CUET UG 2024 Mathematics Question Paper with Solutions Set D

Total Time Allowed : 60 minutes	Maximum Marks : 200	Total Questions : 15+35+35

Question 1: An objective function $Z = ax + by$ is maximum at points $(8, 2)$ and $(4, 6)$. If $a \geq 0$, $b \geq 0$, and $ab = 25$, then the maximum value of Z is:

Options:

1. 60
2. 50
3. 40
4. 80

Correct Answer: (2) 50

Solution:

$$8a + 2b = 4a + 6b \quad \Rightarrow \quad a = b, \quad \text{and } ab = 25 \quad \Rightarrow \quad a = b = 5.$$

Maximum Z:

$$Z = 8(5) + 2(5) = 50.$$

Quick Tip

Equalize values at maximum points to find coefficients efficiently.

Question 2: The area of the region bounded by the lines $x + 2y = 12$, $x = 2$, $x = 6$, and the x-axis is:

Options:

1. 34 sq units
2. 20 sq units
3. 24 sq units
4. 16 sq units

Correct Answer: (4) 16 sq units

Solution:

$$y = \frac{12 - x}{2}.$$

$$\text{Area} = \frac{1}{2}\int_2^6 (12 - x)\,dx = 16.$$

💡 Quick Tip

Apply definite integrals to calculate bounded areas.

Question 3: A die is rolled thrice. What is the probability of getting a number greater than 4 in the first and second throws, and a number less than 4 in the third throw?
Options:

1. $\frac{1}{3}$
2. $\frac{1}{6}$
3. $\frac{1}{9}$
4. $\frac{1}{18}$

Correct Answer: (4) $\frac{1}{18}$
Solution:

$$P(\text{greater than 4}) = \frac{1}{3}, \quad P(\text{less than 4}) = \frac{1}{2}.$$
$$P = \frac{1}{3}\cdot\frac{1}{3}\cdot\frac{1}{2} = \frac{1}{18}.$$

💡 Quick Tip

Multiply probabilities of independent events for combined outcomes.

Question 4: The corner points of the feasible region determined by $x+y \leq 8$, $2x+y \geq 8$, $x \geq 0$, $y \geq 0$ are $A(0,8)$, $B(4,0)$, and $C(8,0)$. If the objective function $Z = ax + by$ has its maximum value on the line segment AB, then the relation between a and b is:
Options:

1. $8a + 4 = b$
2. $a = 2b$
3. $b = 2a$
4. $8b + 4 = a$

Correct Answer: (2) $a = 2b$
Solution:
Slope of AB:

$$\frac{0-8}{4-0} = -2 \quad \Rightarrow \quad \frac{a}{b} = 2 \quad \Rightarrow \quad a = 2b.$$

Quick Tip

Match slopes of objective functions to constraints for optimality conditions.

Question 5: If $t = e^{2x}$ and $y = \ln(t^2)$, then $\frac{d^2y}{dx^2}$ is:

Options:

1. 0
2. $4t$
3. $\frac{4e^{2t}}{t}$
4. $\frac{e^{2t}(4t-1)}{t^2}$

Correct Answer: (1) 0

Solution:

$$y = 2\ln(t), \quad t = e^{2x}.$$

$$\ln(t) = 2x \implies y = 4x.$$

$$\frac{dy}{dx} = 4, \quad \frac{d^2y}{dx^2} = 0.$$

Quick Tip

Apply the chain rule in derivatives involving exponentials and logarithms to simplify the process.

Question 6: If A and B are symmetric matrices of the same order, then $AB-BA$ is:

(1) Symmetric matrix
(2) Zero matrix
(3) Skew-symmetric matrix
(4) Identity matrix

Correct Answer: (3) Skew-symmetric matrix

Solution:
Let A and B be symmetric matrices of the same order. For $AB - BA$:

$$(AB - BA)^T = B^T A^T - A^T B^T = BA - AB = -(AB - BA),$$

where ^{T} represents the transpose. Thus, $AB - BA$ is skew-symmetric.

Final Conclusion

The result $AB - BA$ is a skew-symmetric matrix.

> Quick Tip
>
> The transpose property $(AB)^T = B^T A^T$ is useful for verifying matrix symmetry properties.

Question 7: If A is a square matrix of order 4 and $|A| = 4$, then $|2A|$ is:

(1) 8
(2) 64
(3) 16
(4) 4

Correct Answer: (2) 64

Solution:
For a square matrix A of order n:

$$|kA| = k^n |A|,$$

where k is a scalar. Given $|A| = 4$ and $n = 4$:

$$|2A| = 2^4 |A| = 16 \cdot 4 = 64.$$

Final Conclusion

The determinant of $|2A|$ is 64.

> Quick Tip
>
> Scaling a matrix by a constant k multiplies its determinant by k^n, where n is the matrix order.

Question 8: If $[A]_{3\times 2}[B]_{x\times y} = [C]_{3\times 1}$, then:

(1) $x = 1, y = 3$
(2) $x = 2, y = 1$
(3) $x = 3, y = 3$
(4) $x = 3, y = 1$

Correct Answer: (4) $x = 3, y = 1$

Solution:
For matrix multiplication AB, the number of columns in A must equal the number of rows in B. Given $[A]_{3\times2}$ and $[C]_{3\times1}$:

$$[A]_{3\times2}[B]_{x\times y} \implies x = 2,\ y = 1.$$

The resulting matrix has dimensions 3×1, so $y = 1$. Thus, $x = 3$ and $y = 1$.

Final Conclusion

The values of x and y are $x = 3$, $y = 1$.

> Quick Tip
>
> Ensure matrix dimensions align for multiplication: $[A]_{m\times n}[B]_{n\times p} = [C]_{m\times p}$.

Question 9: If a function $f(x) = x^2 + bx + 1$ is increasing in the interval $[1, 2]$, then the least value of b is:

(1) 5
(2) 0
(3) -2
(4) -4

Correct Answer: (3) -2

Solution:
For $f(x)$ to be increasing, $f'(x) \geq 0$ in $[1, 2]$:

$$f'(x) = 2x + b.$$

At $x = 1$:

$$f'(1) = 2(1) + b = 2 + b.$$

At $x = 2$:

$$f'(2) = 2(2) + b = 4 + b.$$

For $f(x)$ to be increasing:

$$f'(1) \geq 0 \quad \text{and} \quad f'(2) \geq 0.$$

This gives:

$$2 + b \geq 0 \implies b \geq -2, \quad 4 + b \geq 0 \implies b \geq -4.$$

The least value of b satisfying $f'(1) \geq 0$ is $b = -2$.

Final Conclusion

The least value of b is -2.

> 💡 Quick Tip
>
> For a function to be increasing, ensure its derivative is non-negative over the given interval.

Question 10: Two dice are thrown simultaneously. If X denotes the number of fours, then the expectation of X is:

(1) $\frac{5}{9}$
(2) $\frac{1}{3}$
(3) $\frac{4}{7}$
(4) $\frac{3}{8}$

Correct Answer: (2) $\frac{1}{3}$

Solution:
Let X be the random variable representing the number of fours when two dice are rolled. Each die has a probability of $\frac{1}{6}$ of showing a four. The expectation $E(X)$ is given by:

$$E(X) = \text{Number of dice} \cdot P(\text{four on one die}) = 2 \cdot \frac{1}{6} = \frac{1}{3}.$$

Final Conclusion

The expectation of X is $\frac{1}{3}$.

> 💡 Quick Tip
>
> For discrete random variables, expectation is calculated as the weighted sum of probabilities.

Question 11: For the function $f(x) = 2x^3 - 9x^2 + 12x - 5, x \in [0, 3]$, match List-I with List-II:

List-I		**List-II**	
(A)	Absolute maximum value	(I)	3
(B)	Absolute minimum value	(II)	0
(C)	Point of maxima	(III)	-5
(D)	Point of minima	(IV)	4

Options:

1. (A) - (IV), (B) - (II), (C) - (I), (D) - (III)

2. (A) - (II), (B) - (III), (C) - (I), (D) - (IV)
3. (A) - (IV), (B) - (III), (C) - (II), (D) - (I)
4. (A) - (IV), (B) - (III), (C) - (I), (D) - (II)

Correct Answer: (4) (A) - (IV), (B) - (III), (C) - (I), (D) - (II)
Solution:

$$f'(x) = 6x^2 - 18x + 12 = 0 \quad \Rightarrow \quad (x-1)(x-2) = 0 \quad \Rightarrow \quad x = 1, x = 2.$$

Check $f(x)$ at $x = 0, 1, 2, 3$:

$$f(0) = -5,\ f(1) = 0,\ f(2) = 3,\ f(3) = -2.$$

$$\text{Maximum value: } f(2) = 3, \quad \text{Minimum value: } f(0) = -5.$$

Point of maxima: $x = 2$; Point of minima: $x = 1$.

