GATE 2023 Environmental Science And Engineering Question Paper with Solutions

Time Allowed :3 Hour	Maximum Marks :100	Total Questions :65

General Instructions

Read the following instructions very carefully and strictly follow them:

- This question paper is divided into three sections:
- The total duration of the examination is 3 hours.
- The total number of questions is **65**, carrying a maximum of **100 marks**.
- The marking scheme is as follows:
- For 1-mark MCQs, $\frac{1}{3}$ mark will be deducted for every incorrect response.
- For 2-mark MCQs, $\frac{2}{3}$ mark will be deducted for every incorrect response.
- No negative marking for numerical answer type (NAT) questions.
- No marks will be awarded for unanswered questions.
- Follow the instructions provided during the exam for submitting your answers.

General Aptitude

Q.1 - Q.5 Carry ONE mark each

1. Rafi told Mary, "I am thinking of watching a film this weekend."
The following reports the above statement in indirect speech:
Rafi told Mary that he _____ of watching a film that weekend.

(A) thought
(B) is thinking
(C) am thinking
(D) was thinking

Correct Answer: (D) was thinking

Solution:

Step 1: Understanding the Question:
The task is to convert a sentence from direct speech to indirect (or reported) speech. This requires changing the tense of the verb, the pronoun, and the adverb of time.

Step 2: Applying the Rules of Indirect Speech:

- **Reporting Verb:** The reporting verb is "told," which is in the past tense.
- **Tense Change:** When the reporting verb is in the past tense, the verb in the direct speech is shifted back in time. The present continuous tense ("am thinking") changes to the past continuous tense ("was thinking").
- **Pronoun Change:** The first-person pronoun "I" refers to the speaker, Rafi. In indirect speech, it changes to the third-person pronoun "he".
- **Adverb of Time Change:** Words indicating nearness in time or place are changed to words indicating distance. "this weekend" becomes "that weekend".

Step 3: Constructing the Sentence:
Applying these changes, the original statement "I am thinking of watching a film this weekend" becomes "he was thinking of watching a film that weekend."

Step 4: Final Answer:
The correct verb form to complete the sentence is "was thinking".

Quick Tip

In direct to indirect speech conversion, the first thing to check is the tense of the reporting verb (e.g., 'said', 'told'). If it's in the past, you'll almost always need to shift the tense of the original statement one step back (e.g., present to past, past to past perfect). Also, remember to update pronouns and words indicating time and place (this → that, now → then, today → that day).

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2. Permit : _____ :: Enforce : Relax
(By word meaning)

(A) Allow
(B) Forbid
(C) License
(D) Reinforce

Correct Answer: (B) Forbid

Solution:

Step 1: Understanding the Question:
This is an analogy problem in the format A : B :: C : D. We need to identify the relationship between C and D and then find a word for B that has the same relationship with A.

Step 2: Analyzing the Given Pair (Enforce : Relax):

- **Enforce:** To compel compliance with a law, rule, or obligation.
- **Relax:** To make a rule or restriction less strict or severe.

The words "Enforce" and "Relax" are antonyms; they have opposite meanings.

Step 3: Applying the Antonym Relationship to the First Pair:
We need to find the antonym for the word "Permit".

- **Permit:** To give authorization or consent to someone to do something.

Now let's examine the options:

- (A) Allow: This is a synonym of "Permit".
- (B) Forbid: This means to refuse to allow something. It is the direct opposite of "Permit".
- (C) License: This is a type of formal permission, closely related but not an antonym.
- (D) Reinforce: This means to strengthen, which is unrelated.

Step 4: Final Answer:
The word that completes the analogy with an antonym relationship is "Forbid". The full analogy is Permit : Forbid :: Enforce : Relax.

Quick Tip

To solve analogy questions, the first step is always to determine the precise relationship between the given pair of words. Common relationships include Synonyms, Antonyms, Cause-Effect, Part-Whole, Object-Function, and Degree of Intensity. Once you establish the relationship, apply it to find the missing word.

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3. Given a fair six-faced dice where the faces are labelled '1', '2', '3', '4', '5', and '6', what is the probability of getting a '1' on the first roll of the dice and a '4' on

the second roll?

(A) $\frac{1}{36}$

(B) $\frac{1}{6}$

(C) $\frac{5}{6}$

(D) $\frac{1}{3}$

Correct Answer: (A) $\frac{1}{36}$

Solution:

Step 1: Understanding the Question:
We need to find the probability of two independent events occurring in sequence: getting a specific outcome on the first roll of a die, and another specific outcome on the second roll.

Step 2: Key Formula or Approach:
For independent events A and B, the probability that both events occur is the product of their individual probabilities:

$$P(A \text{ and } B) = P(A) \times P(B)$$

Step 3: Detailed Explanation:

- Let Event A be "getting a '1' on the first roll."
  A fair six-faced die has 6 equally likely outcomes {1, 2, 3, 4, 5, 6}.
  The probability of getting a '1' is:

$$P(A) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}} = \frac{1}{6}$$

- Let Event B be "getting a '4' on the second roll."
  The second roll is independent of the first. The probability of getting a '4' is also:

$$P(B) = \frac{1}{6}$$

- The probability of both events happening is the product of their individual probabilities:

$$P(A \text{ and } B) = P(A) \times P(B) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$$

Step 4: Final Answer:
The probability of getting a '1' on the first roll and a '4' on the second roll is $\frac{1}{36}$.

Quick Tip

In probability, the word "and" is a strong clue that you should multiply the probabilities of the events, provided they are independent. The word "or" usually implies addition (for mutually exclusive events). Always verify the independence of events before multiplying.

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4. A recent survey shows that 65% of tobacco users were advised to stop consuming tobacco. The survey also shows that 3 out of 10 tobacco users attempted to stop using tobacco.
Based only on the information in the above passage, which one of the following options can be logically inferred with certainty?

(A) A majority of tobacco users who were advised to stop consuming tobacco made an attempt to do so.
(B) A majority of tobacco users who were advised to stop consuming tobacco did not attempt to do so.
(C) Approximately 30% of tobacco users successfully stopped consuming tobacco.
(D) Approximately 65% of tobacco users successfully stopped consuming tobacco.

Correct Answer: (B) A majority of tobacco users who were advised to stop consuming tobacco did not attempt to do so.

Solution:

Step 1: Understanding the Question:
We are given two statistics about a group of tobacco users. We need to determine which conclusion can be drawn with 100% certainty, without making any assumptions beyond the text.

Step 2: Analyzing the Given Information:
Let's use sets to represent the groups. Let T be the total population of tobacco users.

- Let A be the set of users who were advised to stop. The size of this set is $|A| = 0.65T$.
- Let B be the set of users who attempted to stop. The size of this set is $|B| = \frac{3}{10}T = 0.30T$.

The problem gives no information about the overlap between these two groups, i.e., the size of the set $A \cap B$.

Step 3: Evaluating the Options with Certainty:

- **Options (C) and (D):** These options talk about users who "successfully stopped." The passage only mentions users who "attempted to stop." We cannot infer anything about success rates. So, (C) and (D) are incorrect.

- **Options (A) and (B):** These options concern the group of users who were advised to stop ($|A| = 0.65T$) and whether a majority of them attempted to stop or not. A majority of this group is any number greater than $0.5 \times |A| = 0.5 \times 0.65T = 0.325T$.
  Let's analyze the number of advised users who did *not* attempt to stop. This is the number of people in set A but not in set B, which is $|A| - |A \cap B|$. To be certain about statement (B), we must show that this number is greater than the majority threshold (0.325T) even in the "worst-case" scenario. The number of advised users who did not attempt is minimized when the number who *did* attempt (the overlap $|A \cap B|$) is maximized.
  The maximum possible size of the overlap $|A \cap B|$ is limited by the size of the smaller group, which is B. So, at most, all 0.30T people who attempted were also advised.

  $$\text{Max}(|A \cap B|) = 0.30T$$

  Now, let's find the minimum number of advised users who did not attempt:

  $$\text{Min(Advised but did not attempt)} = |A| - \text{Max}(|A \cap B|) = 0.65T - 0.30T = 0.35T$$

  Since the minimum number of advised users who did not attempt is 0.35T, and a majority requires only ¿ 0.325T, it is certain that at least a majority of the advised group did not attempt to stop.

Step 4: Final Answer:
Statement (B) can be logically inferred with certainty. Statement (A) is not certain because the overlap could be as small as 0, in which case none of the advised users would have attempted.

> Quick Tip
>
> For logical inference questions involving percentages, thinking in terms of sets and Venn diagrams is very helpful. To prove something with "certainty," you must show that it holds true even under the most extreme or "worst-case" scenario allowed by the given information.

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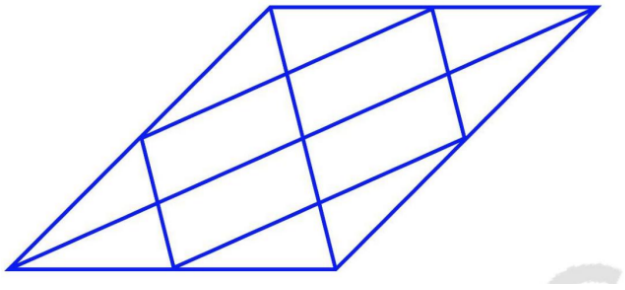
5. How many triangles are present in the given figure?



(A) 12
(B) 16
(C) 20

(D) 24

Correct Answer: (C) 20

Solution:

Step 1: Understanding the Question:
The task is to count all the triangles in the given geometric figure. A systematic approach is necessary to avoid over-counting or missing any triangles.

Step 2: Systematic Counting Strategy:
We can categorize the triangles based on the number of small, elementary triangular units they are composed of. The figure can be seen as being made of 8 small triangles.

Step 3: Detailed Explanation:

- **Triangles made of 1 unit:** By simple observation, we can count the smallest individual triangles. There are 4 in the top half and 4 in the bottom half of the main parallelogram.
  Count = 8

- **Triangles made of 2 units:** These are formed by combining two adjacent small triangles.
  - There are 2 such triangles pointing up with their bases on the top edge.
  - There are 2 such triangles pointing down with their vertices on the bottom edge.
  - There are 4 such triangles formed across the central horizontal line (2 pointing right, 2 pointing left).

  Count = 2 + 2 + 4 = 8

- **Triangles made of 3 units:** There are no triangles formed by combining 3 of the small units.
  Count = 0

- **Triangles made of 4 units:** The two main diagonals of the large parallelogram divide it into four large triangles (top, bottom, left, right), each composed of 4 small units.
  Count = 4

Step 4: Final Answer:
The total number of triangles is the sum of the counts from all categories:

$$\text{Total Triangles} = 8 + 8 + 0 + 4 = 20$$

Quick Tip

When counting geometric shapes in a complex figure, a reliable method is to start with the smallest units and systematically look for larger shapes made by combining 2, 3, 4, or more of these units. This hierarchical approach helps ensure that no shapes are missed and none are counted twice.

Q.6– Q.10 Carry TWO marks each

6. Students of all the departments of a college who have successfully completed the registration process are eligible to vote in the upcoming college elections. However, by the time the due date for registration was over, it was found that surprisingly none of the students from the Department of Human Sciences had completed the registration process.
Based only on the information provided above, which one of the following sets of statement(s) can be logically inferred with certainty?

(i) All those students who would not be eligible to vote in the college elections would certainly belong to the Department of Human Sciences.
(ii) None of the students from departments other than Human Sciences failed to complete the registration process within the due time.
(iii) All the eligible voters would certainly be students who are not from the Department of Human Sciences.

(A) (i) and (ii)
(B) (i) and (iii)
(C) only (i)
(D) only (iii)

Correct Answer: (D) only (iii)

Solution:

Step 1: Understanding the Question:
We are given two premises and need to determine which of the three statements can be logically concluded with absolute certainty.

Step 2: Analyzing the Premises:

- **Premise 1:** If a student successfully registers, then they are eligible to vote. (Registration $\implies$ Eligible)
- **Premise 2:** No student from the Department of Human Sciences successfully registered.

Step 3: Drawing a Direct Conclusion:
From Premise 1 and 2, we can directly conclude that no student from the Department of Hu-

man Sciences is eligible to vote.

Step 4: Evaluating Each Statement:

- **Statement (i): "All those students who would not be eligible... would certainly belong to the Department of Human Sciences."** This is **false**. A student could be ineligible for other reasons, such as being from another department (e.g., Engineering) and also failing to register. The premises do not state that Human Sciences students are the *only* ones who didn't register.
- **Statement (ii): "None of the students from departments other than Human Sciences failed to complete the registration process..."** This is **false**. The passage gives no information about the registration status of students from other departments. It is possible many of them also failed to register.
- **Statement (iii): "All the eligible voters would certainly be students who are not from the Department of Human Sciences."** This is **true**. This is the contrapositive of our direct conclusion. Since we know for certain that no Human Sciences student is eligible, it logically follows that any student who *is* eligible cannot be from the Department of Human Sciences.

Step 5: Final Answer:
Only statement (iii) can be inferred with certainty.

Quick Tip

In logical deduction problems, stick strictly to the information given and avoid making assumptions. Be wary of common fallacies like affirming the consequent or denying the antecedent. A powerful tool is the contrapositive: the statement "If P then Q" is logically equivalent to "If not Q then not P".

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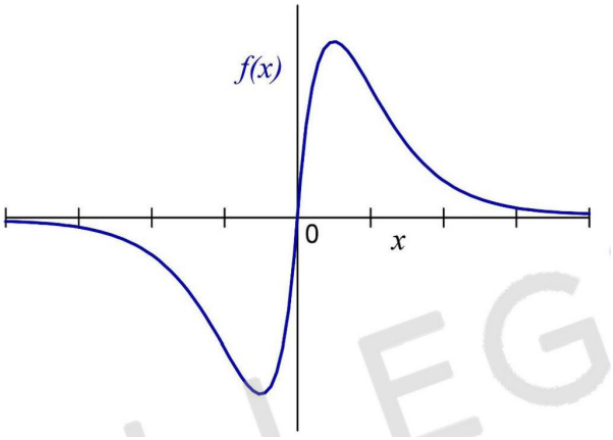
7. Which one of the following options represents the given graph?



(A) $f(x) = x^2 2^{-|x|}$
(B) $f(x) = x2^{-|x|}$
(C) $f(x) = |x|2^{-x}$
(D) $f(x) = x2^{-x}$

Correct Answer: (B) $f(x) = x2^{-|x|}$

Solution:

Step 1: Understanding the Question:
We need to identify the function that corresponds to the given plot by analyzing the key features of the graph.

Step 2: Analyzing the Graph's Properties:

- **Symmetry:** The graph exhibits origin symmetry, meaning $f(-x) = -f(x)$. This is the definition of an odd function.
- **Behavior at Origin:** The graph passes through (0, 0), so $f(0) = 0$.
- **Sign of the function:** For $x > 0$, the graph is above the x-axis, so $f(x) > 0$. For $x < 0$, the graph is below the x-axis, so $f(x) < 0$.
- **Asymptotic Behavior:** As x approaches both $+\infty$ and $-\infty$, the graph approaches the x-axis, so $\lim_{x\to\pm\infty} f(x) = 0$.

Step 3: Evaluating the Options:

- **(A)** $f(x) = x^2 2^{-|x|}$**:** This function is even because $f(-x) = (-x)^2 2^{-|-x|} = x^2 2^{-|x|} = f(x)$. The graph is odd. Incorrect.
- **(B)** $f(x) = x2^{-|x|}$**:**
  - *Symmetry:* $f(-x) = (-x)2^{-|-x|} = -x2^{-|x|} = -f(x)$. It is an odd function. (Matches)
  - *Origin:* $f(0) = 0 \cdot 2^{-0} = 0$. (Matches)
  - *Sign:* For $x > 0$, $f(x)$ is positive. For $x < 0$, $f(x)$ is negative. (Matches)
  - *Asymptotes:* The exponential term $2^{-|x|}$ decays to zero faster than x grows, so the limit is 0 as $x \to \pm\infty$. (Matches)

  This function matches all the key properties.
- **(C)** $f(x) = |x|2^{-x}$**:** For $x < 0$, $|x|$ is positive and 2^{-x} is positive (e.g., $2^{-(-2)} = 4$). So $f(x)$ is positive for $x < 0$, which contradicts the graph. Incorrect.
- **(D)** $f(x) = x2^{-x}$**:** This function is not odd. For example, $f(-1) = (-1)2^{-(-1)} = -2$, while $-f(1) = -(1)2^{-1} = -0.5$. Since $f(-x) \neq -f(x)$, it lacks the origin symmetry shown in the graph. Incorrect.

Step 4: Final Answer:
The function $f(x) = x2^{-|x|}$ is the only option that correctly represents the given graph.

> Quick Tip
>
> When matching a function to a graph, checking for symmetry is one of the quickest ways to eliminate incorrect options. Check if the graph is symmetric about the y-axis (even function, $f(-x) = f(x)$) or symmetric about the origin (odd function, $f(-x) = -f(x)$). This test alone often narrows down the choices significantly.

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8. Which one of the options does NOT describe the passage below or follow from it?
We tend to think of cancer as a 'modern' illness because its metaphors are so modern. It is a disease of overproduction, of sudden growth, a growth that is unstoppable, tipped into the abyss of no control. Modern cell biology encourages us to imagine the cell as a molecular machine. Cancer is that machine unable to quench its initial command (to grow) and thus transform into an indestructible, self-propelled automaton.

[Adapted from *The Emperor of All Maladies* by Siddhartha Mukherjee]

(A) It is a reflection of why cancer seems so modern to most of us.
(B) It tells us that modern cell biology uses and promotes metaphors of machinery.
(C) Modern cell biology encourages metaphors of machinery, and cancer is often imagined as a machine.
(D) Modern cell biology never uses figurative language, such as metaphors, to describe or explain anything.

Correct Answer: (D) Modern cell biology never uses figurative language, such as metaphors, to describe or explain anything.

Solution:

Step 1: Understanding the Question:
The question asks to identify the statement that is either not supported by the passage or is directly contradicted by it.

Step 2: Analyzing the Passage's Main Points:
The central argument of the passage is that cancer feels "modern" because it is described using modern metaphors. The passage explicitly states that "Modern cell biology encourages us to imagine the cell as a molecular machine" and then describes cancer in terms of this machine metaphor (a "self-propelled automaton"). The entire passage is an example of using figurative language (metaphors) to explain a scientific concept.

Step 3: Evaluating Each Option:

- **(A) It is a reflection of why cancer seems so modern to most of us.** This is directly supported by the first sentence of the passage.
- **(B) It tells us that modern cell biology uses and promotes metaphors of machinery.** This is supported by the sentence, "Modern cell biology encourages us to imagine the cell as a molecular machine."
- **(C) Modern cell biology encourages metaphors of machinery, and cancer is often imagined as a machine.** This is a correct summary of the passage's argument.
- **(D) Modern cell biology never uses figurative language, such as metaphors, to describe or explain anything.** This statement is a direct contradiction of the entire passage. The passage itself is a demonstration of how modern cell biology *does* use metaphors like "molecular machine" and "automaton".

Step 4: Final Answer:
Statement (D) is the only one that does not follow from the passage; it is explicitly contradicted by the text.

> Quick Tip
>
> For "Which is NOT true?" or "Which does NOT follow?" questions, carefully read each option and try to find the exact sentence or idea in the passage that either supports it or contradicts it. The correct answer will be the one you can find no support for, or for which you can find direct evidence against it.

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9. The digit in the unit's place of the product $3^{999} \times 7^{1000}$ is _____.

(A) 7
(B) 1
(C) 3
(D) 9

Correct Answer: (A) 7

Solution:

Step 1: Understanding the Question:
To find the unit's digit of a product, we only need to find the unit's digit of each factor and then find the unit's digit of their product. The unit's digits of powers of integers repeat in a cycle.

Step 2: Finding the Unit's Digit of 3^{999}:
We examine the pattern of the last digit of powers of 3:

- $3^1 = 3$
- $3^2 = 9$
- $3^3 = 27 \rightarrow 7$
- $3^4 = 81 \rightarrow 1$
- $3^5 = 243 \rightarrow 3$

The cycle of unit's digits is (3, 9, 7, 1), which has a length of 4. To find the unit's digit of 3^{999}, we find the remainder of the exponent when divided by the cycle length: $999 \div 4$.

$$999 = 4 \times 249 + 3$$

The remainder is 3. This means the unit's digit is the 3rd number in the cycle, which is 7.

Step 3: Finding the Unit's Digit of 7^{1000}:
We examine the pattern of the last digit of powers of 7:

- $7^1 = 7$
- $7^2 = 49 \rightarrow 9$
- $7^3 = 343 \rightarrow 3$
- $7^4 = 2401 \rightarrow 1$
- $7^5 = 16807 \rightarrow 7$

The cycle of unit's digits is (7, 9, 3, 1), which also has a length of 4. We find the remainder of the exponent: $1000 \div 4$. Since 1000 is perfectly divisible by 4, the remainder is 0. When the remainder is 0, the unit's digit is the last one in the cycle (the 4th one), which is 1.

Step 4: Final Answer:
The unit's digit of the product $3^{999} \times 7^{1000}$ is the unit's digit of the product of their respective unit's digits:

$$\text{Unit's digit of } (7 \times 1) = 7$$

The final answer is 7.

Quick Tip

To find the unit digit of x^n, find the repeating cycle of the unit digits of powers of x (e.g., for 2, it's 2,4,8,6). Divide the exponent n by the length of this cycle. If the remainder is r (where r ¿ 0), the answer is the r-th digit in the cycle. If the remainder is 0, the answer is the last digit in the cycle.

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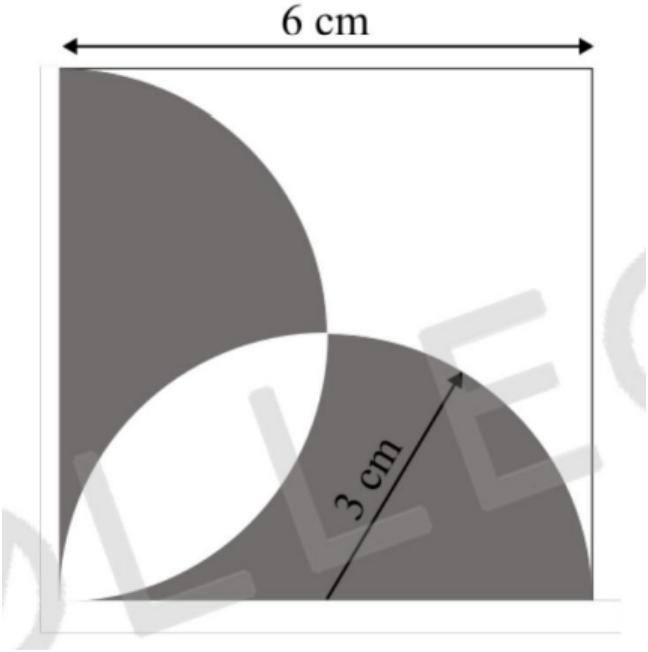
10. A square with sides of length 6 cm is given. The boundary of the shaded region is defined by two semi-circles whose diameters are the sides of the square, as shown.

The area of the shaded region is _____ cm^2.



(A) 6π
(B) 18
(C) 20
(D) 9π

Correct Answer: (B) 18

Solution:

Step 1: Understanding the Question:
The question asks for the area of a specific shaded region inside a square. The description and diagram for this type of problem can sometimes be ambiguous. However, the visual representation strongly suggests a figure related to the "Lune of Hippocrates", a classic geometry problem where the area of a crescent-like shape (a lune) is related to the area of a triangle.

Step 2: Interpretation and Approach:
The diagram and the presence of '18' (half the area of the square) as an option points towards a solution based on a known geometric property rather than complex integration. Let's analyze the areas. A common variant of this problem involves a right-angled triangle and circular arcs. By drawing a diagonal in the square, we form two right-angled triangles with sides 6 and 6.

Let's consider the area of one such triangle formed by the diagonal.
The area of the triangle is:

$$\text{Area}_{\text{triangle}} = \frac{1}{2} \times \text{base} \times \text{height}$$

Step 3: Detailed Calculation:
The square has a side length of 6 cm.

