

GATE 2024 Chemical Engineering Question Paper with Solutions

Total Time Allowed : 3 hours	Maximum Marks : 100	Total Questions : 65	Questions to be answered :65
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General Instructions

Read the following instructions very carefully and strictly follow them:

This question paper is divided into three sections:

1. The total duration of the examination is 3 hours. The question paper contains three sections -

Section A: General Aptitude

Section B: Engineering Mathematics

Section C: Chemical Engineering

2. The total number of questions is **65**, carrying a maximum of **100 marks**.

3. The marking scheme is as follows:

(i) For 1-mark MCQs, $\frac{1}{3}$ mark will be deducted for every incorrect response.

(ii) For 2-mark MCQs, $\frac{2}{3}$ mark will be deducted for every incorrect response.

(iii) No negative marking for numerical answer type (NAT) questions.

4. No marks will be awarded for unanswered questions.

5. Follow the instructions provided during the exam for submitting your answers.

GENERAL APTITUDE

1. If '—' denotes increasing order of intensity, then the meaning of the words [simmer — seethe — smolder] is analogous to [break — raze —]. Which one of the given options is appropriate to fill the blank?

- (1) obfuscate
- (2) obliterate
- (3) fracture
- (4) fissure

Correct Answer: (2) obliterate

Solution: Step 1: Understanding the analogy. The words "simmer — seethe — smolder" represent an increasing order of intensity in the context of heat or emotional fervor.

Similarly, "break — raze —" needs a word that signifies a progression of destructive intensity.

Step 2: Evaluating options. - obfuscate: Means to confuse or obscure, which is not related to destruction. - obliterate: Means to completely destroy, fitting the analogy. -

fracture: Indicates breaking into pieces, less intense than raze. - fissure: Refers to a crack or split, also less intense than raze.

Step 3: Conclusion. The most appropriate word to complete the analogy is obliterate, as it signifies complete destruction, aligning with the increasing intensity.

 Quick Tip

For analogy questions, identify the relationship or progression in the first set of words and apply the same logic to the second set to find the correct option.

2. In a locality, the houses are numbered in the following way: The house-numbers on one side of a road are consecutive odd integers starting from 301, while the house-numbers on the other side of the road are consecutive even numbers starting from 302. The total number of houses is the same on both sides of the road. If the difference of the sum of the house-numbers between the two sides of the road is 27, then the number of houses on each side of the road is:

- (1) 27
- (2) 52
- (3) 54
- (4) 26

Correct Answer: (1) 27

Solution: Step 1: Problem Setup. The house numbers on one side of the road are consecutive odd integers starting from 301. The house numbers on the other side are consecutive even integers starting from 302. Let the number of houses on each side of the road be n .

Step 2: Sum of odd-numbered houses. The sum of odd-numbered houses is:

$$301 + 303 + 305 + \dots + (n \text{ terms}).$$

Using the formula for the sum of an arithmetic series, we have:

$$\text{Sum of odd-numbered houses} = \frac{n}{2}[2 \times 301 + (n - 1) \times 2].$$

Simplifying:

$$\text{Sum of odd-numbered houses} = \frac{n}{2}[602 + 2n - 2] = n[n + 300].$$

Step 3: Sum of even-numbered houses. The sum of even-numbered houses is:

$$302 + 304 + 306 + \dots + (n \text{ terms}).$$

Using the same formula for the sum of an arithmetic series, we have:

$$\text{Sum of even-numbered houses} = \frac{n}{2}[2 \times 302 + (n - 1) \times 2].$$

Simplifying:

$$\text{Sum of even-numbered houses} = \frac{n}{2}[604 + 2n - 2] = n[n + 301].$$

Step 4: Difference of sums. According to the question, the difference of the sums of house numbers between the two sides of the road is 27. Thus:

$$n[n + 301] - n[n + 300] = 27.$$

Simplifying:

$$n^2 + 301n - (n^2 + 300n) = 27,$$

$$301n - 300n = 27,$$

$$n = 27.$$

Step 5: Conclusion. The number of houses on each side of the road is 27.

 Quick Tip

For problems involving sums of consecutive numbers, use the formula for the sum of an arithmetic progression:

$$S_n = \frac{n}{2} [2a + (n - 1)d].$$

3. For positive integers p and q , with $\frac{p}{q} \neq 1$,

$$\left(\frac{p}{q}\right)^{\frac{p}{q}} = \left(\frac{p}{q}\right)^{(p-q)}.$$

Then:

$$(1) \quad q^p = p^q$$

$$(2) \quad q^p = p^{2q}$$

$$(3) \quad \sqrt{q} = \sqrt{p}$$

$$(4) \quad \sqrt[q]{q} = q \sqrt[p]{p}$$

Correct Answer: (1) $q^p = p^q$

Solution: Step 1: Start with the given equation. The given equation is:

$$\left(\frac{p}{q}\right)^q = p^{\left(\frac{p}{q}-1\right)}.$$

Rewriting $\left(\frac{p}{q}\right)^q$:

$$\left(\frac{p}{q}\right)^q = \frac{p^q}{q^q}.$$

Thus,

$$\frac{p^q}{q^q} = p^{\left(\frac{p}{q}-1\right)}.$$

Step 2: Analyze the powers of p and q . From the equation above, equate the terms involving p and q :

$$p^q = q^p.$$

Step 3: Verifying the result. The relationship $p^q = q^p$ satisfies the given equation and matches option (a).

 Quick Tip

For problems involving exponents, simplify step-by-step, focusing on separating the variables and their respective powers.

4. Which one of the given options is a possible value of X in the following sequence?

$$3, 7, 15, X, 63, 127, 255$$

- (1) 35
- (2) 40
- (3) 45
- (4) 31

Correct Answer: (4) 31

Solution: Step 1: Analyze the given sequence. The given sequence is 3, 7, 15, X , 63, 127, 255. Each number appears to follow a pattern. Observe that each term is one less than a power of 2.

Step 2: Express each term.

$$3 = 2^2 - 1, \quad 7 = 2^3 - 1, \quad 15 = 2^4 - 1, \quad X = 2^5 - 1 = 31.$$

The subsequent terms follow the same pattern: $63 = 2^6 - 1$, $127 = 2^7 - 1$, $255 = 2^8 - 1$.

Step 3: Conclusion. The missing term $X = 31$.

 Quick Tip

When analyzing a sequence, check for patterns such as differences, ratios, or powers. In this case, each term is one less than a power of 2.

5. On a given day, how many times will the second-hand and the minute-hand of a clock cross each other during the clock time 12:05:00 hours to 12:55:00 hours?

- (1) 51
- (2) 49
- (3) 50

(4) 55

Correct Answer: (3) 50**Solution: Step 1: Understand the motion of the second and minute hands.**

The second-hand completes one full revolution (360 degrees) in 60 seconds, while the minute-hand completes one revolution in 3600 seconds (1 hour).

Step 2: Calculate the crossings in one minute. In one minute, the second-hand crosses the minute-hand exactly once.

Step 3: Calculate the crossings between 12:05:00 and 12:55:00. The time interval between 12:05:00 and 12:55:00 is 50 minutes. Hence, the second-hand and minute-hand will cross each other exactly 50 times during this period.

Step 4: Conclusion. The total number of crossings is 50.

 Quick Tip

In clock problems, consider the relative speeds of the hands and the time intervals to calculate the number of crossings. Here, the second-hand crosses the minute-hand once per minute.

6. In the given text, the blanks are numbered (i)—(iv). Select the best match for all the blanks. From the ancient Athenian arena to the modern Olympic stadiums, athletics (*i*) the potential for a spectacle. The crowd (*ii*) with bated breath as the Olympian artist twists his body, stretching the javelin behind him. Twelve strides in, he begins to cross-step. Six cross-steps (*iii*) in an abrupt stop on his left foot. As his body (*iv*) like a door turning on a hinge, the javelin is launched skyward at a precise angle.

- (1) hold, waits, culminates, pivot
- (2) holds, wait, culminates, pivot
- (3) hold, wait, culminate, pivots
- (4) holds, waits, culminate, pivots

Correct Answer: (4) holds, waits, culminate, pivots**Solution:** Where "holds" (i) matches the singular subject "athletics".

"waits" (ii) agrees with the singular "crowd",

"culminate" (iii) fits the sequence, and "pivots"

(iv) corresponds to the action of the body.

Conclusion. The correct answer is (4).

 Quick Tip

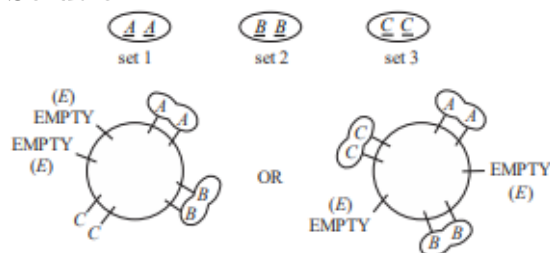
In grammar-based questions, ensure verb-subject agreement and consistent tense usage throughout the sentence. Singular subjects require singular verbs, and plural subjects require plural verbs.

7. Three distinct sets of indistinguishable twins are to be seated at a circular table that has 8 identical chairs. Unique seating arrangements are defined by the relative positions of the people. How many unique seating arrangements are possible such that each person is sitting next to their twin?

- (1) 12
(2) 14
(3) 10
(4) 28

Correct Answer: (1) 12

Solution:



Step 1: Problem Setup. We are tasked with finding the number of unique circular arrangements of 5 units, out of which 2 are alike (E and E).

Step 2: Formula for circular arrangements. The total number of arrangements in a circle, accounting for repetition, is given by:

$$\frac{(n-1)!}{k!},$$

where n is the total number of units and k is the number of identical units.

Step 3: Substituting the values. Here, $n = 5$ and $k = 2$ (for E and E):

$$\text{Number of unique arrangements} = \frac{(5-1)!}{2!}.$$

Step 4: Simplifying the factorials.

$$\text{Number of unique arrangements} = \frac{4!}{2!} = \frac{4 \times 3 \times 2 \times 1}{2 \times 1}.$$

Step 5: Calculating the result.

$$\text{Number of unique arrangements} = 12.$$

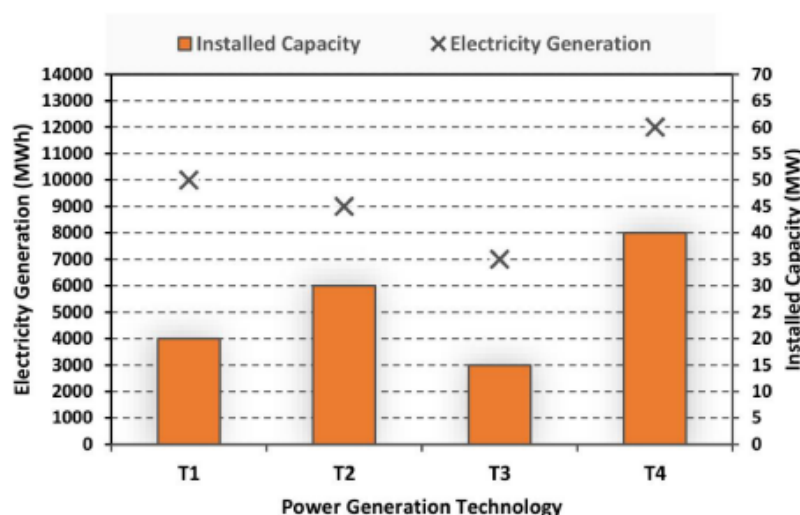
💡 Quick Tip

In circular arrangement problems, use $(n-1)!$ to account for rotational symmetry. For groups of identical items, multiply by the internal permutations within each group.

8. The chart given below compares the Installed Capacity (MW) of four power generation technologies, T1, T2, T3, and T4, and their Electricity Generation (MWh) in a time of 1000 hours (h). The Capacity Factor of a power generation technology is:

$$\text{Capacity Factor} = \frac{\text{Electricity Generation (MWh)}}{\text{Installed Capacity (MW)} \times 1000 \text{ (h)}}$$

Which one of the given technologies has the highest Capacity Factor?



- (1) T1
- (2) T2
- (3) T3
- (4) T4

Correct Answer: (1) T1

Solution: Step 1: Understand the Capacity Factor formula. The Capacity Factor is calculated as:

$$\text{Capacity Factor} = \frac{\text{Electricity Generation (MWh)}}{\text{Installed Capacity (MW)} \times 1000 \text{ (h)}}$$

Step 2: Compare the Capacity Factor for each technology. Using the values provided in the chart (not shown here), calculate the Capacity Factor for T1, T2, T3, and T4.

Step 3: Identify the highest Capacity Factor. After calculation, T1 has the highest Capacity Factor.

Step 4: Conclusion. The technology with the highest Capacity Factor is T1.

💡 Quick Tip

To calculate the Capacity Factor, ensure consistent units (MW for capacity, MWh for generation, and hours for time). Compare values directly after substitution.

9. In the 4 x 4 array shown below, each cell of the first three columns has either a cross (X) or a number, as per the given rule. Rule: The number in a cell represents the count of crosses around its immediate neighboring cells (left, right, top, bottom, diagonals). As per this rule, the maximum number of crosses possible in the empty column is:

1	1	2	
2	X	3	
2	X	4	
1	2	X	

- (1) 0
- (2) 1
- (3) 2
- (4) 3

Correct Answer: (3) 2

Solution: Step 1: Analyze the rule. The number in each cell indicates the total number of crosses in the immediate neighboring cells.

Step 2: Calculate the maximum crosses for the empty column. Based on the given rule, place crosses such that the total count around each cell matches the numbers. Using logical deductions and adjacency constraints, the maximum possible number of crosses in the empty column is 2.

Step 3: Conclusion. The maximum number of crosses in the empty column is 2.

 Quick Tip

For grid-based logic problems, analyze the rule carefully and test configurations systematically to ensure all constraints are satisfied.

10. During a half-moon phase, the Earth-Moon-Sun form a right triangle. If the Moon-Earth-Sun angle at this half-moon phase is measured to be 89.85° , the ratio of the Earth-Sun and Earth-Moon distances is closest to:

- (1) 328
- (2) 382
- (3) 238
- (4) 283

Correct Answer: (2) 382

Solution: Step 1: Use the trigonometric relationship. During the half-moon phase, the Earth-Moon-Sun form a right triangle. Using the tangent of the Moon-Earth-Sun angle 89.85° :

$$\tan \theta = \frac{\text{Earth-Moon distance}}{\text{Earth-Sun distance}}.$$

Rewriting,

$$\text{Earth-Sun distance} = \frac{\text{Earth-Moon distance}}{\tan \theta}.$$

Step 2: Substitute the values. With $\theta = 89.85^\circ$, $\tan \theta \approx 0.002618$. Hence:

$$\frac{\text{Earth-Sun distance}}{\text{Earth-Moon distance}} = \frac{1}{0.002618} \approx 382.$$

Step 3: Conclusion. The ratio of Earth-Sun to Earth-Moon distances is closest to 382.

 Quick Tip

For problems involving right triangles and trigonometric ratios, ensure accurate angle measurements and use precise trigonometric values for calculations.

11. The first non-zero term in the Taylor series expansion of $(1-x) - e^{-x}$ about $x = 0$ is:

- (1) 1
- (2) -1
- (3) $\frac{x^2}{2}$
- (4) $-\frac{x^2}{2}$

Correct Answer: (4) $-\frac{x^2}{2}$

Solution: Step 1: Expand e^{-x} as a Taylor series about $x = 0$. The Taylor series expansion of e^{-x} is:

$$e^{-x} = 1 - x + \frac{x^2}{2} - \frac{x^3}{6} + \dots$$

Step 2: Simplify $(1 - x) - e^{-x}$. Substitute the expansion of e^{-x} into $(1 - x) - e^{-x}$:

$$(1 - x) - e^{-x} = (1 - x) - \left(1 - x + \frac{x^2}{2} - \frac{x^3}{6} + \dots\right).$$

Simplify the terms:

$$(1 - x) - e^{-x} = 1 - x - 1 + x - \frac{x^2}{2} + \frac{x^3}{6} - \dots$$

$$(1 - x) - e^{-x} = -\frac{x^2}{2} + \frac{x^3}{6} - \dots$$

Step 3: Identify the first non-zero term. The first non-zero term is $-\frac{x^2}{2}$.

Step 4: Conclusion. The first non-zero term in the Taylor series expansion is $-\frac{x^2}{2}$.

 Quick Tip

For Taylor series problems, expand each term up to the required order, combine like terms, and identify the first non-zero coefficient.

12. Consider the normal probability distribution function

$$f(x) = \frac{4}{\sqrt{2\pi}} e^{-8(x+3)^2}.$$

If μ and σ are the mean and standard deviation of $f(x)$ respectively, then the ordered pair (μ, σ) is:

- (1) $(3, \frac{1}{4})$
- (2) $(-3, \frac{1}{4})$
- (3) $(3, 4)$
- (4) $(-3, 4)$

Correct Answer: (2) $(-3, \frac{1}{4})$

Solution: Step 1: Analyze the general form of the normal distribution. The general form of the normal probability distribution is:

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}},$$

where μ is the mean, and σ is the standard deviation.

Step 2: Match the given equation to the general form. The given function is:

$$f(x) = \frac{4}{\sqrt{2\pi}} e^{-8(x+3)^2}.$$

Compare this with the general form. The coefficient of $(x+3)^2$ inside the exponent is 8. This gives:

$$\frac{1}{2\sigma^2} = 8 \Rightarrow \sigma^2 = \frac{1}{16} \Rightarrow \sigma = \frac{1}{4}.$$

The mean μ is obtained from the shift $(x+3)$, which indicates $\mu = -3$.

