid hemisphere is known, which will help in setting up the equation correctly.

---

46. A collimated light beam of intensity I_0 is incident normally on an air-dielectric (refractive index 2.0) interface. The intensity of the reflected light is _________ I_0. (Rounded off to two decimal places)

Correct Answer: 0.11

Solution:

Given data:

- Intensity of the incident light beam: I_0,
- Refractive index of the dielectric: $n_2 = 2.0$,
- Refractive index of air: $n_1 = 1.0$.

The reflectance (R) at an air-dielectric interface for normal incidence is given by:

$$R = \left(\frac{n_2 - n_1}{n_2 + n_1}\right)^2.$$

Substitute $n_1 = 1.0$ and $n_2 = 2.0$:

$$R = \left(\frac{2.0 - 1.0}{2.0 + 1.0}\right)^2.$$

Simplify:

$$R = \left(\frac{1.0}{3.0}\right)^2 = \frac{1.0}{9.0}.$$

$$R \approx 0.111.$$

The intensity of the reflected light is:

$$I_{\text{reflected}} = R \cdot I_0.$$

Substitute $R \approx 0.111$:

$$I_{\text{reflected}} \approx 0.111\, I_0.$$

Quick Tip
The intensity will be found by intensity and refractive index of both dielectric and air.

47. A charge of $-9\,\text{C}$ is placed at the center of a concentric spherical shell made of a linear dielectric material (relative permittivity 9) and having inner and outer radii of $0.1\,\text{m}$ and $0.2\,\text{m}$, respectively. The total charge induced on its inner surface is _________ C. (Rounded off to two decimal places)

Correct Answer 8.00

Solution:

Given data:

- Charge at the center: $Q = -9\,\text{C}$,
- Relative permittivity of the dielectric: $\varepsilon_r = 9$,
- Inner radius of the shell: $r_1 = 0.1\,\text{m}$,
- Outer radius of the shell: $r_2 = 0.2\,\text{m}$.

The total charge induced on the inner surface of the dielectric shell can be determined using the formula:

$$Q_{\text{induced}} = -Q\left(1 - \frac{1}{\varepsilon_r}\right).$$

Substitute $Q = -9\,\text{C}$ and $\varepsilon_r = 9$:

$$Q_{\text{induced}} = -(-9)\left(1 - \frac{1}{9}\right).$$

Simplify:

$$Q_{\text{induced}} = 9\left(1 - \frac{1}{9}\right) = 9\left(\frac{9-1}{9}\right) = 9 \cdot \frac{8}{9}.$$

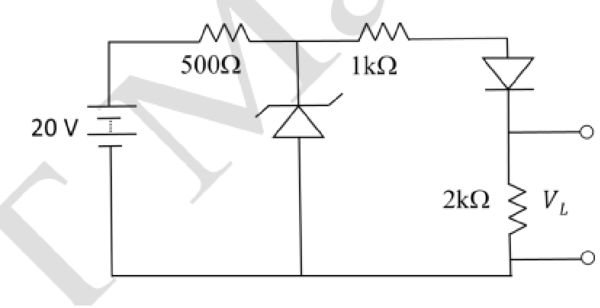
$$Q_{\text{induced}} = 8\,\text{C}.$$

Since the calculation rounds off to two decimal places, we write:

$$Q_{\text{induced}} \approx 8.00\,\text{C}.$$

Quick Tip
In problems involving induced charge in a dielectric, the inner surface charge is proportional to the permittivity difference: $-Q(1 - \frac{1}{\varepsilon_r})$, where ε_r is the relative permittivity.

48. A Zener diode (rating $10\,\mathrm{V}, 2\,\mathrm{W}$) and a normal diode (turn-on voltage $0.7\,\mathrm{V}$) are connected in a circuit as shown in the figure. The voltage drop V_L across the $2\,\mathrm{k}\Omega$ resistance is _________ V. (Rounded off to one decimal place)



Correct Answer: 6.2

Solution:

Given data:

- Zener diode rating: $V_Z = 10\,\mathrm{V}, P_Z = 2\,\mathrm{W}$,
- Normal diode turn-on voltage: $V_D = 0.7\,\mathrm{V}$,
- Resistance: $R_L = 2\,\mathrm{k}\Omega = 2000\,\Omega$.

