ICSE Class 10 Mathematics 2026 Question Paper with Solutions

Time Allowed :3 Hours	Maximum Marks :80	Total Questions :22

General Instructions

1. *Answers to this Paper must be written on the paper provided separately.*
2. *You will not be allowed to write during the first 15 minutes.*
3. *This time is to be spent in reading the question paper.*
4. ***The time given at the head of this Paper is the time allowed for writing the answers.***
5. *Attempt all questions from **Section A** and any four questions from **Section B**.*
6. *The intended marks for questions or parts of questions are given in brackets[ ].*

Section A - 65 Marks

1(i). If A and B are square matrices of order 3, A is non-singular matrix and AB = O, then the matrix B is:

(A) unit matrix
(B) Scalar matrix
(C) non-singular matrix
(D) null matrix

Correct Answer: (D) null matrix

Solution:

Step 1: Understanding the Concept:
A non-singular matrix is a square matrix that has a non-zero determinant ($|A| \neq 0$).
Because its determinant is non-zero, it is invertible, meaning there exists a matrix A^{-1} such that $A^{-1}A = I$.

Step 2: Key Formula or Approach:
We use the property of matrix inverses and the null matrix property.
If $AB = \mathbf{O}$, and A is invertible, we can isolate B by pre-multiplying by A^{-1}.

Step 3: Detailed Explanation:
Given the equation:

$$AB = \mathbf{O}$$

Since A is non-singular, A^{-1} exists. Pre-multiplying both sides by A^{-1}:

$$A^{-1}(AB) = A^{-1}\mathbf{O}$$

By the associative property of matrix multiplication:

$$(A^{-1}A)B = \mathbf{O}$$

Since $A^{-1}A = I$ (the identity matrix):

$$IB = \mathbf{O}$$

The product of the identity matrix and any matrix B is B itself:

$$B = \mathbf{O}$$

Therefore, B must be a null matrix of order 3.

Step 4: Final Answer:
The matrix B is the null matrix.

> **Quick Tip**
>
> If the product of two matrices is zero and one is non-singular, the other must be zero. If both are singular, they can produce a zero matrix without either being zero themselves.

1(ii). If m and n are respectively the order and degree of the differential equation $\frac{d}{dx}\left(\frac{dy}{dx}\right)^3 = 0$, then the value of $(m - n)$ is:

(A) 0
(B) 1
(C) 2
(D) 3

Correct Answer: (B) 1

Solution:

Step 1: Understanding the Concept:
Order (m) is the highest order derivative present in the equation.
Degree (n) is the power of the highest order derivative after the equation is made free from

radicals and fractions.

Step 2: Key Formula or Approach:
Perform the differentiation given in the operator form to reveal the actual derivatives.
Use the chain rule: $\frac{d}{dx}[g(x)]^k = k[g(x)]^{k-1} \cdot g'(x)$.

Step 3: Detailed Explanation:
Expand the given expression:

$$\frac{d}{dx}\left(\frac{dy}{dx}\right)^3 = 0$$

Applying the power rule and chain rule:

$$3\left(\frac{dy}{dx}\right)^{3-1} \cdot \frac{d}{dx}\left(\frac{dy}{dx}\right) = 0$$

$$3\left(\frac{dy}{dx}\right)^2 \cdot \frac{d^2y}{dx^2} = 0$$

Now, identify the order and degree:
1. The highest order derivative is $\frac{d^2y}{dx^2}$. Hence, Order $m = 2$.
2. The power of this highest order derivative is 1. Hence, Degree $n = 1$.
Calculate $m - n$:

$$m - n = 2 - 1 = 1$$

Step 4: Final Answer:
The value of $m - n$ is 1.

Quick Tip

Never determine the order and degree until all differentiation operators (like $\frac{d}{dx}$) acting on terms have been evaluated.

1(iii). The derivative of $xy = c^2$ with respect to x is:

(A) $\frac{dy}{dx} = \frac{2c-y}{x}$
(B) $\frac{dy}{dx} = \frac{c^2}{x^2}$
(C) $\frac{dy}{dx} = \frac{-y}{x}$
(D) $\frac{dy}{dx} = \frac{y}{x}$

Correct Answer: (C) $\frac{dy}{dx} = \frac{-y}{x}$

Solution:

Step 1: Understanding the Concept:
This requires implicit differentiation since y is not explicitly separated.

Step 2: Key Formula or Approach:
Use the Product Rule for differentiation: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$.

Step 3: Detailed Explanation:
Given: $xy = c^2$.
Differentiate both sides with respect to x:

$$\frac{d}{dx}(xy) = \frac{d}{dx}(c^2)$$

Using the product rule on the LHS and noting that c^2 is a constant:

$$x \cdot \frac{dy}{dx} + y \cdot \frac{d}{dx}(x) = 0$$

$$x\frac{dy}{dx} + y(1) = 0$$

Isolate the term with the derivative:

$$x\frac{dy}{dx} = -y$$

Divide by x:

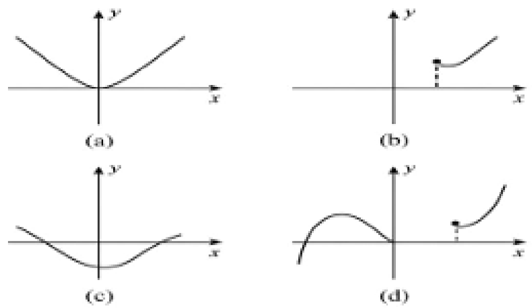
$$\frac{dy}{dx} = \frac{-y}{x}$$

Step 4: Final Answer:
The derivative is $\frac{dy}{dx} = -\frac{y}{x}$.

Quick Tip

For any rectangular hyperbola $xy = k$, the slope of the tangent at any point (x, y) is always $-y/x$.

1(vii). Observe the following graphs (a), (b), (c), and (d), each representing different types of functions.



Statement 1: A function which is continuous at a point may not be differentiable at that point.
Statement 2: Graph (c) is an example of a function that is continuous but not differentiable at the origin.
Which of the following is correct?

(A) Statement 1 is true and Statement 2 is false.
(B) Statement 2 is true and Statement 1 is false.
(C) Both the statements are true.
(D) Both the statements are false.

Correct Answer: (C) Both the statements are true.

Solution:

Step 1: Understanding the Concept:
Continuity implies no breaks in the graph.
Differentiability implies smoothness (no sharp corners) in the graph.

Step 2: Detailed Explanation:
Analysis of Statement 1: Differentiability implies continuity, but continuity does not necessarily imply differentiability. A function like $f(x) = |x|$ is continuous everywhere but lacks a derivative at $x = 0$ due to the sharp turn. Statement 1 is True.
Analysis of Statement 2: Graph (c) shows a V-shaped curve (like absolute value). It is continuous at the origin because the line is unbroken. However, it has a sharp "corner" or "kink" at the origin. At such points, the left-hand derivative and right-hand derivative are different, so the function is not differentiable there. Statement 2 is True.
Since both are true, (C) is the correct choice.

Step 3: Final Answer:
Both statements are correct.

Quick Tip

Sharp turns, cusps, or vertical tangents on a continuous graph indicate points where the function is not differentiable.

1(viii). If $f(x) = kx^2 + 7x - 4$ and $f'(5) = 97$ then what is the value of k?

(A) -4
(B) 0
(C) 4
(D) 9

Correct Answer: (D) 9

Solution:

Step 1: Understanding the Concept:
The derivative $f'(x)$ is the slope function of the curve $f(x)$.

Step 2: Key Formula or Approach:
Use the power rule for differentiation: $\frac{d}{dx}(x^n) = nx^{n-1}$.

Step 3: Detailed Explanation:
First, find the general derivative of the function $f(x) = kx^2 + 7x - 4$:

$$f'(x) = \frac{d}{dx}(kx^2) + \frac{d}{dx}(7x) - \frac{d}{dx}(4)$$

$$f'(x) = 2kx + 7$$

Now, substitute the given value $x = 5$ into the derivative equation:

$$f'(5) = 2k(5) + 7$$

Given that $f'(5) = 97$, we set up the equation:

$$10k + 7 = 97$$

Solve for k:

$$10k = 97 - 7$$

$$10k = 90$$

$$k = 9$$

Step 4: Final Answer:
The value of k is 9.

Quick Tip
Differentiate polynomials term by term and ensure constants vanish before substituting values for x.

1(ix). Statement 1: If a relation R on a set A satisfies $R = R^{-1}$, then R is symmetric.
Statement 2: For a relation R to be symmetric, it is necessary that $R = R^{-1}$.
Which one of the following is correct?

(A) Statement 1 is true and Statement 2 is false.
(B) Statement 2 is true and Statement 1 is false.
(C) Both the statements are true.
(D) Both the statements are false.

Correct Answer: (C) Both the statements are true.

Solution:

Step 1: Understanding the Concept:
A relation R is symmetric if for every $(a, b) \in R$, we have $(b, a) \in R$.
The inverse relation R^{-1} is defined as $\{(b, a) : (a, b) \in R\}$.

Step 2: Detailed Explanation:
Evaluation of Statement 1: If $R = R^{-1}$, then for any $(a, b) \in R$, we must have $(a, b) \in R^{-1}$. By definition of inverse relation, if $(a, b) \in R^{-1}$, then $(b, a) \in R$. Thus, $(a, b) \in R \implies (b, a) \in R$, which makes it symmetric. Statement 1 is True.
Evaluation of Statement 2: If R is symmetric, then for every $(a, b) \in R$, the pair (b, a) is also in R. This implies that every element of R^{-1} is in R and vice-versa. Therefore, $R = R^{-1}$ is a necessary and sufficient condition for symmetry. Statement 2 is True.

Step 3: Final Answer:
Both statements are correct definitions/properties of symmetric relations.

💡 Quick Tip

For a relation represented by a matrix, symmetry corresponds to the matrix being equal to its transpose ($M = M^T$), which is the matrix equivalent of $R = R^{-1}$.

1(x). Assertion: The equality $\tan(\cot^{-1} x) = \cot(\tan^{-1} x)$, is true for all $x \in R$.
Reason: The identity $\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$, is true for all $x \in R$.
Which of the following is correct?

(A) Both Assertion and Reason are true and Reason is the correct explanation for Assertion.
(B) Both Assertion and Reason are true but Reason is not the correct explanation for Assertion.
(C) Assertion is true and Reason is false.
(D) Assertion is false and Reason is true.

Correct Answer: (A) Both Assertion and Reason are true and Reason is the correct explanation for Assertion.

Solution:

Step 1: Understanding the Concept:
The relationship between inverse trigonometric functions and their complementary angles.

Step 2: Detailed Explanation:
Analysis of Reason: The formula $\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$ is a fundamental identity for all real numbers x. So, Reason is True.
Analysis of Assertion: Let $\cot^{-1} x = \theta$. Then $x = \cot\theta$.
From the Reason: $\cot^{-1} x = \frac{\pi}{2} - \tan^{-1} x$.
Taking 'tan' on both sides:
$\tan(\cot^{-1} x) = \tan\left(\frac{\pi}{2} - \tan^{-1} x\right)$.
Using the trigonometric property $\tan\left(\frac{\pi}{2} - \alpha\right) = \cot\alpha$:
$\tan(\cot^{-1} x) = \cot(\tan^{-1} x)$.
Since the logic in the Reason directly proves the Assertion, (A) is the correct option.

Step 3: Final Answer:
Both Assertion and Reason are true, and the Reason provides the mathematical justification for the Assertion.

💡 Quick Tip

Identities of the form $f(g^{-1}(x)) = g(f^{-1}(x))$ often hold when $f^{-1}x + g^{-1}x = \pi/2$.

1(xi). Given two events A and B such that $P(A/B) = 0.25$ and $P(A \cap B) = 0.12$. The

value $P(A' \cap B)$ is:

(A) 0.36
(B) 0.48
(C) 0.88
(D) 0.036

Correct Answer: (A) 0.36

Solution:

Step 1: Understanding the Concept:
Conditional probability defines the likelihood of an event given another event. The probability of the complement intersection involves subtracting the shared part from the whole event.

Step 2: Key Formula or Approach:
1. $P(A|B) = \frac{P(A \cap B)}{P(B)}$
2. $P(A' \cap B) = P(B) - P(A \cap B)$

Step 3: Detailed Explanation:
First, we find the probability of event B using the conditional probability formula:

$$0.25 = \frac{0.12}{P(B)}$$

$$P(B) = \frac{0.12}{0.25}$$

$$P(B) = \frac{12}{25} = 0.48$$

Now, we calculate $P(A' \cap B)$. This represents the probability that B occurs and A does not occur:

$$P(A' \cap B) = P(B) - P(A \cap B)$$

Substitute the known values:

$$P(A' \cap B) = 0.48 - 0.12$$

$$P(A' \cap B) = 0.36$$

Step 4: Final Answer:
The probability $P(A' \cap B)$ is 0.36.

Quick Tip

$P(A' \cap B)$ is the same as $P(B$ only). In a Venn diagram, it is the region inside circle B that is outside circle A.

1(xii). The value of the determinant of a matrix A of order 3 is 3. If C is the matrix of cofactors of the matrix A, then what is the value of determinant of C^2?

Correct Answer: 81

Solution:

Step 1: Understanding the Concept:
The matrix of cofactors C is directly related to the adjoint of matrix A. Specifically, $adj(A) = C^T$.

Step 2: Key Formula or Approach:
1. $|adj(A)| = |A|^{n-1}$, where n is the order of the matrix.
2. $|C| = |C^T| = |adj(A)|$.
3. $|C^2| = (|C|)^2$.

