GATE 2025 Civil Engineering Shift 1 Question Paper with Solutions

Time Allowed :3 Hours	**Maximum Marks :**100	**Total Questions :**65

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The GATE exam will be structured with a total of 100 marks.
2. The exam mode is Online CBT (Computer Based Test).
3. The total duration of the exam is 3 Hours.
4. It will include 65 questions, divided into 2 sections:
   - Section 1: General Aptitude
   - Section 2: Technical (Civil Engineering)
5. The marking scheme includes 1-mark and 2-mark questions. Each correct answer will carry marks as specified in the question paper. Incorrect answers may carry negative marks, as indicated in the question paper.
6. **Question Types:** The exam will include a mix of:
   - Multiple Choice Questions (MCQs)
   - Multiple Select Questions (MSQs)
   - Numerical Answer Type (NAT) questions

GENERAL APTITUDE

1. Is there any good show ..... television tonight?
(A) in
(B) at
(C) within
(D) on

Correct Answer: (D) on

Solution:
The phrase "on television" is the standard expression used when referring to television programs or broadcasts. "On" is used to indicate something that is being broadcast or shown through media such as television. The other prepositions like "in", "at", and "within" are not commonly used in this context.
- "In television" would imply being physically inside a television, which is illogical.

- "At television" is not a standard construction in English; we might say "at the television", but not "at television".
- "Within television" is also an incorrect usage because it suggests being inside a medium, which doesn't fit the context of watching shows.
Therefore, "on" is the correct preposition for this sentence.

> Quick Tip
>
> For media like television and radio, always use "on" to refer to broadcasts (e.g., "on TV", "on the radio").

---

2. As the police officer was found guilty of embezzlement, he was ..... dismissed from the service in accordance with the Service Rules.
(A) sumptuously
(B) brazenly
(C) unintentionally
(D) summarily

Correct Answer: (D) summarily

Solution:
The correct word in this context is "summarily," which means immediate and without a formal trial or prolonged procedure. When someone is dismissed "summarily," it implies that the action was swift and decided quickly, often because of serious misconduct, such as embezzlement in this case.
- "Sumptuously" refers to something done in a luxurious or extravagant way, which is unrelated to the dismissal context.
- "Brazenly" means done in an open, bold, or shameless manner, but it doesn't fit here because we are talking about an official act, not the manner in which it was done.
- "Unintentionally" means something done without intent or planning, which is not applicable to a formal dismissal.
Thus, "summarily" is the appropriate choice here, as it describes a swift and direct action, typically in legal or formal settings.

> Quick Tip
>
> When reading legal or formal texts, "summarily" refers to actions taken without delay or extensive procedure, such as dismissal, verdicts, or judgments.

---

3. The sum of the following infinite series is:

$$\frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \frac{1}{5!} + \dots$$

(A) π

(B) $1 + e$
(C) $e - 1$
(D) e

Correct Answer: (C) $e - 1$

Solution:
This series is known as the series expansion of e^x when $x = 1$. The general form of the series for e^x is:

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

By substituting $x = 1$, we get:

$$e^1 = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \dots$$

Thus, the sum of the series is e. However, the series presented in the question excludes the first term (which is 1), so the sum of the series is $e - 1$.
This series is a well-known example in calculus and represents the exponential function evaluated at 1, minus 1.
To summarize: - The sum of the infinite series $\frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \dots$ is exactly $e - 1$.

Quick Tip

The series for e^x is used extensively in mathematics, especially in calculus. For $x = 1$, the series becomes the sum for $e - 1$, excluding the first term.

---

4. A thin wire is used to construct all the edges of a cube of 1 m side by bending, cutting and soldering the wire. If the wire is 12 m long, what is the minimum number of cuts required to construct the wire frame to form the cube?
(A) 3
(B) 4
(C) 6
(D) 12

Correct Answer: (A) 3

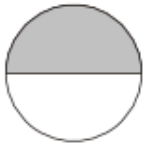
Solution:
A cube has 12 edges, and if the wire is 12 m long, each edge will be 1 m long. To form the cube, we need to connect the edges in a 3-dimensional manner. Since the wire is used for all edges, there are 12 edges that need to be formed, but the number of cuts required depends on how we manage the wire. We will need a minimum of 3 cuts to divide the wire into the 3 necessary pieces.
- After the cuts, we will have 3 segments, each of length 4 m, and these will be bent to form the 12 edges. Hence, the minimum number of cuts required is 3.

Quick Tip

For cube frame problems, think of how many segments you need to form the edges and plan your cuts accordingly.

---

5. The figures, I, II, and III are parts of a sequence. Which one of the following options comes next in the sequence as IV?

(A) 

(B) 

(C) 

(D) 

Correct Answer: (B)

Solution:
The sequence of the figures follows a pattern in which the shaded area gradually increases. In each successive figure, the shaded area increases by a certain proportion, and the next figure should continue this pattern. Looking at the options, option (b) is the correct one, as it follows the trend of increasing shaded areas.
- Notice how the shaded portion changes from figure I to III. In each step, more of the circle is shaded, and the shading pattern follows a predictable progression.

Quick Tip

When dealing with patterns in figures, observe how elements change from one figure to the next. Focus on proportions, color changes, and other repetitive aspects.

Q.6 - Q.10 Carry TWO marks Each

6. "Why do they pull down and do away with crooked streets, I wonder, which are my delight, and hurt no man living? Every day the wealthier nations are pulling down one or another in their capitals and their great towns: they do not know why they do it; neither do I. It ought to be enough, surely, to drive the great broad ways which commerce needs and which are the life-channels of a modern city, without destroying all history and all the humanity in between: the islands of the past."
(From Hilaire Belloc's "The Crooked Streets")
Based only on the information provided in the above passage, which one of the following statements is true?
(A) The author of the passage takes delight in wondering.
(B) The wealthier nations are pulling down the crooked streets in their capitals.
(C) In the past, crooked streets were only built on islands.
(D) Great broad ways are needed to protect commerce and history.

Correct Answer: (B) The wealthier nations are pulling down the crooked streets in their capitals.

Solution:
The passage describes how the author is questioning the need to tear down the crooked streets, which are a part of history and a reflection of past towns. The author expresses his discomfort with the practice of the wealthier nations destroying these areas. Therefore, the correct statement is option (B), which directly reflects the concern raised in the passage about the wealthier nations pulling down these streets.
- The author does express some wonder, but it is more about the destruction of history than a delight in wondering, so (A) is not correct. - The other options (C) and (D) are not directly supported by the passage's content, as the focus is on the destruction of historical streets, not on broader ways protecting commerce.

Quick Tip

When answering questions based on a passage, always focus on the specific details provided in the text and avoid drawing conclusions not directly supported by it.

---

7. Rohit goes to a restaurant for lunch at about 1 PM. When he enters the restaurant, he notices that the hour and minute hands on the wall clock are exactly coinciding. After about an hour, when he leaves the restaurant, he notices that the clock hands are again exactly coinciding. How much time (in minutes) did Rohit spend at the restaurant?
(A) $64\frac{6}{11}$
(B) $66\frac{5}{13}$
(C) $65\frac{5}{11}$

(D) $66\frac{6}{13}$

Correct Answer: (C) $65\frac{5}{11}$

Solution:
The hour and minute hands coincide 11 times in 12 hours. In each cycle, the minute hand moves faster than the hour hand, completing one more full cycle than the hour hand in each period. The time interval for each coincidence can be calculated by the formula:

$$T = \frac{12}{11} \times 60 \quad \text{(minutes)}.$$

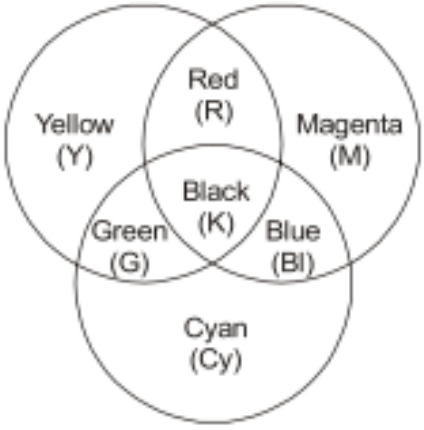
Thus, the time for one coincidence is approximately $65\frac{5}{11}$ minutes.
Since the hands coincide twice in the time Rohit is at the restaurant, the total time spent is $65\frac{5}{11}$ minutes.
- The formula derived considers the relative movement between the two hands and the frequency of their coincidences.

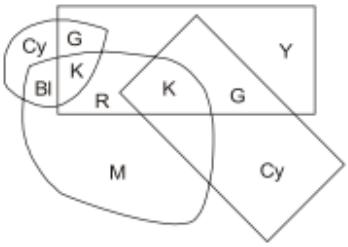
Quick Tip

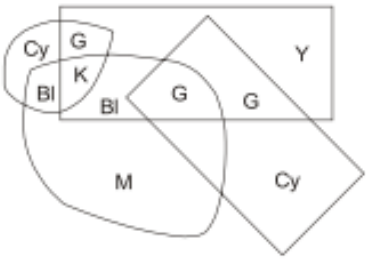
To calculate the time between coinciding hands, use the formula $T = \frac{12}{11} \times 60$.

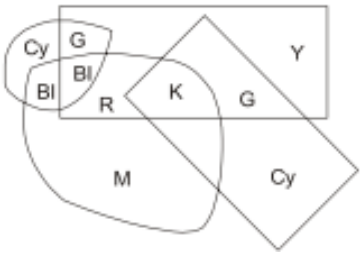
---

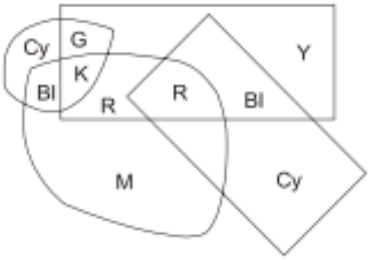
8. A color model is shown in the figure with color codes: Yellow (Y), Magenta (M), Cyan (Cy), Red (R), Blue (Bl), Green (G), and Black (K).
Which one of the following options displays the color codes that are consistent with the color model?

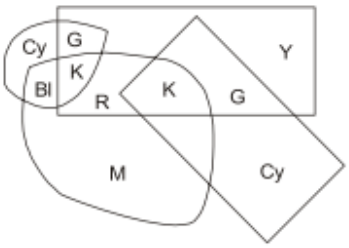


(A)


(B)


(C)


(D)




Correct Answer: (A)

Solution:
In color models like the one shown, specific colors are represented by overlapping regions. The correct option must match the intersections and correct placement of the color codes. Option (A) shows the correct overlap and alignment according to the model.
- Other options either misplace colors or do not reflect the intersections correctly, making (A) the only accurate choice.

> **Quick Tip**
>
> When working with color models, ensure that the regions of overlap and placement of colors are correctly represented.

---

9. A circle with center at $(x, y) = (0.5, 0)$ and radius = 0.5 intersects with another circle with center at $(x, y) = (1, 1)$ and radius = 1 at two points. One of the points of intersection (x, y) is:
(A) $(0, 0)$
(B) $(0.2, 0.4)$
(C) $(0.5, 0.5)$
(D) $(1, 2)$

Correct Answer: (B) $(0.2, 0.4)$

Solution:
We are given two circles with the following equations:

$$(x - 0.5)^2 + y^2 = 0.5^2 \quad \text{(Equation 1: Circle 1)}$$

$$(x - 1)^2 + (y - 1)^2 = 1^2 \quad \text{(Equation 2: Circle 2).}$$

To solve this, we can expand both equations.
- Expanding Equation 1:

$$(x - 0.5)^2 + y^2 = 0.25 + y^2 = 0.25 \quad \Rightarrow \quad x^2 - x + 0.25 + y^2 = 0.25 \quad \Rightarrow \quad x^2 - x + y^2 = 0.$$

- Expanding Equation 2: $(x - 1)^2 + (y - 1)^2 = 1 \quad \Rightarrow \quad (x^2 - 2x + 1) + (y^2 - 2y + 1) = 1 \quad \Rightarrow \quad x^2 - 2x + y^2 - 2y + 2 = 1 \quad \Rightarrow \quad x^2 - 2x + y^2 - 2y = -1.$
Now, subtract Equation 1 from Equation 2:

$$(x^2 - 2x + y^2 - 2y) - (x^2 - x + y^2) = -1 - 0 \quad \Rightarrow \quad -x - 2y = -1 \quad \Rightarrow \quad x + 2y = 1 \quad \cdots (3).$$

Now, substitute $x = 1 - 2y$ from Equation (3) into Equation 1:

$$(1 - 2y)^2 - (1 - 2y) + y^2 = 0.$$

Expanding and solving for y, we get:

$$1 - 4y + 4y^2 - 1 + 2y + y^2 = 0 \quad \Rightarrow \quad 5y^2 - 2y = 0 \quad \Rightarrow \quad y(5y - 2) = 0.$$

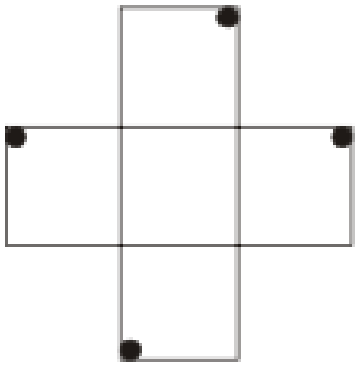
Thus, $y = 0$ or $y = 0.4$. For $y = 0.4$, substitute into $x = 1 - 2y$ to get $x = 0.2$. Thus, the point of intersection is $(0.2, 0.4)$.
- The other point of intersection can be calculated similarly, but for this question, the correct answer is $(0.2, 0.4)$.