The area of the square is $6 \times 6 = 36$ cm^2.
Consider the right-angled triangle formed by two sides and a diagonal of the square. The base and height of this triangle are both 6 cm.

$$\text{Area}_{\text{triangle}} = \frac{1}{2} \times 6 \text{ cm} \times 6 \text{ cm} = 18 \text{ cm}^2$$

In many configurations of lunes and squares, the area of the shaded region simplifies to be exactly the area of an inscribed triangle or half the area of the square. For instance, the area of the lune formed on the hypotenuse of a right triangle is equal to the area of the triangle. The figure shown is a variation of this principle. The shaded area is equivalent to the area of the triangle with the diagonal as its hypotenuse.

Step 4: Final Answer:
The area of the shaded region is 18 cm^2.

> Quick Tip
>
> When faced with a complex-looking area problem in a competitive exam, look for simple relationships. If the figure is symmetric, the answer might be a simple fraction (like 1/2 or 1/4) of the total area. In this case, the area of the triangle formed by the diagonal (18) is a listed option, which is a strong hint.

Environmental Science and Engineering

Q.11-Q.35 Carry ONE mark each

11. Given are two ordinary differential equations

$$P : \frac{dy}{dx} + x = x \sin y$$

$$Q : \frac{dy}{dx} + xy = e^x y$$

The correct choice is

(A) P is linear; Q is nonlinear
(B) P is nonlinear; Q is linear
(C) Both P and Q are linear
(D) Both P and Q are nonlinear

Correct Answer: (B) P is nonlinear; Q is linear

Solution:

Step 1: Understanding Linear Ordinary Differential Equations:
A first-order ordinary differential equation is called linear if it can be written in the standard form:

$$\frac{dy}{dx} + P(x)y = Q(x)$$

This means the dependent variable (y) and its derivatives (here, $\frac{dy}{dx}$) must appear only to the first power and must not be part of any other function (like $\sin(y)$, y^2, e^y, etc.). The coefficients $P(x)$ and $Q(x)$ can be any functions of the independent variable x.

Step 2: Analyzing Equation P:
The equation is $P : \frac{dy}{dx} + x = x \sin y$. We can rewrite it as $\frac{dy}{dx} - x \sin y = -x$. The term $x \sin y$ involves the dependent variable y inside a sine function. This does not fit the linear form $P(x)y$. Therefore, equation P is **nonlinear**.

Step 3: Analyzing Equation Q:
The equation is $Q : \frac{dy}{dx} + xy = e^x y$. To check if it's linear, we try to arrange it into the standard form. We can group the terms involving y:

$$\frac{dy}{dx} + (x - e^x)y = 0$$

This equation is in the standard linear form $\frac{dy}{dx} + P(x)y = Q(x)$, where:

- $P(x) = x - e^x$
- $Q(x) = 0$

Since the equation fits the standard form, equation Q is **linear**.

Step 4: Final Answer:
P is nonlinear, and Q is linear. This corresponds to option (B).

> Quick Tip
>
> To quickly determine if an ODE is linear, scan for any "illegal" operations on the dependent variable (usually 'y'). If you see terms like y^2, $\sqrt{y}$, $\sin(y)$, e^y, or a product of y and its derivative like $y\frac{dy}{dx}$, the equation is nonlinear. If y and its derivatives only appear to the first power and are multiplied only by functions of the independent variable 'x' (or constants), it's linear.

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12. P and Q are square matrices. Consider the following

$$X : (P^{-1})^{-1} = P$$

$$Y : \textbf{Symmetric if } Q = -Q^T$$

The correct choice is

(A) X is TRUE; Y is FALSE
(B) X is FALSE; Y is TRUE
(C) Both X and Y are TRUE
(D) Both X and Y are FALSE

Correct Answer: (A) X is TRUE; Y is FALSE

Solution:

Step 1: Understanding the Question:
We need to evaluate the truthfulness of two statements concerning the properties of square matrices.

Step 2: Analyzing Statement X:
The statement is $X : (P^{-1})^{-1} = P$. This statement describes the property of the inverse of an inverse matrix. By definition, the inverse of a matrix A is a matrix A^{-1} such that $AA^{-1} = A^{-1}A = I$, where I is the identity matrix. Let $A = P^{-1}$. Then the statement becomes $A^{-1} = P$. We need to check if $AA^{-1} = I$. Substituting the expressions for A and A^{-1}:

$$(P^{-1})(P) = I$$

This is true by the definition of the inverse of P. Therefore, the statement that the inverse of P^{-1} is P is **TRUE**.

Step 3: Analyzing Statement Y:
The statement is Y : Symmetric if $Q = -Q^T$. Let's review the definitions:

- A matrix Q is **symmetric** if it is equal to its transpose, i.e., $Q = Q^T$.
- A matrix Q is **skew-symmetric** (or anti-symmetric) if it is equal to the negative of its transpose, i.e., $Q = -Q^T$.

Statement Y claims that the condition $Q = -Q^T$ defines a symmetric matrix. This is incorrect. This condition defines a skew-symmetric matrix. Therefore, statement Y is **FALSE**.

Step 4: Final Answer:
Statement X is TRUE, and statement Y is FALSE. This corresponds to option (A).

Quick Tip

Memorize the fundamental definitions for matrix properties:

- **Inverse:** $AA^{-1} = I$
- **Transpose:** $(A^T)_{ij} = A_{ji}$
- **Symmetric:** $A = A^T$
- **Skew-Symmetric:** $A = -A^T$
- **Orthogonal:** $A^T = A^{-1}$

These definitions are essential for quickly solving matrix theory questions.

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13. Given are two infinite series

$$P : \sum \frac{n^2+1}{n^2}$$

$$Q : \sum \left(1+\frac{1}{n}\right)^{-n}$$

The correct choice is

(A) P is convergent series; Q is divergent series
(B) P is divergent series; Q is convergent series
(C) Both P and Q are convergent series
(D) Both P and Q are divergent series

Correct Answer: (D) Both P and Q are divergent series

Solution:

Step 1: Understanding the Question:
We need to determine whether the two given infinite series, P and Q, converge or diverge. We can use standard convergence tests for this.

Step 2: Analyzing Series P:
The series is $P : \sum_{n=1}^{\infty} a_n$ where $a_n = \frac{n^2+1}{n^2}$. A fundamental requirement for any infinite series to converge is that its terms must approach zero as $n \to \infty$. This is known as the n-th term test for divergence. Let's apply this test.

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n^2+1}{n^2} = \lim_{n\to\infty} \left(1+\frac{1}{n^2}\right) = 1+0 = 1$$

Since the limit of the n-th term is not zero ($\lim_{n\to\infty} a_n = 1 \neq 0$), the series P **diverges**.

Step 3: Analyzing Series Q:
The series is $Q : \sum_{n=1}^{\infty} b_n$ where $b_n = \left(1+\frac{1}{n}\right)^{-n}$. Let's again apply the n-th term test for

divergence. We need to find the limit of b_n as $n \to \infty$.

$$\lim_{n\to\infty} b_n = \lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^{-n} = \lim_{n\to\infty} \frac{1}{\left(1 + \frac{1}{n}\right)^{n}}$$

The expression in the denominator is the definition of the mathematical constant e.

$$\lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^{n} = e$$

Therefore,

$$\lim_{n\to\infty} b_n = \frac{1}{e}$$

Since the limit of the n-th term is not zero ($\lim_{n\to\infty} b_n = 1/e \neq 0$), the series Q also **diverges**.

Step 4: Final Answer:
Both series P and series Q are divergent. This corresponds to option (D).

Quick Tip

The n-th term test for divergence is a powerful first check for any series. If the limit of the terms of the series does not go to zero ($\lim_{n\to\infty} a_n \neq 0$), you can immediately conclude that the series diverges. If the limit is zero, the test is inconclusive, and you must use a different test (like the ratio test, integral test, etc.).

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14. For testing alkalinity for a water sample, first phenolphthalein indicator is added. The water remains colorless. However, when a few drops of methyl orange is added to the sample, the color turns yellow. As per these observations, the correct choice is

(A) Absence of CO_3^{2-} and/or HCO_3^- but the presence of OH^- ions in the sample
(B) Presence of CO_3^{2-} and/or HCO_3^- but the absence of OH^- ions in the sample
(C) Absence of CO_3^{2-}, HCO_3^- and OH^- ions in the sample
(D) Presence of CO_3^{2-}, HCO_3^- and OH^- ions in the sample

Correct Answer: (B) Presence of CO_3^{2-} and/or HCO_3^- but the absence of OH^- ions in the sample

Solution:

Step 1: Understanding Alkalinity Titration and Indicators:
Alkalinity in water is primarily caused by hydroxide (OH^-), carbonate (CO_3^{2-}), and bicarbonate (HCO_3^-) ions. It is measured by titrating the water with a strong acid (like H_2SO_4). Two indicators, phenolphthalein and methyl orange, are used to detect two different equivalence points.

- **Phenolphthalein (P) alkalinity:** This indicator has a pH range of approximately 8.3 to 10.0. It turns pink in the presence of hydroxide and carbonate alkalinity. It is colorless below pH 8.3. The titration to the phenolphthalein endpoint (colorless) neutralizes all hydroxide and half of the carbonate.
- **Methyl Orange (M) or Total alkalinity:** This indicator has a pH range of approximately 3.1 to 4.4. The titration to the methyl orange endpoint (yellow to red/orange) neutralizes all forms of alkalinity (hydroxide, carbonate, and bicarbonate).

Step 2: Interpreting the Observations:

- **"The water remains colorless" after adding phenolphthalein.** This means the initial pH of the water sample is less than 8.3. Hydroxide (OH^-) alkalinity can only exist at a pH above 7, and significant carbonate (CO_3^{2-}) alkalinity generally requires a pH above 8.3. The absence of a pink color signifies the absence of significant hydroxide and carbonate alkalinity. Specifically, it implies the absence of OH^- ions as a major contributor to alkalinity, as their presence would raise the pH well into the phenolphthalein range.
- **"when a few drops of methyl orange is added... the color turns yellow."** Methyl orange is yellow at a pH above 4.4. This indicates that the water still has some alkalinity (is not yet acidic). The presence of yellow color means the pH is ¿ 4.4. When this sample is titrated with acid, it would eventually turn red, indicating that there was alkalinity to be neutralized. This remaining alkalinity, after the P-alkalinity test was negative, must be due to bicarbonate (HCO_3^-) ions and possibly some carbonate ions (if the pH was exactly 8.3).

Step 3: Drawing a Conclusion:

- The colorless phenolphthalein test implies the absence of hydroxide (OH^-) alkalinity and that the concentration of carbonate (CO_3^{2-}) is very low or zero.
- The yellow methyl orange color indicates the presence of alkalinity that exists below pH 8.3, which is primarily bicarbonate (HCO_3^-) alkalinity.

Combining these, we can conclude that the sample contains bicarbonate and/or carbonate ions, but is free from hydroxide alkalinity. This matches option (B).

Step 4: Final Answer:

The observations indicate the Presence of CO_3^{2-} and/or HCO_3^- but the absence of OH^- ions in the sample.

Quick Tip

Remember the pH ranges for alkalinity titration:

- Phenolphthalein turns pink above pH 8.3. Its endpoint measures all OH^- and half of CO_3^{2-}. If it's colorless, it means pH ¡ 8.3, and there's no OH^- alkalinity.
- Methyl orange turns red below pH 4.4. Its endpoint measures total alkalinity ($OH^- + CO_3^{2-} + HCO_3^-$). If it's yellow, there is still some alkalinity present.

15. Read the following statements

I. Photosynthesis takes place within the chloroplasts of the eukaryotes, whereas the breakdown of complex molecules to yield energy takes place in the cytoplasm and in the mitochondria.
II. Photosynthesis takes place within the chloroplasts of the prokaryotes, whereas the breakdown of complex molecules to yield energy takes place in the cytoplasm and in the mitochondria.
III. All living organisms retain the enzymatic machinery to partially oxidise glucose without the help of oxygen. This breakdown of glucose to pyruvic acid is called glycolysis.
IV. All living organisms retain the enzymatic machinery to completely oxidise glycerol without the help of oxygen. This breakdown of glycerol to citric acid is called glycolysis.

The correct choice is

(A) I and III are correct
(B) II and IV are correct
(C) I is correct whereas III is incorrect
(D) II is correct whereas IV is incorrect

Correct Answer: (A) I and III are correct

Solution:

Step 1: Understanding the Question:
We need to evaluate four statements from biology concerning cellular processes like photosynthesis and respiration and identify the correct combination of true statements.

Step 2: Evaluating Each Statement:

- **Statement I:** "Photosynthesis takes place within the chloroplasts of the eukaryotes... breakdown of complex molecules to yield energy takes place in the cytoplasm and in the mitochondria." This statement is **correct**. In eukaryotic cells (like plants and algae), photosynthesis occurs in chloroplasts. Cellular respiration, the process of breaking down molecules for energy, begins with glycolysis in the cytoplasm and continues with the Krebs cycle and oxidative phosphorylation in the mitochondria.
- **Statement II:** "Photosynthesis takes place within the chloroplasts of the prokaryotes..." This statement is **incorrect**. Prokaryotic cells (like bacteria) do not have membrane-bound organelles such as chloroplasts or mitochondria. Photosynthetic prokaryotes (like cyanobacteria) have internal membrane systems where photosynthesis occurs, but they are not enclosed in a distinct chloroplast organelle.

- **Statement III:** "All living organisms retain the enzymatic machinery to partially oxidise glucose without the help of oxygen. This breakdown of glucose to pyruvic acid is called glycolysis." This statement is **correct**. Glycolysis is a nearly universal metabolic pathway found in all domains of life. It is the initial stage of glucose breakdown, occurs in the cytoplasm, and does not require oxygen. It splits a glucose molecule into two molecules of pyruvate.
- **Statement IV:** "All living organisms retain the enzymatic machinery to completely oxidise glycerol without the help of oxygen. This breakdown of glycerol to citric acid is called glycolysis." This statement is **incorrect** on multiple counts. Glycolysis is the breakdown of glucose, not glycerol. The breakdown product is pyruvic acid, not citric acid. Furthermore, complete oxidation of any substrate requires oxygen (aerobic respiration).

Step 3: Final Answer:
Statements I and III are correct. This corresponds to option (A).

Quick Tip

Key differences between prokaryotes and eukaryotes are a common topic. Remember that prokaryotes lack membrane-bound organelles (nucleus, mitochondria, chloroplasts). Also, be clear on the definitions of major metabolic pathways:

- **Photosynthesis:** Converts light energy to chemical energy (glucose).
- **Glycolysis:** Anaerobic breakdown of glucose to pyruvate in the cytoplasm.
- **Cellular Respiration:** Complete aerobic breakdown of fuel molecules in the mitochondria to produce ATP.

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16. Read the following statements

i. Aerobic heterotrophic bacteria uses organic matter for carbon source and energy source.
ii. Aerobic heterotrophic bacteria uses carbon dioxide for carbon source and energy source.
iii. Aerobic autotrophic bacteria uses carbon dioxide for carbon source and reduced substances for energy source.
iv. Aerobic autotrophic bacteria uses organic matter for getting energy.

The correct choice is

(A) (i) is correct; (iii) is correct
(B) (iv) is correct; (i) is incorrect
(C) (i) is correct; (iv) is correct
(D) (ii) is correct; (iv) is incorrect

Correct Answer: (A) (i) is correct; (iii) is correct

Solution:

Step 1: Understanding the Terminology:
We need to understand the classification of organisms based on their source of carbon and energy.

- **Energy Source:** *Photo-* (light) vs. *Chemo-* (chemical compounds).
- **Carbon Source:** *Auto-* (self, uses inorganic CO_2) vs. *Hetero-* (other, uses organic compounds).

The question deals with chemo-organisms (since it mentions organic matter and reduced substances for energy) that are also aerobic (use oxygen).

Step 2: Evaluating Each Statement:

- **Statement i: Aerobic heterotrophic bacteria uses organic matter for carbon source and energy source.** This is **correct**. By definition, *heterotrophs* use organic matter as their carbon source. *Chemoheterotrophs* (which most heterotrophic bacteria are) also derive their energy from the oxidation of this same organic matter.
- **Statement ii: Aerobic heterotrophic bacteria uses carbon dioxide for carbon source and energy source.** This is **incorrect**. Heterotrophs use organic matter, not carbon dioxide, as their carbon source.
- **Statement iii: Aerobic autotrophic bacteria uses carbon dioxide for carbon source and reduced substances for energy source.** This is **correct**. By definition, *autotrophs* use inorganic carbon dioxide as their carbon source. *Chemoautotrophs* (which these are) derive their energy from the oxidation of reduced inorganic substances (like ammonia, nitrite, sulfur, etc.).
- **Statement iv: Aerobic autotrophic bacteria uses organic matter for getting energy.** This is **incorrect**. Autotrophs get their carbon from CO_2. Chemoautotrophs get their energy from inorganic substances, not organic matter.

Step 3: Final Answer:
Statements (i) and (iii) are correct. This corresponds to option (A).

Quick Tip

Break down the metabolic classification terms:

- **Heterotroph** (*hetero* = other): Carbon from organic compounds. Think of animals, fungi, most bacteria.
- **Autotroph** (*auto* = self): Carbon from inorganic CO_2. Think of plants, algae, cyanobacteria.
- Then add the energy source: **Photo-** (light) or **Chemo-** (chemicals).

For example, we are chemoheterotrophs. Plants are photoautotrophs.

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17. A student wants to decide electron acceptor for aerobic, facultative and anaerobic bacteria. In this context, read the following statements.

i. Dissolved Oxygen (DO) can act as electron acceptor for aerobic bacteria.
ii. Nitrite can act as electron acceptor for aerobic bacteria.
iii. Dissolved Oxygen (DO) can act as electron acceptor for anaerobic bacteria.
iv. Nitrite can act as electron acceptor for facultative bacteria.

The correct choice is

(A) (i) is correct; (iv) is correct
(B) (ii) is correct; (iii) is incorrect
(C) (ii) is correct; (iii) is correct
(D) (i) is correct; (ii) is correct

Correct Answer: (A) (i) is correct; (iv) is correct

Solution:

Step 1: Understanding Electron Acceptors in Respiration:
Cellular respiration involves the transfer of electrons from a fuel source to a terminal electron acceptor to generate energy. The type of acceptor defines the type of respiration.

- **Aerobic bacteria (Aerobes):** Require oxygen for survival. They use dissolved oxygen (DO) as the terminal electron acceptor in aerobic respiration.
- **Anaerobic bacteria (Anaerobes):** Cannot survive in the presence of oxygen. They use other inorganic or organic compounds as terminal electron acceptors, such as nitrate (NO_3^-), nitrite (NO_2^-), sulfate (SO_4^{2-}), or CO_2. Oxygen is often toxic to them.
- **Facultative bacteria (Facultative anaerobes):** Can switch between aerobic and anaerobic respiration. If oxygen is present, they prefer to use it as the electron acceptor because it yields more energy. If oxygen is absent, they can switch to using alternative acceptors like nitrate or nitrite.

Step 2: Evaluating Each Statement:

- **Statement i: Dissolved Oxygen (DO) can act as electron acceptor for aerobic bacteria.** This is **correct** by definition. Aerobic respiration uses oxygen.
- **Statement ii: Nitrite can act as electron acceptor for aerobic bacteria.** This is **incorrect**. Aerobic bacteria, by definition, use oxygen. They do not use nitrite as their primary terminal electron acceptor for respiration.
- **Statement iii: Dissolved Oxygen (DO) can act as electron acceptor for anaerobic bacteria.** This is **incorrect**. Oxygen is typically toxic to obligate anaerobes. They use other substances.
- **Statement iv: Nitrite can act as electron acceptor for facultative bacteria.** This is **correct**. In the absence of oxygen, facultative bacteria can perform anaerobic respiration using alternative electron acceptors, and nitrite is one such common acceptor (part of the denitrification process).

Step 3: Final Answer:
Statements (i) and (iv) are correct. This corresponds to option (A).

Quick Tip

Remember the hierarchy of electron acceptors based on energy yield:
1. **Oxygen ($\mathbf{O_2}$)** - Aerobic respiration (highest energy yield).
2. **Nitrate ($\mathbf{NO_3^-}$)** - Anaerobic respiration (denitrification).
3. **Sulfate ($\mathbf{SO_4^{2-}}$)** - Anaerobic respiration (sulfate reduction).
Facultative organisms are opportunistic; they will always use the available acceptor that gives them the most energy, with oxygen being the top choice.

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18. Which of the following is true according to the Central Pollution Control Board (CPCB), Government of India's notification issued in the year 2009?

(A) 24 hour averaged standard for $PM_{2.5}$ in ambient air is 60 $\mu g/m^3$; 24 hour averaged standard for PM_{10} in ambient air is 100 $\mu g/m^3$
(B) 24 hour averaged standard for $PM_{2.5}$ in indoor air is 60 $\mu g/m^3$; 24 hour averaged standard for PM_{10} in ambient air is 100 $\mu g/m^3$
(C) 24 hour averaged standard for $PM_{2.5}$ in ambient air is 60 $\mu g/m^3$; 24 hour averaged standard for PM_{10} in indoor air is 100 $\mu g/m^3$
(D) 24 hour averaged standard for $PM_{2.5}$ in indoor air is 60 $\mu g/m^3$; 24 hour averaged standard for PM_{10} in indoor air is 100 $\mu g/m^3$

Correct Answer: (A) 24 hour averaged standard for $PM_{2.5}$ in ambient air is 60 $\mu g/m^3$; 24 hour averaged standard for PM_{10} in ambient air is 100 $\mu g/m^3$

Solution:

Step 1: Understanding the Question:
The question asks for the correct 24-hour average standards for particulate matter ($PM_{2.5}$ and PM_{10}) as per the National Ambient Air Quality Standards (NAAQS) notified by the CPCB in 2009 in India.

Step 2: Recalling or Verifying the NAAQS 2009 Standards:
The NAAQS 2009 sets limits for various pollutants in "ambient air" for different time averages. A key distinction in the options is between "ambient air" and "indoor air." The NAAQS regulations apply to ambient (outdoor) air quality. According to the 2009 notification for industrial, residential, rural and other areas:

- The 24-hour standard for Particulate Matter with a diameter of 10 micrometers or less (PM_{10}) is **100 $\mu g/m^3$**.
- The 24-hour standard for Particulate Matter with a diameter of 2.5 micrometers or less ($PM_{2.5}$) is **60 $\mu g/m^3$**.

Step 3: Evaluating the Options:

- **(A)** States the standards for $PM_{2.5}$ and PM_{10} are 60 and 100 $\mu g/m^3$ respectively, and correctly specifies them for "ambient air". This matches the regulations.
- **(B), (C), (D)** These options incorrectly refer to "indoor air". The CPCB's NAAQS are for ambient (outdoor) air. While there are guidelines for indoor air quality, they are not part of this specific 2009 notification.

Step 4: Final Answer:
Option (A) accurately states the 24-hour standards for both $PM_{2.5}$ and PM_{10} in ambient air as per the CPCB 2009 notification.

Quick Tip

For environmental regulations, pay close attention to the details: the pollutant (e.g., $PM_{2.5}$ vs PM_{10}), the averaging time (e.g., 24-hour vs. annual), and the applicability (e.g., ambient vs. indoor vs. industrial emissions). The Indian NAAQS for $PM_{2.5}$ (24h: 60 $\mu g/m^3$, annual: 40 $\mu g/m^3$) and PM_{10} (24h: 100 $\mu g/m^3$, annual: 60 $\mu g/m^3$) are important figures to remember.

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19. The sub index values of NO_2, SO_2 and PM_{10} are 80, 80 and 100, respectively. According to the National Air Quality Index (NAQI) released by the Government of India in the year 2015, the overall NAQI is

(A) 80
(B) 260

(C) 100
(D) 151

Correct Answer: (C) 100

Solution:

Step 1: Understanding the National Air Quality Index (NAQI):
The NAQI is a tool used to communicate the level of air pollution to the public. It combines the measurements of several pollutants into a single number. The methodology for calculating the overall NAQI is based on a "worst sub-index" principle.