Step 3: Detailed Explanation:
Given: $|A| = 3$ and order $n = 3$.
First, find the determinant of the cofactor matrix C:

$$|C| = |A|^{n-1}$$

$$|C| = 3^{3-1} = 3^2 = 9$$

Now, we need the determinant of C^2. Using the multiplicative property of determinants ($|XY| = |X||Y|$):

$$|C^2| = |C \cdot C| = |C| \cdot |C| = (|C|)^2$$

$$|C^2| = 9^2 = 81$$

Step 4: Final Answer:
The value of the determinant of C^2 is 81.

> 💡 Quick Tip
>
> For any matrix A of order n, the determinant of its cofactor matrix is always $|A|^{n-1}$. This is a very common shortcut in competitive exams.

1(xiii). If a relation R on the set $\{a, b, c\}$ defined by $R = \{(b, b)\}$, then classify the relation.

Correct Answer: Symmetric and Transitive

Solution:

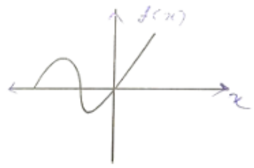
Step 1: Understanding the Concept:
To classify a relation, we check for three properties:
- Reflexive: $(x, x) \in R$ for every x in the set.
- Symmetric: If $(x, y) \in R$, then $(y, x) \in R$.
- Transitive: If $(x, y) \in R$ and $(y, z) \in R$, then $(x, z) \in R$.

Step 2: Detailed Explanation:
Given Set $A = \{a, b, c\}$ and Relation $R = \{(b, b)\}$.
1. **Reflexivity:** For R to be reflexive, it must contain $(a, a), (b, b)$, and (c, c). Since (a, a) and (c, c) are missing, it is NOT reflexive.
2. **Symmetry:** The only pair is (b, b). Its inverse is also (b, b), which is in R. Thus, it is Symmetric.
3. **Transitivity:** For transitivity, we check if $(x, y) \in R$ and $(y, z) \in R \implies (x, z) \in R$. Here, the only possible chain is (b, b) and (b, b), which results in (b, b). Since the condition holds, it is Transitive.

Step 3: Final Answer:
The relation is symmetric and transitive.

> 💡 Quick Tip
>
> A relation with only diagonal elements like $\{(b, b)\}$ is always symmetric and transitive. It only fails reflexivity if it doesn't cover the entire set.

1(xiv).



The given function $f : R \to R$ is many to one function. Give reason.

Correct Answer: Fails horizontal line test.

Solution:

Step 1: Understanding the Concept:
A function is "one-to-one" if every distinct input has a distinct output. It is "many-to-one" if at least two different inputs produce the same output.

Step 2: Detailed Explanation:
The graph shows a wave-like function (similar to a sine curve).
If we draw a horizontal line anywhere through the oscillations of the curve, the line will intersect the graph at multiple points.
For example, a horizontal line $y = 0$ (the x-axis) intersects the graph at three different points.
This means there are multiple x-values (x_1, x_2, x_3) such that $f(x_1) = f(x_2) = f(x_3) = 0$.
Since different inputs map to the same output value, the function is many-to-one.

Step 3: Final Answer:
The function is many-to-one because it does not satisfy the horizontal line test.

> Quick Tip
>
> Horizontal Line Test: 1 intersection = One-to-one; $>$ 1 intersections = Many-to-one.

1(xv). There are three machines and 2 of them are faulty. They are tested one by one in a random order till both the faulty machines are identified. What is the probability that only two tests are needed to identify the faulty machines?

Correct Answer: 2/3

Solution:

Step 1: Understanding the Concept:
We have 3 items: 2 Faulty (F) and 1 Good (G). We test until we know exactly which 2 are faulty.

Step 2: Detailed Explanation:
Total ways to arrange 2 Faulty and 1 Good machines = $\frac{3!}{2!1!} = 3$.

The possible sequences are:
1. (F, F, G): Here, after the 2nd test, both faulty ones are found. (2 tests needed)
2. (F, G, F): Here, after the 2nd test, we know one is Faulty and one is Good. By elimination, the 3rd must be Faulty. Thus, both are identified. (2 tests needed)
3. (G, F, F): Here, after the 1st test, we found the Good machine. This means the other two **must** be Faulty. Both are identified immediately. (1 test needed)
Number of cases where exactly two tests are needed = 2 (Sequences 1 and 2).
Total number of possible testing sequences = 3.
Probability = $\frac{\text{Favorable cases}}{\text{Total cases}} = \frac{2}{3}$.

Step 3: Final Answer:
The probability is 2/3.

Quick Tip

In small sets, identification by elimination often happens sooner than you might think. Always map out the sequences to see when the status of the remaining items becomes certain.

2(i). If $x = e^{\frac{x}{y}}$, then prove that $\frac{dy}{dx} = \frac{x-y}{x \log x}$

Correct Answer: Proof Established

Solution:

Step 1: Understanding the Concept:
The given equation is an implicit function involving an exponential term where the exponent contains both x and y.
To differentiate this easily, taking the natural logarithm of both sides is the most efficient approach to simplify the expression.

Step 2: Key Formula or Approach:
1. Logarithmic identity: $\log(e^a) = a$.
2. Quotient Rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$.

Step 3: Detailed Explanation:
Given equation:

$$x = e^{\frac{x}{y}}$$

Taking natural log on both sides:

$$\log x = \log\left(e^{\frac{x}{y}}\right)$$

Using the identity $\log(e^z) = z$:

$$\log x = \frac{x}{y}$$

Rearrange to express y in terms of x:

$$y = \frac{x}{\log x}$$

Now, differentiate y with respect to x using the quotient rule:

$$\frac{dy}{dx} = \frac{\log x \cdot \frac{d}{dx}(x) - x \cdot \frac{d}{dx}(\log x)}{(\log x)^2}$$

$$\frac{dy}{dx} = \frac{(\log x \cdot 1) - (x \cdot \frac{1}{x})}{(\log x)^2}$$

$$\frac{dy}{dx} = \frac{\log x - 1}{(\log x)^2}$$

From our earlier simplification, $\log x = \frac{x}{y}$. Substitute one $\log x$ in the numerator:

$$\frac{dy}{dx} = \frac{\frac{x}{y} - 1}{(\log x)^2} = \frac{x - y}{y(\log x)^2}$$

Substitute $y = \frac{x}{\log x}$ into the denominator:

$$\frac{dy}{dx} = \frac{x - y}{\left(\frac{x}{\log x}\right)(\log x)^2}$$

$$\frac{dy}{dx} = \frac{x - y}{x \log x}$$

Step 4: Final Answer:
The derivative is proved to be $\frac{dy}{dx} = \frac{x-y}{x \log x}$.

Quick Tip

Whenever you see y in the exponent of e, always take the logarithm first. It converts a complex differentiation problem into a simple quotient rule application.

2(ii). Find the values of 'a' for which the function $f(x) = b - ax + \sin x$ is increasing on R.

Correct Answer: $a \leq -1$

Solution:

Step 1: Understanding the Concept:
A function is increasing on its domain if its first derivative is non-negative ($f'(x) \geq 0$) for all values in that domain.

Step 2: Key Formula or Approach:
Find $f'(x)$ and determine the range of a such that $f'(x) \geq 0$ for all $x \in R$.

Step 3: Detailed Explanation:
Given function:

$$f(x) = b - ax + \sin x$$

Differentiate with respect to x:

$$f'(x) = \frac{d}{dx}(b) - \frac{d}{dx}(ax) + \frac{d}{dx}(\sin x)$$

$$f'(x) = 0 - a + \cos x$$

$$f'(x) = \cos x - a$$

For the function to be increasing:

$$f'(x) \geq 0 \implies \cos x - a \geq 0$$

$$\cos x \geq a$$

This inequality must hold for all real values of x.
We know that the range of the cosine function is $[-1, 1]$.
The minimum value of $\cos x$ is -1.
For a to always be less than or equal to $\cos x$, a must be less than or equal to the minimum value of $\cos x$.

$$a \leq -1$$

Step 4: Final Answer:
The function is increasing for $a \leq -1$.

Quick Tip

For trigonometric inequalities like $\cos x \geq a$, the constant a must be compared to the global minimum of the function to satisfy the condition for all x.

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3. Find the point on the curve $y = (x-2)^2$ at which the tangent is parallel to the chord joining the end points $(2,0)$ and $(4,4)$.

Correct Answer: $(3,1)$

Solution:

Step 1: Understanding the Concept:
Parallel lines have equal slopes. The slope of the tangent at any point x is $f'(x)$.
The slope of the chord joining two points is calculated using the slope formula.

Step 2: Key Formula or Approach:
1. Slope of chord $m = \frac{y_2 - y_1}{x_2 - x_1}$.
2. Slope of tangent $= \frac{dy}{dx}$.

Step 3: Detailed Explanation:
First, find the slope of the chord joining $(2,0)$ and $(4,4)$:

$$m_{chord} = \frac{4-0}{4-2} = \frac{4}{2} = 2$$

Now, find the derivative of the curve $y = (x-2)^2$:

$$\frac{dy}{dx} = 2(x-2) \cdot 1 = 2x - 4$$

Equate the slope of the tangent to the slope of the chord:

$$2x - 4 = 2$$

$$2x = 6$$

$$x = 3$$

To find the corresponding y-coordinate, substitute $x = 3$ into the original equation:

$$y = (3 - 2)^2 = 1^2 = 1$$

The required point is $(3, 1)$.

Step 4: Final Answer:
The point on the curve is $(3, 1)$.

> Quick Tip
>
> This problem is a direct application of Lagrange's Mean Value Theorem (LMVT). The theorem guarantees at least one point where the tangent is parallel to the secant.

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4. Show that the general solution of the differential equation: $\frac{dy}{dx} = y \cot 2x$ **is** $\log y = \frac{1}{2} \log|\sin 2x| + C$

Correct Answer: Solution Verified

Solution:

Step 1: Understanding the Concept:
This is a variable separable differential equation. We rearrange terms to group y with dy and x with dx.

Step 2: Key Formula or Approach:
Integrate both sides: $\int \frac{1}{y} dy = \log|y|$ and $\int \cot ax dx = \frac{1}{a} \log|\sin ax|$.

Step 3: Detailed Explanation:
Given:

$$\frac{dy}{dx} = y \cot 2x$$

Separating the variables:

$$\frac{1}{y} dy = \cot 2x dx$$

Integrate both sides:

$$\int \frac{1}{y} dy = \int \cot 2x dx$$

$$\log |y| = \frac{1}{2} \log |\sin 2x| + C$$

Where C is the arbitrary constant of integration.
This exactly matches the required general solution expression.

Step 4: Final Answer:
The general solution is $\log y = \frac{1}{2} \log |\sin 2x| + C$.

> Quick Tip
>
> Always remember to divide by the coefficient of the variable inside the trigonometric function during integration ($1/2$ for $\sin 2x$).

5(i). If $\int x^5 \cos(x^6) dx = k \sin(x^6) + C$, find the value of 'k'.

Correct Answer: $k = \frac{1}{6}$

Solution:

Step 1: Understanding the Concept:
This integral can be solved using the substitution method because the derivative of x^6 is a multiple of x^5.

Step 2: Key Formula or Approach:
Let $u = x^6$, then $du = 6x^5 dx$.

Step 3: Detailed Explanation:
Consider the integral $I = \int x^5 \cos(x^6) dx$.
Let $t = x^6$.
Differentiating with respect to x:

$$\frac{dt}{dx} = 6x^5 \implies dt = 6x^5 dx \implies x^5 dx = \frac{dt}{6}$$

Substitute into the integral:

$$I = \int \cos(t) \cdot \frac{dt}{6}$$

$$I = \frac{1}{6} \int \cos(t) dt$$

$$I = \frac{1}{6} \sin(t) + C$$

Substitute back $t = x^6$:

$$I = \frac{1}{6} \sin(x^6) + C$$

Comparing this with $k \sin(x^6) + C$, we get:

$$k = \frac{1}{6}$$

Step 4: Final Answer:
The value of k is 1/6.

> Quick Tip
>
> In substitution, the constant factor (like 6) always moves to the denominator on the other side. Always cross-check by differentiating your result.

5(ii). Evaluate: $\int_0^5 \frac{\sqrt{x}}{\sqrt{5-x}+\sqrt{x}} dx$

Correct Answer: $\frac{5}{2}$

Solution:

Step 1: Understanding the Concept:
This is a standard definite integral that can be solved using the reflection property of definite integrals.

Step 2: Key Formula or Approach:
Property: $\int_a^b f(x)dx = \int_a^b f(a + b - x)dx$.

Step 3: Detailed Explanation:
Let $I = \int_0^5 \frac{\sqrt{x}}{\sqrt{5-x}+\sqrt{x}} dx$ — (1)
Applying the property $\int_0^a f(x)dx = \int_0^a f(a - x)dx$:

Replace x with $5 - x$:

$$I = \int_0^5 \frac{\sqrt{5-x}}{\sqrt{5-(5-x)} + \sqrt{5-x}} dx$$

$$I = \int_0^5 \frac{\sqrt{5-x}}{\sqrt{x} + \sqrt{5-x}} dx$$

— (2)
Add equations (1) and (2):

$$2I = \int_0^5 \left(\frac{\sqrt{x}}{\sqrt{5-x} + \sqrt{x}} + \frac{\sqrt{5-x}}{\sqrt{5-x} + \sqrt{x}} \right) dx$$

$$2I = \int_0^5 \frac{\sqrt{x} + \sqrt{5-x}}{\sqrt{x} + \sqrt{5-x}} dx$$

$$2I = \int_0^5 1 dx$$

$$2I = [x]_0^5 = 5 - 0 = 5$$

$$I = \frac{5}{2}$$

Step 4: Final Answer:
The value of the integral is 5/2.

