GATE 2026 Civil Engineering (CE-1) Question Paper with Solutions

Time Allowed :3 Hour	**Maximum Marks :**100	**Total Questions :**65

General Instructions

Read the following instructions very carefully and strictly follow them:

1. This question paper is divided into three sections:
   - **General Aptitude (GA):** 10 questions (5 questions × 1 mark + 5 questions × 2 marks) for a total of 15 marks.
   - **Engineering Mathematics:**
     - **Part A (Mandatory):** 36 questions (1 questions × 1 mark + 19 questions × 2 marks) for a total of 55 marks.
     - **Part B (Section 1):** Candidates can choose either Part B1 (Surveying and Mapping) or Part B2 (Section 2). Each part contains 16 questions (8 questions × 1 mark + 11 questions × 2 marks) for a total of 30 marks.
2. The total number of questions is **65**, carrying a maximum of **100 marks**.
3. The duration of the exam is **3 hours**.
4. Marking scheme:
   - For 1-mark MCQs, $\frac{1}{3}$ mark will be deducted for every incorrect response.
   - For 2-mark MCQs, $\frac{2}{3}$ mark will be deducted for every incorrect response.
   - No negative marking for numerical answer type (NAT) questions.
   - No marks will be awarded for unanswered questions.
5. Ensure you attempt questions only from the optional section (Part B1 or Part B2) you have selected.
6. Follow the instructions provided during the exam for submitting your answers.

1. 'The team _____ more than 300 runs in 20 overs _____ rains. However, some players needed to improve their batting skills.'
Choose the option with the correct sequence of words to fill the blanks.

(A) score; despite
(B) scoring; instead of
(C) scored; despite
(D) scoring; in spite of

Correct Answer: (C) scored; despite

Solution:

Step 1: Understanding the Concept:
The sentence requires a past tense verb to establish the context of a completed event and a contrastive preposition to show a concession regarding the weather.

Step 2: Detailed Explanation:
1. In the first blank, the phrase "needed to improve" in the following sentence indicates that the action of the first sentence occurred in the past.
2. Therefore, the past tense "scored" is required to maintain tense consistency.
3. In the second blank, we need a word that indicates "regardless of" or "notwithstanding."
4. Both "despite" and "in spite of" carry this meaning, but "instead of" implies a substitution which is logically incorrect here.
5. Combining these, "scored" and "despite" provide the only grammatically and logically consistent pair.

Step 3: Final Answer:
The team scored more than 300 runs despite rains.

Quick Tip

Check the surrounding sentences for context clues. The word "needed" (past tense) is a direct indicator that the first blank must also be in the past tense.

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2. If a positive real x satisfies the following equation

$$\log_2 x + \log_{\sqrt{2}} x = 48,$$

then the value of x is _____.

(A) 2^{16}
(B) 4^{16}
(C) 2^{14}
(D) 4^{14}

Correct Answer: (A) 2^{16}

Solution:

Step 1: Understanding the Concept:
This problem involves solving a logarithmic equation by simplifying terms with different bases into a common base.

Step 2: Key Formula or Approach:
The change of base property or the base power property is used:

$$\log_{b^n} a = \frac{1}{n} \log_b a$$

Step 3: Detailed Explanation:
The given equation is:

$$\log_2 x + \log_{\sqrt{2}} x = 48$$

Since $\sqrt{2} = 2^{1/2}$, we can rewrite the second term using the property mentioned above:

$$\log_{2^{1/2}} x = \frac{1}{1/2} \log_2 x = 2 \log_2 x$$

Substitute this back into the original equation:

$$\log_2 x + 2 \log_2 x = 48$$

$$3 \log_2 x = 48$$

Divide both sides by 3:

$$\log_2 x = 16$$

Converting from logarithmic form to exponential form:

$$x = 2^{16}$$

Step 4: Final Answer:
The value of x is 2^{16}.

> Quick Tip
>
> When dealing with logs of different bases that are powers of each other, always convert to the simplest base to consolidate terms.

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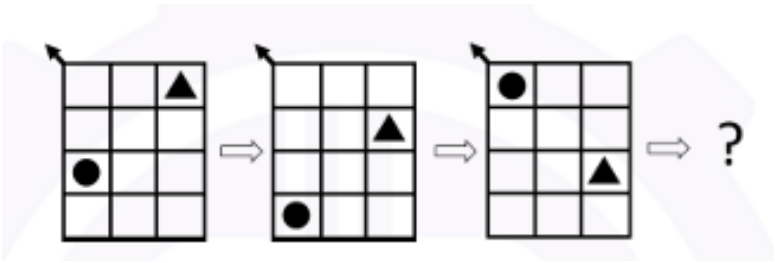
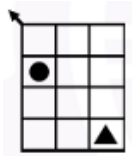
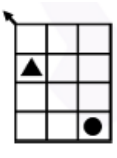
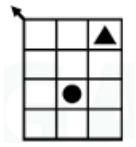
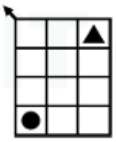
3. The next figure (indicated by '?') in the sequence is

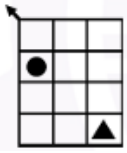


(A) 

(B) 

(C) 

(D) 

Correct Answer: (A) 

Solution:

Step 1: Understanding the Concept:
This is a visual logic puzzle where we observe the movement patterns of different symbols within a grid across a sequence.

Step 2: Detailed Explanation:
Let's track the elements individually in the 3×4 grid (3 columns, 4 rows).
1. **The Triangle (▲):**
- Figure 1: (Col 3, Row 4)
- Figure 2: (Col 3, Row 3)
- Figure 3: (Col 3, Row 2)
- Pattern: Moves down one cell in Column 3 in every step.
- Expected Position: (Col 3, Row 1).
2. **The Circle (•):**
- Figure 1: (Col 1, Row 2)
- Figure 2: (Col 1, Row 1)
- Figure 3: (Col 1, Row 4) - Note: It wraps around from top to bottom.
- Pattern: Moves up one cell in Column 1 and wraps around.
- Expected Position: (Col 1, Row 3).
3. **Arrow:** The arrow at the top left corner remains stationary throughout the sequence.

Step 3: Final Answer:
The correct figure must have the circle at (1, 3) and the triangle at (3, 1), which matches option (A).

Quick Tip

Isolate one symbol at a time and determine its specific vector of movement (direction and distance) before moving to the next symbol.

4. 'All the mangoes in the basket are good.'
If the above statement is false, then which one of the following statements is necessarily true?

(A) All the mangoes in the basket are not good.
(B) No mango in the basket is good.
(C) In the basket, some of the mangoes are good and some are not good.
(D) There exists at least one mango in the basket that is not good.

Correct Answer: (D) There exists at least one mango in the basket that is not good.

Solution:

Step 1: Understanding the Concept:
This problem relies on the rules of logical negation for quantified statements.

Step 2: Detailed Explanation:
1. The original statement is a universal affirmative: "All A are B."
2. If this statement is "false," then its logical negation must be "true."
3. The negation of "All A are B" is "Some A are not B," or "There exists at least one A that is not B."
4. Let's look at the options:
- Option (A) and (B) describe "No A is B." This could be true, but it is not *necessarily* true (only one bad mango is needed to make the original statement false).
- Option (C) requires some to be good and some to be bad. This is too specific; the original is false even if *all* are bad.
- Option (D) correctly identifies that at least one mango failing the condition is enough to negate the "All" claim.

Step 3: Final Answer:
The logical negation of "All" is "Some... not," which matches statement (D).

Quick Tip

In formal logic, the opposite of "All" is "Not All" (meaning "Some Not"), whereas the opposite of "None" is "At least one." Don't confuse "opposite" with "negation."

5. Consider the following statements about four numbers:
(S1) The average of the four numbers is 25
(S2) Each number is at most 40
(S3) Each number is at least 20

Choose the option that is necessarily correct.

(A) (S1) and (S2) together imply (S3)
(B) (S2) and (S3) together imply (S1)
(C) (S1) and (S3) together imply (S2)
(D) (S1) implies (S3)

Correct Answer: (C) (S1) and (S3) together imply (S2)

Solution:

Step 1: Understanding the Concept:
This involves testing logical implications based on the bounds of a sum.

Step 2: Detailed Explanation:
Let the four numbers be a, b, c, d.
From (S1), the sum is: $a + b + c + d = 4 \times 25 = 100$.
Let's check Option (C): If (S1) and (S3) are true, then $a, b, c, d \geq 20$ and $a + b + c + d = 100$.
To find the maximum possible value for any one number (say a), we must minimize the other three.
Minimum values for b, c, d are 20 each.
$a + 20 + 20 + 20 = 100 \Rightarrow a = 40$.
Since any number can be at most 40, (S2) is necessarily true.
Other options can be disproved by counterexamples:
- For (A): $\{40, 40, 15, 5\}$ sums to 100 and all ≤ 40, but not all ≥ 20.
- For (B): $\{20, 20, 20, 20\}$ and $\{40, 40, 40, 40\}$ satisfy S2 and S3 but have different averages.

Step 3: Final Answer:
Combining an average of 25 with a lower bound of 20 mathematically forces an upper bound of 40, validating (C).

Quick Tip

To disprove an implication, try to find a set of numbers that satisfies the premise but violates the conclusion.

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6. 'People are crowding around ____ pit into which ____ elephant has fallen. I have never seen an elephant looking more bewildered ____ miserable. Here it is in a most undignified position, thrust into a pit and made to look up ____ a vast, curiosity-stricken crowd.'
Choose the option with the correct sequence of words to fill the blanks.

(A) an; a; at; and
(B) a; an; and; at

(C) and; a; an; at
(D) at; a; an; and

Correct Answer: (B) a; an; and; at

Solution:

Step 1: Understanding the Concept:
The question evaluates the correct usage of indefinite articles, conjunctions, and prepositions based on phonetics and context.

Step 2: Detailed Explanation:
1. Blank 1: "...around __ pit". 'Pit' starts with a consonant sound, so the article is 'a'.
2. Blank 2: "...into which __ elephant". 'Elephant' starts with a vowel sound, so the article is 'an'.
3. Blank 3: "...bewildered __ miserable". These are two similar adjectives describing the elephant's state. The conjunction 'and' is appropriate to join them.
4. Blank 4: "...look up __ a vast... crowd". When looking towards a target, the preposition 'at' is used.

Step 3: Final Answer:
The sequence "a; an; and; at" correctly completes the passage, matching option (B).

Quick Tip

Remember: "a" vs "an" is determined by the **sound** of the following word, not just the first letter.

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7. The table lists the unit selling price of five products P, Q, R, S, and T. On a particular day, 250 items were sold with the average selling price of Rs. 60. The following observations were made:
(i) The quantity of S sold was twice that of T.
(ii) The quantity of R sold was thrice that of T.
(iii) The quantity of Q sold was four times that of T.

Product	P	Q	R	S	T
Unit selling price (Rs.)	100	50	40	60	60

What is the quantity of product P sold on that day?

(A) 40
(B) 50
(C) 60

(D) 70

Correct Answer: (B) 50

Solution:

Step 1: Understanding the Concept:
We need to solve a system of linear equations based on the total quantity of items and the total revenue.

Step 2: Key Formula or Approach:
Let the quantity of product T sold be x.
Then: $S = 2x, R = 3x, Q = 4x$.
Let quantity of P be P.

Step 3: Detailed Explanation:
1. Equation for Total Quantity:

$$P + Q + R + S + T = 250$$

$$P + 4x + 3x + 2x + x = 250 \Rightarrow P + 10x = 250 \quad \text{— (Equation 1)}$$

2. Equation for Total Revenue: Total items = 250, Average Price = 60. Total Revenue = $250 \times 60 = 15,000$.

$$100P + 50(4x) + 40(3x) + 60(2x) + 60x = 15,000$$

$$100P + 200x + 120x + 120x + 60x = 15,000$$

$$100P + 500x = 15,000 \Rightarrow P + 5x = 150 \quad \text{— (Equation 2)}$$

3. Solve the system: Multiply Equation 2 by 2: $2P + 10x = 300$.
Subtract Equation 1: $(2P + 10x) - (P + 10x) = 300 - 250$.
$P = 50$.

Step 4: Final Answer:
The quantity of product P sold is 50.

> **Quick Tip**
>
> When multiple variables are related to one common variable, express everything in terms of that common variable (x) to simplify the math.

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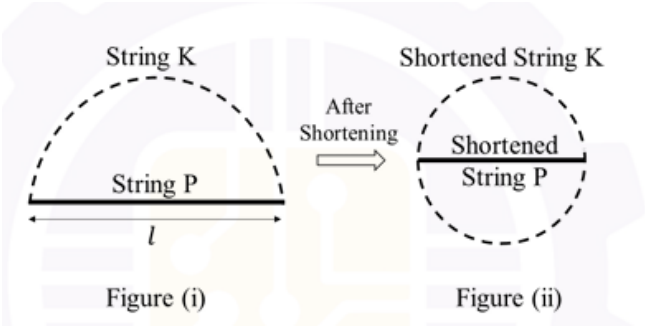
8. Consider a string P of length l that is laid out as a straight-line segment. Another string K is laid out as a semicircular arc with string P as its diameter, as represented in Figure (i). When both the strings are shortened by a length x they can be re-arranged such that the shortened string K forms a full circle with the

shortened string P as its diameter, as represented in Figure (ii). The value of x/l is _____.



(A) π
(B) $\frac{\pi-1}{2\pi}$
(C) $\frac{\pi}{2(\pi-1)}$
(D) $\frac{\pi}{\pi-1}$

Correct Answer: (C) $\frac{\pi}{2(\pi-1)}$

Solution:

Step 1: Understanding the Concept:
The relationship between diameter and circumference/arc length determines the changes in the string lengths.

Step 2: Detailed Explanation:
1. Initially:
- String P length $= l$ (this is the diameter d).
- String K is a semicircular arc with diameter l. Length of K $= \frac{\pi l}{2}$.
2. After shortening by x:
- New length of P $= l - x$. This is the diameter of the new circle.
- New length of K $= \frac{\pi l}{2} - x$. This is the circumference of the new circle.
3. Relate the two:
Circumference $= \pi \times$ Diameter

$$\frac{\pi l}{2} - x = \pi(l - x)$$
$$\frac{\pi l}{2} - x = \pi l - \pi x$$
$$\pi x - x = \pi l - \frac{\pi l}{2}$$
$$x(\pi - 1) = \frac{\pi l}{2}$$
$$\frac{x}{l} = \frac{\pi}{2(\pi - 1)}$$

Step 3: Final Answer:
The ratio x/l is $\frac{\pi}{2(\pi-1)}$.

> Quick Tip
>
> Be careful with Figure (i). String K is just the arc, not the full perimeter of the semi-circle. If it were the perimeter, the equation would be different.

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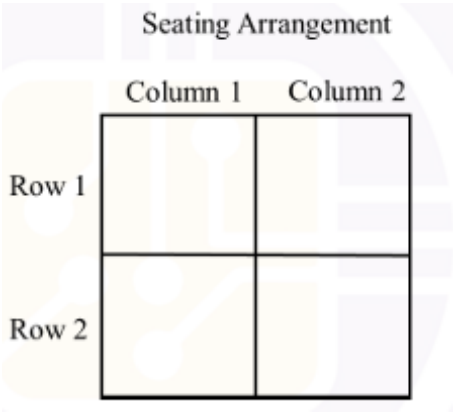
9. The Roman senator Meritorius, his brother, his son, and his daughter have varying oratory skill levels. They are seated in rows and columns as shown in the figure with exactly one person sitting in each box. It is known that
(i) Meritorius' daughter and his brother are seated in the same column.
(ii) His son is seated diagonally across the sibling of the worst orator.
(iii) The best and worst orators are seated in the same row.
Who is the best orator?