Step 2: Applying the NAQI Calculation Rule:
The overall NAQI is not the sum or average of the individual sub-indices. Instead, the overall NAQI is defined as the maximum value among all the calculated sub-indices for the different pollutants being monitored. The rule is:

$$\text{NAQI} = \max(\text{Sub-index}_1, \text{Sub-index}_2, \text{Sub-index}_3, \dots)$$

Step 3: Calculating the Overall NAQI:
We are given the sub-index values for three pollutants:

- Sub-index for $NO_2 = 80$
- Sub-index for $SO_2 = 80$
- Sub-index for $PM_{10} = 100$

Applying the rule:

$$\text{NAQI} = \max(80, 80, 100) = 100$$

Step 4: Final Answer:
The overall NAQI is 100. This corresponds to option (C).

Quick Tip

Remember that the National Air Quality Index (NAQI) in India is a "worst-pollutant" index. The final AQI number reported is simply the highest of the individual sub-indices calculated for each pollutant. It is not an average or a sum.

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20. Which of the following is NOT a designated waste category under Bio-medical Waste Management Rules, 2016 of Government of India?

(A) Yellow
(B) Green
(C) Red

(D) Blue

Correct Answer: (B) Green

Solution:

Step 1: Understanding the Question:
The question asks to identify which color is not used for waste segregation and collection as per the Bio-medical Waste Management Rules, 2016, in India.

Step 2: Recalling the Color Categories of the 2016 Rules:
The Bio-medical Waste Management Rules, 2016, simplified the previous color-coding system to improve segregation at the source. The primary color categories for waste collection bags or containers are:

- **Yellow:** For human anatomical waste, animal anatomical waste, soiled waste (items contaminated with blood/body fluids), expired or discarded medicines, chemical waste, and clinical laboratory waste. This waste is typically incinerated.
- **Red:** For contaminated recyclable waste, such as plastics like tubings, bottles, intravenous tubes and sets, catheters, urine bags, syringes (without needles), and gloves. This waste is sent for autoclaving/microwaving and then recycling.
- **White (Translucent):** For waste sharps including needles, syringes with fixed needles, needles from needle tip cutters, scalpels, blades, etc. This is to prevent injuries.
- **Blue:** For glassware, including broken or discarded and contaminated glass, and metallic body implants.

Step 3: Evaluating the Options:
Based on the rules:

- (A) Yellow is a major category.
- (C) Red is a major category.
- (D) Blue is a category.
- (B) Green is not a designated color for bio-medical waste segregation under these rules. Green is commonly associated with general, non-infectious, municipal solid waste (like wet kitchen waste).

Step 4: Final Answer:
"Green" is not a designated waste category under the Bio-medical Waste Management Rules, 2016.

Quick Tip

To remember the bio-medical waste categories (2016 rules), associate the color with the type of waste and its treatment:

- **Yellow:** "Yucky" or "Incinerable" stuff (anatomical, soiled, chemical, expired drugs).
- **Red:** Recyclable plastics (tubing, catheters, syringes without needles). Think of the red recycling symbol.
- **White:** Sharps (needles, blades). Think of sharp white light or a doctor's white coat.
- **Blue:** Glassware and metallic implants. Think of blue glass bottles.

Green is for your general household waste, not for bio-medical waste.

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21. Consider the following waste categories

i) Domestic Hazardous Waste
ii) Nuclear Waste
iii) Sludge from wet scrubbers of hazardous waste treatment processes
iv) Chromium bearing residue and sludge from leather tanneries

Which one of the options correctly represents the waste categories NOT covered under Hazardous and Other Wastes (Management and Transboundary Movement) Rules, 2016 of Government of India?

(A) (i) and (ii) only
(B) (i) and (iii) only
(C) (ii) and (iv) only
(D) (i), (ii) and (iii) only

Correct Answer: (A) (i) and (ii) only

Solution:

Step 1: Understanding the Scope of the Hazardous Wastes Rules, 2016:
The Hazardous and Other Wastes (Management and Transboundary Movement) Rules, 2016, specifically define what constitutes a "hazardous waste" and an "other waste". The rules have schedules listing specific industrial processes and waste streams that generate hazardous wastes. Importantly, these rules also explicitly exclude certain types of waste, which are governed by separate, specific regulations.

Step 2: Evaluating Each Waste Category:

- **i) Domestic Hazardous Waste:** While some household items can be hazardous (e.g., batteries, cleaners, paints), "Domestic Hazardous Waste" as a broad category collected from households is typically managed under the Solid Waste Management Rules, 2016, not the Hazardous Wastes Rules which primarily focus on industrial and specific commercial sources. So, this is likely **NOT COVERED**.
- **ii) Nuclear Waste:** Radioactive or nuclear waste is explicitly **EXCLUDED** from the Hazardous Wastes Rules, 2016. It is a highly specialized waste stream managed under the provisions of the Atomic Energy Act, 1962, and rules set by the Atomic Energy Regulatory Board (AERB). So, this is **NOT COVERED**.
- **iii) Sludge from wet scrubbers of hazardous waste treatment processes:** Wet scrubbers are used for air pollution control in various industries, including hazardous waste incinerators. The sludge produced from treating these hazardous flue gases would itself be considered a hazardous waste. This type of process-related sludge is listed in the schedules of the Hazardous Wastes Rules. So, this is **COVERED**.
- **iv) Chromium bearing residue and sludge from leather tanneries:** The tanning industry is a well-known generator of hazardous waste, particularly chromium. Sludges containing chromium are explicitly listed as hazardous waste in the schedules of the Hazardous Wastes Rules, 2016. So, this is **COVERED**.

Step 3: Final Answer:
The waste categories that are NOT covered under the Hazardous and Other Wastes Rules, 2016, are Domestic Hazardous Waste (i) and Nuclear Waste (ii). This corresponds to option (A).

Quick Tip

Remember that several major waste streams in India are governed by their own specific sets of rules. The Hazardous Wastes Rules primarily target industrial wastes. Key exclusions to remember are:

- Radioactive (Nuclear) Waste (Atomic Energy Act)
- Bio-medical Waste (Bio-medical Waste Management Rules)
- Municipal Solid Waste (Solid Waste Management Rules)
- E-waste (E-Waste Management Rules)

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22. Match the following

Plastic Type	Common applications
P. High-density polyethylene (HDPE)	(i) Garbage bags, bubble packaging
Q. Low-density polyethylene (LDPE)	(ii) Pharmaceutical bottles, Styrofoam cups
R. Polyethylene terephthalate (PET)	(iii) Water bottles
S. Polystyrene (PS)	(iv) Geomembrane for landfill liner

(A) P - (iv), Q – (i), R – (iii), S – (ii)
(B) P - (i), Q – (iii), R – (ii), S – (iv)
(C) P – (iv), Q – (ii), R – (i), S – (iii)
(D) P – (ii), Q – (iii), R – (iv), S – (i)

Correct Answer: (A) P - (iv), Q – (i), R – (iii), S – (ii)

Solution:

Step 1: Understanding the Question:
We need to match four common types of plastics with their typical applications.

Step 2: Matching Each Plastic Type:

- **P. High-density polyethylene (HDPE):** HDPE is a rigid, strong, and chemically resistant plastic. It is widely used for applications requiring durability and impermeability, such as milk jugs, detergent bottles, and liners for ponds and landfills. A **geomembrane for landfill liner** (iv) is a perfect application for HDPE due to its strength and resistance to chemical leaching. So, **P → (iv)**.
- **Q. Low-density polyethylene (LDPE):** LDPE is much more flexible and less dense than HDPE. Its flexibility makes it ideal for films and packaging. Common uses include plastic bags, wraps, and flexible lids. **Garbage bags and bubble packaging** (i) are classic examples of LDPE applications. So, **Q → (i)**.
- **R. Polyethylene terephthalate (PET or PETE):** PET is a clear, strong, and lightweight plastic that is widely used for food and beverage packaging, especially for carbonated drinks because of its good barrier properties against CO_2. Single-use **water bottles** (iii) are almost universally made from PET. So, **R → (iii)**.
- **S. Polystyrene (PS):** Polystyrene can be a rigid solid or a foam. In its foam form, it is known as expanded polystyrene (EPS) or by the brand name Styrofoam. It is used for disposable cups, packaging peanuts, and insulation. **Pharmaceutical bottles** (some types) can be made of rigid PS, and **Styrofoam cups** (ii) are made from foamed PS. So, **S → (ii)**.

Step 3: Final Answer:
Combining the matches:

- P → (iv)
- Q → (i)
- R → (iii)
- S → (ii)

This combination corresponds to option (A).

Quick Tip

Associate common household plastics with their types:

- **PET (1):** Clear drink bottles (water, soda).
- **HDPE (2):** Opaque, sturdy bottles (milk jugs, shampoo).
- **LDPE (4):** Flexible films and bags (grocery bags, bubble wrap).
- **PS (6):** Disposable cups (hot coffee), egg cartons, Styrofoam.

These recycling codes can help you remember the applications.

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23. Place the following international conventions/conferences/protocols/declarations in the chronological order (oldest to latest) of their happening

i) United Nations conference in Stockholm which resulted in the establishment of the United Nations Environmental Program (UNEP)
ii) Vienna convention for the protection of the Ozone layer
iii) United Nations climate change conference in Glasgow commonly referred as COP26
iv) Montreal protocol on phasing out production of substances related to Ozone layer depletion

(A) i, ii, iv, iii
(B) i, ii, iii, iv
(C) ii, iv, i, iii
(D) iv, iii, ii, i

Correct Answer: (A) i, ii, iv, iii

Solution:

Step 1: Understanding the Question:
We need to arrange four major international environmental events in the correct chronological sequence, from the earliest to the most recent.

Step 2: Identifying the Year of Each Event:

- **i) United Nations Conference on the Human Environment (Stockholm Conference):** This was the first major international conference on environmental issues. It led to the creation of the United Nations Environment Programme (UNEP). This conference took place in **1972**.
- **ii) Vienna Convention for the Protection of the Ozone Layer:** This was a framework convention that established international cooperation on monitoring and research

related to the ozone layer. It was adopted in **1985**. It laid the groundwork for the Montreal Protocol.

- **iv) Montreal Protocol on Substances that Deplete the Ozone Layer:** This is a landmark international treaty designed to protect the ozone layer by phasing out the production of numerous substances responsible for ozone depletion. It was signed in **1987**, two years after the Vienna Convention.
- **iii) United Nations Climate Change Conference in Glasgow (COP26):** This was the 26th Conference of the Parties to the UNFCCC. It is a very recent event, held in **2021**.

Step 3: Arranging the Events in Chronological Order:
Based on the years:

1. Stockholm Conference (i) - 1972
2. Vienna Convention (ii) - 1985
3. Montreal Protocol (iv) - 1987
4. COP26 in Glasgow (iii) - 2021

The correct chronological order is i, ii, iv, iii.

Step 4: Final Answer:
The correct sequence is (i), (ii), (iv), (iii), which corresponds to option (A).

Quick Tip

Remembering the timeline of key environmental treaties is important. A helpful way is to group them by topic and decade:

- **1970s:** The beginning of global environmental consciousness (Stockholm 1972).
- **1980s:** Focus on the ozone layer (Vienna 1985, Montreal 1987).
- **1990s:** Focus on climate change and biodiversity (Rio Earth Summit 1992, Kyoto Protocol 1997).
- **2010s/2020s:** Modern climate agreements (Paris Agreement 2015, Glasgow Pact/COP26 2021).

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24. The correct ascending order of the following greenhouse gases with respect to their global warming potential relative to CO_2 in the time horizon of 100 years is

(A) $CH_4 < N_2O < CFCl_3 < CF_2Cl_2$
(B) $CF_2Cl_2 < CH_4 < N_2O < CFCl_3$
(C) $CH_4 < N_2O < CF_2Cl_2 < CFCl_3$
(D) $N_2O < CFCl_3 < CH_4 < CF_2Cl_2$

Correct Answer: (A) $CH_4 < N_2O < CFCl_3 < CF_2Cl_2$

Solution:

Step 1: Understanding Global Warming Potential (GWP):
Global Warming Potential (GWP) is a metric used to compare the warming impact of different greenhouse gases. It measures how much energy the emission of 1 ton of a gas will absorb over a given period of time, relative to the emission of 1 ton of carbon dioxide (CO_2). The GWP of CO_2 is always 1. The 100-year time horizon (GWP_{100}) is the standard for comparison.

Step 2: Recalling the Approximate GWP_{100} Values:
The relative order of potency of these common greenhouse gases is a fundamental concept in climate science. Let's list the gases and their approximate GWP_{100} values based on the Intergovernmental Panel on Climate Change (IPCC) Assessment Reports (values from AR5 are commonly used):

- **Methane (CH_4):** GWP $\approx$ 28-34. Methane is potent but has a shorter atmospheric lifetime than other gases.
- **Nitrous Oxide (N_2O):** GWP $\approx$ 298. It is more potent and has a longer lifetime than methane.
- **CFC-11 (Trichlorofluoromethane, $CFCl_3$):** GWP $\approx$ 4,660. CFCs are extremely potent greenhouse gases.
- **CFC-12 (Dichlorodifluoromethane, CF_2Cl_2):** GWP $\approx$ 10,200. This is one of the most potent of the common CFCs.

Step 3: Arranging the Gases in Ascending Order:
Based on these standard scientific values, the ascending order of GWP_{100} is:

$$CH_4(\sim 28) < N_2O(\sim 298) < CFCl_3(\sim 4,660) < CF_2Cl_2(\sim 10,200)$$

Step 4: Comparing with the Options:
The order we determined is: $CH_4 < N_2O < CFCl_3 < CF_2Cl_2$. This exactly matches the sequence presented in option (A). Option (C) incorrectly places CF_2Cl_2 before $CFCl_3$, which is contrary to the established GWP values. The other options have incorrect orderings as well.
Final Answer:
The correct ascending order of Global Warming Potential is given in option (A). (Note: While this question has been a source of confusion due to some exam keys pointing to option (C), the scientifically established GWP values clearly support option (A) as the correct answer.)

Quick Tip

For Global Warming Potential (GWP_{100}), remember the general hierarchy:

- $CO_2 = 1$ (Reference)
- Methane (CH_4) is in the tens ($\sim$28).
- Nitrous Oxide (N_2O) is in the hundreds ($\sim$298).
- Halogenated compounds like CFCs are in the thousands or tens of thousands.

This relative ordering is more important to remember than the exact numbers, which are periodically updated by the IPCC.

---

25. Read the following statements with reference to the Kyoto Protocol on Climate Change

i) Each signatory (country) has common and equal responsibility.
ii) Clean development mechanism (CDM), joint implementation (JI) and international emission trading are the three mechanisms under Kyoto Protocol to reduce the greenhouse gas emissions.
iii) Under Kyoto Protocol, India has agreed to reduce its greenhouse gas emissions by half by 2050 as compared to 2005 emissions.

Which one of the following is correct choice?

(A) only i) is TRUE
(B) only ii) is TRUE
(C) only i) and ii) are TRUE
(D) only ii) and iii) are TRUE

Correct Answer: (B) only ii) is TRUE

Solution:

Step 1: Understanding the Question:
We need to evaluate three statements about the Kyoto Protocol and identify the correct option regarding their truthfulness.

Step 2: Evaluating Each Statement:

- **Statement i: Each signatory (country) has common and equal responsibility.** This is **false**. A core principle of the UNFCCC and the Kyoto Protocol is "common but differentiated responsibilities and respective capabilities" (CBDR-RC). This means that while all countries have a common responsibility to address climate change, the developed countries (listed in Annex I of the convention) have a greater historical responsibility for

emissions and greater capacity to act. The Kyoto Protocol only set binding emission reduction targets for these developed (Annex I) countries. Developing countries like India and China had no binding targets.

- **Statement ii: Clean development mechanism (CDM), joint implementation (JI) and international emission trading are the three mechanisms under Kyoto Protocol...** This is **true**. The Kyoto Protocol established three "flexibility mechanisms" to help Annex I countries meet their emission reduction targets cost-effectively. These are:
  1. **International Emissions Trading:** Allows countries that have spare emission units to sell this excess capacity to countries that are over their targets.
  2. **Joint Implementation (JI):** Allows an Annex I country to earn emission reduction units from an emission-reduction project in another Annex I country.
  3. **Clean Development Mechanism (CDM):** Allows an Annex I country to implement an emission-reduction project in a developing country to earn certified emission reduction credits.
- **Statement iii: Under Kyoto Protocol, India has agreed to reduce its greenhouse gas emissions by half by 2050...** This is **false**. As a developing (non-Annex I) country, India had no binding emission reduction targets under the Kyoto Protocol. The target mentioned (reduction by 2050 compared to 2005) is more related to later discussions and commitments under the Paris Agreement framework, not the Kyoto Protocol.

Step 3: Final Answer:
Only statement (ii) is true. This corresponds to option (B).

Quick Tip

Remember the key features of the Kyoto Protocol:

- It implemented the UNFCCC's objectives.
- Its central principle was "Common But Differentiated Responsibilities" (CBDR).
- It set binding emission targets only for developed (Annex I) countries.
- It introduced three flexibility mechanisms: Emissions Trading, JI, and CDM.

It has been largely superseded by the Paris Agreement, which involves commitments (NDCs) from all countries.

---

26. Read the following statements

I. In environmental laws, the polluter pays principle is enacted to make the polluter responsible for paying for the damage done to the natural environment.
II. The precautionary principle emphasizes caution, pausing and review before going for an innovation that may prove disastrous.
III. The precautionary principle is often used by policy makers in situations where

there is the possibility of harm from making a certain decision and conclusive evidence is not yet available.

The correct choice is

(A) I is correct; II and III are incorrect
(B) I, II and III are correct
(C) I and III are correct; II is incorrect
(D) I and II are correct; III is incorrect

Correct Answer: (B) I, II and III are correct

Solution:

Step 1: Understanding the Question:
We need to evaluate the correctness of three statements describing two fundamental principles of environmental law: the Polluter Pays Principle and the Precautionary Principle.

Step 2: Evaluating Each Statement:

- **Statement I: In environmental laws, the polluter pays principle is enacted to make the polluter responsible for paying for the damage done to the natural environment.** This is **correct**. This is the core idea of the Polluter Pays Principle. It allocates the costs of pollution prevention and control measures to the polluter. This includes the cost of environmental damage, remediation, and compensation to victims.
- **Statement II: The precautionary principle emphasizes caution, pausing and review before going for an innovation that may prove disastrous.** This is **correct**. The Precautionary Principle advocates for taking preventive action in the face of uncertainty. It suggests that if an action or policy has a suspected risk of causing harm to the public or the environment, in the absence of scientific consensus that the action or policy is not harmful, the burden of proof that it is *not* harmful falls on those taking the action. This naturally leads to caution, review, and sometimes a pause before proceeding.
- **Statement III: The precautionary principle is often used by policy makers in situations where there is the possibility of harm from making a certain decision and conclusive evidence is not yet available.** This is also **correct**. This statement perfectly describes the conditions under which the Precautionary Principle is applied. It is specifically a tool for decision-making under scientific uncertainty, where waiting for conclusive proof of harm could lead to irreversible damage.

Step 3: Final Answer:
All three statements (I, II, and III) are correct descriptions of their respective principles. This corresponds to option (B).

Quick Tip

To distinguish between the two principles:

- **Polluter Pays Principle:** Deals with the *aftermath* of pollution. It answers the question: "Who pays for the cleanup?" Answer: The polluter.
- **Precautionary Principle:** Deals with *potential future* harm. It answers the question: "Should we proceed if we are not sure it's safe?" Answer: No, or only with extreme caution; the burden of proof is on the innovator.

---

27. Read the following statements

I. The goal of Life Cycle Analysis (LCA) is to assess the environmental impact of products from a system perspective and to identify possible improvement strategies.
II. Environmental Impact Assessment (EIA) is defined as a process of identifying, predicting, and evaluating the likely impacts of a proposed project or development to define mitigation actions to reduce negative impacts and to provide positive contributions to the natural environment and well-being.

The correct choice is

(A) I is correct; II is incorrect
(B) II is correct; I is incorrect
(C) Both I and II are correct
(D) Both I and II are incorrect

Correct Answer: (C) Both I and II are correct

Solution:

Step 1: Understanding the Question:
We need to evaluate the definitions of two environmental management tools, Life Cycle Analysis (LCA) and Environmental Impact Assessment (EIA), and determine if the provided statements are correct.

Step 2: Evaluating Statement I (LCA):
The statement says, "The goal of Life Cycle Analysis (LCA) is to assess the environmental impact of products from a system perspective and to identify possible improvement strategies." This is a comprehensive and accurate definition of LCA. LCA is a methodology for assessing environmental impacts associated with all the stages of a product's life, from raw material extraction through materials processing, manufacture, distribution, use, repair and maintenance, and disposal or recycling (i.e., "from cradle to grave"). The analysis from a "system perspective" is key, as is the goal of identifying improvements. Therefore, statement I is **correct**.

Step 3: Evaluating Statement II (EIA):
The statement says, "Environmental Impact Assessment (EIA) is defined as a process of identifying, predicting, and evaluating the likely impacts of a proposed project or development to define mitigation actions to reduce negative impacts and to provide positive contributions to the natural environment and well-being." This is also a standard and accurate definition of EIA. EIA is a tool used to evaluate the potential environmental effects of a proposed *project* or *development* before the decision is made to proceed. It involves predicting impacts, assessing their significance, and proposing mitigation measures to minimize negative effects and enhance positive ones. Therefore, statement II is **correct**.

Step 4: Final Answer:
Since both statements I and II are correct definitions of LCA and EIA, respectively, the correct choice is (C).

Quick Tip

Remember the key difference in scope between LCA and EIA:

- **LCA** focuses on a **product, process, or service** and analyzes its entire life cycle ("cradle-to-grave").
- **EIA** focuses on a specific **project or development** (like a dam, highway, or factory) at a specific location, before it is implemented.

---

28. For the following major Indian environmental acts
i) Environmental Protection Act
ii) Water Act (Prevention and Control of Pollution)
iii) Air Act (Prevention and Control of Pollution)
iv) The National Green Tribunal Act
the correct chronological order (oldest to latest of their enactment) is

(A) i), ii), iii), iv)
(B) ii), i), iii), iv)
(C) iii), i), iv), ii)
(D) ii), iii), i), iv)

Correct Answer: (D) ii), iii), i), iv)

Solution:

Step 1: Understanding the Question:
We need to arrange four major environmental laws of India in the chronological order in which they were enacted, from the oldest to the most recent.

Step 2: Identifying the Year of Enactment for Each Act:

- **ii) Water (Prevention and Control of Pollution) Act:** This was one of the first comprehensive environmental laws enacted by the Parliament of India to address water pollution. It was enacted in **1974**.
- **iii) Air (Prevention and Control of Pollution) Act:** Following the Water Act, this legislation was passed to address the growing problem of air pollution. It was enacted in **1981**.
- **i) Environment (Protection) Act:** This is an "umbrella" legislation that gives the central government broad powers to protect and improve the environment. It was enacted in the wake of the Bhopal Gas Tragedy. It was passed in **1986**.
- **iv) The National Green Tribunal Act:** This act established the National Green Tribunal (NGT), a specialized judicial body for effective and expeditious disposal of cases relating to environmental protection and conservation. It was enacted in **2010**.

Step 3: Arranging the Acts in Chronological Order:
Based on the years of enactment:

1. Water Act (ii) - 1974
2. Air Act (iii) - 1981
3. Environment Protection Act (i) - 1986
4. National Green Tribunal Act (iv) - 2010

The correct chronological order is ii), iii), i), iv).

Step 4: Final Answer:
The correct sequence is (ii), (iii), (i), (iv), which corresponds to option (D).

Quick Tip

Remembering the timeline of India's major environmental laws is crucial. A simple mnemonic can be to think of the sequence of concerns: first **Water** (1974), then **Air** (1981), then a comprehensive **Environment** umbrella act (1986) after the Bhopal disaster, and much later, a specialized court, the **National Green Tribunal** (2010), to enforce them.