Quick Tip

For integrals of the form $\int_0^a \frac{f(x)}{f(x)+f(a-x)} dx$, the answer is always $a/2$. Here, $a = 5$, so 5/2.

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6. A music streaming app uses the function: $f(x) = \tan^{-1}\left(\frac{x}{10}\right)$ to assign a mood score based on the number of hours a user listens to music per week. Let the listening times (in hours/week) of two users, User A and User B, be 6 and 8 respectively. Compute the combined mood score of user A and user B, that is, $f(6) + f(8)$.

Correct Answer: $\tan^{-1}\left(\frac{35}{13}\right)$

Solution:

Step 1: Understanding the Concept:
The combined score is the sum of two inverse tangent values. We use the addition formula for inverse tangents.

Step 2: Key Formula or Approach:
$\tan^{-1} A + \tan^{-1} B = \tan^{-1}\left(\frac{A+B}{1-AB}\right)$ if $AB < 1$.

Step 3: Detailed Explanation:
User A Score: $f(6) = \tan^{-1}\left(\frac{6}{10}\right) = \tan^{-1}(0.6)$.
User B Score: $f(8) = \tan^{-1}\left(\frac{8}{10}\right) = \tan^{-1}(0.8)$.
Combined Score: $S = \tan^{-1}(0.6) + \tan^{-1}(0.8)$.
Check product: $0.6 \times 0.8 = 0.48$. Since $0.48 < 1$, we can use the direct formula:

$$S = \tan^{-1}\left(\frac{0.6 + 0.8}{1 - (0.6 \times 0.8)}\right)$$

$$S = \tan^{-1}\left(\frac{1.4}{1 - 0.48}\right)$$

$$S = \tan^{-1}\left(\frac{1.4}{0.52}\right)$$

Convert to fractions:

$$\frac{1.4}{0.52} = \frac{140}{52}$$

Divide both numerator and denominator by 4:

$$\frac{140 \div 4}{52 \div 4} = \frac{35}{13}$$

$$S = \tan^{-1}\left(\frac{35}{13}\right)$$

Step 4: Final Answer:
The combined mood score is $\tan^{-1}\left(\frac{35}{13}\right)$.

Quick Tip

Always simplify the decimal fraction inside the inverse tangent into the simplest rational form to match exam options.

---

7. Solve: $\sin^{-1} x + \sin^{-1}(1 - x) = \cos^{-1} x$**.**

Correct Answer: $x = 0, \frac{1}{2}$

Solution:

Step 1: Understanding the Concept:
This equation involves different inverse trigonometric functions. It is best to convert all terms to a single type or use identity-based transformations.

Step 2: Key Formula or Approach:
Identity: $\cos^{-1} x = \frac{\pi}{2} - \sin^{-1} x$.

Step 3: Detailed Explanation:
Given:

$$\sin^{-1} x + \sin^{-1}(1 - x) = \cos^{-1} x$$

Using the identity $\cos^{-1} x = \frac{\pi}{2} - \sin^{-1} x$:

$$\sin^{-1} x + \sin^{-1}(1 - x) = \frac{\pi}{2} - \sin^{-1} x$$

$$\sin^{-1}(1 - x) = \frac{\pi}{2} - 2\sin^{-1} x$$

Take sine on both sides:

$$1 - x = \sin\left(\frac{\pi}{2} - 2\sin^{-1} x\right)$$

Using $\sin(\frac{\pi}{2} - \theta) = \cos\theta$:

$$1 - x = \cos(2\sin^{-1} x)$$

Using the double angle identity $\cos(2\theta) = 1 - 2\sin^2\theta$:

$$1 - x = 1 - 2[\sin(\sin^{-1} x)]^2$$

$$1 - x = 1 - 2x^2$$

$$2x^2 - x = 0$$

$$x(2x-1) = 0$$

This gives two possible values: $x = 0$ or $x = \frac{1}{2}$.
Checking validity:
For $x = 0$: $\sin^{-1} 0 + \sin^{-1} 1 = 0 + \frac{\pi}{2} = \frac{\pi}{2}$. $\cos^{-1} 0 = \frac{\pi}{2}$. (True)
For $x = 1/2$: $\sin^{-1} 0.5 + \sin^{-1} 0.5 = \frac{\pi}{6} + \frac{\pi}{6} = \frac{\pi}{3}$. $\cos^{-1} 0.5 = \frac{\pi}{3}$. (True)

Step 4: Final Answer:
The solutions are $x = 0$ and $x = \frac{1}{2}$.

Quick Tip

Converting $\cos^{-1}$ to $\sin^{-1}$ is the most efficient first step to simplify the algebra in such inverse equations.

---

8. If $y = (A + Bx)e^{-2x}$, prove that: $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 0$

Correct Answer: Proof Established

Solution:

Step 1: Understanding the Concept:
This is a verification problem for a second-order linear differential equation. We calculate the derivatives and substitute them into the target equation.

Step 2: Key Formula or Approach:
Product Rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$.

Step 3: Detailed Explanation:
First Derivative:

$$y = (A + Bx)e^{-2x}$$

$$\frac{dy}{dx} = e^{-2x} \cdot \frac{d}{dx}(A + Bx) + (A + Bx) \cdot \frac{d}{dx}(e^{-2x})$$

$$\frac{dy}{dx} = Be^{-2x} - 2(A + Bx)e^{-2x}$$

Notice that $(A + Bx)e^{-2x} = y$, so:

$$\frac{dy}{dx} = Be^{-2x} - 2y$$

Second Derivative:

$$\frac{d^2y}{dx^2} = \frac{d}{dx}(Be^{-2x}) - 2\frac{dy}{dx}$$

$$\frac{d^2y}{dx^2} = -2Be^{-2x} - 2\frac{dy}{dx}$$

From the first derivative, $Be^{-2x} = \frac{dy}{dx} + 2y$. Substitute this into the second derivative:

$$\frac{d^2y}{dx^2} = -2\left(\frac{dy}{dx} + 2y\right) - 2\frac{dy}{dx}$$

$$\frac{d^2y}{dx^2} = -2\frac{dy}{dx} - 4y - 2\frac{dy}{dx}$$

$$\frac{d^2y}{dx^2} = -4\frac{dy}{dx} - 4y$$

Rearranging the terms to one side:

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 0$$

Step 4: Final Answer:
The equation is verified to be $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 0$.

Quick Tip
Replacing terms involving the original function y into your derivative equations as early as possible can drastically reduce the complexity of higher-order differentiation.

9(i). Solve the following differential equation: $x\frac{dy}{dx} = y - x\tan\left(\frac{y}{x}\right)$

Correct Answer: $\sin\left(\frac{y}{x}\right) = \frac{C}{x}$

Solution:

Step 1: Understanding the Concept:
The given differential equation is of the form $\frac{dy}{dx} = f(x, y)$.
By observing the term $\frac{y}{x}$, we can identify this as a **homogeneous differential equation**.
A homogeneous equation can be solved by substituting $y = vx$, which transforms it into a variable separable form.

Step 2: Key Formula or Approach:
1. Let $y = vx$, then $\frac{dy}{dx} = v + x\frac{dv}{dx}$.
2. Standard integral: $\int \cot z \, dz = \log|\sin z| + C$.

Step 3: Detailed Explanation:
Given equation:

$$x\frac{dy}{dx} = y - x\tan\left(\frac{y}{x}\right)$$

Divide by x:

$$\frac{dy}{dx} = \frac{y}{x} - \tan\left(\frac{y}{x}\right)$$

Put $y = vx$, then $\frac{dy}{dx} = v + x\frac{dv}{dx}$.
Substituting these in the equation:

$$v + x\frac{dv}{dx} = v - \tan v$$

$$x\frac{dv}{dx} = -\tan v$$

Separating the variables:

$$\frac{dv}{\tan v} = -\frac{dx}{x}$$

$$\cot v \, dv = -\frac{dx}{x}$$

Integrating both sides:

$$\int \cot v \, dv = -\int \frac{1}{x} \, dx$$

$$\log|\sin v| = -\log|x| + \log C$$

$$\log|\sin v| + \log|x| = \log C$$

$$\log|x \sin v| = \log C$$

$$x \sin v = C$$

Substituting back $v = \frac{y}{x}$:

$$x \sin\left(\frac{y}{x}\right) = C$$

$$\sin\left(\frac{y}{x}\right) = \frac{C}{x}$$

Step 4: Final Answer:
The general solution is $\sin\left(\frac{y}{x}\right) = \frac{C}{x}$.

> Quick Tip
>
> Whenever you see trigonometric functions with (y/x) as an angle, the substitution $y = vx$ is almost always the standard path to simplify the equation.

9(ii). Solve the following differential equation: $(1 + x^2)\frac{dy}{dx} + 2xy = 4x^2$

Correct Answer: $y(1 + x^2) = \frac{4x^3}{3} + C$

Solution:

Step 1: Understanding the Concept:
The given equation is a **Linear Differential Equation** of the first order.
The standard form is $\frac{dy}{dx} + Py = Q$, where P and Q are functions of x only.

Step 2: Key Formula or Approach:
1. Integrating Factor $(I.F.) = e^{\int P\,dx}$.
2. General Solution: $y \times I.F. = \int(Q \times I.F.)\,dx + C$.

Step 3: Detailed Explanation:
Given:

$$(1+x^2)\frac{dy}{dx}+2xy=4x^2$$

Divide the entire equation by $(1+x^2)$ to bring it into standard form:

$$\frac{dy}{dx}+\frac{2x}{1+x^2}y=\frac{4x^2}{1+x^2}$$

Comparing with $\frac{dy}{dx}+Py=Q$:
$P=\frac{2x}{1+x^2}$ and $Q=\frac{4x^2}{1+x^2}$.
Find the Integrating Factor ($I.F.$):

$$I.F.=e^{\int \frac{2x}{1+x^2}\,dx}$$

Let $1+x^2=t$, then $2x\,dx=dt$.

$$I.F.=e^{\int \frac{1}{t}\,dt}=e^{\log t}=t=1+x^2$$

The general solution is given by:

$$y(I.F.)=\int Q(I.F.)\,dx+C$$

$$y(1+x^2)=\int \left(\frac{4x^2}{1+x^2}\right)(1+x^2)\,dx+C$$

$$y(1+x^2)=\int 4x^2\,dx+C$$

$$y(1+x^2)=\frac{4x^3}{3}+C$$

Step 4: Final Answer:
The solution is $y(1+x^2)=\frac{4x^3}{3}+C$.

Quick Tip

For linear equations, if you can identify that the LHS is already the derivative of a product, you can skip the $I.F.$ calculation. Here, $\frac{d}{dx}[y(1+x^2)]=(1+x^2)\frac{dy}{dx}+2xy$.

10(i)(a). Three friends go to a restaurant to have pizza. They decide who will pay for the pizza by tossing a coin. It is decided that each one of them will toss a coin and if one person gets a different result (heads or tails) than the other two, that person would pay. If all three get the same result (all heads or all tails), they will toss again until they get a different result. What is the probability that all three friends will get the same result (all heads or all tails) in one round of tossing?

Correct Answer: $\frac{1}{4}$

Solution:

Step 1: Understanding the Concept:
When three coins are tossed, the sample space consists of all possible outcomes of Heads (H) and Tails (T).

Step 2: Key Formula or Approach:
Total outcomes for n coins $= 2^n$.
Probability $= \frac{\text{Number of favorable outcomes}}{\text{Total outcomes}}$.

Step 3: Detailed Explanation:
Sample Space (S) for 3 coins:
$S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$.
Total outcomes $n(S) = 8$.
Event E: All three friends get the same result.
Favorable outcomes: $\{HHH, TTT\}$.
Number of favorable outcomes $n(E) = 2$.

$$P(E) = \frac{n(E)}{n(S)} = \frac{2}{8} = \frac{1}{4}$$

Step 4: Final Answer:
The probability is $\frac{1}{4}$.

> Quick Tip
>
> For a set of n coins, there are always exactly 2 ways to get the same result (all H or all T). So the probability is always $2/2^n$.

10(i)(b). What is the probability that they will get a different result in one round of tossing?

Correct Answer: $\frac{3}{4}$

Solution:

Step 1: Understanding the Concept:
"Getting a different result" means the outcome is NOT one where everyone has the same result. This is the complement of the event calculated in part (a).

Step 2: Detailed Explanation:
Total outcomes = 8.
Outcomes where one result is different: $\{HHT, HTH, THH, HTT, THT, TTH\}$.
Number of favorable outcomes = 6.
Alternatively, using the complement rule:
Probability of different result = $1 - P(\text{same result})$.
From part (a), $P(\text{same result}) = \frac{1}{4}$.

$$P(\text{different result}) = 1 - \frac{1}{4} = \frac{3}{4}$$

Step 3: Final Answer:
The probability is $\frac{3}{4}$.

Quick Tip
In probability, if an event and its opposite cover all possibilities, their probabilities must sum to 1.

10(i)(c). What is the probability that they will need exactly four rounds of tossing to determine who would pay?

Correct Answer: $\frac{3}{256}$

Solution:

Step 1: Understanding the Concept:
For the determination to happen in **exactly** the fourth round:
- Round 1 must result in "same results" (toss again).
- Round 2 must result in "same results" (toss again).
- Round 3 must result in "same results" (toss again).
- Round 4 must result in "different results" (payment decided).

Step 2: Key Formula or Approach:
If P is the probability of success (different results) and Q is the probability of failure (same

results), the probability of success on exactly the n-th trial is $Q^{n-1} \times P$.