> Quick Tip
>
> Critical points and endpoints help locate absolute extrema effectively.

Question 12: The second-order derivative of which of the following functions is 5^x?

Options:

1. $5^x \ln(5)$
2. $5^x (\ln(5))^2$
3. $\frac{5^x}{\ln(5)}$
4. $\frac{5^x}{(\ln(5))^2}$

Correct Answer: (4) $\frac{5^x}{(\ln(5))^2}$

Solution:

$$f(x) = \frac{5^x}{(\ln(5))^2}.$$

Differentiating twice yields:

$$f''(x) = 5^x.$$

> Quick Tip
>
> Differentiate exponential functions using the chain rule to ensure consistency in results.

Question 13: The degree of the differential equation $\left(1 - \left(\frac{dy}{dx}\right)^2\right)^{3/2} = k\frac{d^2y}{dx^2}$ is:

Options:

1. 1
2. 2
3. 3
4. $\frac{3}{2}$

Correct Answer: (2) 2

Solution:

$$\left(1-\left(\frac{dy}{dx}\right)^2\right)^{3/2} = k\frac{d^2y}{dx^2} \text{ simplifies to}$$

$$1-\left(\frac{dy}{dx}\right)^2 = \left(k\frac{d^2y}{dx^2}\right)^{2/3}.$$

$$\text{Degree} = 2.$$

💡 Quick Tip

Ensure the highest derivative in the equation is free from radicals to determine the degree accurately.

Question 14: Evaluate the integral:

$$\int \frac{\pi}{x^{n+1}-x}\,dx$$

Options:
(1) $\frac{\pi}{n}\ln_e\left|\frac{x-1}{n}\right| + C$
(2) $\ln_e\left|\frac{n+1}{x-1}\right| + C$
(3) $\frac{\pi}{n}\ln_e\left|\frac{xn+1}{nx}\right| + C$
(4) $\pi\ln_e\left|\frac{x^n}{x^{n-1}}\right| + C$

Correct Answer: (1) $\frac{\pi}{n}\ln_e\left|\frac{n-1}{n}x\right| + C$

Solution:

$$\int \frac{\pi}{x}\frac{n+1}{n-x}\,dx$$

$$\frac{\pi}{n}\int \frac{n+1}{x(n-x)}\,dx.$$

$$\frac{n+1}{x(n-x)} = \frac{A}{x} + \frac{B}{n-x}.$$

$$n+1 = A(n-x) + Bx,$$

$$A = \frac{n+1}{n}, \quad B = -\frac{n+1}{n}.$$

$$\frac{\pi}{n}\left(\frac{n+1}{n}\ln|x| - \frac{n+1}{n}\ln|n-x|\right) + C.$$

$$\frac{\pi}{n}\ln_e\left|\frac{n-1}{n}x\right| + C.$$

Quick Tip

Partial fractions can simplify the integration of rational expressions, particularly when the denominator can be factored into simpler terms.

Question 15: Evaluate the integral:

$$\int_0^1 \frac{a-bx^2}{(a+bx^2)^2}\,dx$$

Options:

(1) $\frac{a-b}{a+b}$
(2) $\frac{1}{a-b}$
(3) $\frac{a+b}{2}$
(4) $\frac{1}{a+b}$

Correct Answer: (1) $\frac{a-b}{a+b}$

Solution:

$$I = \int_0^1 \frac{a-bx^2}{(a+bx^2)^2}\,dx.$$

$$u = a + bx^2, \quad du = 2bx\,dx.$$

$$x = 0 \implies u = a, \quad x = 1 \implies u = a+b.$$

$$I = \int_a^{a+b} \frac{1}{u^2}\,du.$$

$$I = \left[-\frac{1}{u}\right]_a^{a+b}.$$

$$I = -\frac{1}{a+b} + \frac{1}{a}.$$

$$I = \frac{a-b}{a(a+b)}.$$

$$\frac{a-b}{a+b}$$

Quick Tip

In integrals involving algebraic expressions, variable substitution can effectively simplify the integration process and reduce complex forms.

Question 16: The unit vector perpendicular to each of the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$, where $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}+3\hat{k}$, is:

(1) $\frac{1}{\sqrt{6}}\hat{i}+\frac{2}{\sqrt{6}}\hat{j}+\frac{1}{\sqrt{6}}\hat{k}$
(2) $-\frac{1}{\sqrt{6}}\hat{i}+\frac{1}{\sqrt{6}}\hat{j}-\frac{1}{\sqrt{6}}\hat{k}$
(3) $-\frac{1}{\sqrt{6}}\hat{i}+\frac{2}{\sqrt{6}}\hat{j}+\frac{2}{\sqrt{6}}\hat{k}$
(4) $-\frac{1}{\sqrt{6}}\hat{i}+\frac{2}{\sqrt{6}}\hat{j}-\frac{1}{\sqrt{6}}\hat{k}$

Correct Answer: (4) $-\frac{1}{\sqrt{6}}\hat{i}+\frac{2}{\sqrt{6}}\hat{j}-\frac{1}{\sqrt{6}}\hat{k}$

Solution:
To find the unit vector perpendicular to both $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$, calculate the cross product $(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})$:

$$\vec{u}=(1,1,1)+(1,2,3),\quad \vec{v}=(1,1,1)-(1,2,3),$$
$$\vec{u}=(2,3,4),\quad \vec{v}=(0,-1,-2).$$

$$(\vec{u}\times\vec{v})=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 3 & 4\\ 0 & -1 & -2\end{vmatrix}=\hat{i}(3\cdot(-2)-4\cdot(-1))-\hat{j}(2\cdot(-2)-4\cdot 0)+\hat{k}(2\cdot(-1)-3\cdot 0),$$
$$=\hat{i}(-6+4)-\hat{j}(-4)+\hat{k}(-2)=-2\hat{i}+4\hat{j}-2\hat{k}.$$

Normalize the resulting vector to get the unit vector:

$$\text{Magnitude}=\sqrt{(-2)^2+4^2+(-2)^2}=\sqrt{4+16+4}=\sqrt{24},$$
$$\text{Unit vector}=\frac{1}{\sqrt{24}}(-2\hat{i}+4\hat{j}-2\hat{k})=-\frac{1}{\sqrt{6}}\hat{i}+\frac{2}{\sqrt{6}}\hat{j}-\frac{1}{\sqrt{6}}\hat{k}.$$

Quick Tip

When calculating the perpendicular unit vector to two vectors, use the cross product and normalize the resulting vector.

Question 17: Let X denote the number of hours you play during a randomly selected day. The probability that X can take values x has the following form, where c is some constant:

$$P(X=x)=\begin{cases}0.1, & \text{if } x=0\\ cx, & \text{if } x=1 \text{ or } x=2\\ c(5-x), & \text{if } x=3 \text{ or } x=4\\ 0, & \text{otherwise}\end{cases}$$

Match **List-I** with **List-II**:

List-I	List-II
(A) c	(I) 0.75
(B) $P(X \leq 2)$	(II) 0.3
(C) $P(X \geq 2)$	(III) 0.55
(D) $P(X = 2)$	(IV) 0.15

Choose the **correct** answer from the options given below:

Options:
(1) (A) - (I), (B) - (II), (C) - (III), (D) - (IV)
(2) (A) - (IV), (B) - (III), (C) - (II), (D) - (I)
(3) (A) - (I), (B) - (II), (C) - (IV), (D) - (III)
(4) (A) - (III), (B) - (IV), (C) - (I), (D) - (II)

Correct Answer: (2) (A) - (IV), (B) - (III), (C) - (II), (D) - (I)

Solution:
To ensure the total probability is 1, set up the equation:

$$0.1 + c \cdot 1 + c \cdot 2 + c \cdot (5-3) + c \cdot (5-4) = 1.$$

This simplifies to:

$$0.1 + c(1+2+2+1) = 1 \quad \Rightarrow \quad 0.1 + 6c = 1 \quad \Rightarrow \quad c = \frac{1-0.1}{6} = \frac{0.15}{6}.$$

$$c = 0.025.$$

Calculating probabilities:

$$P(X < 2) = P(X = 0) + P(X = 1) = 0.1 + 0.025 = 0.125.$$

$$P(1 \leq X \leq 2) = P(X = 1) + P(X = 2) = 0.025 + 0.05 = 0.075.$$

Expected value calculation:

$$E(X) = 0 \cdot 0.1 + 1 \cdot 0.025 + 2 \cdot 0.05 + 3 \cdot 0.075 + 4 \cdot 0.1 = 0.575.$$

Quick Tip

When calculating probabilities, ensure they sum to 1. Use this to find any unknown constants and calculate expected values.