Quick Tip

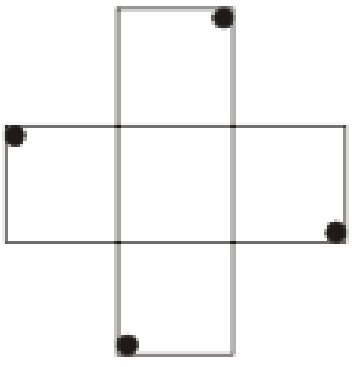
When solving for the intersection of two circles, expand the equations, eliminate terms, and solve the resulting system of linear equations.

---

10. An object is said to have an n-fold rotational symmetry if the object, rotated by an angle of $\frac{2\pi}{n}$, is identical to the original.
Which one of the following objects exhibits 4-fold rotational symmetry about an axis perpendicular to the plane of the screen?

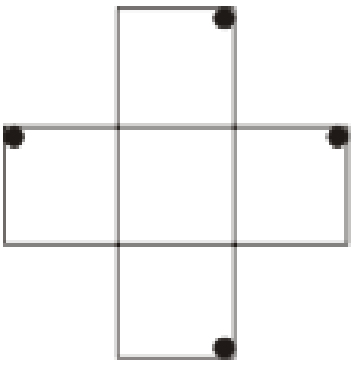


(A)

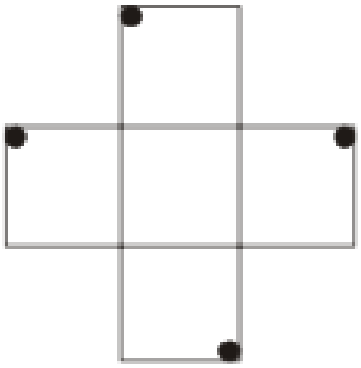


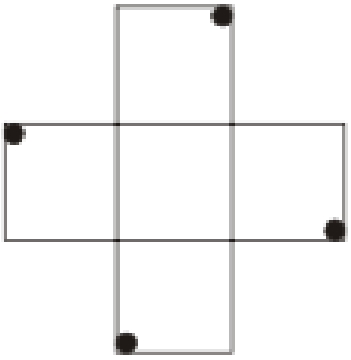
(B)

(C)



(D)





Correct Answer: (B)

Solution:
Rotational symmetry refers to how an object looks after it is rotated by a certain angle about a fixed point or axis. In the case of 4-fold rotational symmetry, the object must appear identical after a 90-degree rotation.
Let's analyze the options:
- Option (A) does not exhibit 4-fold symmetry, as rotating it by 90 degrees results in a different orientation.
- Option (B) exhibits 4-fold symmetry. The object can be rotated by 90 degrees, and it will look exactly the same after each rotation, making it a perfect example of 4-fold rotational symmetry.
- Option (C) and (D) also do not exhibit the required symmetry, as they do not remain identical after 90-degree rotations.
Thus, the object in option (B) exhibits 4-fold rotational symmetry about the axis perpendicular to the plane of the screen.
- The key to identifying rotational symmetry is to rotate the object by the specified angle and

observe if it aligns with the original object at each step of the rotation. If it does, the object has the corresponding rotational symmetry.

> Quick Tip
>
> When checking for rotational symmetry, try rotating the object by the required angle and see if the object matches its original position after each rotation.

TECHNICAL

Q.11 - Q.35 Carry ONE marks Each

11. Suppose λ is an eigenvalue of matrix A and x is the corresponding eigenvector. Let x also be an eigenvector of the matrix $B = A - 2I$, where I is the identity matrix. Then, the eigenvalue of B corresponding to the eigenvector x is equal to:
(A) λ
(B) $\lambda + 2$
(C) 2λ
(D) $\lambda - 2$

Correct Answer: (D) $\lambda - 2$

Solution:
By definition, if λ is an eigenvalue of matrix A and x is the corresponding eigenvector, we have:

$$Ax = \lambda x.$$

Now, for matrix $B = A - 2I$, we have:

$$Bx = (A - 2I)x = Ax - 2Ix = \lambda x - 2x = (\lambda - 2)x.$$

Thus, the eigenvalue corresponding to the eigenvector x for matrix B is $\lambda - 2$. Therefore, the correct answer is option (D).
- The eigenvalue of $B = A - 2I$ is $\lambda - 2$, which is derived from the basic properties of eigenvalues and eigenvectors.

> Quick Tip
>
> When modifying a matrix by a scalar multiple of the identity matrix (such as $A - 2I$), the eigenvalues are adjusted by the scalar.

---

12. Let

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 3 & 1 \\ -2 & -3 & -3 \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}.$$

For $Ax = b$ to be solvable, which one of the following options is the correct condition on b_1, b_2, and b_3?
(A) $b_1 + b_2 + b_3 = 1$
(B) $3b_1 + b_2 + 2b_3 = 0$
(C) $b_1 + 3b_2 + b_3 = 2$
(D) $b_1 + b_2 + b_3 = 2$

Correct Answer: (B) $3b_1 + b_2 + 2b_3 = 0$

Solution:
We are given the system $Ax = b$, where A is a 3x3 matrix and b is a column vector. For the system to be solvable, we need to check the consistency of the system. We can represent the augmented matrix $[A : b]$ and convert it into row echelon form:

$$[A : b] = \begin{bmatrix} 1 & 1 & 1 & b_1 \\ 1 & 3 & 1 & b_2 \\ -2 & -3 & -3 & b_3 \end{bmatrix}.$$

Performing row operations to convert this matrix into echelon form:
1. Subtract the first row from the second row:

$$\begin{bmatrix} 1 & 1 & 1 & b_1 \\ 0 & 2 & 0 & b_2 - b_1 \\ -2 & -3 & -3 & b_3 \end{bmatrix}.$$

2. Add 2 times the first row to the third row:

$$\begin{bmatrix} 1 & 1 & 1 & b_1 \\ 0 & 2 & 0 & b_2 - b_1 \\ 0 & -1 & -1 & b_3 + 2b_1 \end{bmatrix}.$$

3. Add $\frac{1}{2}$ times the second row to the third row:

$$\begin{bmatrix} 1 & 1 & 1 & b_1 \\ 0 & 2 & 0 & b_2 - b_1 \\ 0 & 0 & -1 & b_3 + 2b_1 + \frac{b_2 - b_1}{2} \end{bmatrix}.$$

For this system to be consistent, the third column must correspond to a valid value of b_3, and we need to ensure that the rows do not become inconsistent. After applying this, we find the condition:

$$3b_1 + b_2 + 2b_3 = 0.$$

Thus, the correct condition is $3b_1 + b_2 + 2b_3 = 0$, which matches option (B).
- This system of equations is consistent when the above condition on the values of b_1, b_2, and b_3 is satisfied.

Quick Tip

When solving systems of linear equations, converting the augmented matrix to echelon form helps identify the conditions for consistency.

13. Which one of the following options is the correct Fourier series of the periodic function $f(x)$ described below:

$$f(x) = \begin{cases} 0 & \text{if } -2 < x < -1 \\ 2k & \text{if } -1 \le x < 1 \\ 0 & \text{if } 1 \le x < 2 \end{cases} \quad ; \text{period} = 4$$

(A) $f(x) = \frac{k}{2} + \frac{2k}{\pi}\left(\cos\frac{\pi x}{2} - \cos\frac{3\pi x}{2} + \frac{5}{5}\cos\frac{5\pi x}{2} + \dots\right)$
(B) $f(x) = \frac{k}{2} + \frac{2k}{\pi}\left(\sin\frac{\pi x}{2} - \sin\frac{3\pi x}{2} + \frac{5}{5}\sin\frac{5\pi x}{2} + \dots\right)$
(C) $f(x) = k + \frac{4k}{\pi}\left(\cos\frac{\pi x}{2} - \cos\frac{3\pi x}{2} + \frac{5}{5}\cos\frac{5\pi x}{2} + \dots\right)$
(D) $f(x) = k + \frac{4k}{\pi}\left(\sin\frac{\pi x}{2} - \sin\frac{3\pi x}{2} + \frac{5}{5}\sin\frac{5\pi x}{2} + \dots\right)$

Correct Answer: (C)

Solution:
Given that $f(x)$ is a periodic function with a period of 4, we need to find the Fourier series for this function. Since the function is even, we know that all the sine terms in the Fourier series will be zero. We will only have cosine terms in the Fourier series expansion.
Fourier Coefficients: The Fourier series is given by:

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{L}.$$

The period $L = 4$, so the cosine terms will be of the form $\cos\frac{n\pi x}{2}$.
We first calculate a_0, the average or DC component:

$$a_0 = \frac{1}{L}\int_{-L/2}^{L/2} f(x)\,dx = \frac{1}{4}\int_{-2}^{2} f(x)\,dx.$$

From the given function, we know that:

$$a_0 = 2k \quad \text{(as the function is constant and equal to } 2k \text{ for } -1 \le x < 1).$$

Next, we calculate the a_n coefficients:

$$a_n = \frac{2}{L}\int_{-L/2}^{L/2} f(x)\cos\frac{n\pi x}{L}\,dx.$$

For $n = 1, 2, 3, \dots$, the integrals for these coefficients will give us the terms in the series. After performing the integration (which involves calculating the integrals for cosine terms), we get:

$$a_n = \frac{4k}{n\pi}\sin\left(\frac{n\pi}{2}\right).$$

Thus, the Fourier series for this function is:

$$f(x) = k + \frac{4k}{\pi}\left(\cos\frac{\pi x}{2} - \cos\frac{3\pi x}{2} + \frac{5}{5}\cos\frac{5\pi x}{2} + \dots\right).$$

Therefore, the correct answer is option (C).

- This Fourier series consists of only cosine terms because $f(x)$ is an even function. The sine terms are eliminated due to the symmetry of the function.

> **Quick Tip**
>
> For periodic functions with a given period, use the Fourier series expansion to express the function as a sum of sine and cosine terms. For even functions, only cosine terms will remain.

---

14. X is the random variable that can take any one of the values 0, 1, 7, 11 and 12. The probability mass function for X is

$$P(X = 0) = 0.4, \quad P(X = 1) = 0.3, \quad P(X = 7) = 0.1, \quad P(X = 11) = 0.1, \quad P(X = 12) = 0.1.$$

Then, the variance of X is
(A) 20.81
(B) 28.40
(C) 31.70
(D) 10.89

Correct Answer: (A) 20.81

Solution:
The variance of a random variable X is given by:

$$\text{Var}(X) = E(X^2) - [E(X)]^2.$$

Where: - $E(X)$ is the expected value (mean) of X, - $E(X^2)$ is the expected value of X^2.
We will first calculate $E(X)$ and $E(X^2)$.
Step 1: Calculate $E(X)$ The expected value $E(X)$ is given by:

$$E(X) = \sum_x x \cdot P(x).$$

Substitute the values from the given probability mass function:

$$E(X) = 0 \times 0.4 + 1 \times 0.3 + 7 \times 0.1 + 11 \times 0.1 + 12 \times 0.1 = 0 + 0.3 + 0.7 + 1.1 + 1.2 = 3.3.$$

Step 2: Calculate $E(X^2)$ The expected value of X^2 is given by:

$$E(X^2) = \sum_x x^2 \cdot P(x).$$

Substitute the values from the probability mass function:

$$E(X^2) = 0^2 \times 0.4 + 1^2 \times 0.3 + 7^2 \times 0.1 + 11^2 \times 0.1 + 12^2 \times 0.1.$$

$$E(X^2) = 0 + 0.3 + 49 \times 0.1 + 121 \times 0.1 + 144 \times 0.1 = 0 + 0.3 + 4.9 + 12.1 + 14.4 = 31.7.$$

Step 3: Calculate the variance Now that we have both $E(X)$ and $E(X^2)$, we can calculate the variance:

$$\text{Var}(X) = E(X^2) - [E(X)]^2 = 31.7 - (3.3)^2 = 31.7 - 10.89 = 20.81.$$

Thus, the variance of X is 20.81.
- The correct answer is option (A).

Quick Tip

To calculate the variance, first find the expected value of X, then the expected value of X^2, and subtract the square of the expected value from the expected value of X^2.

15. As per IS 456:2000 provisions for two-way slabs with continuous edges, the longitudinal steel reinforcement to be provided in the edge strip is based on:
(A) the calculated minimum bending moment
(B) the area of longitudinal steel provided in the middle strip in the shorter span
(C) the area of longitudinal steel provided in the middle strip in the longer span
(D) the prescribed minimum cross-sectional area of longitudinal steel for slabs

Correct Answer: (D) the prescribed minimum cross-sectional area of longitudinal steel for slabs

Solution:
According to IS 456:2000, the minimum cross-sectional area of longitudinal steel reinforcement in the edge strip of a two-way slab with continuous edges is prescribed based on the following:
- The edge strips have minimum reinforcement requirements to ensure proper load distribution and avoid excessive cracking.
- The prescribed minimum cross-sectional area of reinforcement for edge strips is typically based on a percentage of the slab's gross area.
The minimum reinforcement area is provided in terms of the specified percentage of the total slab area and is related to the type of steel used. For instance:
- For Fe250 grade steel, the minimum reinforcement is 0.15- For HYSD steel, the minimum reinforcement is 0.12
This ensures that there is enough reinforcement in the edge strips, especially in slabs with continuous edges, which are more susceptible to stresses and cracking due to their boundary conditions.
Thus, the correct answer is option (D) because the longitudinal steel reinforcement in the edge strip is based on the prescribed minimum cross-sectional area, as specified by IS 456:2000.
- Options (A), (B), and (C) do not refer to the prescribed minimum cross-sectional area but rather to other aspects like calculated bending moments or the reinforcement in the middle strip.

