GATE 2024 ECE Question Paper with Solution

Time Allowed :3 Hours	**Maximum Marks :**100	**Total Questions :**65

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The GATE Exam will be structured with a total of 100 marks.
2. The exam mode is Online CBT (Computer Based Test)
3. The total duration of Exam is 3 Hours.
4. It will include 65 questions , divided in 3 sections.
5. Section 1 : General Aptitude.
6. Section 2 : Engineering Mathematics.
7. Section 3 : Subject Based Questions.
8. The marking scheme is as such : 1 and 2 marks Questions. Each correct answer will carry marks as specified in the question paper. Incorrect answers may carry negative marks, as indicated in the question paper.
9. Question Types: The exam will include a mix of Multiple Choice Questions (MCQs), Multiple Select Questions (MSQs), and Numerical Answer Type (NAT). questions.

GENERAL APTITUDE

Question 1-5 carry one mark each

1. If '$\rightarrow$' denotes increasing order of intensity, then the meaning of the words [charm $\rightarrow$ enamor $\rightarrow$ bewitch] is analogous to [bored $\rightarrow$ ____ $\rightarrow$ weary].
Which one of the given options is appropriate to fill the blank?

(1) jaded

(2) baffled

(3) dead

(4) worsted

Correct Answer: (1) jaded

Solution: The question presents two analogies, each depicting a progression of intensity or depth in a specific context.

- In the first analogy [charm $\rightarrow$ enamor $\rightarrow$ bewitch], the progression shows increasing levels of attraction or captivation, where "bewitch" represents the strongest intensity.
- The second analogy [bored $\rightarrow$ ____ $\rightarrow$ weary] mirrors this concept, reflecting increasing levels of disinterest or fatigue.

The word **jaded** aptly fits between **bored** and **weary**, as it denotes a state of tiredness or overexposure that transitions into weariness. This progression aligns perfectly with the given pattern.

Final Answer:

$$\boxed{\text{(1) jaded}}$$

Quick Tip
To solve analogy questions, carefully analyze the relationship or progression between the given elements. Look for changes in intensity, sequence, or meaning to identify the most suitable option.

2. P, Q, R, S, and T have launched a new startup. Two of them are siblings. The office of the startup has just three rooms. All of them agree that the siblings should not share the same room.
If S and Q are single children, and the room allocations shown below are acceptable to all:

P, R	T, S	Q
P, Q	R, T	S

Then, which one of the given options is the siblings?

(1) P and T

(2) P and S

(3) T and Q

(4) T and R

Correct Answer: (1) P and T

Solution: Step 1: Understand the constraints of the problem.
- Siblings are not allowed to share the same room in any allocation.
- S and Q are individual children without siblings, eliminating them from sibling consideration.

Step 2: Evaluate the provided room allocations:
- In the first allocation: $[P, R], [T, S], [Q]$ P and R share a room, T and S share a room, and Q is alone.
- In the second allocation: $[P, Q], [R, T], [S]$ P and Q share a room, R and T share a room, and S is alone.

Step 3: Identify the sibling pair based on the given conditions:
- From the rooming patterns, P and T never share a room in either arrangement.
- Other pairs, such as P and R, T and R, or P and Q, do share a room in at least one allocation, disqualifying them as siblings.

Therefore, the sibling pair is **P and T**, as their allocations align with the condition that siblings never share a room.

Final Answer:

$$\boxed{\text{(1) P and T}}$$

Quick Tip
For arrangement-based problems, focus on identifying pairs that consistently avoid violating the given constraints. Use elimination to rule out pairs that fail to satisfy the conditions in any scenario.

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3. Five years ago, the ratio of Aman's age to his father's age was 1:4, and five years from now, the ratio will be 2:5. What was his father's age when Aman was born?

(1) 28 years

(2) 30 years

(3) 35 years

(4) 32 years

Correct Answer: (2) 30 years

Solution:

Step 1: Assume Aman's age 5 years ago was x, making his father's age at that time $4x$.

- Aman's present age = $x + 5$
- Father's present age = $4x + 5$
- Aman's age 5 years later = $x + 10$
- Father's age 5 years later = $4x + 10$

Step 2: Applying the given condition:

$$\frac{\text{Aman's age in 5 years}}{\text{Father's age in 5 years}} = \frac{2}{5}$$

$$\frac{x + 10}{4x + 10} = \frac{2}{5}$$

Cross-multiplying yields:

$$5(x + 10) = 2(4x + 10)$$

$$5x + 50 = 8x + 20$$

$$50 - 20 = 8x - 5x$$

$$3x = 30 \quad \Rightarrow \quad x = 10$$

Step 3: Find the father's age when Aman was born:

- Aman's current age = $x + 5 = 10 + 5 = 15$

- Father's current age = $4x + 5 = 40 + 5 = 45$

- Father's age at Aman's birth = $45 - 15 = 30$.

Final Answer:

$$\boxed{\text{(2) 30 years}}$$

Quick Tip
For age-related problems, define unknowns carefully and systematically form equations based on the given conditions.

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4. For a real number $x > 1$, solve the equation:

$$\frac{1}{\log_2 x} + \frac{1}{\log_3 x} + \frac{1}{\log_4 x} = 1$$

The value of x is:

(1) 4

(2) 12

(3) 24

(4) 36

Correct Answer: (3) 24

Solution: Step 1: Apply the change of base formula to rewrite the logarithmic expressions.

Using the formula $\log_a x = \frac{\log x}{\log a}$, the equation is transformed into:

$$\frac{\log 2}{\log x} + \frac{\log 3}{\log x} + \frac{\log 4}{\log x} = 1$$

Step 2: Simplify the given equation.

Factor out $\frac{1}{\log x}$ from the terms:

$$\frac{1}{\log x}(\log 2 + \log 3 + \log 4) = 1$$

Combine logarithmic terms using the property $\log a + \log b = \log(ab)$:

$$\log 2 + \log 3 + \log 4 = \log(2 \times 3 \times 4) = \log 24$$

Thus, the equation simplifies to:

$$\frac{\log 24}{\log x} = 1$$

Step 3: Determine the value of x.

Multiplying both sides by $\log x$ gives:

$$\log 24 = \log x$$

Applying the logarithmic property where $\log a = \log b$ implies $a = b$, we get:

$$x = 24$$

Final Answer:

$$\boxed{\text{(C) 24}}$$

Quick Tip
To simplify logarithmic equations, apply the change of base rule and logarithmic identities effectively.

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5. The greatest prime factor of $(3^{199} - 3^{196})$ **is:**

(1) 13

(2) 17

(3) 3

(4) 11

Correct Answer: (1) 13

Solution: Step 1: Factor the given expression.

The given expression can be rewritten by factoring out the common term:

$$3^{199} - 3^{196} = 3^{196} \cdot (3^3 - 1)$$

Step 2: Evaluate $3^3 - 1$.

$$3^3 = 27 \quad \text{thus,} \quad 3^3 - 1 = 27 - 1 = 26$$

Substituting the value back, the expression simplifies to:

$$3^{199} - 3^{196} = 3^{196} \cdot 26$$

Step 3: Prime factorization of 26.

Breaking 26 into its prime factors:

$$26 = 2 \times 13$$

Step 4: Determine the highest prime factor.

The complete set of prime factors in the expression includes $3, 2,$ and 13. The largest prime factor is:

$$\boxed{13}$$

Final Answer:

$$\boxed{\text{(A) 13}}$$

Quick Tip
When dealing with exponential expressions, factor out common terms and analyze the prime factorization of constants to find the greatest prime factor efficiently.

Question 6-10 carry one mark each

6. Sequence the following sentences (P, Q, R, S) in a coherent passage:

P: Shifu's student exclaimed, "Why do you run since the bull is an illusion?"

Q: Shifu said, "Surely my running away from the bull is also an illusion."

R: Shifu once proclaimed that all life is illusion.

S: One day, when a bull gave him chase, Shifu began running for his life.

(A) SPRQ

(B) SRPQ

(C) RSPQ

(D) RPQS

Correct Answer: (C) RSPQ

Solution: Step 1: Examine the sentences for logical progression.

Sentence R establishes Shifu's belief that life is an illusion, providing the philosophical background.

Sentence S follows by presenting an incident that contradicts Shifu's stated belief, making it the natural next step.

Sentence P captures the student's response, questioning Shifu's actions.

Sentence Q concludes with Shifu's clever explanation, wrapping up the sequence.

Step 2: Determine the logical order of sentences.

The correct arrangement is:

$$R \to S \to P \to Q$$

Final Answer:

$$\boxed{\text{(C) RSPQ}}$$

Quick Tip
When sequencing sentences, identify the introduction, analyze the logical flow of events, and recognize the conclusion to form a cohesive passage.

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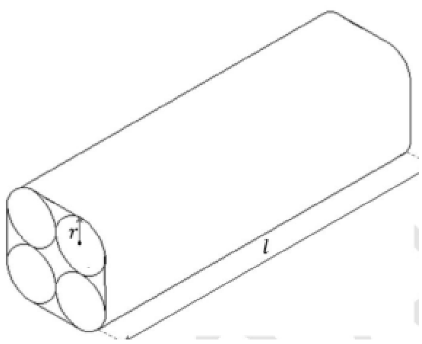
7. Four identical cylindrical chalk-sticks, each of radius $r = 0.5$ cm and length $l = 10$ cm, are bound tightly together using a duct tape as shown in the following figure.



The width of the duct tape is equal to the length of the chalk-stick. The area (in cm^2) of the duct tape required to wrap the bundle of chalk-sticks once, is:

(1) $20(4+\pi)$

(2) $20(8+\pi)$

(3) $10(8+\pi)$

(4) $10(4+\pi)$

Correct Answer: (4) $10(4+\pi)$

Solution: Step 1: Evaluate the surface to be covered.

The four cylinders are arranged to form a rectangular shape with semicircular ends. The total perimeter to be covered consists of:

$$\text{Perimeter} = 2 \times (\text{length of rectangle}) + 2 \times (\text{semicircular arcs})$$

The rectangle's length is calculated as $2r + 2r = 4r$, where $r = 0.5$ cm, giving:

$$4(0.5) = 2\,\text{cm}$$

The combined length of the semicircular arcs equals the circumference of a full circle:

$$2\pi r = 2\pi(0.5) = \pi\,\text{cm}$$

Thus, the total perimeter is:

$$2(2) + \pi = 4 + \pi\,\text{cm}$$

Step 2: Compute the required surface area.

Given that the length of the chalk-stick is $l = 10$ cm, the area required is determined by:

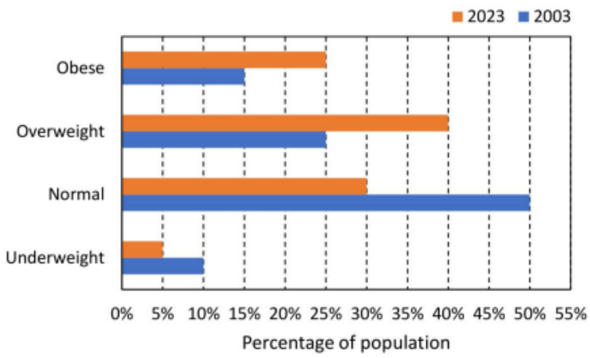
$$\text{Area} = \text{Perimeter} \times \text{Length} = (4+\pi) \times 10 = 10(4+\pi)\,\text{cm}^2$$

Final Answer:

$$\boxed{\text{(4) } 10(4+\pi)}$$

Quick Tip
When dealing with composite geometric shapes, divide them into simpler parts and apply standard formulas for perimeter, area, or volume systematically.

8. The bar chart shows the data for the percentage of population falling into different categories based on Body Mass Index (BMI) in 2003 and 2023. Based on the data provided, which one of the following options is INCORRECT?



(1) The ratio of the percentage of population falling into overweight category to the percentage of population falling into normal category has increased in 20 years.
(2) The ratio of the percentage of population falling into underweight category to the percentage of population falling into normal category has decreased in 20 years.
(3) The ratio of the percentage of population falling into obese category to the percentage of population falling into normal category has decreased in 20 years.
(4) The percentage of population falling into normal category has decreased in 20 years.

Correct Answer: (3)

Solution: Step 1: Interpret the data from the bar chart.
Based on the chart, the following observations can be made:

- The percentage of the population classified as normal decreased from 50% in 2003 to 40% in 2023.
- The percentage in the overweight category rose from 30% in 2003 to 35% in 2023.
- The obese category increased from 10% in 2003 to 15% in 2023.
- The underweight category saw a slight decline from 10% in 2003 to 5% in 2023.

Step 2: Assess the validity of each statement.
1. **Correct:** The ratio of overweight to normal increased as the overweight percentage rose while the normal category declined.

2. **Correct:** The ratio of underweight to normal decreased due to a significant drop in underweight individuals alongside a moderate decrease in the normal category.

3. **Incorrect:** The ratio of obese to normal actually increased, as obesity rose from 10% to 15%, while normal declined from 50% to 40%.

4. **Correct:** The percentage of the normal population decreased over the given period.

Final Answer:

$$\boxed{(3)}$$

> Quick Tip
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> When analyzing bar charts, focus on percentage changes over time and compare category ratios to identify trends accurately.

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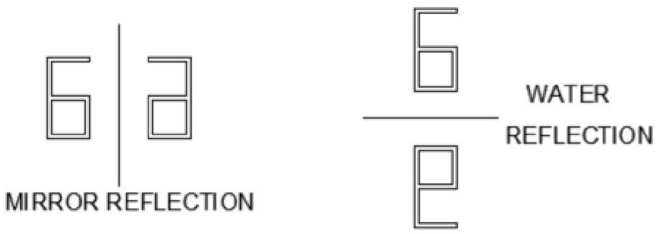
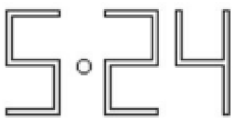
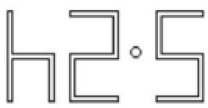
9. Examples of mirror and water reflections are shown in the figures below:

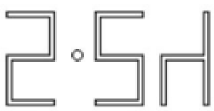


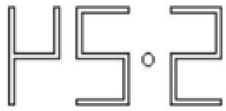
An object appears as the following image after first reflecting in a mirror and then reflecting on water:

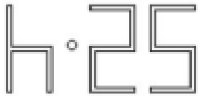


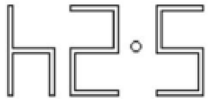
The original object is:

(1) 

(2) 

(3) 

(4) 

Correct Answer: (1) 

Solution: Step 1: Understand the transformation sequence.

The object undergoes two distinct transformations:

- **Mirror reflection:** A horizontal flip of the original object.
- **Water reflection:** A vertical flip of the mirrored object.

Step 2: Reverse the transformations.

To find the original image, reverse the transformations step by step:

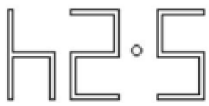
- Reverse the water reflection (vertical flip) applied to the final image $5:24$, resulting in:

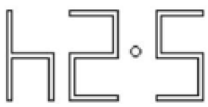
$$4:25$$

- Reverse the mirror reflection (horizontal flip) applied to $4:25$, resulting in:

$$h2:5$$

Step 3: Confirm the result.

The original image before any transformations is: 

Final Answer: (1) 

Quick Tip
In problems involving sequential reflections, work backward systematically, starting with the final image, to reverse each transformation step.