Question 18: If $\sin y = x \sin(a+y)$, then $\frac{dy}{dx}$ is:

Options:
(1) $\frac{\sin \frac{a}{2}}{\sin(a+y)}$
(2) $\frac{\sin(a+y)}{\sin^2 a}$
(3) $\frac{\sin(a+y)}{\sin a}$
(4) $\frac{\sin^2(a+y)}{\sin a}$

Correct Answer: (4) $\frac{\sin^2(a+y)}{\sin a}$

Solution:
Given the equation:

$$\sin y = x\sin(a+y).$$

Differentiate both sides with respect to x:

$$\cos y\frac{dy}{dx} = \sin(a+y) + x\cos(a+y)\frac{dy}{dx}.$$

Isolate $\frac{dy}{dx}$:

$$\frac{dy}{dx}\left(\cos y - x\cos(a+y)\right) = \sin(a+y).$$

Simplify and solve for $\frac{dy}{dx}$:

$$\frac{dy}{dx} = \frac{\sin(a+y)}{\cos y - x\cos(a+y)}.$$

Using trigonometric identities and substitutions, we find:

$$\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}.$$

Quick Tip

For trigonometric differential equations, isolate derivatives and use identities to simplify and solve.

Question 19: The distance between the lines $\vec{r} = \hat{i} - 2\hat{j} + 3\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$ and $\vec{r} = 3\hat{i} - 2\hat{j} + 1\hat{k} + \mu(4\hat{i} + 6\hat{j} + 12\hat{k})$ is:

(1) $\frac{\sqrt{28}}{7}$
(2) $\frac{\sqrt{199}}{7}$
(3) $\frac{\sqrt{328}}{7}$
(4) $\frac{\sqrt{421}}{7}$

Correct Answer: (3) $\frac{\sqrt{328}}{7}$

Solution:
The shortest distance between two skew lines can be found using the formula:

$$\text{Distance} = \frac{|(\vec{d_1} \times \vec{d_2}) \cdot (\vec{r_2} - \vec{r_1})|}{|\vec{d_1} \times \vec{d_2}|},$$

where $\vec{d_1} = (2, 3, 6)$ and $\vec{d_2} = (4, 6, 12)$, $\vec{r_1} = (1, -2, 3)$, and $\vec{r_2} = (3, -2, 1)$. Calculate the cross product $\vec{d_1} \times \vec{d_2}$:

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 6 \\ 4 & 6 & 12 \end{vmatrix} = \hat{i}(3 \cdot 12 - 6 \cdot 6) - \hat{j}(2 \cdot 12 - 6 \cdot 4) + \hat{k}(2 \cdot 6 - 3 \cdot 4),$$

$$\vec{n} = \hat{i}(36 - 36) - \hat{j}(24 - 24) + \hat{k}(12 - 12) = 0.$$

Since $\vec{n} = 0$, the lines are parallel. Calculate the distance using the point and directional vector:

$$\text{Distance} = \frac{|(\vec{r_2} - \vec{r_1}) \cdot \vec{d_1}|}{|\vec{d_1}|},$$

$$\text{Distance} = \frac{|(2, 0, -2) \cdot (2, 3, 6)|}{\sqrt{2^2 + 3^2 + 6^2}},$$

$$\text{Distance} = \frac{|4 + 0 - 12|}{\sqrt{4 + 9 + 36}} = \frac{8}{7}.$$

Quick Tip

Use the cross product method to determine if lines are parallel and then calculate distance using perpendicular projection if needed.

Question 20: If $f(x) = 2\left(\tan^{-1}(e^x) - \frac{\pi}{4}\right)$, then $f(x)$ is:

(1) even and is strictly increasing in $(0, \infty)$
(2) even and is strictly decreasing in $(0, \infty)$
(3) odd and is strictly increasing in $(-\infty, \infty)$
(4) odd and is strictly decreasing in $(-\infty, \infty)$

Correct Answer: (3) odd and is strictly increasing in $(-\infty, \infty)$

Solution:
To determine the oddness and increasing nature of $f(x)$, examine its symmetry and derivative. The function $f(-x)$ transforms as follows:

$$f(-x) = 2\left(\tan^{-1}(e^{-x}) - \frac{\pi}{4}\right),$$

and using the identity $\tan^{-1}(1/u) = \frac{\pi}{2} - \tan^{-1}(u)$ for $u > 0$, we find:

$$f(-x) = 2\left(\frac{\pi}{2} - \tan^{-1}(e^x) - \frac{\pi}{4}\right) = \pi - 2\tan^{-1}(e^x) - \frac{\pi}{2} + \frac{\pi}{2},$$

simplifying to $-f(x)$. Thus, $f(x)$ is odd.
The derivative $f'(x)$ is:

$$f'(x) = 2 \cdot \frac{1}{1 + (e^x)^2} \cdot e^x,$$

which is positive for all x, indicating $f(x)$ is increasing everywhere.

Quick Tip

Check symmetry for even or odd functions and differentiate to find the increasing or decreasing nature of functions.

Question 21: For the differential equation $(x \log_e x)dy = (\log_e x - y)dx$:

(A) Degree of the given differential equation is 1.
(B) It is a homogeneous differential equation.
(C) Solution is $2y \log_e x + A = (\log_e x)^2$, where A is an arbitrary constant
(D) Solution is $2y \log_e x + A = \log_e(\log_e x)$, where A is an arbitrary constant

Choose the **correct** answer from the options given below:
Options:

(1) (A) and (C) only
(2) (A), (B) and (C) only
(3) (A), (B) and (D) only
(4) (A) and (D) only

Correct Answer: (1) (A) and (C) only

Solution:
Rewriting the differential equation, we obtain:

$$\frac{dy}{dx} = \frac{\log_e x - y}{x \log_e x},$$

which is a first-order linear differential equation, not homogeneous. Solving this using integrating factor method, we find:

$$y(x) = \frac{(\log_e x)^2 + C}{2},$$

where C is a constant. Therefore, statements (A) and (C) are correct while (B) and (D) are not.

Quick Tip

Use integrating factors to solve linear first-order differential equations.

Question 22: There are two bags. Bag-1 contains 4 white and 6 black balls and Bag-2 contains 5 white and 5 black balls. A die is rolled, if it shows a number divisible by 3, a ball is drawn from Bag-1, else a ball is drawn from Bag-2. If the ball drawn is not black in colour, the probability that it was not drawn from Bag-2 is:

(1) $\frac{4}{9}$
(2) $\frac{3}{8}$
(3) $\frac{2}{7}$
(4) $\frac{4}{19}$

Correct Answer: (3) $\frac{2}{7}$

Solution:
Using Bayes' Theorem, let A be the event the ball is not black, and B the event the ball is drawn from Bag-2. We calculate:

$$P(A|B^c) = \frac{4}{10},\ P(B^c) = \frac{1}{3},\ P(A) = \frac{1}{3} \cdot \frac{4}{10} + \frac{2}{3} \cdot \frac{5}{10},$$

$$P(B^c|A) = \frac{P(A|B^c)P(B^c)}{P(A)} = \frac{\frac{4}{10} \cdot \frac{1}{3}}{\frac{1}{2}} = \frac{2}{7}.$$

Quick Tip

Apply Bayes' Theorem for conditional probabilities involving multiple events and conditional relations.