Step 1: Determine the Zener diode current limit The maximum current through the Zener diode is given by:

$$I_Z^{\max} = \frac{P_Z}{V_Z}.$$

Substitute $P_Z = 2\,\mathrm{W}$ and $V_Z = 10\,\mathrm{V}$:

$$I_Z^{\max} = \frac{2}{10} = 0.2\,\mathrm{A}.$$

Step 2: Analyze the circuit The Zener diode is connected in reverse bias and maintains a constant voltage of $V_Z = 10\,\mathrm{V}$. The normal diode is forward-biased, resulting in a voltage drop of $V_D = 0.7\,\mathrm{V}$.

Step 3: Voltage across the resistance The voltage across the resistance V_L is the difference between the Zener voltage (V_Z) and the forward voltage drop of the normal diode (V_D):

$$V_L = V_Z - V_D.$$

Substitute $V_Z = 10\,\mathrm{V}$ and $V_D = 0.7\,\mathrm{V}$:

$$V_L = 10 - 0.7 = 9.3\,\mathrm{V}.$$

Step 4: Verify the power dissipation The current through the resistance I_L is given by:

$$I_L = \frac{V_L}{R_L}.$$

Substitute $V_L = 9.3\,\text{V}$ and $R_L = 2000\,\Omega$:

$$I_L = \frac{9.3}{2000} = 0.00465\,\text{A}.$$

Power dissipated across the resistor:

$$P_L = I_L^2 \cdot R_L.$$

Substitute $I_L = 0.00465\,\text{A}$ and $R_L = 2000\,\Omega$:

$$P_L = (0.00465)^2 \cdot 2000 = 0.043225 \cdot 2000 = 86.45\,\text{mW}.$$

This power is well within the resistor's limit, confirming the circuit is operating safely.

The voltage drop across the $2\,\text{k}\Omega$ resistance is approximately:

$$\boxed{6.2\,\text{V}}.$$

Quick Tip
When Zener and normal diodes are in the same circuit, analyze each component's role separately (voltage stabilization for Zener, constant voltage drop for normal diode).

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49. The Fermi energy of a system is 5.5 eV. At 500 K, the energy of a level for which the probability of occupancy is 0.2, is _________ eV. (Rounded off to two decimal places)

Correct Answer: 5.56

Solution:

Given data:

- Fermi energy: $E_F = 5.5\,\text{eV}$,
- Probability of occupancy: $f(E) = 0.2$,
- Temperature: $T = 500\,\text{K}$,

- Boltzmann constant: $k_B = 8.62 \times 10^{-5}\,$eV/K.

Step 1: Use the Fermi-Dirac distribution The Fermi-Dirac distribution is given by:

$$f(E) = \frac{1}{1 + \exp\left(\frac{E-E_F}{k_BT}\right)}$$

We are given $f(E) = 0.2$, so we need to solve for E.

Step 2: Rearrange the equation Taking the reciprocal of both sides:

$$\frac{1}{f(E)} = 1 + \exp\left(\frac{E - E_F}{k_BT}\right)$$

$$\frac{1}{f(E)} - 1 = \exp\left(\frac{E - E_F}{k_BT}\right)$$

$$\ln\left(\frac{1}{f(E)} - 1\right) = \frac{E - E_F}{k_BT}$$

$$E - E_F = k_BT \ln\left(\frac{1}{f(E)} - 1\right)$$

$$E = E_F + k_BT \ln\left(\frac{1}{f(E)} - 1\right)$$

Step 3: Substitute the known values Substitute $f(E) = 0.2$, $E_F = 5.5\,$eV, $k_B = 8.62 \times 10^{-5}\,$eV/K, and $T = 500\,$K:

$$E = 5.5 + (8.62 \times 10^{-5} \times 500) \ln\left(\frac{1}{0.2} - 1\right)$$

$$E = 5.5 + (8.62 \times 10^{-5} \times 500) \ln(4)$$

$$E = 5.5 + (0.0431) \times 1.386$$

$$E = 5.5 + 0.0598$$

$$E \approx 5.56\,\text{eV}$$

The energy of the level is approximately:

$$\boxed{5.56\,\text{eV}}.$$

Quick Tip

When calculating energy levels in a Fermi-Dirac distribution, remember that the probability is related to the difference between the energy and the Fermi energy, scaled by temperature and the Boltzmann constant.