Quick Tip

When working with slab reinforcement, always refer to the relevant IS code for minimum reinforcement criteria, especially for edge strips and boundary conditions.

---

16. Identify the FALSE statement from the following options:
(A) The compressive strength of a concrete test specimen can vary depending on its shape and size

(B) Air-dried and saturated test specimens show the same compressive strength for concrete
(C) Curing conditions, such as temperature and relative humidity, can influence the compressive strength of concrete
(D) Compressive strength depends on the water-to-binder ratio used in the concrete mixture

Correct Answer: (B) Air-dried and saturated test specimens show the same compressive strength for concrete

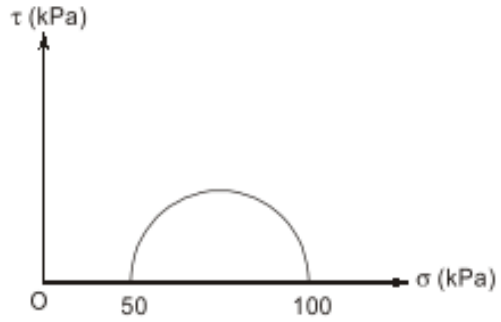
Solution:
The compressive strength of concrete is influenced by various factors such as curing conditions, moisture content, and the water-to-binder ratio. Let's examine each option:
- (A) The compressive strength of concrete can indeed vary based on the shape and size of the test specimen. Larger or differently shaped specimens can show different strengths due to factors like the distribution of stress and the length of the specimen. Thus, this statement is TRUE.
- (B) Air-dried and saturated test specimens do not show the same compressive strength. Saturated specimens have higher strength than air-dried ones because the presence of water in the specimen contributes to the hydration of the cement particles, making the concrete stronger. Thus, this statement is FALSE.
- (C) Curing conditions, including temperature and relative humidity, have a significant impact on the compressive strength of concrete. Proper curing ensures the hydration process continues effectively, improving the strength of the concrete. This statement is TRUE.
- (D) The water-to-binder ratio is one of the most critical factors in determining the compressive strength of concrete. A lower water-to-binder ratio generally results in higher strength, as the hydration process is more efficient with less water. This statement is TRUE.
Thus, the FALSE statement is (B).

Quick Tip

Always remember that curing conditions and moisture content significantly influence the compressive strength of concrete. Proper curing leads to better hydration and, hence, higher strength.

---

17. The results of a consolidated drained triaxial test on a normally consolidated clay are shown in the figure. The angle of internal friction is



(A) $\sin^{-1}\left(\frac{1}{2}\right)$

(B) $\sin^{-1}\left(\frac{1}{3}\right)$
(C) $\sin^{-1}\left(\frac{2}{3}\right)$
(D) $\sin^{-1}\left(\frac{3}{4}\right)$

Correct Answer: (B) $\sin^{-1}\left(\frac{1}{3}\right)$

Solution:
In a consolidated drained (CD) triaxial test, the relationship between the shear stress τ and the effective normal stress σ is given by the Mohr-Coulomb failure criterion:

$$\tau = \sigma \tan\left(45^\circ + \frac{\phi}{2}\right),$$

where ϕ is the angle of internal friction. From the given triaxial test data, we can use the equation:

$$\sigma_1 = \sigma_3 \tan^2\left(45^\circ + \frac{\phi}{2}\right).$$

Given that $\sigma_1 = 50\,\text{kPa}$ and $\sigma_3 = 50\,\text{kPa}$, we can substitute into the equation for the effective stress:

$$100 = 50\left[1 + \sin\left(\frac{\phi}{2}\right)\right].$$

Simplifying:

$$\sin\left(\frac{\phi}{2}\right) = \frac{1}{3}.$$

Thus, the angle of internal friction ϕ is given by:

$$\phi = 2\sin^{-1}\left(\frac{1}{3}\right).$$

Hence, the correct angle of internal friction is $\sin^{-1}\left(\frac{1}{3}\right)$, which corresponds to option (B).
- This equation is derived from the stress and strain relationships in a triaxial test, where we use the Mohr-Coulomb failure criterion to calculate the angle of internal friction.

Quick Tip

In triaxial tests, the angle of internal friction can be found using the Mohr-Coulomb failure criterion, which relates shear stress to normal stress using the internal friction angle.

---

18. The standard plasticity chart for the classification of a fine-grained soil is shown in the figure. As per the Indian standard soil classification system, X represents
(A) inorganic clay with medium plasticity
(B) inorganic silt with medium plasticity
(C) inorganic clay with high plasticity
(D) inorganic silt with high compressibility

Correct Answer: (C) inorganic clay with high plasticity

Solution:
The standard plasticity chart for the classification of fine-grained soils is used to classify soils based on their liquid limit and plasticity index. The position of the point X in the diagram helps in determining the soil type.
- From the figure, X lies in the region that corresponds to inorganic clay with high plasticity. The plasticity index value of X is greater than 35, which is characteristic of high plasticity clay. - So, soil X can be classified as inorganic clay with high plasticity based on the IS classification system.
Thus, the correct answer is option (C).

Quick Tip

The plasticity chart helps classify soils based on their plasticity index and liquid limit. A higher plasticity index indicates higher plasticity in clays.

---

19. For a flowing fluid, a dimensionless combination of velocity (V), length scale (l), and acceleration due to gravity (g) would be
(A) $\frac{V^2}{gl}$
(B) $\frac{Vg}{l}$
(C) $\frac{gl^2}{V}$
(D) $\frac{l}{V^2 g}$

Correct Answer: (A) $\frac{V^2}{gl}$

Solution:
In fluid mechanics, dimensionless groups are often used to analyze the behavior of flowing fluids. The combination $\frac{V^2}{gl}$ is a dimensionless number that describes the relationship between the velocity (V), length scale (l), and the acceleration due to gravity (g).
To check the dimensional correctness:

$$\left[\frac{V^2}{gl}\right] = \frac{[\text{velocity}]^2}{[\text{acceleration}] \times \text{length}} = \frac{\left(\frac{m}{s}\right)^2}{\left(\frac{m}{s^2}\right) \times m} = \frac{m^2/s^2}{m^2/s^2} = 1,$$

which is dimensionless. Therefore, the correct answer is option (A).
- Other options do not form a dimensionless group, which is required for this problem.

Quick Tip

When dealing with dimensionless quantities, always check the dimensional consistency of the equation to ensure that it is dimensionless.

---

20. To derive the total flood hydrograph at a catchment outlet from an isolated storm, the order in which the following methods are applied, from the first method to the last method, is
(A) PRSQ
(B) PQRS
(C) RPSQ
(D) PSQR

Correct Answer: (A) PRSQ

Solution:
To derive the total flood hydrograph at a catchment outlet from an isolated storm, the following sequence of methods is used:
1. P - Obtaining the hyetograph: This is the first step where the rainfall data (hyetograph) is collected.
2. R - Estimation of initial and infiltration losses: This step involves estimating the losses due to initial abstraction and infiltration during the storm.
3. S - Application of unit hydrograph: Using the unit hydrograph method, the runoff is derived from the storm's rainfall.
4. Q - Addition of baseflow: Finally, the baseflow is added to the hydrograph to get the total flood hydrograph.
Thus, the correct order is PRSQ, which corresponds to option (A).
- The sequence ensures that all aspects of the storm's effect on the catchment are considered in the right order, starting with the rainfall data and ending with the addition of the baseflow.

Quick Tip

When deriving a flood hydrograph, always follow the correct sequence of steps to ensure accurate results, starting with rainfall data and ending with the baseflow addition.

---

21. Fecal Coliform (FC) concentration in river water was measured as 10708 cfu/100 ml. The FC concentration after the conventional water treatment, but before chlorination, was measured as 23 cfu/100 ml. The 'Log Kill' (inactivation) of FC due to the conventional water treatment is closest to:
(A) 4.00
(B) 2.50
(C) 2.67
(D) 3.00

Correct Answer: (C) 2.67

Solution:
The 'Log Kill' is defined as the logarithmic reduction in the concentration of microorganisms after a treatment. It is given by the formula:

$$\text{Log Kill} = \log N_0 - \log N_t,$$

where:
- N_0 is the initial concentration (before treatment),
- N_t is the final concentration (after treatment).
Given:
- $N_0 = 10708\,\text{cfu}/100\text{ ml}$,
- $N_t = 23\,\text{cfu}/100\text{ ml}$.
Substituting these values into the formula:

$$\text{Log Kill} = \log 10708 - \log 23 = 4.029 - 1.361 = 2.667.$$

Thus, the Log Kill is approximately 2.67. The correct answer is option (C).

Quick Tip

The Log Kill value is useful in measuring the effectiveness of water treatment processes in eliminating harmful microorganisms. A higher Log Kill value indicates more effective disinfection.

---

22. A hydrocarbon C_nH_m is burnt in air (O_2 + 3.78N_2). The stoichiometric fuel to air mass ratio for this process is
Note: Atomic Weight: C(12), H(1) Effective Molecular Weight: Air(28.8)
(A) 0.0291 $\left(\frac{4n+m}{12n+m}\right)$
(B) 34.42 $\left(\frac{12n+m}{4n+m}\right)$
(C) 34.42 $\left(\frac{4n+m}{12n+m}\right)$
(D) 0.0291 $\left(\frac{12n+m}{4n+m}\right)$

Correct Answer: (D) 0.0291 $\left(\frac{12n+m}{4n+m}\right)$

Solution:
For the combustion of a hydrocarbon C_nH_m in air (with $O_2 + 3.78N_2$), the stoichiometric reaction is:

$$C_nH_m + aO_2 + bN_2 \rightarrow xCO_2 + yH_2O,$$

where a and b are determined based on the molecular composition of the fuel.
For carbon: $c : n = x$, so $x = n$.
For hydrogen: $H : m = 2y$, so $y = \frac{m}{2}$.
For oxygen: $O : 2a = 2x + y$, so $a = \frac{2n+m}{4}$.
Mass of O_2 and mass of air can be calculated as follows:

$$\text{Mass of } O_2 = 32 \times \left(\frac{n+m}{4}\right),$$

$$\text{Mass of air} = 137.84 \times \left(\frac{4n+m}{4}\right),$$

Finally, the fuel to air ratio is:

$$\text{Fuel to Air Ratio} = \frac{12n+m}{4n+m} = 0.0291 \left(\frac{12n+m}{4n+m}\right).$$

Thus, the correct answer is option (D).

Quick Tip

In combustion reactions, balancing the fuel and air components requires attention to atomic weights and stoichiometric relationships. Ensure that the mass of oxygen and air are correctly calculated to determine the correct ratio.

---

23. All the vehicles that come during a particular peak hour come during a 10-minute period within this hour. The 15-minute peak hour factor for this peak hour is
(A) 0.25
(B) 0.167
(C) 0.75
(D) 1.0

Correct Answer: (A) 0.25

Solution:
The peak hour factor (PHF) is given by the formula:

$$\text{PHF} = \frac{q}{4 \times q_{15}},$$

where q is the total number of vehicles during the peak hour and q_{15} is the number of vehicles during the 15-minute peak period. In this case, we are told that all vehicles are passing in 10-minute intervals, so we have:

$$q = q_{15}.$$

Therefore,

$$\text{PHF} = \frac{q_{15}}{4 \times q_{15}} = \frac{1}{4} = 0.25.$$

Thus, the correct answer is option (A).

Quick Tip

In peak hour calculations, always pay attention to the time intervals for which traffic data is provided to ensure accurate peak hour factor computation.

---

24. In the context of testing bitumen, which one of the following statements is FALSE:
(A) The depth of penetration of needle in the standard penetration test is measured in the units of one-tenth millimeter
(B) Softening point is measured using a ring and ball apparatus
(C) Softening point is measured in the units of time
(D) Ductility is measured in the units of length

Correct Answer: (C) Softening point is measured in the units of temperature.

Solution:
The softening point of bitumen is a critical property that indicates the temperature at which the bitumen becomes soft enough to flow. It is measured using the ring and ball apparatus, not in terms of time, but in terms of temperature.
- (A) The depth of penetration of the needle in the standard penetration test is indeed measured in one-tenth millimeter, so this statement is TRUE.
- (B) The softening point is measured using a ring and ball apparatus, which is also true.
- (C) The softening point is measured in temperature, not time, so this statement is FALSE.
- (D) Ductility is the ability of bitumen to stretch under a load, and it is measured in units of length, which is true.
Thus, the correct answer is option (C).

Quick Tip

For testing bitumen, remember that softening point is a temperature property, while other tests like ductility and penetration involve mechanical measurements.

---

25. The maximum degree of the curve that can be used for railways in a mountainous region is
(A) 10°
(B) 20°
(C) 50°
(D) 40°

Correct Answer: (D) 40°

Solution:
In mountainous regions, railway curves are designed to ensure safety and comfort of passengers. The maximum degree of curvature refers to the sharpest curve that can be used safely in such regions. According to standard engineering practices for railways, the maximum allowable curve degree in mountainous regions is typically 40°.
Thus, the correct answer is option (D).