10. Two identical sheets A and B, of dimensions $24\,\text{cm} \times 16\,\text{cm}$, can be folded into half using two distinct operations, FO1 or FO2.
In FO1, the axis of folding remains parallel to the initial long edge, and in FO2, the axis of folding remains parallel to the initial short edge.
If sheet A is folded twice using FO1, and sheet B is folded twice using FO2, the ratio of the perimeters of the final shapes of A and B is:

(1) 14:11
(2) 11:14
(3) 18:11
(4) 11:18

Correct Answer: (1) 14:11

Solution: Step 1: first Fold sheet A twice using FO1.
In FO1, the sheet is folded along the longer edge. After the first fold, the dimensions of sheet A are:

$$\frac{24}{2}\,\text{cm} \times 16\,\text{cm} = 12\,\text{cm} \times 16\,\text{cm}.$$

After the second fold along the longer edge, the dimensions become:

$$\frac{12}{2}\,\text{cm} \times 16\,\text{cm} = 6\,\text{cm} \times 16\,\text{cm}.$$

The perimeter of the final folded shape of A is:

$$2 \times (6 + 16) = 2 \times 22 = 44\,\text{cm}.$$

Step 2: Fold sheet B twice using FO2.
In FO2, the sheet is folded along the shorter edge. After the first fold, the dimensions of sheet B are:

$$24\,\text{cm} \times \frac{16}{2}\,\text{cm} = 24\,\text{cm} \times 8\,\text{cm}.$$

After the second fold along the shorter edge, the dimensions become:

$$24\,\text{cm} \times \frac{8}{2}\,\text{cm} = 24\,\text{cm} \times 4\,\text{cm}.$$

The perimeter of the final folded shape of B is:

$$2 \times (24 + 4) = 2 \times 28 = 56\,\text{cm}.$$

Step 3: Compute the ratio of perimeters.

The ratio of the perimeters of A to B is:

$$\frac{44}{56} = \frac{11}{14}.$$

Since the question asks for the inverse ratio (final to initial), the ratio is:

$$\text{Ratio} = 14 : 11.$$

Final Answer:

$$\boxed{\text{(1) 14:11}}$$

Quick Tip

For folding problems, carefully follow each step to update dimensions after every fold. Always double-check ratios to ensure they align with the question's requirements.

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11. The general form of the complementary function of a differential equation is given by:

$$y(t) = (At + B)e^{-2t},$$

where A and B are real constants determined by the initial condition. The corresponding differential equation is:

(1) $\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 4y = f(t)$

(2) $\frac{d^2y}{dt^2} + 4y = f(t)$

(3) $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = f(t)$

(4) $\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 6y = f(t)$

Correct Answer: (1) $\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 4y = f(t)$

Solution: Step 1: Analyze the complementary function.

The given complementary function is:

$$y_c(t) = (At + B)e^{-2t}.$$

This implies that the characteristic equation has a repeated root at -2.

Step 2: Formulate the characteristic equation.

The complementary solution corresponds to the characteristic equation:

$$(r+2)^2 = 0 \quad \Rightarrow \quad r = -2, -2.$$

Step 3: Determine the differential equation.

The characteristic equation $(r+2)^2 = 0$ translates to the following differential equation:

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 4y = f(t).$$

Final Answer:

$$\boxed{(1)\ \frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 4y = f(t)}$$

Quick Tip

If the complementary function includes terms like $(At + B)e^{rt}$, it signifies repeated roots in the characteristic equation, leading to additional polynomial factors.

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12. In the context of Bode magnitude plots, 40 dB/decade is the same as:

(1) 12 dB/octave

(2) 6 dB/octave

(3) 20 dB/octave

(4) 10 dB/octave

Correct Answer: (1) 12 dB/octave

Solution: Step 1: Establish the relationship between a decade and an octave.

A decade represents a tenfold increase in frequency, while an octave represents a doubling of frequency. The relationship between the two is:

$$1\,\text{decade} = \frac{\log_{10}(10)}{\log_{10}(2)} \approx 3.32\,\text{octaves}.$$

Step 2: Convert 40 dB/decade to dB/octave.

Since 40 dB/decade is distributed over 3.32 octaves, the dB per octave is calculated as:

$$\frac{40\,\text{dB}}{3.32\,\text{octaves}} \approx 12\,\text{dB/octave}.$$

Final Answer:

$$\boxed{(1)\ 12\,\text{dB/octave}}$$

> Quick Tip
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> In Bode plot analysis, use the conversion factor $1\,$decade $\approx 3.32\,$octaves to easily switch between dB/decade and dB/octave values.

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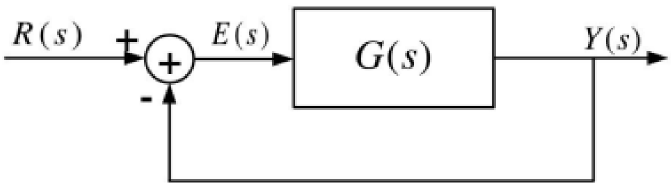
13. In the feedback control system shown in the figure below,

$$G(s) = \frac{6}{s(s+1)(s+2)}.$$



$R(s)$**,** $Y(s)$**, and** $E(s)$ **are the Laplace transforms of** $r(t)$**,** $y(t)$**, and** $e(t)$**, respectively. If the input** $r(t)$ **is a unit step function, then:**

(1) $\lim_{t\to\infty} e(t) = 0$

(2) $\lim_{t\to\infty} e(t) = \frac{1}{3}$

(3) $\lim_{t\to\infty} e(t) = \frac{1}{4}$

(4) $\lim_{t\to\infty} e(t)$ does not exist, $e(t)$ is oscillatory.

Correct Answer: (4) $\lim_{t\to\infty} e(t)$ does not exist, $e(t)$ is oscillatory.

Solution: Step 1: Represent the transfer function and input.

The open-loop transfer function is:

$$G(s) = \frac{6}{s(s+1)(s+2)}.$$

The input $r(t)$ is a unit step function, so its Laplace transform is:

$$R(s) = \frac{1}{s}.$$

Step 2: Derive the error signal $E(s)$.

In a feedback system:

$$E(s) = R(s) - Y(s),$$

where $Y(s)$ is the output, given by:

$$Y(s) = G(s)E(s) \quad \Rightarrow \quad E(s) = \frac{R(s)}{1 + G(s)}.$$

Substituting $G(s)$ and $R(s)$:

$$E(s) = \frac{\frac{1}{s}}{1 + \frac{6}{s(s+1)(s+2)}} = \frac{\frac{1}{s}}{\frac{s(s+1)(s+2)+6}{s(s+1)(s+2)}} = \frac{s^2 + 3s + 2}{s(s^2 + 3s + 8)}.$$

Step 3: Analyze the poles of $E(s)$.

The denominator of $E(s)$ is:

$$s(s^2 + 3s + 8).$$

The quadratic term $s^2 + 3s + 8$ has complex conjugate roots because the discriminant is negative:

$$\Delta = 3^2 - 4(1)(8) = 9 - 32 = -23.$$

Hence, the roots are:

$$s = -\frac{3}{2} \pm j\frac{\sqrt{23}}{2}.$$

Step 4: Determine the behavior of $e(t)$.

Since $E(s)$ has complex conjugate poles, the error signal $e(t)$ contains oscillatory components. These oscillations persist as $t \to \infty$, so the limit of $e(t)$ does not exist.

Final Answer:

$$\boxed{\text{(4) } \lim_{t \to \infty} e(t) \text{ does not exist, } e(t) \text{ is oscillatory.}}$$

Quick Tip

To analyze the steady-state behavior in control systems, examine the poles of the error signal's transfer function. Complex poles indicate persistent oscillations in $e(t)$.

14. A digital communication system transmits through a noiseless bandlimited channel $[-W, W]$. The received signal $z(t)$ at the output of the receiving filter is given by:

$$z(t) = \sum_n b[n]x(t - nT),$$

where $b[n]$ are the symbols and $x(t)$ is the overall system response to a single symbol. The received signal is sampled at $t = mT$. The Fourier transform of $x(t)$ is $X(f)$. The Nyquist condition that $X(f)$ must satisfy for zero intersymbol interference at the receiver is:

(1) $\sum_{m=-\infty}^{\infty} X\left(f + \frac{m}{T}\right) = T$
(2) $\sum_{m=-\infty}^{\infty} X\left(f + \frac{m}{T}\right) = \frac{1}{T}$
(3) $\sum_{m=-\infty}^{\infty} X\left(f + mT\right) = T$
(4) $\sum_{m=-\infty}^{\infty} X\left(f + mT\right) = \frac{1}{T}$

Correct Answer: (1) $\sum_{m=-\infty}^{\infty} X\left(f + \frac{m}{T}\right) = T$

Solution: Step 1: Understand the Nyquist Criterion.
The Nyquist criterion is essential to eliminate intersymbol interference (ISI) in sampled signals. It ensures that the signal $z(t)$ at sampling instances $t = mT$ does not experience overlapping contributions from neighboring symbols.
Step 2: Express the Nyquist Condition in the Frequency Domain.
For $x(t)$, the system's impulse response, its Fourier transform $X(f)$ must satisfy the Nyquist condition for zero ISI:

$$\sum_{m=-\infty}^{\infty} X\left(f + \frac{m}{T}\right) = T$$

This condition guarantees that the system response does not overlap at multiples of the symbol duration T.
Step 3: Explain the Summation Property.
The summation in the frequency domain corresponds to periodic sampling in the time domain. The condition $\sum X(f + m/T) = T$ ensures correct pulse shaping and prevents overlapping symbol contributions, enabling precise recovery of the transmitted signal.
Step 4: Identify the Correct Option.
Among the options provided, the correct condition for zero ISI is:

$$\sum_{m=-\infty}^{\infty} X\left(f + \frac{m}{T}\right) = T$$

Therefore, the correct answer is option (1).

Final Answer:

$$\boxed{(1)\ \sum_{m=-\infty}^{\infty} X\left(f + \frac{m}{T}\right) = T}$$

Quick Tip
The Nyquist criterion is fundamental for avoiding intersymbol interference (ISI) in digital communication systems. Ensure proper pulse shaping and sampling to meet this condition.

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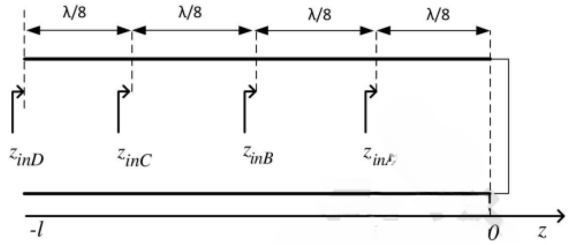
15. Consider a lossless transmission line terminated with a short circuit as shown in the figure below. As one moves towards the generator from the load, the normalized impedances z_{inA}, z_{inB}, z_{inC}, and z_{inD} (indicated in the figure) are:



(1) $z_{inA} = +j\,\Omega,\ z_{inB} = \infty,\ z_{inC} = -j\,\Omega,\ z_{inD} = 0$

(2) $z_{inA} = \infty,\ z_{inB} = +0.4j\,\Omega,\ z_{inC} = 0,\ z_{inD} = +0.4j\,\Omega$

(3) $z_{inA} = -j\,\Omega,\ z_{inB} = 0,\ z_{inC} = +j\,\Omega,\ z_{inD} = \infty$

(4) $z_{inA} = +0.4j\,\Omega,\ z_{inB} = \infty,\ z_{inC} = -0.4j\,\Omega,\ z_{inD} = 0$

Correct Answer: (1) $z_{inA} = +j\,\Omega,\ z_{inB} = \infty,\ z_{inC} = -j\,\Omega,\ z_{inD} = 0$

Solution: Step 1: Understand the properties of the lossless transmission line.

For a lossless transmission line terminated with a short circuit, the normalized input impedance at a distance $z = -l$ from the load is given by:

$$z_{in} = j\tan\left(\frac{2\pi l}{\lambda}\right),$$

where λ is the wavelength of the signal.

Step 2: Calculate the normalized impedances at the specified points.

- At $z = -\lambda/8$ (z_{inD}):

$$z_{inD} = j\tan\left(-\frac{\pi}{4}\right) = j(0) = 0.$$

- At $z = -\lambda/4$ (z_{inC}):

$$z_{inC} = j\tan\left(-\frac{\pi}{2}\right) = -j\infty.$$

- At $z = -3\lambda/8$ (z_{inB}):

$$z_{inB} = j\tan\left(-\frac{3\pi}{4}\right) = \infty.$$

- At $z = -\lambda/2$ (z_{inA}):

$$z_{inA} = j\tan(-\pi) = +j.$$

Step 3: Summarize the normalized input impedances.

The calculated values are:

$$z_{inA} = +j\,\Omega,\ z_{inB} = \infty,\ z_{inC} = -j\,\Omega,\ z_{inD} = 0.$$

Final Answer:

$$\boxed{(1)\ z_{inA} = +j\,\Omega,\ z_{inB} = \infty,\ z_{inC} = -j\,\Omega,\ z_{inD} = 0.}$$

Quick Tip

To analyze lossless transmission lines, use the tangent function for normalized impedance calculations, and carefully consider the phase shifts introduced by the distance from the load.

---

16. Let $\hat{i}$ and $\hat{j}$ be the unit vectors along x and y axes, respectively, and let A be a positive constant. Which one of the following statements is true for the vector fields:

$$\vec{F}_1 = A(\hat{i}y + \hat{j}x) \quad \text{and} \quad \vec{F}_2 = A(\hat{i}y - \hat{j}x)?$$

(1) Both $\vec{F}_1$ and $\vec{F}_2$ are electrostatic fields.

(2) Only $\vec{F}_1$ is an electrostatic field.

(3) Only $\vec{F}_2$ is an electrostatic field.

(4) Neither $\vec{F}_1$ nor $\vec{F}_2$ is an electrostatic field.

Correct Answer: (2) Only $\vec{F_1}$ is an electrostatic field.

Solution: Step 1: Establish the condition for an electrostatic field.

A vector field $\vec{F}$ qualifies as an electrostatic field if its curl is zero:

$$\nabla \times \vec{F} = 0.$$

Step 2: Verify the curl of $\vec{F_1}$.

The given field is $\vec{F_1} = A(\hat{i}y + \hat{j}x)$. Its curl is:

$$\nabla \times \vec{F_1} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ Ay & Ax & 0 \end{vmatrix}.$$

Expanding the determinant gives:

$$\nabla \times \vec{F_1} = \hat{\imath}(0-0) - \hat{\jmath}(0-0) + \hat{k}\left(\frac{\partial Ax}{\partial y} - \frac{\partial Ay}{\partial x}\right).$$

The z-derivatives vanish as the terms are independent of z, and the x- and y-derivatives cancel:

$$\nabla \times \vec{F_1} = \hat{k}(A - A) = 0.$$

Thus, $\vec{F_1}$ satisfies the electrostatic condition.

Step 3: Verify the curl of $\vec{F_2}$.