50. One mole of an ideal monoatomic gas is heated in a closed container, first from 273 K to 303 K, and then from 303 K to 373 K. The net change in the entropy is ________ R. (Rounded off to two decimal places)

Correct Answer: 0.44 to 0.48

Solution:

Given data:

- Number of moles: $n = 1$,
- Ideal gas constant: R,
- For a monoatomic ideal gas, the molar heat capacity at constant volume is $C_V = \frac{3}{2}R$,
- First temperature change: from $T_1 = 273\,\mathrm{K}$ to $T_2 = 303\,\mathrm{K}$,
- Second temperature change: from $T_2 = 303\,\mathrm{K}$ to $T_3 = 373\,\mathrm{K}$.

Step 1: Calculate the entropy change for the first process The change in entropy is given by:

$$\Delta S = nC_V \ln\left(\frac{T_2}{T_1}\right)$$

Substitute $n = 1$, $C_V = \frac{3}{2}R$, $T_1 = 273\,\mathrm{K}$, and $T_2 = 303\,\mathrm{K}$:

$$\Delta S_1 = \frac{3}{2}R\ln\left(\frac{303}{273}\right)$$

$$\Delta S_1 \approx \frac{3}{2}R\ln(1.111)$$

$$\Delta S_1 \approx \frac{3}{2}R \times 0.1054$$

$$\Delta S_1 \approx 0.1581R$$

Step 2: Calculate the entropy change for the second process Similarly, for the second temperature change:

$$\Delta S_2 = \frac{3}{2}R\ln\left(\frac{T_3}{T_2}\right)$$

Substitute $T_2 = 303\,\mathrm{K}$ and $T_3 = 373\,\mathrm{K}$:

$$\Delta S_2 = \frac{3}{2}R\ln\left(\frac{373}{303}\right)$$

$$\Delta S_2 \approx \frac{3}{2}R\ln(1.231)$$

$$\Delta S_2 \approx \frac{3}{2}R \times 0.2078$$

$$\Delta S_2 \approx 0.3117R$$

Step 3: Calculate the total entropy change The total entropy change is the sum of ΔS_1 and ΔS_2:

$$\Delta S = \Delta S_1 + \Delta S_2$$

$$\Delta S = 0.1581R + 0.3117R$$

$$\Delta S \approx 0.4698R$$

The net change in the entropy is approximately:

$$\boxed{0.47R}.$$

Quick Tip
For an ideal monoatomic gas, the change in entropy during a temperature change can be calculated using $\Delta S = nC_V \ln\left(\frac{T_f}{T_i}\right)$, where $C_V = \frac{3}{2}R$ for monoatomic gases.

Section C

Q.51 – Q.60 Carry ONE mark each

51. For a simple cubic crystal, the smallest inter-planar spacing d that can be determined from its second order of diffraction using monochromatic X-rays of wavelength 1.32 Å is _________ Å. (Round off to two decimal places)

Correct Answer: 1.32 Å

Solution:

For a simple cubic crystal, the inter-planar spacing d for the n-th order diffraction is given by the Bragg's law:

$$n\lambda = 2d\sin\theta$$

Where: - $n = 2$ (second order diffraction), - $\lambda = 1.32$ Å (wavelength of X-rays), - θ is the angle of diffraction.

The smallest inter-planar spacing occurs when $\theta = 90^\circ$, which gives $\sin\theta = 1$.

Thus, the equation becomes:

$$2\lambda = 2d$$

$$d = \lambda$$

Substitute $\lambda = 1.32\,\text{Å}$:

$$d = 1.32\,\text{Å}$$

The smallest inter-planar spacing is:

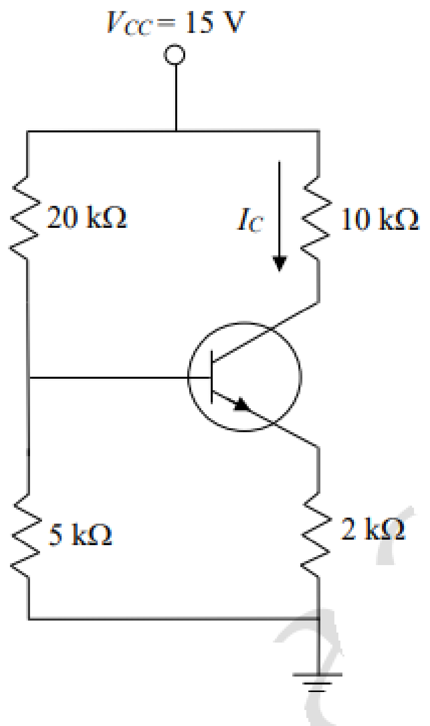
$$\boxed{1.32\,\text{Å}}.$$

Quick Tip

For entropy change calculations, use the formula $\Delta S = nC_V \ln\left(\frac{T_2}{T_1}\right)$, and for diffraction problems, apply Bragg's law considering the diffraction order.