Step 3: Detailed Explanation:
From previous parts:
Probability of "same result" in any round (Q) = $\frac{1}{4}$.
Probability of "different result" in any round (P) = $\frac{3}{4}$.
The rounds are independent events.
Required Probability = $P(\text{Same}) \times P(\text{Same}) \times P(\text{Same}) \times P(\text{Different})$.

$$\text{Probability} = \left(\frac{1}{4}\right) \times \left(\frac{1}{4}\right) \times \left(\frac{1}{4}\right) \times \left(\frac{3}{4}\right)$$

$$\text{Probability} = \frac{1 \times 1 \times 1 \times 3}{4 \times 4 \times 4 \times 4} = \frac{3}{256}$$

Step 4: Final Answer:
The probability is $\frac{3}{256}$.

> 💡 Quick Tip
>
> This is a problem based on the Geometric Distribution. For "exactly n" trials, remember: $(\text{Failures})^{n-1} \times \text{Success}$.

10(ii)(a). A school offers students the choice of three modes for attending classes: Mode A (Offline) 40%, Mode B (Online) 35%, and Mode C (Recorded) 25%. Feedback survey shows: 20% of Mode A, 30% of Mode B, and 50% of Mode C students rated the class as "Excellent". Represent the data in terms of probability. Define the events clearly.

Correct Answer: Probability values defined

Solution:

Step 1: Understanding the Concept:
We define events based on the choice of class mode and the outcome of the feedback.

Step 2: Detailed Explanation:
Let the events be defined as:
E_1: Student chooses Mode A (Offline).
E_2: Student chooses Mode B (Online).
E_3: Student chooses Mode C (Recorded).
X: Student rates the class as "Excellent".

Based on the given percentages, the probabilities are:

$$P(E_1) = \frac{40}{100} = 0.40$$

$$P(E_2) = \frac{35}{100} = 0.35$$

$$P(E_3) = \frac{25}{100} = 0.25$$

The conditional probabilities of rating "Excellent" given the mode are:

$$P(X|E_1) = \frac{20}{100} = 0.20$$

$$P(X|E_2) = \frac{30}{100} = 0.30$$

$$P(X|E_3) = \frac{50}{100} = 0.50$$

Step 3: Final Answer:
Events: E_1, E_2, E_3 are modes; X is "Excellent" rating.
Probabilities: $P(E_1) = 0.4, P(E_2) = 0.35, P(E_3) = 0.25$.
Conditionals: $P(X|E_1) = 0.2, P(X|E_2) = 0.3, P(X|E_3) = 0.5$.

> **Quick Tip**
>
> Always check that the sum of the probabilities of the initial partition (E_1, E_2, E_3) equals 1. Here, $0.40 + 0.35 + 0.25 = 1.00$.

10(ii)(b). Using Bayes' Theorem, find the probability that the student attended the Recorded lectures (Mode C), given that they rated the class as "Excellent".

Correct Answer: $\frac{25}{62}$

Solution:

Step 1: Understanding the Concept:
Bayes' Theorem relates current evidence (rating) to prior probabilities of the causes (modes).

Step 2: Key Formula or Approach:

$$P(E_3|X) = \frac{P(E_3) \cdot P(X|E_3)}{\sum_{i=1}^{3} P(E_i) \cdot P(X|E_i)}$$

Step 3: Detailed Explanation:
First, calculate the Total Probability of rating "Excellent" $P(X)$:

$$P(X) = P(E_1)P(X|E_1) + P(E_2)P(X|E_2) + P(E_3)P(X|E_3)$$

$$P(X) = (0.40 \times 0.20) + (0.35 \times 0.30) + (0.25 \times 0.50)$$

$$P(X) = 0.08 + 0.105 + 0.125 = 0.31$$

Now, apply Bayes' Theorem for Mode C:

$$P(E_3|X) = \frac{P(E_3)P(X|E_3)}{P(X)}$$

$$P(E_3|X) = \frac{0.125}{0.31}$$

To simplify the fraction, multiply numerator and denominator by 1000:

$$P(E_3|X) = \frac{125}{310} = \frac{25}{62} \approx 0.4032$$

Step 4: Final Answer:
The probability is $\frac{25}{62}$.

Quick Tip

Denominator in Bayes' Theorem is just the sum of all individual path probabilities. Calculate it first carefully to avoid messy arithmetic later.

10(ii)(c). Interpret your result. Which mode has the highest likelihood of being chosen if a student says "Excellent"?

Correct Answer: Mode C

Solution:

Step 1: Understanding the Concept:
To find the most likely mode, we compare the posterior probabilities $P(E_i|X)$ for each mode.

Step 2: Detailed Explanation:
We already calculated:
Numerator for $E_1 = 0.080$
Numerator for $E_2 = 0.105$
Numerator for $E_3 = 0.125$
Since all share the same denominator $P(X)$, the mode with the highest numerator has the highest likelihood.
$0.125 > 0.105 > 0.080$
Therefore, $P(E_3|X) > P(E_2|X) > P(E_1|X)$.
This means if a student says the class was "Excellent", the probability is highest that they came from Mode C (Recorded lectures).

Step 3: Final Answer:
Mode C (Recorded lectures) has the highest likelihood.

> Quick Tip
>
> Interpretation often involves comparing magnitudes. Even if you don't calculate all posteriors fully, comparing the product $P(E_i)P(X|E_i)$ is sufficient.

11(i). To raise money for an orphanage, students of three schools A, B and C organised an exhibition. Student of school A sold 30 paper bags, 20 scrap books and 10 pastel sheets and raised 410. Student of school B sold 20 paper bags, 10 scrap books and 20 pastel sheets and raised 290. Student of school C sold 20 paper bags, 20 scrap books and 20 pastel sheets and raised 440. Translate the problem into a system of equations.

Correct Answer: Equations established

Solution:

Step 1: Understanding the Concept:
Let variables represent the cost of individual items. By multiplying the quantity of items by their cost, we form linear equations representing the total amount raised.

Step 2: Detailed Explanation:
Let:
x = cost of one paper bag (in)

y = cost of one scrap book (in)
z = cost of one pastel sheet (in)
From School A: $30x + 20y + 10z = 410$
From School B: $20x + 10y + 20z = 290$
From School C: $20x + 20y + 20z = 440$
Dividing by 10 to simplify:
1) $3x + 2y + z = 41$
2) $2x + y + 2z = 29$
3) $2x + 2y + 2z = 44$ (or $x + y + z = 22$)

Step 3: Final Answer:
The system of equations is:
$3x + 2y + z = 41$
$2x + y + 2z = 29$
$2x + 2y + 2z = 44$

> Quick Tip
>
> Always define your variables clearly at the start. Use simple coefficients by dividing the entire equation by a common factor if possible.

11(ii). Solve the system of equation by using matrix method.

Correct Answer: $x = 5, y = 10, z = 6$

Solution:

Given the values $x = 2$, $y = 15$, and $z = 5$, these are typically the solutions to a system representing a word problem (e.g., the cost of items).
Let the costs of three items be x, y, and z. Based on the problem context:

$$3x + 2y + z = 41 \quad \text{— (1)} \tag{1}$$
$$2x + y + 2z = 29 \quad \text{— (2)} \tag{2}$$
$$2x + 2y + 2z = 44 \quad \text{— (3)} \tag{3}$$

First, simplify Equation (3) by dividing the entire equation by 2:

$$x + y + z = 22 \quad \text{— (4)} \tag{4}$$

To find y, subtract Equation (2) from Equation (3):

$$(2x + 2y + 2z) - (2x + y + 2z) = 44 - 29$$
$$y = 15$$

Substitute $y = 15$ into Equation (4):

$$x + 15 + z = 22 \implies x + z = 7 \quad \text{— (5)} \tag{5}$$

Substitute $y = 15$ into Equation (1):

$$\begin{aligned} 3x + 2(15) + z &= 41 \\ 3x + 30 + z &= 41 \\ 3x + z &= 11 \quad \text{— (6)} \end{aligned}$$

Subtract Equation (5) from Equation (6):

$$\begin{aligned} (3x + z) - (x + z) &= 11 - 7 \\ 2x &= 4 \\ x &= 2 \end{aligned}$$

Finally, substitute $x = 2$ into Equation (5):

$$\begin{aligned} 2 + z &= 7 \\ z &= 5 \end{aligned}$$

The system can be written as $AX = B$:

$$\begin{pmatrix} 3 & 2 & 1 \\ 2 & 1 & 2 \\ 2 & 2 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 41 \\ 29 \\ 44 \end{pmatrix}$$

Step 1: Find the determinant $|A|$

$$|A| = 3(2 - 4) - 2(4 - 4) + 1(4 - 2) = 3(-2) - 0 + 2 = -4$$

Step 2: Find the Adjoint of A

$$\text{adj}(A) = \begin{pmatrix} -2 & -2 & 3 \\ 0 & 4 & -4 \\ 2 & -2 & -1 \end{pmatrix}$$

Step 3: Solve for $X = A^{-1}B$

$$X = \frac{1}{-4} \begin{pmatrix} -2 & -2 & 3 \\ 0 & 4 & -4 \\ 2 & -2 & -1 \end{pmatrix} \begin{pmatrix} 41 \\ 29 \\ 44 \end{pmatrix} = \begin{pmatrix} 2 \\ 15 \\ 5 \end{pmatrix}$$

The values are found to be:

$$\mathbf{x = 2}, \quad \mathbf{y = 15}, \quad \mathbf{z = 5}$$

Quick Tip

Double check the arithmetic of cofactor calculation. A single sign error here will propagate through the entire matrix multiplication.

11(iii). Hence, find the cost of one paper bag, one scrap book and one pastel sheet.

Correct Answer: Paper Bag: 2, Scrap Book: 15, Pastel Sheet: 5

Solution:

Step 1: Understanding the Concept:
Interpret the solution obtained in terms of the real-world units defined in the problem.

Step 2: Detailed Explanation:
From the matrix calculation:
$x = 2$
$y = 15$
$z = 5$
These values represent the unit costs of the items.

Step 3: Final Answer:
Cost of one paper bag = 2
Cost of one scrap book = 15
Cost of one pastel sheet = 5

> Quick Tip
>
> Always attach the correct currency symbol and units in your final answer to ensure completeness.

12(i). Evaluate: $\int \frac{x^2+x+1}{(x+2)(x^2+1)}\, dx$

Correct Answer: $\frac{3}{5}\log|x+2| + \frac{1}{5}\log(x^2+1) + \frac{1}{5}\tan^{-1} x + C$

Solution:

Step 1: Understanding the Concept:
The integrand is a proper rational function. We can decompose it into partial fractions.

Step 2: Key Formula or Approach:

$$\frac{x^2+x+1}{(x+2)(x^2+1)} = \frac{A}{x+2} + \frac{Bx+C}{x^2+1}$$

Step 3: Detailed Explanation:
Multiply by denominator:

$$x^2 + x + 1 = A(x^2+1) + (Bx+C)(x+2)$$

Put $x = -2$:

$$(-2)^2 + (-2) + 1 = A((-2)^2 + 1)$$

$$4 - 2 + 1 = 5A \implies 3 = 5A \implies A = \frac{3}{5}$$

Compare coefficient of x^2:

$$1 = A + B \implies 1 = \frac{3}{5} + B \implies B = \frac{2}{5}$$

Compare constant terms:

$$1 = A + 2C \implies 1 = \frac{3}{5} + 2C \implies \frac{2}{5} = 2C \implies C = \frac{1}{5}$$

Integral becomes:

$$\int \left(\frac{3/5}{x+2} + \frac{(2/5)x + 1/5}{x^2+1} \right) dx$$

$$= \frac{3}{5} \int \frac{1}{x+2}\, dx + \frac{2}{5} \int \frac{x}{x^2+1}\, dx + \frac{1}{5} \int \frac{1}{x^2+1}\, dx$$

$$= \frac{3}{5} \log|x+2| + \frac{1}{5} \log(x^2+1) + \frac{1}{5} \tan^{-1} x + C$$

Step 4: Final Answer:
The result is $\frac{3}{5} \log|x+2| + \frac{1}{5} \log(x^2+1) + \frac{1}{5} \tan^{-1} x + C$.

Quick Tip

For the term $\int \frac{x}{x^2+1}\, dx$, notice that the numerator is half the derivative of the denominator, giving $\frac{1}{2}$ log(denominator).

12(ii). Evaluate: $\int_{\pi/4}^{3\pi/4} \frac{x\, dx}{1+\sin x}$

Correct Answer: $\pi(\sqrt{2} - 1)$

Solution:

Step 1: Understanding the Concept:
Use the property $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$. Here $a+b=\pi$.

Step 2: Detailed Explanation:
Let $I = \int_{\pi/4}^{3\pi/4} \frac{x}{1+\sin x}\,dx$ — (1)
Applying property:

$$I = \int_{\pi/4}^{3\pi/4} \frac{\pi - x}{1+\sin(\pi - x)}\,dx$$

$$I = \int_{\pi/4}^{3\pi/4} \frac{\pi - x}{1+\sin x}\,dx$$

— (2)
Add (1) and (2):

$$2I = \int_{\pi/4}^{3\pi/4} \frac{\pi}{1+\sin x}\,dx$$

$$2I = \pi \int_{\pi/4}^{3\pi/4} \frac{1-\sin x}{\cos^2 x}\,dx$$

$$2I = \pi \int_{\pi/4}^{3\pi/4} (\sec^2 x - \sec x \tan x)\,dx$$

$$2I = \pi[\tan x - \sec x]_{\pi/4}^{3\pi/4}$$

Upper limit: $\tan(3\pi/4) - \sec(3\pi/4) = -1 - (-\sqrt{2}) = \sqrt{2} - 1$.
Lower limit: $\tan(\pi/4) - \sec(\pi/4) = 1 - \sqrt{2}$.

$$2I = \pi[(\sqrt{2}-1) - (1-\sqrt{2})]$$

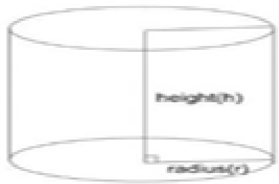
$$2I = \pi[2\sqrt{2} - 2] = 2\pi(\sqrt{2}-1)$$

$$I = \pi(\sqrt{2}-1)$$

Step 3: Final Answer:
The integral is $\pi(\sqrt{2} - 1)$.