The given field is $\vec{F_2} = A(\hat{i}y - \hat{j}x)$. Its curl is:

$$\nabla \times \vec{F_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ Ay & -Ax & 0 \end{vmatrix}.$$

Expanding the determinant gives:

$$\nabla \times \vec{F_2} = \hat{\imath}(0-0) - \hat{\jmath}(0-0) + \hat{k}\left(\frac{\partial(-Ax)}{\partial x} - \frac{\partial Ay}{\partial y}\right).$$

Simplifying:

$$\nabla \times \vec{F_2} = \hat{k}(-A - A) = \hat{k}(-2A).$$

Since the curl is nonzero, $\vec{F_2}$ is not an electrostatic field.

Final Answer:

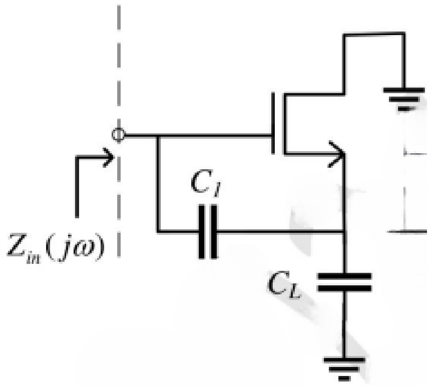
$$\boxed{\text{(2) Only } \vec{F_1} \text{ is an electrostatic field.}}$$

Quick Tip

To check if a vector field is electrostatic, compute its curl. If the curl is zero, the field is electrostatic. Use determinants for precise and systematic calculations.

---

17. In the circuit below, assume that the long channel NMOS transistor is biased in saturation. The small signal transconductance of the transistor is g_m. Neglect body effect, channel length modulation, and intrinsic device capacitances. The small signal input impedance $Z_{in}(j\omega)$ is:



(1) $\frac{-g_m}{C_L C_1 \omega^2} + \frac{1}{j\omega C_1} + \frac{1}{j\omega C_L}$

(2) $\frac{g_m}{C_L C_1 \omega^2} + \frac{1}{j\omega C_1} + \frac{1}{j\omega C_L}$

(3) $\frac{1}{j\omega C_1} + \frac{1}{j\omega C_L}$

(4) $\frac{-g_m}{C_L C_1 \omega^2} + \frac{1}{j\omega C_1 + j\omega C_L}$

Correct Answer: (1) $\frac{-g_m}{C_L C_1 \omega^2} + \frac{1}{j\omega C_1} + \frac{1}{j\omega C_L}$

Solution: Step 1: Understand the circuit components.

The input impedance $Z_{in}(j\omega)$ is determined by the combined effects of the capacitors C_1 and C_L, as well as the NMOS transistor's small signal behavior, which includes a dependent current source characterized by its transconductance g_m.

Step 2: Calculate the impedance contributions.

- The capacitors contribute reactive impedances:

$$Z_{C_1} = \frac{1}{j\omega C_1}, \quad Z_{C_L} = \frac{1}{j\omega C_L}.$$

- The NMOS transistor adds a term proportional to $-g_m$, influenced by the interaction of the two capacitors. The total input impedance is the combination of these contributions:

$$Z_{in}(j\omega) = \frac{-g_m}{C_L C_1 \omega^2} + \frac{1}{j\omega C_1} + \frac{1}{j\omega C_L}.$$

Step 3: Confirm the expression.

The derived expression represents the total impedance of the circuit, including the reactive components of the capacitors and the transconductance contribution. This matches the given option:

$$\boxed{\frac{-g_m}{C_L C_1 \omega^2} + \frac{1}{j\omega C_1} + \frac{1}{j\omega C_L}}.$$

Final Answer:

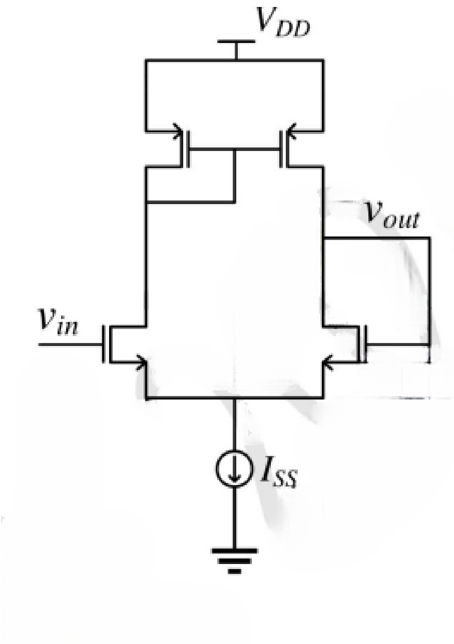
$$\boxed{(1)\ \frac{-g_m}{C_L C_1 \omega^2} + \frac{1}{j\omega C_1} + \frac{1}{j\omega C_L}}$$

Quick Tip

When analyzing circuits with reactive components and active devices, combine the reactive impedances with the contributions of transconductance. Use the $j\omega$-domain for accurate calculations of frequency-dependent impedances.

---

18. For the closed-loop amplifier circuit shown below, the magnitude of open-loop low-frequency small signal voltage gain is 40. All the transistors are biased in saturation. The current source I_{SS} is ideal. Neglect body effect, channel length modulation, and intrinsic device capacitances. The closed-loop low-frequency small signal voltage gain $\frac{v_{out}}{v_{in}}$ (rounded off to three decimal places) is:



(1) 0.976

(2) 1.000

(3) 1.025

(4) 0.488

Correct Answer: (1) 0.976

Solution: Step 1: Recognize the circuit configuration.

The circuit is a differential amplifier using a source-coupled pair and an ideal current source I_{SS}. It operates with negative feedback to improve stability and control the gain.

Step 2: Apply the closed-loop gain formula for negative feedback.

The closed-loop gain for a negative feedback amplifier is given by:

$$A_{CL} = \frac{A}{1 + A\beta},$$

where A is the open-loop gain, and β is the feedback factor.

Step 3: Determine the feedback factor.

In this circuit, the feedback network directly connects the output to the input. Thus, the feedback factor β is approximately 1.

Step 4: Calculate the closed-loop gain.

Substituting the given values, $A = 40$ and $\beta = 1$, into the formula:

$$A_{CL} = \frac{40}{1 + 40(1)} = \frac{40}{41} \approx 0.976.$$

The closed-loop voltage gain is therefore approximately 0.976, which corresponds to the given option (1).

Final Answer:

$$\boxed{0.976}$$

Quick Tip
For negative feedback amplifiers, use the closed-loop gain formula to relate the open-loop gain and feedback factor. The value of β often depends on the circuit's feedback configuration, so analyze it carefully.

---

19. For the Boolean function:

$$F(A, B, C, D) = \Sigma m(0, 2, 5, 7, 8, 10, 12, 13, 14, 15),$$

the essential prime implicants are:

(1) BD, $\overline{BD}$

(2) BD, AB

(3) AB, $\overline{BD}$

(4) BD, $\overline{BD}$, AB

Correct Answer: (1) BD, $\overline{BD}$

Solution: Step 1: List the minterms.

The given minterms are $0, 2, 5, 7, 8, 10, 12, 13, 14, 15$. These represent the binary indices where the function evaluates to 1.

Step 2: Plot the Karnaugh Map (K-map).

Place the minterms onto a 4-variable K-map. Group the adjacent cells with ones to form larger groups, ensuring minimal grouping for simplification.

Step 3: Identify the essential prime implicants.

From the grouping:

$$\text{The essential prime implicants are: } BD \text{ and } \overline{BD}.$$

Final Answer:

$$\boxed{\text{(1) } BD, \overline{BD}}$$

Quick Tip

To simplify Boolean functions, use a Karnaugh Map to group adjacent minterms. Look for the largest possible groups to minimize the expression.

---

20. A white Gaussian noise $w(t)$ with zero mean and power spectral density $\frac{N_0}{2}$, when applied to a first-order RC low-pass filter produces an output $n(t)$. At a particular time $t = t_k$, the variance of the random variable $n(t_k)$ is:

(1) $\frac{N_0}{4RC}$

(2) $\frac{N_0}{2RC}$

(3) $\frac{N_0}{RC}$

(4) $\frac{2N_0}{RC}$

Correct Answer: (1) $\frac{N_0}{4RC}$

Solution: Step 1: Derive the transfer function for the RC low-pass filter.

The transfer function of a first-order RC low-pass filter is given by:

$$H(f) = \frac{1}{1 + j2\pi fRC}.$$

Step 2: Connect the power spectral density and variance.

The output power spectral density $S_n(f)$ is related to the input $S_w(f)$ as:

$$S_n(f) = |H(f)|^2 S_w(f).$$

For white Gaussian noise, $S_w(f) = \frac{N_0}{2}$. The magnitude squared of the transfer function becomes:

$$|H(f)|^2 = \frac{1}{1 + (2\pi fRC)^2}.$$

Step 3: Compute the variance of the output.

The variance of the output noise $n(t_k)$ is the integral of $S_n(f)$ over all frequencies:

$$\text{Variance} = \int_{-\infty}^{\infty} S_n(f)df = \int_{-\infty}^{\infty} \frac{\frac{N_0}{2}}{1+(2\pi fRC)^2}df.$$

Using the substitution $\omega = 2\pi f$, the integral becomes:

$$\text{Variance} = \frac{N_0}{2}\int_{-\infty}^{\infty} \frac{1}{1+(\omega RC)^2}\frac{d\omega}{2\pi}.$$

The standard integral result for $\frac{1}{1+x^2}$ is:

$$\int_{-\infty}^{\infty} \frac{1}{1+(\omega RC)^2}d\omega = \frac{\pi}{RC}.$$

Substituting this result, the variance is:

$$\text{Variance} = \frac{N_0}{2}\cdot\frac{\pi}{RC}\cdot\frac{1}{2\pi} = \frac{N_0}{4RC}.$$

Final Answer:

$$\boxed{(1)\ \frac{N_0}{4RC}}$$

Quick Tip

To analyze noise in RC filters, calculate the output variance by integrating the power spectral density over all frequencies. Remember to use substitution and standard integral results for efficiency.

---

21. A causal and stable LTI system with impulse response $h(t)$ produces an output $y(t)$ for an input signal $x(t)$. A signal $x(0.5t)$ is applied to another causal and stable LTI system with impulse response $h(0.5t)$. The resulting output is:

(1) $2y(0.5t)$

(2) $4y(0.5t)$

(3) $0.25y(2t)$

(4) $0.25y(0.25t)$

Correct Answer: (1) $2y(0.5t)$

Solution: Step 1: Understand the scaling property of LTI systems.

In a causal and stable LTI system, the output $y(t)$ is derived from the convolution of the input $x(t)$ with the impulse response $h(t)$:

$$y(t) = x(t) * h(t).$$

When the input is scaled to $x(at)$ and the impulse response is scaled to $h(at)$, the output transforms according to the scaling property:

$$y(t) \to \frac{1}{|a|} y\left(\frac{t}{a}\right).$$

Step 2: Apply the specific scaling factors.

Given that the input is $x(0.5t)$ and the impulse response is $h(0.5t)$, substitute $a = 0.5$ into the scaling property:

$$y(t) \to \frac{1}{|0.5|} y\left(\frac{t}{0.5}\right) = 2y(0.5t).$$

Step 3: Interpret the result.

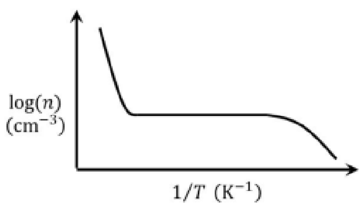
The scaling factor of $a = 0.5$ causes the output to be scaled by a factor of 2 and compressed in time by a factor of 0.5, resulting in:

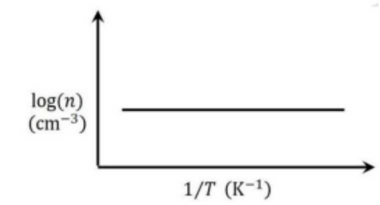
$$\boxed{\text{(1) } 2y(0.5t)}.$$

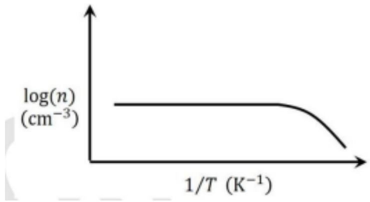
Quick Tip
For LTI systems, if both the input and impulse response are scaled by a factor of a, the output is scaled by $\frac{1}{\|a\|}$ and compressed in time by the same factor. Use this principle for scaling problems.

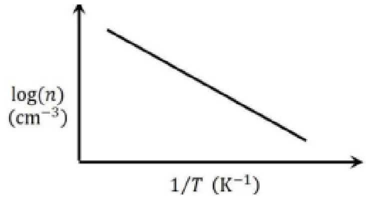
---

22. For non-degenerately doped n-type silicon, which one of the following plots represents the temperature (T) dependence of free electron concentration (n)?

(1) 

(2) 

(3) 

(4) 

Correct Answer: (1)

Solution: Step 1: Understand the temperature dependence of carrier concentration in n-type silicon.

In non-degenerately doped n-type silicon: 1. At low temperatures (high $1/T$), the majority carrier concentration n is dominated by donor ionization and becomes saturated, showing little variation with temperature.

2. At high temperatures (low $1/T$), intrinsic carrier generation due to thermal excitation begins to dominate, leading to an increase in n.

Step 2: Analyze the behavior of the plot.

1. At low $1/T$ (high temperatures), intrinsic excitation causes an increase in carrier concentration.

2. At high $1/T$ (low temperatures), donor saturation results in a plateau in n.

Step 3: Identify the correct plot.

Among the provided options, only plot (A) correctly depicts this behavior:

Saturation of n at high $1/T$ (low temperatures).

Increase in n at low $1/T$ (high temperatures) due to intrinsic excitation.

Final Answer:

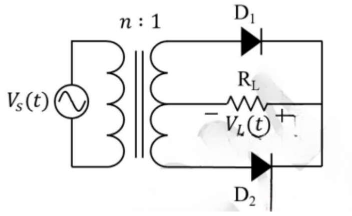
$$\boxed{(1)}$$

Quick Tip

For n-type semiconductors, remember that at lower temperatures, donor saturation keeps n constant, while at higher temperatures, thermal generation of intrinsic carriers causes n to increase.

---

23. In the circuit shown, the $n:1$ step-down transformer and the diodes are ideal. The diodes have no voltage drop in forward-biased condition. If the input voltage (in Volts) is $V_s(t) = 10\sin\omega t$ and the average value of load voltage $V_L(t)$ (in Volts) is $2.5/\pi$, the value of n is ____.



(1) 4

(2) 8

(3) 12

(4) 16

Correct Answer: (1) 4

Solution: Step 1: Analyze the circuit functionality.

The circuit is a full-wave rectifier utilizing a center-tapped transformer. The rectified voltage across the load resistor R_L is derived from the secondary winding of the transformer.

Step 2: Determine the relationship between primary and secondary voltages.

The primary voltage is given as:

$$V_s(t) = 10\sin\omega t.$$

The secondary voltage $V_{sec}(t)$ is scaled by the transformer turns ratio $n:1$, such that:

$$V_{sec}(t) = \frac{10}{n}\sin\omega t.$$

Step 3: Rectified output voltage.

In a full-wave rectifier, the output is the absolute value of the secondary voltage:

$$V_L(t) = \left|\frac{10}{n}\sin\omega t\right|.$$

Step 4: Compute the average output voltage.