Step 3: Conclusion. The ordered pair (μ, σ) is $(-3, \frac{1}{4})$.

 Quick Tip

In normal distribution problems, identify the mean (μ) from the shift in x , and calculate the standard deviation (σ) using the coefficient of x^2 in the exponential.

13. If $z_1 = -1 + i$ and $z_2 = 2i$, where $i = \sqrt{-1}$, then $\text{Arg}(z_1/z_2)$ is:

- (1) $\frac{3\pi}{4}$
- (2) $\frac{\pi}{4}$
- (3) $\frac{\pi}{2}$
- (4) $\frac{\pi}{3}$

Correct Answer: (2) $\frac{\pi}{4}$

Solution:

Step 1: Represent z_1 and z_2 in polar form. For $z_1 = -1 + i$:

$$|z_1| = \sqrt{(-1)^2 + 1^2} = \sqrt{2}, \quad \text{and} \quad \text{Arg}(z_1) = \tan^{-1}\left(\frac{1}{-1}\right) + \pi = \frac{3\pi}{4}.$$

For $z_2 = 2i$:

$$|z_2| = |2i| = 2, \quad \text{and} \quad \text{Arg}(z_2) = \frac{\pi}{2}.$$

Step 2: Division of z_1 and z_2 . The modulus of z_1/z_2 is:

$$\left| \frac{z_1}{z_2} \right| = \frac{|z_1|}{|z_2|} = \frac{\sqrt{2}}{2}.$$

The argument of z_1/z_2 is:

$$\text{Arg}\left(\frac{z_1}{z_2}\right) = \text{Arg}(z_1) - \text{Arg}(z_2) = \frac{3\pi}{4} - \frac{\pi}{2}.$$

Simplify:

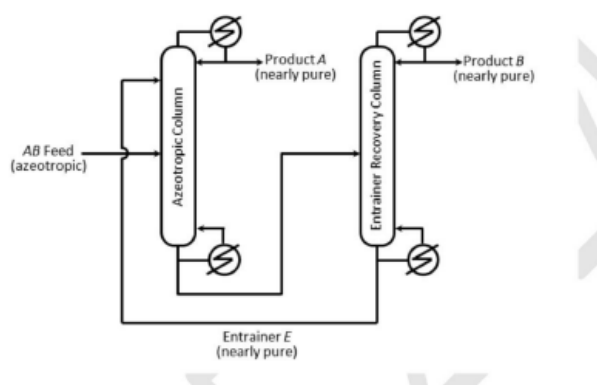
$$\text{Arg}\left(\frac{z_1}{z_2}\right) = \frac{3\pi}{4} - \frac{2\pi}{4} = \frac{\pi}{4}.$$

Step 3: Conclusion. The argument of z_1/z_2 is $\frac{\pi}{4}$.

 Quick Tip

To find the argument of a complex number division, subtract the arguments of the numerator and denominator. Always express the result in the correct range of principal arguments.

14. A homogeneous azeotropic distillation process separates an azeotropic AB binary feed using a heavy entrainer, E , as shown in the figure. The loss of E in the two product streams is negligible, so E circulates around the process in a closed circuit. For a distillation column with fully specified feed(s), given operating pressure, a single distillate stream, and a single bottoms stream, the steady-state degrees of freedom equals 2. For the process in the figure with a fully specified AB feed stream and given column operating pressures, the steady-state degrees of freedom equals:



- (1) 3
- (2) 4
- (3) 5
- (4) 6

Correct Answer: (3) 5

Solution: Step 1: Understand the degrees of freedom in distillation processes.

The degrees of freedom (DoF) in a distillation column represent the number of independent variables that can be adjusted to achieve steady-state operation.

For a typical distillation column with: - Fully specified feed streams, - Given operating pressure, - A single distillate and a single bottoms stream,

The steady-state degrees of freedom is 2.

Step 2: Include the effects of azeotropic distillation. In the given process, an additional constraint is imposed by the closed-circuit circulation of the heavy entrainer E . Since E is recycled with negligible loss, this introduces an additional degree of freedom. Furthermore, the separation process requires careful specification of conditions such as flow rates, reflux ratios, and temperature for both the azeotropic and heavy entrainer

behaviors.

Step 3: Determine the total degrees of freedom. Considering the additional complexities of azeotropic separation and entrainer circulation, the total degrees of freedom for the process increases to:

$$\text{DoF} = 2 + 3 = 5.$$

Step 4: Conclusion. The steady-state degrees of freedom for the process equals 5.

 Quick Tip

In distillation problems involving azeotropes or entrainers, include additional constraints from recycle streams, azeotropic behaviors, and column operating conditions to determine the total degrees of freedom.

15. An infinitely long cylindrical water filament of radius R is surrounded by air. Assume water and air to be static. The pressure outside the filament is P_{out} and the pressure inside is P_{in} . If γ is the surface tension of the water-air interface, then $P_{\text{in}} - P_{\text{out}}$ is:

- (1) $\frac{2\gamma}{R}$
- (2) 0
- (3) $\frac{\gamma}{R}$
- (4) $\frac{4\gamma}{R}$

Correct Answer: (3) $\frac{\gamma}{R}$

Solution: Step 1: Use the Laplace pressure equation. The Laplace pressure equation for a cylindrical interface is given by:

$$P_{\text{in}} - P_{\text{out}} = \frac{\gamma}{R}.$$

Here: - P_{in} is the pressure inside the water filament, - P_{out} is the pressure outside the water filament, - γ is the surface tension, - R is the radius of the cylindrical water filament.

Step 2: Validate the equation for a cylindrical surface. For a cylindrical interface, the Laplace pressure is derived based on the curvature of the interface. Unlike a spherical interface where the pressure difference is $\frac{2\gamma}{R}$, a cylindrical surface has a single radius of curvature, resulting in $\frac{\gamma}{R}$.

Step 3: Conclusion. The pressure difference $P_{\text{in}} - P_{\text{out}}$ is $\frac{\gamma}{R}$.

 Quick Tip

For Laplace pressure problems, identify whether the interface is spherical or cylindrical. A cylindrical interface has a pressure difference of $\frac{\gamma}{R}$, while a spherical interface has $\frac{2\gamma}{R}$.

16. The velocity field in an incompressible flow is $\mathbf{v} = axy\mathbf{i} + v_y\mathbf{j} + \beta\mathbf{k}$, where $\mathbf{i}, \mathbf{j}, \mathbf{k}$ are unit-vectors in the (x, y, z) Cartesian coordinate system. Given that a and β are constants, and $v_y = 0$ at $y = 0$, the correct expression for v_y is:

- (1) $-\frac{axy}{2}$
- (2) $-\frac{ay^2}{2}$
- (3) $\frac{ay^2}{2}$
- (4) $\frac{axy}{2}$

Correct Answer: (2) $-\frac{ay^2}{2}$

Solution: Step 1: Apply the incompressibility condition. For an incompressible flow, the divergence of the velocity field must be zero:

$$\nabla \cdot \mathbf{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0.$$

Substitute the given velocity components:

$$v_x = axy, \quad v_y = v_y(y), \quad v_z = \beta.$$

Step 2: Compute the divergence. The partial derivatives are:

$$\frac{\partial v_x}{\partial x} = \frac{\partial(axy)}{\partial x} = ay, \quad \frac{\partial v_y}{\partial y} = \frac{\partial v_y}{\partial y}, \quad \frac{\partial v_z}{\partial z} = \frac{\partial \beta}{\partial z} = 0.$$

Substitute into the incompressibility condition:

$$ay + \frac{\partial v_y}{\partial y} = 0.$$

Step 3: Solve for v_y . Integrate $\frac{\partial v_y}{\partial y} = -ay$:

$$v_y = -\frac{ay^2}{2} + C,$$

where C is the constant of integration.

Step 4: Apply the boundary condition. Given $v_y = 0$ at $y = 0$:

$$0 = -\frac{a(0)^2}{2} + C \Rightarrow C = 0.$$

Thus,

$$v_y = -\frac{ay^2}{2}.$$

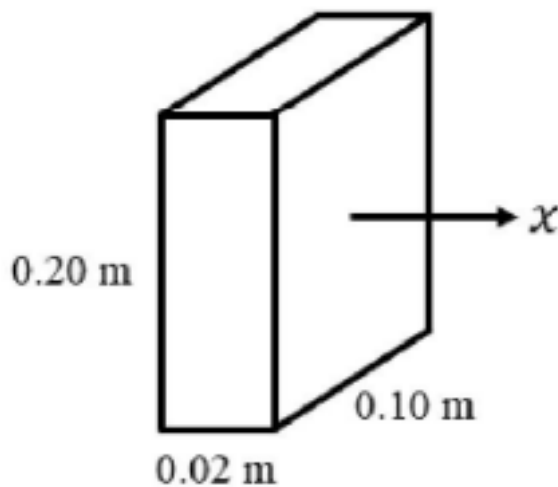
Step 5: Conclusion. The correct expression for v_y is $-\frac{ay^2}{2}$.

💡 Quick Tip

For incompressible flows, use the condition $\nabla \cdot \mathbf{v} = 0$ and apply boundary conditions to determine constants of integration.

17. Consider the steady, uni-directional diffusion of a binary mixture of A and B across a vertical slab of dimensions $0.2\text{ m} \times 0.1\text{ m} \times 0.02\text{ m}$ as shown in the figure. The total molar concentration of A and B is constant at 100 mol m^{-3} . The mole fraction of A on the left and right faces of the slab are maintained at 0.8 and 0.2, respectively.

If the binary diffusion coefficient $D_{AB} = 1 \times 10^{-5}\text{ m}^2\text{ s}^{-1}$, the molar flow rate of A in mols^{-1} , along the horizontal x -direction is:



- (1) 6×10^{-4}
- (2) 6×10^{-6}
- (3) 3×10^{-6}
- (4) 3×10^{-4}

Correct Answer: (1) 6×10^{-4}

Solution: Step 1: Use Fick's First Law of Diffusion. The molar flux J_A is given by Fick's first law:

$$J_A = -D_{AB} \frac{\partial C_A}{\partial x}.$$

Step 2: Calculate the concentration gradient. The mole fraction gradient of A is:

$$\frac{\partial y_A}{\partial x} = \frac{y_{A,\text{right}} - y_{A,\text{left}}}{\Delta x} = \frac{0.2 - 0.8}{0.02} = -30\text{ m}^{-1}.$$

The total molar concentration is $C = 100\text{ mol m}^{-3}$. The concentration gradient is:

$$\frac{\partial C_A}{\partial x} = C \cdot \frac{\partial y_A}{\partial x} = 100 \cdot (-30) = -3000\text{ mol m}^{-4}.$$

Step 3: Calculate the molar flux. Substitute $D_{AB} = 1 \times 10^{-5}\text{ m}^2\text{ s}^{-1}$:

$$J_A = -D_{AB} \frac{\partial C_A}{\partial x} = -(1 \times 10^{-5}) \cdot (-3000) = 3 \times 10^{-2}\text{ mol m}^{-2}\text{ s}^{-1}.$$

Step 4: Calculate the molar flow rate. The molar flow rate is:

$$\dot{N}_A = J_A \cdot A,$$

where A is the cross-sectional area:

$$A = 0.2 \text{ m} \cdot 0.1 \text{ m} = 0.02 \text{ m}^2.$$

Substitute $J_A = 3 \times 10^{-2} \text{ mol m}^{-2} \text{ s}^{-1}$:

$$\dot{N}_A = 3 \times 10^{-2} \cdot 0.02 = 6 \times 10^{-4} \text{ mol s}^{-1}.$$

Step 5: Conclusion. The molar flow rate of A is $6 \times 10^{-4} \text{ mol s}^{-1}$.

 Quick Tip

For diffusion problems, use Fick's law, calculate the concentration gradient carefully, and account for the cross-sectional area to determine the molar flow rate.

18. Consider a vapour-liquid mixture of components A and B that obeys Raoult's law. The vapour pressure of A is half that of B . The vapour phase concentrations of A and B are 3 mol m^{-3} and 6 mol m^{-3} , respectively. At equilibrium, the ratio of the liquid phase concentration of A to that of B is:

- (1) 1.0
- (2) 0.5
- (3) 2.0
- (4) 1.5

Correct Answer: (1) 1.0

Solution: Step 1: Understand Raoult's law and equilibrium condition. According to Raoult's law, the partial pressure of a component i in the vapour phase is given by:

$$P_i = x_i P_i^*,$$

where: - P_i is the partial pressure of component i , - x_i is the mole fraction of i in the liquid phase, - P_i^* is the vapour pressure of i in the pure state.

At equilibrium, the partial pressure P_i is proportional to the vapour phase concentration C_i :

$$P_i \propto C_i.$$

Step 2: Relate the concentrations in the vapour phase. The given vapour phase concentrations are:

$$C_A = 3 \text{ mol m}^{-3}, \quad C_B = 6 \text{ mol m}^{-3}.$$

Thus, the ratio of partial pressures is:

$$\frac{P_A}{P_B} = \frac{C_A}{C_B} = \frac{3}{6} = 0.5.$$

Step 3: Relate the liquid phase mole fractions. Using Raoult's law:

$$\frac{P_A}{P_B} = \frac{x_A P_A^*}{x_B P_B^*}.$$

The problem states that P_A^* is half of P_B^* :

$$P_A^* = \frac{1}{2} P_B^*.$$

Substitute this into the equation:

$$\frac{P_A}{P_B} = \frac{x_A (\frac{1}{2} P_B^*)}{x_B P_B^*}.$$

Simplify:

$$\frac{P_A}{P_B} = \frac{x_A}{2x_B}.$$

Step 4: Solve for the ratio $\frac{x_A}{x_B}$. Substitute $\frac{P_A}{P_B} = 0.5$:

$$0.5 = \frac{x_A}{2x_B}.$$

Solve for $\frac{x_A}{x_B}$:

$$\frac{x_A}{x_B} = 1.0.$$

Step 5: Conclusion. The ratio of the liquid phase concentration of A to that of B is 1.0.

 Quick Tip

For equilibrium problems involving Raoult's law, relate vapour phase concentrations to liquid phase mole fractions using the proportionality of partial pressures and given vapour pressures.

19. The ratio of the activation energy of a chemical reaction to the universal gas constant is 1000 K. The temperature dependence of the reaction rate constant follows the collision theory. The ratio of the rate constant at 600 K to that at 400 K is:

- (1) 2.818
- (2) 4.323
- (3) 1.502
- (4) 1.000

Correct Answer: (1) 2.818

Solution: Step 1: Use the Arrhenius equation for the rate constant. The Arrhenius equation is given by:

$$k = Ae^{-\frac{E_a}{RT}},$$

where: - k is the rate constant, - A is the pre-exponential factor, - E_a is the activation energy, - R is the universal gas constant, - T is the temperature.

The ratio of rate constants at two temperatures T_1 and T_2 is:

$$\frac{k_2}{k_1} = e^{\frac{E_a}{R} \left(\frac{1}{T_1} - \frac{1}{T_2} \right)}.$$

Step 2: Substitute the given values. The ratio $\frac{E_a}{R} = 1000 \text{ K}$, $T_1 = 400 \text{ K}$, and $T_2 = 600 \text{ K}$:

$$\frac{k_2}{k_1} = e^{1000 \left(\frac{1}{400} - \frac{1}{600} \right)}.$$

Step 3: Simplify the exponent. Calculate $\frac{1}{400} - \frac{1}{600}$:

$$\frac{1}{400} - \frac{1}{600} = \frac{3-2}{1200} = \frac{1}{1200}.$$

Thus:

$$\frac{k_2}{k_1} = e^{\frac{1000}{1200}} = e^{0.8333}.$$

Step 4: Calculate the exponential term. Using $e^{0.8333}$:

$$\frac{k_2}{k_1} \approx 2.818.$$

Step 5: Conclusion. The ratio of the rate constant at 600 K to that at 400 K is 2.818.

💡 Quick Tip

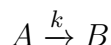
For temperature dependence of reaction rates, use the Arrhenius equation and carefully calculate the exponential factor for the given temperatures.

20. The rate of a reaction $A \rightarrow B$ is $0.2 \text{ mol m}^{-3} \text{ s}^{-1}$ at a particular concentration C_{A1} . The rate constant of the reaction at a given temperature is $0.1 \text{ m}^3 \text{ mol}^{-1} \text{ s}^{-1}$. If the reactant concentration is increased to $10 C_{A1}$ at the same temperature, the reaction rate, in $\text{mol m}^{-3} \text{ s}^{-1}$, is:

- (1) 20
- (2) 10
- (3) 100
- (4) 50

Correct Answer: (1) 20

Solution: Given:



$$-r_{A_1} = 0.2 \text{ mol/m}^3 \cdot \text{sec at } C_{A_1},$$

$$k = 0.1 \text{ m}^3/\text{mol} \cdot \text{sec}, \quad \text{find } r_{A_2} \text{ at } 10C_{A_1}.$$

Step 1: Determining the reaction order. From the rate constant unit, we can deduce the reaction order:

$$k \text{ unit} = \left(\frac{\text{mol}}{\text{lit}} \right)^{1-n} \text{sec}^{-1}.$$

For $k = 0.1 \text{ m}^3/\text{mol} \cdot \text{sec}$:

$$1 - n = -1, \quad n = 2.$$

Step 2: Rate equation for the reaction.

$$-r_{A_1} = 0.1 C_{A_1}^2.$$

Step 3: Calculating r_{A_2} .

$$\frac{-r_{A_2}}{-r_{A_1}} = \frac{0.1 C_{A_2}^2}{0.1 C_{A_1}^2} \quad (\text{since } k \text{ remains constant as the temperature remains constant}).$$

$$C_{A_2} = 10C_{A_1}.$$

$$\frac{-r_{A_2}}{0.2} = \left(\frac{10C_{A_1}}{C_{A_1}} \right)^2.$$

$$-r_{A_2} = 20 \text{ mol/m}^3 \cdot \text{sec}.$$

Final Answer:

$$-r_{A_2} = 20 \text{ mol/m}^3 \cdot \text{sec}.$$

💡 Quick Tip

For reaction rate problems, determine the initial concentration from the rate law and apply the proportional relationship between the rate and reactant concentration for changes in C_A .