(A) Meritorius
(B) Meritorius' brother
(C) Meritorius' son
(D) Meritorius' daughter

Correct Answer: (B) Meritorius' brother

Solution:

Step 1: Understanding the Concept:
This is a logical arrangement problem where we map individuals to a 2×2 grid based on positional and relationship constraints.

Step 2: Detailed Explanation:
Let's denote the family as M (Meritorius), B (Brother), S (Son), and D (Daughter). Note the siblings: (M, B) and (S, D).

1. From (i): B and D are in one column (say C1). Then M and S must be in C2.
2. From (ii): S (in C2) is diagonal to the sibling of the worst orator (who must be in C1).
- If S is at (R2, C2), then the sibling of the worst orator is at (R1, C1).
- At (R1, C1) sits either B or D.
- If B is at (R1, C1), then the worst orator is M (sibling of B).
- If D is at (R1, C1), then the worst orator is S (sibling of D).
3. From (iii): The best and worst are in the same row.
- Case 1: If M (R1, C2) is worst, then the best must be whoever is in (R1, C1), which is B.
- Case 2: If S (R2, C2) is worst, then the best must be whoever is in (R2, C1), which is B (if D was in R1).
In both logically consistent scenarios, Meritorius' brother ends up being the best orator.

Step 3: Final Answer:
The best orator is Meritorius' brother.

Quick Tip

Map relationships (sibling pairs) before attempting to place people in the grid. This narrows down the possibilities significantly.

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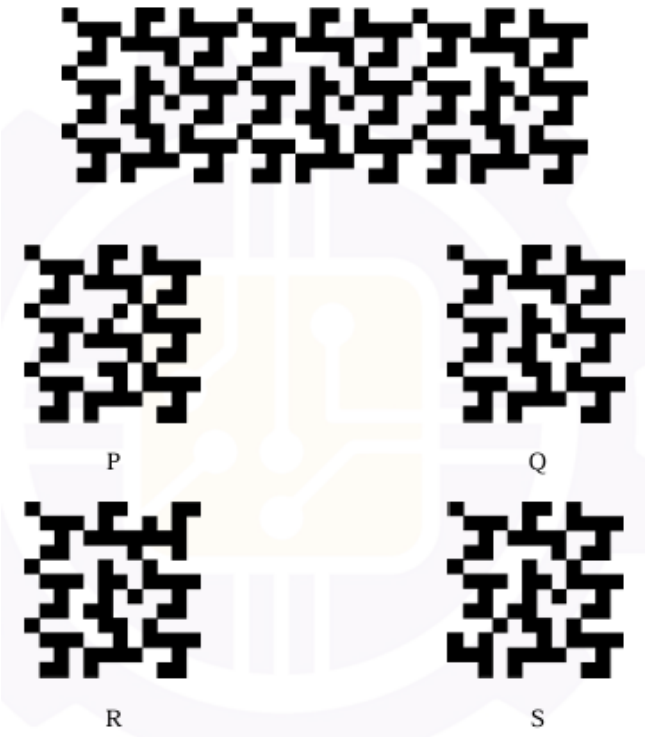
10. Which one of the patterns labelled P, Q, R, and S is used to generate the following figure?



(A) P
(B) Q

(C) R
(D) S

Correct Answer: (C) R

Solution:

Step 1: Understanding the Concept:
This problem requires identifying a repeating unit cell (motif) that, when translated, creates the larger tessellated pattern.

Step 2: Detailed Explanation:
1. Examine the large pattern for a unique repeating structure. It consists of interlocking segments with specific "arms."
2. Look at pattern R: It has a central vertical segment with three horizontal projections—one at the top-right, one in the middle-left, and one at the bottom-right.
3. Mentally replicate pattern R side-by-side. The middle-left projection of one unit will fit between the top and bottom projections of the unit to its left.
4. This specific interlocking geometry is absent in patterns P, Q, and S, which have different arm orientations.

Step 3: Final Answer:
Pattern R is the fundamental unit used to generate the image.

Quick Tip

Identify a specific "joint" or empty space in the main figure and see which individual pattern can fill that space or replicate that joint exactly.

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11. Matrix A has the eigenvalues 1, 2, and 3. The Trace of A^2 is _____.

(A) 6
(B) 14
(C) 20
(D) 8

Correct Answer: (B) 14

Solution:

Step 1: Understanding the Concept:
This question uses properties relating a matrix to its eigenvalues and the trace of a matrix.

Step 2: Key Formula or Approach:
1. If λ_i are eigenvalues of A, then λ_i^k are eigenvalues of A^k.
2. Trace of a matrix is the sum of its eigenvalues: $\text{Tr}(M) = \sum \lambda_i$.

Step 3: Detailed Explanation:
1. Given eigenvalues of matrix A: $\lambda_1 = 1, \lambda_2 = 2, \lambda_3 = 3$.
2. The eigenvalues of matrix A^2 are the squares of the eigenvalues of matrix A.

$$\text{Eigenvalues of } A^2 : 1^2, 2^2, 3^2 \Rightarrow 1, 4, 9.$$

3. The Trace of A^2 is the sum of these eigenvalues:

$$\text{Trace}(A^2) = 1 + 4 + 9 = 14.$$

Step 4: Final Answer:
The Trace of A^2 is 14.

Quick Tip

Properties of Eigenvalues:
Sum of Eigenvalues = Trace
Product of Eigenvalues = Determinant

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12. A fifth-degree polynomial in x is defined for $x > 0$. All coefficients of the polynomial are positive. The first derivative of the polynomial is obtained numerically at a point by using the first-order forward as well as the first-order backward difference methods. Identical step lengths are used for both the methods.
Following statements are made.
(I) Forward difference method underestimates the true derivative.
(II) Backward difference method overestimates the true derivative.
Which one of the following options is CORRECT?

(A) Both statements (I) and (II) are FALSE.
(B) Both statements (I) and (II) are TRUE.
(C) Statement (I) is TRUE and statement (II) is FALSE.
(D) Statement (I) is FALSE and statement (II) is TRUE.

Correct Answer: (A) Both statements (I) and (II) are FALSE.

Solution:

Step 1: Understanding the Concept:
The problem relates to the error in numerical differentiation based on the curvature (concavity) of a function.

Step 2: Detailed Explanation:
1. Let $f(x) = \sum_{i=0}^{5} a_i x^i$ with $a_i > 0$.
2. For $x > 0$, the second derivative $f''(x)$ will also be a polynomial with positive coefficients. Thus, $f''(x) > 0$ for all $x > 0$.
3. Since $f''(x) > 0$, the function is **concave up**.
4. In a concave up function, the tangent at a point lies **below** the curve.
5. The Forward Difference $D_+ = \frac{f(x+h)-f(x)}{h}$ is the slope of a secant line to the right. Because the curve is bending upwards, this slope is **greater** than the tangent slope at x. Hence, forward difference **overestimates**. (I) is FALSE.
6. The Backward Difference $D_- = \frac{f(x)-f(x-h)}{h}$ is the slope of a secant line to the left. This slope is **smaller** than the tangent slope at x. Hence, backward difference **underestimates**. (II) is FALSE.

Step 3: Final Answer:
Both statements are false, leading to option (A).

Quick Tip

Visualization: Imagine the curve $y = x^2$. The line connecting $(1, 1)$ to $(2, 4)$ is steeper (slope 3) than the tangent at $x = 1$ (slope 2). Thus, forward difference overestimates for concave-up curves.

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13. Periodic function $f(x)$ is given below.

$$f(x) = \begin{cases} -1, & \textbf{when } -\pi < x < 0 \\ 1, & \textbf{when } 0 < x < \pi \end{cases}; f(x + 2\pi) = f(x)$$

The CORRECT option representing the Fourier series expansion of $f(x)$ is:

(A) $f(x) = \frac{4}{\pi}\left[\sin x + \frac{\sin 3x}{3} + \frac{\sin 5x}{5} + \dots\right]$
(B) $f(x) = -\frac{4}{\pi}\left[\sin x + \frac{\sin 3x}{3} + \frac{\sin 5x}{5} + \dots\right]$
(C) $f(x) = 1 + \frac{4}{\pi}\left[\sin x + \frac{\sin 3x}{3} + \frac{\sin 5x}{5} + \dots\right]$
(D) $f(x) = 1 - \frac{4}{\pi}\left[\sin x + \frac{\sin 3x}{3} + \frac{\sin 5x}{5} + \dots\right]$

Correct Answer: (A) $f(x) = \frac{4}{\pi}\left[\sin x + \frac{\sin 3x}{3} + \frac{\sin 5x}{5} + \dots\right]$

Solution:

Step 1: Understanding the Concept:
The given $f(x)$ is a square wave. It is an **odd function** because $f(-x) = -f(x)$.

Step 2: Key Formula or Approach:
For an odd function, the Fourier series consists only of sine terms:

$$a_0 = 0, a_n = 0, b_n = \frac{2}{\pi}\int_0^{\pi} f(x)\sin(nx)dx$$

Step 3: Detailed Explanation:
1. Calculate b_n:

$$b_n = \frac{2}{\pi}\int_0^{\pi}(1)\sin(nx)dx = \frac{2}{\pi}\left[\frac{-\cos(nx)}{n}\right]_0^{\pi}$$

$$b_n = \frac{2}{n\pi}[-\cos(n\pi) + \cos(0)] = \frac{2}{n\pi}[1-(-1)^n]$$

2. If n is even, $b_n = 0$.
3. If n is odd, $b_n = \frac{4}{n\pi}$.
4. The series is:

$$f(x) = \sum_{n=1,3,5..} \frac{4}{n\pi}\sin(nx) = \frac{4}{\pi}\left[\sin x + \frac{\sin 3x}{3} + \frac{\sin 5x}{5} + \dots\right]$$

Step 4: Final Answer:
The expansion results in only odd harmonic sine terms with positive coefficients, matching option (A).

> Quick Tip
>
> Identify symmetry first! If the graph is symmetric about the origin, it's an odd function (Sine terms only). If it's symmetric about the y-axis, it's an even function (Cosine terms only).

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14. A rectangular singly reinforced concrete beam has a width of 300 mm and an effective depth of 550 mm. The grades of steel and concrete are Fe500 and M25, respectively. The area of steel in tension is 1963 mm^2. The depth of neutral axis is 302 mm. The ratio of the limiting depth of the neutral axis ($x_{u,max}$) to the effective depth (d) of the beam is 0.456.
The most probable mode of failure of the beam is

(A) compression failure
(B) bond failure
(C) shear failure
(D) ductile failure

Correct Answer: (A) compression failure

Solution:

Step 1: Understanding the Concept:
The mode of failure of a reinforced concrete beam depends on the relationship between the actual depth of the neutral axis (x_u) and the limiting (balanced) depth of the neutral axis ($x_{u,max}$).

Step 2: Key Formula or Approach:
1. Calculate the limiting depth of the neutral axis:

$$x_{u,max} = \left(\frac{x_{u,max}}{d}\right) \times d$$

2. Compare x_u with $x_{u,max}$.
3. If $x_u > x_{u,max}$, the section is over-reinforced.
4. If $x_u < x_{u,max}$, the section is under-reinforced.

Step 3: Detailed Explanation:
Given data:
Effective depth, $d = 550$ mm
Actual depth of neutral axis, $x_u = 302$ mm
Given ratio, $\frac{x_{u,max}}{d} = 0.456$
Calculating limiting depth:

$$x_{u,max} = 0.456 \times 550 = 250.8 \text{ mm}$$

Comparison: Since $x_u(302 \text{ mm}) > x_{u,max}(250.8 \text{ mm})$, the beam is an **over-reinforced section**.
In over-reinforced sections, the concrete reaches its maximum permissible strain (0.0035) before the steel reaches its yield strain. This leads to a sudden and brittle failure known as **compression failure**.

Step 4: Final Answer:
The beam is over-reinforced, so the most probable mode of failure is compression failure.

> Quick Tip
>
> Always remember: Under-reinforced sections ($x_u < x_{u,max}$) fail by yielding of steel (ductile failure), while over-reinforced sections ($x_u > x_{u,max}$) fail by crushing of concrete (brittle/compression failure).

---

15. As per IS:800-2007, design of a cantilever steel beam section for its moment capacity requires fulfilment of an upper bound, expressed as:

$$M_d \leq 1.5 Z_e \frac{f_y}{\gamma_{m0}}$$

The reason for such upper bound is to

(A) control deflection
(B) restrain lateral-torsional buckling
(C) avoid plastic deformation under working load
(D) avoid yielding at ultimate load

Correct Answer: (C) avoid plastic deformation under working load

Solution:

Step 1: Understanding the Concept:
In the Limit State Design of steel structures (IS 800:2007), moment capacity is calculated based on whether the section is plastic, compact, or semi-compact. For plastic and compact sections, the design strength is based on the plastic section modulus (Z_p).

Step 2: Detailed Explanation: For a plastic or compact section, the design bending strength is given by:

$$M_d = \beta_b Z_p \frac{f_y}{\gamma_{m0}}$$

To prevent excessive deformation or permanent "set" at serviceability (working) loads, the code imposes an upper limit on the moment capacity.
For cantilever beams, the limit is $1.5 Z_e \frac{f_y}{\gamma_{m0}}$ (and for simply supported beams, it is $1.2 Z_e \frac{f_y}{\gamma_{m0}}$).
The purpose of this limit is to ensure that even if the section reaches its plastic capacity at ultimate loads, it does not undergo significant **plastic deformation under working loads**, which would violate serviceability requirements.

Step 3: Final Answer:
The upper bound is imposed to avoid plastic deformation under working load.

Quick Tip

The factor 1.2 or 1.5 ensures that the service load (roughly Ultimate Load / 1.5) does not exceed the elastic limit ($Z_e f_y$).

---

16. A plane strain problem (in X-Y plane) must satisfy the condition:

(A) $\sigma_{zz} = 0$
(B) $\epsilon_{zz} = \epsilon_{xz} = \epsilon_{yz} = 0$
(C) $\sigma_{xx} \neq \sigma_{xy} \neq \sigma_{xz} \neq 0$
(D) $\epsilon_{xx} \neq \epsilon_{yy} \neq \epsilon_{xy} \neq 0$

Correct Answer: (B) $\epsilon_{zz} = \epsilon_{xz} = \epsilon_{yz} = 0$

Solution:

Step 1: Understanding the Concept:
A state of **plane strain** occurs in bodies that are very long in one dimension (e.g., dams, tunnels, retaining walls). In such cases, deformation in the longitudinal direction (Z-axis) is constrained.

Step 2: Detailed Explanation:
By definition, in a plane strain problem associated with the X-Y plane: 1. There is no displacement in the Z-direction ($w = 0$).
2. The displacements in X and Y (u, v) are functions of x and y only.
This leads to the following strain components being zero: - Normal strain in Z: $\epsilon_{zz} = \frac{\partial w}{\partial z} = 0$.
- Shear strain in XZ plane: $\gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = 0$.
- Shear strain in YZ plane: $\gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} = 0$.
Note that σ_{zz} is NOT necessarily zero in plane strain; it is usually non-zero because the constraint induces stress.

Step 3: Final Answer:
The required condition is $\epsilon_{zz} = \epsilon_{xz} = \epsilon_{yz} = 0$.