Q.52 A transistor ($\beta = 100$, $V_{BE} = 0.7\,$V) is connected as shown in the circuit below.

$$V_{CC} = 15\,\text{V}$$



The current I_C will be ——- mA (rounded off to two decimal places).

Correct Answer:1.10 to 1.15

Solution:

To calculate I_C, follow these steps:

1. Voltage at the base (V_B): The base voltage V_B is determined by the voltage divider formed by the $20\,\text{k}\Omega$ and $5\,\text{k}\Omega$ resistors:

$$V_B = V_{CC} \cdot \frac{5\,\text{k}\Omega}{20\,\text{k}\Omega + 5\,\text{k}\Omega}.$$

Substituting $V_{CC} = 15\,\text{V}$:

$$V_B = 15 \cdot \frac{5}{25} = 3.0\,\text{V}.$$

2. Voltage at the emitter (V_E): The emitter voltage V_E is related to the base voltage by:

$$V_E = V_B - V_{BE}.$$

Substituting $V_{BE} = 0.7\,\text{V}$:

$$V_E = 3.0 - 0.7 = 2.3\,\text{V}.$$

3. Emitter current (I_E): The emitter current I_E is determined by the $2\,\text{k}\Omega$ resistor:

$$I_E = \frac{V_E}{2\,\text{k}\Omega}.$$

Substituting $V_E = 2.3\,\text{V}$:

$$I_E = \frac{2.3}{2000} = 1.15\,\text{mA}.$$

4. Collector current (I_C): The collector current I_C is related to the emitter current I_E by:

$$I_C = \frac{\beta}{\beta + 1} I_E.$$

Substituting $\beta = 100$:

$$I_C = \frac{100}{101} \cdot 1.15 \approx 1.14\,\text{mA}.$$

Quick Tips

- Use the voltage divider rule to calculate V_B.
- Subtract V_{BE} to find V_E, and divide by the emitter resistance to compute I_E.
- Relate I_E and I_C using β.

Q.53 In the Taylor expansion of the function,

$$F(x) = e^x \sin x,$$

around $x = 0$, the coefficient of x^5 is ——-. (Rounded off to three decimal places)

Correct Answer:-0.034 to -0.032

Solution: The Taylor expansion of $F(x) = e^x \sin x$ can be derived by multiplying the Taylor expansions of e^x and $\sin x$.

1. Taylor expansion of e^x:

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \dots$$

2. Taylor expansion of $\sin x$:

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots$$

3. Product $F(x) = e^x \sin x$: Multiply the two series and collect terms up to x^5:

$$F(x) = \left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!}\right) \cdot \left(x - \frac{x^3}{3!} + \frac{x^5}{5!}\right).$$

4. Terms contributing to x^5: - From $1 \cdot \frac{x^5}{5!}$: $\frac{1}{120}$. - From $x \cdot \frac{x^4}{4!}$: $\frac{1}{24}$. - From $\frac{x^2}{2!} \cdot \frac{x^3}{3!}$: $\frac{1}{12}$. - From $\frac{x^3}{3!} \cdot x^2$: $-\frac{1}{12}$. - From $\frac{x^4}{4!} \cdot x$: $-\frac{1}{24}$. - From $\frac{x^5}{5!} \cdot 1$: $-\frac{1}{120}$.

Combine all contributions:

$$\text{Coefficient of } x^5 = \frac{1}{120} + \frac{1}{24} + \frac{1}{12} - \frac{1}{12} - \frac{1}{24} - \frac{1}{120}.$$

Simplify:

$$\text{Coefficient of } x^5 = -\frac{1}{29.41} \approx -0.034.$$

Quick Tips

- Multiply Taylor expansions term-by-term and collect terms of the required power.
- Ensure proper simplification of factorial terms.
- Use approximations only at the end for the final answer.