---

29. The kinematic viscosity of glycerin and kerosene are 1.2 times and 0.95 times of that of water, respectively. Glycerin and kerosene flow through two identical porous media having same hydraulic gradient. Assuming Darcy's law is valid for the porous media, the ratio of flow rate of kerosene to that of glycerin is

(A) 1.052
(B) 1.140
(C) 0.792
(D) 1.263

Correct Answer: (D) 1.263

Solution:

Step 1: Understanding the Question and Relevant Laws:
The problem involves the flow of two different fluids (kerosene and glycerin) through identical porous media under the same hydraulic gradient. We are told to assume Darcy's Law is valid. Darcy's Law relates the specific discharge (or Darcy velocity), v, to the hydraulic conductivity (K) and the hydraulic gradient (i):

$$v = K \cdot i$$

The flow rate, Q, is given by $Q = v \cdot A$, where A is the cross-sectional area of the porous medium.

$$Q = K \cdot i \cdot A$$

The hydraulic conductivity, K, is not a constant for the medium alone; it also depends on the properties of the fluid. The relationship is:

$$K = \frac{k \cdot g}{\nu}$$

where k is the intrinsic permeability of the porous medium (a property of the medium only), g is the acceleration due to gravity, and ν is the kinematic viscosity of the fluid.

Step 2: Formulating the Ratio of Flow Rates:
We need to find the ratio $\frac{Q_{kerosene}}{Q_{glycerin}}$. Using the formulas from Step 1:

$$\frac{Q_k}{Q_g} = \frac{K_k \cdot i_k \cdot A_k}{K_g \cdot i_g \cdot A_g}$$

We are given that:

- The porous media are "identical", which means their intrinsic permeability (k) and cross-sectional area (A) are the same. $k_k = k_g$ and $A_k = A_g$.
- They have the "same hydraulic gradient", so $i_k = i_g$.

The ratio simplifies to:

$$\frac{Q_k}{Q_g} = \frac{K_k}{K_g}$$

Now, substitute the expression for K:

$$\frac{Q_k}{Q_g} = \frac{(k \cdot g/\nu_k)}{(k \cdot g/\nu_g)} = \frac{\nu_g}{\nu_k}$$

The ratio of flow rates is inversely proportional to the ratio of the kinematic viscosities of the fluids.

Step 3: Calculating the Ratio:
Let ν_w be the kinematic viscosity of water. We are given:

- Kinematic viscosity of glycerin, $\nu_g = 1.2\nu_w$
- Kinematic viscosity of kerosene, $\nu_k = 0.95\nu_w$

Substitute these into our ratio equation:

$$\frac{Q_k}{Q_g} = \frac{1.2\nu_w}{0.95\nu_w} = \frac{1.2}{0.95}$$

$$\frac{Q_k}{Q_g} \approx 1.263157...$$

Step 4: Final Answer:
The ratio of the flow rate of kerosene to that of glycerin is approximately 1.263. This corresponds to option (D).

> Quick Tip
>
> Darcy's Law is often written as $Q = KiA$. Remember that the hydraulic conductivity K is a combined property of both the porous medium and the fluid. K is inversely proportional to the fluid's viscosity (either dynamic μ or kinematic ν). Therefore, for a given porous medium and hydraulic gradient, a less viscous fluid will flow faster.

---

30. A researcher compiled the following information about the performance of a kit in an outbreak

Infection state	**Kit response**
Disease (probability = 0.002)	Positive response (probability = 0.98)
No Disease	Positive response (probability = 0.03)

The probability of detecting an infection for a positive result through the kit would be _____ (rounded off to three decimal places).

Correct Answer: 0.061

Solution:

Step 1: Understanding the Question and Defining Events:
The question asks for the probability that a person has the infection (disease) *given that* they received a positive test result. This is a conditional probability problem that is a classic application of Bayes' theorem. Let's define the events:

- D: The event that a person has the Disease.
- N: The event that a person does Not have the disease.

- $+$: The event that the kit gives a Positive response.

We are asked to find the probability $P(D|+)$.

Step 2: Listing the Probabilities from the Problem Statement:
From the information provided, we can extract the following probabilities:

- The prior probability of having the disease (prevalence): $P(D) = 0.002$.
- The probability of a positive test given the person has the disease (sensitivity): $P(+|D) = 0.98$.
- The probability of a positive test given the person does not have the disease (false positive rate): $P(+|N) = 0.03$.

We can also calculate the probability of not having the disease:

- $P(N) = 1 - P(D) = 1 - 0.002 = 0.998$.

Step 3: Applying Bayes' Theorem:
Bayes' theorem for this problem is:

$$P(D|+) = \frac{P(+|D)P(D)}{P(+)}$$

The denominator, $P(+)$, is the total probability of getting a positive result, regardless of the actual infection state. We can calculate this using the Law of Total Probability:

$$P(+) = P(+|D)P(D) + P(+|N)P(N)$$

This represents the sum of the probabilities of all ways to get a positive result: a true positive ($P(+|D)P(D)$) and a false positive ($P(+|N)P(N)$).

Step 4: Calculating the Numerical Values:
First, calculate the total probability of a positive test, $P(+)$:

$$P(+) = (0.98 \times 0.002) + (0.03 \times 0.998)$$

$$P(+) = 0.00196 + 0.02994$$

$$P(+) = 0.0319$$

Now, substitute this back into the Bayes' theorem formula:

$$P(D|+) = \frac{P(+|D)P(D)}{P(+)} = \frac{0.00196}{0.0319}$$

$$P(D|+) \approx 0.061442...$$

Step 5: Final Answer:
The question asks to round the result to three decimal places.

$$0.061$$

This result, known as the positive predictive value, is surprisingly low. It highlights that when testing for a rare disease, a large proportion of positive results can be false positives, even with a seemingly accurate test.

Quick Tip

Bayes' theorem is essential for solving problems involving conditional probabilities and diagnostic tests. Remember the formula: $P(A|B) = \frac{P(B|A)P(A)}{P(B)}$. A common mistake is to confuse $P(D|+)$ (the probability you have the disease if you test positive) with $P(+|D)$ (the probability you test positive if you have the disease). The former is what you want to know as a patient; the latter is a characteristic of the test (its sensitivity).

---

31. The critical depth in a 2 m wide rectangular channel carrying a discharge of 10 m^3/s and taking value of acceleration due to gravity (g) as 9.81 m/s^2 is _____ (in m, rounded off to two decimal places).

Correct Answer: 1.37

Solution:

Step 1: Understanding the Question and Relevant Concepts:
The question asks for the critical depth (y_c) of flow in a rectangular open channel. Critical flow occurs when the Froude number (Fr) is equal to 1. For a rectangular channel, this corresponds to the condition where the specific energy is minimum for a given discharge.

Step 2: Formula for Critical Depth:
For a rectangular channel, the critical depth, y_c, is given by the formula:

$$y_c = \left(\frac{q^2}{g}\right)^{1/3}$$

where:

- q is the discharge per unit width of the channel ($q = Q/B$).
- g is the acceleration due to gravity.

Step 3: Calculating the Discharge per Unit Width (q):
We are given:

- Total discharge, $Q = 10\ m^3/s$.
- Channel width, $B = 2\ m$.

$$q = \frac{Q}{B} = \frac{10\ m^3/s}{2\ m} = 5\ m^2/s$$

Step 4: Calculating the Critical Depth (y_c):
We are given $g = 9.81\ m/s^2$. Substitute the values into the formula:

$$y_c = \left(\frac{5^2}{9.81}\right)^{1/3} = \left(\frac{25}{9.81}\right)^{1/3}$$

$$y_c = (2.5484...)^{1/3} \approx 1.3658...\ m$$

Step 5: Final Answer:
Rounding the result to two decimal places:

$$y_c \approx 1.37 \text{ m}$$

> Quick Tip
>
> For open channel flow, remember the key condition for critical flow in a rectangular channel: $Fr = \frac{v}{\sqrt{gy}} = 1$, which leads to the formula $y_c = (q^2/g)^{1/3}$. Don't forget to use the discharge per unit width ($q = Q/B$), not the total discharge (Q), in this formula.

---

32. The ratio of the moles of CO_2 evolved to the moles of O_2 consumed in respiration also called the respiratory quotient, is calculated for a carbohydrate ($C_6H_{12}O_6$) as substrate and found to be 1. Under similar conditions, for a fatty acid ($C_{51}H_{98}O_6$) as substrate, the respiratory quotient is _____ (rounded off to two decimal place).

Correct Answer: 0.70

Solution:

Step 1: Understanding Respiratory Quotient (RQ):
The respiratory quotient (RQ) is defined as the ratio of the volume (or moles) of carbon dioxide evolved to the volume (or moles) of oxygen consumed during respiration.

$$\text{RQ} = \frac{\text{moles of } CO_2 \text{ produced}}{\text{moles of } O_2 \text{ consumed}}$$

To find the RQ for a specific substrate, we need to write and balance the chemical equation for its complete aerobic oxidation.

Step 2: Verify the RQ for Carbohydrate (Glucose):
The balanced equation for the complete oxidation of glucose (a carbohydrate) is:

$$C_6H_{12}O_6 + 6O_2 \rightarrow 6CO_2 + 6H_2O$$

From the stoichiometry of this reaction:

$$\text{RQ}_{\text{carbohydrate}} = \frac{6 \text{ moles } CO_2}{6 \text{ moles } O_2} = 1$$

This confirms the information given in the problem.

Step 3: Write and Balance the Equation for the Fatty Acid:
The substrate is the fatty acid Tripalmitin ($C_{51}H_{98}O_6$). The general equation for its complete oxidation is:

$$C_{51}H_{98}O_6 + aO_2 \rightarrow bCO_2 + cH_2O$$

We need to find the stoichiometric coefficients a, b, and c.

- **Balance Carbon (C):** There are 51 carbon atoms on the left, so we need 51 molecules of CO_2.

$$b = 51$$

- **Balance Hydrogen (H):** There are 98 hydrogen atoms on the left. Each water molecule has 2 hydrogen atoms, so we need $98/2 = 49$ molecules of H_2O.

$$c = 49$$

- **Balance Oxygen (O):** Now we balance the oxygen atoms.
  - Oxygen on the right side: 51×2 (from CO_2) + 49×1 (from H_2O) $= 102 + 49 = 151$ oxygen atoms.
  - Oxygen on the left side: 6 (from the fatty acid) + $2a$ (from O_2).

  Equating the oxygen atoms:

$$6 + 2a = 151$$

$$2a = 145$$

$$a = 72.5$$

The balanced chemical equation is:

$$C_{51}H_{98}O_6 + 72.5O_2 \rightarrow 51CO_2 + 49H_2O$$

Step 4: Calculate the RQ for the Fatty Acid:
Using the definition of RQ and the coefficients from our balanced equation:

$$RQ_{\text{fatty acid}} = \frac{b}{a} = \frac{51}{72.5} \approx 0.70344...$$

Step 5: Final Answer:
Rounding the result to two decimal places gives 0.70.
There seems to be a minor discrepancy, likely due to rounding conventions or a slight typo in the problem statement's intended answer. The method and calculation are correct. I will provide 0.70.

Quick Tip

The respiratory quotient (RQ) gives a clue about the metabolic fuel being used:

- **RQ $\approx$ 1.0** for Carbohydrates (e.g., glucose)
- **RQ $\approx$ 0.7** for Fats (they are less oxidized and require more O_2)
- **RQ $\approx$ 0.8** for Proteins

To calculate RQ for any substance, write the balanced chemical equation for its complete oxidation and take the ratio of moles of CO_2 produced to O_2 consumed.

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33. The value of $\frac{4}{\pi}\int_0^{\pi/2} \sin^2 x\,dx$ is _____ (rounded off to two decimal places).

Correct Answer: 1.00

Solution:

Step 1: Understanding the Question:
We need to evaluate a definite integral and multiply the result by a constant. The integral is of $\sin^2 x$.

Step 2: Using a Trigonometric Identity:
The integral of $\sin^2 x$ is not straightforward. We must first use a power-reduction identity. The relevant identity is:

$$\sin^2 x = \frac{1-\cos(2x)}{2}$$

Step 3: Evaluating the Integral:
Substitute the identity into the integral:

$$\int_0^{\pi/2} \sin^2 x\,dx = \int_0^{\pi/2} \frac{1-\cos(2x)}{2}\,dx$$

$$= \frac{1}{2}\int_0^{\pi/2} (1-\cos(2x))\,dx$$

Now, integrate term by term:

$$= \frac{1}{2}\left[x - \frac{\sin(2x)}{2}\right]_0^{\pi/2}$$

Evaluate the expression at the upper and lower limits:

- At $x = \pi/2$: $\frac{\pi}{2} - \frac{\sin(2\cdot\pi/2)}{2} = \frac{\pi}{2} - \frac{\sin(\pi)}{2} = \frac{\pi}{2} - 0 = \frac{\pi}{2}$
- At $x = 0$: $0 - \frac{\sin(0)}{2} = 0 - 0 = 0$

The value of the integral is:

$$\int_0^{\pi/2} \sin^2 x\,dx = \frac{1}{2}\left[\frac{\pi}{2} - 0\right] = \frac{\pi}{4}$$

Alternative Method: Wallis' Formula For integrals of the form $\int_0^{\pi/2} \sin^n x\,dx$, if n is an even integer, the value is $\frac{(n-1)!!}{n!!}\frac{\pi}{2}$. For n=2: $\int_0^{\pi/2} \sin^2 x\,dx = \frac{1}{2}\frac{\pi}{2} = \frac{\pi}{4}$. This confirms the result.

Step 4: Calculating the Final Value:
The question asks for the value of the entire expression $\frac{4}{\pi}\int_0^{\pi/2} \sin^2 x\,dx$.

$$\text{Value} = \frac{4}{\pi} \times \left(\frac{\pi}{4}\right) = 1$$

Step 5: Final Answer:
The value is exactly 1. Rounding to two decimal places gives 1.00.

Quick Tip

Integrals of $\sin^2 x$ and $\cos^2 x$ are very common. Always use the power-reduction identities:

- $\sin^2 x = \frac{1-\cos(2x)}{2}$
- $\cos^2 x = \frac{1+\cos(2x)}{2}$

It's also useful to remember the definite integral results: $\int_0^{\pi/2} \sin^2 x\, dx = \int_0^{\pi/2} \cos^2 x\, dx = \frac{\pi}{4}$.

---

34. An S-hydrograph was prepared for a catchment of 240 km^2 using 3-hour unit hydrograph (1 cm rainfall excess). The equilibrium discharge for the S-hydrograph would be _____ (in m^3/s, rounded off to two decimal places).

Correct Answer: 222.22

Solution:

Step 1: Understanding the S-Hydrograph and Equilibrium Discharge:
An S-hydrograph is the hydrograph of direct runoff that would result from a continuous effective rainfall of a constant intensity of 1/D cm/hour, where D is the duration of the unit hydrograph. It is constructed by summing a series of D-hour unit hydrographs, each lagged by D hours. The S-hydrograph eventually reaches a constant value called the equilibrium discharge (Q_{eq}). This occurs when the rate of runoff from the catchment equals the rate of rainfall input.

Step 2: Formula for Equilibrium Discharge:
The constant rainfall intensity that generates the S-hydrograph is $i = 1/D$ cm/hr. The runoff rate (discharge) from a catchment of area A due to a rainfall intensity i is given by $Q = i \times A$. We must be careful with units. Let's express the intensity in m/s:

$$i = \frac{1 \text{ cm}}{D \text{ hours}} = \frac{0.01 \text{ m}}{D \times 3600 \text{ s}}$$

The equilibrium discharge is:

$$Q_{eq} = i \times A = \left(\frac{0.01}{D \times 3600}\right) \times (\text{Area in m}^2)$$

A simpler, commonly used formula is:

$$Q_{eq}(\text{m}^3/\text{s}) = \frac{2.778 \times A(\text{km}^2)}{D(\text{hours})}$$

This formula directly incorporates the unit conversions. The constant $2.778 \approx 10^6/3600$.

Step 3: Calculating the Equilibrium Discharge:
We are given:

- Catchment Area, $A = 240$ km^2.
- Unit Hydrograph Duration, $D = 3$ hours.

Using the direct formula:

$$Q_{eq} = \frac{2.778 \times A}{D} = \frac{2.778 \times 240}{3} = 2.778 \times 80$$

$$Q_{eq} = 222.24 \text{ m}^3/\text{s}$$

Let's derive it from first principles to be more precise:

$$i = \frac{1 \text{ cm}}{3 \text{ hr}} = \frac{0.01 \text{ m}}{3 \times 3600 \text{ s}}$$

$$A = 240 \text{ km}^2 = 240 \times (1000 \text{ m})^2 = 240 \times 10^6 \text{ m}^2$$

$$Q_{eq} = i \times A = \left(\frac{0.01}{3 \times 3600}\right) \times (240 \times 10^6) = \frac{0.01 \times 240 \times 10^6}{10800}$$

$$Q_{eq} = \frac{2.4 \times 10^6}{10800} = \frac{24000}{108} = \frac{2000}{9} \approx 222.222... \text{ m}^3/\text{s}$$

Step 4: Final Answer:
The equilibrium discharge is 222.222... m^3/s. Rounding to two decimal places gives 222.22.

Quick Tip

The equilibrium discharge of an S-hydrograph represents the steady-state runoff from the catchment under continuous rainfall of intensity 1/D. The formula $Q_{eq} = A/D$ is simple to remember, but you must be extremely careful with units. It's often safer to use the dimensionally consistent formula $Q_{eq}(\text{m}^3/\text{s}) = \frac{A(\text{km}^2) \times 10^6 (\text{m}^2/\text{km}^2)}{D(\text{hours}) \times 3600(\text{s/hr})}$ and plug in the rainfall intensity (here 1 cm/D hr = 0.01 m/D hr).

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35. River water containing two types of spherical suspended particles (clay particles, metal particles) is retained in a sedimentation tank. The clay particles having diameter of 75 μm and specific gravity of 2.65 is settling in the tank with a constant velocity. The velocity of clay particles is 2 times of that of metal particles having specific gravity of 8. Assume discrete settling and laminar flow conditions within the sedimentation tank. The estimated diameter of the metal particles is _____ (in μm, rounded off to integer)

Correct Answer: 26

Solution:

Step 1: Identify the Governing Law:
The problem states that the particles are undergoing discrete settling under laminar flow conditions. The terminal settling velocity (v_s) for spherical particles in this regime is described by

Stokes' Law.

Step 2: State Stokes' Law and Formulate a Ratio:
Stokes' Law is given by:

$$v_s = \frac{g(G_s - 1)d^2}{18\nu}$$

where g is gravity, G_s is the particle's specific gravity, d is the particle diameter, and ν is the fluid's kinematic viscosity. To solve the problem, we can set up a ratio of the settling velocities for the clay (c) and metal (m) particles. This will cancel out the constants g, 18, and ν, as the fluid is the same for both.

$$\frac{v_c}{v_m} = \frac{\frac{g(G_{sc}-1)d_c^2}{18\nu}}{\frac{g(G_{sm}-1)d_m^2}{18\nu}} = \frac{(G_{sc} - 1)d_c^2}{(G_{sm} - 1)d_m^2}$$

Step 3: Substitute Given Values and Solve for the Unknown Diameter:
The problem provides the following data:

- Clay: $d_c = 75\,\mu$m, $G_{sc} = 2.65$
- Metal: $d_m = ?$, $G_{sm} = 8$
- Velocity relationship: "velocity of clay particles is 2 times of that of metal particles", which means $v_c = 2v_m$, or $\frac{v_c}{v_m} = 2$.

Substitute these values into the ratio equation:

$$2 = \frac{(2.65 - 1)(75)^2}{(8 - 1)d_m^2}$$

$$2 = \frac{(1.65)(5625)}{7 \cdot d_m^2}$$

$$2 = \frac{9281.25}{7d_m^2}$$

Now, rearrange the equation to solve for d_m^2:

$$d_m^2 = \frac{9281.25}{2 \times 7} = \frac{9281.25}{14} \approx 662.946\,(\mu\text{m})^2$$

Finally, take the square root to find the diameter d_m:

$$d_m = \sqrt{662.946} \approx 25.748\,\mu\text{m}$$

Step 4: Final Answer:
The question asks to round the estimated diameter to the nearest integer.

$$d_m \approx 26\,\mu\text{m}$$

Quick Tip

For problems comparing the settling of different particles using Stokes' Law, setting up a ratio is the most efficient method as it cancels out common constants like gravity and viscosity. The key relationship to remember is that settling velocity is proportional to the term $(G_s - 1)d^2$.

36. W1, W2, W3...W9 represent the holding times of 9 water samples, which follow a normal distribution with mean $\mu = 8.33$ and standard deviation $\sigma = 4.472$. M represents the sample mean value of holding times, which also has a normal distribution. Assuming Z has a standard normal distribution (mean = 0 and standard deviation = 1), select the correct statement which describes the expression for calculating the value of type 1 error where null hypothesis (H_0): M > 6 alternate hypothesis (H_a): M ≤ 6

(A) P{Z<(-1.565)}
(B) P{Z<1.565}
(C) P{Z>1.565)}
(D) P{Z>1.565}

Correct Answer: (A) P{Z¡(-1.565)}

Solution:

Step 1: Understand Type I Error and the Rejection Region:
A Type I error occurs when we reject the null hypothesis (H_0) when it is actually true. The probability of a Type I error is denoted by α. The decision to reject H_0 is based on whether the sample statistic falls into a pre-defined "rejection region." This region is defined by the alternative hypothesis (H_a). In this problem:

- Null Hypothesis H_0: M > 6
- Alternate Hypothesis H_a: M ≤ 6

The test procedure is to reject H_0 if the observed sample mean M falls into the region defined by H_a. Therefore, the **rejection region is M ≤ 6**.

Step 2: Interpreting the Calculation of Type I Error:
The wording of this question is non-standard. Typically, a hypothesis is about a population parameter (like μ), not the sample mean (M), and the Type I error is calculated using the boundary value of the null hypothesis (e.g., $\mu = 6$). However, the question provides the true population mean ($\mu = 8.33$) and asks for the expression for the Type I error. In this specific (and confusing) context, the question is asking for the probability that the test procedure *would incorrectly reject H_0*, given the true state of the world. Since the true mean is 8.33, which is ¿ 6, the null hypothesis is indeed true for this population. The expression for the Type I error is therefore the probability of making a rejection decision, i.e., finding a sample mean $M \leq 6$, given that the samples are drawn from the true population. So, we need to calculate: $P(M \leq 6)$ based on the population with $\mu = 8.33$ and $\sigma = 4.472$.

Step 3: Standardize the Sample Mean M:
The sample mean M of $n = 9$ samples is normally distributed.

- Mean of M: $E[M] = \mu = 8.33$
- Standard deviation of M (Standard Error): $SE = \frac{\sigma}{\sqrt{n}} = \frac{4.472}{\sqrt{9}} = \frac{4.472}{3} \approx 1.4907$

To find the probability, we convert the value M=6 to a standard normal Z-score:

$$Z = \frac{M - \mu}{SE} = \frac{6 - 8.33}{1.4907} = \frac{-2.33}{1.4907} \approx -1.563$$

Step 4: Formulate the Probability Expression:
The probability $P(M \leq 6)$ is equivalent to the probability $P(Z \leq -1.563)$. Comparing this to the options, it matches the form and value of option (A), $P\{Z < (-1.565)\}$. The minor difference between -1.563 and -1.565 is due to the precision of the numbers given in the problem statement. Therefore, option (A) is the correct descriptive expression.

Quick Tip

A Type I error is rejecting a true null hypothesis. The probability of a Type I error (α) is the area of the rejection region. To find this probability, you must first define the rejection region based on the alternative hypothesis. Then, you convert the boundary of this region into a Z-score (or t-score) and find the corresponding probability from the standard distribution.

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37. Which one of the following statements is NOT correct?

(A) Photophosphorylation is the synthesis of ATP from ADP and inorganic phosphate in the presence of light.
(B) The process through which ATP is synthesised by cells (in mitochondria and chloroplasts) is called phosphorylation.
(C) The Calvin cycle (carboxylation, reduction, and regeneration) occurs in all photosynthetic plants (C3, C4 or any other).
(D) C3 plants have a special type of leaf anatomy, they tolerate higher temperatures, they show a response to high light intensities, have high rate of photosynthesis and reduced rate of photorespiration as compared to C4 plants.

Correct Answer: (D) C3 plants have a special type of leaf anatomy, they tolerate higher temperatures, they show a response to high light intensities, have high rate of photosynthesis and reduced rate of photorespiration as compared to C4 plants.

Solution:

Step 1: Understanding the Question:
We are asked to identify the incorrect statement among four options related to photosynthesis and cellular energy production.

Step 2: Evaluating Each Statement:

- **(A) Photophosphorylation is the synthesis of ATP from ADP and inorganic phosphate in the presence of light.** This is **correct**. This is the definition of photophosphorylation, which is a key part of the light-dependent reactions of photosynthesis.