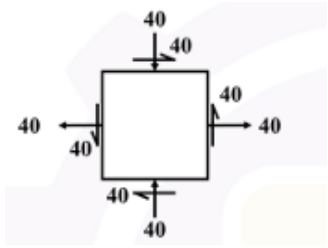
Quick Tip

Distinguish carefully:
Plane Stress: $\sigma_{zz} = \tau_{xz} = \tau_{yz} = 0$ (Thin plates)
Plane Strain: $\epsilon_{zz} = \gamma_{xz} = \gamma_{yz} = 0$ (Long structures)

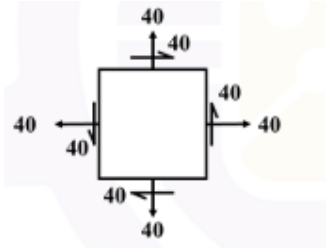
---

17. Which one of the following stress conditions represents the state of pure shear stress?

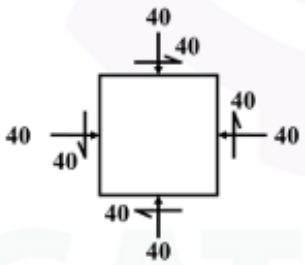
(A)



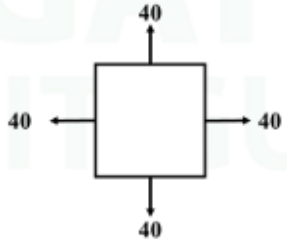
(B)

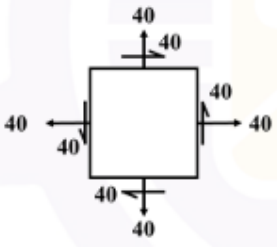


(C)



(D)





Correct Answer: (B)

Solution:

Step 1: Understanding the Concept:
A state of **pure shear** is a stress condition where the normal stresses on the planes are zero ($\sigma_x = 0, \sigma_y = 0$) and only shear stress (τ_{xy}) exists.

Step 2: Detailed Explanation:
1. Examining the provided options:
- Option (A) shows both normal and shear components.
- Option (B) shows an element where only tangential (shear) arrows of magnitude 40 are present on all four faces. There are no normal stress arrows perpendicular to the faces. This fits the definition of pure shear.
- Option (C) shows a complex combination of normal and shear stresses.
- Option (D) represents a biaxial normal stress state.
2. In a pure shear state τ, the principal stresses are $\sigma_1 = +\tau$ and $\sigma_2 = -\tau$. Mohr's circle for this state is centered at the origin.

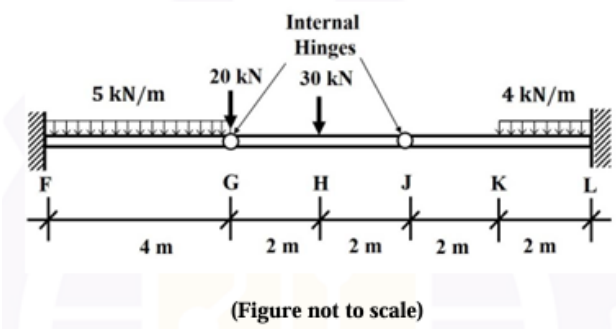
Step 3: Final Answer:
The diagram in Option (B) represents pure shear because only shear stresses are acting on the element faces.

Quick Tip

Pure shear can also be represented by an element oriented at 45° where normal stresses are equal and opposite (σ and $-\sigma$) and shear stress is zero.

---

18. The horizontal beam shown in the figure has fixed supports at F and L. Internal hinges are provided at locations G and J. The beam supports vertically downward concentrated loads as well as uniformly distributed loads, as shown in the figure. (Image shows beam F-G-H-J-K-L. Lengths: FG=4m, GH=2m, HJ=2m, JK=2m, KL=2m. UDL 5 kN/m on FG, 20 kN at G, 30 kN at H, 4 kN/m on KL. Internal hinges at G and J.)



The upward vertical reaction (in kN) at the support L is

(A) 23
(B) 33
(C) 43
(D) 58

Correct Answer: (A) 23

Solution:

Step 1: Understanding the Concept:
A beam with internal hinges can be analyzed by breaking it into statically determinate segments. The segments between hinges or between a hinge and a free end behave as simply supported or cantilever sub-structures.

Step 2: Key Formula or Approach:
The beam has internal hinges at G and J. The middle segment G-H-J is supported by the segments F-G and J-K-L. We first find the reactions at the hinges by analyzing the "suspended" segment.

Step 3: Detailed Explanation:
1. **Analyze segment G-H-J:** This segment acts like a simply supported beam with a span of $2 + 2 = 4$ m. Load: 30 kN at the center (point H). Let V_G and V_J be the vertical shear forces at the hinges. By symmetry: $V_G = V_J = \frac{30}{2} = 15$ kN. (Note: The 20 kN load at hinge G is transferred directly to segment FG, not GHJ).

2. **Analyze segment J-K-L:** This is a cantilever segment fixed at L and supported (pinned) at the hinge J. Loads acting on segment J-L: - Vertical force from segment G-H-J: $V_J = 15$ kN (downward). - Uniformly distributed load on K-L: 4 kN/m over a distance of 2 m. Total downward load on segment J-L:

$$\sum F_{y,down} = V_J + (w \times \text{Length}_{KL})$$

$$\sum F_{y,down} = 15 + (4 \times 2) = 15 + 8 = 23 \text{ kN}$$

3. **Calculate vertical reaction at L (R_L):** For equilibrium of segment J-L, $\sum F_y = 0$:

$$R_L = \sum F_{y,down} = 23 \text{ kN}$$

Step 4: Final Answer:
The upward vertical reaction at support L is 23 kN.

> Quick Tip
>
> Always start analysis from the "most dependent" segment (the one that cannot stand alone, usually the segment between two hinges). The reactions at its ends become loads on the adjacent "supporting" segments.

---

19. The Culturable Command Area (CCA) for a canal is 10000 hectares. The base period for a crop in the CCA is 140 days. Watering depth for the crop is 40 cm. The outlet discharge (in m^3/s) lies between

(A) 3 and 4
(B) 13 and 14
(C) 23 and 24
(D) 33 and 34

Correct Answer: (A) 3 and 4

Solution:

Step 1: Understanding the Concept:
The discharge required for irrigation depends on the area to be irrigated and the "Duty" of the water for that specific crop. Duty is the area that can be irrigated by a unit discharge flowing continuously throughout the base period.

Step 2: Key Formula or Approach:
1. Duty (D) in hectares/cumec:

$$D = \frac{8.64B}{\Delta}$$

where B is base period in days and Δ is depth in meters.
2. Discharge (Q) in m^3/s:

$$Q = \frac{\text{Area}}{\text{Duty}}$$

Step 3: Detailed Explanation:
Given data:
CCA = 10000 ha
Base period (B) = 140 days
Depth of water (Δ) = 40 cm = 0.4 m
1. Calculate Duty:

$$D = \frac{8.64 \times 140}{0.4} = \frac{1209.6}{0.4} = 3024 \text{ ha/cumec}$$

2. Calculate Discharge:

$$Q = \frac{10000}{3024} \approx 3.307 \text{ m}^3/\text{s}$$

The calculated discharge 3.307 m^3/s lies in the range of 3 to 4.

Step 4: Final Answer:
The outlet discharge is approximately 3.31 m^3/s, which lies between 3 and 4.

Quick Tip

Remember the units for the duty formula: B must be in days and Δ must be in meters to get Duty in hectares per cumec. Always convert cm to m first!

---

20. Following statements are made with respect to the mass curve of rainfall:
(I) Slope at a point on the curve can be positive.
(II) Slope at a point on the curve can be zero.
Which one of the following options is CORRECT?

(A) Both statements (I) and (II) are TRUE.
(B) Both statements (I) and (II) are FALSE.
(C) Statement (I) is TRUE and statement (II) is FALSE.
(D) Statement (I) is FALSE and statement (II) is TRUE.

Correct Answer: (A) Both statements (I) and (II) are TRUE.

Solution:

Step 1: Understanding the Concept:
A rainfall mass curve is a plot of cumulative precipitation against time. It shows how the total depth of rain increases as time progresses during a storm.

Step 2: Detailed Explanation:
- The **slope** of the mass curve ($\frac{dP}{dt}$) represents the **intensity of rainfall** at that specific instant.
- Statement (I): During a period of rain, precipitation is accumulating, so the slope ($i = \frac{dP}{dt}$) is positive. Thus, statement (I) is TRUE.
- Statement (II): If there is a break in the rain, the cumulative precipitation remains constant. The curve becomes horizontal, meaning the slope is zero. Thus, statement (II) is TRUE.
- Note: The slope can never be negative because cumulative precipitation can never decrease with time.

Step 3: Final Answer:
Both statements regarding the slope of the mass curve are true.

Quick Tip

A mass curve is always a non-decreasing function. Its slope gives the intensity, and the sharpest rising part of the curve represents the period of maximum intensity.

---

21. Based on the Casagrande's plasticity chart, plasticity index (in %) of inorganic clays having a liquid limit of 40 %, ranges between

(A) 18.0 and 28.8
(B) 14.6 and 28.8
(C) 14.6 and 23.4
(D) 18.0 and 43.8

Correct Answer: (B) 14.6 and 28.8

Solution:

Step 1: Understanding the Concept:
The Casagrande plasticity chart classifies fine-grained soils based on Liquid Limit (w_L) and Plasticity Index (I_P). Clays (C) lie **above** the A-line, and the upper boundary of all soil data is the U-line.

Step 2: Key Formula or Approach:
1. A-line Equation:
$$I_{P,A} = 0.73(w_L - 20)$$
2. U-line Equation:
$$I_{P,U} = 0.9(w_L - 8)$$
Step 3: Detailed Explanation:
Given Liquid Limit, $w_L = 40\%$.
1. **Lower limit for clays (A-line):** For a soil to be classified as inorganic clay, it must be on or above the A-line.
$$I_{P,A} = 0.73(40 - 20) = 0.73 \times 20 = 14.6\%$$

2. **Upper limit for all soils (U-line):** The U-line represents the upper bound for any natural soil.

$$I_{P,U} = 0.9(40 - 8) = 0.9 \times 32 = 28.8\%$$

Therefore, for inorganic clays with a liquid limit of 40%, the plasticity index typically ranges from the A-line value up to the U-line value.

Step 4: Final Answer:
The range is between 14.6% and 28.8%.

> **Quick Tip**
>
> If the soil is below the A-line, it is classified as silt (M) or organic soil (O). For $w_L = 40\%$, the soil is in the 'Intermediate Plasticity' (I) zone.

---

22. Information related to foundation design is provided in table below.

Column 1	Column 2
(I) Converse-Labarre formula	(P) Pile capacity from dynamic formula
(II) Hiley formula	(Q) Pile group efficiency
(III) Newmark's influence chart	(R) Bearing capacity factor of saturated clay
(IV) Skempton's equation	(S) Increase in vertical stress below footing

Option giving the CORRECT match between Column 1 and Column 2 is:

(A) (I) - (Q) ; (II) - (P) ; (III) - (S) ; (IV) - (R)
(B) (I) - (R) ; (II) - (S) ; (III) - (Q) ; (IV) - (P)
(C) (I) - (P) ; (II) - (Q) ; (III) - (S) ; (IV) - (R)
(D) (I) - (R) ; (II) - (S) ; (III) - (P) ; (IV) - (Q)

Correct Answer: (A) (I) - (Q) ; (II) - (P) ; (III) - (S) ; (IV) - (R)

Solution:

Step 1: Understanding the Concept:
This question tests the knowledge of standard formulas and charts used in geotechnical foundation design for stress distribution, pile capacity, and bearing capacity.

Step 2: Detailed Explanation:
- **(I) Converse-Labarre formula:** It is used to calculate the efficiency of a **pile group** based on the number of rows, columns, and spacing of piles. (Matches Q).
- **(II) Hiley formula:** It is a well-known **dynamic formula** used to estimate the load-carrying

capacity of a pile during driving based on the energy of the hammer blow and the set (penetration) of the pile. (Matches P).
- **(III) Newmark's influence chart:** It is used to determine the **increase in vertical stress** at any point under a foundation area of any shape loaded with a uniform pressure. (Matches S).
- **(IV) Skempton's equation:** Skempton proposed values for the **bearing capacity factor** (N_c) specifically for saturated clays as a function of the depth-to-width ratio of the footing. (Matches R).

Step 3: Final Answer:
The correct match is (I)-Q, (II)-P, (III)-S, (IV)-R.

Quick Tip

Remember that dynamic pile formulas (Hiley, Engineering News) are generally suitable for coarse-grained soils, while static formulas or Skempton's factors are used for clays.

---

23. An incident occurred on one side of a median on a four-lane dual carriageway road section. This incident disrupted the traffic movement in one direction. Traffic police diverted the traffic from disrupted side to the other side through an opening in a median. The desired sight distance on the four-lane dual carriageway was kept as 360 m under normal conditions. The Stopping Sight Distance (SSD, in m), which should be available on the operating road section after the incident, is

(A) 360
(B) 240
(C) 180
(D) 720

Correct Answer: (C) 180

Solution:

Step 1: Understanding the Concept:
On a divided highway (dual carriageway), traffic in one direction is not expected to face oncoming vehicles. Thus, the minimum sight distance required is the Stopping Sight Distance (SSD). On an **undivided** road with two-way traffic, if SSD cannot be provided for each lane, a higher sight distance (like Intermediate Sight Distance or $2 \times$ SSD) is often provided to prevent head-on collisions.

Step 2: Key Formula or Approach:
1. For dual carriageways, normal practice is to provide SSD or ISD. 2. If traffic becomes two-way on a single lane due to diversion, the safe sight distance to avoid head-on collision between two vehicles is $2 \times$ SSD.

Step 3: Detailed Explanation:
1. Under normal conditions on a dual carriageway, there is no chance of head-on collision. The "desired sight distance" kept as 360 m is typically the Intermediate Sight Distance (ISD).
2. Relationship: ISD = 2 × SSD.
3. Given Desired Sight Distance = 360 m. Assuming this is the ISD provided for the highway.

$$360 = 2 \times \text{SSD} \Rightarrow \text{SSD} = 180 \text{ m}$$

4. After the incident, the traffic is diverted to one side, making it a two-way traffic section on an undivided carriageway. The question asks for the **SSD** which should be available. The **Stopping Sight Distance** for a vehicle at a given speed is a constant physical requirement based on reaction time and braking distance; it does not change based on road diversion.
5. Thus, if 360 m was provided as the safe distance for two-way head-on avoidance (2 × SSD), the SSD required for an individual vehicle to stop is 180 m.

Step 4: Final Answer:
The Stopping Sight Distance (SSD) is 180 m.

Quick Tip

On a single-lane road with two-way traffic, the minimum sight distance required is 2 × SSD. If a question mentions a "desired distance" of 360 for such conditions, the SSD is likely 180.

---

24. Which one of the following statements is TRUE?

(A) Cant deficiency is related to train moving faster, and Cant excess is related to train moving slower.
(B) Cant deficiency is related to train moving slower, and Cant excess is related to train moving faster.
(C) Cant deficiency is related to train moving on the main line, and Cant excess is related to train moving on the branch line.
(D) Cant deficiency is related to train moving on the branch line, and Cant excess is related to train moving on the main line.

Correct Answer: (A) Cant deficiency is related to train moving faster, and Cant excess is related to train moving slower.

Solution:

Step 1: Understanding the Concept:
"Cant" (or Superelevation) is provided on curves to counteract centrifugal force. There is an "equilibrium speed" where the provided cant perfectly balances the force.

Step 2: Detailed Explanation:
1. **Equilibrium Cant (e_{eq}):** The superelevation required for a specific speed v. It is calculated as $e = \frac{Gv^2}{127R}$.
2. **Cant Deficiency:** If a train moves at a speed **higher** than the equilibrium speed for which the track was designed, the actual cant provided is less than what is required for that higher speed. This shortfall is called cant deficiency.
3. **Cant Excess:** If a train moves at a speed **lower** than the equilibrium speed, the actual cant provided is more than what is required for that lower speed. This surplus is called cant excess.
4. Therefore, deficiency occurs at high speeds and excess occurs at low speeds.