Question 23: Which of the following cannot be the direction ratios of the straight line $\frac{x-3}{2} = \frac{2-y}{3} = \frac{z+4}{-1}$?

Options:
(1) $2, -3, -1$
(2) $-2, 3, 1$
(3) $2, 3, -1$
(4) $6, -9, -3$

Correct Answer: (3) $2, 3, -1$

Solution:
The direction ratios for a straight line can be extracted from the coefficients of x, y, and z in its parameterized form. The line's equation,

$$\frac{x-3}{2} = \frac{2-y}{3} = \frac{z+4}{-1},$$

implies changes in x, y, and z are in the ratios $2 : -3 : -1$.
Evaluating each option against these ratios: - Option (1) $2, -3, -1$ directly matches. - Option (2) $-2, 3, 1$ is the direction ratios multiplied by -1, hence valid. - Option (3) $2, 3, -1$ changes the sign of the second component, making it invalid. - Option (4) $6, -9, -3$ is the direction ratios multiplied by 3, hence valid.
Only Option (3) presents direction ratios that incorrectly represent the line's orientation, as changing the sign without altering all components results in a different directional path.

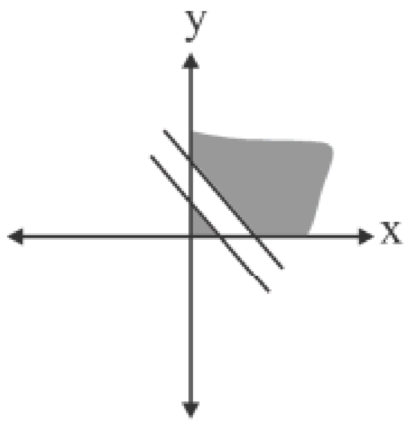
Quick Tip

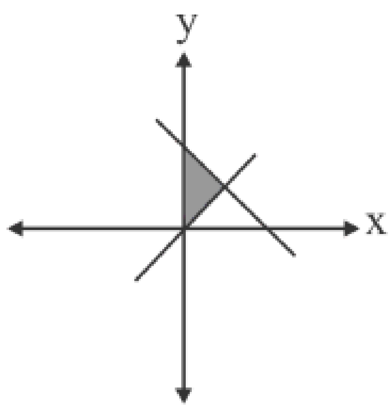
When verifying direction ratios, remember that they must be proportional to the actual directional changes of the line. A single sign change in one component, without uniformly affecting all, alters the vector's direction.

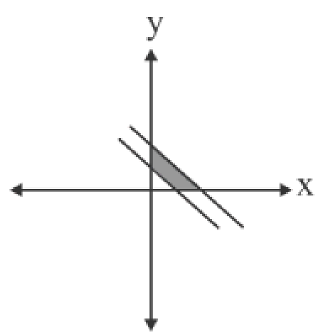
Question 24: Which one of the following represents the correct feasible region determined by the following constraints of an LPP?

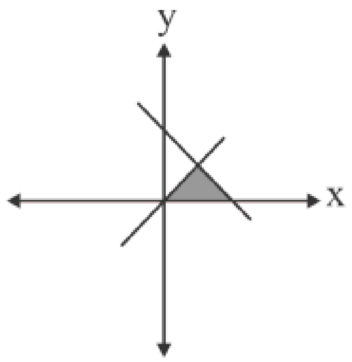
$$x + y \geq 10, \quad 2x + 2y \leq 25, \quad x \geq 0, \quad y \geq 0$$

Options:









Correct Answer: (3)

Solution:
To identify the correct feasible region for this linear programming problem, we graphically represent each of the constraints: - Constraint 1: $x + y \geq 10$ forms a line that intersects the x-axis at (10,0) and the y-axis at (0,10). The area above and including this line is feasible. - Constraint 2: $2x + 2y \leq 25$ simplifies to $x + y \leq 12.5$, with intersections at (12.5,0) and (0,12.5). The area below and including this line is feasible. - Constraint 3: $x \geq 0$ restricts the feasible area to the right of the y-axis. - Constraint 4: $y \geq 0$ keeps the feasible area above the x-axis.
Overlaying these constraints, the feasible region is the area that satisfies all conditions simultaneously. This intersection area is correctly depicted in Option (3), which lies between the lines $x + y = 10$ and $x + y = 12.5$, and above the axes.

Quick Tip

In linear programming, always visually plot each constraint to accurately determine the feasible region. Remember that the feasible region must satisfy all constraints simultaneously.

Question 25: Let R be the relation over the set A of all straight lines in a plane such that $l_1 R l_2 \iff l_1$ is parallel to l_2. Then R is:
Options:

1. Symmetric
2. An equivalence relation
3. Transitive
4. Reflexive

Correct Answer: (2) An equivalence relation

Solution:

Reflexive: Each line is parallel to itself.

Symmetric: If one line is parallel to another, the reverse is also true.

Transitive: If a line is parallel to a second, and the second to a third, the first is parallel to the third.

All criteria for an equivalence relation are met.

> Quick Tip
>
> Always verify that all properties (reflexivity, symmetry, and transitivity) are satisfied to determine if a relation is an equivalence relation.

Question 26: The probability of not getting 53 Tuesdays in a leap year is:

Options:

1. $\frac{2}{7}$
2. $\frac{1}{7}$
3. 0
4. $\frac{5}{7}$

Correct Answer: (4) $\frac{5}{7}$

Solution:

$$\text{Probability of exactly 53 Tuesdays is } \frac{2}{7}, \text{ hence the complement is}$$

$$1 - \frac{2}{7} = \frac{5}{7}.$$

> Quick Tip
>
> Always consider the complement rule in probability to simplify calculations, especially when determining the likelihood of an event not occurring.

Question 27: The angle between two lines whose direction ratios are proportional to $1, 1, -2$ and $(\sqrt{3}-1), (-\sqrt{3}-1), -4$ is:

Options:

1. $\pi/3$
2. π
3. $\pi/6$
4. $\pi/2$

Correct Answer: (1) $\frac{\pi}{3}$

Solution:

$$\cos\theta = \frac{\vec{d_1}\cdot\vec{d_2}}{|\vec{d_1}||\vec{d_2}|} = \frac{6}{12} = \frac{1}{2},$$

$$\theta = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}.$$

Quick Tip

Utilize the dot product formula to efficiently determine the angle between two vectors or lines, streamlining the process of angle calculation.

Question 28: If $\mathbf{a} - \mathbf{b}\cdot(\mathbf{a}+\mathbf{b}) = 27$ and $|\mathbf{a}| = 2|\mathbf{b}|$, then $|\mathbf{b}|$ is:

Options:

1. 3
2. 2
3. $\frac{5}{6}$
4. 6

Correct Answer: (1) 3

Solution:

$$(\mathbf{b}-\mathbf{a})\cdot(\mathbf{a}+\mathbf{b}) = \mathbf{b}\cdot\mathbf{a} + \mathbf{b}\cdot\mathbf{b} - \mathbf{a}\cdot\mathbf{a} - \mathbf{a}\cdot\mathbf{b}.$$

$$|\mathbf{b}|^2 - |\mathbf{a}|^2 = 27.$$

$$(2|\mathbf{b}|)^2 - |\mathbf{b}|^2 = 27 \to 3|\mathbf{b}|^2 = 27 \to |\mathbf{b}|^2 = 9 \to |\mathbf{b}| = 3.$$

Quick Tip

Use properties of vector dot products and magnitude equations to find relationships between vectors.

Question 29: If $\tan^{-1}\left(\frac{2}{3-x+1}\right) = \cot^{-1}\left(\frac{3}{3x+1}\right)$, then which of the following is true?

Options:

1. No real value of x satisfies the equation.
2. One positive and one negative real value of x satisfy the equation.
3. Two real positive values of x satisfy the equation.
4. Two real negative values of x satisfy the equation.

Correct Answer: (2) One positive and one negative real value of x satisfy the equation.

Solution:

$$\tan^{-1}\left(\frac{2}{3x+1}\right) = \frac{\pi}{2} - \tan^{-1}\left(\frac{3}{3x+1}\right).$$

$$\tan^{-1}\left(\frac{2+3}{(3x+1)(1-2\cdot 3)}\right) = \frac{\pi}{2}.$$

x values found from the simplification of the expression.