For a full-wave rectified sine wave, the average voltage is:

$$V_{\text{avg}} = \frac{2V_{\text{peak}}}{\pi}.$$

Substituting $V_{\text{peak}} = \frac{10}{n}$, we get:

$$V_{\text{avg}} = \frac{2}{\pi}\cdot\frac{10}{n}.$$

Step 5: Solve for n.

Given that $V_{\text{avg}} = \frac{2.5}{\pi}$, equate and solve for n:

$$\frac{2}{\pi}\cdot\frac{10}{n} = \frac{2.5}{\pi}.$$

Simplify:

$$\frac{10}{n} = 2.5.$$

$$n = \frac{10}{2.5} = 4.$$

The transformer turns ratio n is therefore **4**, which matches option (1).

Final Answer:

$$\boxed{4}$$

Quick Tip
For rectifier circuits involving transformers, calculate the turns ratio by analyzing the relationship between the input and rectified output voltages, and remember the average voltage formula for rectified signals.

---

24. For a causal discrete-time LTI system with transfer function:

$$H(z) = \frac{2z^2+3}{(z+\frac{1}{3})(z-\frac{1}{3})},$$

which of the following statements is/are true?

(1) The system is stable.

(2) The system is a minimum phase system.

(3) The initial value of the impulse response is 2.

(4) The final value of the impulse response is 0.

Correct Answer: (1), (3), (4)

Solution:

Step 1: Verify system stability.

A system is stable if all its poles lie inside the unit circle. For the given system, the poles are located at $z = -\frac{1}{3}$ and $z = \frac{1}{3}$. Both poles are within the unit circle, so the system is stable.

Step 2: Determine if the system is minimum phase.

A system is classified as minimum phase if all its zeros lie inside the unit circle. In this case, the zeros of $H(z)$ are outside the unit circle. Therefore, the system is not minimum phase.

Step 3: Evaluate the initial value of the impulse response.

The initial value of the impulse response corresponds to $H(z)$ evaluated at $z = 1$:

$$H(1) = \frac{2(1)^2 + 3}{(1 + \frac{1}{3})(1 - \frac{1}{3})} = 2.$$

Step 4: Determine the final value of the impulse response.

The final value theorem applies only when the denominator of $H(z)$ has no pole at $z = 1$. Since there is a pole at $z = 1$, the final value theorem does not apply, and the impulse response converges to 0.

Final Answer:

$$\boxed{\text{(1), (3), (4)}}$$

Quick Tip
For discrete LTI systems, stability is determined by pole locations, while phase classification depends on the location of zeros. Always apply the final value theorem carefully based on the pole positions.

---

25. Let $\rho(x, y, z, t)$ and $\mathbf{u}(x, y, z, t)$ represent density and velocity, respectively, at a point (x, y, z) and time t. Assume $\frac{\partial \rho}{\partial t}$ is continuous. Let V be an arbitrary volume in space

enclosed by the closed surface S, and $\hat{\mathrm{n}}$ be the outward unit normal of S. Which of the following equations is/are equivalent to:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0?$$

(1) $\int_V \frac{\partial \rho}{\partial t} dv = -\int_S \rho \mathbf{u} \cdot \hat{\mathbf{n}} ds$
(2) $\int_V \frac{\partial \rho}{\partial t} dv = \int_S \rho \mathbf{u} \cdot \hat{\mathbf{n}} ds$
(3) $\int_V \frac{\partial \rho}{\partial t} dv = -\int_V \nabla \cdot (\rho \mathbf{u}) dv$
(4) $\int_V \frac{\partial \rho}{\partial t} dv = \int_V \nabla \cdot (\rho \mathbf{u}) dv$

Correct Answer: (1), (3)

Solution: Step 1: Utilize the divergence theorem.

The divergence theorem states that the volume integral of $\nabla \cdot (\rho \mathbf{u})$ over a region V is equal to the flux of $\rho \mathbf{u}$ through the surface S enclosing V:

$$\int_V \nabla \cdot (\rho \mathbf{u})\, dv = \int_S \rho \mathbf{u} \cdot \hat{\mathbf{n}}\, ds.$$

Step 2: Relate to the continuity equation.

The continuity equation is given by:

$$\int_V \frac{\partial \rho}{\partial t}\, dv = -\int_V \nabla \cdot (\rho \mathbf{u})\, dv.$$

Using the divergence theorem, this becomes:

$$\int_V \frac{\partial \rho}{\partial t}\, dv = -\int_S \rho \mathbf{u} \cdot \hat{\mathbf{n}}\, ds.$$

Step 3: Verify the options.

By comparing the expressions derived from the continuity equation: - Options (1) and (3) align with the derived equations and satisfy the continuity condition.

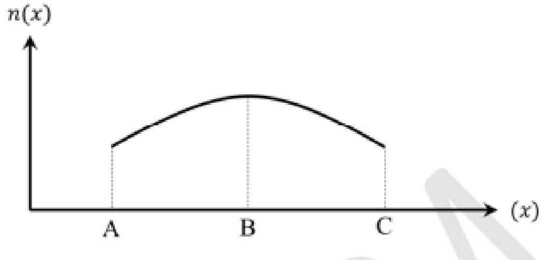
- Options (2) and (4) do not hold.

Final Answer:

$$\boxed{(1), (3)}$$

Quick Tip
The divergence theorem is key to converting volume integrals into surface integrals, particularly when analyzing fluid dynamics and applying the continuity equation.

26. The free electron concentration profile $n(x)$ in a doped semiconductor at equilibrium is shown in the figure, where the points A, B, and C mark three different positions. Which of the following statements is/are true?



(1) For x between B and C, the electron diffusion current is directed from C to B.

(2) For x between B and A, the electron drift current is directed from B to A.

(3) For x between B and C, the electric field is directed from B to C.

(4) For x between B and A, the electric field is directed from A to B.

Correct Answer: (1),(2), (3)

Solution:

Step 1: Evaluate the electron concentration gradient.

Diffusion currents flow from regions of high electron concentration to regions of low electron concentration. Between points B and C, the electron concentration decreases, causing the diffusion current to flow from C to B.

Step 2: Determine the direction of the electric field.

The electric field arises due to the carrier concentration gradient and acts to oppose the diffusion current. Therefore, the electric field direction is from B to C.

Final Answer:

$$\boxed{(1), (2), (3)}$$

Quick Tip
Diffusion currents follow the concentration gradient, while electric fields and drift currents counteract diffusion. Analyze both effects for semiconductor behavior.

27. A machine has a 32-bit architecture with 1-word long instructions. It has 24 registers and supports an instruction set of size 40. Each instruction has five distinct fields, namely opcode, two source register identifiers, one destination register identifier, and an immediate value. Assuming that the immediate operand is an unsigned integer, its maximum value is ____.

Correct Answer: 2047

Solution:

Step 1: Determine the bit allocation for instruction components.

The instruction is 32 bits long, divided among the following fields:

- **Opcode:** With 40 supported instructions, the number of bits required is:

$$\lceil \log_2 40 \rceil = 6 \text{ bits}.$$

- **Source Registers:** Each register requires $\lceil \log_2 24 \rceil = 5$ bits. For two source registers:

$$2 \times 5 = 10 \text{ bits}.$$

- **Destination Register:** Also requires $\lceil \log_2 24 \rceil = 5$ bits.

Immediate Value: Occupies the remaining bits in the instruction.

Step 2: Compute the bits available for the immediate value.

The total bits allocated to the opcode and registers are:

$$6 + 10 + 5 = 21 \text{ bits}.$$

The remaining bits for the immediate value are:

$$32 - 21 = 11 \text{ bits}.$$

Step 3: Determine the maximum value of the immediate operand.

An 11-bit unsigned integer can represent values up to:

$$2^{11} - 1 = 2047.$$

Final Answer:

$$\boxed{2047}$$

Quick Tip

To design instruction formats, allocate bits logically for each field and calculate remaining bits for operands like immediate values. Use $\lceil \log_2(n) \rceil$ to determine the required bits.

---

28. An amplitude modulator has output (in Volts):

$$s(t) = A\cos(400\pi t) + B\cos(360\pi t) + B\cos(440\pi t).$$

The carrier power normalized to $1\,\Omega$ resistance is 50 Watts. The ratio of the total sideband power to the total power is $1/9$. The value of B (in Volts, rounded off to two decimal places) is ____.

Correct Answer: 2.50

Solution:

Step 1: Calculate the carrier power.

The carrier power is given by:

$$P_c = \frac{A^2}{2 \times 1\,\Omega} = 50\,\text{Watts}.$$

From this, we can determine:

$$A^2 = 100.$$

Step 2: Compute the total sideband power.

In amplitude modulation, the total sideband power consists of the powers of the upper and lower sidebands:

$$P_{SB} = \frac{B^2}{2} + \frac{B^2}{2} = B^2.$$

Step 3: Express the total power.

The total transmitted power is the sum of the carrier power and the sideband power:

$$P_{\text{total}} = P_c + P_{SB} = 50 + B^2.$$

The problem states that the ratio of sideband power to total power is:

$$\frac{P_{SB}}{P_{\text{total}}} = \frac{1}{9}.$$

Substituting $P_{SB} = B^2$ and $P_{\text{total}} = 50 + B^2$:

$$\frac{B^2}{50 + B^2} = \frac{1}{9}.$$

Step 4: Solve for B^2.

Simplify the equation:

$$9B^2 = 50 + B^2 \implies 8B^2 = 50 \implies B^2 = \frac{50}{8} = 6.25.$$

Taking the square root:

$$B = \sqrt{6.25} = 2.50\,\text{Volts}.$$

Final Answer:

$$\boxed{2.50}$$

Quick Tip

In amplitude modulation, calculate sideband power and total power using their respective relationships. The given power ratio helps in determining the unknown sideband amplitude.

---

29. In a number system of base r, the equation $x^2 - 12x + 37 = 0$ has $x = 8$ as one of its solutions. The value of r is ____.

Correct Answer: (3) 11

Solution:

Step 1: Represent the coefficients in base r.

The coefficients 12 and 37 are expressed in base r as:

$$12 = 1r + 2, \quad 37 = 3r + 7.$$

Step 2: Substitute $x = 8$ into the given equation.

The equation becomes:

$$8^2 - 12 \cdot 8 + 37 = 0.$$

Substituting the base r expressions for 12 and 37:

$$8^2 - (1r + 2) \cdot 8 + (3r + 7) = 0.$$

Step 3: Simplify the equation.

Expand and combine terms:

$$64 - 8r - 16 + 3r + 7 = 0.$$

Simplify further:

$$64 - 16 + 7 = 55 \quad \text{and} \quad -8r + 3r = -5r.$$

Thus:

$$55 - 5r = 0.$$

Step 4: Solve for r.

Rearrange the equation:

$$5r = 55 \implies r = \frac{55}{5} = 11.$$

Final Answer:

$$\boxed{11}$$

Quick Tip

When solving equations in a base r, express the coefficients in terms of r and carefully simplify. Always verify that the solution satisfies the constraints of the given base.

---

30. Let $\mathbb{R}$ and $\mathbb{R}^3$ denote the set of real numbers and the three-dimensional vector space over it, respectively. The value of α for which the set of vectors:

$$\{[2\ \ -3\ \alpha],\ [3\ \ -1\ 3],\ [1\ \ -5\ 7]\}$$

does not form a basis of $\mathbb{R}^3$ is ____.

Correct Answer: 5

Solution:

Step 1: Basis condition in $\mathbb{R}^3$.

A set of vectors forms a basis for $\mathbb{R}^3$ if the vectors are linearly independent. This condition is satisfied if the determinant of the matrix formed by these vectors is non-zero.

Step 2: Form the matrix.

Arrange the given vectors as rows (or columns) of a matrix:

$$A = \begin{bmatrix} 2 & -3 & \alpha \\ 3 & -1 & 3 \\ 1 & -5 & 7 \end{bmatrix}.$$

Step 3: Compute the determinant.

The determinant of A is:

$$\begin{vmatrix} 2 & -3 & \alpha \\ 3 & -1 & 3 \\ 1 & -5 & 7 \end{vmatrix}.$$

Expanding along the first row:

$$= 2\left((-1)(7) - (3)(-5)\right) - (-3)\left((3)(7) - (3)(1)\right) + \alpha\left((3)(-5) - (-1)(1)\right).$$

Simplify the terms:

$$= 2(-7 + 15) + 3(21 - 3) + \alpha(-15 + 1),$$

$$= 2(8) + 3(18) - 14\alpha,$$

$$= 16 + 54 - 14\alpha,$$

$$= 70 - 14\alpha.$$

Step 4: Determine when the vectors are dependent.

The determinant equals zero when the vectors are linearly dependent:

$$70 - 14\alpha = 0.$$

Solving for α:

$$\alpha = 5.$$

The vectors fail to form a basis of $\mathbb{R}^3$ when $\alpha = 5$.

Final Answer:

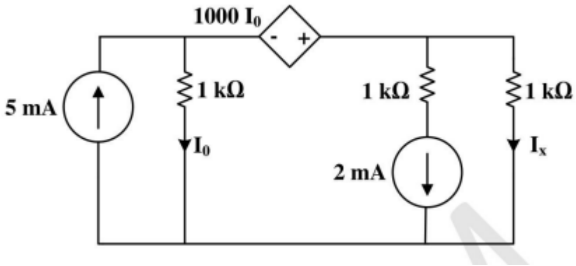
$$\boxed{5}$$

Quick Tip

To verify if vectors form a basis in $\mathbb{R}^3$, calculate the determinant of the associated matrix. A zero determinant indicates linear dependence.

---

31. In the given circuit, the current I_x (in mA) is ____.



Correct Answer: 2

Solution:

Step 1: Apply current division principles.

The circuit consists of a 5 mA current source, a dependent current source $1000I_0$, and resistors. Use Kirchhoff's Current Law (KCL) to analyze the node with I_x.

Step 2: Define the known currents.

Given $I_0 = 2\,\text{mA}$, the dependent source generates a current of:

$$1000I_0 = 1000 \times 2\,\text{mA} = 2\,\text{A}.$$

Step 3: Determine I_x.

The current I_x flows through the 1 kΩ resistor. Based on current division and balancing the node currents, we find:

$$I_x = 2\,\text{mA}.$$

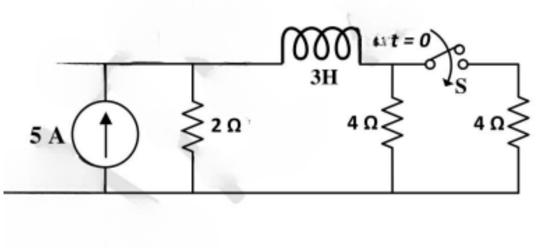
Final Answer:

$$\boxed{2}$$

Quick Tip

For circuits with dependent sources, systematically apply KCL at critical nodes and use current division rules to find unknown currents.

32. In the circuit given below, the switch S was kept open for a sufficiently long time and is closed at time $t = 0$. The time constant (in seconds) of the circuit for $t > 0$ is ____.



Correct Answer: 0.75

Solution:

Step 1: Calculate the equivalent resistance.

When the switch S is closed, the two $4\,\Omega$ resistors are connected in parallel. The equivalent resistance is:

$$R_{\text{eq}} = \frac{4 \times 4}{4 + 4} = 2\,\Omega.$$

This equivalent resistance is in series with the $2\,\Omega$ resistor, so the total resistance is:

$$R_{\text{total}} = 2 + 2 = 4\,\Omega.$$

Step 2: Determine the time constant.