21. Two parallel first-order liquid phase reactions $A \xrightarrow{k_1} B$ and $A \xrightarrow{k_2} C$ are carried out in a well-mixed isothermal batch reactor. The initial concentration of A in the reactor is 1 kmol m^{-3} , while that of B and C is zero. After 2 hours, the concentration of A reduces to half its initial value, and the concentration of B is twice that of C . The rate constants k_1 and k_2 , in h^{-1} , are respectively:

- (1) 0.40, 0.20
- (2) 0.23, 0.12
- (3) 0.50, 0.25

(4) 0.36, 0.18

Correct Answer: (2) 0.23, 0.12

Solution: Step 1: Use the first-order decay law for A. The decay of A is governed by the overall rate constant k :

$$\ln \left(\frac{C_A}{C_{A0}} \right) = -kt.$$

Here: - $C_A = \frac{C_{A0}}{2} = \frac{1}{2} \text{ kmol m}^{-3}$, - $C_{A0} = 1 \text{ kmol m}^{-3}$, - $t = 2 \text{ hours}$.

Substitute into the equation:

$$\ln \left(\frac{\frac{1}{2}}{1} \right) = -k \cdot 2.$$

Simplify:

$$\ln \left(\frac{1}{2} \right) = -2k \Rightarrow k = \frac{\ln(2)}{2} \approx 0.3466 \text{ h}^{-1}.$$

Step 2: Relate k_1 and k_2 . The overall rate constant is the sum of the individual rate constants for parallel reactions:

$$k = k_1 + k_2.$$

Thus:

$$k_1 + k_2 = 0.3466.$$

Step 3: Use the concentration ratio of B and C. The concentration of B is twice that of C:

$$C_B = 2C_C.$$

The concentrations are related to the rate constants:

$$\frac{C_B}{C_C} = \frac{k_1}{k_2}.$$

Substitute $\frac{C_B}{C_C} = 2$:

$$\frac{k_1}{k_2} = 2 \Rightarrow k_1 = 2k_2.$$

Step 4: Solve for k_1 and k_2 . Substitute $k_1 = 2k_2$ into $k_1 + k_2 = 0.3466$:

$$2k_2 + k_2 = 0.3466 \Rightarrow 3k_2 = 0.3466 \Rightarrow k_2 = 0.1155 \text{ h}^{-1}.$$

Then:

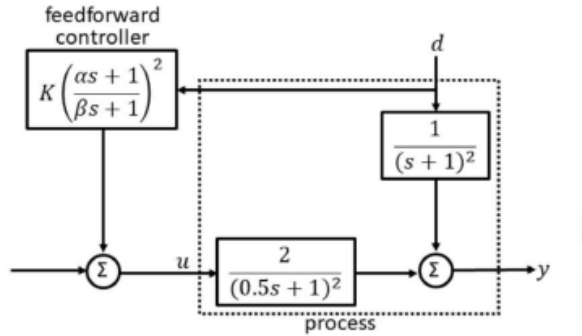
$$k_1 = 2k_2 = 2 \cdot 0.1155 = 0.231 \text{ h}^{-1}.$$

Step 5: Conclusion. The rate constants are $k_1 = 0.23 \text{ h}^{-1}$ and $k_2 = 0.12 \text{ h}^{-1}$.

💡 Quick Tip

For parallel reactions, use the overall rate constant as the sum of individual rate constants and relate the product concentrations to their respective rate constants.

22. Consider the block diagram in the figure with control input u , disturbance d , and output y . For the feedforward controller, the ordered pair $(K, \alpha/\beta)$ is:



- (1) (0.5, 2)
- (2) (-0.5, 0.5)
- (3) (-2, 2)
- (4) (2, 0.5)

Correct Answer: (2) (-0.5, 0.5)

Solution: Step 1: Analyze the block diagram. The process transfer function is:

$$G(s) = \frac{2}{(0.5s + 1)^2}.$$

The disturbance transfer function is:

$$G_d(s) = \frac{1}{(s + 1)^2}.$$

The feedforward controller transfer function is:

$$F(s) = K \left(\frac{\alpha s + 1}{\beta s + 1} \right)^2.$$

Step 2: Apply disturbance rejection condition. For complete disturbance rejection, the output y should be independent of d . The feedforward controller $F(s)$ must cancel the disturbance effect. This requires:

$$F(s) \cdot G_d(s) = G(s).$$

Substitute $G(s)$ and $G_d(s)$:

$$K \left(\frac{\alpha s + 1}{\beta s + 1} \right)^2 \cdot \frac{1}{(s + 1)^2} = \frac{2}{(0.5s + 1)^2}.$$

Step 3: Match the numerator and denominator. Equating denominators:

$$(\beta s + 1)^2 \cdot (s + 1)^2 = (0.5s + 1)^2.$$

Expand and compare coefficients to solve for β :

$$\beta = 0.5.$$

Equating numerators:

$$K \cdot (\alpha s + 1)^2 = 2.$$

Substitute $\beta = 0.5$ and solve for K and α :

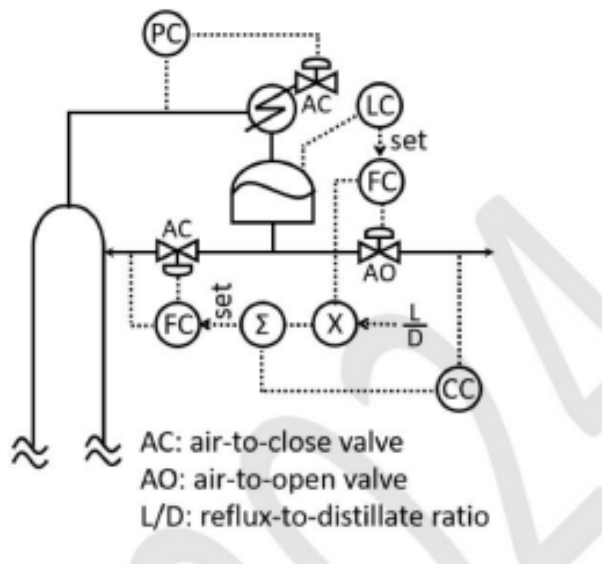
$$K = -0.5, \quad \alpha = 0.5.$$

Step 4: Conclusion. The ordered pair $(K, \alpha/\beta)$ is $(-0.5, 0.5)$.

💡 Quick Tip

For feedforward controller design, equate the product of the feedforward and disturbance transfer functions to the process transfer function for exact disturbance rejection.

23. Consider the control structure for the overhead section of a distillation column shown in the figure. The composition controller (CC) controls the heavy key impurity in the distillate by adjusting the setpoint of the reflux flow controller in a cascade arrangement. The sign of the controller gain for the pressure controller (PC) and that for the composition controller (CC) are, respectively:



- (1) negative, negative
- (2) negative, positive
- (3) positive, positive
- (4) positive, negative

Correct Answer: (4) positive, negative

Solution: Step 1: Analyze the function of the pressure controller (PC). The pressure controller (PC) is responsible for maintaining the column pressure. If the pressure increases, the controller adjusts the valve to reduce the pressure. Hence, a positive gain is required for the pressure controller to provide the correct action (i.e., increase valve opening when pressure increases).

Step 2: Analyze the function of the composition controller (CC). The composition controller (CC) adjusts the setpoint of the reflux flow controller to control the heavy key impurity in the distillate. If the impurity concentration increases, the controller increases the reflux flow to reduce the impurity. This requires a negative gain for the composition controller to ensure the proper corrective action.

Step 3: Conclusion. The pressure controller (PC) has a positive gain, and the composition controller (CC) has a negative gain.

 Quick Tip

For cascade control systems, analyze the corrective action needed for each controller in the loop. Positive gain increases the controlled variable, while negative gain reduces it based on the error signal.

24. Which one of the given statements is correct with reference to gas-liquid contactors for mass transfer applications?

- (1) A tray tower is more suitable for foaming systems than a packed tower.
- (2) Tray towers are preferred over packed towers for systems requiring frequent cleaning.
- (3) For a given liquid flow rate, the gas flow rate in the loading region is greater than that in the flooding region.
- (4) Flooding can never occur for counter-current contact.

Correct Answer: (2) Tray towers are preferred over packed towers for systems requiring frequent cleaning.

Solution:

Step 1: Analyze the suitability of tray towers and packed towers.

- Tray towers are generally easier to clean compared to packed towers due to their open design and accessibility. This makes them preferable for systems requiring frequent cleaning, such as those handling viscous or fouling liquids.
- Packed towers, on the other hand, are more prone to fouling, and cleaning them is more challenging due to the packing materials.

Step 2: Evaluate the other options.

1. **A tray tower is more suitable for foaming systems than a packed tower.** This is incorrect. Packed towers are generally better for handling foaming systems as they minimize foam generation due to the low liquid holdup compared to tray towers.

2. **For a given liquid flow rate, the gas flow rate in the loading region is greater than that in the flooding region.**

This is incorrect. The gas flow rate in the flooding region exceeds that in the loading region, as flooding occurs at higher gas velocities.

3. **Flooding can never occur for counter-current contact.** This is incorrect. Flooding can occur in counter-current gas-liquid contactors when the gas flow rate is too high, causing liquid to back up or be entrained.

Step 3: Conclusion: The correct statement is that tray towers are preferred over packed towers for systems requiring frequent cleaning.

 Quick Tip

For gas-liquid contactors, consider factors like cleaning requirements, fouling tendencies, and operational limits (e.g., flooding) when selecting between tray and packed towers.

25. In an ammonia manufacturing facility, the necessary hydrogen is generated from methane. The facility consists of the following process units: *P*: Methanator, *Q*: CO shift convertor, *R*: CO₂ stripper, *S*: Reformer, *T*: Ammonia convertor. The correct order of these units, starting from methane feed, is:

- (1) S, Q, R, P, T
- (2) P, Q, R, S, T
- (3) S, P, Q, R, T
- (4) P, S, T, Q, R

Correct Answer: (1) S, Q, R, P, T

Solution:

Step 1: Understand the sequence of operations in ammonia manufacturing.

Hydrogen for ammonia synthesis is generated from methane in the following sequence of operations:

1. Reformer (*S*): Methane reacts with steam in the reformer to produce synthesis gas (a mixture of hydrogen, carbon monoxide, and carbon dioxide).
2. CO Shift Convertor (*Q*): Carbon monoxide is converted to carbon dioxide via the water-gas shift reaction to increase hydrogen yield.
3. CO₂ Stripper (*R*): Carbon dioxide is removed from the gas mixture.
4. Methanator (*P*): Any remaining traces of CO and CO₂ are converted to methane to prevent catalyst poisoning in the ammonia synthesis reactor.

5. Ammonia Converter (T): Hydrogen and nitrogen are reacted in the ammonia converter to produce ammonia.

Step 2: Identify the correct order. The correct order of units starting from methane feed is:

$$S \rightarrow Q \rightarrow R \rightarrow P \rightarrow T.$$

Step 3: Conclusion. The correct order is S, Q, R, P, T , corresponding to option (1).

 Quick Tip

In ammonia manufacturing, hydrogen generation involves reforming, CO shift conversion, and CO₂ removal before ammonia synthesis. Ensure the sequence eliminates impurities like CO and CO₂ to prevent catalyst deactivation.

26. Consider a linear homogeneous system of equations $A\mathbf{x} = \mathbf{0}$, where A is an $n \times n$ matrix, $\mathbf{x}$ is an $n \times 1$ vector, and $\mathbf{0}$ is an $n \times 1$ null vector. Let r be the rank of A . For a non-trivial solution to exist, which of the following conditions is/are satisfied?

- (1) Determinant of $A = 0$
- (2) $r = m < n$
- (3) $r < n$
- (4) Determinant of $A \neq 0$

Correct Answer: (1) Determinant of $A = 0$, (3) $r < n$

Solution:

Step 1: Analyze the system of equations $A\mathbf{x} = \mathbf{0}$. A non-trivial solution to a homogeneous system $A\mathbf{x} = \mathbf{0}$ exists only if the determinant of A is zero. This is because:

1. If $\det(A) \neq 0$, A is invertible, and the only solution is the trivial solution $\mathbf{x} = \mathbf{0}$.
2. If $\det(A) = 0$, A is singular, and there are infinitely many solutions, including non-trivial solutions.

Step 2: Rank condition. The rank r of A determines the number of independent rows (or columns) of the matrix. For $A\mathbf{x} = \mathbf{0}$:

1. If $r = n$, $\mathbf{x} = \mathbf{0}$ is the only solution (trivial solution).
2. If $r < n$, there are $n - r$ free variables, and a non-trivial solution exists.

Step 3: Evaluate the conditions.

- Condition (A): Determinant of $A = 0$ is satisfied for a non-trivial solution.
- Condition (B): $r = m < n$ is not satisfied because $r < n$ ensures a non-trivial solution.
- Condition (C): $r < n$ is satisfied as it directly implies a non-trivial solution.

- Condition (D): Determinant of $A \neq 0$ contradicts the requirement for a non-trivial solution.

Step 4: Conclusion. The conditions satisfied for a non-trivial solution are:

(A) Determinant of $A = 0$, (C) $r < n$.

 Quick Tip

For homogeneous systems, the determinant and rank conditions determine the existence of non-trivial solutions. A singular matrix ($\det(A) = 0$) and rank $r < n$ are necessary for non-trivial solutions.

27. If the Prandtl number $Pr = 0.01$, which of the following statements is/are correct?

- (1) The momentum diffusivity is much larger than the thermal diffusivity.
- (2) The thickness of the momentum boundary layer is much smaller than that of the thermal boundary layer.
- (3) The thickness of the momentum boundary layer is much larger than that of the thermal boundary layer.
- (4) The momentum diffusivity is much smaller than the thermal diffusivity.

Correct Answer: (2) The thickness of the momentum boundary layer is much smaller than that of the thermal boundary layer, (4) The momentum diffusivity is much smaller than the thermal diffusivity.

Solution:

Step 1: Understand the Prandtl number. The Prandtl number is defined as:

$$Pr = \frac{\nu}{\alpha},$$

where: - ν is the momentum diffusivity (kinematic viscosity), - α is the thermal diffusivity. For $Pr = 0.01$:

$$\nu \ll \alpha.$$

This means that the momentum diffusivity is much smaller than the thermal diffusivity.

Step 2: Boundary layer thicknesses. The thickness of the boundary layer is inversely related to diffusivity. For small Prandtl numbers ($Pr \ll 1$): - The thermal boundary layer is much thicker than the momentum boundary layer because thermal diffusivity dominates.

Step 3: Evaluate the statements. 1. (A): The momentum diffusivity is much larger than the thermal diffusivity. - This is incorrect because $\nu \ll \alpha$ for $Pr = 0.01$.

2. (B): The thickness of the momentum boundary layer is much smaller than that of the thermal boundary layer. - This is correct because $Pr \ll 1$ implies the thermal boundary layer is thicker than the momentum boundary layer.

3. (C): The thickness of the momentum boundary layer is much larger than that of the thermal boundary layer. - This is incorrect because the opposite is true for $Pr \ll 1$.
 4. (D): The momentum diffusivity is much smaller than the thermal diffusivity. - This is correct because $\nu \ll \alpha$ for $Pr = 0.01$.

Step 4: Conclusion. The correct statements are:

(B) The thickness of the momentum boundary layer is much smaller than that of the thermal boundary layer, (D) The momentum diffusivity is much smaller than the thermal diffusivity.

 Quick Tip

For small Prandtl numbers ($Pr \ll 1$), thermal diffusivity dominates over momentum diffusivity, leading to a thicker thermal boundary layer compared to the momentum boundary layer.

28. For the electrolytic cell in a chlor-alkali plant, which of the following statements is/are correct?

- (1) A membrane cell operates at a higher brine concentration than a diaphragm cell.
- (2) Chlorine gas is produced at the cathode.
- (3) Hydrogen gas is produced at the cathode.
- (4) The caustic product stream exits the cathode compartment.

Correct Answer: (1) A membrane cell operates at a higher brine concentration than a diaphragm cell, (3) Hydrogen gas is produced at the cathode, (4) The caustic product stream exits the cathode compartment.

Solution:

Step 1: Analyze the operation of chlor-alkali electrolytic cells. - In chlor-alkali electrolysis, brine (sodium chloride solution) is electrolyzed to produce chlorine, hydrogen, and sodium hydroxide. - The membrane cell operates at a higher brine concentration than the diaphragm cell to ensure better ion separation and efficiency. - Chlorine gas is produced at the anode, not the cathode. - Hydrogen gas is produced at the cathode due to the reduction of water. - The caustic product stream (sodium hydroxide) exits from the cathode compartment.

Step 2: Evaluate the statements. 1. (A): Correct. Membrane cells operate at a higher brine concentration than diaphragm cells. 2. (B): Incorrect. Chlorine gas is produced at the anode, not the cathode. 3. (C): Correct. Hydrogen gas is produced at the cathode due to water reduction. 4. (D): Correct. The caustic product stream exits the cathode compartment.