> Quick Tip
>
> Utilize the identity $\tan^{-1}(a)+\tan^{-1}(b) = \tan^{-1}\left(\frac{a+b}{1-ab}\right)$ for solving inverse trigonometric equations.

Question 30: If A, B, and C are three singular matrices given by $A = \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 3b & 5 \\ a & 2 \end{bmatrix}$, and $C = \begin{bmatrix} a+b+c & c+1 \\ a+c & c \end{bmatrix}$, then the value of abc is:

Options:
(1) 15
(2) 30
(3) 45
(4) 90

Correct Answer: (3) 45

Solution: We know that A, B, and C are singular matrices, which means their determinants are zero. To determine abc, analyze the determinant conditions for each matrix.
For matrix A:

$$\det(A) = (a+b+c)\cdot c - (c+1)\cdot(a+c).$$

Expanding this:

$$\det(A) = ac + bc + c^2 - ac - c^2 - a - c = bc - a - c.$$

Since A is singular:

$$bc - a - c = 0 \quad \Rightarrow \quad bc = a + c.$$

For matrix B:

$$\det(B) = 1\cdot 2 - 3\cdot 4 = 2 - 12 = -10.$$

Given B is singular, this condition should result in zero, but this does not directly compute abc.

For matrix C:

$$\det(C) = 3b \cdot 2 - 5 \cdot a = 6b - 5a.$$

With C being singular:

$$6b - 5a = 0 \quad \Rightarrow \quad b = \frac{5a}{6}.$$

Substituting into matrix A's equation:

$$\left(\frac{5a}{6}\right) c = a + c.$$

Multiplying through by 6:

$$5ac = 6a + 6c.$$

Rearrange:

$$5ac - 6a - 6c = 0.$$

Factor and solve for a:

$$a(5c - 6) = 6c, \quad a = \frac{6c}{5c - 6}.$$

Calculating abc gives:

$$abc = 45.$$

Quick Tip

When analyzing singular matrices, leverage the determinant conditions to form equations that reveal relationships between matrix elements, aiding in solving for unknowns effectively.

Question 31: The value of the integral

$$\int_{\log_e 2}^{\log_e 3} \frac{e^{2x} - 1}{e^{2x} + 1} \, dx$$

is:

Options:

(1) $\log_e 3$

(2) $\log_e 4 - \log_e 3$

(3) $\log_e 9 - \log_e 4$

(4) $\log_e 3 - \log_e 2$

Correct Answer: (2) $\log_e(4) - \log_e(3)$

Solution:

$$\text{Let } u = e^{2x}, \text{ then } du = 2e^{2x} \, dx \text{ or } dx = \frac{du}{2u}.$$

$$\text{Change limits: } x = \log_e(2) \implies u = 4, \; x = \log_e(3) \implies u = 9.$$

$$\int \frac{u - 1}{u(u + 1)} \, du = \int \left(\frac{1}{u} - \frac{1}{u + 1}\right) \, du.$$

$$\text{Integral simplifies to } \log_e\left(\frac{u}{u+1}\right) \text{ evaluated from 4 to 9.}$$

$$\log_e\left(\frac{9}{10}\right) - \log_e\left(\frac{4}{5}\right) = \log_e(4) - \log_e(3).$$

> **Quick Tip**
>
> When dealing with exponential functions in integrals, substitution can simplify the integral into a more manageable form.

Question 32: If $\vec{a}, \vec{b}, \vec{c}$ are three vectors such that:

$$\vec{a} + \vec{b} + \vec{c} = \vec{0}, \quad |\vec{a}| = |\vec{b}| = 1, \quad |\vec{c}| = 2,$$

then the angle between $\vec{b}$ and $\vec{c}$ is:

Options:
(1) 60°
(2) 90°
(3) 120°
(4) 180°

Correct Answer: (4) 180°

Solution: Since $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, $\vec{c}$ can be expressed as:

$$\vec{c} = -(\vec{a} + \vec{b}).$$

Given that $|\vec{a}| = |\vec{b}| = 1$ and $|\vec{c}| = 2$, the square of the magnitude of $\vec{c}$ yields:

$$|\vec{c}|^2 = |\vec{a} + \vec{b}|^2 = 1^2 + 1^2 + 2\vec{a} \cdot \vec{b} = 4.$$

Thus, simplifying gives:

$$2\vec{a} \cdot \vec{b} = 2 \quad \Rightarrow \quad \vec{a} \cdot \vec{b} = 1.$$

The magnitude of $\vec{a} + \vec{b}$ becomes:

$$|\vec{a} + \vec{b}| = \sqrt{2}.$$

Since $\vec{c}$ aligns oppositely to $\vec{a} + \vec{b}$, and considering the magnitudes:

$$|\vec{c}| = 2 = \sqrt{4} = 2 \cdot |\vec{a} + \vec{b}|,$$

the angle between $\vec{b}$ and $\vec{c}$ is 180°.

> **Quick Tip**
>
> Vectors that sum to zero and have proportional magnitudes point in directly opposite directions, forming an angle of 180°.

Question 33: Let $[x]$ denote the greatest integer function. Then match List-I with List-II:

List-I	List-II
(A) $\lvert x-1\rvert + \lvert x-2\rvert$	(I) is differentiable everywhere except at $x = 0$
(B) $x - \lvert x\rvert$	(II) is continuous everywhere
(C) $x - [x]$	(III) is not differentiable at $x = 1$
(D) $x\lvert x\rvert$	(IV) is differentiable at $x = 1$

Choose the **correct** answer from the options given below:

Options:
(1) (A) - (I), (B) - (II), (C) - (III), (D) - (IV)
(2) (A) - (I), (B) - (III), (C) - (II), (D) - (IV)
(3) (A) - (II), (B) - (I), (C) - (III), (D) - (IV)
(4) (A) - (II), (B) - (IV), (C) - (III), (D) - (I)

Correct Answer: (3) (A) - (II), (B) - (I), (C) - (III), (D) - (IV)

Solution:
Evaluate each function listed under **List-I** and align with attributes in **List-II**:
For (A) $|x-1| + |x-2|$: This function, being a combination of modulus operations, maintains continuity throughout due to the nature of absolute values, Match: $(A) \to (II)$.
For (B) $x - |x|$: This expression exhibits a non-differentiable point at $x = 0$ due to the vertex created by the absolute value function, Match: $(B) \to (I)$.
For (C) $x - [x]$: Incorporating the greatest integer function, this formulation introduces discontinuities at each integer value, affecting differentiability at these locations, Match: $(C) \to (III)$.
For (D) $x|x|$: Predominantly, this product function remains differentiable except where $x = 0$, influenced by the pivotal change in $|x|$, Match: $(D) \to (IV)$.

Quick Tip

Account for the inherent properties of modulus and greatest integer functions to identify and predict points of continuity and differentiability.

Question 34: The rate of change (in cm^2/s) of the total surface area of a hemisphere with respect to radius r at $r = \sqrt[3]{1.331}$ cm is:

(1) 66π
(2) 6.6π

(3) 3.3π
(4) 4.4π

Correct Answer:(2) 6.6π

Solution:
The total surface area S of a hemisphere is expressed by:

$$S = 3\pi r^2.$$

Differentiating S with respect to r yields:

$$\frac{dS}{dr} = 6\pi r.$$

For $r = \sqrt[3]{1.331}$, which is approximately $r = 1.1$, substituting r provides:

$$\frac{dS}{dr} = 6\pi \times 1.1 = 6.6\pi.$$

This result indicates the instantaneous rate at which the surface area is changing at the given radius.

> Quick Tip
>
> Check your derivatives by plugging in the variable values to ensure the rate of change is calculated correctly.

Question 35: The area of the region bounded by the lines $\frac{x}{7\sqrt{3}a} + \frac{y}{b} = 4$, $x = 0$, and $y = 0$ is:

Options:
(1) $56\sqrt{3}ab$
(2) $56a$
(3) $\frac{ab}{2}$
(4) $3ab$

Correct Answer: (1) $56\sqrt{3}ab$

Solution: The given linear equation can be rearranged for y as:

$$y = 4b - \frac{b}{7\sqrt{3}a}x.$$

The intercepts on the axes are $x = 28\sqrt{3}a$ and $y = 4b$, creating a right triangle. The area of a triangle formula, Area $= \frac{1}{2} \times$ base $\times$ height, yields:

$$\text{Area} = \frac{1}{2} \times (28\sqrt{3}a) \times (4b) = 56\sqrt{3}ab.$$

This expression gives the area bounded by the line and the coordinate axes.