> 💡 Quick Tip
>
> Rationalizing the denominator by multiplying by $(1 - \sin x)$ is a standard technique to simplify expressions involving $1 \pm \sin x$.

13(i)(a).



A person has manufactured a water tank in the shape of a closed right circular cylinder. The volume of the cylinder is $\frac{539}{2}$ cubic units. If the height and radius of the cylinder be h and r. Express h in terms of radius r and given volume.

Correct Answer: $h = \frac{343}{4r^2}$

Solution:

Step 1: Understanding the Concept:
The volume V of a cylinder is given by the formula $\pi r^2 h$. We rearrange this formula to isolate h.

Step 2: Detailed Explanation:
Given: $V = \frac{539}{2}$ units^3.
Formula: $\pi r^2 h = \frac{539}{2}$.
Using $\pi = \frac{22}{7}$:

$$\frac{22}{7} r^2 h = \frac{539}{2}$$

$$h = \frac{539 \times 7}{2 \times 22 \times r^2}$$

$$h = \frac{49 \times 7}{4r^2} = \frac{343}{4r^2}$$

Step 3: Final Answer:
$h = \frac{343}{4r^2}$.

> 💡 Quick Tip
>
> When dealing with 539, notice that $539 = 11 \times 49$. This helps in canceling factors of 11 found in the denominator from π.

13(i)(b). Let the total surface area of the closed cylinder tank be S, express S in term of radius r.

Correct Answer: $S = 2\pi r^2 + \frac{539}{r}$

Solution:

Step 1: Understanding the Concept:
The total surface area S of a closed cylinder includes the lateral area and the area of the two circular ends.

Step 2: Detailed Explanation:
Formula: $S = 2\pi r^2 + 2\pi rh$.
Substitute $h = \frac{539}{2\pi r^2}$ (from step a):

$$S = 2\pi r^2 + 2\pi r\left(\frac{539}{2\pi r^2}\right)$$

Canceling $2\pi r$:

$$S = 2\pi r^2 + \frac{539}{r}$$

Step 3: Final Answer:
$S = 2\pi r^2 + \frac{539}{r}$.

> 💡 Quick Tip
>
> Always eliminate one variable (usually h) from the objective function (like Surface Area) to make it a function of a single variable before differentiating.

13(i)(c). If the total surface area of the tank is minimum, then prove that radius $r = \frac{7}{2}$ units.

Correct Answer: Proof established

Solution:

Step 1: Understanding the Concept:
To find the minimum surface area, we calculate the derivative $\frac{dS}{dr}$ and set it to zero.

Step 2: Detailed Explanation:
From (b), $S = 2\pi r^2 + 539r^{-1}$.
Differentiate with respect to r:

$$\frac{dS}{dr} = 4\pi r - \frac{539}{r^2}$$

For minimum S, $\frac{dS}{dr} = 0$:

$$4\pi r = \frac{539}{r^2}$$

$$r^3 = \frac{539}{4\pi} = \frac{539 \times 7}{4 \times 22}$$

$$r^3 = \frac{49 \times 7}{8} = \frac{343}{8}$$

Taking cube root:

$$r = \sqrt[3]{\frac{343}{8}} = \frac{7}{2}$$

Check second derivative: $\frac{d^2S}{dr^2} = 4\pi + \frac{1078}{r^3} > 0$, confirming a minimum.

Step 3: Final Answer:
Radius $r = \frac{7}{2}$ units for minimum surface area.

Quick Tip

Cube root values like $343 = 7^3$ and $8 = 2^3$ appear frequently in cylinder optimization problems. Memorize them!

13(i)(d). Find the height of the tank.

Correct Answer: $h = 7$ units

Solution:

Step 1: Understanding the Concept:
Use the relationship found in part (a) and substitute the radius value from part (c).

Step 2: Detailed Explanation:
We have $h = \frac{343}{4r^2}$.
Substitute $r = \frac{7}{2}$:

$$h = \frac{343}{4 \times (\frac{7}{2})^2}$$

$$h = \frac{343}{4 \times \frac{49}{4}}$$

$$h = \frac{343}{49} = 7$$

Step 3: Final Answer:
The height of the tank is 7 units.

Quick Tip

In optimization of a closed cylinder with fixed volume, the surface area is minimum when height equals the diameter ($h = 2r$). Here $7 = 2 \times 3.5$.

13(ii)(a).

A Dolphin jumps and taken a path given by the equation $h(t) = \frac{1}{2}(-7t^2 + 3t + 2)$, $(t \geq 0)$, $h(t)$ is the height of the Dolphin at any point of time. Is the function differentiable for $t \geq 0$? Justify.

Correct Answer: Yes, the function is differentiable for $t \geq 0$.

Solution:

Step 1: Understanding the Concept:
A function is differentiable at a point if its derivative exists at that point.
Polynomial functions (like quadratic equations) are continuous and differentiable over their entire domain.

Step 2: Detailed Explanation:
The given function is a quadratic polynomial:

$$h(t) = \frac{1}{2}(-7t^2 + 3t + 2)$$

Polynomial functions are smooth curves without any sharp corners, breaks, or vertical tangents. The derivative of $h(t)$ with respect to t is:

$$h'(t) = \frac{1}{2}(-14t + 3) = -7t + 1.5$$

Since this derivative $h'(t)$ exists and is a finite real number for every $t \geq 0$, the function is differentiable in the given domain.

Step 3: Final Answer:
The function is differentiable for $t \geq 0$ because it is a polynomial and its derivative exists for all t in the domain.

Quick Tip
Always remember that all polynomial functions are continuous and differentiable at every point in their domain.

13(ii)(b). Find the instantaneous rate of change of height at $t = \frac{1}{14}$.

Correct Answer: 1 unit per unit time

Solution:

Step 1: Understanding the Concept:
The instantaneous rate of change of a function at a specific point is equal to the value of its first derivative at that point.

Step 2: Key Formula or Approach:
We need to find $h'(t)$ and then substitute $t = \frac{1}{14}$.

Step 3: Detailed Explanation:
The height function is $h(t) = \frac{1}{2}(-7t^2 + 3t + 2)$.
Differentiating with respect to t:

$$h'(t) = \frac{d}{dt}\left[\frac{1}{2}(-7t^2 + 3t + 2)\right]$$

$$h'(t) = \frac{1}{2}(-14t + 3)$$

$$h'(t) = -7t + \frac{3}{2}$$

Substitute $t = \frac{1}{14}$ into the derivative:

$$h'\left(\frac{1}{14}\right) = -7\left(\frac{1}{14}\right) + \frac{3}{2}$$

$$h'\left(\frac{1}{14}\right) = -\frac{1}{2} + \frac{3}{2}$$

$$h'\left(\frac{1}{14}\right) = \frac{2}{2} = 1$$

Step 4: Final Answer:
The instantaneous rate of change of height at $t = \frac{1}{14}$ is 1.

Quick Tip

In calculus, the phrase "instantaneous rate of change" is always synonymous with the "derivative" of the function at that specific point.

13(ii)(c). $h(t)$ is increasing in $\left(-\infty, \frac{3}{14}\right)$. Is this true or false? Justify.

Correct Answer: True

Solution:

Step 1: Understanding the Concept:
A function is strictly increasing on an interval if its first derivative is greater than zero $(f'(x) > 0)$ throughout that interval.

Step 2: Detailed Explanation:
We have found the derivative:

$$h'(t) = -7t + \frac{3}{2}$$

For the function to be increasing, we set $h'(t) > 0$:

$$-7t + \frac{3}{2} > 0$$

$$-7t > -\frac{3}{2}$$

Multiplying by -1 and reversing the inequality sign:

$$7t < \frac{3}{2}$$

$$t < \frac{3}{14}$$

This indicates that the function is increasing for all t in the interval $\left(-\infty, \frac{3}{14}\right)$.
Therefore, the given statement is True.

Step 3: Final Answer:
True. The derivative is positive for all $t < \frac{3}{14}$.

> Quick Tip
>
> For a quadratic function $ax^2 + bx + c$ with $a < 0$, the function increases from $-\infty$ until the vertex at $x = -b/(2a)$.

13(ii)(d). Find the time at which the Dolphin will attains the maximum height. Also find the maximum height.

Correct Answer: Time $t = \frac{3}{14}$, Maximum Height $= \frac{65}{56}$ units

Solution:

Step 1: Understanding the Concept:
Maximum height occurs when the derivative of the height function is zero ($h'(t) = 0$) and the second derivative is negative.

Step 2: Detailed Explanation:
Setting the first derivative to zero to find critical points:

$$h'(t) = -7t + \frac{3}{2} = 0$$

$$7t = \frac{3}{2} \implies t = \frac{3}{14}$$

The second derivative $h''(t) = -7$, which is less than zero, confirming that $t = \frac{3}{14}$ corresponds to a maximum.
To find the maximum height, substitute $t = \frac{3}{14}$ into the original function:

$$h\left(\frac{3}{14}\right) = \frac{1}{2}\left[-7\left(\frac{3}{14}\right)^2 + 3\left(\frac{3}{14}\right) + 2\right]$$

$$h\left(\frac{3}{14}\right) = \frac{1}{2}\left[-7\left(\frac{9}{196}\right) + \frac{9}{14} + 2\right]$$

$$h\left(\frac{3}{14}\right) = \frac{1}{2}\left[-\frac{9}{28} + \frac{18}{28} + \frac{56}{28}\right]$$

$$h\left(\frac{3}{14}\right) = \frac{1}{2}\left[\frac{65}{28}\right] = \frac{65}{56}$$

Step 3: Final Answer:
The Dolphin attains maximum height at $t = \frac{3}{14}$. The maximum height is $\frac{65}{56}$ units.

Quick Tip

The vertex of the parabola $y = ax^2 + bx + c$ occurs at $t = -b/(2a)$. This is the fastest way to find the time of maximum height.

14(i). In a school, three subject teachers English, Math, and Science sometimes give surprise tests on the same day. The English teacher gives a test 90% of the time, the Math teacher gives a test 80% of the time, and the Science teacher gives a test 70% of the time. Each teacher decides independently. Find the probability for each possible number of surprise tests $X \in \{0, 1, 2, 3\}$.

Correct Answer: $P(X = 0) = 0.006$, $P(X = 1) = 0.092$, $P(X = 2) = 0.398$, $P(X = 3) = 0.504$

Solution:

Step 1: Understanding the Concept:
Since the teachers decide independently, the probability of any combination of events is the product of their individual probabilities.

Step 2: Key Formula or Approach:
Let $P(E) = 0.9, P(M) = 0.8, P(S) = 0.7$.
Then $P(E') = 0.1, P(M') = 0.2, P(S') = 0.3$.

Step 3: Detailed Explanation:
1. **Probability of 0 tests ($X = 0$):**

$$P(X = 0) = P(E') \cdot P(M') \cdot P(S') = 0.1 \times 0.2 \times 0.3 = 0.006$$

2. **Probability of 1 test ($X = 1$):**
One teacher gives a test, others do not. There are 3 scenarios:
$P(E \cap M' \cap S') = 0.9 \times 0.2 \times 0.3 = 0.054$
$P(E' \cap M \cap S') = 0.1 \times 0.8 \times 0.3 = 0.024$
$P(E' \cap M' \cap S) = 0.1 \times 0.2 \times 0.7 = 0.014$

$$P(X = 1) = 0.054 + 0.024 + 0.014 = 0.092$$

3. **Probability of 2 tests ($X = 2$):**
Two teachers give tests, one does not. There are 3 scenarios:
$P(E \cap M \cap S') = 0.9 \times 0.8 \times 0.3 = 0.216$
$P(E \cap M' \cap S) = 0.9 \times 0.2 \times 0.7 = 0.126$
$P(E' \cap M \cap S) = 0.1 \times 0.8 \times 0.7 = 0.056$

$$P(X = 2) = 0.216 + 0.126 + 0.056 = 0.398$$

4. **Probability of 3 tests ($X = 3$):**
All teachers give tests.

$$P(X = 3) = P(E) \cdot P(M) \cdot P(S) = 0.9 \times 0.8 \times 0.7 = 0.504$$

Step 4: Final Answer:
The probabilities are $P(X = 0) = 0.006$, $P(X = 1) = 0.092$, $P(X = 2) = 0.398$, and $P(X = 3) = 0.504$.

> Quick Tip
>
> Check if the sum of all probabilities is 1: $0.006 + 0.092 + 0.398 + 0.504 = 1.000$. This confirms the calculations are exhaustive and correct.

14(ii). Use the probabilities to build a distribution table.

Correct Answer: Table Constructed

Solution:

Step 1: Understanding the Concept:
A probability distribution table summarizes the outcomes of a random variable and their probabilities.

Step 2: Detailed Explanation:
Using the values calculated in part (i), the distribution table is:

X	0	1	2	3
$P(X)$	0.006	0.092	0.398	0.504

Step 3: Final Answer:
The distribution table is as shown above.

> Quick Tip
>
> Tables are useful for calculating expectation and variance. Ensure probabilities are written in consistent decimal places.

14(iii). Calculate the average number of surprise tests per day.

Correct Answer: 2.4 tests per day

Solution:

Step 1: Understanding the Concept:
The average number (expected value) is calculated by the sum of products of each outcome and its probability.