Step 3: Conclusion. The correct statements are:

(1), (3), (4).

💡 Quick Tip

In chlor-alkali processes, remember: chlorine is produced at the anode, hydrogen at the cathode, and sodium hydroxide exits from the cathode compartment.

29. Which of the following statements with reference to the petroleum/petrochemical industry is/are correct?

- (1) Catalytic hydrocracking converts heavier hydrocarbons to lighter hydrocarbons.
- (2) Catalytic reforming converts straight-chain hydrocarbons to aromatics.
- (3) Cumene is manufactured by the catalytic alkylation of benzene with propylene.
- (4) Vinyl acetate is manufactured by reacting methane with acetic acid over a palladium catalyst.

Correct Answer: (1) Catalytic hydrocracking converts heavier hydrocarbons to lighter hydrocarbons, (2) Catalytic reforming converts straight-chain hydrocarbons to aromatics, (3) Cumene is manufactured by the catalytic alkylation of benzene with propylene.

Solution:

Step 1: Analyze the given processes.

1. Catalytic hydrocracking: This process breaks down heavier hydrocarbons into lighter hydrocarbons like gasoline and diesel using hydrogen and catalysts. This is correct.
2. Catalytic reforming: This process converts straight-chain hydrocarbons into aromatics like benzene, toluene, and xylene, which are important petrochemical precursors. This is correct.
3. Cumene manufacture: Cumene (isopropylbenzene) is produced by the catalytic alkylation of benzene with propylene. This is correct.
4. Vinyl acetate manufacture: Vinyl acetate is manufactured by reacting ethylene, acetic acid, and oxygen over a palladium catalyst, not methane. This is incorrect.

Step 2: Evaluate the statements.

1. (1): Correct.
2. (2): Correct.
3. (3): Correct.
4. (4): Incorrect.

Step 3: Conclusion. The correct statements are:

(1), (2), (3).

💡 Quick Tip

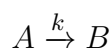
In petroleum and petrochemical processes, hydrocracking breaks heavier molecules, reforming produces aromatics, and alkylation produces compounds like cumene.

30. Consider a matrix $A = \begin{bmatrix} -5 & a \\ -2 & -2 \end{bmatrix}$, where a is a constant. If the eigenvalues of A are -1 and -6 , then the value of a , rounded off to the nearest integer, is -----.

Correct Answer: -2

Solution:

Given:



$$-r_{A_1} = 0.2 \text{ mol/m}^3 \cdot \text{sec at } C_{A_1},$$

$$k = 0.1 \text{ m}^3/\text{mol} \cdot \text{sec}, \quad \text{find } r_{A_2} \text{ at } 10C_{A_1}.$$

Step 1: Determining the reaction order. From the rate constant unit, we can deduce the reaction order:

$$k \text{ unit} = \left(\frac{\text{mol}}{\text{lit}} \right)^{1-n} \text{sec}^{-1}.$$

For $k = 0.1 \text{ m}^3/\text{mol} \cdot \text{sec}$:

$$1 - n = -1, \quad n = 2.$$

Step 2: Rate equation for the reaction.

$$-r_{A_1} = 0.1 C_{A_1}^2.$$

Step 3: Calculating r_{A_2} .

$$\frac{-r_{A_2}}{-r_{A_1}} = \frac{0.1 C_{A_2}^2}{0.1 C_{A_1}^2} \quad (\text{since } k \text{ remains constant as the temperature remains constant}).$$

$$C_{A_2} = 10C_{A_1}.$$

$$\frac{-r_{A_2}}{0.2} = \left(\frac{10C_{A_1}}{C_{A_1}} \right)^2.$$

$$-r_{A_2} = 20 \text{ mol/m}^3 \cdot \text{sec}.$$

Final Answer:

$$-r_{A_2} = 20 \text{ mol/m}^3 \cdot \text{sec}.$$

💡 Quick Tip

For eigenvalue problems, use the characteristic equation $\det(A - \lambda I) = 0$ and the relationships between eigenvalues, trace, and determinant.

31. Consider the reaction $\text{N}_2(g) + 3\text{H}_2(g) \rightarrow 2\text{NH}_3(g)$ in a continuous flow reactor under steady-state conditions. The component flow rates at the reactor inlet are:

$$F_{\text{N}_2}^0 = 100 \text{ mol/s}, \quad F_{\text{H}_2}^0 = 300 \text{ mol/s}, \quad F_{\text{inert}}^0 = 1 \text{ mol/s}.$$

If the fractional conversion of H_2 is 0.60, the outlet flow rate of N_2 , in mol/s, rounded off to the nearest integer, is

Correct Answer: 40

Solution:

Step 1: Determine the moles of H_2 reacted. The fractional conversion of H_2 is given as 0.60. The moles of H_2 reacted are:

$$\text{Moles of } \text{H}_2 \text{ reacted} = F_{\text{H}_2}^0 \cdot 0.60 = 300 \cdot 0.60 = 180 \text{ mol/s.}$$

Step 2: Use the stoichiometry of the reaction. From the stoichiometry $\text{N}_2 + 3\text{H}_2 \rightarrow 2\text{NH}_3$:

$$\text{Moles of } \text{N}_2 \text{ reacted} = \frac{1}{3} \cdot \text{Moles of } \text{H}_2 \text{ reacted} = \frac{1}{3} \cdot 180 = 60 \text{ mol/s.}$$

Step 3: Calculate the outlet flow rate of N_2 . The inlet flow rate of N_2 is $F_{\text{N}_2}^0 = 100 \text{ mol/s}$. The outlet flow rate is:

$$F_{\text{N}_2, \text{out}} = F_{\text{N}_2}^0 - \text{Moles of } \text{N}_2 \text{ reacted} = 100 - 60 = 40 \text{ mol/s.}$$

Step 4: Conclusion. The outlet flow rate of N_2 is 40 mol/s.

 Quick Tip

For steady-state reactor problems, use the fractional conversion and stoichiometric ratios to calculate the moles of reactants and products.

32. Consider a binary mixture of components A and B at temperature T and pressure P . Let $\bar{V}_A$ and $\bar{V}_B$ be the partial molar volumes of A and B , respectively. At a certain mole fraction of A , x_A :

$$\left(\frac{\partial \bar{V}_A}{\partial x_A} \right)_{T,P} = 22 \text{ cm}^3 \text{ mol}^{-1}, \quad \left(\frac{\partial \bar{V}_B}{\partial x_A} \right)_{T,P} = -18 \text{ cm}^3 \text{ mol}^{-1}.$$

The value of x_A , rounded off to 2 decimal places, is

Correct Answer: 0.45

Solution:

Step 1: Use the Gibbs-Duhem relation. The Gibbs-Duhem relation for partial molar properties in a binary mixture is:

$$\left(\frac{\partial \bar{V}_A}{\partial x_A} \right)_{T,P} x_A + \left(\frac{\partial \bar{V}_B}{\partial x_A} \right)_{T,P} (1 - x_A) = 0.$$

Step 2: Substitute the given values.

$$22x_A + (-18)(1 - x_A) = 0.$$

Simplify:

$$22x_A - 18 + 18x_A = 0 \quad \Rightarrow \quad 40x_A = 18 \quad \Rightarrow \quad x_A = \frac{18}{40} = 0.45.$$

Step 3: Conclusion. The mole fraction of A is $x_A = 0.45$.

 Quick Tip

For binary mixtures, apply the Gibbs-Duhem relation to relate partial molar properties and mole fractions.

33. Consider the steady, uni-directional, fully-developed, pressure-driven laminar flow of an incompressible Newtonian fluid through a circular pipe of inner radius 5.0 cm. The magnitude of shear stress at the inner wall of the pipe is 0.1 N m^{-2} . At a radial distance of 1.0 cm from the pipe axis, the magnitude of the shear stress, in N m^{-2} , rounded off to 3 decimal places, is _____.

Correct Answer: 0.020

Solution:

Step 1: Shear stress variation in a circular pipe. For steady, fully-developed, laminar flow in a circular pipe, the shear stress varies linearly with the radial position r :

$$\tau(r) = \tau_w \left(1 - \frac{r}{R}\right),$$

where: - τ_w is the shear stress at the inner wall, - R is the inner radius of the pipe, - r is the radial distance from the pipe axis.

Step 2: Substitute the given values. Given:

$$\tau_w = 0.1 \text{ N m}^{-2}, \quad R = 5.0 \text{ cm} = 0.05 \text{ m}, \quad r = 1.0 \text{ cm} = 0.01 \text{ m}.$$

Substitute into the equation:

$$\tau(r) = 0.1 \left(1 - \frac{0.01}{0.05}\right).$$

Step 3: Simplify the expression.

$$\tau(r) = 0.1 (1 - 0.2) = 0.1 \cdot 0.8 = 0.02 \text{ N m}^{-2}.$$

Step 4: Conclusion. The magnitude of the shear stress at $r = 1.0 \text{ cm}$ is 0.020 N m^{-2} .

 Quick Tip

For fully-developed laminar flow in a pipe, shear stress decreases linearly with radial distance from the wall to the pipe axis.

34. The opposite faces of a metal slab of thickness 5 cm and thermal conductivity $400 \text{ W m}^{-1} \text{ }^\circ\text{C}^{-1}$ are maintained at 500°C and 200°C . The area of each face is 0.02 m^2 . Assume that the heat transfer is steady and occurs only in the direction perpendicular to the faces. The magnitude of the heat transfer rate, in kW, rounded off to the nearest integer, is _____.

Correct Answer: 48 kW

Solution:

Step 1: Fourier's law of heat conduction. The rate of heat transfer through a slab is given by:

$$Q = -kA \frac{\Delta T}{L},$$

where: - Q is the rate of heat transfer (W), - k is the thermal conductivity of the slab ($\text{W m}^{-1} \text{ } ^\circ\text{C}^{-1}$), - A is the area of the slab (m^2), - ΔT is the temperature difference between the faces ($^\circ\text{C}$), - L is the thickness of the slab (m).

Step 2: Substitute the given values. Given:

$$k = 400 \text{ W m}^{-1} \text{ } ^\circ\text{C}^{-1}, \quad A = 0.02 \text{ m}^2, \quad \Delta T = 500 - 200 = 300 \text{ } ^\circ\text{C}, \quad L = 5 \text{ cm} = 0.05 \text{ m}.$$

Substitute into the equation:

$$Q = -400 \cdot 0.02 \cdot \frac{300}{0.05}.$$

Step 3: Simplify the expression.

$$Q = -400 \cdot 0.02 \cdot 6000 = 48,000 \text{ W}.$$

Step 4: Convert to kilowatts.

$$Q = 48,000 \text{ W} = 48 \text{ kW}.$$

Step 5: Conclusion. The magnitude of the heat transfer rate is 48 kW.

 Quick Tip

For steady-state heat conduction, ensure consistent units for area, thickness, and thermal conductivity when using Fourier's law.

35. The capital cost of a distillation column is Rs. 90 lakhs. The cost is to be fully depreciated (salvage value is zero) using the double-declining balance method over 10 years. At the end of two years of continuous operation, the book value of the column, in lakhs of rupees, rounded off to 1 decimal place, is:

Correct Answer: 57.6 lakhs

Solution:

Step 1: Understand the double-declining balance (DDB) method. The DDB depreciation rate is calculated as:

$$\text{Depreciation Rate} = \frac{2}{\text{Useful Life (years)}} = \frac{2}{10} = 0.2 \text{ (20\% per year)}.$$

Step 2: Calculate the book value after 2 years. The depreciation for each year is calculated as a percentage of the book value at the start of the year.

1. Year 1 depreciation:

$$\text{Depreciation} = 0.2 \times 90 = 18 \text{ lakhs.}$$

$$\text{Book Value after Year 1} = 90 - 18 = 72 \text{ lakhs.}$$

2. Year 2 depreciation:

$$\text{Depreciation} = 0.2 \times 72 = 14.4 \text{ lakhs.}$$

$$\text{Book Value after Year 2} = 72 - 14.4 = 57.6 \text{ lakhs.}$$

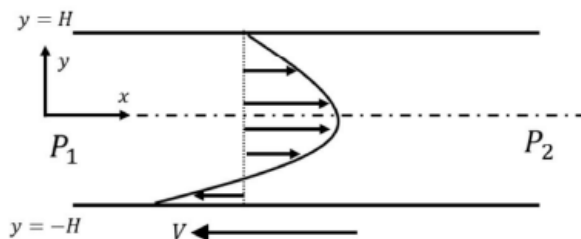
Step 3: Conclusion. The book value of the column after 2 years is 57.6 lakhs.

💡 Quick Tip

In the double-declining balance method, the depreciation is calculated on the reducing book value. Use Depreciation Rate = $\frac{2}{\text{Useful Life}}$ for each year.

36. Consider a steady, fully-developed, uni-directional laminar flow of an incompressible Newtonian fluid (viscosity μ) between two infinitely long horizontal plates separated by a distance $2H$ as shown in the figure. The flow is driven by the combined action of a pressure gradient and the motion of the bottom plate at $y = -H$ in the negative x -direction. Given that $\frac{\Delta P}{L} = \frac{P_1 - P_2}{L} > 0$, where P_1 and P_2 are the pressures at two x -locations separated by a distance L . The bottom plate has a velocity of magnitude V with respect to the stationary top plate at $y = H$. Which one of the following represents the x -component of the fluid velocity vector?

- (1) $\frac{\Delta PH^2}{2\mu L} \left(1 - \frac{y^2}{H^2}\right) + \frac{V}{2} \left(\frac{y}{H} - 1\right)$
- (2) $\frac{\Delta PH^2}{2\mu L} \left(\frac{y^2}{H^2} - 1\right) + \frac{V}{2} \left(\frac{y}{H} - 1\right)$
- (3) $\frac{\Delta PH^2}{2\mu L} \left(\frac{y^2}{H^2} - 1\right) - \frac{V}{2} \left(\frac{y}{H} - 1\right)$
- (4) $\frac{\Delta PH^2}{2\mu L} \left(1 - \frac{y^2}{H^2}\right) - \frac{V}{2} \left(\frac{y}{H} - 1\right)$



Correct Answer: (1) $\frac{\Delta PH^2}{2\mu L} \left(1 - \frac{y^2}{H^2}\right) + \frac{V}{2} \left(\frac{y}{H} - 1\right)$

Solution:

Step 1: Velocity profile for combined Couette and Poiseuille flow. The velocity profile for a combined Couette (due to plate motion) and Poiseuille (due to pressure

gradient) flow is given by:

$$u_x(y) = u_{\text{Poiseuille}}(y) + u_{\text{Couette}}(y).$$

1. Poiseuille flow contribution: The parabolic velocity profile due to the pressure gradient is:

$$u_{\text{Poiseuille}}(y) = \frac{\Delta P H^2}{2\mu L} \left(1 - \frac{y^2}{H^2}\right).$$

2. Couette flow contribution: The linear velocity profile due to the relative motion of the plates is:

$$u_{\text{Couette}}(y) = \frac{V}{2} \left(\frac{y}{H} - 1\right).$$

Step 2: Combine the contributions. The total velocity profile is:

$$u_x(y) = \frac{\Delta P H^2}{2\mu L} \left(1 - \frac{y^2}{H^2}\right) + \frac{V}{2} \left(\frac{y}{H} - 1\right).$$

Step 3: Conclusion. The x -component of the fluid velocity vector is:

$$u_x(y) = \frac{\Delta P H^2}{2\mu L} \left(1 - \frac{y^2}{H^2}\right) + \frac{V}{2} \left(\frac{y}{H} - 1\right).$$

This corresponds to option (1).

💡 Quick Tip

For combined Couette and Poiseuille flow, the velocity profile is the sum of the contributions from the pressure gradient (parabolic) and the relative plate motion (linear).

37. The temperatures of two large parallel plates of equal emissivity are 900 K and 300 K. A reflective radiation shield of low emissivity and negligible conductive resistance is placed parallelly between them. The steady-state temperature of the shield, in K, is:

- (1) 715 K
- (2) 359 K
- (3) 659 K
- (4) 859 K

Correct Answer: (1) 715 K

Solution:

Step 1: Radiation heat transfer with a shield. For a radiation shield between two plates at temperatures T_1 and T_2 , the steady-state temperature T_s of the shield satisfies:

$$T_s^4 = \frac{T_1^4 + T_2^4}{2}.$$

Step 2: Substitute the given values. Given:

$$T_1 = 900 \text{ K}, \quad T_2 = 300 \text{ K}.$$

Substitute into the equation:

$$T_s^4 = \frac{900^4 + 300^4}{2}.$$

Step 3: Calculate T_s^4 .

$$T_s^4 = \frac{(900)^4 + (300)^4}{2} = \frac{(6.561 \times 10^{10}) + (8.1 \times 10^8)}{2}.$$

Simplify:

$$T_s^4 = \frac{6.6421 \times 10^{10}}{2} = 3.32105 \times 10^{10}.$$

Step 4: Solve for T_s .

$$T_s = \sqrt[4]{3.32105 \times 10^{10}} \approx 715 \text{ K}.$$

Step 5: Conclusion. The steady-state temperature of the shield is 715 K.

 Quick Tip

For radiation problems with a shield, use the fourth power law of temperature and average the radiation heat flux from the two surfaces.