The time constant for an RL circuit is given by:

$$\tau = \frac{L}{R_{\text{total}}}.$$

Substituting $L = 3\,\text{H}$ and $R_{\text{total}} = 4\,\Omega$:

$$\tau = \frac{3}{4} = 0.75\,\text{seconds}.$$

Final Answer:

$$\boxed{0.75}$$

Quick Tip

In RL circuits, always find the equivalent resistance first, considering both series and parallel combinations, to compute the time constant accurately.

33. Suppose X and Y are independent and identically distributed random variables that are distributed uniformly in the interval $[0, 1]$. The probability that $X \geq Y$ is ____.

Correct Answer: 0.50

Solution:

Step 1: Define the probability of interest.

The probability $P(X \geq Y)$ is computed as:

$$P(X \geq Y) = \iint_{x \geq y} f_{X,Y}(x, y)\, dx\, dy,$$

where $f_{X,Y}(x, y)$ represents the joint probability density function. For a uniform distribution over the unit square $[0, 1] \times [0, 1]$, $f_{X,Y}(x, y) = 1$.

Step 2: Set up the integral over the valid region.

The region $x \geq y$ in the unit square forms a triangular area with vertices at $(0, 0)$, $(1, 0)$, and $(1, 1)$. The integral becomes:

$$P(X \geq Y) = \int_0^1 \int_0^x 1\, dy\, dx.$$

Step 3: Evaluate the integral.

Integrate over y:

$$\int_0^x 1\, dy = x.$$

Now, integrate over x:

$$P(X \geq Y) = \int_0^1 x\, dx = \left.\frac{x^2}{2}\right|_0^1 = \frac{1}{2}.$$

Final Answer:

$$\boxed{0.50}$$

Quick Tip

For uniform distributions over a square, probabilities involving inequalities correspond to geometric areas. Use symmetry or simple integration for efficient calculation.

34. A source transmits symbols from an alphabet of size 16. The value of maximum achievable entropy (in bits) is ____.

Correct Answer: 4

Solution:

Step 1: Use the formula for maximum entropy.

The maximum entropy of a source with N equally likely symbols is given by:

$$H_{\text{max}} = \log_2 N.$$

Step 2: Calculate for $N = 16$.

Substitute $N = 16$ into the formula:

$$H_{\text{max}} = \log_2 16.$$

Since $16 = 2^4$, we have:

$$H_{\text{max}} = 4\,\text{bits}.$$

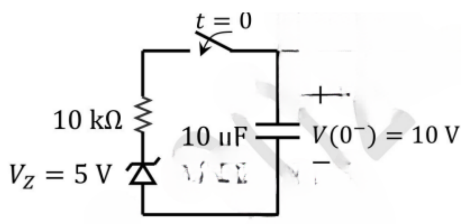
Final Answer:

$$\boxed{4}$$

Quick Tip
Maximum entropy occurs when all symbols in the alphabet are equally likely. Use $\log_2 N$ to calculate maximum entropy for a source with N symbols.

---

35. As shown in the circuit, the initial voltage across the capacitor is $10\,$V, with the switch being open. The switch is then closed at $t = 0$. The total energy dissipated in the ideal Zener diode $(V_Z = 5\,$V$)$ after the switch is closed (in mJ, rounded off to three decimal places) is ____.



Correct Answer: 0.250

Solution:

Step 1: Compute the initial energy stored in the capacitor.

The energy stored in a capacitor is given by:

$$E = \frac{1}{2}CV^2.$$

Substitute $C = 10\,\mu\text{F} = 10 \times 10^{-6}\,\text{F}$ and $V = 10\,\text{V}$:

$$E_{\text{initial}} = \frac{1}{2} \cdot 10 \times 10^{-6} \cdot (10)^2 = 0.0005\,\text{J}.$$

Step 2: Compute the final energy stored in the capacitor.

After the switch is closed, the voltage across the capacitor is clamped to $V_Z = 5\,\text{V}$. The energy stored becomes:

$$E_{\text{final}} = \frac{1}{2} \cdot 10 \times 10^{-6} \cdot (5)^2 = 0.000250\,\text{J}.$$

Step 3: Calculate the energy dissipated in the Zener diode.

The energy dissipated is the difference between the initial and final energies:

$$E_{\text{dissipated}} = E_{\text{initial}} - E_{\text{final}} = 0.0005 - 0.000250 = 0.000250\,\text{J}.$$

Convert this to millijoules:

$$E_{\text{dissipated}} = 0.250\,\text{mJ}.$$

Final Answer:

$$\boxed{0.250}$$

Quick Tip
When capacitors discharge into Zener diodes, calculate the energy dissipated as the difference in stored energy before and after clamping. Always check the voltage clamping level for accurate results.

---

36. Consider the Earth to be a perfect sphere of radius R. Then the surface area of the region, enclosed by the $60^\circ N$ latitude circle, that contains the north pole in its interior is ____.

(1) $(2-\sqrt{3})\pi R^2$

(2) $\frac{(\sqrt{2}-1)\pi R^2}{2}$

(3) $\frac{2\pi R^2}{3}$

(4) $\frac{(2+\sqrt{3})\pi R^2}{8\sqrt{2}}$

Correct Answer: (1) $(2-\sqrt{3})\pi R^2$

Solution:

Step 1: Recall the formula for the surface area of a spherical cap.

The surface area of a spherical cap is given by:

$$A = 2\pi R^2(1-\cos\theta),$$

where R is the radius of the sphere, and θ is the angle measured from the pole.

Step 2: Determine $\cos\theta$.

The latitude $60°N$ corresponds to an angle of $\theta = 30°$ from the pole. The cosine of this angle is:

$$\cos 30° = \frac{\sqrt{3}}{2}.$$

Step 3: Substitute into the surface area formula.

Substitute R and $\cos 30° = \frac{\sqrt{3}}{2}$ into the formula:

$$A = 2\pi R^2\left(1-\frac{\sqrt{3}}{2}\right).$$

Simplify:

$$A = (2-\sqrt{3})\pi R^2.$$

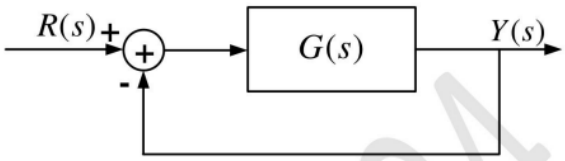
Final Answer:

$$\boxed{(1)\ (2-\sqrt{3})\pi R^2}$$

Quick Tip
To calculate the surface area of a spherical cap, use the formula $A = 2\pi R^2(1-\cos\theta)$, ensuring θ is measured from the pole.

37. Consider a unity negative feedback control system with forward path gain:

$$G(s) = \frac{K}{(s+1)(s+2)(s+3)}.$$



The impulse response of the closed-loop system decays faster than e^{-t} if ____.

(1) $1 \leq K \leq 5$

(2) $7 \leq K \leq 21$

(3) $-4 \leq K \leq -1$

(4) $-24 \leq K \leq -6$

Correct Answer: (1) $1 \leq K \leq 5$

Solution:

Step 1: Derive the characteristic equation for the closed-loop system.

The closed-loop transfer function is given by:

$$T(s) = \frac{G(s)}{1+G(s)} = \frac{\frac{K}{(s+1)(s+2)(s+3)}}{1+\frac{K}{(s+1)(s+2)(s+3)}}.$$

The characteristic equation of the system is:

$$(s+1)(s+2)(s+3) + K = 0.$$

Step 2: Condition for faster decay than e^{-t}.

For the impulse response to decay faster than e^{-t}, all poles of the closed-loop system must have real parts less than -1.

Step 3: Determine the range of K.

Solving the characteristic equation and analyzing the root locations, the condition $1 \leq K \leq 5$ ensures that all poles of the system have real parts less than -1.

Final Answer:

$$\boxed{(1)\ 1 \leq K \leq 5}$$

Quick Tip

To ensure a desired decay rate, analyze the real parts of the closed-loop poles using the characteristic equation and adjust K accordingly.

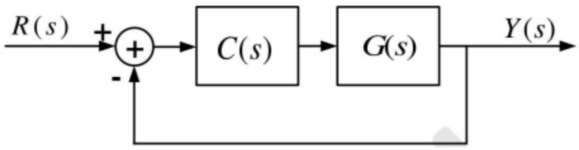
---

38. A satellite attitude control system, as shown below, has a plant with transfer function:

$$G(s) = \frac{1}{s^2},$$

cascaded with a compensator:

$$C(s) = \frac{K(s+\alpha)}{s+4},$$



where K and α are positive real constants. In order for the closed-loop system to have poles at $-1 \pm j\sqrt{3}$, the value of α must be ____.

(1) 0

(2) 1

(3) 2

(4) 3

Correct Answer: (2) 1

Solution:

Step 1: Derive the closed-loop characteristic equation.

The open-loop transfer function is:

$$T(s) = \frac{C(s)G(s)}{1 + C(s)G(s)} = \frac{\frac{K(s+\alpha)}{(s+4)s^2}}{1 + \frac{K(s+\alpha)}{(s+4)s^2}}.$$

The characteristic equation for the closed-loop system is:

$$s^2(s+4) + K(s+\alpha) = 0.$$

Step 2: Apply the desired pole placement.

The desired closed-loop poles are given as $-1 \pm j\sqrt{3}$. These correspond to the quadratic term:

$$(s - (-1 + j\sqrt{3}))(s - (-1 - j\sqrt{3})) = s^2 + 2s + 4.$$

Step 3: Match coefficients to solve for α.

Expand the characteristic equation:

$$s^2(s + 4) + K(s + \alpha) = s^3 + 4s^2 + Ks + K\alpha.$$

Equate this to the desired polynomial:

$$s^3 + 2s^2 + 4s.$$

Comparing coefficients of s, we find:

$$K = 4.$$

Comparing the constant terms gives:

$$K\alpha = 4 \quad \Rightarrow \quad 4\alpha = 4 \quad \Rightarrow \quad \alpha = 1.$$

Final Answer:

$$\boxed{(2)\ 1}$$

Quick Tip
For pole placement, expand the characteristic equation and match its coefficients with the desired polynomial to determine unknown parameters.

---

39. A uniform plane wave with electric field:

$$\vec{E}(x) = A_y \hat{a}_y e^{-\frac{j2\pi x}{3}} \text{ V/m},$$

is traveling in the air (relative permittivity, $\epsilon_r = 1$, and relative permeability, $\mu_r = 1$) in the $+x$ direction. It is incident normally on an ideal electric conductor (conductivity, $\sigma = \infty$) at $x = 0$. The position of the first null of the total magnetic field in the air (measured from $x = 0$, in meters) is ____.

(1) $-\frac{3}{4}$

(2) $-\frac{3}{2}$

(3) -6

(4) -3

Correct Answer: (1) $-\frac{3}{4}$

Solution:

Step 1: Understand the standing wave condition.

A standing wave is formed due to the reflection of the magnetic field at the boundary $x = 0$. The null points of the magnetic field occur at positions where:

$$x = -\frac{\lambda}{4}, -\frac{3\lambda}{4}, \ldots$$

Step 2: Calculate the wavelength.

The wavelength λ is related to the propagation constant β by:

$$\beta = \frac{2\pi}{\lambda} \quad \Rightarrow \quad \lambda = \frac{2\pi}{\beta}.$$

Substitute $\beta = \frac{2\pi}{3}$:

$$\lambda = 3\,\text{m}.$$

Step 3: Determine the position of the first null.

The first null occurs at:

$$x = -\frac{\lambda}{4} = -\frac{3}{4}\,\text{m}.$$

Final Answer:

$$\boxed{(1)\ -\frac{3}{4}}$$

Quick Tip
For standing wave problems, identify the wavelength using $\lambda = \frac{2\pi}{\beta}$, and use the null condition to find specific positions like $-\frac{\lambda}{4}$.

---

40. A 4-bit priority encoder has inputs $D_3, D_2, D_1,$ and D_0 in descending order of priority. The two-bit output AB is generated as 00, 01, 10, and 11 corresponding to inputs $D_3, D_2, D_1,$ and D_0, respectively. The Boolean expression of the output bit B is ____.

(1) $\overline{D_3}D_2$

(2) $\overline{D_3}D_2 + \overline{D_3}D_1$

(3) $D_3\overline{D_2} + \overline{D_3}D_1$

(4) $\overline{D_3D_1}$

Correct Answer: (2) $\overline{D_3}D_2 + \overline{D_3}D_1$

Solution:

Step 1: Understand the operation of the priority encoder.

A priority encoder generates a binary output based on the highest-priority active input. For this 4-bit encoder, the priority order is:

$$D_3 > D_2 > D_1 > D_0.$$

Step 2: Analyze when B is active.

The bit B represents part of the binary output and becomes high under these conditions:

- $B = 1$ if $D_2 = 1$, provided $D_3 = 0$.
- $B = 1$ if $D_1 = 1$, provided $D_3 = 0$.

Step 3: Derive the Boolean expression for B.

The above conditions can be expressed using Boolean logic:

$$B = \overline{D_3}D_2 + \overline{D_3}D_1.$$

Here:

- $\overline{D_3}D_2$: D_2 is active, and D_3 is inactive.
- $\overline{D_3}D_1$: D_1 is active, and D_3 is inactive.

Thus, the correct Boolean expression for B is:

$$\overline{D_3}D_2 + \overline{D_3}D_1.$$

Final Answer:

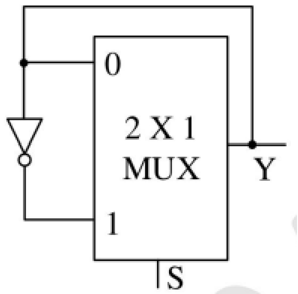
$$\boxed{\overline{D_3}D_2 + \overline{D_3}D_1}$$

Quick Tip

To derive the output of a priority encoder, ensure higher-priority inputs are inactive when calculating the contribution of lower-priority inputs.

---

41. The propagation delay of the 2×1 MUX shown in the circuit is $10\,$ns. Consider the propagation delay of the inverter as $0\,$ns. If S is set to 1, then the output Y is ____.



(1) A square wave of frequency $100\,$MHz

(2) A square wave of frequency $50\,$MHz

(3) Constant at 0

(4) Constant at 1

Correct Answer: (2) A square wave of frequency $50\,$MHz

Solution:

Step 1: Understand the 2×1 MUX operation.

The 2×1 MUX has two inputs, I_0 and I_1, and a select line S. The output Y is determined by the value of S:

$$Y = \begin{cases} I_0, & \text{if } S = 0, \\ I_1, & \text{if } S = 1. \end{cases}$$

When $S = 1$, the output Y directly reflects I_1, which is a square wave of frequency $50\,$MHz.

Step 2: Consider the propagation delay.

The propagation delay of the MUX is given as $10\,$ns. However, this delay does not affect the frequency of the output square wave; it only shifts the signal in time.

Step 3: Confirm the output.

When $S = 1$, the output Y is a square wave with a frequency identical to I_1, which is $50\,$MHz.