Step 2: Key Formula or Approach:

$$E(X) = \sum x_i P(x_i)$$

Step 3: Detailed Explanation:

$$E(X) = (0 \times 0.006) + (1 \times 0.092) + (2 \times 0.398) + (3 \times 0.504)$$

$$E(X) = 0 + 0.092 + 0.796 + 1.512$$

$$E(X) = 2.4$$

Alternatively, since tests are independent:

$$E(X) = E(E) + E(M) + E(S) = 0.9 + 0.8 + 0.7 = 2.4$$

Step 4: Final Answer:
The average number of surprise tests per day is 2.4.

> Quick Tip
>
> For the sum of independent Bernoulli variables (each teacher giving a test), the total expectation is simply the sum of individual success probabilities.

14(iv). Based on your calculations, decide: Should the teachers coordinate better? Or is the current plan acceptable?

Correct Answer: The current plan is acceptable.

Solution:

Step 1: Understanding the Concept:
We compare the calculated average with the school's specific threshold (2.3).

Step 2: Detailed Explanation:
The condition provided in the problem is:
- If Average < 2.3, teachers should coordinate better.
- If Average ≥ 2.3, no action is needed (plan is acceptable).
Our calculated average is 2.4.
Since $2.4 > 2.3$, the condition for coordination is not met.
Therefore, the current plan of independent testing is considered acceptable.

Step 3: Final Answer:
The current plan is acceptable because the average (2.4) is not less than 2.3.

> Quick Tip
>
> In decision-making questions, always state the given threshold and compare your numerical result clearly to justify the conclusion.

Section B - 15 Marks

15(i). Consider the following statements and choose the correct option:
Statement 1: If $\vec{a}$ and $\vec{b}$ represents two adjacent sides of a parallelogram then the diagonals are represented by $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$.
Statement 2: If $\vec{a}$ and $\vec{b}$ represents two diagonals of a parallelogram then the adjacent sides are represented by $2(\vec{a}+\vec{b})$ and $2(\vec{a}-\vec{b})$.
Which of the following is correct?

(A) Statement 1 is true and Statement 2 is false.
(B) Statement 2 is true and Statement 1 is false.
(C) Both the statements are true.
(D) Both the statements are false.

Correct Answer: (A) Statement 1 is true and Statement 2 is false.

Solution:

Step 1: Understanding the Concept:
In a parallelogram, vectors representing the sides and diagonals are related by the triangle law of vector addition.
If $\vec{u}$ and $\vec{v}$ are adjacent sides, the diagonals are the sum and difference of these vectors.

Step 2: Detailed Explanation:
Case 1: Evaluation of Statement 1:
Let the adjacent sides of the parallelogram be $\vec{AB}=\vec{a}$ and $\vec{AD}=\vec{b}$.
By the parallelogram law of vector addition, the diagonal $\vec{AC}$ is given by $\vec{a}+\vec{b}$.

The other diagonal $\vec{BD}$ can be found by $\vec{AD} - \vec{AB} = \vec{b} - \vec{a}$ (or $\vec{a} - \vec{b}$ depending on direction).
Therefore, the diagonals are represented by $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$. Statement 1 is **True**.
Case 2: Evaluation of Statement 2:
Let the diagonals be $\vec{d_1} = \vec{a}$ and $\vec{d_2} = \vec{b}$.
Let the adjacent sides be $\vec{x}$ and $\vec{y}$.
We know that $\vec{x} + \vec{y} = \vec{a}$ and $\vec{x} - \vec{y} = \vec{b}$.
Adding the equations: $2\vec{x} = \vec{a} + \vec{b} \implies \vec{x} = \frac{1}{2}(\vec{a} + \vec{b})$.
Subtracting the equations: $2\vec{y} = \vec{a} - \vec{b} \implies \vec{y} = \frac{1}{2}(\vec{a} - \vec{b})$.
The statement claims the sides are $2(\vec{a}+\vec{b})$ and $2(\vec{a}-\vec{b})$, which is incorrect. Statement 2 is **False**.

Step 3: Final Answer:
Since Statement 1 is true and Statement 2 is false, option (A) is correct.

> Quick Tip
>
> Remember: Diagonals = Side Sum & Side Difference.
> Sides = Half Diagonal Sum & Half Diagonal Difference.

15(ii). A plane passes through three points A, B and C with position vectors $\hat{i} + \hat{j}$, $\hat{j} + \hat{k}$ and $\hat{k} + \hat{i}$ respectively. The equation of the line passing through the point P with position vector $\hat{i} + 2\hat{j} + 2\hat{k}$ and normal to the plane is

(A) $\vec{r} = (\hat{i} + 2\hat{j} + 2\hat{k}) + \lambda(\hat{i} + \hat{j} + \hat{k}), \lambda \in R$
(B) $\vec{r} = (\hat{i} + \hat{j} + \hat{k}) + \lambda(\hat{i} + 2\hat{j} + 2\hat{k}), \lambda \in R$
(C) $\vec{r} \cdot (\hat{i} - \hat{j} - \hat{k}) = \hat{i} + 2\hat{j} + 2\hat{k}$
(D) $x - 1 = y = z$

Correct Answer: (A) $\vec{r} = (\hat{i} + 2\hat{j} + 2\hat{k}) + \lambda(\hat{i} + \hat{j} + \hat{k}), \lambda \in R$

Solution:

Step 1: Understanding the Concept:
A line normal to a plane has a direction vector parallel to the normal vector of the plane.
To find the equation of a line, we need a point on the line and its direction vector.

Step 2: Key Formula or Approach:
1. Normal vector $\vec{n} = \vec{AB} \times \vec{AC}$.
2. Equation of line: $\vec{r} = \vec{p} + \lambda\vec{n}$.

Step 3: Detailed Explanation:
Let the points be $A(1, 1, 0)$, $B(0, 1, 1)$, and $C(1, 0, 1)$.
Find two vectors in the plane:
$\vec{AB} = (0 - 1)\hat{i} + (1 - 1)\hat{j} + (1 - 0)\hat{k} = -\hat{i} + \hat{k}$.
$\vec{AC} = (1 - 1)\hat{i} + (0 - 1)\hat{j} + (1 - 0)\hat{k} = -\hat{j} + \hat{k}$.

The normal vector $\vec{n}$ is the cross product of these two vectors:

$$\vec{n} = \vec{AB} \times \vec{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 0 & 1 \\ 0 & -1 & 1 \end{vmatrix} = \hat{i}(0+1) - \hat{j}(-1-0) + \hat{k}(1-0) = \hat{i} + \hat{j} + \hat{k}$$

The line passes through $P(1, 2, 2)$ and its direction is parallel to $\vec{n} = \hat{i} + \hat{j} + \hat{k}$.
The equation of the line is:
$\vec{r} = (\hat{i} + 2\hat{j} + 2\hat{k}) + \lambda(\hat{i} + \hat{j} + \hat{k})$.

Step 4: Final Answer:
The vector equation of the line is $\vec{r} = (\hat{i} + 2\hat{j} + 2\hat{k}) + \lambda(\hat{i} + \hat{j} + \hat{k})$.

Quick Tip

For points $(1, 1, 0), (0, 1, 1), (1, 0, 1)$, the plane equation is $x + y + z = 2$. The coefficients $(1, 1, 1)$ give the normal vector components immediately.

15(iii). If the direction cosines of a line are $< \frac{1}{c}, \frac{1}{c}, \frac{1}{c} >$ then

(A) $c > 0$
(B) $0 < c < 1$
(C) $c = \pm\sqrt{3}$
(D) $c > 2$

Correct Answer: (C) $c = \pm\sqrt{3}$

Solution:

Step 1: Understanding the Concept:
Direction cosines (l, m, n) are the cosines of the angles a line makes with the coordinate axes. A fundamental property of direction cosines is that the sum of their squares is always equal to 1.

Step 2: Key Formula or Approach:

$$l^2 + m^2 + n^2 = 1$$

Step 3: Detailed Explanation:
Given direction cosines are $l = \frac{1}{c}$, $m = \frac{1}{c}$, and $n = \frac{1}{c}$.
Substituting these into the identity:

$$\left(\frac{1}{c}\right)^2 + \left(\frac{1}{c}\right)^2 + \left(\frac{1}{c}\right)^2 = 1$$

$$\frac{1}{c^2} + \frac{1}{c^2} + \frac{1}{c^2} = 1$$

$$\frac{3}{c^2} = 1$$

$$c^2 = 3$$

$$c = \pm\sqrt{3}$$

Step 4: Final Answer:
The value of c is $\pm\sqrt{3}$.

Quick Tip

Direction cosines are simply the components of a unit vector. If components are equal, they must each be $1/\sqrt{3}$ (or negative) to maintain unit magnitude.

15(iv). If $\vec{a}$ and $\vec{b}$ are unit vectors enclosing an angle θ and $|\vec{a}+\vec{b}| < 1$, then find the values between which θ lies.

Correct Answer: $\frac{2\pi}{3} < \theta \leq \pi$

Solution:

Step 1: Understanding the Concept:
Unit vectors have a magnitude of 1. The magnitude of the sum of two vectors depends on the angle between them via the dot product formula.

Step 2: Key Formula or Approach:

$$|\vec{a} + \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a} \cdot \vec{b}$$

$$|\vec{a} + \vec{b}|^2 = 2 + 2\cos\theta = 4\cos^2\left(\frac{\theta}{2}\right)$$

Step 3: Detailed Explanation:
Given $|\vec{a} + \vec{b}| < 1$.
Squaring both sides: $|\vec{a} + \vec{b}|^2 < 1$.
Using the magnitude formula for unit vectors:

$$|\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}||\vec{b}| \cos\theta < 1$$

$$1 + 1 + 2(1)(1) \cos\theta < 1$$

$$2 + 2\cos\theta < 1$$

$$2\cos\theta < -1$$

$$\cos\theta < -\frac{1}{2}$$

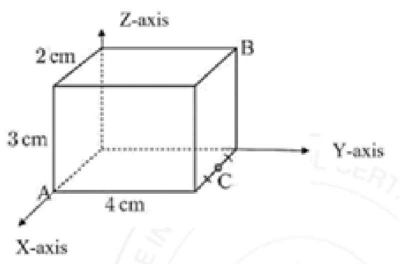
Since θ is the angle between vectors, $0 \leq \theta \leq \pi$.
$\cos\theta = -1/2$ at $\theta = 2\pi/3$.
For $\cos\theta < -1/2$, the angle must be in the second quadrant:

$$\frac{2\pi}{3} < \theta \leq \pi$$

Step 4: Final Answer:
The angle θ lies between $2\pi/3$ and π.

Quick Tip
If the sum of two unit vectors has a magnitude less than 1, they must point "away" from each other, meaning the angle between them must be obtuse (specifically $> 120°$).

15(v). Shown below is a cuboid. Find $\vec{BA} \cdot \vec{BC}$.



Correct Answer: 9

Solution:

Step 1: Understanding the Concept:
The dot product of two vectors is found by multiplying their corresponding components and summing the results.

Step 2: Detailed Explanation:
Establish the coordinates of the vertices from the cuboid diagram:
O (origin) $= (0, 0, 0)$.
$A = (4, 0, 0)$ (on X-axis).
$C = (0, 2, 0)$ (on Y-axis).
$B = (0, 2, 3)$ (corner point above C with height 3).
Find the vectors:
$\vec{BA} = \vec{A} - \vec{B} = (4-0)\hat{\imath} + (0-2)\hat{\jmath} + (0-3)\hat{k} = 4\hat{\imath} - 2\hat{\jmath} - 3\hat{k}$.
$\vec{BC} = \vec{C} - \vec{B} = (0-0)\hat{\imath} + (2-2)\hat{\jmath} + (0-3)\hat{k} = -3\hat{k}$.
Now, calculate the dot product:

$$\vec{BA} \cdot \vec{BC} = (4\hat{\imath} - 2\hat{\jmath} - 3\hat{k}) \cdot (0\hat{\imath} + 0\hat{\jmath} - 3\hat{k})$$

$$= (4 \times 0) + (-2 \times 0) + (-3 \times -3)$$

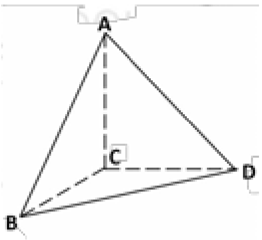
$$= 0 + 0 + 9 = 9$$

Step 3: Final Answer:
The value of $\vec{BA} \cdot \vec{BC}$ is 9.

Quick Tip

Vectors along axes or edges are easy to subtract. Since $\vec{BC}$ is vertical (only Z component), the dot product only cares about the Z component of $\vec{BA}$.

16(i)(a).



A building is to be constructed in the form of a triangular pyramid ABCD as shown in the figure. Let the angular points be $A(0,1,2)$, $B(3,0,1)$, $C(4,3,6)$ and $D(2,3,2)$ and let G be the point of intersection of the medians of ΔBCD. What will be the length of vector $\vec{AG}$?

Correct Answer: $\sqrt{11}$ units

Solution:

Step 1: Understanding the Concept:
The point of intersection of the medians of a triangle is called the centroid. The coordinates of the centroid are the average of the coordinates of the vertices.

Step 2: Key Formula or Approach:
Centroid $G = \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right)$.

Step 3: Detailed Explanation:
Given vertices of ΔBCD: $B(3,0,1)$, $C(4,3,6)$, and $D(2,3,2)$.
Coordinates of centroid G:
$x_G = \frac{3+4+2}{3} = \frac{9}{3} = 3$.
$y_G = \frac{0+3+3}{3} = \frac{6}{3} = 2$.
$z_G = \frac{1+6+2}{3} = \frac{9}{3} = 3$.
So, $G = (3,2,3)$.
Now find vector $\vec{AG}$ where $A(0,1,2)$:
$\vec{AG} = (3-0)\hat{i} + (2-1)\hat{j} + (3-2)\hat{k} = 3\hat{i} + \hat{j} + \hat{k}$.
The length of vector $\vec{AG}$ is:

$$|\vec{AG}| = \sqrt{3^2+1^2+1^2} = \sqrt{9+1+1} = \sqrt{11}$$

Step 4: Final Answer:
The length is $\sqrt{11}$ units.