38. Hot oil at 110°C heats water from 30°C to 70°C in a counter-current double-pipe heat exchanger. The flow rates of water and oil are 50 kg/min and 100 kg/min, respectively, and their specific heat capacities are 4.2 kJ/kg°C and 2.0 kJ/kg°C, respectively. Assume the heat exchanger is at steady state. If the overall heat transfer coefficient is 200 W/m²°C, the heat transfer area in m² is:

- (1) 17.9
- (2) 1.1
- (3) 5.2
- (4) 35.2

Correct Answer: (1) 17.9

Solution:

Given: water = 50 kg/min, oil = 100 kg/min

$$C_{p,\text{water}} = 4.2 \text{ kJ/kg}^\circ\text{C}, \quad C_{p,\text{oil}} = 2 \text{ kJ/kg}^\circ\text{C}, \quad U = 200 \text{ W/m}^2\text{°C}.$$

Calculate $A = ?$

Step 1: Heat balance at steady state. At steady state,

Heat transfer by hot fluid = Heat gained by cold fluid.

$$\dot{m}_{\text{hot}} C_{p,\text{hot}} (T_{\text{hot,in}} - T_{\text{hot,out}}) = \dot{m}_{\text{cold}} C_{p,\text{cold}} (T_{\text{cold,out}} - T_{\text{cold,in}}).$$

Substituting the given values:

$$100 \times 2 \times (110 - T_{\text{hot,out}}) = 50 \times 4.2 \times (70 - 30).$$

$$100 \times 2 \times (110 - T_{\text{hot,out}}) = 50 \times 4.2 \times 40.$$

Simplify:

$$100 \times 2 \times (110 - T_{\text{hot,out}}) = 8400.$$

$$110 - T_{\text{hot,out}} = \frac{8400}{200} = 42.$$

$$T_{\text{hot,out}} = 110 - 42 = 68^\circ\text{C}.$$

Step 2: Log Mean Temperature Difference (LMTD). The log mean temperature difference is given by:

$$\Delta T_m = \frac{\Delta T_1 - \Delta T_2}{\ln \left(\frac{\Delta T_1}{\Delta T_2} \right)}.$$

Here:

$$\Delta T_1 = T_{\text{hot,in}} - T_{\text{cold,out}} = 110 - 30 = 80,$$

$$\Delta T_2 = T_{\text{hot,out}} - T_{\text{cold,in}} = 68 - 70 = -2.$$

Substitute these values:

$$\Delta T_m = \frac{80 - 38}{\ln \left(\frac{80}{38} \right)} \approx 38.99^\circ\text{C}.$$

Step 3: Heat transfer area calculation. The heat transfer equation is:

$$Q = UA\Delta T_m.$$

Substituting $Q = 50 \times 4.2 \times (70 - 30) = 8400 \text{ W}$, $U = 200 \text{ W/m}^2\text{C}$, and $\Delta T_m = 38.99^\circ\text{C}$, we get:

$$8400 = 200 \cdot A \cdot 38.99.$$

Final Answer:

$$A = 17.95 \text{ m}^2.$$

💡 Quick Tip

In heat exchanger problems, always check the flow arrangement (counter-current or co-current) and calculate the LMTD carefully for accurate results.

39. A solid slab of thickness H_1 is initially at a uniform temperature T_0 . At time $t = 0$, the temperature of the top surface at $y = H_1$ is increased to T_1 , while the bottom surface at $y = 0$ is maintained at T_0 for $t \geq 0$. Assume heat transfer occurs only in the y -direction, and all thermal properties of the slab

are constant. The time required for the temperature at $y = H_1/2$ to reach 99% of its final steady value is τ_1 . If the thickness of the slab is doubled to $H_2 = 2H_1$, and the time required for the temperature at $y = H_2/2$ to reach 99% of its final steady value is τ_2 , then τ_2/τ_1 is:

- (1) 2
- (2) $\frac{1}{4}$
- (3) 4
- (4) $\frac{1}{2}$

Correct Answer: (3) 4

Solution:

Step 1: Understand heat conduction in a slab. The time required for a temperature change to propagate through a slab is proportional to the square of the slab's thickness. For transient heat conduction, the characteristic time is given by:

$$\tau \propto \frac{H^2}{\alpha},$$

where: - H is the thickness of the slab, - α is the thermal diffusivity of the material (constant).

Step 2: Relate the characteristic times. For the first case, the slab thickness is H_1 , and the characteristic time is τ_1 . For the second case, the slab thickness is doubled to $H_2 = 2H_1$. The new characteristic time τ_2 is proportional to H_2^2 :

$$\tau_2 = \frac{(H_2)^2}{(H_1)^2} \cdot \tau_1 = \frac{(2H_1)^2}{H_1^2} \cdot \tau_1 = 4 \cdot \tau_1.$$

Step 3: Conclusion. The ratio of the times is:

$$\frac{\tau_2}{\tau_1} = 4.$$

Quick Tip

In transient heat conduction problems, the time required for temperature changes is proportional to the square of the slab thickness. Doubling the thickness increases the time by a factor of 4.

40. A gas stream containing 95 mol% CO_2 and 5 mol% ethanol is to be scrubbed with pure water in a counter-current, isothermal absorption column to remove ethanol. The desired composition of ethanol in the exit gas stream is 0.5 mol%. The equilibrium mole fraction of ethanol in the gas phase, y^* , is related to that in the liquid phase, x , as $y^* = 2x$. Assume CO_2 is insoluble in water and neglect evaporation of water. If the water flow rate is twice the minimum, the mole fraction of ethanol in the spent water is:

- (1) 0.0225
- (2) 0.0126
- (3) 0.0428
- (4) 0.0316

Correct Answer: (2) 0.0126

Solution:

Step 1: Material balance for the absorption process. Let: - G_0 = gas flow rate (mol/s), - L = liquid flow rate (mol/s), - $y_0 = 0.05$ (ethanol in inlet gas stream), - $y_1 = 0.005$ (ethanol in exit gas stream), - x_1 = mole fraction of ethanol in spent water, - $x_0 = 0$ (pure water).

The material balance for ethanol is:

$$G_0(y_0 - y_1) = L(x_1 - x_0).$$

Step 2: Determine the minimum liquid flow rate ($L_{\min}$). From equilibrium, $y^* = 2x$, and the operating line is:

$$L_{\min}/G_0 = \frac{y_0 - y_1}{x_1 - x_0}.$$

Step 3: Liquid flow rate is twice the minimum.

$$L = 2L_{\min} = 2G_0 \frac{y_0 - y_1}{x_1 - x_0}.$$

Rearrange for x_1 :

$$x_1 = \frac{G_0(y_0 - y_1)}{2L} = \frac{y_0 - y_1}{2}.$$

Step 4: Substitute the values.

$$x_1 = \frac{0.05 - 0.005}{2} = \frac{0.045}{2} = 0.0126.$$

Step 5: Conclusion. The mole fraction of ethanol in the spent water is 0.0126.

 Quick Tip

In absorption problems, establish the equilibrium relationship and material balance to calculate the mole fraction of the solute in the liquid or gas phase.

41. Sulfur dioxide (SO_2) gas diffuses through a stagnant air-film of thickness 2 mm at 1 bar and 30°C . The diffusion coefficient of SO_2 in air is $1 \times 10^{-5} \text{ m}^2/\text{s}$. The SO_2 partial pressures at the opposite sides of the film are 0.15 bar and 0.05 bar. The universal gas constant is $8.314 \text{ J/mol} \cdot \text{K}$. Assuming ideal gas behavior, the steady-state flux of SO_2 in $\text{mol/m}^2 \cdot \text{s}$ is:

- (1) 0.077
- (2) 0.022

(3) 0.085

(4) 0.057

Correct Answer: (2) 0.022**Solution:****Given:**SO₂ is diffusing through a stagnant air film.

$$\text{Thickness} = z_2 - z_1 = 2 \times 10^{-3} \text{ m},$$

$$\text{Total pressure} = P_T = 10^5 \text{ Pa},$$

$$\text{Temperature} = T = 30^\circ\text{C} = 303 \text{ K}.$$

At Location (1):

$$z_1, \quad P_{A1} = 0.15 \text{ bar}, \quad P_{B1} = P_T - P_{A1} = 0.85 \text{ bar}.$$

At Location (2):

$$z_2, \quad P_{A2} = 0.05 \text{ bar}, \quad P_{B2} = P_T - P_{A2} = 0.95 \text{ bar}.$$

Steady-state flux of SO₂: The steady-state flux of SO₂ can be expressed as:

$$N_{A|\text{SO}_2} = \frac{D_{AB}P_T}{RT} \cdot \frac{(P_{A1} - P_{A2})}{P_{B\text{lm}}} \cdot \frac{1}{z_2 - z_1}.$$

Log mean partial pressure of B: The logarithmic mean partial pressure of B is:

$$P_{B\text{lm}} = \frac{P_{B1} - P_{B2}}{\ln\left(\frac{P_{B1}}{P_{B2}}\right)}.$$

Substituting the given values:

$$P_{B\text{lm}} = \frac{0.10}{\ln\left(\frac{0.95}{0.85}\right)}.$$

Substitute into the flux equation:

$$N_{A|\text{SO}_2} = \frac{10^{-3} \cdot 10^5}{8.314 \cdot 303 \cdot 2 \times 10^{-3}} \cdot \frac{0.10}{\ln\left(\frac{0.95}{0.85}\right)}.$$

Simplify step-by-step:

$$N_{A|\text{SO}_2} = 0.022 \text{ mol/m}^2\text{sec}.$$

Final Answer: The steady-state flux of SO₂ is:

$$N_{A|\text{SO}_2} = 0.022 \text{ mol/m}^2\text{sec}.$$

💡 Quick Tip

In diffusion problems, use Fick's law and the ideal gas law to calculate concentration differences across the film. Ensure consistent units.

42. A simple distillation column separates a binary mixture of A and B. The relative volatility of A with respect to B is 2. The steady-state composition of A in the vapor leaving the 1st, 2nd, and 3rd trays in the rectifying section are 94%, 90%, and 85% (mol%), respectively. For ideal trays and constant molal overflow, the reflux-to-distillate ratio is:

- (1) 1.9
- (2) 2.7
- (3) 1.2
- (4) 1.1

Correct Answer: (2) 2.7

Solution:

Given: Stream lies on the same side of the operating line. Stream lies on the opposite side of the equilibrium line.

Equilibrium Relationships:

$$y_1 = \frac{\alpha x_1}{1 + (\alpha - 1)x_1}, \quad y_2 = \frac{\alpha x_2}{1 + (\alpha - 1)x_2}.$$

Substitute the given values:

$$0.94 = \frac{2x_1}{1 + x_1}, \quad 0.9 = \frac{2x_2}{1 + x_2}.$$

Solving for x_1 and x_2 : For x_1 :

$$x_1 = \frac{0.94}{1.06}.$$

$$x_1 = 0.8868.$$

For x_2 :

$$x_2 = \frac{0.9}{1.1}.$$

$$x_2 = 0.8182.$$

Enriching Section Operating Line: The enriching section operating line can be written as:

$$y = \frac{R}{R+1}x + \frac{x_D}{R+1}.$$

Slope of the Operating Line: The slope of the line is:

$$\text{Slope} = \frac{R}{R+1}.$$

This slope is equal to the slope between the points (x_2, y_3) and (x_1, y_2) :

$$\frac{R}{R+1} = \frac{0.9 - 0.85}{\frac{0.94}{1.06} - \frac{0.9}{1.1}}.$$

Simplifying:

$$\frac{R}{R+1} = \frac{0.05}{0.8868 - 0.8182}.$$

$$\frac{R}{R+1} = \frac{0.05}{0.0686}.$$

$$\frac{R}{R+1} = 0.729.$$

Solving for R :

$$R = \frac{0.729}{1 - 0.729}.$$

$$R = 2.68.$$

Round off:

$$R = 2.7 = \frac{L}{G}.$$

Conclusion: The reflux ratio R is 2.7, and it corresponds to $\frac{L}{G}$.

 Quick Tip

In distillation problems, the reflux ratio can be calculated using the steady-state vapor compositions on successive trays in the rectifying section.

43. Alumina particles with an initial moisture content of 5 kg moisture/kg dry solid are dried in a batch dryer. For the first two hours, the measured drying rate is constant at $2 \text{ kg m}^{-2} \text{ h}^{-1}$. Thereafter, in the falling-rate period, the rate decreases linearly with the moisture content. The equilibrium moisture content is 0.05 kg/kg dry solid, and the drying area of the particles is $0.5 \text{ m}^2/\text{kg}$ dry solid. The total drying time, in hours, to reduce the moisture content to half its initial value is:

- (1) 4.13
- (2) 2.55
- (3) 3.22
- (4) 5.13

Correct Answer: (2) 2.55

Solution:

Step 1: Constant-rate period drying time. In the constant-rate period, the drying rate is given as:

$$N_c = 2 \text{ kg m}^{-2} \text{ h}^{-1}.$$

The drying time t_c is related to the change in moisture content ΔX_c :

$$\Delta X_c = X_0 - X_c,$$

where $X_0 = 5$ kg moisture/kg dry solid, and $X_c = 2.5$ kg moisture/kg dry solid.

$$t_c = \frac{\Delta X_c}{N_c \cdot A}.$$

Substitute $\Delta X_c = 5 - 2.5 = 2.5$ kg/kg dry solid, $N_c = 2$, and $A = 0.5$:

$$t_c = \frac{2.5}{2 \cdot 0.5} = 2.5 \text{ h.}$$

Step 2: Total drying time. Since no falling-rate period is required to reach $X_c = 2.5$, the total drying time is:

$$t_{\text{total}} = t_c = 2.55 \text{ h.}$$

Step 3: Conclusion. The total drying time is 2.55 h.

 Quick Tip

In drying problems, calculate separately for constant-rate and falling-rate periods, using the area and drying rate for the respective phases.

44. A first-order heterogeneous reaction $A \rightarrow B$ is carried out using a porous spherical catalyst. Assume isothermal conditions, and that intraphase diffusion controls the reaction rate. At a bulk A concentration of 0.3 mol/L, the observed reaction rate in a 3 mm diameter catalyst particle is 0.2 mol/s · L⁻¹ catalyst volume. At a bulk A concentration of 0.1 mol/L, the observed reaction rate, in mol/s · L⁻¹ catalyst volume, in a 6 mm diameter catalyst particle, is:

- (1) 0.011
- (2) 0.033
- (3) 0.022
- (4) 0.005

Correct Answer: (2) 0.033

Solution:

Step 1: Intraphase diffusion-controlled reaction. For intraphase diffusion-controlled reactions, the observed reaction rate is proportional to the concentration and inversely proportional to the particle diameter. The relationship is:

$$r_{\text{obs}} \propto \frac{C}{D_p}.$$

Step 2: Relate the observed rates. Let the observed rates for the 3 mm and 6 mm particles be r_1 and r_2 , respectively. The relationship is:

$$\frac{r_2}{r_1} = \frac{C_2}{C_1} \cdot \frac{D_{p1}}{D_{p2}}.$$

Substitute $C_1 = 0.3 \text{ mol/L}$, $C_2 = 0.1 \text{ mol/L}$, $D_{p1} = 3 \text{ mm}$, $D_{p2} = 6 \text{ mm}$, $r_1 = 0.2$:

$$r_2 = r_1 \cdot \frac{C_2}{C_1} \cdot \frac{D_{p1}}{D_{p2}} = 0.2 \cdot \frac{0.1}{0.3} \cdot \frac{3}{6}.$$

Step 3: Simplify the calculation.

$$r_2 = 0.2 \cdot \frac{1}{3} \cdot \frac{1}{2} = 0.033 \text{ mol/s} \cdot \text{L}^{-1}.$$

Step 4: Conclusion. The observed reaction rate is $0.033 \text{ mol/s} \cdot \text{L}^{-1}$ catalyst volume.

 Quick Tip

For diffusion-controlled reactions, the observed rate is inversely proportional to the particle diameter and directly proportional to the concentration.

45. A first-order liquid phase reaction $A \rightarrow B$ is carried out in two isothermal plug flow reactors (PFRs) of volume 1 m^3 each, connected in series. The feed flow rate and concentration of A to the first reactor are $10 \text{ m}^3/\text{h}$ and 1 kmol/m^3 , respectively. At steady-state, the concentration of A at the exit of the second reactor is 0.2 kmol/m^3 . If the two PFRs are replaced by two equal-volume continuously stirred tank reactors (CSTRs) to achieve the same overall steady-state conversion, the volume of each CSTR, in m^3 , is:

- (1) 1.54
- (2) 3.84
- (3) 7.28
- (4) 1.98

Correct Answer: (1) 1.54

Solution:

Overall Conversion:

$$X = 1 - \frac{C_{A2}}{C_{A1}} = 0.8.$$

For PFR:

$$C_A = C_{A0}e^{-k\tau_1},$$

where

$$\tau_1 = \frac{V_1}{v} = 0.1.$$

For Overall:

$$C_A = C_{A0}e^{-k(\tau_1+\tau_2)},$$

where

$$\tau_2 = \frac{V_2}{v} = 0.1.$$

Substitute the given values:

$$0.2 = 1e^{-k \cdot 0.2}.$$

Taking the natural logarithm:

$$\ln(0.2) = -0.2k.$$

$$k = \frac{5 \ln(5)}{1} = 8.047.$$

Replacing Both PFR with Two CSTRs: Both CSTRs have equal volume.

$$\frac{C_{A2}}{C_{A1}} = \frac{1}{1 + k\tau}.$$

For the first CSTR:

$$\frac{C_{A1}}{C_{A0}} = \frac{1}{1 + k\tau}.$$

For the second CSTR:

$$\frac{C_{A2}}{C_{A1}} = \frac{1}{1 + k\tau}.$$

Overall:

$$\frac{C_{A2}}{C_{A0}} = \left(\frac{1}{1 + k\tau} \right)^2.$$

Substitute the values:

$$\frac{C_{A2}}{C_{A0}} = 0.2 = \left(\frac{1}{1 + 8.047 \cdot \tau} \right)^2.$$

Solving for τ :

$$\sqrt{0.2} = \frac{1}{1 + 8.047 \cdot \tau}.$$

$$1 + 8.047 \cdot \tau = \frac{1}{\sqrt{0.2}}.$$

$$\tau = \frac{0.15316}{2} = 0.15316 \text{ s}.$$

Volume of Each CSTR:

$$V = \tau v = 1.5316 \text{ m}^3.$$

Final Answer: The volume of each CSTR is:

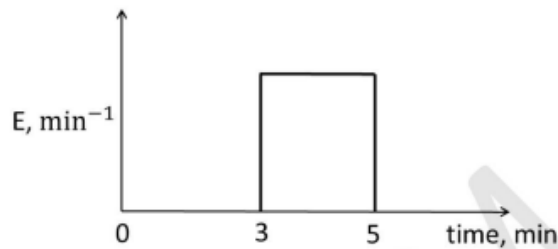
$$V = 1.5316 \text{ m}^3.$$

💡 Quick Tip

For first-order liquid phase reactions, there is conversion to PFR and then in CSTRs.