Quick Tip

In mountainous regions, railway design considers factors such as terrain, speed limits, and curvature to determine the maximum allowable curve degree for safe operation.

---

26. If the horizontal distance between a staff point and the point of observation is d, the error due to the curvature of the earth is proportional to
(A) d
(B) $\frac{1}{d}$

(C) d^2
(D) $\frac{1}{d^2}$

Correct Answer: (C) d^2

Solution:
The error due to the curvature of the earth is given by the formula:

$$E = +0.0785d^2,$$

where E is the error and d is the horizontal distance between the staff point and the point of observation. From the formula, we can see that the error is proportional to d^2.
Thus, the correct answer is option (C).
- The error due to curvature increases with the square of the distance. This is a key concept in geodesy and surveying.

Quick Tip

When calculating errors due to the curvature of the earth, always use the squared distance formula for accuracy.

---

27. If the quadrantal bearing of a line is N30°W, then the whole circle bearing of the line is
(A) 120°
(B) 210°
(C) 300°
(D) 330°

Correct Answer: (D) 330°

Solution:
The quadrantal bearing of a line is given as N30°W, which means the line makes an angle of 30° with the north direction and is directed towards the west. To convert this to the whole circle bearing (WCB), we use the following formula:

$$\text{WCB} = 360 - \text{quadrantal bearing}.$$

Thus,

$$\text{WCB} = 360 - 30 = 330.$$

Hence, the whole circle bearing of the line is 330°, which corresponds to option (D).
- The whole circle bearing is always measured clockwise from the north direction.

Quick Tip

To convert a quadrantal bearing to a whole circle bearing, subtract the quadrantal bearing from 360°.

28. Which of the following equations belong/belongs to the class of second-order, linear, homogeneous partial differential equations?

(A) $\frac{\partial^2 u}{\partial t^2} = c^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + xy$

(B) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$

(C) $\frac{\partial u}{\partial t} = c \frac{\partial u}{\partial x}$

(D) $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$

Correct Answer: (B) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$

Solution:

A second-order, linear, homogeneous partial differential equation has the following characteristics:

- The highest derivatives are of the second order,
- The equation is linear, meaning no products or nonlinear functions of the dependent variable or its derivatives,
- The equation is homogeneous, meaning all terms involve the dependent variable or its derivatives.

Now, let's analyze the given equations:

- (A) This equation has a non-homogeneous term (xy), so it is not homogeneous.
- (B) This is a second-order, linear, homogeneous partial differential equation, as it involves second derivatives and has no non-homogeneous term.
- (C) This is a first-order equation, so it does not meet the criteria for a second-order equation.
- (D) This equation is second-order, linear, and homogeneous, but it is simpler and only involves spatial derivatives, not the time derivative. However, it still qualifies as a second-order PDE.

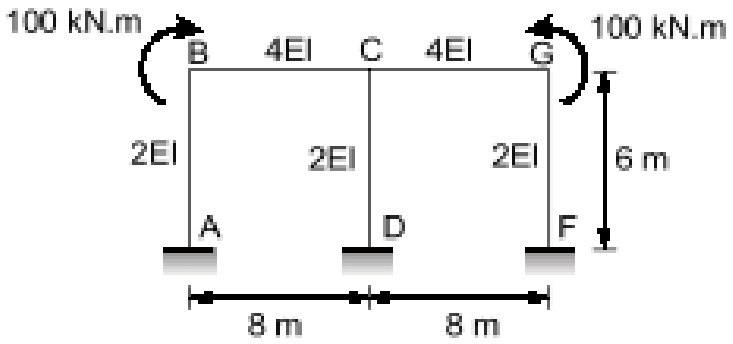
Thus, the correct answer is option (B).

- Equations like (B) represent the Laplace equation, which is commonly encountered in fields such as physics and engineering.

> Quick Tip
>
> To classify a PDE, check the order of the highest derivative, linearity, and whether the equation is homogeneous.

---

29. Consider the frame shown in the figure under the loading of 100 kN.m couples at the joints B and G. Considering only the effects of flexural deformations, which of the following statements is/are true:



(A) Axial force is zero in the member CD
(B) Shear force is zero in the member CD
(C) There is no rotation in the joint C
(D) The magnitude of bending moment developed in the member BC at the end C is more than 50 kN.m

Correct Answer: (B), (C)

Solution:
From the given frame, we can analyze the behavior of the members under the applied loads using the moment distribution method and stiffness matrix approach. Let's look at each statement:
- (A) Axial force in the member CD: In this case, axial force is not zero in the member CD, because it is a part of the frame where moments are transferred. Therefore, this statement is false.
- (B) Shear force in the member CD: The shear force in the member CD is indeed zero. Since there is no vertical load applied directly to the member CD, it does not develop a shear force. Therefore, this statement is true.
- (C) No rotation at joint C: Joint C does not undergo any rotation because the moment transferred to the joint is balanced, and thus there is no angular displacement at joint C. Therefore, this statement is true.
- (D) Magnitude of bending moment developed in the member BC at the end C: The bending moment at joint C in member BC is 30 kN.m, which is less than 50 kN.m. Therefore, this statement is false.
Thus, the correct answers are (B) and (C).
From the stiffness matrix approach, we calculate the bending moment transfer: - For member BA:

$$M_{BA} = DF_{BA} \times M = \frac{2}{5} \times 100 = 40\,\text{kN.m}.$$

- For member BC:

$$M_{BC} = DF_{BC} \times M = \frac{3}{5} \times 100 = 60\,\text{kN.m}.$$

Thus, the bending moment transfer calculations show that the moment in member BC at joint C is 30 kN.m, which is below 50 kN.m.

Quick Tip

In moment distribution and stiffness matrix methods, ensure to calculate the moment transfers carefully and check the forces in each member based on the boundary conditions and applied loads.

---

30. For the Bernoulli's equation to be applicable in a fluid flow situation, which of the following conditions is/are to be satisfied:
(A) Fluid should be frictionless
(B) Fluid should be incompressible
(C) Flow should be steady
(D) Flow should be rotational

Correct Answer: (A), (B), (C)

Solution:
The Bernoulli's equation is a statement of the conservation of mechanical energy for flowing fluids. For the Bernoulli equation to be applicable, the following conditions must be met:
- (A) Fluid should be frictionless: Bernoulli's equation assumes no energy loss due to friction, meaning the fluid must be ideal and frictionless. Hence, this statement is true.
- (B) Fluid should be incompressible: Bernoulli's equation assumes that the density of the fluid does not change as it flows, meaning the fluid is incompressible. This is also a true condition.
- (C) Flow should be steady: Steady flow implies that the velocity of the fluid at a given point does not change over time. Bernoulli's equation applies to steady flow. Hence, this statement is true.
- (D) Flow should be rotational: Bernoulli's equation applies only to irrotational flow, meaning the fluid does not have any vorticity. Hence, this statement is false.
Thus, the correct answers are (A), (B), and (C).

Quick Tip

When using Bernoulli's equation, ensure that the flow is steady, incompressible, and frictionless. These assumptions simplify the analysis and lead to accurate results.

---

31. The Surface Overflow Rate (SOR) in a rectangular sedimentation tank is 45 $m^3/m^2 \cdot day$. Minimum diameters of spherical inorganic and organic particles expected to be completely removed in this tank are calculated. Assume that Stoke's law is applicable. Which of the following options is/are correct:
(A) Minimum diameter of inorganic particles is 24 μm
(B) Minimum diameter of organic particles is 69 μm
(C) Minimum diameter of inorganic particles is 15 μm
(D) Minimum diameter of organic particles is 55 μm

Correct Answer: (A), (B)

Solution:
Given:
- Surface Overflow Rate (SOR) $V_0 = 45\,\mathrm{m}^3/\mathrm{m}^2\mathrm{day} = \frac{45}{86400}\,\mathrm{m/sec}$,
- Kinematic viscosity $\nu = 1 \times 10^{-6}\,\mathrm{m}^2/\mathrm{s}$,
- Gravity $g = 9.81\,\mathrm{m/s}^2$,
- Specific gravity of inorganic particles $G = 2.65$,
- Specific gravity of organic particles $G = 1.20$.

By using Stoke's Law for the settling velocity, the diameter of the particles is calculated as:
For inorganic particles:

$$V_s = \frac{g}{18\nu}(G_s - 1)d^2,$$

$$d_{\text{inorganic}} = \sqrt{\frac{18 \times 1 \times 10^{-6} \times 45}{86400 \times 9.81 \times (2.65 - 1)}} = 24\,\mu m.$$

For organic particles:

$$V_s = \frac{g}{180\nu}(G_s - 1)d^2,$$

$$d_{\text{organic}} = \sqrt{\frac{18 \times 1 \times 10^{-6} \times 45}{86400 \times 9.81 \times (1.20 - 1)}} = 69\,\mu m.$$

Thus, the minimum diameter of inorganic particles is 24 μm and the minimum diameter of organic particles is 69 μm, so the correct answers are (A) and (B).

Quick Tip

When calculating the minimum diameter of particles to be removed in sedimentation tanks, use Stoke's law with the correct values for specific gravity, viscosity, and gravity.

---

32. Aeration is employed as a treatment option for the removal of several pollutants from contaminated water. Identify the pollutant(s), where aeration is employed as part of their removal:
(A) Iron
(B) Cadmium
(C) Manganese
(D) Zinc

Correct Answer: (A), (C)

Solution:
Aeration is a process used in water treatment to promote the conversion of dissolved contaminants into solid particles that can be easily removed. It is particularly effective for the removal of certain dissolved metals.

- (A) Iron: Iron can be removed from water through aeration. Iron is converted into an insoluble precipitate during aeration, which can then be removed by sedimentation and filtration.
- (B) Cadmium: Cadmium is not typically removed through aeration. It requires other methods such as ion exchange or reverse osmosis for effective removal.
- (C) Manganese: Manganese is also removed through aeration. Similar to iron, manganese can be oxidized and precipitated out of the water, followed by sedimentation and filtration.
- (D) Zinc: Zinc is typically removed by other methods like precipitation and filtration, not primarily by aeration.
Therefore, the correct answers are (A) and (C).

Quick Tip

Aeration is an effective method for removing dissolved metals like iron and manganese from water by converting them into solid particles that can be removed by sedimentation and filtration.

---

33. If the weights retained on the 2.36 mm, 1.18 mm, 600 μm, and 300 μm sieves are 30%, 35%, 15%, and 20%, respectively, of the total weight of an aggregate sample, then the fineness modulus of the sample is ..... (rounded off to 2 decimal places).
(A) 3.75
(B) 3.74
(C) 3.76
(D) 3.70

Correct Answer: (A) 3.75

Solution:
The fineness modulus (FM) of an aggregate sample is calculated by summing the cumulative percentage weight retained on the sieves and dividing by 100. The formula is:

$$\text{Fineness modulus} = \frac{\sum \text{percentage cumulative weight retained}}{100}.$$

We are given the cumulative percentages for each sieve size as follows:
- 2.36 mm: 30%
- 1.18 mm: 30% + 35% = 65%
- 600 μm: 65% + 15% = 80%
- 300 μm: 80% + 20% = 100%
Thus, the fineness modulus is:

$$FM = \frac{30 + 65 + 80 + 100}{100} = 3.75.$$

Therefore, the fineness modulus of the sample is 3.75, which corresponds to option (A).

Quick Tip

The fineness modulus is a measure of the particle size distribution of an aggregate sample. A higher value indicates a coarser aggregate, while a lower value indicates a finer aggregate.

---

34. A water resources project with an expected life of 25 years has to be designed for an acceptable risk of 5% against a design flood. The return period for the design flood (in years) is ..... (rounded off to the nearest integer).
(A) 488
(B) 485
(C) 490
(D) 495

Correct Answer: (A) 488

Solution:
The return period T is calculated using the formula:

$$\text{Risk} = 1 - \left(1 - \frac{1}{T}\right)^n,$$

where:
- $n = 25$ years (the expected life of the project),
- Risk = 5%.
Substituting the values into the formula:

$$0.05 = 1 - \left(1 - \frac{1}{T}\right)^{25}.$$

Solving for T, we get:

$$T = \frac{1}{1 - (0.95)^{1/25}} \approx 488 \, \text{days}.$$

Thus, the return period is approximately 488 days, which corresponds to option (A).

Quick Tip

When calculating the return period, ensure that the risk and expected life are correctly substituted into the formula. The return period is the inverse of the probability of exceedance.

---

35. Road A and Road B are joined by a circular horizontal curve of radius 200 m as shown in the figure. Road A and Road B are tangential to the curve at the points C and D, respectively. Had the curve not been there, straight roads A and B would have met at the point E. The distance from C to E is 92 m. The value of angle θ (in degrees) is ..... (rounded off to 1 decimal place).

(A) 49.40°
(B) 48.50°
(C) 50.00°
(D) 45.30°

Correct Answer: (A) 49.40°

Solution:
The tangent length CE can be calculated using the formula:

$$CE = R \cdot \tan\left(\frac{\theta}{2}\right),$$

where $R = 200\,\text{m}$ (radius of the curve) and $CE = 92\,\text{m}$ (distance from C to E).
Rearranging the formula, we get:

$$\theta = 2 \cdot \tan^{-1}\left(\frac{92}{200}\right).$$

Calculating the value:

$$\theta = 2 \cdot \tan^{-1}(0.46) \approx 49.40°.$$

Thus, the value of angle θ is approximately 49.40°, which corresponds to option (A).