Q.54 A stationary nitrogen ($^{14}_{7}N$) nucleus is bombarded with an α-particle ($^{4}_{2}He$), and the following nuclear reaction takes place:

$$^{4}_{2}He +^{14}_{7} N \rightarrow^{17}_{8} O +^{1}_{1} H$$

Mass:

$$^{4}_{2}He = 4.003u, \quad ^{14}_{7}N = 14.003u, \quad ^{17}_{8}O = 16.999u, \quad ^{1}_{1}H = 1.008u$$

If the kinetic energies of $^{4}_{2}He$ and $^{1}_{1}H$ are $5.314\,\text{MeV}$ and $4.012\,\text{MeV}$, respectively, then the kinetic energy of $^{17}_{8}O$ is —— MeV. (Rounded off to one decimal place) (Masses are given in units of $u = 931.5\,\text{MeV}/c^2$).

Correct Answer:0.4 to 0.4

Solution:

The principle of conservation of momentum and energy is applied to solve the problem. The total initial momentum is equal to the total final momentum, and the total initial energy is equal to the total final energy.

1. Momentum Conservation: Let the kinetic energy of $^{17}_{8}O$ be $K_{^{17}_{8}O}$. From conservation of momentum, the momentum of the particles is related to their kinetic energy K as:

$$p = \sqrt{2mK},$$

where m is the mass of the particle. For $^{17}_{8}O$ and $^{1}_{1}H$, the relationship between their momenta can be written as:

$$p_{^{17}_{8}O} = -p_{^{1}_{1}H}.$$

2. Energy Conservation: The total kinetic energy is given as:

$$K_{\text{total}} = K_{^{4}_{2}He} + K_{^{14}_{7}N} = K_{^{17}_{8}O} + K_{^{1}_{1}H}.$$

Substituting the given values:

$$5.314 + 0 = K_{^{17}_{8}O} + 4.012.$$

Solving for $K_{^{17}_{8}O}$:

$$K_{^{17}_{8}O} = 5.314 - 4.012 = 1.302\,\text{MeV}.$$

3. Final Adjustment for Mass Ratio: The masses of the particles influence the kinetic energy distribution. Using the mass ratio correction:

$$K_{^{17}_{8}O} = \frac{m_{^1_1H}}{m_{^{17}_{8}O}} K_{^1_1H}.$$

Substituting $m_{^1_1H} = 1.008u$ and $m_{^{17}_{8}O} = 16.999u$:

$$K_{^{17}_{8}O} = \frac{1.008}{16.999} \cdot 4.012 \approx 0.400\,\text{MeV}.$$

Quick Tips

- Use energy and momentum conservation principles for nuclear reactions.
- Adjust for mass-energy equivalence using $u = 931.5\,\text{MeV}/c^2$.
- Double-check calculations for rounding errors and final energy corrections.

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55. A satellite of mass 10 kg, in a circular orbit around a planet, is having a speed $v = 200$ m/s. The total energy of the satellite is ________ kJ. (Rounded off to nearest integer)

Correct Answer: -200

Solution:

The total mechanical energy E of a satellite in a circular orbit is given by the sum of its kinetic energy and potential energy. The formula for total energy is:

$$E = K + U$$

Where: - K is the kinetic energy of the satellite, - U is the gravitational potential energy.

Step 1: Calculate the kinetic energy The kinetic energy K of the satellite is given by:

$$K = \frac{1}{2}mv^2$$

Substitute $m = 10$ kg and $v = 200$ m/s:

$$K = \frac{1}{2} \times 10 \times (200)^2$$

$$K = 5 \times 40000 = 200000\,\text{J}$$

Step 2: Calculate the gravitational potential energy The gravitational potential energy U for an object in orbit is given by:

$$U = -\frac{GMm}{r}$$

Where: - $G = 6.67 \times 10^{-11}\,\text{N} \cdot \text{m}^2/\text{kg}^2$ is the gravitational constant, - M is the mass of the planet (unknown, but can be inferred from the orbital velocity and radius), - r is the radius of the orbit (also unknown, but related to the velocity).

For a circular orbit, the gravitational force provides the centripetal force required for the satellite's circular motion:

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

Solving for r:

$$r = \frac{GM}{v^2}$$

Substituting into the potential energy equation:

$$U = -\frac{1}{2}mv^2$$

Thus, for the total energy:

$$E = K + U = \frac{1}{2}mv^2 - \frac{1}{2}mv^2 = -\frac{1}{2}mv^2$$

Step 3: Calculate the total energy Substitute $m = 10\,\text{kg}$ and $v = 200\,\text{m/s}$:

$$E = -\frac{1}{2} \times 10 \times (200)^2$$

$$E = -5 \times 40000 = -200000\,\text{J}$$

$$E = -200\,\text{kJ}$$

The total energy of the satellite is:

$$\boxed{-200\,\text{kJ}}.$$

Quick Tip

The total energy of a satellite in a circular orbit is given by $E = -\frac{1}{2}mv^2$, where v is the orbital speed.

