GATE 2026 ST Question Paper with Solutions

Time Allowed :3 Hour	Maximum Marks :100	Total Questions :65

General Instructions

Read the following instructions very carefully and strictly follow them:

1. This question paper is divided into three sections:
   - **General Aptitude (GA):** 10 questions (5 questions $\times$ 1 mark + 5 questions $\times$ 2 marks) for a total of 15 marks.
   - **Engineering Mathematics:**
     - **Part A (Mandatory):** 36 questions (1 questions $\times$ 1 mark + 19 questions $\times$ 2 marks) for a total of 55 marks.
     - **Part B (Section 1):** Candidates can choose either Part B1 (Surveying and Mapping) or Part B2 (Section 2). Each part contains 16 questions (8 questions $\times$ 1 mark + 11 questions $\times$ 2 marks) for a total of 30 marks.
2. The total number of questions is **65**, carrying a maximum of **100 marks**.
3. The duration of the exam is **3 hours**.
4. Marking scheme:
   - For 1-mark MCQs, $\frac{1}{3}$ mark will be deducted for every incorrect response.
   - For 2-mark MCQs, $\frac{2}{3}$ mark will be deducted for every incorrect response.
   - No negative marking for numerical answer type (NAT) questions.
   - No marks will be awarded for unanswered questions.
5. Ensure you attempt questions only from the optional section (Part B1 or Part B2) you have selected.
6. Follow the instructions provided during the exam for submitting your answers.

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1. Let X be a random variable having discrete uniform distribution on $\{1, 3, 5, 7, \ldots, 99\}$. Then $E(X \mid X$ is not a multiple of 15$)$ equals

(A) $\frac{2365}{47}$
(B) $\frac{2365}{50}$
(C) 50
(D) 47

Correct Answer: (A) $\frac{2365}{47}$

Solution:
Step 1: Understanding the Concept:

The discrete uniform distribution means each element in the set has an equal probability of occurring.
The set $S = \{1, 3, 5, \ldots, 99\}$ consists of odd integers from 1 to 99.
We need to calculate the conditional expectation $E(X \mid A)$, where A is the event that X is not a multiple of 15.

Step 2: Key Formula or Approach:
1. Identify the total number of elements in S.
2. Identify the elements in S that are multiples of 15 and remove them.
3. The conditional expectation is given by:

$$E(X \mid A) = \frac{\sum_{x \in A} x \cdot P(X = x)}{P(A)} = \frac{\sum_{x \in A} x}{n(A)}$$

where $n(A)$ is the number of elements in set A.

Step 3: Detailed Explanation:
The set S contains odd numbers: $1, 3, 5, \ldots, 2n - 1 = 99 \implies 2n = 100 \implies n = 50$.
Total elements in S is 50.
The sum of the first n odd integers is n^2.

$$\text{Sum}(S) = 50^2 = 2500$$

Multiples of 15 in the range $[1, 99]$ are $\{15, 30, 45, 60, 75, 90\}$.
Since S only contains odd numbers, we only consider the odd multiples: $\{15, 45, 75\}$.
The number of such multiples is 3.
Number of elements in A (not multiples of 15) is $n(A) = 50 - 3 = 47$.
Sum of elements in A is:

$$\text{Sum}(A) = \text{Sum}(S) - (15 + 45 + 75) = 2500 - 135 = 2365$$

Applying the formula for expectation:

$$E(X \mid A) = \frac{2365}{47}$$

Step 4: Final Answer:
The value of $E(X \mid X \text{ is not a multiple of } 15)$ is $\frac{2365}{47}$.

> Quick Tip
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> For the sum of arithmetic progressions like odd numbers, always use n^2. To find the count of multiples of k in a set of odds, list them out if the range is small to avoid errors with even multiples.

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2. In a testing of hypothesis problem, which one of the following statements is true?

(A) The probability of the Type-I error cannot be higher than the probability of the Type-II error
(B) Type-II error occurs if the test accepts the null hypothesis when the null hypothesis is actually false
(C) Type-I error occurs if the test rejects the null hypothesis when the null hypothesis is actually false
(D) The sum of the probability of the Type-I error and the probability of the Type-II error should be 1

Correct Answer: (B) Type-II error occurs if the test accepts the null hypothesis when the null hypothesis is actually false

Solution:
Step 1: Understanding the Concept:
In hypothesis testing, we make decisions about a null hypothesis (H_0) based on sample data. Two types of errors can occur:
1. Type-I Error (α): Rejecting H_0 when H_0 is true.
2. Type-II Error (β): Failing to reject (accepting) H_0 when H_0 is false (i.e., H_1 is true).

Step 2: Detailed Explanation:
- Statement (A) is false because α and β are independent of each other in a fixed-sample size setting (though they are inversely related when changing thresholds). One can be larger than the other.
- Statement (B) is the textbook definition of a Type-II error: accepting H_0 when it should have been rejected.
- Statement (C) describes a correct decision (rejecting a false hypothesis), not an error. A Type-I error is rejecting a true hypothesis.
- Statement (D) is false; $\alpha + \beta$ does not necessarily equal 1.