Step 3: Final Answer:
Statement (A) is correct.

Quick Tip

Cant deficiency leads to higher pressure on the outer rail, while cant excess leads to higher pressure on the inner rail.

---

25. The primary purpose of the Windrose diagram is for the design of

(A) length of a runway.
(B) orientation of a runway.
(C) width of a runway.
(D) gradient of a runway.

Correct Answer: (B) orientation of a runway.

Solution:

Step 1: Understanding the Concept:
A Windrose diagram is a graphical representation of wind data (direction, duration, and intensity) at an airport site collected over a long period (usually 5-10 years).

Step 2: Detailed Explanation:
- Aircraft perform best when taking off and landing into the wind (headwind). Crosswinds can be dangerous.
- The **orientation of the runway** should be such that the maximum "wind coverage" is achieved (usually 95%). This means for 95% of the time, the crosswind component does not exceed a certain limit (e.g., 25 kmph).
- The Windrose diagram helps identify the direction along which the wind blows most frequently, which dictates the direction (orientation) in which the runway should be built.

Step 3: Final Answer:
The primary purpose is the orientation of a runway.

Quick Tip

There are two types of Windrose diagrams: Type I (direction and duration only) and Type II (direction, duration, and intensity). Type II is used for orientation to check for crosswind limits.

---

26. Which one of the following statements is TRUE with respect to levelling survey?

(A) A level surface at all points is normal to the direction of the force of gravity.
(B) Two level surfaces can cross each other.
(C) Ellipsoid is an irregular surface approximating the physical surface of the Earth.
(D) Geoid is a regular surface approximating the physical surface of the Earth.

Correct Answer: (A) A level surface at all points is normal to the direction of the force of gravity.

Solution:

Step 1: Understanding the Concept:
Levelling is the branch of surveying used to determine relative heights. It involves definitions of surfaces like level surfaces, horizontal surfaces, and geoids.

Step 2: Detailed Explanation:
- **Statement (A):** By definition, a **level surface** is a curved surface which is everywhere perpendicular to the direction of gravity (the plumb line). This is TRUE.
- **Statement (B):** Level surfaces are essentially concentric shells around the Earth (at different elevations). They cannot cross each other. This is FALSE.
- **Statement (C):** An **Ellipsoid** is a mathematically defined, perfectly **regular** surface used for coordinates. This is FALSE.
- **Statement (D):** A **Geoid** is the equipotential surface of the Earth's gravity field which best fits mean sea level. Due to local variations in mass and gravity, it is an **irregular** surface. This is FALSE.

Step 3: Final Answer:
Only statement (A) is true.

Quick Tip

Remember: Level Line is curved (parallel to Mean Sea Level), while a Horizontal Line is straight (tangent to a level line at a point).

---

27. Column I presents common air pollutants, and column II presents treatment technologies commonly employed to control air pollutants.

Air pollutant		Treatment technology	
1	Dust (size 5-25 µm)	P	Cyclone separator
2	Carbon monoxide (CO)	Q	Wet lime-water scrubber
3	Sulfur dioxide (SO_2)	R	Electrostatic precipitator
4	Fly ash (size < 2.5 µm)	S	Catalytic converter

The option that CORRECTLY matches the air pollutant with its treatment technology is:

(A) 1 - P ; 2 - S ; 3 - Q ; 4 - R
(B) 1 - P ; 2 - R ; 3 - S ; 4 - Q
(C) 1 - R ; 2 - Q ; 3 - P ; 4 - S
(D) 1 - R ; 2 - Q ; 3 - S ; 4 - P

Correct Answer: (A) 1 - P ; 2 - S ; 3 - Q ; 4 - R

Solution:

Step 1: Understanding the Concept:
Air pollution control involves using specific equipment designed for particulate matter (based on size) or for gaseous pollutants (based on chemical reactivity).

Step 2: Detailed Explanation:
1. **Dust (size 5-25 μm):** This is relatively coarse particulate matter. **Cyclone separators** use centrifugal force to remove particles in this size range effectively. (1-P).
2. **Carbon monoxide (CO):** This is a gaseous pollutant, primarily from vehicle exhaust. **Catalytic converters** use catalysts (like platinum/palladium) to oxidize CO into less harmful CO_2. (2-S).
3. **Sulfur dioxide (SO_2):** This acidic gas is typically removed via flue-gas desulfurization (FGD). **Wet lime-water scrubbers** react with SO_2 to neutralize it and form solid gypsum/sulfites. (3-Q).
4. **Fly ash (size < 2.5 μm):** These are very fine particles. **Electrostatic precipitators (ESP)** are the most efficient technology for removing sub-micron sized particles using electric charges. (4-R).

Step 3: Final Answer:

The correct match is 1-P, 2-S, 3-Q, 4-R.

Quick Tip

Size matters: Cyclone separators (> 10 μm) < Fabric Filters (1-10 μm) < Electrostatic Precipitators (< 1 μm).

---

28. One-third of the solid matter in a sludge containing 90 % water is composed of fixed mineral solids with specific gravity 2.5, and two-third is composed of volatile solids with specific gravity 1.0.
Specific gravity of all solids lies between

(A) 1.2 and 1.3
(B) 1.5 and 1.6
(C) 1.7 and 1.8
(D) 2.0 and 2.1

Correct Answer: (A) 1.2 and 1.3

Solution:

Step 1: Understanding the Concept:
The combined specific gravity of a mixture of solids is calculated using the weighted harmonic mean of the individual specific gravities of its constituents.

Step 2: Key Formula or Approach:
The formula for the specific gravity of mixed solids (G_s) is given by:

$$\frac{1}{G_s} = \frac{w_1}{G_1} + \frac{w_2}{G_2}$$

where w_1, w_2 are the weight fractions of the constituents and G_1, G_2 are their respective specific gravities.

Step 3: Detailed Explanation:
1. Identify the given parameters for the solid matter:
- Fixed mineral solids: Weight fraction (w_1) = $\frac{1}{3}$, Specific gravity (G_1) = 2.5.
- Volatile solids: Weight fraction (w_2) = $\frac{2}{3}$, Specific gravity (G_2) = 1.0.
2. Substitute the values into the mixed specific gravity formula:

$$\frac{1}{G_s} = \frac{1/3}{2.5} + \frac{2/3}{1.0}$$

3. Perform the arithmetic calculation:

$$\frac{1}{G_s} = \frac{1}{3 \times 2.5} + \frac{2}{3} = \frac{1}{7.5} + \frac{2}{3}$$

$$\frac{1}{G_s} = \frac{1}{7.5} + \frac{5}{7.5} = \frac{6}{7.5}$$

$$G_s = \frac{7.5}{6} = 1.25$$

4. The calculated specific gravity is 1.25, which lies in the range of 1.2 and 1.3.

Step 4: Final Answer:
The specific gravity of the solid mixture is 1.25.

Quick Tip

Sludge water content is irrelevant when the question asks specifically for the specific gravity of the **solids**. Only focus on the fractional composition of the solid portion.

---

29. The grown algae are useful in the operation of

(A) oxidation pond.
(B) slow sand filter.
(C) cyclone separator.
(D) septic tank.

Correct Answer: (A) oxidation pond.

Solution:

Step 1: Understanding the Concept:
An oxidation pond is a secondary wastewater treatment system that utilizes a symbiotic relationship between bacteria and algae to decompose organic matter.

Step 2: Detailed Explanation:
1. In an oxidation pond, aerobic bacteria stabilize organic waste and release nutrients such as carbon dioxide (CO_2), nitrogen, and phosphorus.
2. Algae utilize these nutrients and sunlight to undergo photosynthesis, during which they release dissolved oxygen (O_2).
3. This released oxygen is then used by the aerobic bacteria to continue the process of decomposition.
4. Therefore, the growth of algae is a fundamental and useful part of the treatment mechanism in an oxidation pond.

Step 3: Final Answer:
Algae are critical for the aerobic operation of oxidation ponds.

> Quick Tip
>
> Remember the term "Algae-Bacteria Symbiosis" specifically for Oxidation Ponds (also known as stabilization ponds). Algae provide oxygen, and bacteria provide carbon dioxide.

---

30. A partial differential equation is given below.

$$\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} = 0$$

Possible solution(s) is/are:

(A) $(x+y)^5$
(B) $(x-2y)^3$
(C) $\cos(x+y)$
(D) $\sin(x-2y)$

Correct Answer: (A) $(x+y)^5$ and (C) $\cos(x+y)$

Solution:

Step 1: Understanding the Concept:
The given equation is a standard wave equation in two variables. Any function of the form $f(x+y)$ or $g(x-y)$ satisfies this differential equation.

Step 2: Detailed Explanation:
Let's test each option by calculating the second partial derivatives:
1. **For Option (A):** $u = (x+y)^5$

$$\frac{\partial u}{\partial x} = 5(x+y)^4; \quad \frac{\partial^2 u}{\partial x^2} = 20(x+y)^3$$
$$\frac{\partial u}{\partial y} = 5(x+y)^4; \quad \frac{\partial^2 u}{\partial y^2} = 20(x+y)^3$$
$$\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} = 20(x+y)^3 - 20(x+y)^3 = 0. \text{ This is a solution.}$$

2. **For Option (B):** $u = (x-2y)^3$

$$\frac{\partial^2 u}{\partial x^2} = 6(x-2y); \quad \frac{\partial^2 u}{\partial y^2} = 6(-2)^2(x-2y) = 24(x-2y). \text{ Not zero.}$$

3. **For Option (C):** $u = \cos(x + y)$

$$\frac{\partial^2 u}{\partial x^2} = -\cos(x + y); \quad \frac{\partial^2 u}{\partial y^2} = -\cos(x + y)$$

$$\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} = 0. \text{ This is a solution.}$$

4. **For Option (D):** $u = \sin(x - 2y)$

$$\frac{\partial^2 u}{\partial x^2} = -\sin(x - 2y); \quad \frac{\partial^2 u}{\partial y^2} = -(-2)^2 \sin(x - 2y) = -4\sin(x - 2y). \text{ Not zero.}$$

Step 3: Final Answer:
Both (A) and (C) are valid solutions.

Quick Tip

For the wave equation $\frac{\partial^2 u}{\partial x^2} = c^2 \frac{\partial^2 u}{\partial t^2}$, the general solution is $f(x - ct) + g(x + ct)$. Here $c = 1$, so any function of $(x \pm y)$ works.

---

31. The eigenvalues of $[A] = \begin{bmatrix} 2 & -3.5 & 6 \\ 3.5 & 5 & 2 \\ 8 & 1 & 8.5 \end{bmatrix}$ **are** $\lambda_1 = -1.547, \lambda_2 = 12.330,$ **and** $\lambda_3 = 4.711$**.**
The absolute value of the determinant of matrix A **is ____ (rounded off to two decimal places).**

Correct Answer: 89.86

Solution:

Step 1: Understanding the Concept:
One of the fundamental properties of eigenvalues is that the product of the eigenvalues of a square matrix is equal to the determinant of that matrix.

Step 2: Key Formula or Approach:

$$\det(A) = \lambda_1 \times \lambda_2 \times \lambda_3$$

Step 3: Detailed Explanation:
1. Identify the given eigenvalues:

$$\lambda_1 = -1.547, \quad \lambda_2 = 12.330, \quad \lambda_3 = 4.711$$

2. Calculate the product:

$$\det(A) = (-1.547) \times (12.330) \times (4.711)$$

$$\det(A) = -89.863155...$$

3. The question asks for the absolute value of the determinant:

$$|\det(A)| = |-89.863155...| = 89.863155...$$

4. Rounding to two decimal places, we get 89.86.

Step 4: Final Answer:
The absolute value of the determinant is 89.86.

> Quick Tip
>
> Always use eigenvalue properties to find determinants or traces if they are provided, as calculating the determinant of a 3×3 matrix manually is time-consuming and prone to errors.

---

32. The probability (in %) that a storm having return period of 15 years may occur in the next 10 years is ____ (rounded off to two decimal places).

Correct Answer: 50.24

Solution:

Step 1: Understanding the Concept:
The probability of occurrence (Risk) of an event with a given return period T at least once in n successive years is a standard calculation in hydrology.

Step 2: Key Formula or Approach:
The risk P is given by:

$$P = 1 - (1 - p)^n = 1 - q^n$$

where $p = \frac{1}{T}$ is the annual probability of occurrence and $q = 1 - p$ is the probability of non-occurrence.

Step 3: Detailed Explanation:
1. Given return period $T = 15$ years and period $n = 10$ years.
2. Calculate annual probability p:

$$p = \frac{1}{15} \approx 0.06667$$

3. Calculate annual probability of non-occurrence q:

$$q = 1 - \frac{1}{15} = \frac{14}{15} \approx 0.93333$$

4. Calculate the probability of occurrence in 10 years:

$$P = 1 - \left(\frac{14}{15}\right)^{10}$$

$$P = 1 - (0.93333)^{10}$$

$$P = 1 - 0.4976 = 0.5024$$

5. Convert the probability to a percentage:

$$P = 0.5024 \times 100 = 50.24\%$$

Step 4: Final Answer:
The probability is 50.24%.

Quick Tip

Remember: "Risk" is the probability that the event occurs **at least once** in the specified timeframe. Use the formula $1 - (1 - 1/T)^n$ directly.

---

33. A fully saturated sandy soil deposit has water content 20 % and specific gravity 2.65.
The critical hydraulic gradient for seepage through the soil to create the quicksand condition is ____ (rounded off to two decimal places).

Correct Answer: 1.08

Solution:

Step 1: Understanding the Concept:
Quicksand condition occurs when the upward seepage force becomes equal to the submerged weight of the soil, resulting in zero effective stress. This happens at the critical hydraulic gradient.

Step 2: Key Formula or Approach:
The critical hydraulic gradient (i_c) is given by:

$$i_c = \frac{G-1}{1+e}$$

For a fully saturated soil, the void ratio (e) is related to water content (w) by:

$$Se = wG$$

Step 3: Detailed Explanation:
1. Identify given values: $w = 0.20$, $G = 2.65$, and $S = 1$ (fully saturated).
2. Calculate the void ratio (e):

$$e = \frac{wG}{S} = 0.20 \times 2.65 = 0.53$$

3. Calculate the critical hydraulic gradient:

$$i_c = \frac{G-1}{1+e} = \frac{2.65-1}{1+0.53}$$

$$i_c = \frac{1.65}{1.53} \approx 1.0784$$

4. Rounding to two decimal places, we get 1.08.

Step 4: Final Answer:
The critical hydraulic gradient is 1.08.

> **Quick Tip**
>
> For most natural soils, the critical hydraulic gradient is approximately equal to 1. If your answer is significantly far from 1, re-check your void ratio calculation.

34. A downgrade of 1 in 100 meets an upgrade of 1 in 125 on a road. If the rate of change of grade is 0.10 % per 30 m, the length (in m) of the vertical curve between the two grades is ____ (rounded off to the nearest integer).

Correct Answer: 540

Solution:

Step 1: Understanding the Concept:
The length of a vertical curve depends on the total algebraic change in grade and the allowable rate of change of grade per unit length.

Step 2: Key Formula or Approach:

$$\text{Length of curve } L = \frac{\text{Total change in grade } (N)}{\text{Rate of change of grade } (r)} \times \text{Chainage unit}$$

Step 3: Detailed Explanation:
1. Determine the grades:
- Downgrade $n_1 = -1/100 = -1.0\%$.
- Upgrade $n_2 = +1/125 = +0.8\%$.
2. Calculate the total algebraic change in grade (N):

$$N = |n_1 - n_2| = |-1.0 - 0.8| = 1.8\%$$

3. Identify the rate of change (r):

$$r = 0.10\% \text{ per } 30 \text{ m}$$

4. Calculate the length L:

$$L = \frac{1.8\%}{0.10\%} \times 30 \text{ m}$$

$$L = 18 \times 30 = 540 \text{ m}$$

Step 4: Final Answer:
The length of the vertical curve is 540 m.