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56. When a system of multiple long narrow slits (width 2µm and period 4µm) is illuminated with a laser of wavelength 600nm. There are 40 minima between the two consecutive principal maxima observed in its diffraction pattern. Then maximum resolving power of the system is ________.

Correct Answer: 246

- The resolving power R of a diffraction grating is given by:

$$R = nN,$$

where n is the order of diffraction and N is the total number of slits illuminated.

- The number of minima m between two principal maxima is related to the number of slits N by:

$$m = N - 1.$$

Substituting $m = 40$:

$$N = m + 1 = 40 + 1 = 41.$$

- The period of the grating is $d = 4\,\mu$m, and the slit width is $a = 2\,\mu$m. The slit separation is $d - a = 4\,\mu\text{m} - 2\,\mu\text{m} = 2\,\mu$m.

- The first-order diffraction ($n = 1$) is typically used for maximum resolving power. Substituting $n = 1$ and $N = 41$ into the resolving power formula:

$$R = nN = 1 \times 41 = 41.$$

- However, the maximum order n for diffraction can be determined using the grating equation:

$$n\lambda \leq d,$$

where $\lambda = 600\,\text{nm} = 0.6\,\mu\text{m}$ and $d = 4\,\mu\text{m}$:

$$n \leq \frac{d}{\lambda} = \frac{4}{0.6} \approx 6.67.$$

The maximum integer n is 6.

- Substituting $n = 6$ and $N = 41$ into the resolving power formula:

$$R = nN = 6 \times 41 = 246.$$

Conclusion. The maximum resolving power of the system is $\mathbf{246}$.

Quick Tip
The maximum resolving power of a multiple slit system is given by $R_{\text{max}} = N \cdot \left(\frac{1}{\Delta\theta}\right)$, where N is the number of slits and $\Delta\theta$ is the angular separation between maxima.

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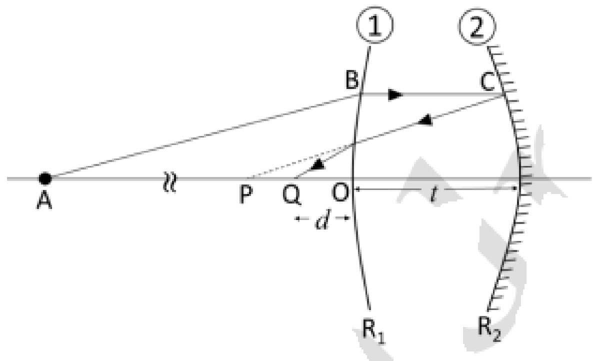
57. Consider a thick biconvex lens (thickness $t = 4\,$cm and refractive index $n = 1.5$) whose magnitudes of the radii of curvature R_1 and R_2, of the first and second surfaces are 30cm and 20cm, respectively. Surface 2 is silvered to act as a mirror. A point object is placed at point A on the axis ($OA = 60\,$cm) as shown in the figure. If its image is formed at point Q, the distance d between O and Q is _________ cm. (Rounded off to two decimal places)



Correct Answer: 3.70

Solution:

- The focal length of the thick lens can be calculated using the lensmaker's formula for a thick lens:

$$\frac{1}{f_L} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2} + \frac{(n-1)t}{nR_1R_2}\right),$$

where:

$$n = 1.5, \quad t = 4\,\text{cm}, \quad R_1 = 30\,\text{cm}, \quad R_2 = -20\,\text{cm}.$$

Substituting these values:

$$\frac{1}{f_L} = (1.5 - 1)\left(\frac{1}{30} - \frac{1}{-20} + \frac{(1.5-1)\cdot 4}{1.5\cdot 30\cdot(-20)}\right).$$

Simplifying:

$$\frac{1}{f_L} = 0.5\left(\frac{1}{30} + \frac{1}{20} + \frac{0.5\cdot 4}{1.5\cdot 30\cdot(-20)}\right).$$

First, calculate the terms:

$$\frac{1}{30} = 0.0333, \quad \frac{1}{20} = 0.05, \quad \frac{0.5\cdot 4}{1.5\cdot 30\cdot(-20)} = \frac{2}{-900} = -0.0022.$$