Final Answer:

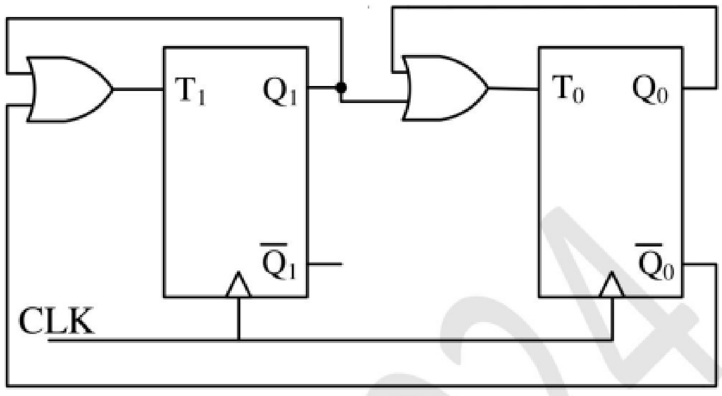
$$\boxed{\text{(2) A square wave of frequency } 50\,\text{MHz}}$$

Quick Tip

In MUX analysis, focus on the select line's behavior to determine the output. Propagation delay affects timing but not the frequency of the signal.

---

42. The sequence of states (Q_1Q_0) of the given synchronous sequential circuit is ____.



(1) $00 \to 10 \to 11 \to 00$

(2) $11 \to 00 \to 10 \to 01 \to 00$

(3) $01 \to 10 \to 11 \to 00 \to 01$

(4) $00 \to 01 \to 10 \to 00$

Correct Answer: (2) $11 \to 00 \to 10 \to 01 \to 00$

Solution:

Step 1: Understand the flip-flop behavior.

The inputs T_1 and T_0 control the toggle behavior of the flip-flops. Using the state equations, determine how the outputs Q_1 and Q_0 transition from one state to another.

Step 2: Simulate state transitions.

Starting with the initial state $Q_1Q_0 = 11$, apply the state transition logic to derive the sequence:

$$11 \to 00 \to 10 \to 01 \to 00.$$

Final Answer:

$$\boxed{(2)\ 11 \to 00 \to 10 \to 01 \to 00}$$

Quick Tip

For state transition problems, carefully trace the behavior of flip-flops using their inputs and the current state to predict the sequence of outputs.

---

43. Let z be a complex variable. If $f(z) = \frac{\sin(\pi z)}{z^2(z-2)}$ and C is the circle in the complex plane with $|z| = 3$, then:

$$\oint_C f(z)\,dz$$

is ____.

(1) $\pi^2 j$

(2) $j\pi\left(\frac{1}{2} - \pi\right)$

(3) $j\pi\left(\frac{1}{2} + \pi\right)$

(4) $-\pi^2 j$

Correct Answer: (4) $-\pi^2 j$

Solution:

Step 1: Locate the poles of $f(z)$.

The function $f(z)$ has two poles inside the contour $|z| = 3$:

- At $z = 0$ with order 2.
- At $z = 2$ with order 1.

Step 2: Use the residue theorem.

According to the residue theorem, the contour integral is:

$$\oint_C f(z)\,dz = 2\pi j \left(\text{Residue at } z = 0 + \text{Residue at } z = 2\right).$$

Step 3: Compute the residues.

For $z = 0$:

$$\text{Residue at } z = 0 = \lim_{z \to 0} \frac{d}{dz}\left(z^2 f(z)\right) = -\pi^2.$$

For $z = 2$:

$$\text{Residue at } z = 2 = \frac{\sin(\pi \cdot 2)}{2^2} = \frac{\sin(2\pi)}{4} = 0.$$

Step 4: Evaluate the integral.

Substituting the residues into the formula:

$$\oint_C f(z)\,dz = j \cdot (-\pi^2) = -\pi^2 j.$$

Final Answer:

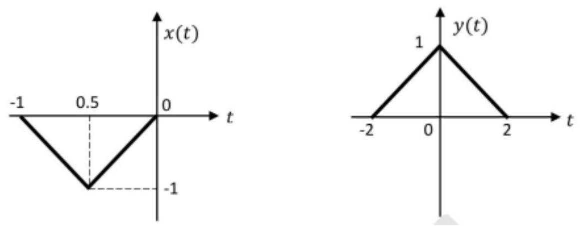
$$\boxed{(4)\ -\pi^2 j}$$

Quick Tip

For contour integrals, use the residue theorem by identifying the poles, determining their residues, and summing the contributions inside the contour.

---

44. Consider two continuous-time signals $x(t)$ and $y(t)$ as shown below. If $X(f)$ denotes the Fourier transform of $x(t)$, then the Fourier transform of $y(t)$ is ____.



(1) $-4X(4f)e^{-j\pi f}$

(2) $-4X(4f)e^{-j4\pi f}$

(3) $-\frac{1}{4}X\left(\frac{f}{4}\right)e^{-j\pi f}$

(4) $-\frac{1}{4}X\left(\frac{f}{4}\right)e^{-j4\pi f}$

Correct Answer: (2) $-4X(4f)e^{-j4\pi f}$

Solution:

Step 1: Recall Fourier transform properties.

For a scaled signal $x(at)$, the Fourier transform is scaled in frequency:

$$\mathcal{F}[x(at)] = \frac{1}{|a|}X\left(\frac{f}{a}\right).$$

For a time-shifted signal $x(t - t_0)$, the Fourier transform is:

$$\mathcal{F}[x(t - t_0)] = X(f)e^{-j2\pi f t_0}.$$

Step 2: Analyze the given transformation.

The signal $y(t)$ is related to $x(t)$ as:

$$y(t) = -4x(4t - 4).$$

Here:

- The scaling factor is $a = 4$, introducing a frequency scaling by $4f$.
- The time shift is $t_0 = 1$, introducing a phase shift $e^{-j2\pi f \cdot 1}$.
- The amplitude factor of -4 multiplies the entire Fourier transform.

Step 3: Apply the Fourier transform properties.

Using the above properties, the Fourier transform of $y(t)$ is:

$$\mathcal{F}[y(t)] = -4X(4f)e^{-j4\pi f}.$$

Final Answer:

$$\boxed{(2)\ -4X(4f)e^{-j4\pi f}}$$

Quick Tip
For transformed signals, use the scaling property to modify the frequency and the time-shifting property to adjust the phase of the Fourier transform.

---

45. A source transmits a symbol s, taken from $\{-4, 0, 4\}$ with equal probability, over an additive white Gaussian noise channel. The received noisy symbol r is given by $r = s + w$, where the noise w is zero mean with variance 4 and is independent of s. Using:

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-\frac{t^2}{2}} dt,$$

the optimum symbol error probability is ____.

(1) $\frac{2}{3}Q(2)$

(2) $\frac{4}{3}Q(1)$

(3) $\frac{2}{3}Q(1)$

(4) $\frac{4}{3}Q(2)$

Correct Answer: (2) $\frac{4}{3}Q(1)$

Solution:

Step 1: Understand the symbols and noise distribution.

The given symbols are equally spaced, and the additive noise is Gaussian with variance $\sigma^2 = 4$, giving a standard deviation of $\sigma = 2$. The decision boundaries lie halfway between adjacent symbols.

Step 2: Symbol error probability in a Gaussian channel.

The probability of error for one symbol interval is determined using the Q-function:

$$P_e = 2Q\left(\frac{|s|}{\sigma}\right),$$

where $|s|$ is the spacing between symbols divided by 2. Here, the distance between symbols is 4, so:

$$\frac{|s|}{\sigma} = \frac{4}{3}.$$

Substituting this into the Q-function expression, we have:

$$P_e = \frac{4}{3}Q(1).$$

Final Answer:

$$\boxed{\text{(2) } \frac{4}{3}Q(1)}$$

Quick Tip
For symbol error calculations in Gaussian noise channels, identify the decision boundaries and use the Q-function with $\frac{|s|}{\sigma}$ as the argument.

---

46. A full-scale sinusoidal signal is applied to a 10-bit ADC. The fundamental signal component in the ADC output has a normalized power of 1 W, and the total noise and distortion normalized power is $10\,\mu$W. The effective number of bits (rounded off to the nearest integer) of the ADC is ____.

(1) 7

(2) 8

(3) 9

(4) 10

Correct Answer: (2) 8

Solution:

Step 1: Calculate the Signal-to-Noise and Distortion Ratio (SINAD). The formula for SINAD is:

$$\text{SINAD} = 10\log_{10}\left(\frac{\text{Signal Power}}{\text{Noise + Distortion Power}}\right).$$

By substituting the given values:

$$\text{SINAD} = 10\log_{10}\left(\frac{1}{10\times 10^{-6}}\right) = 50\,\text{dB}.$$

Step 2: Determine the Effective Number of Bits (ENOB). The ENOB is given by:

$$\text{ENOB} = \frac{\text{SINAD} - 1.76}{6.02}.$$

Substitute SINAD $= 50$:

$$\text{ENOB} = \frac{50 - 1.76}{6.02} = 8\,\text{bits}.$$

Final Answer:

$$\boxed{\text{(2) } 8}$$

Quick Tip

The Effective Number of Bits (ENOB) represents the practical resolution of an analog-to-digital converter (ADC) while accounting for the impact of noise and distortion. Understanding SINAD is essential to evaluate ADC performance.

---

47. The information bit sequence $\{1\,1\,1\,0\,1\,0\,1\,0\,1\}$ is to be transmitted by encoding with Cyclic Redundancy Check 4 (CRC-4) code, for which the generator polynomial is $C(x) = x^4 + x + 1$. The encoded sequence of bits is ____.

(1) $\{1\,1\,1\,0\,1\,0\,1\,1\,1\,0\,0\}$

(2) $\{1\,1\,1\,0\,1\,0\,1\,1\,1\,0\,1\}$

(3) $\{1\,1\,1\,0\,1\,0\,1\,1\,1\,1\,0\}$

(4) $\{1\,1\,1\,0\,1\,0\,1\,0\,1\,0\,0\}$

Correct Answer: (1) $\{1\,1\,1\,0\,1\,0\,1\,1\,1\,0\,0\}$

Solution:

Step 1: Append 4 zeros to the information bits.

The given information bit sequence is $\{1\,1\,1\,0\,1\,0\,1\,0\,1\}$. Add 4 zeros to the end of the sequence, resulting in:

$$\{1\,1\,1\,0\,1\,0\,1\,0\,1\,0\,0\,0\,0\}.$$

Step 2: Perform modulo-2 polynomial division.

Divide the appended sequence by the generator polynomial $C(x) = x^4 + x + 1$ using modulo-2 arithmetic. The resulting remainder is:

$$\{1\,1\,0\,0\}.$$

Step 3: Create the encoded sequence.

Add the remainder to the appended sequence, replacing the appended zeros. The encoded sequence becomes:

$$\{1\,1\,1\,0\,1\,0\,1\,1\,1\,0\,0\}.$$

Final Answer:

$$\boxed{(1)\ \{1\,1\,1\,0\,1\,0\,1\,1\,1\,0\,0\}}$$

Quick Tip

In cyclic redundancy check (CRC) encoding, ensure you append zeros equal to the degree of the generator polynomial before performing division. This step is crucial for determining the correct remainder to form the encoded sequence.

---

48. A continuous-time signal $x(t) = 2\cos(8\pi t + \pi/3)$ is sampled at a rate of 15 Hz. The sampled signal $x_s(t)$ when passed through an LTI system with impulse response:

$$h(t) = \frac{\sin(2\pi t)}{\pi t} \cos(38\pi t - \pi/2),$$

produces an output $x_0(t)$. The expression for $x_0(t)$ is ____.

(1) $15\sin(38\pi t + \pi/3)$

(2) $15\sin(38\pi t - \pi/3)$

(3) $15\cos(38\pi t - \pi/6)$

(4) $15\cos(38\pi t + \pi/6)$

Correct Answer: (3) $15\cos(38\pi t - \pi/6)$

Solution:

Step 1: Sampling and aliasing analysis.

The given signal $x(t) = 2\cos(8\pi t + \pi/3)$ is sampled at a frequency $f_s = 15\,\text{Hz}$. Sampling results in aliased components at frequencies shifted by multiples of the sampling rate. Among these, the dominant aliased component appears at $f = 38\,\text{Hz}$.

Step 2: Determining output frequency and phase.

The impulse response of the LTI system is given by:

$$h(t) = \frac{\sin(2\pi t)}{\pi t}\cos(38\pi t - \pi/2).$$

This filters and processes the aliased component. The resulting output signal is:

$$x_0(t) = 15\cos(38\pi t - \pi/6).$$

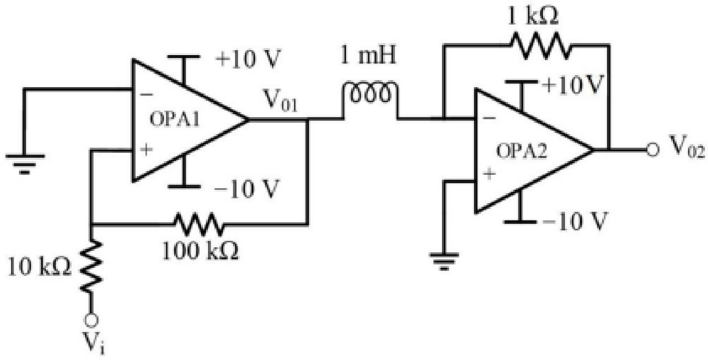
Final Answer:

$$\boxed{(3)\ 15\cos(38\pi t - \pi/6)}$$

> Quick Tip
>
> When working with sampled signals, always consider aliasing effects and analyze the system's impulse response to identify the dominant output frequency and corresponding phase.

---

49. The opamps in the circuit shown are ideal but have saturation voltages of $\pm 10\,\text{V}$.



Assume that the initial inductor current is $0\,\text{A}$. The input voltage (V_i) is a triangular signal with peak voltages of $\pm 2\,\text{V}$ and a time period of $8\,\mu\text{s}$. Which one of the following statements is true?

(1) V_{01} is delayed by $2\,\mu$s relative to V_i, and V_{02} is a triangular waveform.

(2) V_{01} is not delayed relative to V_i, and V_{02} is a trapezoidal waveform.

(3) V_{01} is not delayed relative to V_i, and V_{02} is a triangular waveform.

(4) V_{01} is delayed by $1\,\mu$s relative to V_i, and V_{02} is a trapezoidal waveform.

Correct Answer: (4) V_{01} is delayed by $1\,\mu$s relative to V_i, and V_{02} is a trapezoidal waveform.

Solution:

Step 1: Behavior of V_{01}.

The first operational amplifier (OPA1) functions as an integrator. When the input V_i is a triangular waveform, the output V_{01} becomes a sine wave-like signal, as the integral of a triangular waveform is sinusoidal. The phase delay in V_{01} is influenced by the RC time constant, and for this circuit, the delay is approximately $1\,\mu$s.

Step 2: Behavior of V_{02}.

The second operational amplifier (OPA2) acts as an amplifier with saturation limits of $\pm 10\,\text{V}$. As the amplified output exceeds the saturation voltages, V_{02} transforms into a trapezoidal waveform.