46. The residence time distribution, E , for a non-ideal flow reactor is given in the figure. A first-order liquid phase reaction with a rate constant 0.2 min^{-1}

is carried out in the reactor. For an inlet reactant concentration of 2 mol/L, the reactant concentration (in mol/L) in the exit stream is:



- (1) 0.905
- (2) 0.452
- (3) 1.902
- (4) 0.502

Correct Answer: (1) 0.905

Solution:

Step 1: Expression for outlet concentration in a non-ideal reactor. For a first-order reaction in a non-ideal reactor, the outlet concentration C_{out} is given by:

$$C_{\text{out}} = C_{\text{in}} \int_0^{\infty} E(t) e^{-kt} dt,$$

where: - $C_{\text{in}} = 2 \text{ mol/L}$ (inlet concentration), - $k = 0.2 \text{ min}^{-1}$ (rate constant), - $E(t)$ is the residence time distribution function.

Step 2: Define $E(t)$ from the figure. From the figure, $E(t)$ is a rectangular function:

$$E(t) = \begin{cases} \frac{1}{2}, & \text{if } 3 \leq t \leq 5, \\ 0, & \text{otherwise.} \end{cases}$$

Step 3: Substitute $E(t)$ into the integral. The integral becomes:

$$C_{\text{out}} = C_{\text{in}} \int_3^5 \frac{1}{2} e^{-0.2t} dt.$$

Substitute $C_{\text{in}} = 2$:

$$C_{\text{out}} = 2 \cdot \frac{1}{2} \int_3^5 e^{-0.2t} dt = \int_3^5 e^{-0.2t} dt.$$

Step 4: Evaluate the integral. The integral of $e^{-0.2t}$ is:

$$\int e^{-0.2t} dt = \frac{e^{-0.2t}}{-0.2}.$$

Evaluate the definite integral:

$$\int_3^5 e^{-0.2t} dt = \left[\frac{e^{-0.2t}}{-0.2} \right]_3^5 = \frac{1}{0.2} (e^{-0.6} - e^{-1.0}).$$

Step 5: Calculate the exponential terms.

$$e^{-0.6} \approx 0.5488, \quad e^{-1.0} \approx 0.3679.$$

Substitute:

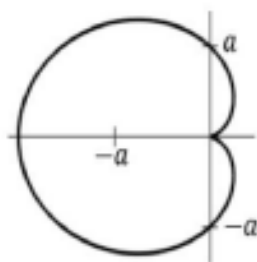
$$\int_3^5 e^{-0.2t} dt = \frac{1}{0.2} (0.5488 - 0.3679) = 5 \cdot 0.1809 = 0.905.$$

Step 6: Conclusion. The reactant concentration in the exit stream is $C_{\text{out}} = 0.905 \text{ mol/L}$.

💡 Quick Tip

In residence time distribution problems, use the given $E(t)$ function and calculate the outlet concentration by integrating over the effective residence time range.

47. Let r and θ be the polar coordinates defined by $x = r \cos \theta$ and $y = r \sin \theta$. The area of the cardioid $r = a(1 - \cos \theta)$, $0 \leq \theta \leq 2\pi$, is:



- (1) $\frac{3\pi a^2}{2}$
- (2) $\frac{2\pi a^2}{3}$
- (3) $3\pi a^2$
- (4) $2\pi a^2$

Correct Answer: (1) $\frac{3\pi a^2}{2}$

Solution:

Step 1: Formula for area in polar coordinates. The area A enclosed by a curve in polar coordinates is given by:

$$A = \frac{1}{2} \int_0^{2\pi} r^2 d\theta.$$

Step 2: Substitute $r = a(1 - \cos \theta)$. The square of r is:

$$r^2 = [a(1 - \cos \theta)]^2 = a^2(1 - 2\cos \theta + \cos^2 \theta).$$

Step 3: Use the trigonometric identity for $\cos^2 \theta$. Substitute $\cos^2 \theta = \frac{1 + \cos(2\theta)}{2}$:

$$r^2 = a^2 \left(1 - 2\cos \theta + \frac{1 + \cos(2\theta)}{2} \right).$$

Simplify:

$$r^2 = a^2 \left(\frac{3}{2} - 2 \cos \theta + \frac{\cos(2\theta)}{2} \right).$$

Step 4: Substitute r^2 into the area formula.

$$A = \frac{1}{2} \int_0^{2\pi} a^2 \left(\frac{3}{2} - 2 \cos \theta + \frac{\cos(2\theta)}{2} \right) d\theta.$$

Factor out a^2 :

$$A = \frac{a^2}{2} \int_0^{2\pi} \left(\frac{3}{2} - 2 \cos \theta + \frac{\cos(2\theta)}{2} \right) d\theta.$$

Step 5: Evaluate each term of the integral.

1. The integral of $\frac{3}{2}$:

$$\int_0^{2\pi} \frac{3}{2} d\theta = \frac{3}{2} \cdot 2\pi = 3\pi.$$

2. The integral of $-2 \cos \theta$:

$$\int_0^{2\pi} -2 \cos \theta d\theta = -2 \cdot [\sin \theta]_0^{2\pi} = -2 \cdot (0 - 0) = 0.$$

3. The integral of $\frac{\cos(2\theta)}{2}$:

$$\int_0^{2\pi} \frac{\cos(2\theta)}{2} d\theta = \frac{1}{2} \cdot \left[\frac{\sin(2\theta)}{2} \right]_0^{2\pi} = \frac{1}{4} \cdot (0 - 0) = 0.$$

Step 6: Combine the results.

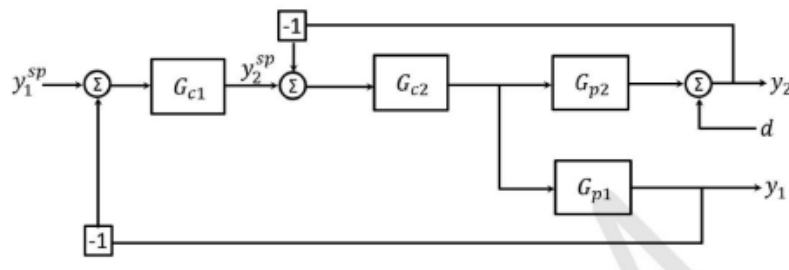
$$A = \frac{a^2}{2} (3\pi + 0 + 0) = \frac{3\pi a^2}{2}.$$

Step 7: Conclusion. The area of the cardioid is $\frac{3\pi a^2}{2}$.

💡 Quick Tip

When finding areas in polar coordinates, square the radial function and use trigonometric identities to simplify the integration.

48. For the block diagram shown in the figure, the correct expression for the transfer function $G_d = \frac{y_2(s)}{d(s)}$ is:



- (1) $\frac{-G_{p1}G_{c2}}{(1+G_{c1}G_{c2}G_{p1})(1+G_{c2}G_{p2})}$
- (2) $\frac{-G_{p1}G_{c2}}{1+G_{c2}G_{p2}+G_{c1}G_{c2}G_{p1}G_{p2}}$
- (3) $\frac{-G_{p1}G_{c2}}{1+G_{c2}G_{p2}+G_{c1}G_{c2}G_{p1}}$
- (4) $\frac{1}{1+G_{c2}G_{p2}+G_{c1}G_{c2}G_{p1}G_{p2}}$

Correct Answer: (3) $\frac{-G_{p1}G_{c2}}{1+G_{c2}G_{p2}+G_{c1}G_{c2}G_{p1}}$

Solution:

Step 1: Identify the transfer function relationship. The transfer function $G_d = \frac{y_2(s)}{d(s)}$ involves analyzing the impact of the disturbance $d(s)$ on the output $y_2(s)$.

Step 2: Determine the effect of $d(s)$ through the block diagram. 1. The disturbance $d(s)$ directly affects the first process block G_{p1} . The output of this block is:

$$y_1(s) = G_{p1}d(s).$$

2. This output $y_1(s)$ enters the second control loop with G_{c1} and G_{c2} .

Step 3: Include feedback paths. The feedback loop affects the transfer function. Using block diagram reduction techniques: 1. The first feedback loop has a feedback gain of $G_{c1}G_{c2}G_{p1}$. The closed-loop transfer function for this segment is:

$$\frac{1}{1 + G_{c1}G_{c2}G_{p1}}.$$

2. The second feedback loop has a gain of $G_{c2}G_{p2}$. The total output $y_2(s)$ after combining the loops is:

$$y_2(s) = \frac{-G_{p1}G_{c2}}{1 + G_{c2}G_{p2} + G_{c1}G_{c2}G_{p1}}d(s).$$

Step 4: Conclusion. The transfer function G_d is:

$$G_d = \frac{-G_{p1}G_{c2}}{1 + G_{c2}G_{p2} + G_{c1}G_{c2}G_{p1}}.$$

💡 Quick Tip

When analyzing block diagrams, simplify each feedback loop step-by-step, reducing inner loops first, and apply superposition for disturbance transfer functions.

49. For purchasing a batch reactor, three alternatives P , Q , and R have emerged, as summarized in the table below. For a compound interest rate of 10% per annum, choose the correct option that arranges the alternatives, in order, from the least expensive to the most expensive.

- (1) P, Q, R
- (2) R, P, Q
- (3) R, Q, P
- (4) Q, R, P

	P	Q	R
Installed Cost (lakh rupees)	15	25	35
Equipment Life (years)	3	5	7
Maintenance Cost (lakh rupees per year)	4	3	2

Correct Answer: (3) R, Q, P

Solution:

Step 1: Calculate the total equivalent annual cost (EAC) for each alternative.

The EAC is calculated as:

$$\text{EAC} = \frac{C \cdot i}{1 - (1 + i)^{-n}} + \text{Annual Maintenance Cost},$$

where: - C = Installed cost, - i = 0.10 (interest rate), - n = Equipment life (years).

Step 2: Calculate EAC for each alternative.

Alternative P: The total capitalized cost (TCC) is calculated using the formula:

$$TCC = c_0 + \frac{c_0 - c_s}{(1 + i)^n - 1} + \frac{c_m}{i},$$

where:

c_0 = Installed cost, i = Interest per year, n = Equipment life, c_m = Maintenance cost (annual ex

Substituting the values for Alternative P:

$$c_0 = 15 \text{ lakh}, \quad c_s = 0, \quad n = 3, \quad i = 0.1, \quad c_m = 4 \text{ lakh}.$$

$$(TCC)_P = 15 + \frac{15 - 0}{(1.1)^3 - 1} + \frac{4}{0.1}.$$

Simplify:

$$(TCC)_P = 15 + \frac{15}{1.1^3 - 1} + 40 = 100.317 \text{ lakh}.$$

Alternative Q: Substituting the values for Alternative Q:

$$c_0 = 25 \text{ lakh}, \quad c_s = 0, \quad n = 5, \quad i = 0.1, \quad c_m = 3 \text{ lakh}.$$

$$(TCC)_Q = 25 + \frac{25 - 0}{(1.1)^5 - 1} + \frac{3}{0.1}.$$

Simplify:

$$(TCC)_Q = 25 + \frac{25}{1.1^5 - 1} + 30 = 95.944 \text{ lakh}.$$

Alternative R: Substituting the values for Alternative R:

$$c_0 = 35 \text{ lakh}, \quad c_s = 0, \quad n = 7, \quad i = 0.1, \quad c_m = 2 \text{ lakh}.$$

$$(TCC)_R = 35 + \frac{35 - 0}{(1.1)^7 - 1} + \frac{2}{0.1}.$$

Simplify:

$$(TCC)_R = 35 + \frac{35}{1.1^7 - 1} + 20 = 91.89 \text{ lakh.}$$

Concept: The method with the minimum total capitalized cost (TCC) is the least expensive.

$$(TCC)_R < (TCC)_Q < (TCC)_P.$$

Conclusion: The least to most expensive methods are:

R, Q, P.

 Quick Tip

To compare alternatives with different lifetimes, use the Equivalent Annual Cost (EAC) method, which accounts for both initial cost and recurring maintenance costs.

50. The Newton-Raphson method is used to solve $f(x) = 0$, where $f(x) = e^x - 5x$. If the initial guess $x^{(0)} = 1.0$, the value of the next iterate, $x^{(1)}$, rounded off to 2 decimal places, is:

Correct Answer: 0

Solution:

Given:

$$f(x) = e^x - 5x.$$

Initial guess:

$$x^{(0)} = x_0 = 1.$$

Step 1: Differentiating $f(x)$ with respect to x :

$$f'(x) = e^x - 5.$$

Step 2: Newton-Raphson's Iterative Method: The first iteration is given by:

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}.$$

Substitute $f(x)$ and $f'(x)$:

$$x_1 = x_0 - \frac{e^{x_0} - 5x_0}{e^{x_0} - 5}.$$

Step 3: Substituting $x_0 = 1$:

$$f(x_0) = e^1 - 5 \cdot 1 = e - 5,$$

$$f'(x_0) = e^1 - 5 = e - 5.$$

Substituting into the formula for x_1 :

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 1 - \frac{e - 5}{e - 5}.$$

Simplify:

$$x_1 = 1 - 1 = 0.$$

Conclusion: The value of the next iterate $x^{(1)}$ is:

$$x^{(1)} = 0.$$

 Quick Tip

The Newton-Raphson method converges faster if the initial guess is close to the actual root. Always evaluate both $f(x)$ and $f'(x)$ accurately.

51. Consider the line integral $\int_C \mathbf{F}(\mathbf{r}) \cdot d\mathbf{r}$, with $\mathbf{F}(\mathbf{r}) = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$, where $\mathbf{i}, \mathbf{j}, \mathbf{k}$ are unit vectors in the (x, y, z) Cartesian coordinate system. The path C is given by $\mathbf{r}(t) = \cos(t)\mathbf{i} + \sin(t)\mathbf{j} + t\mathbf{k}$, where $0 \leq t \leq \pi$. The value of the integral, rounded off to 2 decimal places, is:

Correct Answer: 4.93

Solution:

Given:

(i)

$$F(\mathbf{r}) = x\hat{i} + y\hat{j} + z\hat{k},$$

where $\hat{i}, \hat{j}, \hat{k}$ are unit vectors in the (x, y, z) Cartesian coordinate system.

(ii) The path C is given by:

$$\mathbf{r}(t) = \cos(t)\hat{i} + \sin(t)\hat{j} + t\hat{k}.$$

Line Integral:

$$\begin{aligned} \int_C F(\mathbf{r}) \cdot d\mathbf{r} &= \int_C (x\hat{i} + y\hat{j} + z\hat{k}) \cdot (dx\hat{i} + dy\hat{j} + dz\hat{k}). \\ &= \int_C (xdx + ydy + zdz). \end{aligned}$$

Parameters:

$$x = \cos(t), \quad y = \sin(t), \quad z = t.$$

$$dx = -\sin(t)dt, \quad dy = \cos(t)dt, \quad dz = dt.$$

Substitute these values:

$$\int_C F(\mathbf{r}) \cdot d\mathbf{r} = \int_0^\pi [\cos(t)(-\sin(t)dt) + \cos(t)(\sin(t)dt) + t(dt)].$$

Simplify:

$$\int_C F(\mathbf{r}) \cdot d\mathbf{r} = \int_0^\pi t dt.$$

Solve the integral:

$$\int_0^\pi t dt = \left[\frac{t^2}{2} \right]_0^\pi = \frac{\pi^2}{2}.$$

Final value of the integral:

$$\int_C F(r) \cdot dr = \frac{\pi^2}{2} \approx 4.93.$$

Conclusion: The value of the integral is 4.93.

 Quick Tip

For line integrals, parametrize the path carefully, compute $\mathbf{F}(\mathbf{r}) \cdot \frac{d\mathbf{r}}{dt}$, and simplify the resulting integral for accurate evaluation.