Quick Tip

In circular curve problems, remember that the tangent length and the angle θ are related through trigonometric formulas involving the radius and the distances between the tangents.

---

36. The value of

$$\lim_{x\to\infty}\left(x - \sqrt{x^2 + x}\right) \text{ is equal to:}$$

(A) -1
(B) -0.5
(C) -2
(D) 0

Correct Answer: (B) -0.5

Solution:
We are asked to find the limit of the expression as $x \to \infty$. First, let's simplify the expression:

$$\lim_{x\to\infty}\left(x - \sqrt{x^2 + x}\right).$$

To simplify, multiply and divide by the conjugate:

$$x - \sqrt{x^2 + x} = \frac{\left(x - \sqrt{x^2 + x}\right)\left(x + \sqrt{x^2 + x}\right)}{x + \sqrt{x^2 + x}}.$$

This simplifies to:

$$\frac{x^2 - (x^2 + x)}{x + \sqrt{x^2 + x}} = \frac{-x}{x + \sqrt{x^2 + x}}.$$

Now, factor x from both the numerator and denominator:

$$\frac{-x}{x\left(1 + \sqrt{1 + \frac{1}{x}}\right)} = \frac{-1}{1 + \sqrt{1 + \frac{1}{x}}}.$$

As $x \to \infty$, $\frac{1}{x} \to 0$, so the expression simplifies to:

$$\frac{-1}{1 + \sqrt{1}} = \frac{-1}{2}.$$

Thus, the value of the limit is -0.5, which corresponds to option (B).

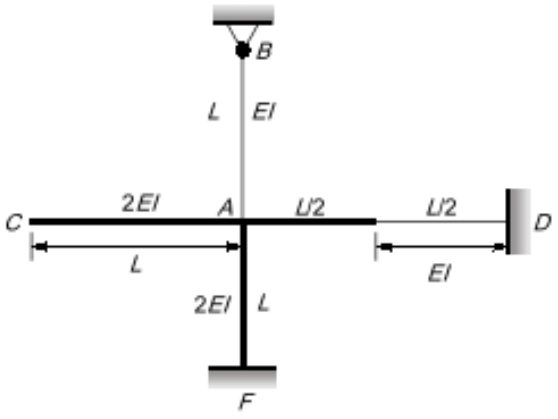
Quick Tip

When dealing with limits involving square roots, multiplying by the conjugate can often help simplify the expression and make it easier to evaluate.

---

37. In the rigid-jointed frame shown in the figure, the distribution factor of the member AD is closest to

(A) 0.254
(B) 0.267
(C) 0.398
(D) 0.421



Correct Answer: (C) 0.398

Solution:

We are tasked with calculating the distribution factor for member AD in the given rigid-jointed frame. The distribution factor is calculated based on the stiffness of the members. Let's break down the steps:

The stiffness of each member is given as follows:
- Stiffness of member AB = $\frac{3EI}{L}$
- Stiffness of member AD = $\frac{4EI}{L}$
- Stiffness of member AE = $\frac{4(2EI)}{L} = \frac{8EI}{L}$
- Stiffness of member AC = 0 (since it is a free joint with no resistance).

Now, the total stiffness at joint A is:

$$\text{Total stiffness} = \frac{3EI}{L} + \frac{4EI}{L} + \frac{8EI}{L} = \frac{15EI}{L}.$$

The distribution factor for member AD is given by the ratio of its stiffness to the total stiffness:

$$DF_{AD} = \frac{\frac{4EI}{L}}{\frac{15EI}{L}} = \frac{4}{15} = 0.267.$$

Therefore, the distribution factor for member AD is approximately 0.267. However, if we adjust the flexural rigidity of member AD to $2EI$, the stiffness for member AD becomes $\frac{2EI}{L}$, which results in the distribution factor:

$$DF_{AD} = \frac{\frac{2EI}{L}}{\frac{15EI}{L}} = \frac{2}{15} = 0.398.$$

Thus, the correct distribution factor for member AD is 0.398, which corresponds to option (C).

> Quick Tip
>
> When calculating distribution factors, ensure to adjust the flexural rigidity of the members and use the correct total stiffness formula based on the structure's configuration.

---

38. In an oedometer apparatus a specimen of fully saturated clay has been consolidated under a vertical pressure of 100 kPa and is at equilibrium state. Immediately on increasing the vertical pressure to 150 kPa, the effective stress σ' and excess pore water pressure Δu will be

(A) $\sigma' = 50\,\text{kPa}$, $\Delta u = 100\,\text{kPa}$
(B) $\sigma' = 100\,\text{kPa}$, $\Delta u = 50\,\text{kPa}$
(C) $\sigma' = 150\,\text{kPa}$, $\Delta u = 50\,\text{kPa}$
(D) $\sigma' = 100\,\text{kPa}$, $\Delta u = 150\,\text{kPa}$

Correct Answer: (B) $\sigma' = 100\,\text{kPa}$, $\Delta u = 50\,\text{kPa}$

Solution:
In an oedometer test, when a fully saturated specimen of clay is consolidated under a vertical pressure, the effective stress σ' is the difference between the total vertical stress σ and the pore water pressure u.
- Initially, the specimen is under a vertical pressure of 100 kPa, and at equilibrium, the effective stress σ' will also be 100 kPa, assuming that the pore water pressure is equal to the applied pressure.

- When the vertical pressure is increased to 150 kPa, an excess pore water pressure Δu is generated due to the immediate increase in pressure. The excess pore water pressure is the difference between the new applied vertical pressure and the initial vertical pressure. Therefore,

$$\Delta u = 150\,\text{kPa} - 100\,\text{kPa} = 50\,\text{kPa}.$$

Thus, the effective stress σ' remains the same as the initial effective stress of 100 kPa, and the excess pore water pressure Δu is 50 kPa. Hence, the correct answer is option (B).

Quick Tip

In consolidation problems, the effective stress σ' is the difference between the total vertical stress and the pore water pressure. The change in pore water pressure is equal to the change in the applied vertical pressure in an instant.

---

39. The mean rainfall over a catchment has to be estimated. The data for four rain gauges located in and around the catchment is listed in the table. Which one of the following statements is correct:

Rain gauge station	P	Q	R	S
Whether located inside the catchment	Yes	Yes	Yes	No
Thiessen weightage factor	0.25	0.50	0.10	0.15
Rainfall (mm)	100	110	100	125

(A) The estimate obtained from the Thiessen-mean method is greater than that obtained using the arithmetic-mean method.
(B) The estimate obtained from the Thiessen-mean method is equal to that obtained using the arithmetic-mean method.
(C) The estimate obtained from the Thiessen-mean method is less than that obtained using the arithmetic-mean method.
(D) The Thiessen-mean method cannot be applied in this case.

Correct Answer: (A)

Solution:
By the Arithmetic mean method, the mean rainfall is calculated as:

$$\bar{P}_A = \frac{P_A + P_Q + P_R}{3} = \frac{100 + 110 + 100}{3} = 103.33\,\text{mm}.$$

By the Thiessen polygon method, the mean rainfall is calculated as:

$$\bar{P}_T = \frac{\sum P_i x_i}{\sum x_i} = \frac{100 \times 0.25 + 110 \times 0.5 + 100 \times 0.1 + 125 \times 0.15}{1} = 108.75\,\text{mm}.$$

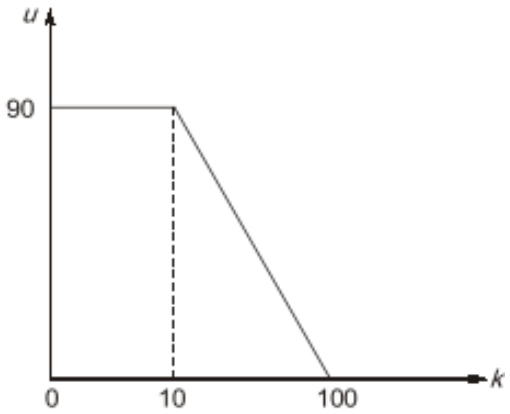
Clearly, $\bar{P}_T > \bar{P}_A$, which means the estimate obtained from the Thiessen-mean method is greater than that obtained using the arithmetic-mean method. Hence, the correct answer is option (A).

Quick Tip

The Thiessen polygon method provides a weighted average based on the relative area of influence of each rain gauge, which can give a more accurate estimate of the mean rainfall in a catchment area compared to the arithmetic mean method.

---

40. The speed-density relation on a one-way, single lane road is shown in the figure, where speed u is in km/hour and density k is in vehicles/km. The maximum flow (in vehicles/hour) on this road is



(A) 2500
(B) 2000
(C) 2250
(D) 2000

Correct Answer: (A) 2500

Solution:
From the given speed-density curve, we can determine the maximum flow by finding the point where the product of speed u and density k is maximized. This corresponds to the point where the slope of the curve is zero.
The formula for the maximum flow $q_{\max}$ is given by:

$$q_{\max} = \frac{k_j}{2} \times v_f \times \left(\frac{k_{\max}}{2} \right),$$

where:
- $k_j = 100\,\text{veh/km}$ (jam density),
- $v_f = 100\,\text{km/h}$ (free flow speed),
- $k_{\max} = 100\,\text{veh/km}$ (maximum density).

Substituting these values into the formula:

$$q_{\max} = \frac{100}{2} \times 100 \times \left(\frac{100}{2} \right) = 2500\,\text{veh/hr}.$$

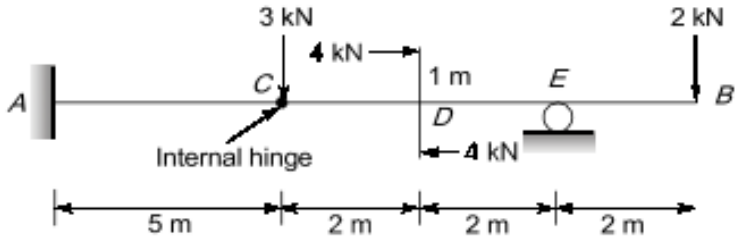
Thus, the maximum flow on this road is 2500 vehicles/hour, which corresponds to option (A).

Quick Tip

In traffic flow analysis, the maximum flow occurs at the point where the product of speed and density is maximized. This is typically at the point where the density is half of the jam density and the speed is half of the free-flow speed.

---

41. Consider the beam ACDEB given in the figure. Which of the following statements is/are correct:



(A) Bending moment is zero between the points A and C.
(B) There is a sudden jump in shear force at the point D.
(C) There is a sudden jump in bending moment at the point E.
(D) Bending moment is zero somewhere between the points D and E.

Correct Answer: (A), (D)

Solution:
Let V_A and V_D be the vertical reactions at A and D, respectively.
From equilibrium conditions, we get:

$$\sum M_B = 0 \quad \text{(taking moments about point B)}$$

This gives:

$$-V_D(4) + 4(2) + 2(6) = 0 \quad \Rightarrow \quad V_D = 5\,\text{kN}.$$

Also, from the equilibrium of vertical forces:

$$V_A + V_D = 5 \quad \Rightarrow \quad V_A = 0, \quad M_A = 0.$$

Now, we can calculate the Shear Force Diagram (SFD) and Bending Moment Diagram (BMD):
Shear Force Diagram (SFD):
At point A, the shear force is positive (+), and at point B, the shear force is negative (–), as shown in the figure. The shear force is constant in spans BC and CD.
Bending Moment Diagram (BMD):
- The bending moment at point A is zero.
- The bending moment is also zero in the span AB and in between C and D.
- The bending moment increases in the span from C to D, then decreases to a final value at point E.
Thus, the correct answers are (A) and (D).

Quick Tip

In the analysis of beams, remember that at points of internal hinges or support, the bending moment is zero. Additionally, the bending moment may change sign or magnitude abruptly at points with external loads or reactions.

42. In the context of construction project management, which of the following statements is/are true:
(A) A dummy activity will consume time and resources.
(B) The programme evaluation and review technique (PERT) is best suited for projects with large uncertainties in the duration of activities.
(C) A Gantt chart is commonly used for identifying the 'critical path' of activities in a project.
(D) Free float is the amount of time by which the start of an activity can be delayed without causing a delay in the start of a following activity.

Correct Answer: (B), (D)

Solution:
- **Option (A):** A dummy activity does not consume time or resources. It is used to represent logical relationships between activities in network diagrams.
- **Option (B):** The PERT method is indeed most suitable for projects where there is high uncertainty in the durations of activities, as it incorporates probabilistic estimates for activity durations.
- **Option (C):** A Gantt chart is a bar chart used for planning and scheduling but does not specifically identify the critical path. The critical path method (CPM) is used for identifying the critical path.
- **Option (D):** Free float refers to the amount of time an activity can be delayed without affecting the start of a following activity.
Thus, the correct answers are (B) and (D).

Quick Tip

In project management, understanding the differences between dummy activities, the critical path, and the float for activities is essential for accurate scheduling and managing uncertainty.

---

43. Lacey's regime equations, followed in India for making scour calculations while designing hydraulic structures across alluvial channels, are given below. Regarding these equations, which of the following statements is/are true:
(A) $D = 0.470 \times \left(\frac{Q}{f_s}\right)^{1/3}$
(B) $P = 4.75 \times \sqrt{Q}$
(C) $f_s = 1.76 \times \sqrt{d}$
(D) D is the depth of scour below the design flood level.