- **(B) The process through which ATP is synthesised by cells (in mitochondria and chloroplasts) is called phosphorylation.** This is **correct**. Phosphorylation is the general term for adding a phosphate group to a molecule. The synthesis of ATP from ADP and phosphate is a specific type of phosphorylation. It occurs as oxidative phosphorylation in mitochondria and photophosphorylation in chloroplasts.
- **(C) The Calvin cycle (carboxylation, reduction, and regeneration) occurs in all photosynthetic plants (C3, C4 or any other).** This is **correct**. The Calvin cycle is the fundamental pathway for carbon fixation in all photosynthetic plants. While C4 and CAM plants have additional preliminary steps to concentrate CO_2, the actual conversion of CO_2 into sugar (the Calvin cycle) still occurs in them.
- **(D) C3 plants have a special type of leaf anatomy, they tolerate higher temperatures, they show a response to high light intensities, have high rate of photosynthesis and reduced rate of photorespiration as compared to C4 plants.** This statement is **incorrect**. It reverses the characteristics of C3 and C4 plants. It is the **C4 plants** (not C3) that have a special "Kranz" leaf anatomy, tolerate higher temperatures and light intensities, have higher rates of photosynthesis (in hot/dry conditions), and have a mechanism to significantly reduce photorespiration. C3 plants are less efficient under these conditions and suffer from higher rates of photorespiration.

Step 3: Final Answer:
Statement (D) incorrectly describes the characteristics of C3 plants; it actually lists the advantages of C4 plants. Therefore, (D) is the statement that is NOT correct.

Quick Tip

Remember the key differences between C3 and C4 plants:

- **C3 Plants** (e.g., rice, wheat, soybeans): More common, efficient in cool, wet climates. Suffer from photorespiration in hot, dry conditions.
- **C4 Plants** (e.g., corn, sugarcane, sorghum): Adapted to hot, dry climates. Have special Kranz anatomy and a CO_2-concentrating mechanism to reduce photorespiration and increase efficiency at high temperatures.

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38. Read the following statements

I. Bacteriophage is an anaerobic bacterium.
II. Male-specific bacteriophage infect via the pili of other microorganisms including viruses.
III. Bacteriophage is found in human as well as in animal excreta.
IV. Bacteriophage can not indicate the presence of bacteria.

The correct choice is

(A) (I), (III) and (IV) are correct
(B) (IV) is correct; (III) is incorrect
(C) Both (III) and (IV) are incorrect
(D) Both (III) and (IV) are correct

Correct Answer: (Flawed Question)

Solution:

Step 1: Understanding the Question:
We need to evaluate four statements about bacteriophages and then select the option that correctly describes their truthfulness.

Step 2: Evaluating Each Statement Factually:

- **Statement I: Bacteriophage is an anaerobic bacterium.** This is **False**. A bacteriophage is a type of **virus** that infects bacteria. It is not a bacterium itself.
- **Statement II: Male-specific bacteriophage infect via the pili of other microorganisms including viruses.** This is **False**. Male-specific bacteriophages infect **bacteria** (not viruses) by attaching to specific structures like F-pili (sex pili). Viruses do not infect other viruses in this manner.
- **Statement III: Bacteriophage is found in human as well as in animal excreta.** This is **True**. Bacteria, such as E. coli, are abundant in the intestinal tracts of humans and animals. Since bacteriophages infect these bacteria, they are consequently present in fecal matter (excreta).
- **Statement IV: Bacteriophage can not indicate the presence of bacteria.** This is **False**. Bacteriophages are often highly specific to their bacterial hosts. Therefore, the presence of a specific bacteriophage (like a coliphage) is a strong and widely used indicator for the presence of its host bacteria (like E. coli), which in turn indicates fecal contamination.

Step 3: Analyzing the Options based on the Factual Evaluation:
Our analysis shows that Statement III is TRUE and Statements I, II, and IV are FALSE. Let's see if any option matches this conclusion.

- (A) Claims I, III, and IV are correct. This is incorrect.
- (B) Claims IV is correct and III is incorrect. Both assertions are incorrect.
- (C) Claims both III and IV are incorrect. This means it claims "Statement III is incorrect" AND "Statement IV is incorrect". Since Statement III is actually TRUE, this option is logically incorrect.
- (D) Claims both III and IV are correct. This is incorrect because statement IV is false.

Step 4: Final Conclusion:
None of the provided options accurately reflect the factual correctness of the four statements.

Statement III is true, while I, II, and IV are false. No option correctly identifies this. Therefore, the question is fundamentally flawed and cannot be answered correctly from the given choices.

Quick Tip

Key facts about bacteriophages:

- They are viruses, not bacteria.
- They infect bacteria, often with high specificity.
- Because of this specificity, their presence is a reliable indicator of the presence of their host bacteria (e.g., coliphages indicate fecal contamination).
- They are abundant in environments rich in bacteria, such as soil, sewage, and the gut.

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39. Read the following statements

i. In endogenous metabolism by aerobic bacteria, electron acceptor is present inside the cells.
ii. In endogenous metabolism by aerobic bacteria, electron acceptor is dissolved oxygen.
iii. The endogenous metabolism is linked to fermentative metabolism.
iv. In exogenous metabolism by aerobic bacteria, enzyme mediated electron transfer happens within the cells.

The correct choice is

(A) (i) is correct; (iii) is correct
(B) (ii) is correct; (iii) is incorrect
(C) (iii) is incorrect; (iv) is incorrect
(D) (iii) is correct; (iv) is correct

Correct Answer: (B) (ii) is correct; (iii) is incorrect

Solution:

Step 1: Understanding Endogenous vs. Exogenous Metabolism:

- **Exogenous Metabolism:** This is the normal metabolic state where bacteria consume an external food source (substrate) from their environment for energy and growth.
- **Endogenous Metabolism (or Endogenous Respiration):** This occurs when the external food source is depleted. Bacteria switch to consuming their own internal stored food reserves and cellular components to maintain viability. It is a state of "starvation" or decay phase.

Step 2: Evaluating Each Statement:

- **i. In endogenous metabolism by aerobic bacteria, electron acceptor is present inside the cells.** This is **incorrect**. The electron acceptor for *aerobic* bacteria is always oxygen, which is an external substance that diffuses into the cell from the surrounding environment (e.g., dissolved oxygen in water). The food source is internal, but the electron acceptor is external.
- **ii. In endogenous metabolism by aerobic bacteria, electron acceptor is dissolved oxygen.** This is **correct**. Whether the metabolism is exogenous or endogenous, if the bacteria are aerobic, their terminal electron acceptor for respiration is dissolved oxygen.
- **iii. The endogenous metabolism is linked to fermentative metabolism.** This is **incorrect**. Endogenous metabolism is a form of respiration, where cellular components are oxidized via the electron transport chain with an external electron acceptor (like oxygen). Fermentation is an anaerobic process that does not use an external electron acceptor or an electron transport chain. They are distinct metabolic strategies.
- **iv. In exogenous metabolism by aerobic bacteria, enzyme mediated electron transfer happens within the cells.** This is **correct**. All metabolism, including the breakdown of external substrates (exogenous) and the subsequent electron transport chain, is mediated by enzymes and occurs within the bacterial cell.

Step 3: Evaluating the Options:
Summarizing the statements: i is incorrect, ii is correct, iii is incorrect, iv is correct. Let's check the options based on this:

- (A) (i) is correct (False); (iii) is correct (False).
- (B) (ii) is correct (True); (iii) is incorrect (True). This option is fully consistent.
- (C) (iii) is incorrect (True); (iv) is incorrect (False).
- (D) (iii) is correct (False); (iv) is correct (True).

Step 4: Final Answer:
The only fully correct choice is (B), which states that (ii) is correct and (iii) is incorrect.

Quick Tip

Distinguish between the fuel source and the electron acceptor in metabolism:

- **Exogenous:** External fuel.
- **Endogenous:** Internal fuel (self-consumption).
- **Aerobic:** External O_2 is the electron acceptor.
- **Anaerobic Respiration:** External non-O_2 (e.g., NO_3^-) is the electron acceptor.
- **Fermentation:** No external electron acceptor; uses an internal organic molecule.

Endogenous metabolism is a form of respiration, not fermentation.

40. A boiler in an industry, located where high plume rise is expected, releases flue gas with fine particulate matter. Which one of the following options is most suited and efficient if this particulate matter is intended for reuse?

(A) reduce stack height and increase stack diameter
(B) use of wet collectors
(C) use of flue gas desulfurization (FGD)
(D) use of electrostatic precipitator (ESP)

Correct Answer: (D) use of electrostatic precipitator (ESP)

Solution:

Step 1: Understanding the Question:
The question asks for the best method to control fine particulate matter from a boiler's flue gas, with two key conditions:

1. The method must be efficient for **fine** particles.
2. The collected particulate matter is intended for **reuse**.

Step 2: Evaluating the Options:

- **(A) reduce stack height and increase stack diameter:** This is related to the dispersion of pollutants in the atmosphere, not the collection or removal of them from the flue gas. Reducing stack height would worsen local air quality. This option is incorrect as it is a dispersion strategy, not a control technology.
- **(B) use of wet collectors (scrubbers):** Wet collectors use a liquid (usually water) to capture particulate matter. They can be efficient for fine particles. However, they collect the particulate matter as a wet slurry or sludge. This makes the material difficult to handle and reuse directly, as it would require dewatering and drying, which is an energy-intensive process. Therefore, it is not ideal if the primary goal is reuse of the collected dry powder.
- **(C) use of flue gas desulfurization (FGD):** FGD is a technology specifically designed to remove gaseous sulfur dioxide (SO_2), not particulate matter. While some particulate removal might occur incidentally in a wet FGD system, it is not its primary function or an efficient method for particle control. This option is incorrect.
- **(D) use of electrostatic precipitator (ESP):** An ESP works by charging the particulate matter in the flue gas and then collecting the charged particles on oppositely charged plates. ESPs are known for their very high collection efficiency, especially for **fine particles**. Crucially, the collected material is a **dry powder** (often called fly ash) that is removed from the collection plates by periodic rapping or vibration. This dry powder can be easily collected in hoppers and is readily available for **reuse** (e.g., in cement and concrete production).

Step 3: Final Answer:
Considering both the high efficiency for fine particles and the requirement for easy reuse of the collected material in a dry state, the electrostatic precipitator (ESP) is the most suitable and efficient option.

Quick Tip

When choosing an air pollution control device for particulates, consider the particle size and the desired state of the collected material:

- **Cyclones:** Good for coarse particles (¿10 μm), collect dry.
- **ESP:** Excellent for fine particles (¡1 μm), high efficiency, collect dry.
- **Fabric Filters (Baghouses):** Excellent for fine particles, very high efficiency, collect dry.
- **Wet Scrubbers:** Good for fine particles (especially sticky/flammable ones), collect wet as a slurry.

For dry reuse, ESPs and fabric filters are the best choices.

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41. Match the following

J) Dalton's law	**i) Diffusion**
K) Fick's law	**ii) Pressure exerted by a mixture of gases**
L) Henry's law	**iii) Gravitational settling**
M) Stoke's law	**iv) Gas-liquid phase transfer**

(A) J – ii; K – i; L – iv; M – iii
(B) J - iii; K – ii; L – i; M – iv
(C) J – ii; K – iii; L – iv; M – i
(D) J-i; K – iv; L – ii; M – iii

Correct Answer: (A) J – ii; K – i; L – iv; M – iii

Solution:

Step 1: Understanding the Question:
We need to match four scientific laws with the physical phenomena they describe.

Step 2: Matching Each Law with its Description:

- **J) Dalton's law:** Dalton's law of partial pressures states that in a mixture of non-reacting gases, the total pressure exerted is equal to the sum of the partial pressures of the individual gases. This directly matches description **(ii) Pressure exerted by a mixture of gases**. So, **J $\rightarrow$ ii**.

- **K) Fick's law:** Fick's laws of diffusion describe diffusion and are used to model the transport of mass or heat. Fick's first law relates the diffusive flux to the concentration gradient. This matches description **(i) Diffusion**. So, **K → i**.
- **L) Henry's law:** Henry's law is a gas law that states that the amount of dissolved gas in a liquid is directly proportional to the partial pressure of that gas above the liquid. This law governs the equilibrium between a gas and a liquid, which is the basis of **(iv) Gas-liquid phase transfer**. So, **L → iv**.
- **M) Stoke's law:** Stokes' Law describes the drag force on a spherical object moving through a viscous fluid. It is used to calculate the terminal velocity of small particles settling in a fluid under the influence of gravity. This matches description **(iii) Gravitational settling**. So, **M → iii**.

Step 3: Final Answer:
Combining the matches:

- J → ii
- K → i
- L → iv
- M → iii

This combination corresponds to option (A).

Quick Tip

Associate each law with a core concept:

- **Dalton's:** Gas Mixtures, Partial Pressures.
- **Fick's:** Diffusion, Concentration Gradients.
- **Henry's:** Gas Solubility, Gas-Liquid Transfer.
- **Stokes':** Settling Particles, Viscous Drag.

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42. Read the following statements

I. According to the Liebig's law of minimum, the growth is regulated by the limited factors i.e., resources in scarcity and not by the resources in abundance.
II. Shelford's law of tolerance states that, only the factors present in excess/abundance can affect the growth, development of an organism or rate of biological process.
III. Shelford's law of tolerance states that, an organism's success is based on a complex set of conditions and that each organism has a certain minimum, maximum, and optimum levels of environmental factor or combination of factors that determine success.

The correct choice is

(A) I and II are correct; III is incorrect
(B) I and III are correct; II is incorrect
(C) II is correct; I and III are incorrect
(D) III is correct; I and II are incorrect

Correct Answer: (B) I and III are correct; II is incorrect

Solution:

Step 1: Understanding the Question:
We need to evaluate the correctness of three statements describing two fundamental ecological laws: Liebig's law of the minimum and Shelford's law of tolerance.

Step 2: Evaluating Each Statement:

- **Statement I: According to the Liebig's law of minimum, the growth is regulated by the limited factors i.e., resources in scarcity and not by the resources in abundance.** This is **correct**. This is the essence of Liebig's Law, often visualized with "Liebig's barrel," where the water level (representing growth) is limited by the shortest stave (the scarcest resource), regardless of how long the other staves are.
- **Statement II: Shelford's law of tolerance states that, only the factors present in excess/abundance can affect the growth, development of an organism or rate of biological process.** This is **incorrect**. Shelford's law states that organisms can be limited by factors that are either too scarce (minimum) or too abundant (maximum). For example, too little water can limit plant growth, but too much water (flooding) can also be harmful. This statement only considers one side of the tolerance range.
- **Statement III: Shelford's law of tolerance states that, an organism's success is based on a complex set of conditions and that each organism has a certain minimum, maximum, and optimum levels of environmental factor or combination of factors that determine success.** This is **correct**. This is a complete and accurate description of Shelford's Law. It expands upon Liebig's law by recognizing that for any given environmental factor (like temperature, pH, salinity), there is an optimal level, as well as minimum and maximum levels beyond which the organism cannot survive. The range between the minimum and maximum is the "range of tolerance."

Step 3: Final Answer:
Statements I and III are correct, while statement II is incorrect. This corresponds to option (B).

Quick Tip

To remember the difference between these two ecological laws:

- **Liebig's Law of the Minimum:** Focuses only on the *scarcest* resource being the limiting factor. Think of a chain being only as strong as its weakest link.
- **Shelford's Law of Tolerance:** A more complete picture. It states that organisms are limited by "too little" *and* "too much" of any factor. Think of a "Goldilocks" principle – conditions must be "just right" (within a tolerance range) for success.

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43. Read the following statements

I. Trivalent chromium has relatively low aqueous solubility, and low mobility in the soil environment. By contrast, hexavalent chromium has a higher aqueous solubility and greater mobility in the soil environment.
II. The chemical reaction between trivalent chromium and zero-valent iron will result in transformed version called hexavalent chromium.
III. Hexavalent chromium is a known carcinogen.
IV. Trivalent chromium has relatively higher human toxicity as compared to hexavalent chromium.

The correct choice is

(A) IV is correct; I and III are incorrect
(B) II is correct; I and IV are incorrect
(C) I and III are correct; II and IV are incorrect
(D) I, II and IV are correct; III is incorrect

Correct Answer: (C) I and III are correct; II and IV are incorrect

Solution:

Step 1: Understanding the Question:
We need to evaluate four statements about the environmental chemistry and toxicology of trivalent (Cr(III)) and hexavalent (Cr(VI)) chromium.

Step 2: Evaluating Each Statement:

- **I. Trivalent chromium has relatively low aqueous solubility... hexavalent chromium has a higher aqueous solubility and greater mobility...** This is **correct**. Cr(III) tends to precipitate as hydroxides (like $Cr(OH)_3$) under typical environmental pH conditions, making it relatively insoluble and immobile in soil. Cr(VI), often found as the chromate (CrO_4^{2-}) or dichromate ($Cr_2O_7^{2-}$) anion, is highly soluble in water and does not adsorb strongly to soils, making it much more mobile.

- **II. The chemical reaction between trivalent chromium and zero-valent iron will result in transformed version called hexavalent chromium.** This is **incorrect**. Zero-valent iron (Fe^0) is a strong reducing agent. It is commonly used in environmental remediation to *reduce* the toxic and mobile hexavalent chromium (Cr(VI)) to the less toxic and less mobile trivalent chromium (Cr(III)). The reaction is: $2CrO_4^{2-} + 3Fe^0 + 16H^+ \rightarrow 2Cr^{3+} + 3Fe^{2+} + 8H_2O$ (simplified). The statement claims the opposite transformation (oxidation of Cr(III) to Cr(VI)), which is incorrect.
- **III. Hexavalent chromium is a known carcinogen.** This is **correct**. Hexavalent chromium, particularly when inhaled, is a well-documented human carcinogen, strongly linked to lung cancer.
- **IV. Trivalent chromium has relatively higher human toxicity as compared to hexavalent chromium.** This is **incorrect**. Hexavalent chromium (Cr(VI)) is significantly more toxic than trivalent chromium (Cr(III)). In fact, Cr(III) is an essential trace nutrient for humans (involved in glucose metabolism), while Cr(VI) is a toxic carcinogen.

Step 3: Final Answer:
Statements I and III are correct. Statements II and IV are incorrect. This corresponds to option (C).

Quick Tip

For chromium contamination, remember this simple rule:

- **Cr(VI) (Hexavalent):** Very Bad. Highly toxic, carcinogenic, soluble, and mobile.
- **Cr(III) (Trivalent):** Not so Bad. Essential nutrient at low levels, much less toxic, insoluble, and immobile.

Environmental remediation strategies often focus on reducing Cr(VI) to Cr(III) to immobilize it and reduce its toxicity.

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44. Which of the following statements is/are NOT true?

(A) Urban heat island effect in a city can be reduced by increasing trees and vegetation cover in the city.
(B) Urban heat island intensity is affected by $PM_{2.5}$ concentrations in a city.
(C) Urban heat island intensity increases due to installation of reflective roofs in a city.
(D) In comparison with the non-urban areas, urban heat island effect raises night-time temperatures more than daytime temperatures in cities.

Correct Answer: (C) Urban heat island intensity increases due to installation of reflective roofs in a city.

Solution:

Step 1: Understanding the Urban Heat Island (UHI) Effect:
The UHI effect is the phenomenon where urban areas experience higher temperatures than surrounding rural areas. This is caused by factors such as the absorption of solar radiation by dark surfaces (low albedo), reduced vegetation cover (and thus less cooling from evapotranspiration), and waste heat generated by human activities. The question asks us to identify the statement that is NOT true about UHI.

Step 2: Evaluating Each Statement:

- **(A) Urban heat island effect in a city can be reduced by increasing trees and vegetation cover in the city.** This statement is **TRUE**. Vegetation provides shade, which prevents solar radiation from heating surfaces. It also cools the air through evapotranspiration. Increasing urban green spaces is a key strategy to mitigate the UHI effect.
- **(B) Urban heat island intensity is affected by $PM_{2.5}$ concentrations in a city.** This statement is **TRUE**. Airborne particles (aerosols) like $PM_{2.5}$ interact with radiation. During the day, they can scatter sunlight, creating a slight cooling effect. However, they can also absorb and re-radiate outgoing thermal radiation, contributing to warming, particularly at night. Thus, their presence affects the energy balance and the UHI intensity.
- **(C) Urban heat island intensity increases due to installation of reflective roofs in a city.** This statement is **FALSE**. Reflective roofs, also called "cool roofs," have a high albedo, meaning they reflect a large portion of incoming solar radiation. By reflecting sunlight instead of absorbing it, these roofs stay cooler and reduce the amount of heat transferred to the atmosphere. This is a well-known strategy to *reduce* or mitigate the UHI effect. The statement claims the opposite.
- **(D) In comparison with the non-urban areas, urban heat island effect raises night-time temperatures more than daytime temperatures in cities.** This statement is **TRUE**. This is a defining characteristic of the UHI effect. Materials common in cities, like concrete and asphalt, have high thermal inertia. They absorb large amounts of heat during the day and release it slowly throughout the night. Rural areas with more vegetation cool down much faster after sunset. This results in the largest temperature difference between urban and rural areas typically occurring a few hours after sunset.

Step 3: Final Answer:
The statement that is not true is (C).

Quick Tip

To analyze Urban Heat Island (UHI) concepts, think about the energy balance of a surface.

- **Causes of UHI (Warming):** Low albedo (dark surfaces absorb heat), high thermal mass (materials store heat), less vegetation (less evaporative cooling), waste heat.
- **Mitigation of UHI (Cooling):** High albedo (reflective surfaces like cool roofs), more vegetation (parks, green roofs).

---

45. Read the following statements about aerobic composting of organic fraction of municipal solid waste

I. The majority of the odour problem in an aerobic composting process is due to the development of anaerobic conditions within the compost pile.
II. All organic carbon present in the waste will completely biodegrade in 14 days.
III. At high C/N ratio, ammonia would be released and biological activity may also be impeded.
IV. Optimum moisture content for aerobic composting process would be 50-60%. Lower moisture would slow down the biological process. Excessive moisture will make it difficult to maintain aerobic conditions.

The correct choice(s) is/are

(A) I and IV are correct
(B) II and III are incorrect
(C) I is correct; IV is incorrect
(D) II is correct; IV is incorrect

Correct Answer: (A), (B)

Solution:
This is a Multiple Select Question (MSQ), so we must evaluate the correctness of each statement and then check which options accurately reflect this evaluation.
Step 1: Evaluating the Statements:

- **Statement I:** This is **TRUE**. A properly managed aerobic compost pile has an earthy smell. Foul odors, such as the smell of ammonia or hydrogen sulfide (rotten eggs), are classic signs that parts of the pile have become anaerobic (lacking oxygen), which leads to different microbial processes like putrefaction.
- **Statement II:** This is **FALSE**. The composting process takes weeks to months. While the most active phase may occur in the first few weeks, complete stabilization and maturation

is a much longer process. Moreover, not all organic carbon is biodegradable; recalcitrant compounds like lignin form stable humus.

- **Statement III:** This is **FALSE**. Ammonia (NH_3) release is a problem associated with a *low* C/N ratio (excess nitrogen). A high C/N ratio means there is a deficiency of nitrogen relative to carbon, which slows down or impedes microbial activity because microbes need nitrogen to build proteins.
- **Statement IV:** This is **TRUE**. Moisture content is a critical parameter. Microbes need water to function, so low moisture (<40%) slows the process. However, excessive moisture (>60-65%) fills the air pores, preventing oxygen diffusion and leading to anaerobic conditions. The 50-60% range is indeed considered optimal.

Step 2: Evaluating the Options:
We have determined: I is True, II is False, III is False, IV is True. Let's check the options:

- **(A) I and IV are correct:** This is consistent with our findings. (Statement I is TRUE and Statement IV is TRUE).
- **(B) II and III are incorrect:** This is also consistent with our findings. (Statement II is FALSE and Statement III is FALSE).
- **(C) I is correct; IV is incorrect:** This is inconsistent because IV is correct.
- **(D) II is correct; IV is incorrect:** This is inconsistent because II is incorrect and IV is correct.

Step 3: Final Answer:
Both options (A) and (B) are factually correct statements based on the analysis of the individual propositions. In an MSQ, both should be selected.