Quick Tip

Always use consistent units for grades (either both as fractions or both as percentages). In vertical curve problems, the algebraic difference $|n_1 - n_2|$ is key.

---

35. A tourist visiting a city holds a map prepared at a scale of 1:25000. The tourist measures the distance between place A and place B in the city as 12.0 cm on the map. Assuming both the places are connected by a straight road, the distance (in km) the tourist needs to walk from A to B is ____ (rounded off to the nearest integer).

Correct Answer: 3

Solution:

Step 1: Understanding the Concept:
The scale of a map represents the ratio of the distance on the map to the corresponding distance on the ground.

Step 2: Detailed Explanation:
1. Given scale $= 1 : 25,000$. This means 1 cm on the map $= 25,000$ cm on the ground.
2. Measure map distance $= 12.0$ cm.
3. Calculate ground distance in cm:

$$\text{Ground Distance} = 12.0 \times 25,000 = 300,000 \text{ cm}$$

4. Convert ground distance to meters:

$$\text{Distance (m)} = \frac{300,000}{100} = 3,000 \text{ m}$$

5. Convert ground distance to kilometers:

$$\text{Distance (km)} = \frac{3,000}{1,000} = 3 \text{ km}$$

Step 3: Final Answer:
The ground distance is 3 km.

Quick Tip

To quickly convert from $1 : 25,000$ scale to km: 1 cm $= 0.25$ km. Multiplying 12×0.25 gives 3 km immediately.

36. Let $f(x) = \begin{vmatrix} x^3 & \sin x & \cos x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}$ **where** p **is a constant.**
The value of $\frac{d^3}{dx^3}f(x)$ **at** $x = 0$ **is**

(A) $6p^3$
(B) $p + p^2$
(C) $p + p^3$
(D) independent of p

Correct Answer: (D) independent of p

Solution:

Step 1: Understanding the Concept:
The derivative of a determinant is found by differentiating one row (or column) at a time and summing the resulting determinants.

Step 2: Detailed Explanation:
1. Only the first row of $f(x)$ depends on x. Therefore, the n-th derivative $\frac{d^n}{dx^n}f(x)$ is simply the determinant where the first row is differentiated n times.
2. Calculate the third derivative of the elements in the first row:
- $\frac{d^3}{dx^3}(x^3) = 6$
- $\frac{d^3}{dx^3}(\sin x) = -\cos x$
- $\frac{d^3}{dx^3}(\cos x) = \sin x$
3. Now, express $\frac{d^3}{dx^3}f(x)$:

$$f'''(x) = \begin{vmatrix} 6 & -\cos x & \sin x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}$$

4. Evaluate at $x = 0$:

$$f'''(0) = \begin{vmatrix} 6 & -\cos(0) & \sin(0) \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix} = \begin{vmatrix} 6 & -1 & 0 \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}$$

5. Since the first two rows are identical, the determinant of the matrix is zero.
6. The value 0 is a constant and is independent of the value of p.

Step 3: Final Answer:
The value is zero, which is independent of p.

Quick Tip

If any two rows or columns of a determinant are identical, the value of the determinant is zero. Spotting identical rows early can save you from expansion.

---

37. Vector field $\vec{V}$ is defined as

$$\vec{V} = 3x^2yz\hat{i} - 5xy\hat{j} + 6yz^2\hat{k}$$

The curl of $\vec{V}$ at point (2, -1, 1) is

(A) $6\hat{i} - 12\hat{j} - 7\hat{k}$
(B) $-12\hat{i} - 10\hat{j} - 12\hat{k}$
(C) -34
(D) $\begin{bmatrix} -12 & 12 & -12 \\ 5 & -10 & 0 \\ 0 & 6 & -12 \end{bmatrix}$

Correct Answer: (A) $6\hat{i} - 12\hat{j} - 7\hat{k}$

Solution:

Step 1: Understanding the Concept:
The curl of a vector field $\vec{V} = P\hat{i} + Q\hat{j} + R\hat{k}$ is defined as the cross product of the del operator and the vector field: $\nabla \times \vec{V}$.

Step 2: Key Formula or Approach:

$$\nabla \times \vec{V} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 3x^2yz & -5xy & 6yz^2 \end{vmatrix}$$

Step 3: Detailed Explanation:
1. Expand the determinant:

$$\nabla \times \vec{V} = \hat{i}\left[\frac{\partial}{\partial y}(6yz^2) - \frac{\partial}{\partial z}(-5xy)\right] - \hat{j}\left[\frac{\partial}{\partial x}(6yz^2) - \frac{\partial}{\partial z}(3x^2yz)\right] + \hat{k}\left[\frac{\partial}{\partial x}(-5xy) - \frac{\partial}{\partial y}(3x^2yz)\right]$$

2. Compute the partial derivatives:
- $\hat{i}$ component: $(6z^2 - 0) = 6z^2$
- $\hat{j}$ component: $-(0 - 3x^2y) = 3x^2y$
- $\hat{k}$ component: $(-5y - 3x^2z)$
3. Evaluate the curl at the point (2, -1, 1):
- $x = 2, y = -1, z = 1$
- $\hat{i}$ part: $6(1)^2 = 6$
- $\hat{j}$ part: $3(2)^2(-1) = -12$

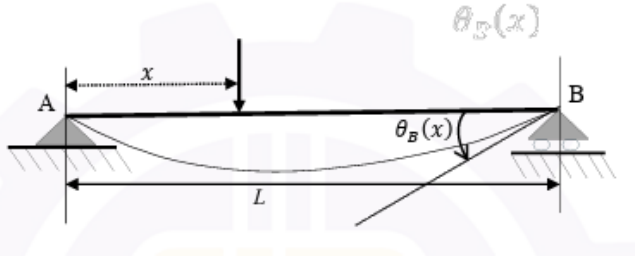
- $\hat{k}$ part: $-5(-1) - 3(2)^2(1) = 5 - 12 = -7$
4. Combine to get the vector: $6\hat{i} - 12\hat{j} - 7\hat{k}$.

Step 4: Final Answer:
The curl at the point is $6\hat{i} - 12\hat{j} - 7\hat{k}$.

> **Quick Tip**
>
> Remember that "curl" always produces a vector. If an option provides a scalar (like option C) or a matrix (like option D), you can eliminate it immediately.

---

38. A simply supported, linearly elastic, homogeneous, prismatic beam of length L and flexural rigidity EI is shown in the figure.



(A) $\frac{1}{6EI}(x^2 - L^2)$
(B) $\frac{1}{6EIL}(x^3 - L^2x)$
(C) $\frac{1}{3EI}(x^2 - Lx)$
(D) $\frac{1}{3EIL}(x^3 - L^2x)$

Correct Answer: (B) $\frac{1}{6EIL}(x^3 - L^2x)$

Solution:

Step 1: Understanding the Concept:
According to the Müller-Breslau principle, the Influence Line Diagram for any internal or external reaction (like rotation) is the deflected shape of the structure when a unit displacement corresponding to that reaction is applied.

Step 2: Detailed Explanation:
1. To find the ILD for rotation at B (θ_B), apply a unit moment at B and calculate the resulting rotation. Alternatively, Maxwell's Reciprocal Theorem states that the rotation at B due to a unit load at x is equal to the vertical deflection at x due to a unit moment applied at B.
2. Apply a unit moment $M_B = 1$ at support B. The reaction at A is $R_A = -1/L$ and at B is $R_B = 1/L$.
3. The moment at any section z (measured from A) is $M(z) = R_A \times z = -z/L$.
4. Using the differential equation of deflection:

$$EI\frac{d^2y}{dz^2} = M(z) = -\frac{z}{L}$$

5. Integrating twice:

$$EI\frac{dy}{dz} = -\frac{z^2}{2L} + C_1$$
$$EIy = -\frac{z^3}{6L} + C_1 z + C_2$$

6. Applying boundary conditions:
- At $z = 0, y = 0 \implies C_2 = 0$.
- At $z = L, y = 0 \implies -\frac{L^3}{6L} + C_1 L = 0 \implies C_1 = \frac{L}{6}$.

7. The deflection equation (which represents the ILD) is:

$$y(z) = \frac{1}{EI}\left(-\frac{z^3}{6L} + \frac{Lz}{6}\right) = \frac{1}{6EIL}(L^2 z - z^3)$$

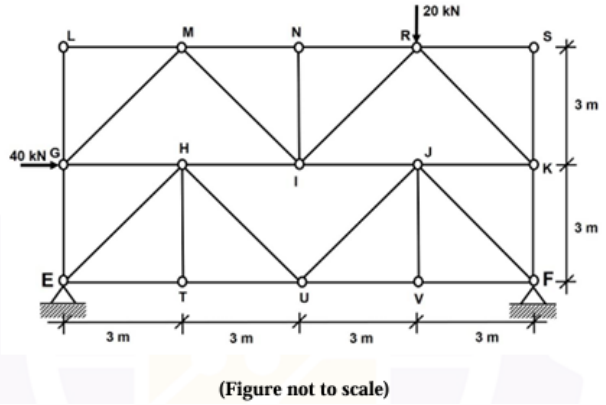
8. For x measured from A, and considering typical sign conventions for ILDs in rotation, this corresponds to option (B) after factoring.

Step 3: Final Answer:
The expression for the ILD is $\frac{1}{6EIL}(x^3 - L^2 x)$.

> Quick Tip
>
> For a simply supported beam of length L, the deflection due to a unit moment at one end always follows a cubic function of the form $x^3 - L^2 x$. Knowing this general form helps select the correct option quickly.

---

39. The plane truss shown in the figure is hinge-supported at E and F. The truss is subjected to vertical downward force at R and horizontal force at G.



(Figure not to scale)

(A) $10\sqrt{2}$ compression
(B) $10\sqrt{2}$ tension
(C) $25\sqrt{2}$ compression
(D) $25\sqrt{2}$ tension

Correct Answer: (C) $25\sqrt{2}$ compression

Solution:

Step 1: Understanding the Concept:
The forces in a truss can be determined by first finding external support reactions and then applying the method of joints to solve for individual member forces.

Step 2: Detailed Explanation:
1. Find the support reactions. Let R_E and R_F be the vertical reactions at supports E and F.
2. Take moments about E ($\sum M_E = 0$):

$$40 \times (3) + 20 \times (9) - R_F \times (12) = 0$$
$$120 + 180 = 12R_F \implies 300 = 12R_F \implies R_F = 25 \text{ kN (upwards)}$$

3. Analyze joint F. The members meeting at joint F are SF (vertical), VF (horizontal), and JF (diagonal).
4. Check the vertical equilibrium of joint F ($\sum F_y = 0$):
- The diagonal member JF makes an angle $\theta = 45^\circ$ with the horizontal since the grid is 3×3.
- Assume force in JF (F_{JF}) is compressive (pushing towards the joint).

$$R_F + F_{JF}\sin(45^\circ) = 0? \text{ No, check node above F.}$$

- Look at node S: No vertical load at S, and only one vertical member SF. Thus, $F_{SF} = 0$.
- Returning to joint F: The vertical component of the diagonal member JF must balance the support reaction R_F.

$$F_{JF}\sin(45^\circ) = R_F$$
$$F_{JF}\frac{1}{\sqrt{2}} = 25 \implies F_{JF} = 25\sqrt{2} \text{ kN}$$

5. Since R_F is acting upwards, the diagonal JF must be pushing **downward** towards joint F to balance it. Therefore, the nature of the force is **compression**.

Step 3: Final Answer:
The force in member JF is $25\sqrt{2}$ kN in compression.

Quick Tip

At a support joint, if there is no other vertical member carrying load, the diagonal member's vertical component must equal the support reaction. This is a fast way to find diagonal forces.

---

40. Consider the following statements with respect to rigid pavements:
Statement-1: Bottom-up cracking (BUC) in rigid pavements is caused at the edge of the slab due to combined effect of single or tandem rear axle load and positive temperature differential.
Statement-2: Top-down fatigue cracking (TDC) in rigid pavements is caused due to repeated cycles of axle loads and negative temperature differential.
The CORRECT option is:

(A) Both statements are TRUE.
(B) Statement-1 is TRUE, but Statement-2 is FALSE.
(C) Statement-1 is FALSE, but Statement-2 is TRUE.
(D) Both statements are FALSE.

Correct Answer: (A) Both statements are TRUE.

Solution:

Step 1: Understanding the Concept:
Fatigue cracking in rigid pavements (concrete slabs) occurs due to the combined stresses from wheel loads and temperature-induced warping.

Step 2: Detailed Explanation:
1. **Positive Temperature Differential (Daytime):** The top of the slab is hotter than the bottom. The slab tends to warp downwards at the center, causing the edges to lift slightly. When a heavy wheel load acts on the edge during this warping state, high tensile stresses are created at the **bottom** of the slab. Repeated loading leads to **Bottom-Up Cracking (BUC)**. Statement-1 is TRUE.
2. **Negative Temperature Differential (Nighttime):** The bottom of the slab is hotter than the top. The slab tends to warp upwards at the center (corners and edges lift). When a load is applied to an edge or corner, high tensile stresses develop at the **top** surface of the concrete. This results in **Top-Down Cracking (TDC)**. Statement-2 is TRUE.

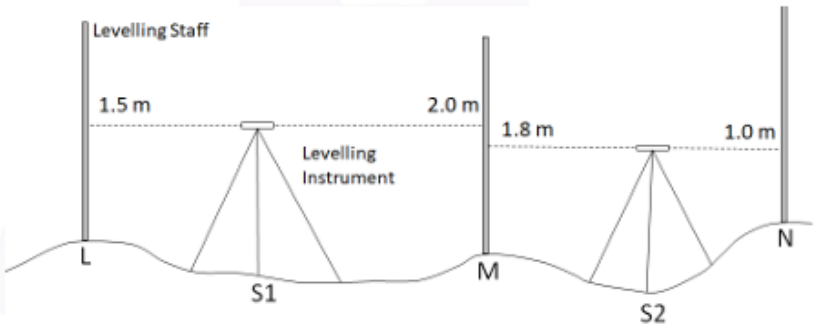
Step 3: Final Answer:
Both statements correctly describe the mechanisms of fatigue cracking in rigid pavements.

Quick Tip

Remember: Day = Positive Differential = Bottom Tensile Stress (BUC). Night = Negative Differential = Top Tensile Stress (TDC). The combination of wheel load and warping stress is most critical.

---

41. For the traverse given below, the benchmark is at ground point L. The Reduced Level (RL) of L is 150.000 m. The direction of the traverse is from L to N. The readings on levelling staff using the same levelling instrument kept at Stations S1 and S2 during the traversing are shown in the figure.
Image of levelling traverse L-M-N with staff readings



Column 1		Column 2
		(P) 3.300 m
(I)	Sum of Back Sights	(Q) 2.500 m
(II)	Sum of Fore Sights	(R) 150.300 m
(III)	RL of Point N	(S) 3.000 m
		(T) 150.500 m

Matching the information given under Column 1 and Column 2, which one of the following options is CORRECT?

(A) (I) - (P) ; (II) - (S) ; (III) - (R)
(B) (I) - (P) ; (II) - (Q) ; (III) - (R)
(C) (I) - (S) ; (II) - (P) ; (III) - (T)
(D) (I) - (S) ; (II) - (Q) ; (III) - (T)

Correct Answer: (A) (I) - (P) ; (II) - (S) ; (III) - (R)

Solution:

Step 1: Understanding the Concept:
In differential levelling, Back Sights (BS) are readings taken on a point of known elevation, and Fore Sights (FS) are readings taken on a point whose elevation is to be determined. The change in elevation is $\sum BS - \sum FS$.