Step 3: Final Answer:
Statement (B) correctly defines the Type-II error.

> Quick Tip
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> Mnemonic: Type I is a "False Positive" (Rejecting Truth). Type II is a "False Negative" (Accepting Falsehood).

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3. Let $\{W(t)\}_{t\geq 0}$ be a standard Brownian motion. Which one of the following statements is NOT true?

(A) $E[W(7)] = 0$
(B) $E[W(5)W(9)] = 7$
(C) $2W(1)$ is normally distributed with mean 0 and variance 4
(D) $E[W(5) \mid W(3) = 3] = 3$

Correct Answer: (B) $E[W(5)W(9)] = 7$

Solution:
Step 1: Understanding the Concept:
A standard Brownian motion $W(t)$ has the following properties:
1. $W(0) = 0$.
2. $W(t) \sim N(0, t)$.
3. Independent increments: $W(t) - W(s)$ is independent of $W(s)$ for $t > s$.
4. Covariance: $\text{Cov}(W(s), W(t)) = E[W(s)W(t)] = \min(s, t)$.

Step 2: Detailed Explanation:
Checking the options:
(A) $E[W(7)] = 0$: Since $W(t)$ has mean 0 for all t, this is true.
(B) $E[W(5)W(9)]$: Using the covariance formula, $E[W(5)W(9)] = \min(5, 9) = 5$. The statement claims it is 7, which is false.
(C) $2W(1)$: Since $W(1) \sim N(0, 1)$, then $2W(1) \sim N(2 \cdot 0, 2^2 \cdot 1) = N(0, 4)$. This is true.
(D) $E[W(5) \mid W(3) = 3]$: By properties of increments:

$$E[W(5) \mid W(3)] = E[W(5) - W(3) + W(3) \mid W(3)] = E[W(5) - W(3)] + W(3)$$

Since increments have mean 0, $E[W(5) - W(3)] = 0$. So, $E[W(5) \mid W(3) = 3] = 0 + 3 = 3$. This is true.

Step 3: Final Answer:
Statement (B) is the one that is NOT true.

> Quick Tip
>
> Remember the covariance of Brownian motion is always the smaller time value: $E[W(s)W(t)] = \min(s, t)$. This is a very common shortcut in stochastic process questions.

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4. Two fair dice, one having red and another having blue color, are tossed independently once. Let A be the event that the die having red colour will show 5 or 6. Let B be the event that the sum of the outcomes will be 7 and let C be the event that the sum of the outcomes will be 8. Then which one of the following statements is true?

(A) A and B are independent as well as A and C are independent
(B) A and B are independent, but A and C are not independent
(C) A and C are independent, but A and B are not independent
(D) Neither A and B are independent, nor A and C are independent

Correct Answer: (B) A and B are independent, but A and C are not independent

Solution:
Step 1: Understanding the Concept:

Two events X and Y are independent if $P(X \cap Y) = P(X)P(Y)$.
Let the outcome of the red die be R and the blue die be B_l. Total outcomes $= 6 \times 6 = 36$.

Step 2: Key Formula or Approach:
1. Calculate probabilities for A, B, and C.
2. Identify the intersection sets and their probabilities.
3. Test the independence condition.

Step 3: Detailed Explanation:
- Event A: $R \in \{5, 6\}$. Sample points: $\{(5, 1), \ldots, (5, 6), (6, 1), \ldots, (6, 6)\}$. Total 12 points. $P(A) = \frac{12}{36} = \frac{1}{3}$.

- Event B: $R + B_l = 7$. Sample points: $\{(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)\}$. Total 6 points. $P(B) = \frac{6}{36} = \frac{1}{6}$.

- Event C: $R + B_l = 8$. Sample points: $\{(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)\}$. Total 5 points. $P(C) = \frac{5}{36}$.

- Check A and B: $A \cap B = \{(5, 2), (6, 1)\}$. Total 2 points. $P(A \cap B) = \frac{2}{36} = \frac{1}{18}$. $P(A)P(B) = \frac{1}{3} \cdot \frac{1}{6} = \frac{1}{18}$. Since $P(A \cap B) = P(A)P(B)$, A and B are independent.

- Check A and C: $A \cap C = \{(5, 3), (6, 2)\}$. Total 2 points. $P(A \cap C) = \frac{2}{36} = \frac{1}{18} = \frac{6}{108}$. $P(A)P(C) = \frac{1}{3} \cdot \frac{5}{36} = \frac{5}{108}$. Since $P(A \cap C) \neq P(A)P(C)$, A and C are not independent.

Step 4: Final Answer:
Events A and B are independent, but A and C are not.

Quick Tip

Independence is often counter-intuitive. In dice problems, a fixed sum of 7 is unique because it contains one outcome for every possible value of a single die, often making it independent of specific die outcomes.