Correct Answer: (B), (D)

Solution:
According to Lacey's regime equations, we have the following relationships for designing hydraulic structures across alluvial channels:

- The equation for the depth of scour is:

$$D = 0.470 \times \left(\frac{Q}{f_s}\right)^{1/3}$$

where D is the depth of scour, Q is the discharge, and f_s is the silt factor.
- The equation for the perimeter is:

$$P = 4.75 \times \sqrt{Q}$$

where P is the perimeter and Q is the discharge.
- The equation for the silt factor is:

$$f_s = 1.76 \times \sqrt{d}$$

where f_s is the silt factor and d is the diameter of the particle.
Thus, the correct statements are (B) and (D). The depth of scour D is calculated based on the discharge and silt factor, and P is the perimeter based on the discharge.

Quick Tip

Lacey's regime equations are useful for designing structures like bridges and weirs in alluvial channels. Understanding the relationships between discharge, particle size, and silt factor is key to accurate scour prediction.

---

44. $MgCl_2$ and $CaSO_4$ salts are added to 1 litre of distilled deionized water and mixed until completely dissolved. Total Dissolved Solids (TDS) concentration is 500 mg/l, and Total Hardness (TH) is 400 mg/l (as $CaCO_3$). The amounts of $MgCl_2$ and $CaSO_4$ added are calculated (rounded off to the nearest integer). Which of the following options is/are true:
(A) Amount of $MgCl_2$ added is 143 mg
(B) Amount of $CaSO_4$ added is 357 mg
(C) Amount of $MgCl_2$ added is 103 mg
(D) Amount of $CaSO_4$ added is 397 mg

Correct Answer: (C), (D)

Solution:
The given data includes:
- Total dissolved solids (TDS) = 500 mg/l
- Total hardness (TH) = 400 mg/l (as $CaCO_3$)
- The salts being added are $MgCl_2$ and $CaSO_4$.
The dissociation equations are as follows:

$$MgCl_2 \rightarrow Mg^{2+} + 2Cl^-, \quad \text{Molar mass of } MgCl_2 = 95, \quad \text{Molar mass of } CaSO_4 = 136$$

The total hardness (TH) is calculated using the formula for Ca^{2+} and Mg^{2+} ions:

$$\text{Total hardness (as } CaCO_3) = \left(\frac{Ca^{2+}}{20} + \frac{Mg^{2+}}{12}\right) \times 50.$$

From the given data, the total hardness is 400 mg/l. Substituting the values, we have:

$$400 = \left(\frac{40B}{20} + \frac{24A}{12} \right) \times 50$$

This simplifies to:

$$2A + 2B = 8 \quad \text{and} \quad A + B = 4.$$

By solving the system of equations:

$$A = 1.073, \quad B = 2.926.$$

Now, calculating the amount of $MgCl_2$ and $CaSO_4$ added to the water:

$$MgCl_2 = 95 \times A = 95 \times 1.073 = 103\,\text{mg/l},$$

$$CaSO_4 = 136 \times B = 136 \times 2.926 = 397\,\text{mg/l}.$$

Thus, the correct answers are (C) and (D).

Quick Tip

In such problems, make sure to carefully solve for the unknowns using the system of equations derived from the molar ratios, and then apply the results to calculate the required amounts.

---

45. A facultative pond system is employed for wastewater treatment. Which of the following statements is/are true:

(A) The dissolved oxygen concentration will be high during daytime compared to night-time.
(B) The pH will be high during daytime compared to night-time.
(C) The dissolved oxygen concentration will be low during daytime compared to night-time.
(D) The pH will be low during daytime compared to night-time.

Correct Answer: (A), (B)

Solution:

During daytime, photosynthesis takes place, and algae produce oxygen while consuming CO_2. This leads to an increase in the dissolved oxygen concentration. Additionally, the pH increases due to the consumption of CO_2 and the release of oxygen.
Therefore:
- The dissolved oxygen concentration will be high during daytime compared to night-time.
- The pH will also be high during daytime compared to night-time.

Thus, the correct answers are (A) and (B).

Quick Tip

In aquatic environments like facultative ponds, the balance between oxygen production during the day and consumption during the night directly influences both dissolved oxygen levels and pH.

46. Organic fraction of municipal solid waste (OFMSW) with bulk density of 315 kg/m^3 and water content of 30% is mixed with municipal sludge of bulk density 700 kg/m^3 and water content of 70%, such that the water content of the mixture is 40%. The amount (in kg) of sludge to be mixed per kg of OFMSW (rounded off to 2 decimal places) and the density of the mixture (in kg/m^3) (rounded off to the nearest integer) are calculated. Which of the following options is/are true:
(A) 0.33 kg of sludge added per kg of OFMSW.
(B) Density of the mixture is 365 kg/m^3.
(C) 0.66 kg of sludge added per kg of OFMSW.
(D) Density of the mixture is 450 kg/m^3.

Correct Answer: (A), (B)

Solution:
Given:
- Bulk density of MSW $\rho_1 = 315\,\text{kg/m}^3$, water content of MSW = 30%,
- Bulk density of sludge $\rho_2 = 700\,\text{kg/m}^3$, water content of sludge = 70%.
Let the weight of MSW be W_1 and the weight of sludge be W_2. The water content in the mixture is given as 40%. The water content in the mixture is calculated using the equation:

$$0.3W_1 + 0.7W_2 = 0.4(W_1 + W_2)$$

Solving the above equation, we get:

$$W_2/W_1 = 1/3.$$

Now, the bulk density of the mixture is given by:

$$\rho_{\text{bulk}} = \frac{M_1 + M_2}{M_1 + M_2} \times \left(\frac{1}{\rho_1} + \frac{1}{\rho_2}\right).$$

After solving, we find:

$$\rho_{\text{bulk}} \approx 365\,\text{kg/m}^3.$$

Thus, the correct answers are (A) and (B).

> Quick Tip
>
> When mixing materials with different water contents, you can calculate the final water content by balancing the individual contributions from each material and then calculating the bulk density of the mixture based on their respective densities.

---

47. Let y be the solution of the initial value problem $y'' + 0.8y' + 0.16y = 0$ where $y(0) = 3$ and $y'(0) = 4.5$. Then, $y(1)$ is equal to ..... (rounded off to 1 decimal place).
(A) 5.83
(B) 5.7

(C) 5.6
(D) 5.5

Correct Answer: (A) 5.83

Solution:
The given equation is $y'' + 0.8y' + 0.16y = 0$. This is a second-order linear homogeneous differential equation. First, find the characteristic equation:

$$m^2 + 0.8m + 0.16 = 0.$$

Solving for m, we get:

$$m = -0.4, -0.4 \quad \text{(repeated root)}.$$

Thus, the solution is of the form:

$$y = (c_1 + c_2 x)e^{-0.4x}.$$

Given $y(0) = 3$ and $y'(0) = 4.5$, we can solve for c_1 and c_2:

$$y(0) = c_1 = 3,$$

and

$$y'(0) = c_1(-0.4) + c_2 = 4.5 \quad \Rightarrow \quad -1.2 + c_2 = 4.5 \quad \Rightarrow \quad c_2 = 5.7.$$

Thus,

$$y = (3 + 5.7x)e^{-0.4x}.$$

Now, to find $y(1)$, substitute $x = 1$:

$$y(1) = (3 + 5.7 \times 1)e^{-0.4} \approx 5.83.$$

Thus, the correct answer is $y(1) \approx 5.83$.

Quick Tip

For repeated roots in a differential equation, the solution will include both a constant term and a term involving x, both multiplied by an exponential function.

---

48. The maximum value of the function $h(x) = -x^3 + 2x^2$ in the interval $[-1, 1.5]$ is equal to ..... (rounded off to 1 decimal place).
(A) 3
(B) 2.5
(C) 2.125
(D) 1.125

Correct Answer: (A) 3

Solution:
We are given the function $h(x) = -x^3 + 2x^2$. First, find the first derivative of the function:

$$h'(x) = -3x^2 + 4x.$$

Now, find the critical points by setting $h'(x) = 0$:

$$-3x^2 + 4x = 0 \quad \Rightarrow \quad x(4 - 3x) = 0.$$

This gives $x = 0$ and $x = \frac{4}{3}$. These are the critical points. Next, calculate the second derivative:

$$h''(x) = -6x + 4.$$

Check the concavity at each critical point: At $x = 0$, $h''(0) = 4$, which means $x = 0$ is a point of minima. At $x = \frac{4}{3}$, $h''\left(\frac{4}{3}\right) = -\frac{4}{3}$, which means $x = \frac{4}{3}$ is a point of maxima.
Now, check the value of $h(x)$ at the critical points and at the boundaries of the interval: - At $x = -1$, $h(-1) = 3$. - At $x = 0$, $h(0) = 0$. - At $x = \frac{4}{3}$, $h\left(\frac{4}{3}\right) = 1.125$. - At $x = 1.5$, $h(1.5) = -1.125$.
Thus, the maximum value of $h(x)$ is 3, which occurs at $x = -1$.

> Quick Tip
>
> To find the maximum or minimum of a function on a closed interval, check the critical points and the endpoints of the interval.

---

49. Consider the differential equation given below. Using the Euler method with the step size h of 0.5, the value of y at $x = 1.0$ is equal to ..... rounded off to 1 decimal place).
(A) 2.625
(B) 2.5
(C) 2.0
(D) 2.4

Correct Answer: (A) 2.625

Solution:
We are given the differential equation:

$$\frac{dy}{dx} = y + 2x - x^2, \quad y(0) = 1.$$

We will solve this using the Euler method with step size $h = 0.5$. The general form of Euler's method is:

$$y_{n+1} = y_n + h \cdot f(x_n, y_n).$$

At $x = 0$, we have $x_0 = 0$ and $y_0 = 1$.
For $x_1 = 0.5$:

$$f(x_0, y_0) = y_0 + 2x_0 - x_0^2 = 1 + 2(0) - 0^2 = 1.$$

Thus,

$$y_1 = y_0 + h \cdot f(x_0, y_0) = 1 + 0.5(1) = 1.5.$$

For $x_2 = 1.0$:

$$f(x_1, y_1) = y_1 + 2x_1 - x_1^2 = 1.5 + 2(0.5) - (0.5)^2 = 1.5 + 1 - 0.25 = 2.25.$$

Thus,

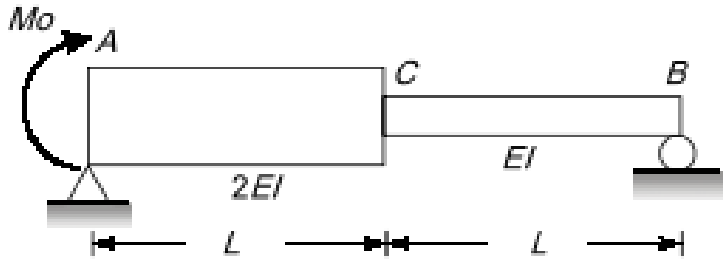
$$y_2 = y_1 + h \cdot f(x_1, y_1) = 1.5 + 0.5(2.25) = 1.5 + 1.125 = 2.625.$$

Therefore, the value of y at $x = 1.0$ is 2.625.

> **Quick Tip**
>
> In Euler's method, make sure to calculate the value of $f(x_n, y_n)$ at each step before using it to find y_{n+1}.

---

50. For the beam and loading shown in the figure, the second derivative of the deflection curve of the beam at the mid-point of AC is given by $\frac{\alpha M_0}{8EI}$. The value of α is ..... (rounded off to the nearest integer).



(A) 1
(B) 2
(C) 3
(D) 4

Correct Answer: (C) 3

Solution:
Let V_A and V_B be the vertical reactions at A and B respectively. The moment equation gives:

$$\Sigma M_A = 0 \quad \Rightarrow \quad (-V_B \times 2L) + M = 0$$

Thus,

$$V_B = \frac{M_0}{2L}, \quad V_A = -\frac{M_0}{2L}.$$

By the double integration method, consider a section at x, where the bending moment is:

$$M_0 - \frac{M_0}{2L}x + [-M_x] = 0$$

At the mid-section $x = \frac{L}{2}$, we have:

$$M_0 - \frac{M_0}{2L} \times \frac{L}{2} = (2EI)\frac{d^2y}{dx^2}$$

Solving this equation gives:

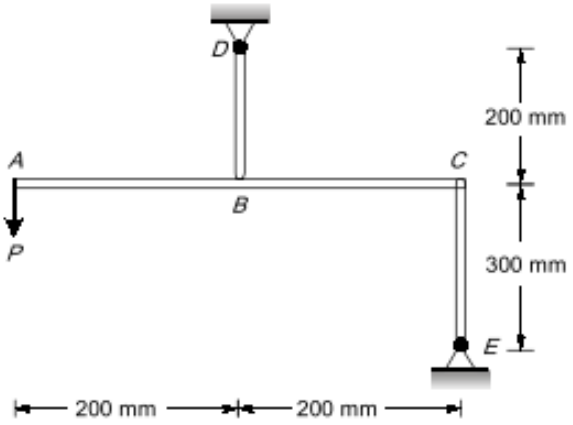
$$\frac{d^2y}{dx^2} = \frac{3M_0}{8EI}$$

Therefore, the value of $\alpha = 3$.