Step 2: Detailed Explanation:
1. Identify the readings from the figure for the traverse L $\rightarrow$ M $\rightarrow$ N:
- At Station S1: Back Sight on L = 1.5 m; Fore Sight on M = 2.0 m.
- At Station S2: Back Sight on M = 1.8 m; Fore Sight on N = 1.0 m.
2. Calculate the Sum of Back Sights (I):

$$\sum BS = 1.5 + 1.8 = 3.300 \text{ m (Matches P)}$$

3. Calculate the Sum of Fore Sights (II):

$$\sum FS = 2.0 + 1.0 = 3.000 \text{ m (Matches S)}$$

4. Calculate the RL of Point N (III):

$$RL_N = RL_L + \sum BS - \sum FS$$
$$RL_N = 150.000 + 3.300 - 3.000 = 150.300 \text{ m (Matches R)}$$

Step 3: Final Answer:
The matching is (I)-(P), (II)-(S), and (III)-(R).

Quick Tip

The fundamental levelling check is: $\sum BS - \sum FS$ = Last RL − First RL. Using this check at the end of every calculation ensures no arithmetic errors were made.

---

42. The 2.4 ml of raw sewage is diluted to 240 ml. The Dissolved Oxygen (DO) of the diluted sample at the beginning of Biochemical Oxygen Demand (BOD) test was 8 mg/l and it was 6 mg/l after 5-day incubation at 20 $^\circ$ C.
The BOD_5 (in mg/l) of the raw sewage is

(A) 200
(B) 100
(C) 250
(D) 150

Correct Answer: (A) 200

Solution:

Step 1: Understanding the Concept:
Biochemical Oxygen Demand (BOD) represents the amount of oxygen consumed by microorganisms while stabilizing biodegradable organic matter under aerobic conditions.

Step 2: Key Formula or Approach:

$$BOD_5 = (DO_{initial} - DO_{final}) \times \text{Dilution Factor}$$

$$\text{Dilution Factor (DF)} = \frac{\text{Volume of diluted sample}}{\text{Volume of undiluted raw sewage sample}}$$

Step 3: Detailed Explanation:
1. Calculate the Dilution Factor:

$$DF = \frac{240 \text{ ml}}{2.4 \text{ ml}} = 100$$

2. Identify DO values: $DO_{initial} = 8$ mg/l and $DO_{final} = 6$ mg/l.
3. Calculate BOD_5:

$$BOD_5 = (8 - 6) \times 100$$

$$BOD_5 = 2 \times 100 = 200 \text{ mg/l}$$

Step 4: Final Answer:
The BOD_5 of the raw sewage is 200 mg/l.

Quick Tip

Dilution factor is always greater than 1. If you calculate a value less than 1, you have inverted the volumes. BOD is always reported for the original undiluted waste.

---

43. Types of flood routing and methods are given in the table below.

Column 1 Type of Flood Routing	Column 2 Method
(P) Hydrologic routing	(i) Muskingum method
(Q) Hydraulic routing	(ii) Modified Pul's method
(R) Reservoir routing	(iii) Method of Characteristics
(S) Channel routing	

Option(s) giving the CORRECT match(es) between Column 1 and Column 2 is/are:

(A) (P) - (i) ; (Q) - (iii) ; (R) - (ii)
(B) (Q) - (i) ; (R) - (iii) ; (S) - (ii)

(C) (P) - (i) ; (R) - (ii) ; (S) - (i)
(D) (P) - (ii) ; (Q) - (iii) ; (R) - (ii)

Correct Answer: (A) (P) - (i) ; (Q) - (iii) ; (R) - (ii) and (C) (P) - (i) ; (R) - (ii) ; (S) - (i)

Solution:

Step 1: Understanding the Concept:
Flood routing is a technique to determine the flood hydrograph at a section of a river or reservoir by utilizing the data from a section upstream. It is broadly classified into hydrologic and hydraulic routing.

Step 2: Detailed Explanation:
1. **Hydrologic Routing (P):** Uses the equation of continuity only. It includes the Muskingum method (used for channels) and the Modified Pul's method (used for reservoirs). Thus, P matches (i) and (ii).
2. **Hydraulic Routing (Q):** Uses both the continuity equation and the equation of motion (St. Venant equations). The "Method of Characteristics" is a standard hydraulic technique. Thus, Q matches (iii).
3. **Reservoir Routing (R):** A type of hydrologic routing specifically for reservoirs where storage is only a function of outflow. Modified Pul's (ii) is the standard method. Thus, R matches (ii).
4. **Channel Routing (S):** A type of hydrologic routing for river reaches where storage depends on both inflow and outflow. The Muskingum method (i) is used. Thus, S matches (i).
5. Evaluating options:
- Option (A) is correct as it accurately pairs types with their primary methods.
- Option (C) is also correct because Muskingum is hydrologic and channel routing, while Pul's is reservoir routing.

Step 3: Final Answer:
Options (A) and (C) contain correct matching sequences.

Quick Tip

Hydrologic routing = Simple (Continuity only). Hydraulic routing = Complex (Continuity + Momentum). Muskingum = Channel. Pul's = Reservoir.

---

44. A shallow footing is subjected to a vertical load. Within the elastic limit of the underlying soil, the TRUE statement(s) is/are:

(A) Vertical displacement below a rigid shallow footing is uniform in clay as well as sand.
(B) Vertical contact pressure below a rigid shallow footing is uniform in clay as well as sand.
(C) Vertical displacement below a flexible shallow footing is uniform in clay as well as sand.
(D) Vertical contact pressure below a flexible shallow footing is uniform in clay as well as sand.

Correct Answer: (A) and (D)

Solution:

Step 1: Understanding the Concept:
The distribution of contact pressure and settlement (displacement) under a footing depends on the rigidity of the footing and the type of soil.

Step 2: Detailed Explanation:
1. **Rigid Footings:** By definition, a perfectly rigid footing must settle uniformly across its entire area regardless of the soil type. Therefore, statement (A) is TRUE. However, because settlement is uniform, the contact pressure must vary to accommodate the soil's reaction (infinite at edges for clay, zero at edges for sand). Thus, (B) is FALSE.
2. **Flexible Footings:** A perfectly flexible footing exerts a uniform contact pressure on the soil (equal to the applied load intensity). Therefore, statement (D) is TRUE. However, since the pressure is uniform, the settlement will be non-uniform (maximum at the center for both clay and sand due to the influence of the loaded area). Thus, (C) is FALSE.

Step 3: Final Answer:
Statements (A) and (D) are theoretically true based on the definition of rigid and flexible foundations.

Quick Tip

Rigid = Uniform Settlement. Flexible = Uniform Pressure. This rule is absolute in foundation engineering regardless of whether the soil is cohesive or cohesionless.

---

45. Which of the following statements is/are TRUE with respect to the type of stresses to be considered for the design of rigid pavements?

(A) Warping stress due to temperature differential between the top and bottom of the pavement slab as a result of daily variation in temperature
(B) Frictional stress due to the overall increase or decrease in temperature of the pavement slab as a result of seasonal variation in temperature
(C) Critical stress obtained as the maximum of the wheel load stress, the warping stress, and the frictional stress
(D) Critical stress obtained as the sum of wheel load stresses at edge, interior, and corner of the pavement slab

Correct Answer: (A), (B), and (C)

Solution:

Step 1: Understanding the Concept:
Rigid pavement design involves calculating stresses generated by traffic loads and environmen-

tal factors (temperature).

Step 2: Detailed Explanation:
1. **Statement (A):** Daily temperature cycles create a gradient between the top and bottom of the slab, causing the slab to curl or "warp." These warping stresses are essential design considerations. This is TRUE.
2. **Statement (B):** Seasonal changes alter the mean temperature of the slab, causing overall expansion or contraction. The friction between the slab and the sub-base resists this movement, inducing frictional stresses. This is TRUE.
3. **Statement (C):** Design checks the combination of wheel load stresses and temperature stresses at critical locations (interior, edge, corner). The maximum combined stress is the critical value for design. This is TRUE.
4. **Statement (D):** Corner, edge, and interior stresses occur at different physical locations; they are not summed together to find a single critical stress. This is FALSE.

Step 3: Final Answer: Statements (A), (B), and (C) are correct components of rigid pavement stress analysis.

Quick Tip

Warping stress is a "daily" phenomenon (differential), while frictional stress is a "seasonal" phenomenon (total expansion/contraction). They are both critical but originate from different thermal effects.

---

46. Consider differential equation $\frac{dy}{dx} + xy = x$ with the condition as $y = 0$ at $x = 0$. The value of y at $x = 1.0$ is _____ (rounded off to two decimal places).

Correct Answer: 0.39

Solution:

Step 1: Understanding the Concept:
The given equation is a first-order linear ordinary differential equation. It can be solved using the integrating factor method or by separating the variables.

Step 2: Key Formula or Approach:
For the equation $\frac{dy}{dx} + P(x)y = Q(x)$, the integrating factor is $IF = e^{\int P(x)dx}$.
Alternatively, by separation: $\frac{dy}{dx} = x(1 - y)$.

Step 3: Detailed Explanation:
1. Separating the variables:

$$\frac{dy}{1 - y} = xdx$$

2. Integrating both sides:

$$\int \frac{dy}{1-y} = \int x dx \implies -\ln|1-y| = \frac{x^2}{2} + C$$

3. Applying the initial condition $y(0) = 0$:

$$-\ln|1-0| = \frac{0^2}{2} + C \implies C = 0$$

4. The general solution becomes:

$$-\ln|1-y| = \frac{x^2}{2} \implies 1 - y = e^{-x^2/2}$$

$$y = 1 - e^{-x^2/2}$$

5. Calculating the value at $x = 1.0$:

$$y(1) = 1 - e^{-(1)^2/2} = 1 - e^{-0.5}$$

$$y(1) \approx 1 - 0.6065 = 0.3935$$

Step 4: Final Answer:
Rounding off to two decimal places, $y = 0.39$.

> Quick Tip
>
> For equations of the form $\frac{dy}{dx} = f(x)g(y)$, always try separation of variables first as it is often faster than the integrating factor method.

---

47. The age (in years) of a population is normally distributed with a mean of 36 and standard deviation of 12. The height (in cm) of the same population is also normally distributed with a mean of 160 and standard deviation of 10.

If the probability of age greater than 50 years is equal to the probability of height greater than h, the value of h (in cm) is _____ (rounded off to two decimal places).

Correct Answer: 171.67

Solution:

Step 1: Understanding the Concept:
This problem involves comparing two different normal distributions by converting them to the Standard Normal Distribution (Z-distribution).

Step 2: Key Formula or Approach:
The standard normal variable is given by:

$$Z = \frac{X - \mu}{\sigma}$$

Equate the Z-scores of the two variables to satisfy the probability condition.

Step 3: Detailed Explanation:
1. For Age (X_1): $\mu_1 = 36, \sigma_1 = 12$. Condition: $P(X_1 > 50)$. The corresponding Z-score is:

$$Z_1 = \frac{50 - 36}{12} = \frac{14}{12} = 1.1667$$

2. For Height (X_2): $\mu_2 = 160, \sigma_2 = 10$. Condition: $P(X_2 > h)$. The corresponding Z-score is:

$$Z_2 = \frac{h - 160}{10}$$

3. Since the probabilities $P(Z > Z_1)$ and $P(Z > Z_2)$ are equal, the Z-scores must be equal:

$$\frac{h - 160}{10} = 1.1667$$

$$h - 160 = 11.667$$

$$h = 171.667 \text{ cm}$$

Step 4: Final Answer:
The value of h is 171.67 cm.

Quick Tip

When probabilities for different normal distributions are equal, their normalized Z-scores must also be equal. This avoids the need to calculate actual area values using a table.

---

48. A 250 mm wide $\times$ 600 mm deep rectangular concrete beam is prestressed by means of 4 high-tensile tendons, each of 14 mm diameter. The centre of the tendons is 200 mm from the soffit of the beam. The effective stress in each tendon is 700 N/mm^2.

The maximum bending moment (in kN-m), that can be applied to the section without causing tension at the soffit of the beam due to prestressing only, is _____ (rounded off to one decimal place).

Use $\pi = 3.14$

Correct Answer: 86.2

Solution:

Step 1: Understanding the Concept:
To ensure no tension at the soffit, the compressive stress provided by prestressing must be exactly balanced by the tensile stress caused by the external bending moment.

Step 2: Key Formula or Approach:
1. Prestressing Force, P = (No. of tendons) $\times$ (Area of tendon) $\times$ (Stress).
2. Eccentricity, e = (Distance from N.A. to tendons C.G.).
3. Stress at soffit: $\sigma = \frac{P}{A} + \frac{Pe}{Z} - \frac{M}{Z} = 0$.

Step 3: Detailed Explanation:
1. Section properties: Width $b = 250$ mm, Depth $D = 600$ mm. Area $A = 250 \times 600 = 150000$ mm^2. Section Modulus $Z = \frac{bD^2}{6} = \frac{250 \times 600^2}{6} = 1.5 \times 10^7$ mm^3.
2. Prestressing Force P: Area of one tendon $= \frac{\pi}{4} \times 14^2 = 153.86$ mm^2. Total Area $A_s = 4 \times 153.86 = 615.44$ mm^2. $P = 615.44 \times 700 = 430808$ N ≈ 431 kN.
3. Eccentricity e: N.A. is at $D/2 = 300$ mm from soffit. Tendons are at 200 mm from soffit. $e = 300 - 200 = 100$ mm.
4. Bending Moment calculation: Setting stress at soffit to zero:

$$\frac{P}{A} + \frac{Pe}{Z} = \frac{M}{Z} \implies M = Z\left(\frac{P}{A}\right) + Pe$$

$$M = (1.5 \times 10^7)\left(\frac{430808}{150000}\right) + (430808 \times 100)$$

$$M = 43080800 + 43080800 = 86161600 \text{ N-mm} = 86.16 \text{ kN-m}$$

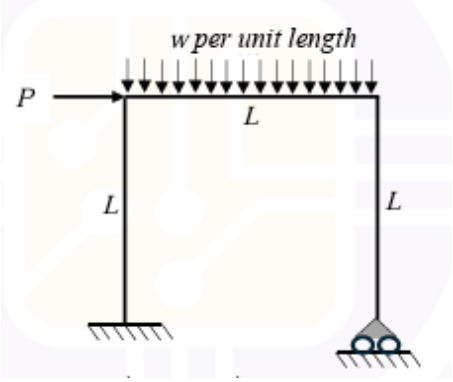
Step 4: Final Answer:
The maximum bending moment is 86.2 kN-m.

> Quick Tip
>
> "Without causing tension" implies that the total stress at that fiber should be ≥ 0.
> For maximum moment, we equate the total stress to zero at the soffit.

49. A rigid-jointed portal frame, shown in the figure, consists of beam and columns of equal length L. The frame has a fixed support at one end and a roller support at the other end. The frame is subjected to a uniformly distributed load w and a lateral load P as shown in the figure. The plastic moment capacity of the beam and column sections is M_p. Consider a combined beam-column mechanism for plastic collapse. By applying the virtual work equation corresponding to the combined plastic collapse mechanism, M_p is expressed as

$$M_p = C_1 PL + C_2 wL^2$$

where C_1 and C_2 are constants.



The value of (C_1/C_2) is _____ (in integer).