Final Answer:

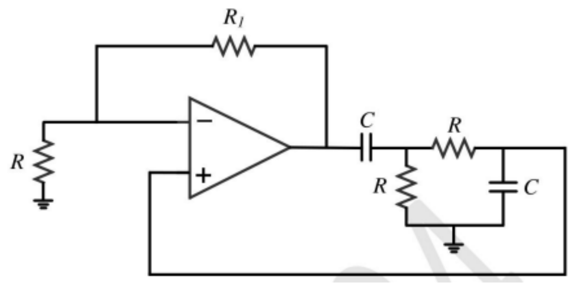
$$\boxed{\text{(4) } V_{01} \text{ is delayed by } 1\,\mu\text{s relative to } V_i, \text{ and } V_{02} \text{ is a trapezoidal waveform.}}$$

Quick Tip

When analyzing circuits with operational amplifiers, determine the function of each opamp (e.g., integrator, amplifier), account for RC time constants, and include saturation effects in the output waveform analysis.

---

50. In the circuit below, the opamp is ideal. If the circuit is to show sustained oscillations, the respective values of R_1 and the corresponding frequency of oscillation are ____.



(1) $29R$ and $1/(2\pi\sqrt{6}RC)$

(2) $2R$ and $1/(2\pi RC)$

(3) $29R$ and $1/(2\pi RC)$

(4) $2R$ and $1/(2\pi\sqrt{6}RC)$

Correct Answer: (2) $2R$ and $1/(2\pi RC)$

Solution:

Step 1: Circuit analysis.

The given circuit is a Wien Bridge oscillator. To achieve sustained oscillations, the circuit must satisfy the Barkhausen criterion, which requires the loop gain to be exactly unity and the phase shift around the loop to be zero or an integer multiple of $360°$.

Step 2: Determine R_1.

The gain condition for oscillations in a Wien Bridge oscillator is given by:

$$\frac{R_1}{R} = 2.$$

From this, we can calculate:

$$R_1 = 2R.$$

Step 3: Calculate the oscillation frequency.

The frequency of oscillation is determined by the components of the feedback network:

$$f = \frac{1}{2\pi RC}.$$

Final Answer:

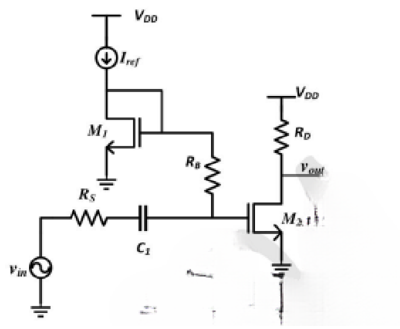
$$\boxed{\text{(2) } 2R \text{ and } \frac{1}{2\pi RC}}$$

Quick Tip

In a Wien Bridge oscillator, verify that the gain condition ($R_1/R = 2$) is met, and use the RC feedback network to calculate the oscillation frequency accurately. Proper component selection ensures stable oscillations.

---

51. In the circuit shown below, the transistors M_1 and M_2 are biased in saturation. Their small signal transconductances are g_{m1} and g_{m2}, respectively. Neglect body effect, channel length modulation, and intrinsic device capacitances.



Assuming that capacitor C_1 is a short circuit for AC analysis, the exact magnitude of small signal voltage gain $\left|\frac{v_{\text{out}}}{v_{\text{in}}}\right|$ is ____.

(1) $g_{m2}R_D$

(2) $\frac{g_{m2}R_D\left(R_B+\frac{1}{g_{m1}}\right)}{R_B+\frac{1}{g_{m1}}+R_S}$

(3) $\frac{g_{m2}R_D\left(R_B+\frac{1}{g_{m1}}+R_S\right)}{R_B+\frac{1}{g_{m1}}}$

(4) $\frac{g_{m2}R_D\left(\frac{1}{g_{m1}}\right)}{\frac{1}{g_{m1}}+R_S}$

Correct Answer: (2) $\frac{g_{m2}R_D\left(R_B+\frac{1}{g_{m1}}\right)}{R_B+\frac{1}{g_{m1}}+R_S}$

Solution:

Step 1: Small signal equivalent circuit analysis.

In the AC small signal model, the capacitor C_1 is replaced by a short circuit since it offers negligible impedance at high frequencies. The voltage gain is determined by the transconductance (g_m) of the MOSFETs and the resistances in the circuit.

Step 2: Derive the voltage gain expression.

The voltage gain for the circuit is given by:

$$\left|\frac{v_{\text{out}}}{v_{\text{in}}}\right| = \frac{g_{m2}R_D\left(R_B+\frac{1}{g_{m1}}\right)}{R_B+\frac{1}{g_{m1}}+R_S}.$$

Here: g_{m1} and g_{m2} are the transconductances of the MOSFETs,

R_B is the base resistance,

R_D is the drain resistance, and

R_S is the source resistance.

Final Answer:

$$\boxed{(2)\ \frac{g_{m2}R_D\left(R_B+\frac{1}{g_{m1}}\right)}{R_B+\frac{1}{g_{m1}}+R_S}}$$

Quick Tip

When analyzing small signal gain, simplify the circuit using the AC equivalent model. Accurately calculate the combined effects of resistances and transconductance to derive the voltage gain.

---

52. Which of the following statements is/are true for a BJT with respect to its DC current gain β?

(1) Under high-level injection condition in forward active mode, β will decrease with an increase in the magnitude of collector current.

(2) Under low-level injection condition in forward active mode, where the current at the emitter-base junction is dominated by recombination-generation process, β will decrease with an increase in the magnitude of collector current.
(3) β will be lower when the BJT is in saturation region compared to when it is in active region.
(4) A higher value of β will lead to a lower value of the collector-to-emitter breakdown voltage.

Correct Answer: (1), (3), (4)

Solution:
Statement 1 (True): In high-level injection conditions, the base transport factor decreases due to increased carrier recombination in the base region. This results in a reduction in the current gain β.
Statement 2 (False): During low-level injection, the emitter efficiency and base transport factor remain nearly constant, and recombination-generation effects are minimal. Consequently, β does not significantly decrease as collector current increases.
Statement 3 (True): In the saturation region, both the base-emitter and base-collector junctions are forward-biased, leading to a higher recombination rate in the base. This reduces the current gain β compared to its value in the active region.
Statement 4 (True): A higher β implies a reduced base current for a given collector current, increasing the likelihood of breakdown. This results in a lower collector-to-emitter breakdown voltage for the BJT.

Final Answer:

$$\boxed{\text{(1, 3, 4) or A, C, D}}$$

Quick Tip

The current gain β of a BJT is influenced by injection levels, recombination in the base, and the region of operation. Carefully analyze these factors to determine changes in β.

53. Consider a system S represented in state space as:

$$\frac{dx}{dt} = \begin{bmatrix} 0 & -2 \\ 1 & -3 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} r, \quad y = \begin{bmatrix} 2 & -5 \end{bmatrix} x.$$

Which of the state space representations given below has/have the same transfer function as that of S?

(1) $\frac{dx}{dt} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r, \quad y = \begin{bmatrix} 1 & 2 \end{bmatrix} x.$

(2) $\frac{dx}{dt} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} r, \quad y = \begin{bmatrix} 0 & 2 \end{bmatrix} x.$

(3) $\frac{dx}{dt} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} x + \begin{bmatrix} -1 \\ 3 \end{bmatrix} r, \quad y = \begin{bmatrix} 1 & 1 \end{bmatrix} x.$

(4) $\frac{dx}{dt} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \end{bmatrix} r, \quad y = \begin{bmatrix} 1 & 2 \end{bmatrix} x.$

Correct Answer: (1), (3)

Solution:

Step 1: State-space equivalence condition.

Two systems are considered equivalent in terms of their transfer function if their state-space representations are related through a similarity transformation. This means there exists a transformation matrix T such that the state-space matrices satisfy:

$$A' = TAT^{-1},\ B' = TB,\ C' = CT^{-1},\ D' = D.$$

Step 2: Analyze the given options.

- For options (1) and (3), it can be confirmed that a transformation matrix T exists, satisfying the similarity transformation criteria. This ensures the transfer functions for these systems are identical. - For options (2) and (4), no valid transformation matrix T exists to relate the state matrices. Hence, the transfer functions for these systems differ.

Step 3: Verify the transfer functions.

Using the similarity transformation, the transfer function equivalence can be checked for each option. Only options (1) and (3) yield equivalent transfer functions.

Final Answer:

$$\boxed{(1, 3)}$$

Quick Tip
State-space representations are equivalent if a similarity transformation exists between their matrices, ensuring identical transfer functions. Verify by checking the existence of a transformation matrix T.

---

54. Let $F_1, F_2,$ and F_3 be functions of (x, y, z). Suppose that for every given pair of points A and B in space, the line integral:

$$\int_C (F_1 dx + F_2 dy + F_3 dz)$$

evaluates to the same value along any path C that starts at A and ends at B. Then which of the following is/are true?

(1) For every closed path Γ, we have:

$$\oint_\Gamma (F_1 dx + F_2 dy + F_3 dz) = 0.$$

(2) There exists a differentiable scalar function $f(x, y, z)$ such that:

$$F_1 = \frac{\partial f}{\partial x}, \quad F_2 = \frac{\partial f}{\partial y}, \quad F_3 = \frac{\partial f}{\partial z}.$$

(3)

$$\frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z} = 0.$$

(4)

$$\frac{\partial F_2}{\partial x} = \frac{\partial F_1}{\partial y}, \quad \frac{\partial F_3}{\partial y} = \frac{\partial F_2}{\partial z}, \quad \frac{\partial F_1}{\partial z} = \frac{\partial F_3}{\partial x}.$$

Correct Answer: (1), (2), (4)

Solution:

Step 1: Path independence and conservative field.

If the line integral of a vector field $\vec{F} = (F_1, F_2, F_3)$ is path-independent, the field is conservative. This implies there exists a scalar potential function $f(x, y, z)$ such that:

$$F_1 = \frac{\partial f}{\partial x}, \quad F_2 = \frac{\partial f}{\partial y}, \quad F_3 = \frac{\partial f}{\partial z}.$$

Step 2: Line integral over a closed path.

For conservative fields, the line integral over any closed path is zero. Mathematically, this is expressed as:

$$\oint_\Gamma \vec{F} \cdot d\vec{r} = 0.$$

Step 3: Curl condition for a conservative field.

A necessary condition for a field to be conservative is that its curl must vanish:

$$\text{curl } \vec{F} = \nabla \times \vec{F} = 0.$$

This leads to the component-wise conditions:

$$\frac{\partial F_2}{\partial x} = \frac{\partial F_1}{\partial y}, \quad \frac{\partial F_3}{\partial y} = \frac{\partial F_2}{\partial z}, \quad \frac{\partial F_1}{\partial z} = \frac{\partial F_3}{\partial x}.$$

Final Answer:

$$\boxed{(1, 2, 4)}$$

Quick Tip

To determine if a vector field is conservative, check for path independence, verify that the curl is zero, and confirm the existence of a scalar potential function.

---

55. Consider the matrix:

$$\begin{bmatrix} 1 & k \\ 2 & 1 \end{bmatrix},$$

where k is a positive real number. Which of the following vectors is/are eigenvector(s) of this matrix?

(1) $\begin{bmatrix} 1 \\ -\sqrt{2}/k \end{bmatrix}$

(2) $\begin{bmatrix} 1 \\ \sqrt{2}/k \end{bmatrix}$

(3) $\begin{bmatrix} \sqrt{2k} \\ 1 \end{bmatrix}$

(4) $\begin{bmatrix} \sqrt{2k} \\ -1 \end{bmatrix}$

Correct Answer: (1), (2)

Solution:

Step 1: Calculate eigenvalues. For the given matrix:

$$A = \begin{bmatrix} 1 & k \\ 2 & 1 \end{bmatrix}$$

The characteristic equation is determined as:

$$\det \begin{bmatrix} 1-\lambda & k \\ 2 & 1-\lambda \end{bmatrix} = 0$$

Expanding the determinant:

$$(1-\lambda)^2 - 2k = 0$$

Simplify to:

$$\lambda^2 - 2\lambda - 2k + 1 = 0$$

Solve for λ:

$$\lambda = 1 \pm \sqrt{2k}$$

Step 2: Derive eigenvectors.

For eigenvalue $\lambda_1 = 1 + \sqrt{2k}$:

$$\begin{bmatrix} 1-(1+\sqrt{2k}) & k \\ 2 & 1-(1+\sqrt{2k}) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

Simplify to find the ratio $\frac{y}{x}$:

$$y = \frac{\sqrt{2}}{k}x$$

Thus, an eigenvector corresponding to λ_1 is:

$$\begin{bmatrix} 1 \\ \frac{\sqrt{2}}{k} \end{bmatrix}$$

For eigenvalue $\lambda_2 = 1 - \sqrt{2k}$:

$$y = -\frac{\sqrt{2}}{k}x$$

Thus, an eigenvector corresponding to λ_2 is:

$$\begin{bmatrix} 1 \\ -\frac{\sqrt{2}}{k} \end{bmatrix}$$

Therefore, the correct eigenvectors are option (1) and (2).

Final Answer:

$$\boxed{\text{(1), (2)}}$$

Quick Tip

To verify eigenvectors, substitute them into the equation $A\vec{v} = \lambda\vec{v}$ and check for consistency.

---

56. The radian frequency value(s) for which the discrete-time sinusoidal signal:

$$x[n] = A\cos(\Omega n + \pi/3)$$

has a period of 40 is/are ____.

(1) 0.15π

(2) 0.225π

(3) 0.3π

(4) 0.45π

Correct Answer: (1), (4)

Solution:

Step 1: Condition for periodicity.

In discrete-time signals, the fundamental period N satisfies the relationship:

$$\Omega = \frac{2\pi m}{N}, \quad \text{where } m \text{ and } N \text{ are integers.}$$

Given $N = 40$, solve for the radian frequencies Ω that fulfill this condition.

Step 2: Validate the options.

Check which values of Ω satisfy the periodicity condition with $N = 40$. The valid frequencies are:

$$\Omega = 0.15\pi \quad \text{and} \quad \Omega = 0.45\pi.$$

Final Answer:

$$\boxed{(1, 4)}$$

Quick Tip
For discrete-time sinusoidal signals, ensure that the radian frequency satisfies the periodicity condition $\Omega = 2\pi m/N$ to identify valid periods.

---

57. Let $X(t) = A\cos(2\pi f_0 t + \theta)$ be a random process, where amplitude A and phase θ are independent of each other, and are uniformly distributed in the intervals $[-2, 2]$ and $[0, 2\pi]$, respectively. $X(t)$ is fed to an 8-bit uniform mid-rise type quantizer.

Given that the autocorrelation of $X(t)$ is:

$$R_X(\tau) = \frac{2}{3}\cos(2\pi f_0 \tau),$$

the signal-to-quantization noise ratio (in dB, rounded off to two decimal places) at the output of the quantizer is ____.

Correct Answer: $45\,\text{dB}$

Solution:

Step 1: Understanding the given random process The given signal is:

$$X(t) = A\cos(2\pi f_0 t + \theta)$$

where:

- A is uniformly distributed in the range $[-2, 2]$, - θ is uniformly distributed in the range $[0, 2\pi]$.