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5. Let X be a random variable taking only two values, 1 and 2. Let $M_X(t)$ be the moment generating function of X. If the expectation of X is $\frac{10}{7}$, then the fourth derivative of $M_X(t)$ evaluated at 0 equals

(A) $\frac{52}{7}$
(B) $\frac{67}{7}$
(C) $\frac{48}{7}$
(D) $\frac{60}{7}$

Correct Answer: (A) $\frac{52}{7}$

Solution:
Step 1: Understanding the Concept:
The k-th derivative of the Moment Generating Function $M_X(t)$ evaluated at $t = 0$ gives the k-th raw moment of X:

$$M_X^{(k)}(0) = E[X^k]$$

Step 2: Key Formula or Approach:
1. Find the probability distribution of X. 2. Use the given mean to solve for probabilities. 3. Calculate the fourth moment $E[X^4]$.

Step 3: Detailed Explanation:
Let $P(X = 1) = p$ and $P(X = 2) = 1 - p$. Given $E[X] = \frac{10}{7}$:

$$E[X] = 1 \cdot p + 2 \cdot (1 - p) = \frac{10}{7}$$

$$p + 2 - 2p = \frac{10}{7} \implies 2 - p = \frac{10}{7}$$

$$p = 2 - \frac{10}{7} = \frac{4}{7}$$

So, $P(X = 1) = \frac{4}{7}$ and $P(X = 2) = 1 - \frac{4}{7} = \frac{3}{7}$.
Now, we need to find $M_X^{(4)}(0) = E[X^4]$:

$$E[X^4] = 1^4 \cdot P(X = 1) + 2^4 \cdot P(X = 2)$$

$$E[X^4] = 1 \cdot \left(\frac{4}{7}\right) + 16 \cdot \left(\frac{3}{7}\right)$$

$$E[X^4] = \frac{4}{7} + \frac{48}{7} = \frac{52}{7}$$

Step 4: Final Answer:
The value of the fourth derivative at zero is $\frac{52}{7}$.

> Quick Tip
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> Remember the identity $M_X^{(n)}(0) = E[X^n]$. It saves time from explicitly calculating the derivative of an exponential function.

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6. Let A be a 3×3 real matrix and let I_3 be the 3×3 identity matrix. Which one of the following statements is NOT true?

(A) If the row-reduced echelon form of A is I_3, then zero is not an eigenvalue of A
(B) If zero is not an eigenvalue of A, then the row-reduced echelon form of A is I_3
(C) If A has three distinct eigenvalues, then the row-reduced echelon form of A is I_3
(D) If the system of equations $Ax = b$ has a solution for every 3×1 real column vector b, then the row-reduced echelon form of A is I_3

Correct Answer: (C) If A has three distinct eigenvalues, then the row-reduced echelon form of A is I_3

Solution:
Step 1: Understanding the Concept:
A 3×3 matrix A is invertible if and only if:
1. Its RREF is I_3.
2. $\det(A) \neq 0$.
3. Zero is not an eigenvalue.
4. The system $Ax = b$ has a unique solution for all b.

Step 2: Detailed Explanation:
- Statement (A) is true: RREF being I_3 means the matrix is invertible, so $\det(A) \neq 0$, which implies zero is not an eigenvalue.
- Statement (B) is true: No zero eigenvalue means $\det(A) \neq 0$, hence A is invertible, and its RREF must be I_3.
- Statement (C) is false: Having three distinct eigenvalues (e.g., $0, 1, 2$) does not guarantee invertibility. If one of the distinct eigenvalues is zero, the matrix is singular and its RREF will not be I_3.
- Statement (D) is true: If $Ax = b$ has a solution for all b, the mapping is surjective. For a square matrix, this implies it is bijective (invertible), so its RREF is I_3.

Step 3: Final Answer:
Statement (C) is not necessarily true.

> Quick Tip
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> Always check if "zero" is included in the set of eigenvalues. A matrix can have many distinct eigenvalues and still be non-invertible if one of them is zero.

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7. A cube is to be cut into 8 pieces of equal size and shape. Here, each cut should be straight and it should not stop till it reaches the other end of the cube. The minimum number of such cuts required is

(A) 3
(B) 4
(C) 7
(D) 8

Correct Answer: (A) 3

Solution:
Step 1: Understanding the Concept:
We want to maximize the number of pieces with a minimum number of cuts.
To get pieces of equal size and shape from a cube, we typically cut along the planes parallel to

the faces.

Step 2: Detailed Explanation:
If we make n_x, n_y, and n_z cuts along the three orthogonal axes:
The number of pieces produced is $(n_x + 1)(n_y + 1)(n_z + 1)$.
We want this product to be 8.
The most efficient way to reach a product of 8 is by setting:

$$(1 + 1)(1 + 1)(1 + 1) = 2 \times 2 \times 2 = 8$$

This requires: - 1 cut in the X-direction. - 1 cut in the Y-direction. - 1 cut in the Z-direction.
Total cuts $= 1 + 1 + 1 = 3$.

Step 3: Final Answer:
The minimum number of cuts is 3.

Quick Tip

To maximize pieces for N cuts, distribute cuts as evenly as possible among the three dimensions. For a target of P pieces, try to find factors of P that are close to each other.