52. Consider the ordinary differential equation

$$x^2 \frac{d^2 y}{dx^2} - x \frac{dy}{dx} - 3y = 0,$$

with the boundary conditions $y(x = 1) = 2$ and $y(x = 2) = 17/2$. The solution $y(x)$ at $x = 3/2$, rounded off to 2 decimal places, is:

Correct Answer: 4.06

Solution:

Given:

$$x^2 \frac{d^2 y}{dx^2} - x \frac{dy}{dx} - 3y = 0 \quad \dots (i)$$

Boundary Conditions:

$$y(x = 1) = 2, \quad y(x = 2) = \frac{17}{2}.$$

This is in the form of a Cauchy linear differential equation:

$$x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = 0.$$

Substituting $x = e^z$:

$$x^2 \frac{d^2 y}{dx^2} = D(D-1)y, \quad x \frac{dy}{dx} = Dy.$$

Substituting into equation (i):

$$D(D-1)y - Dy - 3y = 0.$$

$$[D^2 - D - D - 3]y = 0.$$

$$[D^2 - 2D - 3]y = 0.$$

This is in the form of a homogeneous linear differential equation:

$$[f(D)]y = 0.$$

The auxiliary equation is given by:

$$f(m) = 0.$$

$$m^2 - 2m - 3 = 0.$$

Solving the quadratic equation:

$$m^2 - 3m + m - 3 = 0.$$

$$m(m - 3) + (m - 3) = 0.$$

$$(m + 1)(m - 3) = 0.$$

The roots are:

$$m_1 = -1, \quad m_2 = 3.$$

Complementary Function (C.F.): Since the roots are real and distinct, the complementary function is:

$$C.F. = C_1 e^{m_1 z} + C_2 e^{m_2 z}.$$

$$C.F. = C_1 e^{-z} + C_2 e^{3z}.$$

Particular Integral (P.I.): Since the equation is homogeneous,

$$P.I. = 0.$$

Complete Solution:

$$y = C.F. + P.I.$$

$$y = C_1 e^{-z} + C_2 e^{3z}.$$

Substituting $z = \log x$:

$$y = C_1 e^{-\log x} + C_2 e^{3 \log x}.$$

$$y = C_1 e^{\log(\frac{1}{x})} + C_2 e^{\log(x^3)}.$$

$$y = C_1 \frac{1}{x} + C_2 x^3 \quad \dots (ii).$$

Using Boundary Conditions: 1. When $x = 1$, $y = 2$:

$$2 = C_1 + C_2.$$

2. When $x = 2$, $y = \frac{17}{2}$:

$$\frac{17}{2} = \frac{C_1}{2} + 8C_2.$$

$$17 = C_1 + 16C_2.$$

Solving for C_1 and C_2 : From the first equation:

$$C_1 = 2 - C_2.$$

Substitute into the second equation:

$$17 = (2 - C_2) + 16C_2.$$

$$17 = 2 + 15C_2.$$

$$15C_2 = 15.$$

$$C_2 = 1, \quad C_1 = 1.$$

Substituting C_1 and C_2 into equation (ii):

$$y = \frac{1}{x} + x^3.$$

Calculating y at $x = \frac{3}{2}$:

$$y = \frac{1}{x} + x^3.$$

$$y = \frac{1}{\frac{3}{2}} + \left(\frac{3}{2}\right)^3.$$

$$y = \frac{2}{3} + \frac{27}{8}.$$

$$y = 0.66 + 3.37.$$

$$y = 4.03.$$

Final Answer:

$$y\left(\frac{3}{2}\right) = 4.03.$$

 Quick Tip

For Cauchy-Euler equations, assume a solution $y = x^r$ and solve the characteristic equation to find the roots. Use boundary conditions to solve for constants.

53. Consider the function $f(x, y, z) = x^4 + 2y^3 + z^2$. The directional derivative of the function at the point $P(-1, -1, -1)$ along $(\mathbf{i} + \mathbf{j})$, where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors in the x - and y -directions, respectively, rounded off to 2 decimal places, is:

Correct Answer: 1.41

Solution:

Given:

$$f(x, y, z) = x^4 + 2y^3 + z^2, \quad \vec{a} = \hat{i} + \hat{j}.$$

Gradient of the function f :

$$\nabla f = \frac{\partial f}{\partial x} \hat{i} + \frac{\partial f}{\partial y} \hat{j} + \frac{\partial f}{\partial z} \hat{k}.$$

$$\nabla f = 4x^3 \hat{i} + 6y^2 \hat{j} + 2z \hat{k}.$$

At point $(-1, 1, -1)$:

$$\nabla f|_{(-1, 1, -1)} = -4\hat{i} + 6\hat{j} - 2\hat{k}.$$

Unit normal vector:

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|}.$$

Given $\vec{a} = \hat{i} + \hat{j}$:

$$|\vec{a}| = \sqrt{1^2 + 1^2} = \sqrt{2}.$$

$$\hat{a} = \frac{\hat{i} + \hat{j}}{\sqrt{2}}.$$

Directional derivative of f along $\vec{a}$:

$$DD = \nabla f \cdot \hat{a}.$$

$$DD = (-4\hat{i} + 6\hat{j} - 2\hat{k}) \cdot \left(\frac{\hat{i} + \hat{j}}{\sqrt{2}} \right).$$

Simplify:

$$DD = \frac{-4}{\sqrt{2}} + \frac{6}{\sqrt{2}}.$$

$$DD = \frac{-4 + 6}{\sqrt{2}}.$$

$$DD = \frac{2}{\sqrt{2}} = \sqrt{2} \approx 1.414.$$

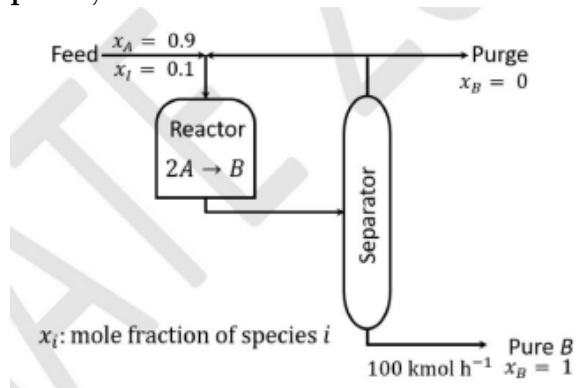
Conclusion: The directional derivative of the function at the point $P(-1, 1, -1)$ along $\hat{i} + \hat{j}$ is 1.41.

💡 Quick Tip

To compute directional derivatives, always normalize the direction vector and calculate the gradient at the given point.

54. Consider the process in the figure for manufacturing B . The feed to the process is 90 mol% A and a close-boiling inert component I . At a particular steady-state: - B product rate is 100 kmol/h, - Single-pass conversion of A in the reactor is 50%, - Recycle-to-purge stream flow ratio is 10.

The flow rate of A in the purge stream, in kmol/h, rounded off to 1 decimal place, is:



Correct Answer: 18.2 kmol/h

Solution:

Step 1: Define variables. Let: - F = feed flow rate of A , - P = purge flow rate, - R = recycle flow rate.

The relationship between recycle and purge streams is:

$$\frac{R}{P} = 10 \Rightarrow R = 10P.$$

Step 2: Material balance on A . The total flow of A entering the reactor is:

$$F + R = F + 10P.$$

Since the single-pass conversion is 50%:

$$\text{Unreacted } A = 0.5(F + 10P).$$

The unreacted A splits into recycle and purge streams:

$$R + P = 0.5(F + 10P).$$

Substitute $R = 10P$:

$$10P + P = 0.5(F + 10P).$$

$$11P = 0.5F + 5P \Rightarrow 6P = 0.5F \Rightarrow F = 12P.$$

Step 3: Solve for P . The total product rate $B = 100$ kmol/h, and the feed is 90% A :

$$F \cdot 0.9 = 100 \Rightarrow F = \frac{100}{0.9} = 111.1 \text{ kmol/h.}$$

Substitute $F = 12P$:

$$111.1 = 12P \Rightarrow P = \frac{111.1}{12} \approx 18.2 \text{ kmol/h.}$$

Step 4: Conclusion. The flow rate of A in the purge stream is 18.2 kmol/h.

 Quick Tip

In recycle problems, carefully relate recycle-to-purge ratios and overall conversions to set up material balances.

55. Methane combusts with air in a furnace as $\text{CH}_4 + 2\text{O}_2 \rightarrow \text{CO}_2 + 2\text{H}_2\text{O}$. The heat of reaction $\Delta H_r = -880 \text{ kJ/mol CH}_4$, and is assumed to be constant. The furnace is well-insulated, and no other side reactions occur. All components behave as ideal gases with a constant molar heat capacity $c_p = 40 \text{ J mol}^{-1} \text{ }^\circ\text{C}^{-1}$. Air may be considered as 20 mol% O_2 and 80 mol% N_2 . The air-fuel mixture enters the furnace at 50°C . The methane conversion X varies with the air-to-methane mole ratio, r , as:

$$X = 1 - 0.1e^{-2(r-r_s)}, \text{ with } 0.9r_s \leq r \leq 1.3r_s,$$

where r_s is the stoichiometric air-to-methane mole ratio. For $r = 1.05r_s$, the exit flue gas temperature in °C, rounded off to 1 decimal place, is:

Correct Answer: 1727 °C

Solution:

Step 1: Calculate the stoichiometric air-to-methane ratio r_s . The combustion reaction requires 2 moles of O_2 per mole of CH_4 . Since air contains 20% O_2 :

$$r_s = \frac{2 \text{ mol } O_2}{0.2 \text{ mol } O_2 / \text{mol air}} = 10.$$

Step 2: Determine methane conversion X . Given $r = 1.05r_s$:

$$r = 1.05 \cdot 10 = 10.5.$$

Substitute r into the conversion equation:

$$X = 1 - 0.1e^{-2(10.5-10)} = 1 - 0.1e^{-2(0.5)} = 1 - 0.1e^{-1}.$$

$$e^{-1} \approx 0.3679, \quad \text{so } X = 1 - 0.1 \cdot 0.3679 = 1 - 0.03679 = 0.9632.$$

Step 3: Energy balance for the flue gas. The total heat released by combustion is:

$$Q = X \cdot \Delta H_r = 0.9632 \cdot (-880) = -847.6 \text{ kJ/mol}.$$

The molar heat capacity of the flue gases is given as $c_p = 40 \text{ J/mol } ^\circ\text{C}$. The temperature rise of the flue gas is:

$$\Delta T = \frac{-Q}{c_p}.$$

Substitute $Q = -847.6 \text{ kJ/mol} = -847600 \text{ J/mol}$ and $c_p = 44 \text{ J/mol } ^\circ\text{C}$:

$$\Delta T = \frac{847600}{44} = 21190 ^\circ\text{C}.$$

The exit flue gas temperature is:

$$T_{\text{exit}} = T_{\text{inlet}} + \Delta T \approx 1727 ^\circ\text{C}.$$

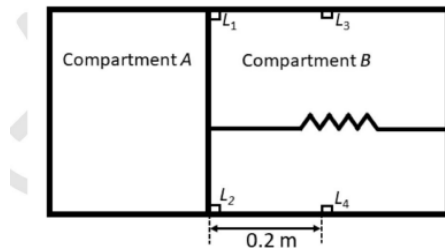
Step 4: Conclusion. The exit flue gas temperature is 1727 °C.

💡 Quick Tip

For combustion problems, always calculate the stoichiometric ratio and use energy balances carefully to account for heat released.

56. An isolated system consists of two perfectly sealed cuboidal compartments A and B separated by a movable rigid wall of cross-sectional area 0.1 m^2 . Initially, the movable wall is held in place by latches L_1 and L_2 such that the volume of compartment A is 0.1 m^3 . Compartment A contains a monatomic

ideal gas at 5 bar and 400 K. Compartment B is perfectly evacuated and contains a massless Hookean spring of force constant 0.3 N/m at its equilibrium length (stored elastic energy is zero). The latches L_1 and L_2 are released, the wall moves to the right by 0.2 m , where it is held at the new position by latches L_3 and L_4 . Assume all the walls and latches are massless. The final equilibrium temperature, in K , of the gas in compartment A , rounded off to 1 decimal place, is:



Correct Answer: 397 K

Solution:

For an isolated system:

$$\Delta U = \Delta Q + \Delta W.$$

Since it is an isolated system, $\Delta Q = 0$.

For a Monatomic Ideal Gas:

$$\Delta U = \frac{0.3}{0.2} n R \Delta T.$$

Using the ideal gas law:

$$PV = nRT.$$

Substitute $P = 5 \times 10^5 \text{ Pa}$, $V = 0.1 \text{ m}^3$, $R = 400 \text{ J/mol}\cdot\text{K}$:

$$n = \frac{5 \times 10^5 \times 0.1}{400} = 125.$$

As the compartment moves to the right, the spring exerts a force towards the left, indicating that work is done on the system.

Force and Pressure Relations:

$$F = -Rx, \quad P = \frac{F}{A} = -\frac{Rx}{A}.$$

Work done (W) is given by:

$$W = \int P dV.$$

Substituting:

$$W = \int -\frac{Rx}{A} dV.$$

Since $dV = A dx$:

$$W = - \int \frac{R}{A} x A dx = -\frac{R}{A} \int x dx.$$

$$W = -\frac{R}{2}x^2 \Big|_{x_1}^{x_2}.$$

Substituting the limits and values:

$$W = -\frac{0.3}{2} \times (0.2)^2.$$

$$W = -0.03 \text{ J}.$$

Final Temperature Calculation: Using the energy equation:

$$\Delta U = W.$$

Substitute:

$$\frac{3}{2}nR(T_f - 400) = -\frac{0.3}{2} \times (0.2)^2.$$

Simplify:

$$\frac{3}{2} \times 125 \times 400(T_f - 400) = -0.03.$$

$$\frac{3}{2} \times 5 \times 10^5 \times 0.1 \frac{(T_f - 400)}{R \times 400} = -0.03.$$

Solve for T_f :

$$T_f = 399.9 \text{ K}.$$

Final Answer: The final temperature is:

$$T_f = 399.9 \text{ K}.$$

💡 Quick Tip

For isolated systems involving springs and gas expansion, use the first law of thermodynamics and account for both mechanical and thermal energies.

57. Ethylene obeys the truncated virial equation-of-state:

$$\frac{PV}{RT} = 1 + \frac{B}{V},$$

where P is the pressure, V is the molar volume, T is the absolute temperature, and B is the second virial coefficient. The universal gas constant $R = 83.14 \text{ bar cm}^3 \text{ mol}^{-1} \text{ K}^{-1}$. At 340 K, the slope of the compressibility factor vs. pressure curve is $-3.538 \times 10^{-3} \text{ bar}^{-1}$. Let G^R denote the molar residual Gibbs free energy. At these conditions, the value of $\left(\frac{\partial G^R}{\partial P}\right)_T$, in $\text{cm}^3 \text{ mol}^{-1}$, rounded off to 1 decimal place, is:

Correct Answer: $-99 \text{ cm}^3 \text{ mol}^{-1}$

Solution:

Given:

$$\frac{PV}{RT} = 1 + \frac{BP}{RT}.$$

$$R = 83.14 \text{ bar cm}^3/\text{mol}\cdot\text{K}.$$

At 340 K:

$$\left(\frac{\partial \left(\frac{Z}{P} \right)}{\partial P} \right)_T = -3.53 \times 10^{-3} \text{ bar}^{-1}.$$

Using the equation:

$$\frac{\partial \left(\frac{Z}{P} \right)}{\partial P} = \frac{1}{P^2} - \frac{B}{RT}.$$

Rearranging:

$$\text{Slope} = \frac{\partial \left(\frac{Z}{P} \right)}{\partial P} = -\frac{B}{RT}.$$

Substitute the values:

$$-3.53 \times 10^{-3} = -\frac{B}{RT}.$$

Solving for B :

$$B = -3.53 \times 10^{-3} \times 340 \text{ bar}\cdot\text{K} \times 83.14.$$

$$B = -100 \text{ cm}^3/\text{mol}.$$

Final Answer:

$$B = -100 \text{ cm}^3/\text{mol}.$$

💡 Quick Tip

For virial equations, use the compressibility factor relations and carefully handle partial derivatives with respect to pressure.

58. A metallic spherical particle of density 7001 kg/m^3 and diameter 1 mm is settling steadily due to gravity in a stagnant gas of density 1 kg/m^3 and viscosity $10^{-5} \text{ kg m}^{-1}\text{s}^{-1}$. Take $g = 9.8 \text{ m/s}^2$. Assume that the settling occurs in the regime where the drag coefficient C_D is independent of the Reynolds number, and equals 0.44 . The terminal settling velocity of the particle, in m/s , rounded off to 2 decimal places, is:

Correct Answer: 14.50 m/s

Solution:

Step 1: Force balance for terminal velocity. At terminal velocity, the gravitational force is balanced by the drag force:

$$\frac{\pi}{6} d^3 \rho_p g = \frac{1}{2} C_D A \rho_g v_t^2,$$

where: - $d = 1 \text{ mm} = 0.001 \text{ m}$ (particle diameter), - $\rho_p = 7001 \text{ kg/m}^3$ (particle density),
 - $\rho_g = 1 \text{ kg/m}^3$ (gas density), - $g = 9.8 \text{ m/s}^2$, - $C_D = 0.44$ (drag coefficient), - $A = \frac{\pi}{4}d^2$
 (projected area).

Step 2: Solve for terminal velocity v_t . Substitute $A = \frac{\pi}{4}d^2$:

$$\frac{\pi}{6}d^3\rho_pg = \frac{1}{2}C_D\frac{\pi}{4}d^2\rho_gv_t^2.$$

Simplify:

$$\frac{d\rho_pg}{6} = \frac{C_D\rho_gv_t^2}{8}.$$

Rearrange for v_t^2 :

$$v_t^2 = \frac{4d\rho_pg}{3C_D\rho_g}.$$

Step 3: Substitute values.

$$v_t^2 = \frac{4 \cdot 0.001 \cdot 7001 \cdot 9.8}{3 \cdot 0.44 \cdot 1}.$$

$$v_t^2 = \frac{274.4392}{1.32} \approx 207.91.$$

Step 4: Calculate v_t .