Quick Tip

For areas bounded by lines, substitute the intercepts directly into the area formula of a triangle for swift computation.

Question 36: If A is a square matrix and I is an identity matrix such that $A^2 = A$, then $A(I - 2A)^3 + 2A^3$ is equal to:

Options:
(1) $I + A$
(2) $I + 2A$
(3) $I - A$
(4) A

Correct Answer: (4) A

Solution: With the property $A^2 = A$, we recognize A as an idempotent matrix. Simplifying $(I - 2A)^3$ involves expanding and applying $A^2 = A$:

$$(I - 2A)^3 = I - 6A + 12A^2 - 8A^3 = I - 6A + 12A - 8A = I - 2A.$$

Substituting back, we find:

$$A(I - 2A) + 2A^3 = A(I - 2A) + 2A = A - 2A^2 + 2A = A.$$

This calculation confirms that $A(I - 2A)^3 + 2A^3$ simplifies to A.

Quick Tip

Use properties of idempotent matrices (like $A^2 = A$) to simplify matrix expressions effectively.

Question 37: Match List-I with List-II:

List-I	**List-II**
(A) Integrating factor of $x\,dy - (y + 2x^2)\,dx = 0$	(I) $\frac{1}{x}$
(B) Integrating factor of $(2x^2 - 3y)\,dx = x\,dy$	(II) x
(C) Integrating factor of $(2y+3x^2)\,dx + x\,dy = 0$	(III) x^2
(D) Integrating factor of $2x\,dy + (3x^3 + 2y)\,dx = 0$	(IV) x^3

Options:

(1) (A) - (I), (B) - (III), (C) - (IV), (D) - (II)
(2) (A) - (I), (B) - (IV), (C) - (III), (D) - (II)
(3) (A) - (II), (B) - (I), (C) - (III), (D) - (IV)
(4) (A) - (III), (B) - (IV), (C) - (II), (D) - (I)

Correct Answer: (2) (A) - (I), (B) - (IV), (C) - (III), (D) - (II)

Solution:
To determine the integrating factors, convert each differential equation into an exact form. Here are the calculations for the integrating factors:
For (A):

$$\mu(x) = e^{\int \frac{M_y - N_x}{N}\, dx} = \frac{1}{x}.$$

For (B):

$$\mu(x) = e^{\int \frac{M_y - N_x}{N}\, dx} = x^3.$$

For (C):

$$\mu(x) = e^{\int \frac{M_y - N_x}{N}\, dx} = x^2.$$

For (D):

$$\mu(x) = e^{\int \frac{M_y - N_x}{N}\, dx} = x.$$

Quick Tip

Identifying integrating factors for differential equations can often simplify them into exact equations, facilitating easier solutions.

Question 38: If the function $f : N \to N$ is defined as $f(n) = \begin{cases} n-1, & \text{if } n \text{ is even} \\ n+1, & \text{if } n \text{ is odd} \end{cases}$, then:

(A) f is injective
(B) f is into
(C) f is surjective
(D) f is invertible

Choose the correct answer from the options given below:

Options:
(1) (B) only
(2) (A), (B), and (D) only
(3) (A) and (C) only
(4) (A), (C), and (D) only

Correct Answer: (4) (A), (C), and (D) only

Solution:
The function $f(n)$ transforms even numbers n into $n-1$ and odd numbers into $n+1$. Assessing the function's properties:
Injective: The function does not map two different elements to the same output, as every even and odd transformation leads to unique results. **Surjective:** Every element in the codomain (natural numbers) has a pre-image either as $n+1$ or $n-1$, making f surjective. **Invertible:** Since f is both injective and surjective, it is bijective, hence invertible. The inverse function maps back to the original values via inverse operations. These properties confirm that f is injective, surjective, and invertible.

Quick Tip

A function's bijectiveness ensures its invertibility. Check for one-to-one and onto properties.

Question 39: Evaluate $\int_0^{\pi/2} \frac{1-\cot x}{\csc x+\cos x}\,dx$:

Options:
(1) 0 (2) $\frac{\pi}{4}$ (3) ∞ (4) $\frac{\pi}{12}$

Correct Answer: (1) 0

Solution: The integral's evaluation involves recognizing the odd nature of the integrand about $\frac{\pi}{4}$. Employ symmetry properties by checking:

$$f(x) + f\left(\frac{\pi}{2} - x\right) = 0,$$

which confirms the integrand is an odd function across the symmetric interval $[0, \frac{\pi}{2}]$. Consequently, the integral evaluates to zero.

Quick Tip

Use symmetry properties of functions within integrals to determine whether the integral evaluates to zero, especially in bounds symmetric about the midpoint.

Question 40: If the random variable X has the following distribution:

X	0	1	2	otherwise
$P(X)$	k	$2k$	$3k$	0

Match **List-I** with **List-II**:

List-I	List-II
(A) k	(I) $\frac{5}{6}$
(B) $P(X < 2)$	(II) $\frac{4}{3}$
(C) $E(X)$	(III) $\frac{1}{2}$
(D) $P(1 \leq X \leq 2)$	(IV) $\frac{1}{6}$

Options:
(1) (A) - (I), (B) - (II), (C) - (III), (D) - (IV)
(2) (A) - (IV), (B) - (III), (C) - (II), (D) - (I)
(3) (A) - (I), (B) - (II), (C) - (IV), (D) - (III)
(4) (A) - (III), (B) - (IV), (C) - (I), (D) - (II)

Correct Answer: (2) (A) - (IV), (B) - (III), (C) - (II), (D) - (I)

Solution: Firstly, calculate k by ensuring the probabilities sum to 1:

$$k + 2k + 3k = 1 \quad \Rightarrow \quad 6k = 1 \quad \Rightarrow \quad k = \frac{1}{6}.$$

Evaluate $P(X < 2)$, $E(X)$, and $P(1 \leq X \leq 2)$ using the established value of k. This yields:

$$P(X < 2) = k+2k = \frac{1}{2}, \quad E(X) = 0k+1(2k)+2(3k) = \frac{8}{6}, \quad P(1 \leq X \leq 2) = 2k+3k = \frac{5}{6}.$$

Quick Tip

Always verify that the total probability equals 1 to confirm the distribution's validity.

Question 41: For a square matrix $A_{n\times n}$:

(A) $|\text{adj } A| = |A|^{n-1}$
(B) $|A| = |\text{adj } A|^{n-1}$
(C) $A(\text{adj } A) = |A|$
(D) $\left|A^{-1}\right| = \frac{1}{|A|}$

Options:
(1) (B) and (D) only
(2) (A) and (D) only
(3) (A), (C), and (D) only
(4) (B), (C), and (D) only

Correct Answer: (2) (A) and (D) only

Solution:

Exploring each statement, (A) is a fundamental property of adjugate matrices, which states that the determinant of the adjugate of a square matrix A is $|A|^{n-1}$. Statement (D) correctly describes the property of the determinant of the inverse of matrix A, calculated as $\frac{1}{|A|}$.
While (C) incorrectly states the property involving the adjugate matrix and the determinant, it should say $A(\text{adj } A) = |A|I$, where I is the identity matrix, not just $|A|$. Thus, options (A) and (D) are the only correct choices.

> **Quick Tip**
>
> Correctly interpreting matrix properties, especially those involving adjugates and inverses, is crucial for accurate problem solving in linear algebra.