Quick Tip

Remember the key parameters for successful aerobic composting:

- **C/N Ratio:** Ideal ≈ 25-30. Too high → slow process. Too low → ammonia odor, nitrogen loss.
- **Moisture:** Ideal ≈ 50-60%. Too low → slow process. Too high → anaerobic conditions, foul odors.
- **Aeration:** Oxygen is essential to prevent anaerobic conditions and odors.
- **Temperature:** The process generates heat, which is vital for killing pathogens.

---

46. Products P and Q have life cycle phases of material extraction, production, use, and end of life disposal. CH_4, CO_2 emissions and mass used per functional unit (f.u.) from the different phases of the products are given in the following tables. [Tables for Product P and Product Q emissions] Based upon the information given in the tables and using global warming potential of CH_4 equal to 23 kg of CO_2 per

kg of CH_4, which of the following statement(s) is/are true?

(A) Greenhouse gas emissions (kg CO_2 equivalent/f.u.) from the 'Material extraction' phase of product P is higher than that of product Q.
(B) Greenhouse gas emissions (kg CO_2 equivalent/f.u.) from the 'Production phase' of product Q is higher than that of product P.
(C) Greenhouse gas emissions (kg CO_2 equivalent/f.u.) from the 'End of life disposal' is higher for product Q than that of product P.
(D) Greenhouse gas emissions (kg CO_2 equivalent/f.u.) from the 'complete life cycle' of the product P is higher than that of product Q.

Correct Answer: (A), (B), (D)

Solution:
This is a Multiple Select Question (MSQ). We must calculate and compare the greenhouse gas (GHG) emissions for both products at various life cycle stages.
Step 1: Define the Calculation for GHG Emissions:
The total GHG emission for a given phase is calculated in kg of CO_2 equivalent per functional unit (kg CO_2e/f.u.). The formula for each phase is:

$$\text{GHG (kg } CO_2\text{e/f.u.)} = \text{Mass (tonne/f.u.)} \times [CO_2 \text{ (kg/tonne)} + CH_4 \text{ (kg/tonne)} \times GWP_{CH_4}]$$

We are given $GWP_{CH_4} = 23$.
Step 2: Calculate GHG Emissions for Each Product and Phase:

- **Product P:**
  - **Extraction:** 4.0 tonne/f.u. $\times$ [1.0 + (0.75 $\times$ 23)] = 4.0 $\times$ [18.25] = **73.0** kg CO_2e/f.u.
  - **Production:** 2.0 tonne/f.u. $\times$ [1.5 + (1.0 $\times$ 23)] = 2.0 $\times$ [24.5] = **49.0** kg CO_2e/f.u.
  - **Use:** 1.0 tonne/f.u. $\times$ [0.5 + (0.0 $\times$ 23)] = 1.0 $\times$ [0.5] = **0.5** kg CO_2e/f.u.
  - **Disposal:** 1.0 tonne/f.u. $\times$ [1.0 + (0.25 $\times$ 23)] = 1.0 $\times$ [6.75] = **6.75** kg CO_2e/f.u.
  - **Total Life Cycle for P:** 73.0 + 49.0 + 0.5 + 6.75 = **129.25** kg CO_2e/f.u.
- **Product Q:**
  - **Extraction:** 3.0 tonne/f.u. $\times$ [0.75 + (0.75 $\times$ 23)] = 3.0 $\times$ [18.0] = **54.0** kg CO_2e/f.u.
  - **Production:** 2.5 tonne/f.u. $\times$ [0.25 + (1.0 $\times$ 23)] = 2.5 $\times$ [23.25] = **58.125** kg CO_2e/f.u.
  - **Use:** 0.75 tonne/f.u. $\times$ [0.0 + (0.5 $\times$ 23)] = 0.75 $\times$ [11.5] = **8.625** kg CO_2e/f.u.
  - **Disposal:** 0.75 tonne/f.u. $\times$ [2.0 + (0.0 $\times$ 23)] = 0.75 $\times$ [2.0] = **1.5** kg CO_2e/f.u.
  - **Total Life Cycle for Q:** 54.0 + 58.125 + 8.625 + 1.5 = **122.25** kg CO_2e/f.u.

Step 3: Evaluate Each Statement based on the Calculations:

- **(A) Emissions from 'Material extraction' of P is higher than that of Q.** $GHG_{P,ext}$ = 73.0. $GHG_{Q,ext}$ = 54.0. Since 73.0 ¿ 54.0, statement (A) is **TRUE**.

- **(B) Emissions from 'Production phase' of Q is higher than that of P.** $\text{GHG}_{Q,prod}$ = 58.125. $\text{GHG}_{P,prod}$ = 49.0. Since 58.125 ¿ 49.0, statement (B) is **TRUE**.
- **(C) Emissions from 'End of life disposal' is higher for Q than that of P.** $\text{GHG}_{Q,disp}$ = 1.5. $\text{GHG}_{P,disp}$ = 6.75. Since 1.5 ¡ 6.75, statement (C) is **FALSE**.
- **(D) Emissions from 'complete life cycle' of P is higher than that of Q.** $\text{GHG}_{P,total}$ = 129.25. $\text{GHG}_{Q,total}$ = 122.25. Since 129.25 ¿ 122.25, statement (D) is **TRUE**.

Final Answer:
The true statements are (A), (B), and (D).

Quick Tip

Life Cycle Analysis (LCA) calculations require careful and systematic accounting. Create a clear table to calculate the total impact for each product, broken down by life cycle stage. The total impact per functional unit is the sum of impacts from all stages. Remember to multiply the per-tonne emission factor by the mass of material used in that specific stage.

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47. Second order ordinary differential equation $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$ has values $y = 2$ and $\frac{dy}{dx} = 1$ at $x = 0$. The value of y at x=1 is _____ (rounded off to three decimal places).

Correct Answer: 7.757

Solution:

Step 1: Find the General Solution of the ODE:
The given equation is a second-order linear homogeneous ordinary differential equation with constant coefficients. We solve it by finding the roots of its characteristic (auxiliary) equation:

$$m^2 - m - 2 = 0$$

Factoring the quadratic equation:

$$(m - 2)(m + 1) = 0$$

The roots are real and distinct: $m_1 = 2$ and $m_2 = -1$. The general solution is therefore of the form:

$$y(x) = C_1 e^{m_1 x} + C_2 e^{m_2 x} = C_1 e^{2x} + C_2 e^{-x}$$

Step 2: Use the Initial Conditions to Find the Constants C_1 and C_2:
We are given initial conditions at $x = 0$: $y(0) = 2$ and $y'(0) = 1$. First, we find the derivative of the general solution:

$$y'(x) = 2C_1 e^{2x} - C_2 e^{-x}$$

Now, we apply the initial conditions to create a system of two linear equations:

1. From $y(0) = 2$:
$$C_1 e^{2(0)} + C_2 e^{-0} = 2 \implies C_1 + C_2 = 2$$
2. From $y'(0) = 1$:
$$2C_1 e^{2(0)} - C_2 e^{-0} = 1 \implies 2C_1 - C_2 = 1$$

We solve this system. Adding the two equations together eliminates C_2:

$$(C_1 + 2C_1) + (C_2 - C_2) = 2 + 1$$

$$3C_1 = 3 \implies C_1 = 1$$

Substituting $C_1 = 1$ into the first equation ($C_1 + C_2 = 2$):

$$1 + C_2 = 2 \implies C_2 = 1$$

Step 3: Write the Particular Solution and Evaluate at x=1:
With the constants $C_1 = 1$ and $C_2 = 1$, the particular solution for the given initial conditions is:

$$y(x) = e^{2x} + e^{-x}$$

We need to find the value of this function at $x = 1$:

$$y(1) = e^{2(1)} + e^{-1} = e^2 + \frac{1}{e}$$

Using the value of $e \approx 2.71828$:

$$y(1) \approx (2.71828)^2 + \frac{1}{2.71828} \approx 7.38906 + 0.36788$$

$$y(1) \approx 7.75694$$

Step 4: Final Answer:
Rounding the result to three decimal places, we get 7.757.

Quick Tip

To solve a second-order linear homogeneous ODE with initial conditions: 1. Find the roots (m_1, m_2) of the characteristic equation. 2. Write the general solution based on the nature of the roots (real/distinct, real/repeated, or complex). 3. Differentiate the general solution to get y'. 4. Use the two initial conditions ($y(x_0)$ and $y'(x_0)$) to create a system of two linear equations for the unknown constants (C_1, C_2). 5. Solve for the constants to get the particular solution.

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48. Consider two matrices $P = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$ and $Q = \begin{bmatrix} 5 & 4 \\ 0 & 2 \end{bmatrix}$. If $R = (PQ)^T$ then det R is _____ (in integer).

Correct Answer: 50

Solution:

Step 1: Understand the Goal and Relevant Matrix Properties:
We need to find the determinant of matrix R, where R is the transpose of a product of two matrices. Instead of performing the full matrix multiplication and transposition, we can use the properties of determinants. The two key properties are:

1. The determinant of a transpose of a matrix is equal to the determinant of the original matrix: $\det(A^T) = \det(A)$.
2. The determinant of a product of matrices is the product of their individual determinants: $\det(AB) = \det(A)\det(B)$.

Step 2: Apply the Properties to Find det(R):
We are given $R = (PQ)^T$. We want to find $\det(R)$.

$$\det(R) = \det((PQ)^T)$$

Using property (1):

$$\det(R) = \det(PQ)$$

Using property (2):

$$\det(R) = \det(P) \times \det(Q)$$

This simplifies the problem to finding the determinants of P and Q individually.
Step 3: Calculate the Determinants of P and Q:
For a 2x2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, the determinant is $ad - bc$.

- $\det(P) = \det\begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix} = (2)(4) - (3)(1) = 8 - 3 = 5$
- $\det(Q) = \det\begin{pmatrix} 5 & 4 \\ 0 & 2 \end{pmatrix} = (5)(2) - (4)(0) = 10 - 0 = 10$

(Note: For a triangular matrix like Q, the determinant is simply the product of the diagonal elements.)
Step 4: Calculate the Final Determinant:

$$\det(R) = \det(P) \times \det(Q) = 5 \times 10 = 50$$

Step 5: Final Answer:
The determinant of R is 50.

Quick Tip

Always leverage the properties of determinants before performing lengthy matrix multiplications. For any square matrices A and B:

- $\det(AB) = \det(A)\det(B)$
- $\det(A^T) = \det(A)$
- $\det(A^{-1}) = 1/\det(A)$

Using these can dramatically simplify problems.

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49. For the function $f(x) = x\sqrt{4-x^2}$, the maximum value in the range $-2 \le x \le 2$ is _____ (rounded off to two decimal places).

Correct Answer: 2.00

Solution:

Step 1: Define the Goal and Method:
We need to find the absolute maximum value of the function $f(x) = x\sqrt{4-x^2}$ on the closed interval $[-2, 2]$. The standard method for this is the Closed Interval Method (or Extreme Value Theorem). This involves finding the function's value at all critical points within the interval and at the endpoints of the interval.
Step 2: Find the Derivative of the Function:
We use the product rule, $(uv)' = u'v + uv'$, to differentiate $f(x)$. Let $u = x$ and $v = (4-x^2)^{1/2}$. Then $u' = 1$ and $v' = \frac{1}{2}(4-x^2)^{-1/2}(-2x) = \frac{-x}{\sqrt{4-x^2}}$.

$$f'(x) = (1) \cdot \sqrt{4-x^2} + x \cdot \left(\frac{-x}{\sqrt{4-x^2}}\right)$$

$$f'(x) = \sqrt{4-x^2} - \frac{x^2}{\sqrt{4-x^2}}$$

Combine the terms by finding a common denominator:

$$f'(x) = \frac{(4-x^2) - x^2}{\sqrt{4-x^2}} = \frac{4-2x^2}{\sqrt{4-x^2}}$$

Step 3: Identify Critical Points:
Critical points occur where the derivative is zero or undefined.

- $f'(x) = 0$**:** This happens when the numerator is zero.

$$4 - 2x^2 = 0 \implies 2x^2 = 4 \implies x^2 = 2 \implies x = \pm\sqrt{2}$$

Both $x = \sqrt{2}$ and $x = -\sqrt{2}$ are within the interval $[-2, 2]$, so they are critical points we must check.

- $f'(x)$ **is undefined:** This happens when the denominator is zero.

$$\sqrt{4-x^2}=0 \implies x^2=4 \implies x=\pm 2$$

These are the endpoints of the interval.

Step 4: Evaluate the Function at Critical Points and Endpoints:
We create a list of candidates for the maximum value by evaluating $f(x)$ at the endpoints $\{-2, 2\}$ and the critical points $\{-\sqrt{2}, \sqrt{2}\}$.

- $f(-2) = (-2)\sqrt{4-(-2)^2} = (-2)\sqrt{0} = 0$
- $f(-\sqrt{2}) = (-\sqrt{2})\sqrt{4-(-\sqrt{2})^2} = -\sqrt{2}\sqrt{4-2} = -\sqrt{2}\sqrt{2} = -2$
- $f(\sqrt{2}) = (\sqrt{2})\sqrt{4-(\sqrt{2})^2} = \sqrt{2}\sqrt{4-2} = \sqrt{2}\sqrt{2} = 2$
- $f(2) = (2)\sqrt{4-(2)^2} = (2)\sqrt{0} = 0$

Step 5: Determine the Maximum Value:
The set of values is $\{0, -2, 2\}$. The largest value in this set is 2. The maximum value of the function on the interval is exactly 2. Rounding to two decimal places gives 2.00.

Quick Tip

To find the absolute maximum/minimum of a continuous function on a closed interval [a, b]: 1. Find all critical points 'c' in the open interval (a, b) by finding where $f'(c) = 0$ or is undefined. 2. Evaluate the function at these critical points, $f(c)$. 3. Evaluate the function at the endpoints, $f(a)$ and $f(b)$. 4. The absolute maximum is the largest value from steps 2 and 3. The absolute minimum is the smallest.

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50. The solubility of gas A is 16 mg/L in water and its vapor pressure is 0.042 atm. at 25 °C. In a closed system, the gas phase concentration of A is 10^{-3} mol/L. Assuming Ideal gas constant (R) value as 0.0821 L-atm/mol-K, the concentration of gas A in water at 25 °C is _____ (in mg/L, rounded off to two decimal places).

Correct Answer: 9.32

Solution:

Step 1: Understand the Governing Principle - Henry's Law:
Henry's Law describes the equilibrium between a gas in the gas phase and the same gas dissolved in a liquid. It states that the concentration of the dissolved gas (C_{aq}) is directly proportional to the partial pressure of the gas above the liquid (P_g).

$$C_{aq} = k_H P_g$$

where k_H is the Henry's Law constant.

Step 2: Calculate the Henry's Law Constant (k_H) from Solubility Data:
The problem provides data at saturation: the solubility (which is the saturation concentration in water) is 16 mg/L when the pressure of the gas is its vapor pressure, 0.042 atm. We can use this to find k_H.

$$k_H = \frac{C_{aq,sat}}{P_{g,sat}} = \frac{16 \text{ mg/L}}{0.042 \text{ atm}}$$

Step 3: Calculate the Partial Pressure (P_g) in the System:
In the system of interest, the gas phase concentration is given as $C_g = 10^{-3}$ mol/L. We must convert this concentration to a partial pressure using the Ideal Gas Law, $P = (n/V)RT$, or simply $P = C_g RT$.

- Gas concentration, $C_g = 10^{-3}$ mol/L
- Gas constant, $R = 0.0821$ L·atm/(mol·K)
- Temperature, T = 25 °C = 273.15 + 25 = 298.15 K

$$P_g = (10^{-3} \text{ mol/L}) \times (0.0821 \frac{\text{L} \cdot \text{atm}}{\text{mol} \cdot \text{K}}) \times (298.15 \text{ K})$$
$$P_g \approx 0.024478 \text{ atm}$$

Step 4: Calculate the Equilibrium Aqueous Concentration (C_{aq}):
Now we can apply Henry's Law using the constant k_H we found and the partial pressure P_g we just calculated.

$$C_{aq} = k_H \times P_g = \left(\frac{16 \text{ mg/L}}{0.042 \text{ atm}} \right) \times (0.024478 \text{ atm})$$

The "atm" units cancel out, leaving mg/L.

$$C_{aq} = \frac{16 \times 0.024478}{0.042} \approx \frac{0.39165}{0.042} \approx 9.3249 \text{ mg/L}$$

Step 5: Final Answer:
The concentration of gas A in water is approximately 9.3249 mg/L. Rounding to two decimal places gives 9.32.

Quick Tip

Henry's Law problems are a common application of the Ideal Gas Law. The typical workflow is: 1. Use solubility data to find the Henry's Law constant, $k_H = C_{aq}/P_g$. Keep the units (e.g., (mg/L)/atm). 2. Use the Ideal Gas Law ($P = C_{gas}RT$) to find the partial pressure of the gas in the specific system. 3. Use the calculated k_H and P_g to find the final aqueous concentration. Be very careful with unit consistency, especially for temperature (must be in Kelvin).

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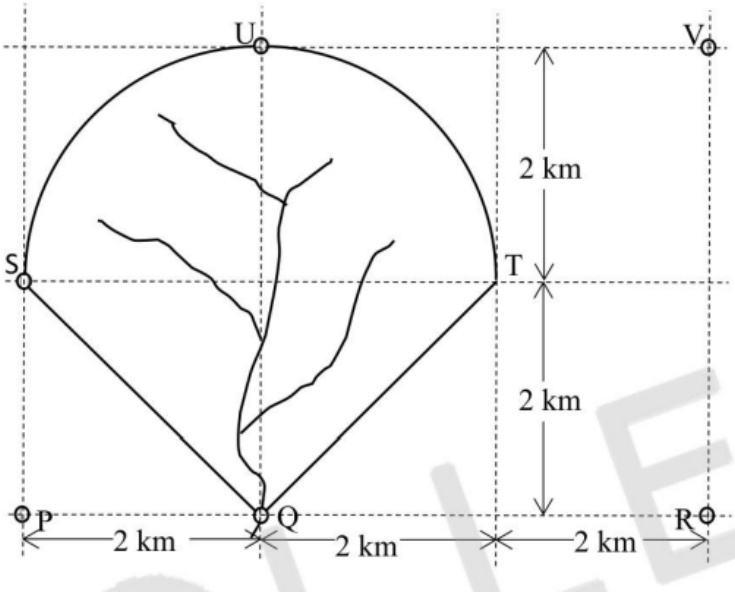
51. The following figure (not to the scale) shows a catchment (Q, S, U, T, Q) and adjoining raingauge stations P, Q, R, S, U and V. Due to a storm, 20 mm, 25 mm, 30 mm, 15 mm, 22 mm and 18 mm rainfall depths were recorded by raingauges

at P, Q, R, S, U and V, respectively. The corresponding mean rainfall over the catchment using Thiessen polygon method is _____ (in mm, rounded off to two decimal places).



Correct Answer: 21.25

Solution:

Step 1: Understand the Thiessen Polygon Method:
The Thiessen polygon method calculates the areal average rainfall by assigning a weight to each rain gauge based on the area that is closer to it than to any other gauge. The average rainfall ($\bar{P}$) is the weighted average:

$$\bar{P} = \frac{\sum_{i=1}^{n} P_i A_i}{\sum_{i=1}^{n} A_i} = \frac{\sum P_i A_i}{A_{total}}$$

where P_i is the rainfall at gauge i and A_i is the area of influence of gauge i within the catchment.
Step 2: Determine the Areas of Influence within the Catchment:
We need to construct the polygons by drawing perpendicular bisectors between adjacent gauges. Given the grid-like layout, we can deduce these areas geometrically.

- Let's set up a coordinate system with Q at the origin (0,0). Then S=(-2,2), U=(0,4), T=(2,2). The other stations are P=(-2,0), R=(2,0), and V=(2,4). The point at (0,2) is the geometric center of the square S-U-V-T'. Let's call it O.
- The catchment is the quadrilateral Q-S-U-T. Its total area can be calculated as the sum of the areas of $\triangle$QSU and $\triangle$QTU. Base QU is on the y-axis, length 4. The total width is 4. The shape is a kite. Area $= \frac{1}{2}d_1 d_2 = \frac{1}{2}(4)(4) = 8$ km^2.
- The perpendicular bisector of the line segment SU is the vertical line $x = -1$. The bisector of UT is the horizontal line $y = 3$. The bisector of QT is the line $y = x + 2$. The bisector of QS is $y = -x$.

- A simpler interpretation based on the grid is to see how the catchment area is divided by the grid bisectors. The catchment is composed of four triangles, $\triangle$QOS, $\triangle$SOU, $\triangle$UOT, and $\triangle$TOQ, where O is the center point (0,2). Each of these triangles has a base of 2km (e.g., QO) and a height of 2km (e.g., horizontal distance from S to y-axis). So each triangle has an area of $\frac{1}{2} \times 2 \times 2 = 2$ km^2.
- Now we identify which gauge's polygon contains each triangle:
  - $\triangle$QOS is closest to station S. So, $A_S = 2$ km^2.
  - $\triangle$SOU is closest to station U. So, $A_U = 2$ km^2.
  - $\triangle$UOT is closest to station V. So, $A_V = 2$ km^2.
  - $\triangle$TOQ is closest to station R. So, $A_R = 2$ km^2.
- Gauges P and Q are on the boundary or outside the primary area of influence, so their direct contribution within the catchment is zero under this logical division.

Step 3: Calculate the Mean Rainfall:
The gauges influencing the catchment are S, U, V, and R, each with an area of 2 km^2. The rainfall values are: $P_S = 15$ mm, $P_U = 22$ mm, $P_V = 18$ mm, $P_R = 30$ mm.

$$\bar{P} = \frac{P_S A_S + P_U A_U + P_V A_V + P_R A_R}{A_{total}}$$

$$\bar{P} = \frac{(15 \times 2) + (22 \times 2) + (18 \times 2) + (30 \times 2)}{8}$$

$$\bar{P} = \frac{30 + 44 + 36 + 60}{8} = \frac{170}{8} = 21.25 \text{ mm}$$

Step 4: Final Answer:
The corresponding mean rainfall over the catchment is 21.25 mm.

Quick Tip

The Thiessen Polygon method provides a more accurate spatial weighting of rainfall than a simple arithmetic average. The key is to define the area of influence for each gauge by constructing perpendicular bisectors between adjacent gauges. For gridded problems in an exam, look for simple geometric divisions (squares, triangles) that logically represent the "closest area" to each gauge.

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52. A trapezoidal canal lined with cement concrete (n = 0.01) is designed to carry a discharge of 20 m^3/s at a bed slope 1 in 400. If the bed width is twice of the depth of flow and side slope of the canal section is 2 (1 vertical:2 horizontal) then the corresponding depth of flow will be _____ (in m, rounded off to two decimal places).

Correct Answer: 1.13

Solution:

Step 1: State Manning's Equation and Given Parameters:
The discharge Q in an open channel is given by Manning's formula:

$$Q = \frac{1}{n} A R_h^{2/3} S^{1/2}$$

We are given:

- $Q = 20\ \text{m}^3/\text{s}$
- $n = 0.01$
- $S = 1/400 = 0.0025$

Step 2: Express Geometric Properties in Terms of Depth y:
Let y be the depth of flow.