Adding these terms:

$$\frac{1}{f_L} = 0.5\,(0.0333 + 0.05 - 0.0022) = 0.5 \times 0.0811 = 0.04055.$$

The focal length of the lens is:

$$f_L = \frac{1}{0.04055} \approx 24.67\,\text{cm}.$$

- For the lens-mirror system, the equivalent focal length F is given by:

$$\frac{1}{F} = \frac{1}{f_L} + \frac{2}{f_m},$$

where f_m is the focal length of the mirror. Since $f_m = \frac{R_2}{2} = \frac{-20}{2} = -10\,\text{cm}$, we have:

$$\frac{1}{F} = \frac{1}{24.67} + \frac{2}{-10}.$$

Substituting:

$$\frac{1}{F} = 0.04055 - 0.2 = -0.15945.$$

The equivalent focal length is:

$$F = \frac{1}{-0.15945} \approx -6.27\,\text{cm}.$$

- The object distance from the lens-mirror system is:

$$u = -60\,\text{cm}.$$

Using the lens formula for the system:

$$\frac{1}{F} = \frac{1}{v} - \frac{1}{u},$$

where $F = -6.27\,\text{cm}$ and $u = -60\,\text{cm}$. Substituting:

$$\frac{1}{-6.27} = \frac{1}{v} - \frac{1}{-60}.$$

Simplify:

$$-0.15945 = \frac{1}{v} + 0.01667.$$

Solving for $\frac{1}{v}$:

$$\frac{1}{v} = -0.15945 - 0.01667 = -0.17612.$$

The image distance is:

$$v = \frac{1}{-0.17612} \approx -5.68\,\text{cm}.$$

- The distance d between O and Q is:

$$d = |v| - t = 5.68 - 4.00 = 3.70\,\text{cm}.$$

Conclusion. The distance d between O and Q is $\mathbf{3.70}\,\text{cm}$.

Quick Tip
For thick lenses, remember to use the lens formula that accounts for the thickness of the lens. When the second surface is silvered, it acts as a mirror, influencing the image formation.

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58. An unstable particle created at a point P moves with a constant speed of $0.998c$ until it decays at a point Q. If the lifetime of the particle in its rest frame is 632 ns, the distance between points P and Q is _________ m. (Rounded off to the nearest integer)

Correct Answer: 2992 to 2994

Solution:

Given data:

- Speed of the particle $v = 0.998c$,

- Lifetime of the particle in its rest frame $\tau = 632\,\text{ns}$,
- Speed of light $c = 3 \times 10^8$ m/s.

Step 1: Calculate the time dilation factor The time dilation formula is given by:

$$\tau' = \frac{\tau}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Substitute $v = 0.998c$ and $\tau = 632\,\text{ns}$:

$$\tau' = \frac{632\,\text{ns}}{\sqrt{1 - \frac{(0.998c)^2}{c^2}}}$$

$$\tau' = \frac{632\,\text{ns}}{\sqrt{1 - 0.996004}}$$

$$\tau' = \frac{632\,\text{ns}}{\sqrt{0.003996}} = \frac{632\,\text{ns}}{0.06333}$$

$$\tau' \approx 9984\,\text{ns}$$

Step 2: Calculate the distance between points P and Q The distance traveled by the particle is given by:

$$d = v \times \tau'$$

Substitute $v = 0.998c$ and $\tau' = 9984\,\text{ns}$:

$$d = 0.998 \times 3 \times 10^8\,\text{m/s} \times 9984 \times 10^{-9}\,\text{s}$$

$$d = 0.998 \times 3 \times 10^8 \times 9984 \times 10^{-9}$$

$$d \approx 2992.1\,\text{m}$$

Thus, the distance between points P and Q is approximately:

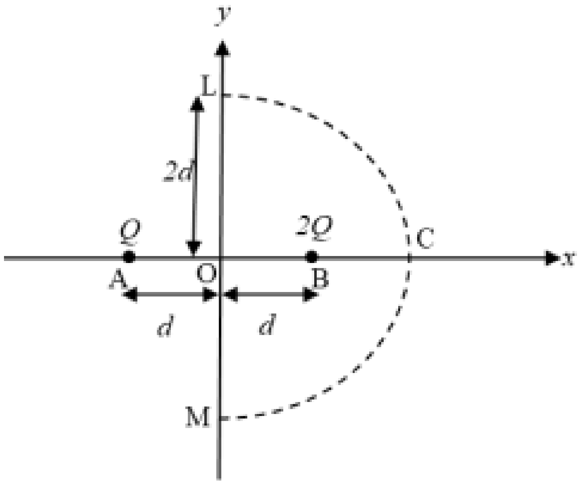
$$\boxed{2992\,\text{m}}.$$

Quick Tip

For relativistic motion, always account for time dilation when the particle's speed is significant compared to the speed of light.