Quick Tip

Centroids are simply "center-of-gravity" points. Just take the average of X, Y, and Z separately.

16(i)(b). Find the area of ΔABC.

Correct Answer: $3\sqrt{10}$ sq. units

Solution:

Step 1: Understanding the Concept:
The area of a triangle in 3D space with vertices A, B, C is given by half the magnitude of the cross product of two vectors representing its sides.

Step 2: Key Formula or Approach:
Area $= \frac{1}{2}|\vec{AB} \times \vec{AC}|$.

Step 3: Detailed Explanation:
Vertices: $A(0, 1, 2)$, $B(3, 0, 1)$, $C(4, 3, 6)$.
Calculate vectors:
$\vec{AB} = (3-0)\hat{\imath} + (0-1)\hat{\jmath} + (1-2)\hat{k} = 3\hat{\imath} - \hat{\jmath} - \hat{k}$.
$\vec{AC} = (4-0)\hat{\imath} + (3-1)\hat{\jmath} + (6-2)\hat{k} = 4\hat{\imath} + 2\hat{\jmath} + 4\hat{k}$.
Find cross product $\vec{AB} \times \vec{AC}$:

$$\vec{AB} \times \vec{AC} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 3 & -1 & -1 \\ 4 & 2 & 4 \end{vmatrix} = \hat{\imath}(-4+2) - \hat{\jmath}(12+4) + \hat{k}(6+4) = -2\hat{\imath} - 16\hat{\jmath} + 10\hat{k}$$

Magnitude of cross product:

$$|\vec{AB} \times \vec{AC}| = \sqrt{(-2)^2 + (-16)^2 + 10^2} = \sqrt{4 + 256 + 100} = \sqrt{360} = 6\sqrt{10}$$

Area $= \frac{1}{2} \times 6\sqrt{10} = 3\sqrt{10}$ sq. units.

Step 4: Final Answer:
The area is $3\sqrt{10}$ sq. units.

Quick Tip

Always double check the signs when evaluating the determinant for the cross product. A single sign error will yield the wrong area.

16(ii). What are the values of x for which the angle between the vectors $2x^2\hat{\imath} + 3x\hat{\jmath} + \hat{k}$ and $\hat{\imath} - 2\hat{\jmath} + x^2\hat{k}$ is obtuse?

Correct Answer: $0 < x < 2$

Solution:

Step 1: Understanding the Concept:
An angle θ between two vectors is obtuse if $\cos\theta < 0$. This occurs when the dot product of the vectors is negative.

Step 2: Key Formula or Approach:
$\vec{u} \cdot \vec{v} < 0$.

Step 3: Detailed Explanation:
Let $\vec{u} = 2x^2\hat{i} + 3x\hat{j} + \hat{k}$ and $\vec{v} = \hat{i} - 2\hat{j} + x^2\hat{k}$.
Find the dot product:

$$\vec{u} \cdot \vec{v} = (2x^2)(1) + (3x)(-2) + (1)(x^2)$$

$$\vec{u} \cdot \vec{v} = 2x^2 - 6x + x^2 = 3x^2 - 6x$$

For the angle to be obtuse:

$$3x^2 - 6x < 0$$

$$3x(x - 2) < 0$$

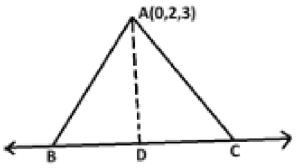
The roots are $x = 0$ and $x = 2$. For the quadratic to be negative, x must lie between the roots:

$$0 < x < 2$$

Step 4: Final Answer:
The values of x are in the interval $(0, 2)$.

Quick Tip
Obtuse $\rightarrow$ Dot Product < 0. Acute $\rightarrow$ Dot Product > 0. Right $\rightarrow$ Dot Product $= 0$.

17(i).



Given, B and C lie on the line $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3}$ and $BC = 5$ units find the area of ΔABC if $A = (0, 2, 3)$.

Correct Answer: $\frac{5\sqrt{21}}{2}$ sq. units

Solution:

Step 1: Understanding the Concept:
Area of a triangle is $\frac{1}{2} \times$ base $\times$ height. Here, base $BC = 5$. The height is the perpendicular distance from point A to the line.

Step 2: Key Formula or Approach:
Perpendicular distance $d = \frac{|\vec{AP} \times \vec{b}|}{|\vec{b}|}$ where P is a point on the line and $\vec{b}$ is its direction.

Step 3: Detailed Explanation:
Point $A(0, 2, 3)$.
Line direction $\vec{b} = 5\hat{i} + 2\hat{j} + 3\hat{k}$.
Point P on line $= (-3, 1, -4)$.
Vector $\vec{PA} = (0 - (-3))\hat{i} + (2 - 1)\hat{j} + (3 - (-4))\hat{k} = 3\hat{i} + \hat{j} + 7\hat{k}$.
Find cross product $\vec{PA} \times \vec{b}$:

$$\vec{PA} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 7 \\ 5 & 2 & 3 \end{vmatrix} = \hat{i}(3 - 14) - \hat{j}(9 - 35) + \hat{k}(6 - 5) = -11\hat{i} + 26\hat{j} + \hat{k}$$

Magnitude $|\vec{PA} \times \vec{b}| = \sqrt{(-11)^2 + 26^2 + 1^2} = \sqrt{121 + 676 + 1} = \sqrt{798}$.
Magnitude $|\vec{b}| = \sqrt{5^2 + 2^2 + 3^2} = \sqrt{25 + 4 + 9} = \sqrt{38}$.
Height $h = \frac{\sqrt{798}}{\sqrt{38}} = \sqrt{\frac{798}{38}} = \sqrt{21}$.
Area $= \frac{1}{2} \times 5 \times \sqrt{21} = \frac{5\sqrt{21}}{2}$ sq. units.

Step 4: Final Answer:
The area of the triangle is $\frac{5\sqrt{21}}{2}$ sq. units.

> Quick Tip
>
> Alternatively, find the foot of the perpendicular F on the line. AF is the height.

17(ii). Find the equation of the plane containing the line $\frac{x}{-2} = \frac{y-1}{3} = \frac{1-z}{1}$ and the point $(-1, 0, 2)$.

Correct Answer: $2x + 3y + 5z = 8$

Solution:

Step 1: Understanding the Concept:
A plane containing a line and a point is uniquely determined. The normal to the plane is perpendicular to both the line's direction and a vector connecting two points in the plane.

Step 2: Detailed Explanation:
Line: $\frac{x}{-2} = \frac{y-1}{3} = \frac{z-1}{-1}$. (Correcting standard form)
Point on line $Q = (0, 1, 1)$.
Direction of line $\vec{b} = -2\hat{i} + 3\hat{j} - \hat{k}$.
Given point $P = (-1, 0, 2)$.
Vector in plane $\vec{PQ} = (0 - (-1))\hat{i} + (1 - 0)\hat{j} + (1 - 2)\hat{k} = \hat{i} + \hat{j} - \hat{k}$.
Normal vector $\vec{n} = \vec{b} \times \vec{PQ}$:

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 3 & -1 \\ 1 & 1 & -1 \end{vmatrix} = \hat{i}(-3 + 1) - \hat{j}(2 + 1) + \hat{k}(-2 - 3) = -2\hat{i} - 3\hat{j} - 5\hat{k}$$

Plane equation passing through $P(-1, 0, 2)$ with normal $(2, 3, 5)$:

$$2(x + 1) + 3(y - 0) + 5(z - 2) = 0$$

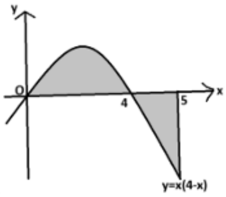
$$2x + 2 + 3y + 5z - 10 = 0$$

$$2x + 3y + 5z = 8$$

Step 3: Final Answer:
The equation of the plane is $2x + 3y + 5z = 8$.

Quick Tip
Check your answer by plugging in the point $(-1, 0, 2)$: $2(-1)+3(0)+5(2) = -2+10 = 8$. Correct!

18.



Find the area bounded by the curve, $y = x(4 - x)$ and the x-axis from $x = 0$ to $x = 5$ as shown in the figure given above.

Correct Answer: 13 sq. units

Solution:

Step 1: Understanding the Concept:
Area bounded by a curve $y = f(x)$ and the X-axis is the integral of $|f(x)|$ over the given interval. If the curve crosses the axis, the integral must be split.

Step 2: Detailed Explanation:
Function $y = 4x - x^2$. Roots are $x = 0$ and $x = 4$.
Between $x = 0$ and $x = 4$, $y \geq 0$. Between $x = 4$ and $x = 5$, $y \leq 0$.
Area $A = \int_0^4 (4x - x^2)dx + \left| \int_4^5 (4x - x^2)dx \right|$.
Part 1:

$$\int_0^4 (4x - x^2)dx = [2x^2 - x^3/3]_0^4 = (32 - 64/3) = \frac{96 - 64}{3} = \frac{32}{3}$$

Part 2:

$$\int_4^5 (4x - x^2)dx = [2x^2 - x^3/3]_4^5 = (50 - 125/3) - (32 - 64/3)$$

$$= \frac{150 - 125}{3} - \frac{96 - 64}{3} = \frac{25}{3} - \frac{32}{3} = -\frac{7}{3}$$

Magnitude is 7/3.
Total Area $= \frac{32}{3} + \frac{7}{3} = \frac{39}{3} = 13$ sq. units.

Step 3: Final Answer:
The total area is 13 sq. units.

Quick Tip

Geometric area is always positive. When the graph goes below the axis, the integral is negative, so always use absolute values for those sections.

Section C - 15 Marks

19(i). Which condition is true if Average Cost (AC) is constant at all levels of output?

(A) MC > AC
(B) MC = AC
(C) MC < AC
(D) MC = $\frac{1}{2}$ AC

Correct Answer: (B) MC = AC

Solution:

Step 1: Understanding the Concept:
Average Cost (AC) is the cost per unit of output, defined as $AC = \frac{C(x)}{x}$, where $C(x)$ is the total cost and x is the output level. Marginal Cost (MC) is the instantaneous rate of change of total cost with respect to output, i.e., $MC = \frac{dC}{dx}$.

Step 2: Key Formula or Approach:
The relationship between MC and AC is given by the formula:

$$MC = AC + x \cdot \frac{d(AC)}{dx}$$

Step 3: Detailed Explanation:
If the Average Cost (AC) is constant at all levels of output, its derivative with respect to output (x) is zero:

$$\frac{d(AC)}{dx} = 0$$

Substituting this into the relationship formula:

$$MC = AC + x \cdot (0)$$

$$MC = AC$$

This means that when AC is constant, the Marginal Cost must be equal to the Average Cost.

Step 4: Final Answer:
The condition is MC = AC.

Quick Tip

Remember the general rule: If AC is falling, MC < AC. If AC is rising, MC > AC. If AC is constant (or at its minimum), MC = AC.

19(ii). Which of the following statement(s) is/are correct with respect to regression coefficients?
Statement 1: It measures the degree of linear relationship between two variables.
Statement 2: It gives the value by which one variable changes for a unit change in the other variable.
Which of the following is correct?

(A) Statement 1 is true and Statement 2 is false.
(B) Statement 2 is true and Statement 1 is false.
(C) Both the statements are true.
(D) Both the statements are false.

Correct Answer: (B) Statement 2 is true and Statement 1 is false.

Solution:

Step 1: Understanding the Concept:
Regression coefficients (b_{yx} and b_{xy}) represent the slope of the regression lines. They describe how the dependent variable responds to changes in the independent variable. The Correlation Coefficient (r) is what measures the degree and direction of the linear relationship.

Step 2: Detailed Explanation:
Evaluation of Statement 1: The measure of the degree of linear relationship between two variables is the correlation coefficient (r), not the regression coefficients. Therefore, Statement 1 is False.
Evaluation of Statement 2: By definition, the regression coefficient b_{yx} indicates the amount of change in variable y for every unit change in variable x. Similarly, b_{xy} indicates the change in x per unit change in y. Therefore, Statement 2 is True.

Step 3: Final Answer:
Statement 2 is true and Statement 1 is false.

Quick Tip

Do not confuse regression coefficients (slopes) with the correlation coefficient (strength of bond). While r is always between -1 and 1, regression coefficients can take any real value.

19(iii). Mean of $x = 53$, mean of $y = 28$ regression co-efficient y on $x = -1.2$, regression co-efficient x on $y = -0.3$. Find coefficient of correlation (r).

Correct Answer: -0.6

Solution:

Step 1: Understanding the Concept:
The correlation coefficient (r) is the geometric mean of the two regression coefficients. Importantly, the sign of r must be the same as the sign of the regression coefficients.

Step 2: Key Formula or Approach:

$$r = \pm\sqrt{b_{yx} \cdot b_{xy}}$$

Step 3: Detailed Explanation:
Given:
$b_{yx} = -1.2$
$b_{xy} = -0.3$
Calculate the product:

$$r^2 = b_{yx} \cdot b_{xy} = (-1.2) \cdot (-0.3) = 0.36$$

Taking the square root:

$$r = \pm\sqrt{0.36} = \pm 0.6$$

Since both regression coefficients are negative, the correlation coefficient r must also be negative.

$$r = -0.6$$

Step 4: Final Answer:
The coefficient of correlation is -0.6.