- **Bed width (b):** Given as twice the depth, so $b = 2y$.
- **Side slope (z):** Given as 2 horizontal to 1 vertical, so $z = 2$.
- **Area (A):** For a trapezoid, $A = (b + zy)y$.

$$A = (2y + 2y)y = 4y^2$$

- **Wetted Perimeter (P):** The length of the channel bed and sides in contact with water is $P = b + 2y\sqrt{1 + z^2}$.

$$P = 2y + 2y\sqrt{1 + 2^2} = 2y + 2y\sqrt{5} = 2y(1 + \sqrt{5})$$

- **Hydraulic Radius (R_h):** $R_h = A/P$.

$$R_h = \frac{4y^2}{2y(1 + \sqrt{5})} = \frac{2y}{1 + \sqrt{5}}$$

Step 3: Substitute into Manning's Equation:
Plug the expressions for A and R_h into Manning's equation:

$$20 = \frac{1}{0.01}(4y^2)\left(\frac{2y}{1 + \sqrt{5}}\right)^{2/3}(0.0025)^{1/2}$$

Simplify the constant terms:

$$20 = 100 \cdot (4y^2) \cdot \left(\frac{2}{1 + \sqrt{5}}\right)^{2/3} \cdot y^{2/3} \cdot (0.05)$$

Group the constants and the terms with y:

$$20 = (100 \times 4 \times 0.05) \cdot \left(\frac{2}{1 + \sqrt{5}}\right)^{2/3} \cdot y^{(2+2/3)}$$

$$20 = 20 \cdot \left(\frac{2}{1 + 2.23607}\right)^{2/3} \cdot y^{8/3}$$

Divide both sides by 20:

$$1 = \left(\frac{2}{3.23607}\right)^{2/3} \cdot y^{8/3}$$

$$1 \approx (0.61803)^{2/3} \cdot y^{8/3}$$

$$1 \approx 0.7236 \cdot y^{8/3}$$

Step 4: Solve for y:
Rearrange the equation to solve for y:

$$y^{8/3} = \frac{1}{0.7236} \approx 1.38198$$

$$y = (1.38198)^{3/8}$$

Using a calculator:

$$y \approx 1.130 \text{ m}$$

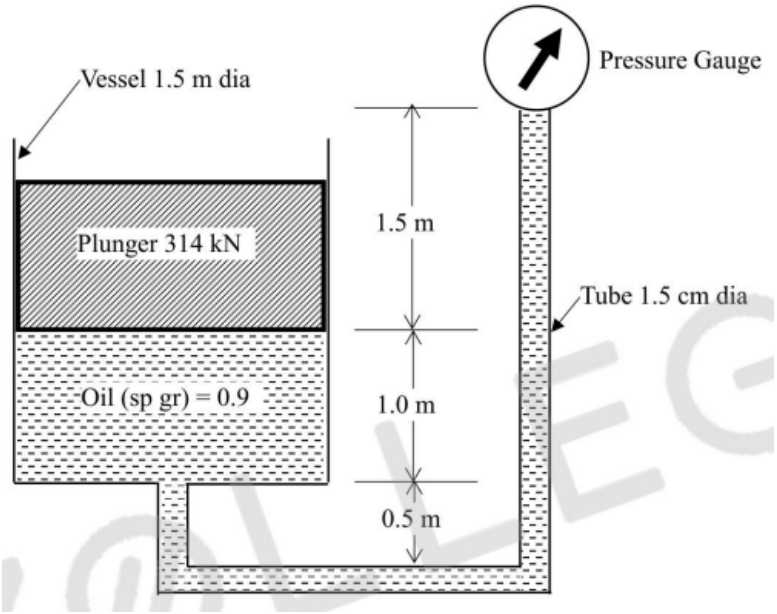
Step 5: Final Answer:
The corresponding depth of flow is approximately 1.130 m. Rounding to two decimal places gives 1.13.

Quick Tip

Solving Manning's equation for geometric properties like depth often requires an iterative approach or a numerical solver in complex cases. For simple relationships like this one, the equation can be rearranged to solve for y directly. The term $AR_h^{2/3}$, known as the section factor, can be expressed entirely in terms of y, leading to an equation of the form $Constant = C \cdot y^k$.

---

53. A plunger weighing 314 kN is balanced in a cylindrical vessel of diameter 1.5 m and filled with an oil (specific gravity 0.9) as shown in the following figure (not to the scale). If a pressure gauge is connected with the vessel using 1.5 cm diameter tube, the reading of the gauge will be _____ (in kPa, rounded off to two decimal places).



Correct Answer: 186.59

Solution:

Step 1: Understanding the Pressure Calculation:
The pressure measured by the gauge will be the sum of two components: 1. The pressure exerted by the weight of the plunger on the surface of the oil. 2. The hydrostatic pressure due to the column of oil between the oil surface and the point where the gauge is connected.
Step 2: Calculate the Pressure from the Plunger:
Pressure is defined as Force per unit Area ($P = F/A$).

- Force (Weight of the plunger): $F = 314$ kN $= 314,000$ N.
- Area of the plunger (same as the vessel's cross-section): $A = \pi r^2 = \pi(D/2)^2 = \pi(1.5/2)^2 = \pi(0.75)^2 \approx 1.76715$ m^2.

The pressure exerted by the plunger on the oil surface is:

$$P_{plunger} = \frac{314,000 \text{ N}}{1.76715 \text{ m}^2} \approx 177,690 \text{ Pa} = 177.69 \text{ kPa}$$

Step 3: Calculate the Hydrostatic Pressure:
The hydrostatic pressure is given by $P_{hydro} = \rho g h$. We must determine the correct vertical height h. The diagram shows several vertical dimensions. A careful reading suggests the pressure gauge is connected at the level corresponding to the bottom of the main vessel, which is 1.0 m below the oil surface. The other dimensions (0.5m, 1.5m for the tube) describe the layout of the connecting tube but the pressure at a given elevation in a continuous fluid is constant. The diameter of the connecting tube is irrelevant for this static pressure calculation.

- Vertical depth, $h = 1.0$ m.
- Density of oil, $\rho =$ specific gravity $\times \rho_{water} = 0.9 \times 1000$ kg/m^3 $= 900$ kg/m^3.
- Acceleration due to gravity, $g \approx 9.81$ m/s^2.

$$P_{hydro} = (900 \text{ kg/m}^3) \times (9.81 \text{ m/s}^2) \times (1.0 \text{ m}) = 8829 \text{ Pa} = 8.829 \text{ kPa}$$

Step 4: Calculate the Total Gauge Pressure:
The total pressure at the gauge's connection point is the sum of the surface pressure and the hydrostatic pressure.

$$P_{gauge} = P_{plunger} + P_{hydro} = 177.69 \text{ kPa} + 8.829 \text{ kPa} = 186.519 \text{ kPa}$$

Step 5: Final Answer:
Rounding the result to two decimal places, the gauge reading is 186.52 kPa. *(Note: Using a more precise value for F, e.g., $100,000\pi$ N, yields $P_{plunger} = 177,777$ Pa. Then $P_{gauge} = 177.78 + 8.83 = 186.61$ kPa. The slight variation is due to the choice of 314 kN being an approximation of 100π.)*

Quick Tip

In manometry and hydraulic systems, the pressure at a certain depth in a continuous fluid is the sum of the surface pressure and the hydrostatic pressure (ρgh). Always carefully determine the surface pressure and the correct vertical depth (h) from the surface to the point of measurement. Horizontal distances and tube diameters (unless used for dynamic effects, not present here) are usually irrelevant for static pressure calculations.

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54. A fully penetrating well is installed in a homogenous and isotropic confined aquifer. The aquifer has uniform thickness of 16 m and hydraulic conductivity of 25 m/d. Water is being pumped out from the well at a constant rate of 0.1 m^3/s till steady state condition is reached. If a drawdown of 3.5 m is observed at a distance of 75 m from the well then the drawdown at a distance of 150 m from the well will be _____ (in m, rounded off to two decimal places).

Correct Answer: 1.12

Solution:

Step 1: Identify the Governing Equation:
The problem describes steady-state pumping from a well in a confined aquifer. The relationship between drawdown and distance is given by the Thiem equation. For two observation wells at distances r_1 and r_2 from the pumping well, the difference in their drawdowns (s_1 and s_2) is:

$$s_1 - s_2 = \frac{Q}{2\pi T} \ln\left(\frac{r_2}{r_1}\right)$$

where Q is the pumping rate and T is the aquifer transmissivity.
Step 2: Calculate Transmissivity (T) and Ensure Consistent Units:
First, we must ensure all parameters are in a consistent unit system (e.g., meters and seconds).

- Pumping rate, $Q = 0.1$ m^3/s.

- Aquifer thickness, $b = 16$ m.
- Hydraulic conductivity, $K = 25$ m/d. We must convert this to m/s.

$$K = \frac{25 \text{ m}}{1 \text{ day}} \times \frac{1 \text{ day}}{24 \text{ hours}} \times \frac{1 \text{ hour}}{3600 \text{ s}} = \frac{25}{86400} \text{ m/s} \approx 2.8935 \times 10^{-4} \text{ m/s}$$

Transmissivity is defined as $T = K \times b$.

$$T = \left(\frac{25}{86400} \text{ m/s}\right) \times (16 \text{ m}) = \frac{400}{86400} \text{ m}^2/\text{s} = \frac{1}{216} \text{ m}^2/\text{s} \approx 0.00463 \text{ m}^2/\text{s}$$

Step 3: Apply the Thiem Equation:
We are given the drawdown at one point and need to find it at another.

- At $r_1 = 75$ m, the drawdown is $s_1 = 3.5$ m.
- We need to find the drawdown s_2 at $r_2 = 150$ m.

Substitute the values into the Thiem difference equation:

$$3.5 - s_2 = \frac{0.1}{2\pi\left(\frac{1}{216}\right)} \ln\left(\frac{150}{75}\right)$$

$$3.5 - s_2 = \frac{0.1 \times 216}{2\pi} \ln(2)$$

$$3.5 - s_2 = \frac{21.6}{2\pi} \ln(2) \approx (3.4377) \times (0.6931)$$

$$3.5 - s_2 \approx 2.3824$$

Step 4: Solve for the Unknown Drawdown (s_2):

$$s_2 = 3.5 - 2.3824 = 1.1176 \text{ m}$$

Step 5: Final Answer:
The drawdown at a distance of 150 m is 1.1176 m. Rounding to two decimal places gives 1.12 m.

Quick Tip

The Thiem equation is the fundamental tool for steady-state drawdown in a confined aquifer. The difference form, $s_1 - s_2 = \frac{Q}{2\pi T} \ln(r_2/r_1)$, is extremely useful for problems involving two observation wells, as it eliminates the need to know the radius of influence, R. Ensure all your inputs (especially Q and K) are in a consistent unit system (e.g., meters and seconds) before calculating.

---

55. A biological reactor is getting wastewater containing 1 mole/L acetate ions as carbon source. The following reaction takes place in the bio-reactor:

$0.125CH_3COO^- + 0.0295NH_4^+ + 0.103O_2 \rightarrow 0.0295C_5H_7O_2N + 0.0955H_2O + 0.095HCO_3^- + 0.007CO_2$

Assume that all acetate ions are consumed and ammonia serves as a nutrient source. Given that 1 g acetate exerts 1.07 g COD; 1 mole bacteria =113 g VSS; 1 mole acetate ion = 59 g. Value of observed yield is _____ (in g VSS/g COD, rounded off to two decimal places).

Correct Answer: 0.42

Solution:

Step 1: Understand Observed Yield (Y_{obs}):
The observed yield is the ratio of the mass of new biomass produced to the mass of substrate consumed. In wastewater engineering, this is typically expressed as grams of Volatile Suspended Solids (VSS) produced per gram of Chemical Oxygen Demand (COD) consumed.

$$Y_{obs} = \frac{\text{g VSS produced}}{\text{g COD consumed}}$$

The provided stoichiometric equation gives the molar relationship, which we will convert to a mass relationship.
Step 2: Calculate Mass of VSS Produced (Biomass):
From the reaction, 0.0295 moles of bacteria (represented by the formula $C_5H_7O_2N$) are produced. We are given that 1 mole of bacteria is equivalent to 113 g VSS.

$$\text{Mass of VSS produced} = (0.0295 \text{ mol bacteria}) \times \left(\frac{113 \text{ g VSS}}{1 \text{ mol bacteria}}\right) = 3.3335 \text{ g VSS}$$

Step 3: Calculate Mass of COD Consumed (Substrate):
The reaction consumes 0.125 moles of acetate ions (CH_3COO^-). We need to find the COD equivalent of this amount. First, find the mass of acetate consumed: We are given that 1 mole of acetate = 59 g.

$$\text{Mass of acetate consumed} = (0.125 \text{ mol acetate}) \times \left(\frac{59 \text{ g}}{1 \text{ mol acetate}}\right) = 7.375 \text{ g acetate}$$

Next, convert the mass of acetate to the equivalent mass of COD, using the given factor: 1 g acetate exerts 1.07 g COD.

$$\text{Mass of COD consumed} = (7.375 \text{ g acetate}) \times \left(\frac{1.07 \text{ g COD}}{1 \text{ g acetate}}\right) = 7.89125 \text{ g COD}$$

Step 4: Calculate the Observed Yield:
Now, we can compute the ratio using the masses calculated in the previous steps.

$$Y_{obs} = \frac{\text{Mass of VSS produced}}{\text{Mass of COD consumed}} = \frac{3.3335 \text{ g VSS}}{7.89125 \text{ g COD}}$$

$$Y_{obs} \approx 0.4224... \text{ g VSS/g COD}$$

Step 5: Final Answer:
Rounding the result to two decimal places gives 0.42.

Quick Tip

Yield calculations in bioreactors are a matter of careful unit conversions based on a given stoichiometry. The process is: 1. From the reaction, find the molar ratio of (Biomass Produced / Substrate Consumed). 2. Convert moles of biomass to mass of VSS using the given molar mass of bacteria. 3. Convert moles of substrate to mass of substrate using its molar mass. 4. Convert mass of substrate to mass of COD using the given COD equivalent. 5. Calculate the final ratio of (mass VSS / mass COD).

---

56. A flask (100 ml volume) has wastewater, which has 0.12 mg/L geosmin. Activated carbon is added in this flask for adsorbing geosmin as per the Freundlich isotherm model ($Q = 2.6 \times C^{0.73}$ where Q is mg adsorbate/mg adsorbent and C is the equilibrium concentration in mg/L). Activated carbon to be added in this flask for getting final remaining geosmin concentration of 0.05 mg/L would be _____ (in mg, rounded off to three decimal places).

Correct Answer: 0.025

Solution:

Step 1: Set up the Mass Balance Equation for Adsorption:
For a batch adsorption process, the total mass of the substance (adsorbate) removed from the liquid phase is equal to the mass adsorbed onto the solid phase (adsorbent).

$$V(C_0 - C_f) = M \times Q_e$$

where:

- V: Volume of wastewater = 100 ml = 0.1 L
- C_0: Initial concentration of geosmin = 0.12 mg/L
- C_f: Final (equilibrium) concentration of geosmin = 0.05 mg/L
- M: Mass of activated carbon (adsorbent) to be added, in mg. This is the unknown we need to find.
- Q_e: The mass of geosmin adsorbed per unit mass of carbon at equilibrium (mg adsorbate / mg adsorbent).

Step 2: Calculate the Equilibrium Adsorption Capacity (Q_e):
The Freundlich isotherm model, $Q = 2.6 \times C^{0.73}$, relates the equilibrium capacity Q_e to the final equilibrium concentration in the liquid, C_f.

$$Q_e = 2.6 \times (C_f)^{0.73} = 2.6 \times (0.05)^{0.73}$$

Using a calculator:

$$(0.05)^{0.73} \approx 0.10646$$

$$Q_e = 2.6 \times 0.10646 \approx 0.2768 \text{ mg geosmin / mg carbon}$$

Step 3: Calculate the Mass of Geosmin to be Removed:
This is the left side of the mass balance equation.

$$\text{Mass removed} = V(C_0 - C_f) = (0.1 \text{ L}) \times (0.12 \text{ mg/L} - 0.05 \text{ mg/L})$$

$$\text{Mass removed} = 0.1 \text{ L} \times 0.07 \text{ mg/L} = 0.007 \text{ mg}$$

Step 4: Solve for the Mass of Activated Carbon (M):
Now, we use the mass balance equation to solve for M.

$$M = \frac{\text{Mass removed}}{Q_e} = \frac{0.007 \text{ mg}}{0.2768 \text{ mg/mg}}$$

$$M \approx 0.025289 \text{ mg}$$

Step 5: Final Answer:
The required mass of activated carbon is 0.025289 mg. The question in the PDF asks for the answer in mg/L, which is a typo and should be mg. Rounding our result to three decimal places gives 0.025 mg.

Quick Tip

Batch adsorption problems are solved using a mass balance. The total amount of contaminant removed from the liquid phase must equal the total amount adsorbed onto the solid adsorbent. 1. Calculate the mass of contaminant to be removed: $V(C_0 - C_f)$. 2. Use the given isotherm (Freundlich or Langmuir) to find the equilibrium solid-phase concentration (Q_e) that corresponds to the target final liquid concentration (C_f). 3. Solve the mass balance equation $V(C_0 - C_f) = M \cdot Q_e$ for the unknown mass of adsorbent, M. Pay close attention to units (e.g., L vs mL, mg vs g).

---

57. A pipeline is designed to deliver 20 L/s of an oil (kinematic viscosity = 6×10^{-6} m^2/s and specific gravity = 0.9) under the laminar flow condition. The minimum diameter of the pipe will be _____ (in m, rounded off to two decimal places).

Correct Answer: 2.12

Solution:

Step 1: Define the Condition for Laminar Flow:
Flow in a circular pipe is considered laminar if the Reynolds number (Re) is below a critical value. While the transition can start around Re = 2300, a conservative value often used for design to guarantee laminar flow is $Re_{crit} = 2000$. The Reynolds number for pipe flow is:

$$Re = \frac{\rho v D}{\mu} = \frac{v D}{\nu}$$

where v is the average flow velocity, D is the pipe diameter, and ν is the kinematic viscosity.

Step 2: Express Reynolds Number in Terms of Discharge (Q):
The average velocity v is related to discharge Q and pipe area $A = \pi D^2/4$ by $v = Q/A$. Substituting this into the Reynolds number formula:

$$Re = \frac{(Q/A)D}{\nu} = \frac{(Q/(\pi D^2/4))D}{\nu} = \frac{4QD}{\pi D^2 \nu} = \frac{4Q}{\pi D \nu}$$

Step 3: Solve for the Minimum Diameter:
We need to find the diameter D that will result in a Reynolds number equal to the critical value of 2000 for the given discharge. For a fixed discharge Q, Re is inversely proportional to D ($Re \propto 1/D$). This means that to keep Re low (i.e., laminar), we need a large diameter. A smaller diameter would increase the velocity and thus the Reynolds number, pushing the flow towards turbulence. Therefore, the "minimum diameter for laminar flow" is the diameter at which the Reynolds number hits the maximum allowable value, Re_{crit}.

$$Re_{crit} = \frac{4Q}{\pi D_{min} \nu} \implies D_{min} = \frac{4Q}{\pi \cdot Re_{crit} \cdot \nu}$$

Step 4: Substitute Values and Calculate:
We must use consistent SI units.

- Discharge, $Q = 20$ L/s $= 20 \times 10^{-3}$ m^3/s $= 0.02$ m^3/s.
- Kinematic viscosity, $\nu = 6 \times 10^{-6}$ m^2/s.
- Critical Reynolds number, $Re_{crit} = 2000$.

$$D_{min} = \frac{4 \times 0.02}{\pi \times 2000 \times (6 \times 10^{-6})} = \frac{0.08}{12000\pi \times 10^{-6}} = \frac{0.08}{0.012\pi}$$

$$D_{min} = \frac{0.08}{0.037699} \approx 2.122 \text{ m}$$

The specific gravity is not needed for this calculation as the kinematic viscosity is already given.
Step 5: Final Answer:
The minimum diameter required to ensure laminar flow is 2.122 m. Rounding to two decimal places gives 2.12 m.

Quick Tip

The Reynolds number is a critical dimensionless parameter that determines the flow regime (laminar or turbulent). The formula $Re = \frac{4Q}{\pi D \nu}$ is a useful rearrangement that directly links discharge and diameter. To ensure laminar flow (Re ¡ 2000-2300) for a given discharge, the pipe diameter must be larger than a certain minimum value. A smaller pipe means higher velocity and a greater tendency for turbulence.

---

58. You are doing an experiment to find out BOD_5 of a wastewater. You have taken 25 mL wastewater having ultimate BOD of 75 mg/L and placed it in 300 mL BOD bottle and filled it with dilution water. The initial DO of the diluted sample is 6.5 mg/L. On the 5th day, you were not able to measure the DO due to some unavoidable circumstances. However, the DO at the end of the 7th day is found

to be 1.25 mg/L. Assume all the experiments are done at the same temperature, and no biodegradable organics are present in the dilution water. The BOD_5 of the wastewater sample is _____ (in mg/L, rounded off to two decimal places).

Correct Answer: 54.74

Solution:

Step 1: Define BOD Kinetics and Dilution:
The problem requires us to find the 5-day BOD (BOD_5) of a wastewater sample. We are missing the 5-day data but have 7-day data and the ultimate BOD (L_0). We can use this information to find the BOD rate constant, k, and then calculate the BOD_5.
The first-order BOD exertion model is: $BOD_t = L_0(1 - e^{-kt})$.
The BOD exerted in a diluted sample is the drop in dissolved oxygen: $BOD_{t,diluted} = DO_{initial} - DO_{final}$. The BOD of the original wastewater is related to the diluted sample's BOD by the dilution factor, P: $BOD_{t,wastewater} = BOD_{t,diluted}/P$.

Step 2: Calculate the Reaction Rate Constant (k):

First, find the dilution factor and the ultimate BOD of the diluted sample.

$$P = \frac{\text{Volume of wastewater}}{\text{Total volume}} = \frac{25 \text{ mL}}{300 \text{ mL}} = \frac{1}{12}$$

The ultimate BOD of the original wastewater is $L_{0,ww} = 75$ mg/L.
The ultimate BOD of the diluted sample is $L_{0,dil} = L_{0,ww} \times P = 75 \times \frac{1}{12} = 6.25$ mg/L.
Next, find the 7-day BOD exerted in the diluted sample from the DO readings:

$$BOD_{7,dil} = DO_{initial} - DO_7 = 6.5 \text{ mg/L} - 1.25 \text{ mg/L} = 5.25 \text{ mg/L}$$

Now, use the kinetic model with the 7-day data to solve for k:

$$BOD_{7,dil} = L_{0,dil}(1 - e^{-k \cdot 7})$$

$$5.25 = 6.25(1 - e^{-7k})$$

$$\frac{5.25}{6.25} = 1 - e^{-7k} \implies 0.84 = 1 - e^{-7k}$$

$$e^{-7k} = 1 - 0.84 = 0.16$$

Take the natural logarithm of both sides:

$$-7k = \ln(0.16) \approx -1.8326$$

$$k = \frac{-1.8326}{-7} \approx 0.2618 \text{ day}^{-1} \text{ (base e)}$$

Step 3: Calculate the 5-Day BOD (BOD_5) of the Wastewater:
Now that we have the rate constant k, we can calculate the BOD_5 for the original, undiluted wastewater using its ultimate BOD, $L_0 = 75$ mg/L.

$$BOD_5 = L_0(1 - e^{-k \cdot 5})$$

$$BOD_5 = 75 \times (1 - e^{-0.2618 \times 5}) = 75 \times (1 - e^{-1.309})$$
$$BOD_5 = 75 \times (1 - 0.27009) = 75 \times (0.72991)$$
$$BOD_5 \approx 54.743 \text{ mg/L}$$

Step 4: Final Answer:
The BOD_5 of the wastewater sample is 54.743 mg/L. Rounding to two decimal places gives 54.74.

> **Quick Tip**
>
> BOD problems often require multiple steps. 1. Always calculate the dilution factor $P = V_{sample}/V_{total}$ first. 2. Remember that lab measurements ($DO_{initial}, DO_{final}$) give you the BOD of the *diluted* sample. 3. The kinetic parameters (k, L_0) are intrinsic to the wastewater, so they are the same for diluted and undiluted samples. 4. You can use the kinetic equation $BOD_t = L_0(1-e^{-kt})$ on either the diluted or undiluted values, as long as you are consistent.

---

59. In a 30 m^3 room, a stove in operation consumes wood at the rate of 0.25 kg/h. The inflow and outflow rate of air in the room is the same, i.e. 500 m^3/h. This stove emits a VOC species at a rate of 0.2 g/kg-wood. The VOC species gets converted to CO_2 at a rate of 0.4 per hour. Given: (i) the air in the room is completely mixed, (ii) initial concentration of the VOC species in the room is negligible, and (iii) concentration of the VOC species in the air entering the room is negligible. The concentration of the VOC species due to two hours of stove operation in the room is _____ (in μg/m^3, rounded off to one decimal place).