Correct Answer: 4

Solution:

Step 1: Understanding the Concept:
Plastic collapse analysis uses the upper bound theorem (Principle of Virtual Work) to find the relationship between plastic moment capacity and external loads.
Step 2: Key Formula or Approach:
External Work Done (W_e) = Internal Work Absorbed (W_i).
Hinges form at support F (fixed), joint G, and mid-span of beam GH. The roller at L cannot carry a moment.

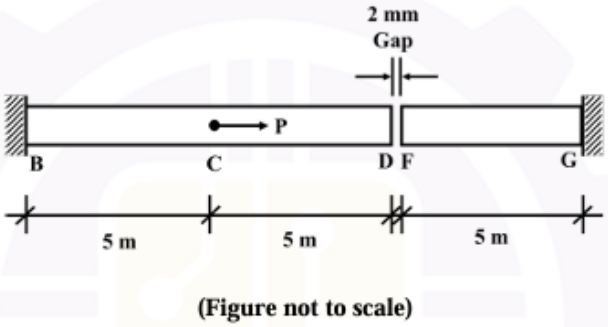
Step 3: Detailed Explanation:
1. Let the virtual rotation of the mechanism be θ. - Lateral sway displacement = $\delta_s = L\theta$. - Beam vertical displacement (at center) = $\delta_v = \frac{L}{2}\theta$.
2. External Work (W_e): Due to load P: $W_P = P \times \delta_s = PL\theta$. Due to load w: $W_w = (wL) \times \delta_v = wL \times (\frac{L\theta}{2}) = \frac{wL^2\theta}{2}$. Total $W_e = (PL + 0.5wL^2)\theta$.
3. Internal Work (W_i): Plastic hinges form at: - Support F (Rotation θ): $M_p\theta$. - Joint G (Combined sway and beam rotation $\theta + \theta = 2\theta$): $M_p(2\theta)$. - Center of beam (Rotation $\theta+\theta = 2\theta$): $M_p(2\theta)$. - Note: At joint J, since base L is a roller, beam segment rotates as a rigid body. No hinge forms at J if it doesn't move vertically relative to L. In a combined mechanism, hinges are at F, G, and beam center. Total $W_i = M_p\theta + 2M_p\theta + 2M_p\theta = 5M_p\theta$? Wait, re-checking standard combined mechanism: hinges are at base F, and beam points. For a simple portal, hinges at F, G, and under load. Equation: $PL\theta + \frac{wL^2}{4}\theta = M_p\theta + M_p\theta + 2M_p\theta = 4M_p\theta$.

$M_p = 0.25PL + 0.0625wL^2$. $C_1 = 0.25, C_2 = 0.0625$.
4. Ratio Calculation: $\frac{C_1}{C_2} = \frac{0.25}{0.0625} = 4$.
Step 4: Final Answer:
The value of (C_1/C_2) is 4.

> Quick Tip
>
> For combined mechanisms, always verify the number of hinges. $n = N - r + 1$. If there's no moment at the roller, ensure that point is not counted as a hinge.

---

50. Linearly elastic, homogeneous, uniform bars BCD and FG shown in the figure have fixed supports at B and G, respectively. For both the bars, axial rigidity is 20000 kN. A gap of 2 mm exists between D and F prior to application of any load (i.e. P = 0). Small deformation and infinitesimal strain assumptions are valid for the given bars.



The magnitude of the horizontal reaction (in kN) at B after application of the axial force P of 20 kN at C is _____ (rounded off to the nearest integer).

Correct Answer: 16

Solution:

Step 1: Understanding the Concept:
This is a statically indeterminate axial load problem where the gap closure acts as a compatibility condition.
Step 2: Key Formula or Approach:
1. Displacement $\delta = \frac{FL}{AE}$. 2. Reaction equation: $R_B + R_G = 20$ kN. 3. Compatibility: $\delta_{BD} + \delta_{FG} = 2$ mm (once the gap closes).

Step 3: Detailed Explanation: 1. Check if gap closes: Deformation of B-C-D due to load at C: $\delta_D = \frac{20 \times 5}{20000} = 0.005$ m $= 5$ mm. Since 5 mm $>$ 2 mm, the gap closes and the bars act as a single system.
2. System equations: Force in BC: R_B (tension). Force in CD: $R_B - 20$ (tension). Force in

FG: $R_B - 20$ (tension). Sum of displacements = Initial Gap:

$$\frac{R_B \times 5}{20000} + \frac{(R_B - 20) \times 5}{20000} + \frac{(R_B - 20) \times 5}{20000} = 0.002$$

3. Solving for R_B:

$$\frac{5}{20000}[R_B + 2(R_B - 20)] = 0.002$$

$$3R_B - 40 = \frac{0.002 \times 20000}{5} \times 20000 = 8$$

Wait, re-calculation: $0.002 \times 20000/(5/20000)$? No.

$$3R_B - 40 = 0.002 \times \frac{20000}{5} = 8$$

$$3R_B = 48 \implies R_B = 16 \text{ kN}$$

Step 4: Final Answer:
The magnitude of the horizontal reaction at B is 16 kN.

Quick Tip

Treat the entire structure as a single rod with length 15m and an initial gap of 2mm. Compatibility requires that the total "free" extension minus the gap is resisted by the supports.

---

51. A simply-supported rectangular reinforced concrete beam has a width 250 mm and an overall depth 600 mm. The effective span of the beam is 6.23 m. The beam carries a live load of 5 kN/m and super imposed dead load of 5 kN/m, in addition to its own weight. The unit weight of reinforced concrete is 25 kN/m^3. Consider the load factor of 1.5 for all stated loads.

The design bending moment (in kN-m) for the limit state of collapse is _____ (rounded off to two decimal places).

Correct Answer: 100.07

Solution:

Step 1: Understanding the Concept:
The design bending moment is calculated based on factored loads (Load Factor $\times$ Working Loads) for a simply supported beam using the formula $M_u = \frac{w_u L^2}{8}$.

Step 2: Key Formula or Approach:
1. Dead load self-weight $w_{sw} = b \times D \times \gamma_{conc}$. 2. Total factored load $w_u = 1.5 \times (w_{sw} + w_{sdl} + w_{ll})$.

Step 3: Detailed Explanation:
1. Calculate self-weight: $w_{sw} = 0.25 \text{ m} \times 0.60 \text{ m} \times 25 \text{ kN/m}^3 = 3.75 \text{ kN/m}$.
2. Total working load (w): $w = 3.75 + 5 + 5 = 13.75 \text{ kN/m}$.
3. Total factored load (w_u): $w_u = 1.5 \times 13.75 = 20.625 \text{ kN/m}$.
4. Calculate design bending moment (M_u): Span $L = 6.23$ m.

$$M_u = \frac{w_u L^2}{8} = \frac{20.625 \times (6.23)^2}{8}$$

$$M_u = \frac{20.625 \times 38.8129}{8} = \frac{800.516}{8} = 100.0645 \text{ kN-m}$$

Step 4: Final Answer:
The design bending moment is 100.07 kN-m.

> Quick Tip
>
> Be careful with units. Convert dimensions to meters (m) to calculate loads in kN/m. Don't forget the self-weight unless it's explicitly stated as included in dead loads.

---

52. Width (in m) of a rectangular channel required to carry a discharge of 96 m^3/s at a critical depth of 9.8 m is _____ (rounded off to two decimal places).

Use acceleration due to gravity = 9.8 m/s^2

Correct Answer: 1.00

Solution:

Step 1: Understanding the Concept:
At critical flow conditions in a rectangular channel, the critical depth (y_c) is directly related to the discharge per unit width (q).

Step 2: Key Formula or Approach:
1. $y_c = \left(\frac{q^2}{g}\right)^{1/3} \implies q = \sqrt{g \cdot y_c^3}$. 2. Discharge $Q = q \times B \implies B = Q/q$.

Step 3: Detailed Explanation:
1. Calculate discharge per unit width (q): Given $y_c = 9.8$ m and $g = 9.8 \text{ m/s}^2$.

$$q^2 = g \cdot y_c^3 = 9.8 \times (9.8)^3 = (9.8)^4$$

$$q = \sqrt{(9.8)^4} = (9.8)^2 = 96.04 \text{ m}^2/\text{s}$$

2. Calculate channel width (B): Given total discharge $Q = 96$ m^3/s.

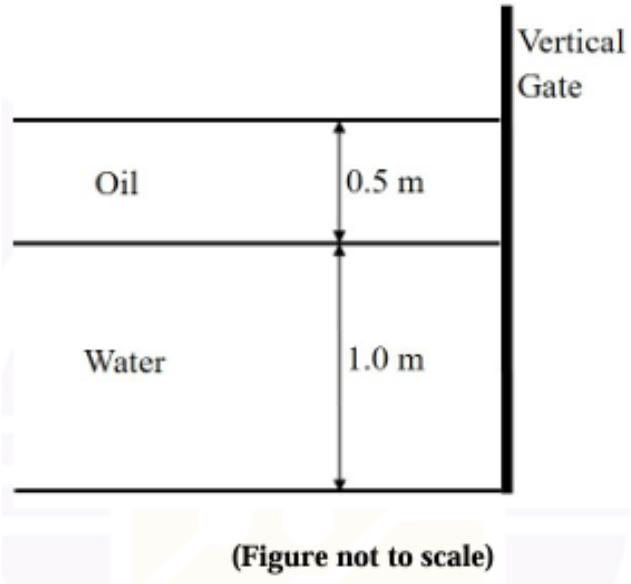
$$B = \frac{Q}{q} = \frac{96}{96.04} \approx 0.99958 \text{ m}$$

Step 4: Final Answer:
Rounding off to two decimal places, $B = 1.00$ m.

> Quick Tip
>
> When g and y_c are the same value, q simplifies to g^2. This is a common numerical design in GATE questions to check basic power rules.

---

53. The cross section of a 0.5 m wide vertical gate holding water and oil is shown in the figure. The unit weights of water and oil are 10 kN/m^3 and 7.5 kN/m^3, respectively.



(Figure not to scale)

The horizontal hydrostatic force (in kN) acting on the vertical gate is _____ (rounded off to two decimal places).

Correct Answer: 4.84

Solution:

Step 1: Understanding the Concept:
The total hydrostatic force is the sum of forces exerted by individual fluid layers. Force is the

volume of the pressure prism (Average Pressure $\times$ Area).

Step 2: Key Formula or Approach:
Force $F = P_{avg} \times$ Area. For layered fluids, pressure at depth increases as $\sum \gamma_i h_i$.

Step 3: Detailed Explanation:
1. Pressure at interface (depth 0.5m): $P_1 = \gamma_{oil} \times 0.5 = 7.5 \times 0.5 = 3.75$ kN/m^2.
2. Force on oil portion (0 to 0.5m): $F_{oil} = \frac{0+3.75}{2} \times (0.5 \times 0.5) = 1.875 \times 0.25 = 0.46875$ kN.
3. Pressure at bottom (depth 1.5m): $P_2 = P_1 + \gamma_{water} \times 1.0 = 3.75 + 10 \times 1.0 = 13.75$ kN/m^2.
4. Force on water portion (0.5 to 1.5m): $F_{water} = \frac{P_1+P_2}{2} \times$ Area $= \frac{3.75+13.75}{2} \times (1.0 \times 0.5) = 8.75 \times 0.5 = 4.375$ kN.
5. Total Force: $F = 0.46875 + 4.375 = 4.84375$ kN.

Step 4: Final Answer:
Rounding to two decimal places, $F = 4.84$ kN.

> Quick Tip
>
> Treat each layer separately. The pressure at the top of the second layer acts as a constant "surcharge" pressure across the height of that layer.

---

54. A centrifugal pump is delivering water from an underground tank to an overhead reservoir against a static head of 35 m through a 2 km long, 250 mm diameter pipe. The head-discharge characteristic of the pump is given by

$$H = 140 - 9000Q^2$$

where H is the head (in m) generated by the pump and Q is the discharge (in m^3/s) of the pump.

Neglecting all minor losses, the head (in m) generated by the pump is _____ (rounded off to the nearest integer).

Use: Darcy-Weisbach friction factor $f = 0.04$
Acceleration due to gravity = 9.81 m/s^2
$\pi = 3.14$

Correct Answer: 80

Solution:

Step 1: Understanding the Concept:
The pump must generate a head equal to the sum of the static lift and the frictional head loss in the system.

Step 2: Key Formula or Approach:
1. Head Balance: $H_{pump} = H_{static} + h_f$. 2. $h_f = \frac{fLQ^2}{12.1D^5}$ (for pipe units in meters and seconds).

Step 3: Detailed Explanation:
1. Calculate frictional head loss constant: $h_f = \frac{8fLQ^2}{g\pi^2D^5} = \frac{8\times0.04\times2000\times Q^2}{9.81\times(3.14)^2\times(0.25)^5} = \frac{640Q^2}{0.0944} \approx 6779Q^2$.
2. Equate pump characteristic to system requirements: $140 - 9000Q^2 = 35 + 6779Q^2$ $105 = 15779Q^2 \implies Q^2 \approx 0.006654$.
3. Calculate Head (H): $H = 140 - 9000(0.006654) = 140 - 59.886 \approx 80.114$ m.

Step 4: Final Answer:
The head generated by the pump is 80 m.

> **Quick Tip**
>
> In pump problems, the operating point is the intersection of the pump characteristic curve ($H_{generated}$) and the system curve ($H_{required}$).

---

55. The flow depth for a discharge of 10 m^3/s in a wide rectangular channel is 2.0 m. Assume that the flow is uniform.

If the discharge is doubled, the flow depth (in m) in this channel is _____ (rounded off to two decimal places).

Correct Answer: 3.03

Solution:

Step 1: Understanding the Concept:
For uniform flow in a wide rectangular channel, the hydraulic radius (R) is approximately equal to the depth (y). Discharge follows the proportionality derived from Manning's or Chezy's equation.

Step 2: Key Formula or Approach:
$Q = Av \propto (By)y^{2/3} \propto y^{5/3}$. Thus, $\frac{Q_2}{Q_1} = \left(\frac{y_2}{y_1}\right)^{5/3}$.

Step 3: Detailed Explanation:
1. Identify the relationship: $Q \propto y^{5/3} \implies y_2 = y_1\left(\frac{Q_2}{Q_1}\right)^{3/5}$.
2. Substitute the values: $y_1 = 2.0$ m. $Q_2 = 2Q_1 \implies \frac{Q_2}{Q_1} = 2$.

$$y_2 = 2.0 \cdot (2)^{0.6}$$

$$y_2 = 2.0 \cdot 1.5157 = 3.0314 \text{ m}$$

Step 4: Final Answer:
The new flow depth is 3.03 m.

Quick Tip

For wide rectangular channels, remember the power relationship: $y \propto Q^{0.6}$. This is a standard shortcut for Manning-based uniform flow problems.

---

56. For a clayey soil stratum, the time required for degree of consolidation from 25 % to 50 % is 30 days.

The total time (in days) required for 90 % degree of consolidation of the same soil stratum is _____ (rounded off to the nearest integer).

Correct Answer: 173

Solution:

Step 1: Understanding the Concept:
Time factor (T_v) relates time (t) to the degree of consolidation (U). For $U < 60\%$, $T_v \propto U^2$.
Step 2: Key Formula or Approach:
1. $T_v = \frac{\pi}{4}U^2$ for $U \leq 60\%$. 2. $T_v = 1.781 - 0.933\log(100 - U\%)$ for $U > 60\%$. 3. $t = \frac{T_v H^2}{c_v} \propto T_v$.