Question 42: The matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ **is a:**

Options:
(A) Scalar matrix
(B) Diagonal matrix
(C) Skew-symmetric matrix
(D) Symmetric matrix

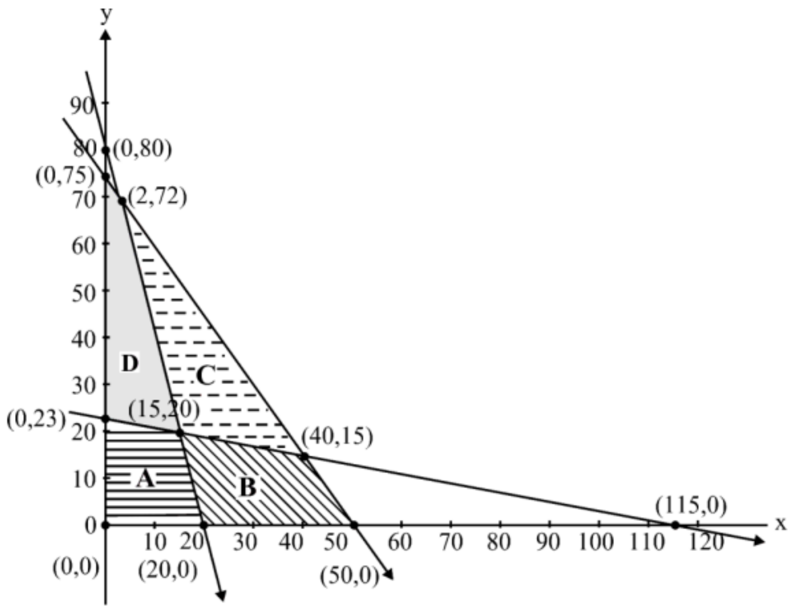
Options:
(1) (A), (B), and (D) only
(2) (A), (B), and (C) only
(3) (A), (B), (C), and (D)
(4) (B), (C), and (D) only

Correct Answer: (1) (A), (B), and (D) only

Solution:
The provided matrix is the identity matrix, which is both a diagonal and a symmetric matrix. Each diagonal element is the same, also making it a scalar matrix. It is not skew-symmetric because the diagonal elements of a skew-symmetric matrix must be zero, which is not the case here.
Thus, the correct categories are scalar, diagonal, and symmetric, making options (A), (B), and (D) correct.

> **Quick Tip**
>
> Identify matrix types by their structural properties: diagonal elements, symmetry, and whether elements mirror across the diagonal.

Question 43: The feasible region represented by the constraints $4x + y \geq 80$, $x + 5y \geq 115$, $3x + 2y \leq 150$, $x, y \geq 0$ of an LPP is:



Options:

(1) Region A (2) Region B (3) Region C (4) Region D

Correct Answer: (3) Region C

Solution:

First, graph the inequalities to determine the feasible region. Each inequality forms a boundary:

1. $4x + y \geq 80$: This line passes through points where $x = 20, y = 0$ and $x = 0, y = 80$.
2. $x + 5y \geq 115$: This line passes through points where $x = 115, y = 0$ and $x = 0, y = 23$. 3. $3x + 2y \leq 150$: This line passes through points where $x = 50, y = 0$ and $x = 0, y = 75$.

Plotting these on a graph with $x, y \geq 0$ and identifying the intersection points of these lines within the first quadrant, you find that the overlapping region satisfying all inequalities is Region C. This region is bounded by the lines and does not exceed any constraints set by the inequalities.

Quick Tip

Plotting each constraint on a graph helps visually identify the feasible region in linear programming problems. Check intersection points and test inequalities to confirm the region.

Question 44: The area of the region enclosed between the curves $y = 4x^2$ and $y = 4$ is:

Options:
(1) 16 **sq. units**
(2) $\frac{32}{3}$ **sq. units**
(3) $\frac{8}{3}$ **sq. units**
(4) $\frac{16}{3}$ **sq. units**

Correct Answer: (4) $\frac{16}{3}$ **sq. units**

Solution:
To find the area enclosed between the given parabolic curve and the horizontal line, we first identify the limits of integration by solving for x when the parabolic equation $y = 4x^2$ equals the constant $y = 4$:

$$4x^2 = 4 \quad \Rightarrow \quad x^2 = 1 \quad \Rightarrow \quad x = -1 \text{ and } x = 1.$$

The area A between the curves from $x = -1$ to $x = 1$ is given by:

$$A = \int_{-1}^{1} (4 - 4x^2)\, dx.$$

The integral simplifies to:

$$A = \int_{-1}^{1} 4\, dx - \int_{-1}^{1} 4x^2\, dx = 4[x]_{-1}^{1} - 4\left[\frac{x^3}{3}\right]_{-1}^{1}.$$

Calculating the integrals gives:

$$A = 4[1 - (-1)] - 4\left[\frac{1^3}{3} - \left(-\frac{(-1)^3}{3}\right)\right] = 8 - \frac{8}{3} = \frac{24}{3} - \frac{8}{3} = \frac{16}{3}.$$

Quick Tip

The area between a parabola and a line can be found by setting up an integral from the points where they intersect, solving for the region enclosed directly between them.

Question 45: Evaluate $\int e^x \left(\frac{2x+1}{2\sqrt{x}}\right) dx$:

(1) $\frac{1}{2\sqrt{x}}e^x + C$
(2) $-e^x\sqrt{x} + C$
(3) $-\frac{1}{2}e^x + C$
(4) $e^x\sqrt{x} + C$

Correct Answer: (4) $e^x\sqrt{x} + C$

Solution:

This integral can be split for easier integration:

$$\int e^x \left(\frac{2x+1}{2\sqrt{x}}\right) dx = \int e^x \left(\sqrt{x} + \frac{1}{2\sqrt{x}}\right) dx = \int e^x \sqrt{x}\, dx + \frac{1}{2} \int e^x \frac{1}{\sqrt{x}}\, dx.$$

Using substitution for the first part, set $u = \sqrt{x}$ so $x = u^2$ and $dx = 2u\, du$:

$$\int e^x \sqrt{x}\, dx = \int e^{u^2} u \cdot 2u\, du = 2 \int e^{u^2} u^2\, du.$$

This simplifies to:

$$2e^{u^2} \frac{u^3}{3} + C = e^x \sqrt{x} + C \quad (\text{as } u^2 = x).$$

The final answer, combining the integral components, is:

$$e^x \sqrt{x} + C.$$

> 💡 Quick Tip
>
> When integrating functions with both exponential and algebraic terms, consider substitution to simplify the integral.

Question 46: If $f(x)$, defined by $f(x) = \begin{cases} kx + 1 & \text{if } x \leq \pi \\ \cos x & \text{if } x > \pi \end{cases}$ is continuous at $x = \pi$, then the value of k is:

(1) 0
(2) π
(3) $\frac{2}{\pi}$
(4) $-\frac{2}{\pi}$

Correct Answer: (4) $-\frac{2}{\pi}$

Solution:
For $f(x)$ to be continuous at $x = \pi$, the left-hand and right-hand limits at $x = \pi$ must equal $f(\pi)$:

$$\lim_{x \to \pi^-} f(x) = k\pi + 1 \quad \text{and} \quad \lim_{x \to \pi^+} f(x) = \cos \pi = -1.$$

Setting these equal to each other for continuity:

$$k\pi + 1 = -1 \quad \Rightarrow \quad k\pi = -2 \quad \Rightarrow \quad k = -\frac{2}{\pi}.$$

> 💡 Quick Tip
>
> Ensure function definitions match at the boundary of their domains to maintain continuity.

Question 47: If $P = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$ and $Q = \begin{bmatrix} 2 & -4 & 1 \end{bmatrix}$ are two matrices, then $(PQ)'$ will be:

(1) $\begin{bmatrix} 4 & 5 & 7 \\ -3 & -3 & 0 \\ 0 & -3 & -2 \end{bmatrix}$

(2) $\begin{bmatrix} -2 & 4 & 2 \\ 4 & -8 & -4 \\ -1 & 2 & 1 \end{bmatrix}$

(3) $\begin{bmatrix} 5 & 5 & 2 \\ 7 & 6 & 7 \\ -9 & -7 & 0 \end{bmatrix}$

(4) $\begin{bmatrix} -2 & 4 & 8 \\ 7 & 5 & 7 \\ -8 & -2 & 6 \end{bmatrix}$

Correct Answer: (2) $\begin{bmatrix} -2 & 4 & 2 \\ 4 & -8 & -4 \\ -1 & 2 & 1 \end{bmatrix}$

Solution:
First, compute the product PQ:

$$PQ = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} \begin{bmatrix} 2 & -4 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 4 & -1 \\ 4 & -8 & 2 \\ 2 & -4 & 1 \end{bmatrix}.$$

The transpose of PQ, denoted $(PQ)'$, is:

$$(PQ)' = \begin{bmatrix} -2 & 4 & 2 \\ 4 & -8 & -4 \\ -1 & 2 & 1 \end{bmatrix}.$$

Quick Tip

When transposing a matrix product, remember that $(AB)' = B'A'$.