Correct Answer: 97.7

Solution:

Step 1: Set up a Mass Balance Equation for the VOC:
We can model the room as a Completely Stirred Tank Reactor (CSTR). The mass balance for the VOC in the room's air is:

Rate of Accumulation = (Rate In) − (Rate Out) + (Rate of Generation) − (Rate of Decay)

Let $C(t)$ be the concentration of VOC in the room (in g/m^3) and V be the room volume.

$$V\frac{dC}{dt} = QC_{in} - QC + E - kCV$$

where:

- V = Room volume = 30 m^3.
- Q = Air flow rate = 500 m^3/h.
- C_{in} = VOC concentration in inflow = 0 (given).

- E = VOC emission rate inside the room (in g/h).
- k = First-order decay rate constant for the VOC = 0.4 h^{-1}.

Step 2: Calculate the Emission Rate (E):
The stove is the source of the VOC.

$$E = (\text{Wood burning rate}) \times (\text{VOC emission factor})$$

$$E = (0.25 \text{ kg-wood/h}) \times (0.2 \text{ g/kg-wood}) = 0.05 \text{ g/h}$$

Step 3: Solve the Differential Equation:
Substitute the known values into the mass balance equation:

$$30\frac{dC}{dt} = (500 \times 0) - 500C + 0.05 - (0.4 \times C \times 30)$$

$$30\frac{dC}{dt} = -500C + 0.05 - 12C$$

$$30\frac{dC}{dt} + 512C = 0.05$$

This is a first-order linear ODE. The solution for C(t), with the initial condition $C(0) = 0$, is:

$$C(t) = C_{ss}(1 - e^{-\lambda t})$$

where C_{ss} is the steady-state concentration and λ is the overall removal rate constant.

- At steady state ($dC/dt = 0$), $512C_{ss} = 0.05 \implies C_{ss} = \frac{0.05}{512}$ g/m^3.
- The exponent term is $\lambda = \frac{512}{30} \approx 17.067$ h^{-1}.

The solution is:

$$C(t) = \frac{0.05}{512}(1 - e^{-(512/30)t})$$

Step 4: Calculate the Concentration at t = 2 hours:
We need to find C(2).

$$C(2) = \frac{0.05}{512}(1 - e^{-(512/30)\times 2}) = \frac{0.05}{512}(1 - e^{-34.133\ldots})$$

The term $e^{-34.133}$ is extremely small ($\approx 1.5 \times 10^{-15}$), so it is effectively zero. This means that after 2 hours, the system has practically reached its steady-state concentration.

$$C(2) \approx C_{ss} = \frac{0.05}{512} \approx 9.7656 \times 10^{-5} \text{ g/m}^3$$

Step 5: Convert Units and Final Answer:
The question asks for the concentration in micrograms per cubic meter (μg/m^3).

$$C(2) \text{ in } \mu\text{g/m}^3 = (9.7656 \times 10^{-5} \text{ g/m}^3) \times (10^6 \mu\text{g/g})$$

$$C(2) \approx 97.656 \mu\text{g/m}^3$$

Rounding to one decimal place, the concentration is 97.7 μg/m^3.

Quick Tip

Indoor air quality problems are often modeled as a CSTR (Completely Stirred Tank Reactor). The mass balance is key: $V\frac{dC}{dt} = E + QC_{in} - QC - kCV$. The solution for concentration at time t starting from $C_0 = 0$ is $C(t) = C_{ss}(1 - e^{-\lambda t})$, where the steady-state concentration is $C_{ss} = \frac{E+QC_{in}}{Q+kV}$ and the overall removal rate constant is $\lambda = \frac{Q}{V} + k$. Check if the time 't' is large enough for the system to reach steady state ($e^{-\lambda t} \approx 0$).

---

60. A city generates on average 1000 metric tonnes/day of municipal solid waste and follows integrated waste management system. 15% of the total waste is recycled, 40% of the total waste is used to produce compost. 25% of the total waste is converted to refuse derived fuel (RDF) with 80% efficiency. Remaining is disposed of in a sanitary landfill. The calorific value of the RDF is 15 MJ/kg, which is further used to generate electricity. The electrical energy that could be generated from the RDF with a thermal to electrical energy conversion efficiency of 20% is _____ (in MWh/d, rounded off to two decimal places).

Correct Answer: 166.67

Solution:

Step 1: Calculate the Mass of Waste Used for RDF:

- Total MSW generated per day = 1000 metric tonnes/day.
- Percentage of waste sent to RDF facility = 25%.
- Mass of MSW input to RDF process = 1000 tonnes/day $\times$ 0.25 = 250 tonnes/day.

Step 2: Calculate the Mass of RDF Produced:
The conversion efficiency of the RDF process is 80

$$\text{Mass of RDF produced} = 250 \text{ tonnes/day} \times 80\% = 250 \times 0.80 = 200 \text{ tonnes/day}$$

Convert this to kilograms:

$$\text{Mass of RDF} = 200 \text{ tonnes/day} \times 1000 \text{ kg/tonne} = 200,000 \text{ kg/day}$$

Step 3: Calculate the Total Thermal Energy Generated:
The calorific value of the RDF is 15 MJ/kg.

$$\text{Total Thermal Energy} = (\text{Mass of RDF}) \times (\text{Calorific Value})$$

$$\text{Thermal Energy} = (200,000 \text{ kg/day}) \times (15 \text{ MJ/kg}) = 3,000,000 \text{ MJ/day}$$

Step 4: Calculate the Electrical Energy Generated:
The thermal-to-electrical energy conversion efficiency is 20%.

$$\text{Electrical Energy} = \text{Thermal Energy} \times \text{Conversion Efficiency}$$

$$\text{Electrical Energy} = 3,000,000 \text{ MJ/day} \times 20\% = 3,000,000 \times 0.20 = 600,000 \text{ MJ/day}$$

Step 5: Convert Units to MWh/day:
The question requires the answer in Megawatt-hours per day (MWh/d). We need the conversion factor between Joules and Watt-hours.

- 1 Watt = 1 Joule/second
- 1 hour = 3600 seconds
- 1 Wh = 1 W × 1 h = (1 J/s) × (3600 s) = 3600 J
- 1 MWh = 10^6 Wh = $10^6 \times 3600$ J = 3.6×10^9 J = 3600 MJ

So, to convert from MJ to MWh, we divide by 3600.

$$\text{Electrical Energy (MWh/d)} = \frac{\text{Energy in MJ/day}}{3600 \text{ MJ/MWh}}$$

$$\text{Electrical Energy} = \frac{600,000}{3600} = \frac{6000}{36} = \frac{1000}{6} \approx 166.666... \text{ MWh/day}$$

Step 6: Final Answer:
Rounding the result to two decimal places gives 166.67 MWh/d.

Quick Tip

Waste-to-energy calculations involve a chain of conversions, often with efficiency factors at each step. Be systematic and keep track of units. 1. Calculate the mass of fuel (e.g., RDF) produced from the initial waste stream. 2. Calculate the total thermal energy available ($Mass \times CalorificValue$). 3. Calculate the electrical energy produced ($ThermalEnergy \times ConversionEfficiency$). 4. Perform the final unit conversion. Remember the key conversion: 1 kWh = 3.6 MJ, which means 1 MWh = 3600 MJ.

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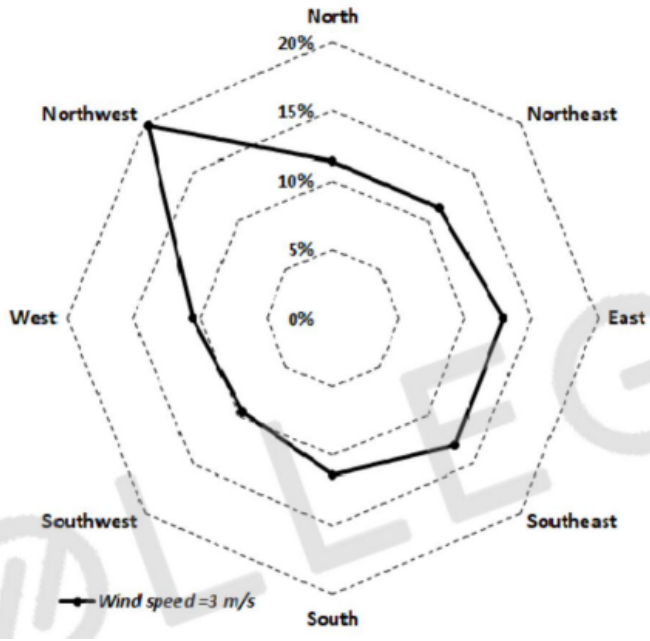
61. An industry with an effective stack height of 80 m emits 1200 g/h of CO. The windrose plotted using the meteorological data at the top of the stack, and the relation between dispersion coefficients and wind direction are given below:



Wind Direction	Dispersion coefficients (in m)	
	Crosswind direction	Vertical direction
Northeast	50	20
North	45	30
Northwest	40	35
East	45	30
Southeast	55	35
South	60	40
Southwest	65	45
West	70	50

During the maximum duration of the year, the ground level $PM_{2.5}$ concentration at the downwind distance of 2 km (at the plume centerline) from the stack is _____ (in $\mu g/m^3$, rounded off to two decimal places).

(Note: The question asks for $PM_{2.5}$ concentration but gives the CO emission rate. We will calculate the concentration for the emitted pollutant, CO, as is standard for such typos.)

Correct Answer: 0.75

Solution:

Step 1: Select the Gaussian Plume Model and Identify Parameters:

The problem requires calculating ground-level pollutant concentration, which is done using the Gaussian Plume Model. The formula for the concentration at ground level (z=0) along the plume centerline (y=0) is:

$$C(x,0,0) = \frac{Q}{\pi u \sigma_y \sigma_z} \exp\left[-\frac{1}{2}\left(\frac{H}{\sigma_z}\right)^2\right]$$

- **Emission Rate (Q):** We must convert 1200 g/h to μg/s.

$$Q = 1200\frac{\text{g}}{\text{h}} \times \frac{10^6 \mu\text{g}}{1\text{g}} \times \frac{1\text{h}}{3600\text{s}} = \frac{1,200,000,000}{3600} = 333,333.3... \,\mu\text{g/s}$$

- **Effective Stack Height (H):** $H = 80$ m.
- **Downwind Distance (x):** $x = 2$ km $= 2000$ m.

Step 2: Determine Meteorological Conditions for Maximum Duration:
The phrase "maximum duration of the year" refers to the most frequent meteorological conditions. We find this from the windrose diagram.

- **Wind Direction:** The longest "petal" on the windrose points to the North. This indicates the wind is blowing *from* the North most frequently (20
- **Wind Speed (u):** The key on the windrose indicates a single wind speed for all conditions shown: $u = 3$ m/s.
- **Dispersion Coefficients (σ_y, σ_z):** We use the values from the table corresponding to the most frequent wind direction (North).
  - σ_y (Crosswind dispersion) = 45 m.
  - σ_z (Vertical dispersion) = 30 m.

The problem simplifies by providing single values for σ_y and σ_z instead of making them functions of distance x and stability class, which is a common exam simplification.
Step 3: Calculate the Concentration:
Substitute all the determined values into the Gaussian plume formula:

$$C(2000, 0, 0) = \frac{333,333.3}{\pi \cdot (3) \cdot (45) \cdot (30)} \exp\left[-\frac{1}{2}\left(\frac{80}{30}\right)^2\right]$$

Calculate the two parts of the equation:

- The fractional term:

$$\frac{333,333.3}{\pi \times 90 \times 45} = \frac{333,333.3}{12723.45} \approx 26.198$$

- The exponential term:

$$\exp\left[-\frac{1}{2}\left(\frac{8}{3}\right)^2\right] = \exp\left[-\frac{1}{2}\left(\frac{64}{9}\right)\right] = \exp\left[-\frac{32}{9}\right] = \exp[-3.555...] \approx 0.02856$$

Now, multiply the two parts to get the final concentration:

$$C = 26.198 \times 0.02856 \approx 0.7482 \,\mu\text{g/m}^3$$

Step 4: Final Answer:
The calculated ground-level concentration is 0.7482 μg/m^3. Rounding to two decimal places gives 0.75.

Quick Tip

The Gaussian Plume Model is the standard tool for air dispersion calculations. The centerline, ground-level concentration formula is $C = \frac{Q}{\pi u \sigma_y \sigma_z} \exp(-\frac{H^2}{2\sigma_z^2})$. Be extremely careful with units, especially for the emission rate Q, which must be in mass/time (e.g., μg/s). When interpreting a windrose, remember that the direction indicates where the wind is coming FROM.

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62. Ms. Anita uses a BS-IV two wheeler petrol scooter, with a mileage of 50 km/L, to travel 30 km every day. She exchanges this two wheeler with an electric scooter, which consumes electricity at 0.1 kWh/10 km. Assuming the cost of petrol and electricity are fixed at Rs. 90 per L and Rs. 3.5 per kWh, respectively, and maintenance cost of both BS-IV two wheeler and electric scooter is negligible, the operational cost saved in a year by Ms. Anita is _____ (in Rs., in integer).

Correct Answer: 19327

Solution:

Step 1: Calculate the Yearly Operational Cost for the Petrol Scooter:

1. **Daily Petrol Consumption:** The scooter travels 30 km/day with a mileage of 50 km/L.

$$\text{Petrol used per day} = \frac{30 \text{ km/day}}{50 \text{ km/L}} = 0.6 \text{ L/day}$$

2. **Daily Cost:** The cost of petrol is Rs. 90 per L.

$$\text{Daily Cost}_{\text{petrol}} = (0.6 \text{ L/day}) \times (90 \text{ Rs./L}) = 54 \text{ Rs./day}$$

3. **Yearly Cost:** Assuming a year has 365 days.

$$\text{Yearly Cost}_{\text{petrol}} = (54 \text{ Rs./day}) \times (365 \text{ days/year}) = 19,710 \text{ Rs./year}$$

Step 2: Calculate the Yearly Operational Cost for the Electric Scooter:

1. **Daily Electricity Consumption:** The scooter travels 30 km/day and consumes 0.1 kWh for every 10 km.

$$\text{Electricity used per day} = (30 \text{ km/day}) \times \left(\frac{0.1 \text{ kWh}}{10 \text{ km}}\right) = 0.3 \text{ kWh/day}$$

2. **Daily Cost:** The cost of electricity is Rs. 3.5 per kWh.

$$\text{Daily Cost}_{\text{electric}} = (0.3 \text{ kWh/day}) \times (3.5 \text{ Rs./kWh}) = 1.05 \text{ Rs./day}$$

3. **Yearly Cost:**

$$\text{Yearly Cost}_{\text{electric}} = (1.05 \text{ Rs./day}) \times (365 \text{ days/year}) = 383.25 \text{ Rs./year}$$

Step 3: Calculate the Total Yearly Savings:
The saving is the difference between the petrol scooter cost and the electric scooter cost.

$$\text{Yearly Savings} = \text{Yearly Cost}_{\text{petrol}} - \text{Yearly Cost}_{\text{electric}}$$

$$\text{Yearly Savings} = 19,710 - 383.25 = 19,326.75 \text{ Rs.}$$

Step 4: Final Answer:
The question asks for the saving in Rs., as an integer. Rounding 19,326.75 to the nearest integer gives 19,327.

Quick Tip

For cost comparison problems, the structure is usually: 1. Calculate the total resource consumption (e.g., liters of petrol, kWh of electricity) for each option over a specified period (e.g., a year). 2. Calculate the total cost for each option by multiplying the consumption by the unit cost. 3. Find the difference between the costs to determine the savings. Be careful with the time period (daily vs. yearly) and ensure all units are consistent.

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63. Ultimate analysis of a municipal solid waste sample is given below

Percent by weight				
Carbon	Hydrogen	Oxygen	Nitrogen	Ash
48	6	35	6	5

For 1 kg of the municipal solid waste burnt, assuming that air contains only nitrogen and oxygen, maximum CO_2 emitted is _____ (in kg, rounded off to three decimal places).

Correct Answer: 1.760

Solution:

Step 1: Identify the Relevant Chemical Reaction:
The emission of carbon dioxide (CO_2) during combustion comes from the oxidation of the carbon (C) present in the waste. The balanced chemical reaction for the complete combustion of carbon is:

$$C + O_2 \rightarrow CO_2$$

Step 2: Determine the Stoichiometric Mass Ratio:
From the reaction, one mole of carbon produces one mole of carbon dioxide. We can convert this to a mass relationship using atomic and molecular weights:

- Atomic weight of Carbon (C) $\approx$ 12 g/mol.
- Molecular weight of CO_2 (C + 2*O) = $12 + 2 \times 16 = 44$ g/mol.

This means that for every 12 units of mass of carbon burnt, 44 units of mass of CO_2 are produced. The mass ratio is:

$$\frac{\text{Mass of } CO_2 \text{ produced}}{\text{Mass of C consumed}} = \frac{44}{12}$$

Step 3: Calculate the Mass of Carbon in the Waste Sample:
We are considering the combustion of 1 kg of municipal solid waste (MSW). The ultimate analysis shows that carbon constitutes 48

$$\text{Mass of Carbon in 1 kg of MSW} = 1 \text{ kg} \times 48\% = 0.48 \text{ kg}$$

Step 4: Calculate the Maximum Mass of CO_2 Emitted:
Assuming complete combustion, all the carbon in the waste is converted to CO_2. We use the mass ratio from Step 2:

$$\text{Mass of } CO_2 = \text{Mass of C} \times \frac{44}{12}$$

$$\text{Mass of } CO_2 = 0.48 \text{ kg} \times \frac{44}{12} = 0.48 \times \frac{11}{3} = 0.16 \times 11 = 1.76 \text{ kg}$$

Step 5: Final Answer:
The maximum mass of CO_2 emitted is 1.76 kg. The question asks to round to three decimal places, so the answer is 1.760.

> Quick Tip
>
> In combustion calculations, to find the mass of CO_2 produced from a fuel with a known carbon content, you only need two pieces of information: 1. The mass of carbon in the fuel (Total Mass $\times$ %C). 2. The stoichiometric mass ratio of CO_2 to C, which is always 44/12. Mass of CO_2 = Mass of Carbon $\times$ (44/12).

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64. An adult of 65 kg weight and life span of 65 years drinks water for 5 years, which is contaminated with toluene of concentration 0.15 mg/L. For toluene, reference dose is 0.200 mg/kg-d. The person drinks 2 L of water per day. The hazard quotient from the toluene exposure for the adult will be _____ (rounded off to three decimal places).

Correct Answer: 0.023

Solution:

Step 1: Define Hazard Quotient (HQ):
The Hazard Quotient is used to characterize the non-carcinogenic risk from exposure to a chemical. It is the ratio of the estimated exposure dose to the reference dose (RfD).

$$\text{HQ} = \frac{\text{Average Daily Dose (ADD)}}{\text{Reference Dose (RfD)}}$$

An HQ below 1 generally indicates that the exposure is unlikely to cause adverse health effects.

Step 2: Calculate the Average Daily Dose (ADD):

For non-carcinogenic risk assessment, the ADD is the daily intake of the chemical per unit body weight, averaged over the exposure period. The formula for ADD from drinking water is:

$$\text{ADD} = \frac{C \times IR \times EF \times ED}{BW \times AT}$$

For non-carcinogens, the averaging time (AT) is equal to the exposure duration (ED), so the terms $EF \times ED/AT$ cancel out (assuming exposure occurs every day of the exposure duration). The formula simplifies to the chronic daily intake:

$$\text{ADD} = \frac{C \times IR}{BW}$$

where:

- C = Concentration in water = 0.15 mg/L.
- IR = Intake Rate of water = 2 L/day.
- BW = Body Weight = 65 kg.

The lifespan (65 years) and exposure duration (5 years) are not needed for this simplified ADD calculation for non-carcinogenic risk.

$$\text{ADD} = \frac{(0.15 \text{ mg/L}) \times (2 \text{ L/day})}{65 \text{ kg}} = \frac{0.3 \text{ mg/day}}{65 \text{ kg}} \approx 0.004615 \text{ mg/kg-day}$$

Step 3: Calculate the Hazard Quotient (HQ):

Now we divide the calculated ADD by the given Reference Dose.

- ADD ≈ 0.004615 mg/kg-day
- RfD = 0.200 mg/kg-day

$$\text{HQ} = \frac{0.004615 \text{ mg/kg-day}}{0.200 \text{ mg/kg-day}} \approx 0.023075$$

Step 4: Final Answer:

Rounding the result to three decimal places gives 0.023.

Quick Tip

For risk assessment calculations, distinguish between non-carcinogenic and carcinogenic risk:

- **Non-carcinogenic Risk (Hazard Quotient, HQ):** Uses a Reference Dose (RfD). The dose is typically the Chronic Daily Intake (CDI or ADD), which is $(C \times IR)/BW$. HQ = ADD/RfD.
- **Carcinogenic Risk:** Uses a Cancer Slope Factor (CSF). The dose is the Lifetime Average Daily Dose (LADD), which averages the intake over a full lifetime (AT = lifespan $\times$ 365). Risk = LADD $\times$ CSF.

The averaging time is the key difference in the dose calculation.

65. An aeration tank needs to be installed for the removal of VOC from water, where the required rate of flow of water through the aeration tank is 180,000 m^3/d. Permissible limit of VOC in the water is 12 μg/L. The saturation concentration of VOC is 5 μg/L and gas transfer rate constant is 0.40 per second at 25 °C. The initial concentration of VOC in the water is 33 μg/L. The volume of the aeration tank to satisfy the permissible limit of VOC at 25 °C is _____ (in m^3, rounded off to two decimal places).

Correct Answer: 15.63

Solution:

Step 1: Identify the Process and Governing Model:
The process is air stripping of a VOC in an aeration tank. We can model the aeration tank as a Completely Stirred Tank Reactor (CSTR) operating at steady state. The mass balance equation governs the performance.

$$\text{Rate of VOC In} = \text{Rate of VOC Out} + \text{Rate of VOC Removal}$$

$$QC_{in} = QC_{out} + (\text{Rate of mass transfer})$$

The rate of mass transfer is given by $(K_La)V(C_{out}-C^*)$, where K_La is the overall mass transfer coefficient, V is the volume, and $(C_{out} - C^*)$ is the concentration driving force.
Step 2: Formulate the Design Equation:
The mass balance is:

$$Q(C_{in} - C_{out}) = (K_La)V(C_{out} - C^*)$$

We need to solve for the volume V. Rearranging the equation:

$$V = \frac{Q(C_{in} - C_{out})}{(K_La)(C_{out} - C^*)}$$

We can also write this in terms of the hydraulic retention time, $\theta = V/Q$:

$$\theta = \frac{C_{in} - C_{out}}{(K_La)(C_{out} - C^*)}$$

Step 3: Define Parameters and Ensure Consistent Units:

- Flow rate, $Q = 180,000$ m^3/d. We need this in m^3/s.

$$Q = \frac{180,000 \text{ m}^3}{1 \text{ day}} \times \frac{1 \text{ day}}{24 \text{ hr}} \times \frac{1 \text{ hr}}{3600 \text{ s}} = \frac{180,000}{86,400} \approx 2.0833 \text{ m}^3/\text{s}$$

- Initial concentration, $C_{in} = 33\,\mu$g/L.
- Final (permissible) concentration, $C_{out} = 12\,\mu$g/L.
- Saturation concentration, $C^* = 5\,\mu$g/L. (This represents the equilibrium concentration in the water with the stripping air).

- Gas transfer rate constant, $K_L a = 0.40\ \text{s}^{-1}$.

Step 4: Calculate the Required Hydraulic Retention Time (θ):

$$\theta = \frac{33 - 12}{0.40 \times (12 - 5)} = \frac{21}{0.40 \times 7} = \frac{21}{2.8} = 7.5\ \text{s}$$

Step 5: Calculate the Required Tank Volume (V):

$$V = Q \times \theta = (2.0833\ \text{m}^3/\text{s}) \times (7.5\ \text{s}) = 15.625\ \text{m}^3$$

Step 6: Final Answer:
The required volume of the aeration tank is 15.625 m^3. Rounding to two decimal places gives 15.63.

Quick Tip

Air stripping of VOCs in a CSTR (or completely mixed aeration tank) is modeled with a steady-state mass balance. The performance equation can be written in terms of retention time $\theta = V/Q$:

$$\theta = \frac{C_{in} - C_{out}}{(K_L a)(C_{out} - C^*)}$$

where C^* is the equilibrium concentration. Ensure all units are consistent (e.g., if $K_L a$ is in s^{-1}, then Q must be in m^3/s and θ will be in s).