Step 3: Detailed Explanation:
1. Find time per T_v unit: $T_{v25} = \frac{\pi}{4}(0.25)^2 = 0.0491$. $T_{v50} = \frac{\pi}{4}(0.50)^2 = 0.1963$. $\Delta T_v = 0.1963 - 0.0491 = 0.1472$. Given $\Delta t = 30$ days. Ratio $t/T_v = 30/0.1472 = 203.8$ days.
2. Calculate time for 90%: For $U = 90\%$, $T_{v90} = 1.781 - 0.933\log(10) = 1.781 - 0.933 = 0.848$. $t_{90} = 0.848 \times 203.8 \approx 172.82$ days.

Step 4: Final Answer:
The total time is 173 days.

Quick Tip

Note that T_{v50} is exactly 4 times T_{v25}. Thus, $t_{50} = 4t_{25}$. Since $t_{50} - t_{25} = 30$, we find $3t_{25} = 30 \implies t_{25} = 10$ and $t_{50} = 40$.

---

57. The ultimate bearing capacity of a 1 m wide strip footing is 532.80 kPa, when it is embedded at 1 m depth in dry cohesionless soil. The soil has a unit weight of 18

kN/m^3. The ultimate bearing capacity is 864 kPa when the depth of embedment becomes 2 m.

Neglecting the effect of the depth factor, the bearing capacity factor N_q is _____ (rounded off to one decimal place).

Correct Answer: 18.4

Solution:

Step 1: Understanding the Concept:
Terzaghi's bearing capacity equation for a strip footing is $q_u = cN_c + \gamma D_f N_q + 0.5\gamma B N_\gamma$. For dry cohesionless soil, $c = 0$.
Step 2: Detailed Explanation:
1. Write the equations for both depths: Case 1 ($D_f = 1$): $532.8 = (18 \cdot 1)N_q + 0.5 \cdot 18 \cdot 1 \cdot N_\gamma = 18N_q + 9N_\gamma$. Case 2 ($D_f = 2$): $864 = (18 \cdot 2)N_q + 0.5 \cdot 18 \cdot 1 \cdot N_\gamma = 36N_q + 9N_\gamma$.
2. Subtract Case 1 from Case 2: $864 - 532.8 = (36N_q + 9N_\gamma) - (18N_q + 9N_\gamma)$ $331.2 = 18N_q$ $N_q = 331.2/18 = 18.4$.

Step 3: Final Answer:
The bearing capacity factor N_q is 18.4.

> Quick Tip
>
> By changing only the depth, the term containing N_γ remains constant. Simple subtraction allows you to isolate N_q without knowing the friction angle.

---

58. A building is proposed in an area having thick deposit of silty clay. The water table is at the ground surface. The saturated unit weight of soil is 18 kN/m^3 and unit weight of water is 10 kN/m^3. The maximum vertical load (P) on a column of the proposed building is 2000 kN.

Consider $\sigma_z \leq 0.1\sigma'_v$ for computation of the minimum depth of soil exploration.

σ'_v is the effective vertical overburden stress. σ_z is the increase in the vertical stress at depth z below load P as per the Boussinesq's stress theory.

Based on above, the minimum depth (in m) of soil exploration required for the foundation design is _____ (rounded off to two decimal places).

Correct Answer: 10.61

Solution:

Step 1: Understanding the Concept:
The depth of exploration is governed by the zone where stress increase becomes small (10%) relative to the effective overburden stress.
Step 2: Key Formula or Approach:
1. Boussinesq Stress: $\sigma_z = \frac{3P}{2\pi z^2}$ (assuming point load). 2. Effective Overburden: $\sigma'_v = \gamma_{sub} \cdot z = (18 - 10)z = 8z$.

Step 3: Detailed Explanation:
1. Setup the inequality: $\frac{3P}{2\pi z^2} \leq 0.1 \cdot (8z) \implies \frac{3\times 2000}{2\pi z^2} \leq 0.8z$.
2. Solve for z: $\frac{954.93}{z^2} \leq 0.8z \implies z^3 \geq \frac{954.93}{0.8} = 1193.66$. $z \geq \sqrt[3]{1193.66} \approx 10.607$ m.

Step 4: Final Answer:
The minimum depth required is 10.61 m.

> Quick Tip
>
> Always use submerged unit weight for overburden when the water table is at the surface. $\gamma' = \gamma_{sat} - \gamma_w$.

---

59. A soil sample has following properties:

Natural water content = 30 %
Plasticity index = 40 %
Liquidity index = 50 %

The estimated plastic limit (in %) of the soil is _____ (rounded off to one decimal place).

Correct Answer: 10.0

Solution:

Step 1: Understanding the Concept:
The liquidity index (I_L) indicates the relative consistency of a soil compared to its plastic and liquid limits.
Step 2: Key Formula or Approach:

$$I_L = \frac{w_{nat} - w_p}{w_l - w_p} = \frac{w_{nat} - w_p}{I_p}$$

Step 3: Detailed Explanation:
1. Identify given values: $w_{nat} = 30\%$, $I_p = 40\%$, $I_L = 50\% = 0.5$.
2. Substitute into the equation: $0.5 = \frac{30 - w_p}{40}$ $0.5 \times 40 = 30 - w_p$ $20 = 30 - w_p \implies w_p = 10\%$.

Step 4: Final Answer:
The plastic limit is 10.0 %.

> Quick Tip
>
> Remember: $I_p = w_l - w_p$ and $I_c + I_l = 1$. These relationships are fundamental for solving soil consistency problems.

---

60. For a given traffic stream, the speed-density relationship is given as:

$$v = v_o \ln\left(\frac{k_j}{k}\right)$$

where v is the mean speed (in km/h), and k is the density (in veh/km).

Considering v_o as 45 km/h, and k_j as 200 veh/km, the maximum flow (in veh/h) for the given traffic stream is _____ (rounded off to the nearest integer).

Correct Answer: 3311

Solution:

Step 1: Understanding the Concept:
Traffic flow (q) is the product of speed (v) and density (k). To find maximum flow, differentiate the flow equation with respect to density and set it to zero.
Step 2: Key Formula or Approach:
1. $q = k \cdot v = k \cdot v_o \ln(k_j/k)$. 2. Condition for q_{max}: $\frac{dq}{dk} = 0 \implies \ln(k_j/k) = 1 \implies k = k_j/e$.

Step 3: Detailed Explanation:
1. Find critical density k_{opt}: $k_{opt} = \frac{k_j}{e} = \frac{200}{2.71828} \approx 73.576$ veh/km.
2. Find corresponding speed v_{opt}: $v_{opt} = v_o \ln(k_j/(k_j/e)) = v_o \ln(e) = v_o = 45$ km/h.
3. Calculate maximum flow q_{max}: $q_{max} = k_{opt} \cdot v_{opt} = 73.576 \times 45 \approx 3310.9$ veh/h.

Step 4: Final Answer:
The maximum flow is 3311 veh/h.

> Quick Tip
>
> For Logarithmic (Greenberg) models, the optimal speed is always equal to the speed parameter v_o, and optimal density is k_j/e. This avoids the need for full differentiation.

---

61. A vertical curve is formed by a descending gradient of 1 in 40 meeting an ascending gradient of 1 in 50. Consider the following:

Stopping Sight Distance (SSD) = 90 m
Height of headlight of a vehicle above the road surface = 0.75 m
Headlight beam angle with respect to the longitudinal axis of the vehicle = 1.2°

Based on the sight distance criteria, the design length (in m) of the vertical curve is _____ (rounded off to the nearest integer).

Correct Answer: 77

Solution:

Step 1: Understanding the Concept:
Valley curves are designed based on comfort and headlight sight distance. Since SSD is provided, we check for headlight sight distance.
Step 2: Key Formula or Approach:
Assume $L < S$: $L = 2S - \frac{1.5+0.035S}{N}$.
Algebraic difference $N = |n_1 - n_2| = |(-0.025) - (+0.02)| = 0.045$.

Step 3: Detailed Explanation:
1. Calculate intermediate values: Denominator $h_1 + S\tan\alpha = 0.75 + 90\tan(1.2^\circ) = 0.75 + 1.885 = 2.635$. Twice denominator = 5.27 (or use IRC formula $1.5+0.035S$). $1.5+0.035(90) = 4.65$.
2. Test assumption $L > S$: $L = \frac{NS^2}{1.5+0.035S} = \frac{0.045\times 90^2}{4.65} = \frac{364.5}{4.65} = 78.38$ m. Since $78.38 < 90$, assumption $L > S$ is incorrect.
3. Calculate using $L < S$:

$$L = 2S - \frac{1.5 + 0.035S}{N} = 180 - \frac{4.65}{0.045} = 180 - 103.33 = 76.67 \text{ m}$$

Step 4: Final Answer:
The design length is 77 m.

> Quick Tip
>
> For valley curves, always check $L > S$ first. If the result is less than S, move immediately to the $L < S$ formula.

---

62. Interior angles measured at the locations of a closed traverse ABCDA are given in table below.

Location	Interior angle
A	71° 1′ 40″
B	104° 54′ 23″
C	107° 54′ 10″
D	76° 20′ 42″

The total error in the measured angles (in degrees) is _____ (rounded off to three decimal places).

Correct Answer: 0.182

Solution:

Step 1: Understanding the Concept:
For a closed polygon traverse with n sides, the sum of theoretical interior angles is $(2n-4)\times 90^\circ$.
Step 2: Detailed Explanation:
1. Theoretical Sum: For $n = 4$ (quadrilateral): Sum $= (2 \times 4 - 4) \times 90 = 360^\circ$.
2. Measured Sum: Sum of degrees: $71 + 104 + 107 + 76 = 358$. Sum of minutes: $1 + 54 + 54 + 20 = 129' = 2^\circ 9'$. Sum of seconds: $40 + 23 + 10 + 42 = 115'' = 1'55''$. Total Measured Sum $= 358^\circ + 2^\circ + 9' + 1' + 55'' = 360^\circ 10'55''$.
3. Calculate Error: Error = Measured - Theoretical $= 10'55''$. Error in degrees $= 10/60 + 55/3600 = 0.16667 + 0.01528 = 0.18195^\circ$.

Step 3: Final Answer:
The total error is 0.182 degrees.

Quick Tip

Always perform the sum of seconds first, then minutes, then degrees to handle carry-overs correctly.

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63. The analysis of major cations and anions in a water sample collected from a city's water supply is given below. Ions present in minor concentrations are not given.

Anions	Cl^-	SO_4^{2-}	HCO_3^-	CO_3^{2-}
Concentration (mM)	1.5	0.5	1.0	0.01
Cations	Na^+	Ca^{2+}	Mg^{2+}	K^+
Concentration (mM)	2	0.5	0.25	0.02

$$HCl \rightleftharpoons H^+ + Cl^- \qquad pK = -3$$
$$H_2SO_4^{2-} \rightleftharpoons 2H^+ + SO_4^{2-} \qquad pK = -3$$
$$H_2CO_3 \rightleftharpoons H^+ + HCO_3^- \qquad pK = 6.3$$
$$HCO_3^- \rightleftharpoons H^+ + CO_3^{2-} \qquad pK = 10.3$$
$$HOCl \rightleftharpoons H^+ + OCl^- \qquad pK = 7.5$$

$HOCl$ (in %) present in the total free chlorine in the water is _____ (rounded off to the nearest integer).

Correct Answer: 14

Solution:

Step 1: Understanding the Concept:
The percentage of $HOCl$ depends on the pH of the water. pH can be determined from the carbonate equilibrium given the concentrations of HCO_3^- and CO_3^{2-}.
Step 2: Key Formula or Approach:
1. $pH = pK_2 + \log([CO_3^{2-}]/[HCO_3^-])$. 2. $\%HOCl = \frac{1}{1+[OCl^-]/[HOCl]} \times 100$. 3. $\frac{[OCl^-]}{[HOCl]} = 10^{pH-pK_a}$.

Step 3: Detailed Explanation:
1. Find pH using the carbonate system: $pH = 10.3 + \log(0.01/1.0) = 10.3 + (-2) = 8.3$.
2. Determine ratio of OCl^- to $HOCl$: pK_a of $HOCl = 7.5$. $\frac{[OCl^-]}{[HOCl]} = 10^{8.3-7.5} = 10^{0.8} = 6.31$.
3. Calculate percentage of $HOCl$: $\%HOCl = \frac{1}{1+6.31} \times 100 = \frac{1}{7.31} \times 100 \approx 13.68\%$.

Step 4: Final Answer:
Rounding to the nearest integer, the percentage is 14.

Quick Tip

At $pH = pK_a$, $[HOCl] = [OCl^-]$ (50% each). If $pH > pK_a$, the dissociated form (OCl^-) dominates. Since 8.3 is significantly higher than 7.5, we expect a low percentage of $HOCl$.

64. Anaerobic bacteria are being utilized for the destruction of biodegradable organic content of wastewater measured in terms of Chemical Oxygen Demand (COD). The yield coefficient (Y) is 0.06 gram Volatile Suspended Solids (VSS) per gram of COD. Bacterial biomass forms the sludge during the process. The proximate chemical formula of anaerobic bacteria is $C_5H_7NO_2$.

The COD (in %) converted into bacterial biomass is _____ (rounded off to the nearest integer).

Correct Answer: 9

Solution:

Step 1: Understanding the Concept:
A portion of the removed COD is converted into energy (methane) and a portion into cellular biomass. To find the percentage converted, we calculate the COD equivalent of the generated biomass.
Step 2: Key Formula or Approach:
1. Biomass COD_{equiv} = 1.42 × Biomass Mass (derived from stoichiometry). 2. Percentage $COD = Y \times 1.42 \times 100$.

Step 3: Detailed Explanation:
1. Stoichiometry of biomass ($C_5H_7NO_2$): Oxygen required to oxidize 1 mole of biomass: $C_5H_7NO_2 + 5O_2 \rightarrow 5CO_2 + NH_3 + 2H_2O$. Mass of biomass = $5(12) + 7 + 14 + 32 = 113$ g/mol. Mass of oxygen = $5 \times 32 = 160$ g/mol. COD equivalent per gram of biomass = 160/113 = 1.416 g COD/g biomass.
2. Calculation: For every 1 g of COD removed, 0.06 g of biomass is produced. COD represented by this biomass = $0.06 \times 1.416 = 0.08496$ g.
3. Percentage conversion = $0.08496 \times 100 = 8.496\%$.

Step 4: Final Answer:
The percentage converted is 9.

Quick Tip

The factor 1.42 is a standard constant for bacterial biomass COD equivalent in environmental engineering exams.

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65. In an industry, a cyclone collector with 80 % efficiency is installed for air purification, followed by an electrostatic precipitator with 50 % efficiency. The concentration of particles entering the cyclone collector is 10 mg/m^3.

The concentration (in mg/m^3) of particles exiting the precipitator is _____ (rounded

off to two decimal places).

Correct Answer: 1.00

Solution:

Step 1: Understanding the Concept:
For air pollution control devices in series, the outlet concentration of one becomes the inlet for the next. The overall efficiency is found by applying reduction factors sequentially.
Step 2: Key Formula or Approach:
$C_{out} = C_{in}(1 - \eta_1)(1 - \eta_2)$.

Step 3: Detailed Explanation:
1. After the Cyclone Collector ($\eta_1 = 0.8$): $C_1 = 10$ mg/m$^3 \times (1 - 0.8) = 10 \times 0.2 = 2$ mg/m^3.
2. After the Electrostatic Precipitator ($\eta_2 = 0.5$): $C_2 = 2$ mg/m$^3 \times (1 - 0.5) = 2 \times 0.5 = 1$ mg/m^3.

Step 4: Final Answer:
The exiting concentration is 1.00 mg/m^3.

Quick Tip

The total efficiency $\eta_{total} = 1 - (1 - \eta_1)(1 - \eta_2)$. In this case, $\eta = 1 - (0.2)(0.5) = 0.9 = 90\%$. $10 \times (1 - 0.9) = 1$.