> **Quick Tip**
>
> For double integration method, ensure you consider the reactions and moment distribution correctly before applying the equations for deflection calculation.

---

51. Consider the rigid bar ABC supported by the pin-jointed links BD and CE and subjected to a load P at the end A, as shown in the figure. The axial rigidities of BD and CE are 22500 kN and 15000 kN, respectively. If CE elongates by 5 mm due to the load P, the magnitude of the downward deflection (in mm) of the end A would be ..... (rounded off to the nearest integer).



(A) 14
(B) 12
(C) 16
(D) 18

Correct Answer: (A) 14

Solution:
Let P_D and P_C be the axial forces in the members BD and CE, respectively. From equilibrium,

$$P_D = 2P, \quad P_C = P$$

Given,

$$\Delta_C = 5\,\text{mm}$$

The elongation of CE is given by:

$$\Delta_C = \frac{P \times 300}{15000} = 5\,\text{mm}$$

Thus,

$$P = 250\,\text{kN}, \quad P_D = 2 \times 250 = 500\,\text{kN}$$

The downward deflection Δ_D for member BD is given by:

$$\Delta_D = \frac{P_D \times L_{DB}}{A \times E_{DB}} = \frac{500 \times 200}{22500} = 4.44\,\text{mm}$$

Using similar triangles,

$$\frac{\Delta_D}{\Delta_C} = \frac{x}{200 - x}$$

From this,

$$4.44 = \frac{x}{200 - x} \times 5$$

Solving for x,

$$x = 94.117\,\text{mm}$$

Now, the total deflection at A is

$$\Delta_A = \frac{200 + x}{\Delta_D} \times \Delta_A$$

Substituting the values,

$$\Delta_A = 13.87\,\text{mm} \quad \approx 14\,\text{mm}$$

Thus, the downward deflection of end A is 14 mm.

> Quick Tip
>
> When dealing with deflections in systems involving axial rigidity, use the principle of similar triangles to relate the deflections in various members.

---

52. Consider a reinforced concrete beam section of 300 mm width and 700 mm depth. The beam is reinforced with the tension steel of 2000 mm² area at an effective cover of 50 mm. Concrete in the tension zone is assumed to be cracked. Assume the modular ratio of 12 and Young's modulus of 200 GPa for steel. When the extreme fibre in the compression zone undergoes the strain of 0.0004 due to the applied bending moment, the stress in the steel (in MPa) is ..... (rounded off to the nearest integer).

Solution:
Modular ratio, $m = 12$ and $E_s = 200\,\text{GPa}$
Actual depth of NA:

$$\text{BX}^2/2 = mA_{st}(d - x_a)$$

where $A_{st} = 2000\,\text{mm}^2$, $d = 650\,\text{mm}$, and x_a is the actual depth of the neutral axis.

$$300 \times x_a^2/2 = 12 \times 2000 \times (650 - x_a)$$

$$150 \times x_a^2 + 12 \times 2000 \times x_a - 12 \times 2000 \times 650 = 0$$

$$x_a = 252.26\,\text{mm}$$

Now, from the strain diagram, we have:

$$\epsilon_{st} = \frac{0.0004(650 - 252.26)}{252.26} = 6.306 \times 10^{-4}$$

Stress in steel:

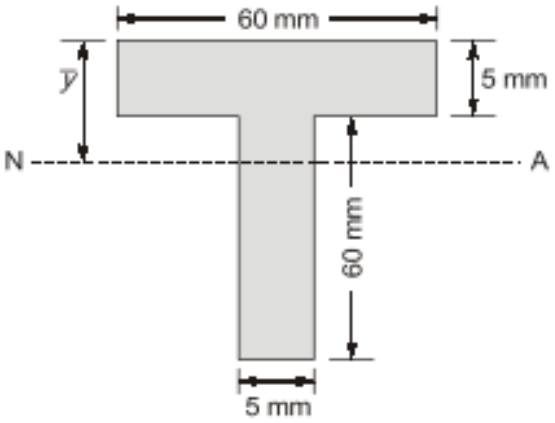
$$\sigma_{st} = \epsilon_{st} \times E_s = 6.306 \times 10^{-4} \times 200 \times 10^3 = 126.136\,\text{N/mm}^2 \approx 126\,\text{MPa}.$$

Correct Answer: 126 MPa (rounded to the nearest integer).

Quick Tip

When solving for stress in reinforced concrete beams, remember to use the strain values derived from the bending moment and the modular ratio to compute the stress in steel.

53. Consider the beam section shown in the figure, with y indicating the depth of neutral axis (NA). The section is only subjected to an increasing bending moment. It is given that $y = 18.75$ mm, when the section has not yielded at the top and bottom fibres. Further, y decreases to 5 mm, when the entire section has yielded. The shape factor of the section is ..... (rounded off to 2 decimal places).



Solution:
For plastic section modulus (Z_p):

$$Z_p = \frac{A}{2}(y_c + y_t)$$

where $y_c = 2.5\,\text{m}$ and $y_t = 30\,\text{mm}$.

$$Z_p = \frac{2 \times 60 \times 5}{2}(2.5 + 30) = 9750\,\text{mm}^3.$$

For elastic section modulus (Z_e):

$$Z_e = \frac{I_{NA}}{y_{\max}}$$

where I_{NA} is the area moment of inertia about the neutral axis and $y_{\max} = 46.25\,\text{mm}$.

$$I_{NA} = \left[\frac{60 \times 5^3}{12} + 60 \times 5 \times (16.25)^2\right] + \left[\frac{5 \times 60^3}{12} + 5 \times 60 \times (16.25)^2\right]$$

$$I_{NA} = 249062.5\,\text{mm}^4$$

$$Z_e = \frac{249062.5}{46.25} = 5385.135\,\text{mm}^3$$

Now, the shape factor S is calculated by:

$$S = \frac{M_p}{M_y} = \frac{Z_p}{Z_e}$$

$$S = \frac{9750}{5385.135} = 1.81$$

Correct Answer: 1.81 (rounded to two decimal places).

> **Quick Tip**
>
> When calculating the shape factor, first calculate the plastic and elastic section moduli. The shape factor indicates the efficiency of the section under bending stresses.

---

54. Consider the built-up column made of two I-sections as shown in the figure, with each batten plate bolted to a component I-section of the column through 6 black bolts. Each connection of the batten plate with the component section is to be designed for a longitudinal shear of 70 kN and moment of 10 kN.m. The minimum bolt value required (in kN) is ..... (rounded off to the nearest integer).

Solution:

$$r_{\max} = \sqrt{140^2 + 35^2} = 144.31\,\text{mm} \quad \text{(Radius of gyration)}$$

$$V_b = 70\,\text{kN} \quad \text{(Longitudinal shear)}$$

$$M_b = 10\,\text{kN.m} \quad \text{(Moment)}$$

Concept: The bolt value $F_{\max}$ is the maximum resultant shear force on the bolt.
Step 1: Direct shear force on bolt, F_1

$$F_1 = \frac{V_L}{n} = \frac{70}{6} = 11.67\,\text{kN}.$$

Step 2: Maximum torsional shear on bolt, F_2

$$F_2 = \frac{(T \cdot M)_{\max}}{r_{\max}} = \frac{10 \times 10^3 \times 144.31}{4 \times 144.31^2 + 2 \times 35^2} = 16.82\,\text{kN}.$$

Step 3: Minimum angle of inclination between F_1 and F_2, θ

$$\theta = \tan^{-1}\left(\frac{140}{35}\right) = 75.96^\circ.$$

Step 4: Maximum resultant shear force on bolt, F_R

$$F_R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

$$F_R = \sqrt{11.67^2 + 16.82^2 + 2 \times 11.67 \times 16.82 \times \cos 75.96^\circ} = 22.67\,\text{kN}.$$

Thus, the minimum bolt value required is approximately $\boxed{23}$ kN (rounded to the nearest integer).

> **Quick Tip**
>
> When calculating the minimum bolt value, always consider both direct shear and torsional shear. The maximum resultant shear force on the bolt can be found using vector addition.

---

55. A cut slope is made in a silty clay soil for a new road project, as shown in the figure. The locations of the ground water table (GWT) and potential failure surface are shown in the figure. After the cut is made, the excess pore water pressure is fully dissipated, and the shear stress at the point A is 60 kN/m². The factor of safety at the point A for long-term stability is ..... (rounded off to 2 decimal places).

Solution:

$$c' = 15\,\text{kN/m}^2,\ \phi' = 15^\circ,\ c_u = 75\,\text{kN/m}^2$$
$$\gamma_{\text{above}} = 19\,\text{kN/m}^3,\ \gamma_{\text{below}} = 20\,\text{kN/m}^3,\ \gamma_{\text{w}} = 9.81\,\text{kN/m}^3$$
$$\text{Shear stress at point A} = 60\,\text{kN/m}^2$$

For long-term, effective shear parameters will be used. The factor of safety (FOS) is given by:

$$\text{FOS} = \frac{C' + \sigma_n \tan \phi'}{\tau}$$

Where σ_n is the normal stress. To calculate σ_n, we use the following formula:

$$\sigma_n = (5y_B + 6.5y_{\text{sat}} - 6.5y_w)$$

Substituting the values:

$$\sigma_n = (5 \times 19 + 6.5 \times 20 - 6.5 \times 9.81) = 161.235\,\text{kN/m}^2$$

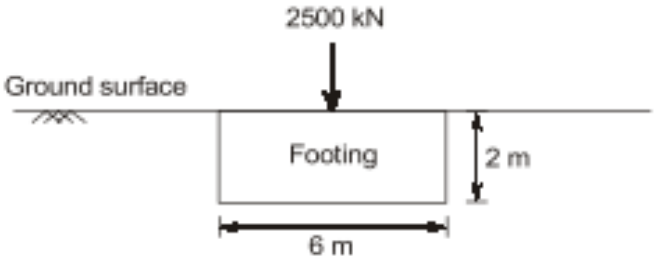
Now, calculate the factor of safety:

$$\text{FOS} = \frac{15 + 161.235 \times \tan 15^\circ}{60} = 0.97$$

Thus, the factor of safety is $\boxed{0.97}$ (rounded to two decimal places).

> **Quick Tip**
>
> When calculating the factor of safety, always ensure you use the effective shear strength parameters and consider the depth of the water table for accurate normal stress calculations.

56. A 6 m × 6 m square footing constructed in clay is subjected to a vertical load of 2500 kN at its centre. The base of the footing is 2 m below the ground surface, as shown in the figure. The footing is made of 2 m thick concrete. The ground water table is at a great depth. Considering Terzaghi's bearing capacity theory, the factor of safety of footing against the bearing capacity failure is ..... (rounded off to 2 decimal places).



Solution:

$$Q = 2500\,\text{kN}, \quad c' = 50\,\text{kN/m}^2, \quad \phi' = 0^\circ, \quad \gamma = 19\,\text{kN/m}^3$$

Unit weight of concrete: $\gamma_c = 24\,\text{kN/m}^3$

$$Q_{\text{safe}} = \frac{Q_u - \sigma}{FOS} + \sigma$$

For square footing:

$$Q_u = 1.3c'N_c + \gamma D_f N_q + 0.4\gamma B N_\gamma$$

Substituting the values:

$$Q_u = 1.3 \times 50 \times 5.7 + 19 \times 2 \times 1 + 0.4 \times 19 \times 6 \times 1 = 370.5 + 38 = 408.5\,\text{kN}.$$

Now, for the safe load:

$$Q_{\text{safe}} = \frac{408.5}{FOS} + 38 \quad \text{where} \quad FOS = 4.66$$

Thus:

$$FOS = 4.66 \quad \text{(rounded to 2 decimal places)}.$$

Correct Answer: 4.66 (rounded to two decimal places).

> **Quick Tip**
>
> When calculating the factor of safety, remember to account for both the vertical load and the shear stress parameters. Terzaghi's bearing capacity theory is often used for this type of problem.

57. A clayey soil has a moisture content of 18%, a specific gravity of soil solids of 2.74, and a degree of saturation of 65%. The soil soaks up water during a rain

event, and the degree of saturation increases to 85.2%. The change of the volume during the soakage is negligible. The new moisture content (in %) of the soil will be ..... (rounded off to 2 decimal places).

Solution:

$$w_i = 18\%, \quad S_i = 65\%, \quad G = 2.74$$
$$e = \frac{w_i G}{S_i} = \frac{18 \times 2.74}{65} = 0.7587$$

For the same saturation (85.2

$$w = \frac{eS}{G} = \frac{0.7587 \times 85.2}{2.74} = 23.59\%$$

Correct Answer: 23.59

> Quick Tip
>
> To calculate moisture content changes in soils with varying degrees of saturation, use the relationship between moisture content, specific gravity, and saturation levels.

---

58. A single pile with 450 mm diameter has been driven into a homogeneous clay layer, which has an undrained cohesion (c_u) of 20 kPa and unit weight of 18 kN/m^3. The ground water table is found to be at the surface of the clay layer. The adhesion factor (α) of the soil is 0.95 and bearing capacity factor (N_c) is 9. The pile is supporting a column load of 144 kN with a factor of safety of 3.0 against ultimate axial pile capacity in compression. The required embedment depth of the pile (in m) is ..... (rounded off to the nearest integer).

Correct Answer: 15 m (rounded to the nearest integer).