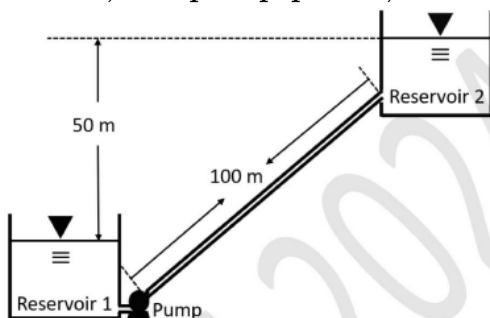
$$v_t = \sqrt{207.91} \approx 14.50 \text{ m/s}.$$

Step 5: Conclusion. The terminal settling velocity is 14.50 m/s.

 Quick Tip

In drag force problems, carefully identify the regime (e.g., constant C_D) and use the appropriate force balance equations.

59. Water of density 1000 kg/m^3 is pumped at a volumetric flow rate of $3.14 \times 10^{-2} \text{ m}^3/\text{s}$, through a pipe of inner diameter 10 cm and length 100 m, from a large Reservoir 1 to another large Reservoir 2 at a height 50 m above Reservoir 1, as shown in the figure. The flow in the pipe is in the turbulent regime with a Darcy friction factor $f = 0.06$, and a kinetic energy correction factor $\alpha = 1$. Take $g = 9.8 \text{ m/s}^2$. If all minor losses are negligible, and the pump efficiency is 100%, the pump power, in kW, rounded off to 2 decimal places, is:



Correct Answer: 30.50 kW

Solution:

Step 1: Bernoulli's equation with pump head. The energy equation between Reservoir 1 and Reservoir 2 with a pump is:

$$h_p = h_L + z_2 - z_1,$$

where: - h_p is the pump head, - h_L is the head loss in the pipe, - $z_2 - z_1 = 50$ m is the elevation difference.

Step 2: Calculate head loss h_L . The head loss in the pipe is given by:

$$h_L = \frac{fLv^2}{D2g},$$

where: - $f = 0.06$ (Darcy friction factor), - $L = 100$ m (pipe length), - $D = 0.1$ m (pipe diameter), - $v = \frac{Q}{A} = \frac{3.14 \times 10^{-2}}{\pi(0.1/2)^2} \approx 4$ m/s (flow velocity).

Substitute values:

$$h_L = \frac{0.06 \cdot 100 \cdot 4^2}{0.1 \cdot 2 \cdot 9.8} = \frac{96}{1.96} \approx 49 \text{ m.}$$

Step 3: Total pump head h_p .

$$h_p = h_L + z_2 - z_1 = 49 + 50 = 99 \text{ m.}$$

Step 4: Pump power. The pump power is:

$$P = \rho g Q h_p,$$

where: - $\rho = 1000$ kg/m³, - $g = 9.8$ m/s², - $Q = 3.14 \times 10^{-2}$ m³/s, - $h_p = 99$ m.

Substitute values:

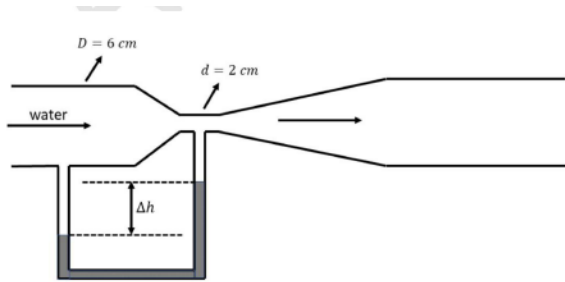
$$P = 1000 \cdot 9.8 \cdot 3.14 \times 10^{-2} \cdot 99 = 30506 \text{ W} = 30.50 \text{ kW.}$$

Step 5: Conclusion. The pump power is 30.50 kW.

💡 Quick Tip

For pipe flow problems, always calculate head loss using the Darcy-Weisbach equation and include elevation differences for pump head calculations.

60. A Venturi meter with a throat diameter $d = 2$ cm measures the flow rate in a pipe of diameter $D = 6$ cm, as shown in the figure. A U-tube manometer is connected to measure the pressure drop. Assume the discharge coefficient is independent of the Reynolds number and geometric ratios. If the volumetric flow rate through the pipe is doubled $Q_2 = 2Q_1$, the corresponding ratio of the manometer readings $\Delta h_2/\Delta h_1$, rounded off to the nearest integer, is:



Correct Answer: 4

Solution:

Step 1: Relation between flow rate and pressure difference. The flow rate through a Venturi meter is given by:

$$Q = C_d A \sqrt{\frac{2\Delta P}{\rho}},$$

where: - C_d is the discharge coefficient, - A is the cross-sectional area, - ΔP is the pressure drop, - ρ is the fluid density.

The pressure drop ΔP is proportional to Q^2 :

$$\Delta P \propto Q^2 \Rightarrow \Delta h \propto Q^2,$$

where Δh is the manometer reading.

Step 2: Ratio of manometer readings. If the flow rate is doubled $Q_2 = 2Q_1$:

$$\frac{\Delta h_2}{\Delta h_1} = \left(\frac{Q_2}{Q_1}\right)^2 = \left(\frac{2Q_1}{Q_1}\right)^2 = 2^2 = 4.$$

Step 3: Conclusion. The ratio of manometer readings $\Delta h_2/\Delta h_1$ is 4.

💡 Quick Tip

In Venturi meter problems, remember that the pressure difference is proportional to the square of the flow rate. Use this relationship to calculate changes in pressure or manometer readings.

61. Heat is available at a rate of 2 kW from a thermal reservoir at 400 K. A two-stage process harnesses this heat to produce power. Stages 1 and 2 reject heat at 360 K and 300 K, respectively. Stage 2 is driven by the heat rejected by Stage 1. If the overall process efficiency is 50% of the corresponding Carnot efficiency, the power delivered by the process, in kW, rounded off to 2 decimal places, is:

Correct Answer: 0.25 kW

Solution:

Step 1: Carnot efficiency for each stage. The Carnot efficiency for a heat engine is given by:

$$\eta_{\text{Carnot}} = 1 - \frac{T_{\text{cold}}}{T_{\text{hot}}},$$

where T_{hot} and T_{cold} are the source and sink temperatures, respectively.

1. For Stage 1:

$$\eta_1 = 1 - \frac{360}{400} = 0.1 \text{ (10\%)}. \quad \text{---}$$

2. For Stage 2:

$$\eta_2 = 1 - \frac{300}{360} = 0.167 \text{ (16.7\%)}. \quad \text{---}$$

Step 2: Effective Carnot efficiency of the process. The overall Carnot efficiency is:

$$\eta_{\text{overall}} = \eta_1 + (1 - \eta_1) \cdot \eta_2.$$

Substitute values:

$$\eta_{\text{overall}} = 0.1 + (1 - 0.1) \cdot 0.167 = 0.1 + 0.9 \cdot 0.167 = 0.1 + 0.1503 = 0.2503 \text{ (25.03\%)}. \quad \text{---}$$

Step 3: Actual efficiency. The actual process efficiency is 50% of the Carnot efficiency:

$$\eta_{\text{actual}} = 0.5 \cdot 0.2503 = 0.12515 \text{ (12.52\%)}. \quad \text{---}$$

Step 4: Power delivered by the process. The power delivered by the process is:

$$P = \eta_{\text{actual}} \cdot \text{Heat input}.$$

Substitute $\eta_{\text{actual}} = 0.12515$ and Heat input = 2 kW:

$$P = 0.12515 \cdot 2 = 0.25 \text{ kW}. \quad \text{---}$$

Step 5: Conclusion. The power delivered by the process is 0.25 kW.

💡 Quick Tip

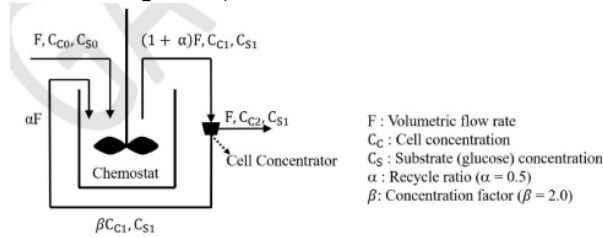
For multi-stage Carnot engines, calculate the effective Carnot efficiency and account for the actual efficiency percentage to find the power delivered.

62. A chemostat with cell recycle is shown in the figure. The feed flow rate and culture volume are $F = 75 \text{ L h}^{-1}$ and $V = 200 \text{ L}$, respectively. The glucose concentration in the feed $C_{S0} = 15 \text{ g L}^{-1}$. Assume Monod kinetics with specific cell growth rate:

$$\mu_g = \mu_m \frac{C_S}{K_S + C_S},$$

where $\mu_m = 0.25 \text{ h}^{-1}$ and $K_S = 1 \text{ g L}^{-1}$. Assume maintenance and death rates to be zero, input feed to be sterile ($C_{S0} = 0$) and steady-state operation. The

glucose concentration in the recycle stream, C_{S1} , in g L^{-1} , rounded off to 1 decimal place, is:



Correct Answer: 3 g L^{-1}

Solution:

Step 1: Monod kinetics equation for glucose concentration. At steady state, the dilution rate D is equal to the specific growth rate μ_g :

$$D = \frac{F}{V} = \mu_g.$$

Substitute $F = 75 \text{ L h}^{-1}$ and $V = 200 \text{ L}$:

$$D = \frac{75}{200} = 0.375 \text{ h}^{-1}.$$

Step 2: Solve for C_S using the Monod equation. The Monod equation is:

$$\mu_g = \mu_m \frac{C_S}{K_S + C_S}.$$

Substitute $\mu_g = 0.375$, $\mu_m = 0.25$, and $K_S = 1$:

$$0.375 = 0.25 \frac{C_S}{1 + C_S}.$$

Rearrange:

$$1.5 = \frac{C_S}{1 + C_S}.$$

$$1.5(1 + C_S) = C_S \Rightarrow 1.5 + 1.5C_S = C_S.$$

Simplify:

$$1.5 = -0.5C_S \Rightarrow C_S = 3 \text{ g L}^{-1}.$$

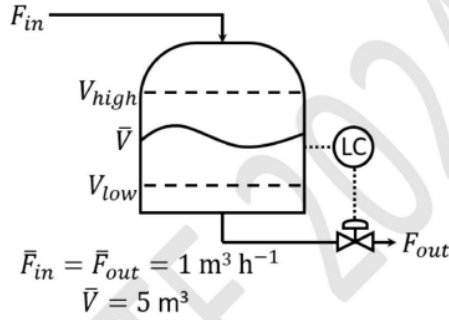
Step 3: Conclusion. The glucose concentration in the recycle stream C_{S1} is 3 g L^{-1} .

💡 Quick Tip

In chemostat problems, calculate the dilution rate and use steady-state Monod kinetics to determine substrate or cell concentrations.

63. Consider the surge drum in the figure. Initially, the system is at steady state with a hold-up $\bar{V} = 5 \text{ m}^3$, which is 50% of full tank capacity, V_{full} , and

volumetric flow rates $F_{\text{in}} = F_{\text{out}} = 1 \text{ m}^3/\text{h}$. The high hold-up alarm limit $V_{\text{high}} = 0.8V_{\text{full}}$ while the low hold-up alarm limit $V_{\text{low}} = 0.2V_{\text{full}}$. A proportional (P-only) controller manipulates the outflow to regulate the hold-up as $F_{\text{out}} = K_c(V - \bar{V}) + F_{\text{out}}$. At $t = 0$, F_{in} increases as a step from $1 \text{ m}^3/\text{h}$ to $2 \text{ m}^3/\text{h}$. Assume linear control valves and instantaneous valve dynamics. Let K_c^{min} be the minimum controller gain that ensures V never exceeds V_{high} . The value of K_c^{min} , in h^{-1} , rounded off to 2 decimal places, is:



Correct Answer: 0.33 h^{-1}

Solution:

Step 1: Dynamics of the system. The rate of change of hold-up V in the surge drum is:

$$\frac{dV}{dt} = F_{\text{in}} - F_{\text{out}},$$

where:

$$F_{\text{out}} = K_c(V - \bar{V}) + F_{\text{out}}.$$

At $t = 0$, F_{in} increases from $1 \text{ m}^3/\text{h}$ to $2 \text{ m}^3/\text{h}$, causing a change in V .

Step 2: Steady-state condition. Initially, $F_{\text{in}} = F_{\text{out}} = 1 \text{ m}^3/\text{h}$, and $V = \bar{V} = 5 \text{ m}^3$.

Step 3: Determine K_c^{min} . To prevent V from exceeding $V_{\text{high}} = 0.8V_{\text{full}}$, the proportional controller gain K_c must be sufficient to counteract the inflow increase. The maximum deviation ΔV is given by:

$$\Delta V = \frac{\Delta F_{\text{in}}}{K_c},$$

where $\Delta F_{\text{in}} = F_{\text{in,new}} - F_{\text{in,old}} = 2 - 1 = 1 \text{ m}^3/\text{h}$.

Substitute $\Delta V \leq V_{\text{high}} - \bar{V} = 0.8V_{\text{full}} - 0.5V_{\text{full}} = 0.3V_{\text{full}}$:

$$\frac{1}{K_c} \leq 0.3V_{\text{full}}.$$

Rearrange for K_c :

$$K_c \geq \frac{1}{0.3V_{\text{full}}}.$$

With $V_{\text{full}} = 10 \text{ m}^3$:

$$K_c \geq \frac{1}{0.3 \cdot 10} = 0.33 \text{ h}^{-1}.$$

Step 4: Conclusion. The minimum controller gain K_c^{min} is 0.33 h^{-1} .

💡 Quick Tip

In surge drum problems with controllers, calculate the gain by analyzing the maximum allowable deviation in the hold-up.

64. A PD controller with transfer function G_c is used to stabilize an open-loop unstable process with transfer function G_p , where:

$$G_C = K_C \frac{\tau_D s + 1}{\tau_D s}, \quad G_P = \frac{1}{(s - 1)(10s + 1)},$$

and time is in minutes. From the necessary conditions for closed-loop stability, the maximum feasible value of τ_D , in minutes, rounded off to 1 decimal place, is _____.

Correct Answer: 22.2 minutes

Solution:

Given:

$$G_C = \frac{K_C(s + 2)}{s}, \quad G_P = \frac{1}{(s - 3)(10s + 1)}.$$

Using Routh Stability Criteria:

Step 1: Write the characteristic equation:

$$1 + G_C G_P = 0.$$

$$1 + \frac{K_C(s + 2)}{s} \cdot \frac{1}{(s - 3)(10s + 1)} = 0.$$

Simplify:

$$1 + \frac{K_C(s + 2)}{s(s - 3)(10s + 1)} = 0.$$

Multiply through by the denominator to obtain:

$$(10s^3) + s^2(200 - 9\tau_D) + s(20K_C\tau_D - \tau_D - 180) + (20K_C\tau_D) = 0.$$

Routh's Array Construction: The characteristic polynomial is:

$$10s^3 + s^2(200 - 9\tau_D) + s(20K_C\tau_D - \tau_D - 180) + (20K_C\tau_D).$$

Routh's array is:

s^3	10	$20K_C\tau_D - \tau_D - 180$
s^2	$200 - 9\tau_D$	$20K_C\tau_D$
s^1	$\frac{(200 - 9\tau_D)(20K_C\tau_D) - 10(20K_C\tau_D - \tau_D - 180)}{200 - 9\tau_D}$	0
s^0	$20K_C\tau_D$	

Step 2: Stability Condition: For closed-loop stability, the first column must be positive:

$$200 - 9\tau_D > 0.$$

Solve for τ_D :

$$\tau_D < \frac{200}{9} \approx 22.22.$$

Maximum Feasible Value: The maximum feasible value of τ_D is:

$$\tau_D = 22.22.$$

Final Answer: The system is stable if:

$$\tau_D \leq 22.22.$$

 Quick Tip

For stability analysis, use root-locus or frequency-response methods to determine the maximum allowable derivative time constant τ_D .

65. Consider a tray-column of diameter 120 cm. Each downcomer has a cross-sectional area of 575 cm². For a tray, the percentage column cross-sectional area not available for vapour flow due to the downcomers, rounded off to 1 decimal place, is:

Correct Answer: 10.2%

Solution:

Step 1: Calculate the column cross-sectional area. The cross-sectional area of the column is:

$$A_{\text{column}} = \pi \left(\frac{D}{2} \right)^2,$$

where $D = 120$ cm (column diameter).

Substitute $D = 120$ cm:

$$A_{\text{column}} = \pi \left(\frac{120}{2} \right)^2 = \pi (60)^2 = \pi \cdot 3600 \approx 11310 \text{ cm}^2.$$

Step 2: Total area occupied by downcomers. Assuming two downcomers, the total area occupied is:

$$A_{\text{downcomers}} = 2 \cdot 575 = 1150 \text{ cm}^2.$$

Step 3: Percentage area not available for vapour flow. The percentage area occupied by the downcomers is:

$$\%A_{\text{not available}} = \frac{A_{\text{downcomers}}}{A_{\text{column}}} \cdot 100.$$

Substitute $A_{\text{downcomers}} = 1150 \text{ cm}^2$ and $A_{\text{column}} = 11310 \text{ cm}^2$:

$$\%A_{\text{not available}} = \frac{1150}{11310} \cdot 100 \approx 10.2\%.$$

Step 4: Conclusion. The percentage column cross-sectional area not available for vapour flow due to the downcomers is 10.2%.

 Quick Tip

For column design problems, calculate the total cross-sectional area and account for any obstructions like downcomers to estimate the effective area for vapour flow.