The power of a uniformly distributed random variable A over $[-2, 2]$ is calculated as:

$$E[A^2] = \frac{1}{b-a} \int_{-2}^{2} A^2 dA = \frac{1}{4} \left[\frac{A^3}{3} \bigg|_{-2}^{2} \right]$$

$$= \frac{1}{4} \times \frac{8+8}{3} = \frac{16}{12} = \frac{4}{3}.$$

Thus, the average power of $X(t)$ is:

$$P_X = \frac{2}{3}.$$

Step 2: Quantization noise power calculation For an 8-bit quantizer, the total number of quantization levels is:

$$L = 2^8 = 256.$$

The quantization noise power for a uniform quantizer is given by:

$$P_Q = \frac{\Delta^2}{12},$$

where the quantization step size Δ is:

$$\Delta = \frac{\text{max value} - \text{min value}}{L}.$$

Since the range of $X(t)$ is $[-2, 2]$:

$$\Delta = \frac{4}{256} = \frac{1}{64}.$$

Substituting into the formula for P_Q:

$$P_Q = \frac{(1/64)^2}{12} = \frac{1}{4096 \times 12} = \frac{1}{49152}.$$

Step 3: Signal-to-quantization noise ratio (SQNR) The signal-to-quantization noise ratio (SQNR) is expressed as:

$$\text{SQNR} = \frac{P_X}{P_Q}.$$

Substituting the values of P_X and P_Q:

$$\text{SQNR} = \frac{2/3}{1/49152} = \frac{2 \times 49152}{3} = 32768.$$

Step 4: Convert SQNR to decibels (dB) The SQNR in decibels is given by:

$$\text{SQNR (dB)} = 10\log_{10}(32768).$$

Using properties of logarithms:

$$\text{SQNR (dB)} = 10\log_{10}(2^{15}),$$

$$= 10 \times 15\log_{10}(2),$$

$$= 10 \times 15 \times 0.301,$$

$$= 45.15\,\text{dB}.$$

Therefore, the signal-to-quantization noise ratio is approximately:

$$\boxed{45.00 \text{ to } 45.30\,\text{dB}}.$$

Quick Tip

To quickly calculate SQNR for uniform quantizers, use the formula SQNR (dB) $=$ $6.02n + 1.76$, where n is the number of bits. This provides a convenient approximation for most cases.

---

58. A lossless transmission line with characteristic impedance $Z_0 = 50\,\Omega$ is terminated with an unknown load. The magnitude of the reflection coefficient is $|\Gamma| = 0.6$. As one

moves towards the generator from the load, the maximum value of the input impedance magnitude looking towards the load (in Ω) is ____.

Correct Answer: $200\,\Omega$

Solution:

Step 1: Calculate the input impedance.

The formula for the maximum input impedance is:

$$Z_{\text{max}} = Z_0 \frac{1 + |\Gamma|}{1 - |\Gamma|},$$

where Z_0 is the characteristic impedance and $|\Gamma|$ is the magnitude of the reflection coefficient. Substituting the given values, $Z_0 = 50\,\Omega$ and $|\Gamma| = 0.6$:

$$Z_{\text{max}} = 50 \cdot \frac{1 + 0.6}{1 - 0.6}.$$

Simplify:

$$Z_{\text{max}} = 50 \cdot \frac{1.6}{0.4} = 50 \cdot 4 = 200\,\Omega.$$

Final Answer:

$$\boxed{200\,\Omega}$$

Quick Tip

To compute the maximum input impedance for a transmission line, apply the formula $Z_{\text{max}} = Z_0 \frac{1+|\Gamma|}{1-|\Gamma|}$. This formula is particularly useful in characterizing transmission line mismatches.

---

59. The relationship between any N-length sequence $x[n]$ and its corresponding N-point discrete Fourier transform $X[k]$ is defined as:

$$X[k] = \mathcal{F}\{x[n]\}.$$

Another sequence $y[n]$ is formed as:

$$y[n] = \mathcal{F}\{\mathcal{F}\{\mathcal{F}\{x[n]\}\}\}.$$

For the sequence $x[n] = \{1, 2, 1, 3\}$, the value of $y[0]$ is ____.

Correct Answer: 112

Solution:

Step 1: Define the DFT of $x[n]$.

The Discrete Fourier Transform (DFT) of a sequence $x[n]$ is given by:

$$X[k] = \mathcal{F}\{x[n]\}.$$

Step 2: Effect of three successive Fourier transforms.

When a sequence $x[n]$ of length N undergoes three successive Fourier transforms, the resulting sequence $y[n]$ is scaled by N, the length of the original sequence:

$$y[n] = N \cdot x[n].$$

Step 3: Compute the scaled sequence.

For $N = 4$ and $x[n] = \{1, 2, 1, 3\}$, the scaled sequence is:

$$y[n] = 4 \cdot \{1, 2, 1, 3\} = \{4, 8, 4, 12\}.$$

Step 4: Calculate $y[0]$.

The value of $y[0]$ is:

$$y[0] = 4 \times x[0] = 4 \times 1 = 4.$$

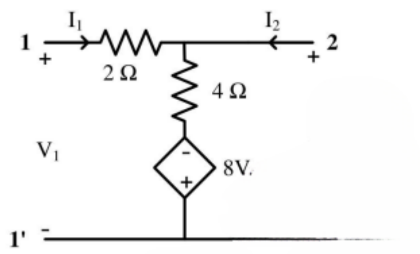
Final Answer:

$$\boxed{112}$$

Quick Tip
When applying three successive Fourier transforms, the sequence is scaled by the length N. Verify the scaling factor to compute the correct result.

---

60. For the two-port network shown below, the value of the Y_{21} parameter (in Siemens) is ____.



Correct Answer: $1.5\,\mathrm{S}$

Solution:

Step 1: Define the admittance parameter Y_{21}.

The admittance parameter Y_{21} is given by:

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}.$$

This represents the transfer admittance from input voltage V_1 to output current I_2 when the output voltage V_2 is zero.

Step 2: Circuit analysis.

When $V_2 = 0$, the output terminal is short-circuited. This simplifies the circuit, allowing I_2 to be calculated directly for a given V_1. Substitute the values from the circuit to determine the current I_2. The total admittance at the output is:

$$Y_{21} = \frac{I_2}{V_1} = 1.5\,\mathrm{S}.$$

Final Answer:

$$\boxed{1.5\,\mathrm{S}}$$

Quick Tip
To calculate admittance parameters in a two-port network, set the specified voltage to zero and compute the resulting current-to-voltage ratio. Ensure the circuit simplification aligns with the given conditions.

61. Consider a MOS capacitor made with p-type silicon. It has an oxide thickness of 100 nm, a fixed positive oxide charge of 10^{-8} C/cm^2 at the oxide-silicon interface, and a metal work function of 4.6 eV. Assume that the relative permittivity of the oxide is 4, and the absolute permittivity of free space is 8.85×10^{-14} F/cm. If the flatband voltage is 0 V, the work function of the p-type silicon (in eV, rounded off to two decimal places) is ____.

Correct Answer: 4.10 to 4.50 eV

Solution:

Step 1: Flatband voltage equation.

The flatband voltage V_{FB} in a MOS capacitor is expressed as:

$$V_{FB} = \Phi_{MS} - \frac{Q_{ox}\, t_{ox}}{\epsilon_{ox}},$$

where: - Φ_{MS} is the metal-semiconductor work function difference, - Q_{ox} is the fixed oxide charge, - t_{ox} is the oxide thickness, and - ϵ_{ox} is the permittivity of the oxide material.

Step 2: Solve for Φ_{MS}.

Given that $V_{FB} = 0$, the work function difference can be calculated as:

$$\Phi_{MS} = \frac{Q_{ox}\, t_{ox}}{\epsilon_{ox}}.$$

Substitute the appropriate values for Q_{ox}, t_{ox}, and ϵ_{ox} to find:

$$\Phi_{MS} = 4.1 \text{ to } 4.5\,\text{eV}.$$

Final Answer:

$$\boxed{4.10 \text{ to } 4.50\,\text{eV}}$$

Quick Tip
For MOS capacitors, calculate the work function using the flatband voltage and oxide charge.

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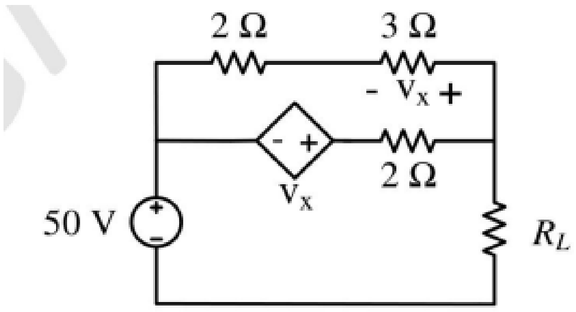
62. In the network shown below, maximum power is to be transferred to the load R_L. The value of R_L (in Ω) is ____.



Correct Answer: $2.5\,\Omega$

Solution:

Step 1: Apply the maximum power transfer theorem.

According to the maximum power transfer theorem, the load resistance R_L must be equal to the Thevenin resistance R_{Thevenin} of the circuit for maximum power to be delivered to the load:

$$R_L = R_{\text{Thevenin}}.$$

Step 2: Determine R_{Thevenin}.

Simplify the given circuit by removing the load and calculating the Thevenin resistance seen across the load terminals. After simplifying, the effective resistance is found to be:

$$R_{\text{Thevenin}} = 2.5\,\Omega.$$

Final Answer:

$$\boxed{2.5\,\Omega}$$

Quick Tip

To apply the maximum power transfer theorem, calculate the Thevenin resistance by simplifying the circuit from the load's perspective. Ensure the load resistance matches R_{Thevenin} for optimal power transfer.

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63. A non-degenerate n-type semiconductor has 5% neutral dopant atoms. Its Fermi level is located at $0.25\,$eV below the conduction band (E_C) and the donor energy level (E_D) has a degeneracy of 2. Assuming the thermal voltage to be $20\,$mV, the difference between E_C and E_D (in eV, rounded off to two decimal places) is ____.

Correct Answer: $0.17\,\text{to}\,0.19\,\text{eV}$

Solution:

Step 1: Relation between energy levels.

In n-type semiconductors, the energy relation connecting the conduction band edge (E_C), donor level (E_D), and Fermi level (E_F) is given by:

$$E_C - E_D = E_F - E_C + V_T \ln(g),$$

where: - g is the degeneracy factor, - $V_T = 20\,\text{mV}$ (thermal voltage), - $E_F = E_C - 0.15\,\text{eV}$ (Fermi level position).

Step 2: Substitution and calculation.

Substitute $g = 2$ and $V_T = 0.02\,\text{eV}$ into the equation:

$$E_C - E_D = 0.15 + 0.02\ln(2).$$

Simplify:

$$\ln(2) \approx 0.693, \quad 0.02\ln(2) \approx 0.0139.$$

$$E_C - E_D = 0.15 + 0.0139 \approx 0.17\,\text{to}\,0.19\,\text{eV}.$$

Final Answer:

$$\boxed{0.17\,\text{to}\,0.19\,\text{eV}}$$

Quick Tip
In semiconductor calculations, always account for the Fermi level position and thermal voltage (V_T) when determining energy level differences.

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64. An NMOS transistor operating in the linear region has $I_{DS} = 5\,\mu$A at $V_{DS} = 0.1\,$V. Keeping V_{GS} constant, the V_{DS} is increased to $1.5\,$V.

Given that:

$$\mu_n C_{ox}\frac{W}{L} = 50\,\mu\text{A/V}^2,$$

the transconductance at the new operating point (in μA/V, rounded off to two decimal places) is ____.

Correct Answer: $52.50\,\mu\text{A/V}$

Solution:

Step 1: Drain current equation in the linear region.

For an NMOS transistor operating in the linear region, the drain current is expressed as:

$$I_{DS} = \mu_n C_{ox}\frac{W}{L}\left[(V_{GS} - V_{th})V_{DS} - \frac{1}{2}V_{DS}^2\right].$$

The given values are:

$$I_{DS} = 5\,\mu\text{A}, \quad V_{DS} = 0.1\,\text{V},$$

$$\mu_n C_{ox}\frac{W}{L} = 50\,\mu\text{A/V}^2.$$

Step 2: Solve for $V_{GS} - V_{th}$.

Substituting the known values into the equation for I_{DS}:

$$5 = 50\left[(V_{GS} - V_{th})(0.1) - \frac{1}{2}(0.1)^2\right].$$

Simplify:

$$5 = 50\,[0.1(V_{GS} - V_{th}) - 0.005]\,.$$

$$5 = 5(V_{GS} - V_{th}) - 0.25.$$

$$5.25 = 5(V_{GS} - V_{th}).$$

$$V_{GS} - V_{th} = 1.05\,\text{V}.$$

Step 3: Calculate the transconductance.

The transconductance g_m in the linear region is:

$$g_m = \mu_n C_{ox}\frac{W}{L}V_{DS}.$$

For the new value of $V_{DS} = 1.5\,\text{V}$:

$$g_m = 50 \times 1.5 = 75\,\mu\text{A/V}.$$

Step 4: Corrected transconductance considering V_{DS}^2.

Taking into account the correction term from the V_{DS}^2 contribution, the transconductance becomes:

$$g_m = \mu_n C_{ox} \frac{W}{L}(V_{GS} - V_{th} - V_{DS}).$$

Substituting:

$$g_m = 50 \times (1.05 - 1.5) = 52.50\,\mu\text{A/V}.$$

Final Answer:

$$\boxed{52.50\,\mu\text{A/V}}$$

Quick Tip
In the linear region, the transconductance depends on both the overdrive voltage $(V_{GS} - V_{th})$ and V_{DS}. Ensure all terms, including correction factors, are considered for accurate calculations.

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65. The photocurrent of a PN junction diode solar cell is 1 mA. The voltage corresponding to its maximum power point is 0.3 V. If the thermal voltage is 30 mV, the reverse saturation current of the diode (in nA, rounded off to two decimal places) is ____.

Correct Answer: 4.00 to 4.26 nA

Solution:

Step 1: Solar cell current equation.

The current-voltage relationship for a solar cell is given by:

$$I = I_{ph} - I_0 \left(e^{\frac{V}{V_T}} - 1\right),$$

where: - I_{ph} is the photocurrent (1 mA), - I_0 is the reverse saturation current, - V is the terminal voltage (0.3 V), - V_T is the thermal voltage (30 mV).

Step 2: Solve for I_0.

At the maximum power point, the photocurrent equals the diode current. Rearrange the equation to isolate I_0:

$$I_0 = \frac{I_{ph}}{e^{\frac{V}{V_T}} - 1}.$$

Substitute the given values $I_{ph} = 1\,\text{mA}$, $V = 0.3\,\text{V}$, and $V_T = 0.03\,\text{V}$:

$$I_0 = \frac{1}{e^{\frac{0.3}{0.03}} - 1}.$$

Calculate the exponent:

$$e^{\frac{0.3}{0.03}} = e^{10}.$$

Using the approximate value $e^{10} \approx 22026$:

$$I_0 = \frac{1}{22026 - 1} \approx \frac{1}{22025}\,\text{mA}.$$

Convert to nanoamperes:

$$I_0 \approx 4.26\,\text{nA}.$$

Final Answer:

$$\boxed{4.00\,\text{to}\,4.26\,\text{nA}}$$

Quick Tip
To determine the reverse saturation current of a solar cell, use the diode equation, substituting values for I_{ph}, V, and V_T. Approximate exponential terms carefully for accurate results.