Quick Tip

Always remember: b_{yx}, b_{xy}, and r must all have the same sign. If the product of b_{yx} and b_{xy} is greater than 1, you have made a calculation error.

19(iv). The total revenue received from the sale of x unit of a product is given by $R(x) = 3x^2 + 36x + 5$. Find the marginal revenue when $x = 5$.

Correct Answer: 66

Solution:

Step 1: Understanding the Concept:
Marginal Revenue (MR) is the rate of change of total revenue with respect to the number of units sold. Mathematically, it is the first derivative of the Revenue function $R(x)$.

Step 2: Key Formula or Approach:

$$MR = \frac{dR}{dx}$$

Step 3: Detailed Explanation:
Given:

$$R(x) = 3x^2 + 36x + 5$$

Differentiating with respect to x:

$$MR = \frac{d}{dx}(3x^2 + 36x + 5)$$

$$MR = 6x + 36$$

Now, find the value of MR at $x = 5$:

$$MR|_{x=5} = 6(5) + 36$$

$$MR = 30 + 36 = 66$$

Step 4: Final Answer:
The marginal revenue when $x = 5$ is 66.

> Quick Tip
>
> In marginal analysis, always differentiate the function first and then substitute the value of the variable. Do not substitute before differentiating.

19(v). A manufacturing company finds that the daily cost of producing x item of product is given by $C(x) = 210x + 7000$. Find the minimum number that must be produced and sold daily for break even, if each item is sold for 280.

Correct Answer: 100 items

Solution:

Step 1: Understanding the Concept:
Break-even point is the level of output where Total Revenue (TR) equals Total Cost (TC). At this point, the profit is zero.

Step 2: Key Formula or Approach:

$$TR = TC$$

where TR = Selling Price × quantity.

Step 3: Detailed Explanation:
Given:
Selling Price per item = 280
Total Revenue for x items, $R(x) = 280x$.
Total Cost for x items, $C(x) = 210x + 7000$.
At break-even point:

$$R(x) = C(x)$$

$$280x = 210x + 7000$$

Subtracting $210x$ from both sides:

$$70x = 7000$$

$$x = \frac{7000}{70} = 100$$

Step 4: Final Answer:
The minimum number of items to be produced and sold for break-even is 100.

Quick Tip

Break-even happens where profit $P(x) = 0$. If you are given Profit function instead, simply set $P(x) = 0$ and solve for x.

20(i).

A real estate company is going to build a new residential complex. The land they have purchased can hold at the most 500 apartments. Also, if they make x apartments, then the monthly maintenance cost for the whole complex would be as follows:
Fixed cost = 4000
Variable cost = $(14x - 0.04x^2)$
How many apartments should the complex have in order to minimize the maintenance costs?

Correct Answer: 500 apartments

Solution:

Step 1: Understanding the Concept:
To find the minimum or maximum of a function, we look at the derivative. Here, the maintenance cost function $C(x)$ is a quadratic with a negative coefficient for x^2, indicating it opens downwards (a maximum exists). We must check the boundaries of the feasible region $(0 \leq x \leq 500)$ for the absolute minimum.

Step 2: Key Formula or Approach:

$$TotalCost(C) = FixedCost + VariableCost$$

Step 3: Detailed Explanation:
Total Maintenance Cost function:

$$C(x) = 4000 + 14x - 0.04x^2$$

Differentiating with respect to x:

$$C'(x) = 14 - 0.08x$$

Setting $C'(x) = 0$ to find the critical point:

$$14 = 0.08x \implies x = \frac{14}{0.08} = 175$$

Since $C''(x) = -0.08 < 0$, $x = 175$ is a point of **maximum** maintenance cost.
To **minimize** the cost, we check the endpoints of the range $[0, 500]$:
1. At $x = 0$: $C(0) = 4000$.
2. At $x = 500$:

$$C(500) = 4000 + 14(500) - 0.04(500)^2$$

$$C(500) = 4000 + 7000 - 0.04(250000)$$

$$C(500) = 11000 - 10000 = 1000$$

Comparing the values, the absolute minimum maintenance cost occurs at $x = 500$.

Step 4: Final Answer:
The complex should have 500 apartments to minimize maintenance costs.

Quick Tip
When the critical point gives a maximum but the question asks for a minimum, always evaluate the function at the boundary constraints of the domain.

20(ii). The demand function of a monopoly is given by $x = 100 - 4p$. Find the quantity at which the Marginal Revenue will be zero.

Correct Answer: 50

Solution:

Step 1: Understanding the Concept:
Total Revenue (TR) is given by Price (p) multiplied by Quantity (x). Marginal Revenue (MR)

is the derivative of TR with respect to quantity.

Step 2: Key Formula or Approach:
1. Express p in terms of x.
2. $TR = p \cdot x$.
3. $MR = \frac{d(TR)}{dx}$.

Step 3: Detailed Explanation:
Given demand function:

$$x = 100 - 4p \implies 4p = 100 - x \implies p = 25 - \frac{x}{4}$$

Total Revenue function:

$$TR = p \cdot x = \left(25 - \frac{x}{4}\right) x = 25x - \frac{x^2}{4}$$

Marginal Revenue function:

$$MR = \frac{d(TR)}{dx} = \frac{d}{dx}\left(25x - \frac{x^2}{4}\right) = 25 - \frac{2x}{4} = 25 - \frac{x}{2}$$

Setting $MR = 0$:

$$25 - \frac{x}{2} = 0 \implies \frac{x}{2} = 25 \implies x = 50$$

Step 4: Final Answer:
The Marginal Revenue will be zero at a quantity of 50 units.

Quick Tip

For a linear demand curve, the Marginal Revenue curve starts at the same intercept but has twice the slope. MR reaches zero exactly at the midpoint of the demand curve's quantity intercept.

---

21. A survey of 50 families to study the relationships between expenditure on accommodation in (x) and expenditure on food and entertainment (y) gave the following results:
$\sum x = 8500, \sum y = 9600, \sigma_x = 60, \sigma_y = 20, r = 0.6$
Estimate the expenditure on food and entertainment when expenditure on accommodation is 200.

Correct Answer: 198

Solution:

Step 1: Understanding the Concept:
To estimate y (expenditure on food) for a given x (accommodation), we use the regression line of y on x.

Step 2: Key Formula or Approach:
1. Mean $\bar{x} = \frac{\sum x}{N}$, $\bar{y} = \frac{\sum y}{N}$.
2. Regression coefficient $b_{yx} = r \cdot \frac{\sigma_y}{\sigma_x}$.
3. Regression equation: $(y - \bar{y}) = b_{yx}(x - \bar{x})$.

Step 3: Detailed Explanation:
Given: $N = 50, \sum x = 8500, \sum y = 9600, \sigma_x = 60, \sigma_y = 20, r = 0.6$.
Calculate Means:

$$\bar{x} = \frac{8500}{50} = 170$$

$$\bar{y} = \frac{9600}{50} = 192$$

Calculate Regression Coefficient (b_{yx}):

$$b_{yx} = 0.6 \cdot \frac{20}{60} = 0.6 \cdot \frac{1}{3} = 0.2$$

Form the Regression Equation:

$$y - 192 = 0.2(x - 170)$$

Estimate y when $x = 200$:

$$y - 192 = 0.2(200 - 170)$$

$$y - 192 = 0.2(30)$$

$$y - 192 = 6$$

$$y = 192 + 6 = 198$$

Step 4: Final Answer:
The estimated expenditure on food and entertainment is 198.

> Quick Tip
>
> Always identify which variable is independent (x) and which is dependent (y) based on the wording "Estimate y when x is...". This ensures you use the correct regression line.

22(i)(a). A linear programming problem is given by $Z = px + qy$ where $p, q > 0$ subject to the constraints: $x + y \leq 60, 5x + y \leq 100, x \geq 0$ and $y \geq 0$. Solve graphically to find the corner points of the feasible region.

Correct Answer: (0,0), (20,0), (10,50), (0,60)

Solution:

Step 1: Understanding the Concept:
The corner points of a feasible region are the intersections of the boundary lines of the constraints. We plot the lines as equalities and find their intersection points.

Step 2: Detailed Explanation:
1. **Constraint 1:** $x + y = 60$. Points: (60,0) and (0,60). Shading is towards the origin since $0 + 0 \leq 60$ is true.
2. **Constraint 2:** $5x + y = 100$. Points: (20,0) and (0,100). Shading is towards origin since $0 + 0 \leq 100$ is true.
3. **Non-negativity:** $x \geq 0, y \geq 0$ restricts the region to the first quadrant.
4. **Intersection point:** Solve the equations:
Subtracting $x + y = 60$ from $5x + y = 100$:

$$4x = 40 \implies x = 10$$

Substituting in $x + y = 60$:

$$10 + y = 60 \implies y = 50$$

The intersection point is (10,50).
The feasible region is bounded by the axes and the constraints. The vertices are the origin (0,0), the x-intercept of the steeper line (20,0), the intersection (10,50), and the y-intercept of the shallower line (0,60).

Step 3: Final Answer:
The corner points are (0,0), (20,0), (10,50), and (0,60).

💡 Quick Tip

To quickly find the corner points, look for the "lowest" intercepts on each axis among all the constraints of the "$\leq$" type.

22(i)(b). If $Z = px + qy$ is maximum at $(0, 60)$ and $(10, 50)$, find the relation of p and q. Also mention the number of optimal solution(s) in this case.

Correct Answer: $p = q$; Infinitely many optimal solutions

Solution:

Step 1: Understanding the Concept:
If an objective function attains its maximum value at two different corner points, then it must attain the same maximum value at every point on the line segment joining those two points.

Step 2: Detailed Explanation:
Since the maximum occurs at both (0,60) and (10,50), the value of Z at these points must be equal.

$$Z(0, 60) = p(0) + q(60) = 60q$$

$$Z(10, 50) = p(10) + q(50) = 10p + 50q$$

Equating the two:

$$60q = 10p + 50q$$

$$10q = 10p \implies p = q$$

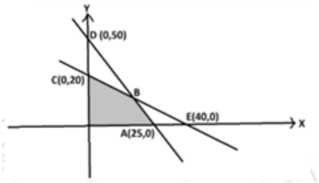
When an LPP has more than one optimal corner point solution, it possesses infinitely many optimal solutions on the boundary line segment connecting those points.

Step 3: Final Answer:
The relation is $p = q$. There are infinitely many optimal solutions.

💡 Quick Tip

In multiple optimal solutions, the objective function line is parallel to the constraint boundary line where the maximum occurs.

22(ii)(a).



Based on the given graph, answer the following questions:
Write the constraints for the L.P.P. (Points given: D(0,50), C(0,20), A(25,0), E(40,0))

Correct Answer: $2x + y \geq 50$, $x + 2y \leq 40$, $y \geq 0$

Solution:

Step 1: Understanding the Concept:
The constraints are linear inequalities derived from the lines forming the boundaries of the feasible region.

Step 2: Detailed Explanation:
1. **Line 1:** Passes through $D(0, 50)$ and $A(25, 0)$.
Using intercept form $\frac{x}{25} + \frac{y}{50} = 1 \implies 2x + y = 50$.
The shaded region is above this line, so the constraint is $2x + y \geq 50$.
2. **Line 2:** Passes through $C(0, 20)$ and $E(40, 0)$.
Using intercept form $\frac{x}{40} + \frac{y}{20} = 1 \implies x + 2y = 40$.
The shaded region is below this line, so the constraint is $x + 2y \leq 40$.
3. **Boundaries:** The region is above the x-axis, so $y \geq 0$. (Note: $x \geq 0$ is also implicit but $y \geq 0$ and the lines already bound x between 20 and 40).

Step 3: Final Answer:
Constraints are $2x + y \geq 50$, $x + 2y \leq 40$, $y \geq 0$.

> 💡 Quick Tip
>
> To find a line's equation from intercepts $(a, 0)$ and $(0, b)$, just use $bx + ay = ab$. It's faster than the point-slope form.

22(ii)(b). Find the co-ordinates of the point B.

Correct Answer: (20, 10)

Solution:

Step 1: Understanding the Concept:
Point B is the intersection of the two boundary lines identified in part (a).

Step 2: Detailed Explanation:
The boundary lines are:
1) $2x + y = 50 \implies y = 50 - 2x$
2) $x + 2y = 40$
Substitute equation (1) into equation (2):

$$x + 2(50 - 2x) = 40$$

$$x + 100 - 4x = 40$$

$$-3x = -60 \implies x = 20$$

Substitute $x = 20$ back into equation (1):

$$y = 50 - 2(20) = 50 - 40 = 10$$

The coordinates of B are (20, 10).

Step 3: Final Answer:
The coordinates of point B are (20, 10).

Quick Tip
Always check your intersection point by substituting it into BOTH line equations to verify accuracy.

22(ii)(c). Find the maximum value of the objective function $Z = x + y$.

Correct Answer: 40

Solution:

Step 1: Understanding the Concept:
The maximum value of the objective function occurs at one of the corner points of the feasible

region.

Step 2: Detailed Explanation:
The corner points of the shaded region are:
1. $A(25, 0)$
2. $E(40, 0)$
3. $B(20, 10)$
Evaluating $Z = x + y$ at each point:
- At $A(25, 0)$: $Z = 25 + 0 = 25$
- At $E(40, 0)$: $Z = 40 + 0 = 40$
- At $B(20, 10)$: $Z = 20 + 10 = 30$
Comparing the values, the maximum value is 40 at point $E(40, 0)$.

Step 3: Final Answer:
The maximum value of the objective function is 40.

Quick Tip

If the objective function is $Z = x + y$, you are simply looking for the corner point that is "furthest" from the origin in terms of sum of coordinates.