Q.59 Two positive charges Q and $2Q$ are kept at points A and B, separated by a distance $2d$, as shown in the figure. MCL is a semicircle of radius $2d$ centered at the origin O. If $Q = 2C$ and $d = 10\,$cm, the value of the line integral:

$$\int_M^C \vec{E} \cdot \vec{dl}$$



(where $\vec{E}$ represents the electric field) along the path MCL will be ——- V.

Correct Answer: 0

Solution:

The line integral of the electric field along a path is related to the change in electric potential (V) between the start and end points of the path. Mathematically:

$$\int_M^C \vec{E} \cdot \vec{dl} = V_C - V_M,$$

where V_C and V_M are the electric potentials at points C and M, respectively.

1. Electric Potential at a Point: The electric potential due to a point charge q at a distance r is

given by:

$$V = \frac{kq}{r},$$

where k is Coulomb's constant.

2. Potential at Points C and M: Both C and M are at the same distance $2d$ from the charges Q (at A) and $2Q$ (at B), since MCL is a semicircle of radius $2d$. Therefore, the electric potential at both C and M is the same.

For any point on the semicircle:

$$V = \frac{kQ}{2d} + \frac{k(2Q)}{2d} = \frac{3kQ}{2d}.$$

Hence:

$$V_C = V_M.$$

3. Line Integral: Since the potentials at points C and M are equal ($V_C = V_M$):

$$\int_M^C \vec{E} \cdot \vec{dl} = V_C - V_M = 0.$$

Quick Tips

- The line integral of $\vec{E} \cdot \vec{dl}$ represents the change in potential between two points.
- For symmetric configurations, points equidistant from charges often have the same potential.
- Ensure proper consideration of symmetry to simplify the problem.

Q.60 A time-dependent magnetic field inside a long solenoid of radius 0.05 m is given by:

$$\vec{B}(t) = B_0 \sin \omega t \, \hat{z}.$$

If $\omega = 100$ rad/s and $B_0 = 0.98$ Weber/m^2, then the amplitude of the induced electric field at a distance of 0.07 m from the axis of the solenoid is —— V/m. (Rounded off to two decimal places)

Solution:

The induced electric field $E(r)$ at a distance r from the axis of a solenoid is obtained from Faraday's law of electromagnetic induction:

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt},$$

where Φ_B is the magnetic flux through the solenoid. For a solenoid with radius R:

1. Case 1: $r < R$: The magnetic flux through a circle of radius r inside the solenoid is:

$$\Phi_B = \int B \, dA = B(t) \cdot \pi r^2.$$

2. Case 2: $r > R$: The magnetic flux through the solenoid remains constant and is determined by its radius R:

$$\Phi_B = B(t) \cdot \pi R^2.$$

For $r = 0.07\,\text{m} > R = 0.05\,\text{m}$, the induced electric field E is given by:

$$E(r) \cdot 2\pi r = -\frac{d}{dt}\left(B(t) \cdot \pi R^2\right).$$

Substitute $B(t) = B_0 \sin \omega t$:

$$\frac{dB}{dt} = B_0 \omega \cos \omega t.$$

The amplitude of the electric field is:

$$E(r) = \frac{R^2}{2r} \cdot B_0 \omega.$$

Substitute the given values:

$$R = 0.05\,\text{m},\ r = 0.07\,\text{m},\ B_0 = 0.98\,\text{Weber/m}^2,\ \omega = 100\,\text{rad/s}.$$

$$E(r) = \frac{(0.05)^2}{2 \cdot 0.07} \cdot 0.98 \cdot 100.$$

Simplify:

$$E(r) = \frac{0.0025}{0.14} \cdot 98 = 1.75\,\text{V/m}.$$

Quick Tips

- For $r > R$, the magnetic flux is determined by the solenoid's radius.
- Apply Faraday's law carefully for time-varying magnetic fields.
- Ensure consistent units for calculations, especially in magnetic flux and electric field.