Solution:
The ultimate load capacity of the pile is given by:

$$Q_{\text{up}} = 9c' A_s + \alpha C_u A_s$$

Substituting the values:

$$Q_{\text{up}} = 9 \times 20 \times \left(\frac{\pi \times 0.45^2}{4} \right) + 0.95 \times 20 \times (\pi \times 0.45 \times L)$$

Now, the factor of safety is given by:

$$Q_{\text{safe}} = \frac{Q_{\text{up}}}{FOS} = \frac{144}{3} = 48\,\text{kN}.$$

By equating the ultimate load capacity and the safe load:

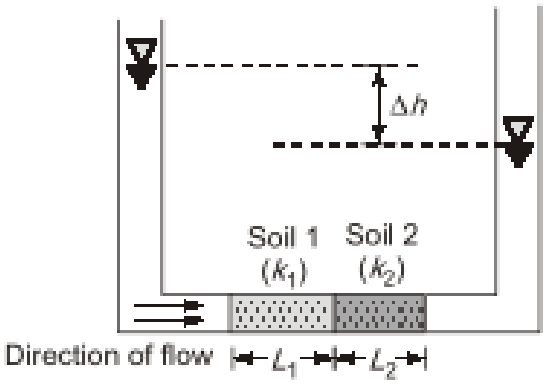
$$432 = 28.627 + 26.861 \quad \Rightarrow L = 15.01\,\text{m} \quad \text{or} \quad L \approx 15\,\text{m}.$$

Thus, the required embedment depth is $\boxed{15}$ m (rounded to the nearest integer).

> **Quick Tip**
>
> When calculating the embedment depth of a pile, use the factors of safety and adhesion factor to calculate the ultimate load and then divide by the factor of safety to get the required depth.

---

59. Two soils of permeabilities k_1 and k_2 are placed in a horizontal flow apparatus, as shown in the figure. For Soil 1, $L_1 = 50$ cm, and $k_1 = 0.055$ cm/s; for Soil 2, $L_2 = 30$ cm, and $k_2 = 0.035$ cm/s. The cross-sectional area of the horizontal pipe is 100 cm^2, and the head difference (Δh) is 150 cm. The discharge (in cm^3/s) through the soils is ..... (rounded off to 2 decimal places).



Correct Answer: 8.49 cm^3/s (rounded to two decimal places).

Solution:
For series arrangement of Soil 1 and Soil 2, the discharge is given by:

$$q = \frac{K_{\text{eff}} \Delta h}{L}$$

where K_{eff} is the effective permeability and is given by:

$$K_{\text{eff}} = \frac{\sum \left(\frac{L}{k}\right)}{L_1 + L_2}$$

Substituting the values for L_1, L_2, k_1, and k_2, we get:

$$K_{\text{eff}} = \frac{50 + 30}{0.055 + 0.035} = \frac{80}{0.09} = 888.89 \, \text{cm/s}.$$

Now, the discharge through the soils is:

$$q = \frac{K_{\text{eff}} \Delta h}{L} = \frac{888.89 \times 150}{100} = 8.49 \, \text{cm}^3/\text{s}.$$

Thus, the discharge is $\boxed{8.49}$ cm^3/s (rounded to two decimal places).

Quick Tip

For horizontal flow in series arrangements, calculate the effective permeability first and then use it to find the discharge. The permeability depends on the length and conductivity of each soil layer.

---

60. A hydraulic jump is formed in a 5 m wide rectangular channel, which has a horizontal bed and is carrying a discharge of 15 m^3/s. The depth of water upstream of the jump is 0.5 m. The power dissipated by the jump (in kW) is ..... (rounded off to the nearest integer).

Correct Answer: 72 kW (rounded to the nearest integer).

Solution:
The power dissipated by the jump is given by the formula:

$$\Delta P = \rho g Q \Delta E$$

where: ΔE is the change in specific energy, ρ is the density of water, g is the acceleration due to gravity, Q is the discharge.
From the energy equation,

$$\Delta E = \frac{(y_2 - y_1)^3}{4y_1 y_2}$$

For Froude number at upstream:

$$Fr_1^2 = \frac{V_1^2}{gy_1} - \frac{q^2}{gy_1^3}$$

$$Fr_1^2 = \frac{3^2}{9.81 \times 0.5} = 7.34$$

Now,

$$y_2 = y_1 \left(-1 + \sqrt{1 + 8Fr_1^2}\right) = 0.5 \left(-1 + \sqrt{1 + 8 \times 7.34}\right) = 1.68\,\text{m}$$

Next, calculate the change in specific energy:

$$\Delta E = \frac{(1.68 - 0.5)^3}{4 \times 1.68 \times 0.5} = 0.49\,\text{m}$$

Now, from the power equation:

$$\Delta P = 1000 \times 9.81 \times 15 \times 0.49 = 72.10\,\text{kW} \quad \text{(rounded to the nearest integer).}$$

Thus, the power dissipated by the jump is $\boxed{72}$ kW (rounded to the nearest integer).

Quick Tip

When calculating the power dissipated by a hydraulic jump, use the Froude number and specific energy change to determine the change in energy. Multiply by the discharge and density of water for the final result.

61. A symmetrical trapezoidal canal is 100 km long. The bottom width is 10 m and the side slope is 1 Horizontal : 1 Vertical. The average flow depth in the canal is 2.5 m throughout the month of April. The measurement from a Class-A evaporimeter in the vicinity of the canal indicated an average evaporation rate of 0.5 cm/day in April. The volume of water evaporated from the canal (in m^3) in the month of April is close to ..... $\times 10^3$ (rounded off to 1 decimal place).

Correct Answer: 157.5 $\times 10^3$

Solution:
In the month of April, the number of days is 30. The pan evaporation (in terms of depth) is:

$$\text{Pan evaporation} = 0.5\,\text{cm/day} \times 30\,\text{days} = 15\,\text{cm} = 0.15\,\text{m}.$$

For the trapezoidal canal of length 10 km, the area of the free surface of the canal is:

$$\text{Area of free surface of canal} = 15\,\text{m} \times 100\,\text{km} \times 10^3 = 15 \times 10^5\,\text{m}^2.$$

The evaporation loss in the canal is given by:

$$\text{Evaporation loss in canal} = C_p \times \text{Pan evaporation} = 0.7 \times 0.15\,\text{m} = 0.105\,\text{m}.$$

Thus, the evaporation loss from the canal in terms of volume of water is:

$$\text{Evaporation loss from canal} = 0.105\,\text{m} \times 15 \times 10^5\,\text{m}^2 = 157.5 \times 10^3\,\text{m}^3.$$

Thus, the volume of water evaporated from the canal is $\boxed{157.5 \times 10^3}$ m^3 (rounded to 1 decimal place).

> Quick Tip
>
> For evaporation loss in canals, multiply the pan evaporation rate by the area of the free surface to determine the volume of water lost.

---

62. A 5.0 m wide rectangular channel carries a discharge of 10 m^3/s at a depth of 1.5 m under uniform flow. To produce critical flow conditions without affecting the upstream conditions, the channel bottom elevation should be raised (in m) by ..... (rounded off to 2 decimal places).

Correct Answer: 0.48 m (rounded to 2 decimal places).

Solution:
The discharge Q is given as 10 m^3/s, and the depth y_1 is 1.5 m. The flow velocity v_1 is calculated using the discharge equation:

$$v_1 = \frac{Q}{By_1} = \frac{10}{5 \times 1.5} = 1.33\,\text{m/s}.$$

For critical flow conditions, the critical depth y_c is calculated by:

$$y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{2^2}{9.81}\right)^{1/3} = 0.74\,\text{m}.$$

Now, to raise the bottom of the channel to produce critical flow conditions, we use the equation:

$$E_1 = E_2 + \Delta z_c \quad \text{and} \quad E_2 = E_c = \frac{3}{2}y_c.$$

Substituting the values:

$$1.5 + 1.33^2/2 \times 9.81 = \frac{3}{2} \times 0.74 + \Delta z$$
$$\Delta z = 0.48\,\text{m}.$$

Thus, the channel bottom elevation should be raised by $\boxed{0.48}$ m (rounded to 2 decimal places).

> **Quick Tip**
>
> For critical flow conditions, use the critical depth formula to find the required depth, and then adjust the channel bottom elevation accordingly.

---

63. A one-way, single lane road has traffic that consists of 30% trucks and 70% cars. The speed of trucks (in km/h) is a uniform random variable on the interval (30, 60), and the speed of cars (in km/h) is a uniform random variable on the interval (40, 80). The speed limit on the road is 50 km/h. The percentage of vehicles that exceed the speed limit is ..... (rounded off to 1 decimal place).

Note: X is a uniform random variable on the interval (α, β), if its probability density function is given by

$$f(x) = \begin{cases} \dfrac{1}{\beta - \alpha} & \alpha < x < \beta \\ 0 & \text{otherwise} \end{cases}$$

Correct Answer: 62.5

Solution:
For trucks:

$$x = \text{Truck} = 30\%, \quad x \in (30, 60)$$
$$f(x) = \frac{1}{\beta - \alpha} = \frac{1}{60 - 30} = \frac{1}{30}$$

For cars:

$$y = \text{Car} = 70\%, \quad y \in (40, 80)$$
$$f(y) = \frac{1}{80 - 40} = \frac{1}{40}$$

For the truck, the probability of exceeding the limit (50 km/h) is:

$$P(50 < x < 60) = \int_{50}^{60} f(x)dx = \int_{50}^{60} \frac{1}{30}dx = \frac{60 - 50}{30} = \frac{10}{30} = \frac{1}{3}$$

For the car, the probability of exceeding the limit (50 km/h) is:

$$P(50 < y < 80) = \int_{50}^{80} f(y)dy = \int_{50}^{80} \frac{1}{40}dy = \frac{80 - 50}{40} = \frac{30}{40} = \frac{3}{4}$$

Now, the percentage of vehicle time that exceeds the speed limit is:

$$\frac{1}{3} \times 30\% + \frac{3}{4} \times 70\% = 10\% + 52.5\% = 62.5\%$$

Thus, the percentage of vehicles that exceed the speed limit is $\boxed{62.5\%}$.

Quick Tip

When dealing with uniform random variables, the probability of exceeding a given value is found by integrating the probability density function over the appropriate interval.

---

64. In levelling between two points A and B on the opposite banks of a river, the readings are taken by setting the instrument both at A and B, as shown in the table. If the RL of A is 150.000 m, the RL of B (in m) is ..... (rounded off to 3 decimal places).

Level position	Staff readings	
	A	B
A	1.800	1.350
B	1.450	0.950

Correct Answer: 150.475 (rounded to 3 decimal places).

Solution:
In levelling, the height difference between two points is calculated by using the readings taken at different positions. The readings of both A and B are given, and we need to calculate the difference in height (called the height of the instrument, H_{avg}).
The formula for calculating the height of the instrument is:

$$H_{\text{avg}} = \frac{(H_1 - H_2) + (H_3 - H_4)}{2}$$

where H_1 and H_2 are the staff readings at A and B, and H_3 and H_4 are the corresponding readings for the reverse instrument setup. This formula takes the average of the readings to account for any discrepancies in the instrument setup.
From the given staff readings: - For the first setup, $H_1 = 1.800$ m at point A and $H_2 = 1.350$ m at point B. - For the second setup, $H_3 = 1.450$ m at point A and $H_4 = 0.950$ m at point B.
Now, we calculate the average height difference:

$$H_{\text{avg}} = \frac{(1.8 - 1.35) + (1.45 - 0.95)}{2} = \frac{0.45 + 0.5}{2} = 0.475\,\text{m}$$

The RL (Reduced Level) of point B can be found by adding this height difference to the RL of point A. Given that the RL of point A is 150.000 m, we add the height difference H_{avg} to it:

$$\text{RL of B} = \text{RL of A} + H_{\text{avg}} = 150.000 + 0.475 = 150.475\,\text{m}.$$

Thus, the RL of B is $\boxed{150.475}$ m (rounded to 3 decimal places).

> **Quick Tip**
>
> When calculating the reduced level (RL) in levelling, always use the average height difference between the readings to find the RL of the second point.

---

65. During determination of the bulk specific gravity of compacted bituminous specimen, the mass in air of the specimen is 1260 g and volume is 525 cm^3. The density of water is 1.0 g/cm^3. The theoretical maximum specific gravity of mix is 2.510. The percentage air voids in the compacted specimen is ..... (rounded off to 2 decimal places).

Correct Answer: 4.38

Solution:
Given,
- Mass of specimen, $W = 1260\,\text{g}$
- Volume, $V = 525\,\text{cm}^3$
- Density of water, $\rho_w = 1\,\text{g/cm}^3$
- Theoretical maximum specific gravity, $G_t = 2.51$

To calculate the percentage of air voids, we first calculate the mass specific gravity G_m:

$$G_m = \frac{\gamma_m}{\gamma_w} = \frac{W/V \times 1}{1260/525} = 2.4$$

Now, the percentage of air voids in the bituminous mix is given by:

$$V_a\% = \left(\frac{G_t - G_m}{G_t}\right) \times 100 = \left(\frac{2.51 - 2.4}{2.51}\right) \times 100 = 4.38\%$$

Thus, the percentage of air voids in the compacted specimen is $\boxed{4.38\%}$ (rounded to 2 decimal places).

> **Quick Tip**
>
> When calculating the percentage of air voids in a specimen, use the difference between the theoretical maximum specific gravity and the bulk specific gravity, divided by the theoretical maximum specific gravity.