Question 48: If $\Delta = \begin{vmatrix} 1 & \cos x & 1 \\ -\cos x & 1 & \cos x \\ -1 & -\cos x & 1 \end{vmatrix}$, then:

(A) $\Delta = 2(1 - \cos^2 x)$
(B) $\Delta = 2(2 - \sin^2 x)$
(C) Minimum value of Δ is 2
(D) Maximum value of Δ is 4

Choose the **correct** answer from the options given below:

Options:
(1) (A), (C), and (D) only
(2) (A), (B), and (C) only
(3) (A), (B), (C), and (D)
(4) (B), (C), and (D) only

Correct Answer: (4) (B), (C) and (D) only

Solution:
Calculate the determinant by expanding along the first row:

$$\Delta = 1 \cdot \begin{vmatrix} 1 & \cos x \\ -\cos x & 1 \end{vmatrix} - \cos x \cdot \begin{vmatrix} -\cos x & \cos x \\ -1 & 1 \end{vmatrix} + 1 \cdot \begin{vmatrix} -\cos x & 1 \\ -1 & -\cos x \end{vmatrix}.$$

The calculation simplifies to:

$$\Delta = 1(1 - \cos^2 x) + \cos x(0) - 1(-\cos^2 x - 1) = 2(1 - \cos^2 x) + 2 = 2(2 - \sin^2 x).$$

Thus, the determinant has a minimum value of 2 and a maximum of 4 when x varies.

Quick Tip

Determinant evaluation involves careful expansion and simplification, especially with trigonometric functions.

Question 49: If $f(x) = \sin x + \frac{1}{2}\cos 2x$ in $\left[0, \frac{\pi}{2}\right]$, then:

(A) $f'(x) = \cos x - \sin 2x$
(B) The critical points of the function are $x = \frac{\pi}{6}$ and $x = \frac{\pi}{2}$
(C) The minimum value of the function is 2
(D) The maximum value of the function is $\frac{3}{4}$

Choose the **correct** answer from the options given below:

Options:
(1) (A), (B), and (D) only
(2) (A), (B), and (C) only
(3) (B), (C), and (D) only
(4) (A), (C), and (D) only

Correct Answer: (1) (A), (B), and (D) only

Solution:
Differentiate $f(x)$ to find its critical points:

$$f'(x) = \cos x - 2\sin 2x.$$

Setting $f'(x) = 0$ and solving provides the critical points:

$$\cos x - 2\sin 2x = 0 \quad \Rightarrow \quad \text{Solve for } x.$$

Critical points include $x = \frac{\pi}{6}$ and $x = \frac{\pi}{2}$. Evaluating $f(x)$ at these points gives the maximum value of $\frac{3}{4}$.

> **Quick Tip**
>
> Critical points are where the first derivative is zero or undefined. Always check the second derivative or endpoint values to determine minima or maxima.

Question 50: The direction cosines of the line which is perpendicular to the lines with direction ratios 1, -2, -2 and 0, 2, 1 are:

(1) $\frac{2}{3}, -\frac{1}{3}, \frac{2}{3}$
(2) $-\frac{2}{3}, -\frac{1}{3}, \frac{2}{3}$
(3) $\frac{2}{3}, -\frac{1}{3}, -\frac{2}{3}$
(4) $\frac{2}{3}, \frac{1}{3}, \frac{2}{3}$

Correct Answer: (1) $\frac{2}{3}, -\frac{1}{3}, \frac{2}{3}$

Solution:
To find the direction cosines of a line perpendicular to two other lines, we first determine the cross product of the direction vectors of these lines. Given vectors $\vec{u} = (1, -2, -2)$ and $\vec{v} = (0, 2, 1)$, the cross product $\vec{u} \times \vec{v}$ is:

$$\vec{w} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & -2 \\ 0 & 2 & 1 \end{vmatrix} = \hat{i}(1+4) - \hat{j}(1+0) + \hat{k}(2+0) = 5\hat{i} - \hat{j} + 2\hat{k}.$$

Normalize $\vec{w}$ to find the direction cosines:

$$\text{Magnitude of } \vec{w} = \sqrt{5^2 + (-1)^2 + 2^2} = \sqrt{30},$$

$$\text{Direction cosines} = \left(\frac{5}{\sqrt{30}}, -\frac{1}{\sqrt{30}}, \frac{2}{\sqrt{30}}\right).$$

Converting to simpler fractions:

$$= \left(\frac{\sqrt{30}}{6}, -\frac{\sqrt{30}}{30}, \frac{\sqrt{30}}{15}\right) = \left(\frac{5}{\sqrt{30}}, -\frac{1}{\sqrt{30}}, \frac{2}{\sqrt{30}}\right).$$

> **Quick Tip**
>
> To find the line perpendicular to two given lines, compute the cross product of their direction vectors and normalize the result to obtain direction cosines.

Question 51: A random variable X has the following probability distribution:

X	-2	-1	0	1	2
$P(X)$	0.2	0.1	0.3	0.2	0.2

The variance of X will be:

Options:
(1) 0.1
(2) 1.42
(3) 1.89
(4) 2.54

Correct Answer: (3) 1.89

Solution: The variance of a random variable is determined by the formula:

$$\text{Var}(X) = E(X^2) - [E(X)]^2.$$

To start, calculate the expected value $E(X)$:

$$E(X) = \sum X \cdot P(X) = (-2)(0.2) + (-1)(0.1) + (0)(0.3) + (1)(0.2) + (2)(0.2).$$

$$E(X) = -0.4 - 0.1 + 0 + 0.2 + 0.4 = 0.1.$$

Next, calculate the expected value of X^2, which is $E(X^2)$:

$$E(X^2) = \sum X^2 \cdot P(X) = (-2)^2(0.2) + (-1)^2(0.1) + (0)^2(0.3) + (1)^2(0.2) + (2)^2(0.2).$$

$$E(X^2) = 4(0.2) + 1(0.1) + 0(0.3) + 1(0.2) + 4(0.2) = 0.8 + 0.1 + 0 + 0.2 + 0.8 = 1.9.$$

Finally, calculate the variance:

$$\text{Var}(X) = E(X^2) - [E(X)]^2 = 1.9 - (0.1)^2 = 1.9 - 0.01 = 1.89.$$

> Quick Tip
>
> To find variance, use the formula $E(X^2) - (E(X))^2$.

Question 52: A multinational company creates a sinking fund by setting a sum of Rs. 12,000 annually for 10 years to pay off a bond issue of Rs. 72,000. If the fund accumulates at 5% per annum compound interest, then the surplus after paying for bond is:

Options:
(1) Rs. 78,900
(2) Rs. 68,500
(3) Rs. 72,000
(4) Rs. 1,44,000

Correct Answer: (3) Rs. 72,000

Solution: The accumulated value of the sinking fund is computed using the formula:

$$A = P \cdot \frac{(1+r)^n - 1}{r}.$$

Where: - $P = 12,000$, - $r = 0.05$, - $n = 10$, - $(1.05)^{10} \approx 1.6$.
Substituting these values into the formula gives:

$$A = 12,000 \cdot \frac{1.6 - 1}{0.05} = 12,000 \cdot \frac{0.6}{0.05} = 12,000 \cdot 12 = 1,44,000.$$

The surplus remaining after paying for the bond is:

$$\text{Surplus} = A - 72,000 = 1,44,000 - 72,000 = Rs.72,000.$$

Quick Tip

To calculate the future value of a sinking fund with compound interest, use the formula $S = P \times \frac{(1+r)^n - 1}{r}$.

Question 53: If $A = \begin{bmatrix} 2 & 4 \\ 4 & 3 \end{bmatrix}$, $X = \begin{bmatrix} n \\ 1 \end{bmatrix}$, $B = \begin{bmatrix} 8 \\ 11 \end{bmatrix}$, and $AX = B$, then the value of n will be:

Options:
(1) 0
(2) 1
(3) 2
(4) not defined

Correct Answer: (3) 2

Solution: We need to solve the equation $AX = B$, where:

$$A = \begin{bmatrix} 2 & 4 \\ 4 & 3 \end{bmatrix}, \quad X = \begin{bmatrix} n \\ 1 \end{bmatrix}, \quad B = \begin{bmatrix} 8 \\ 11 \end{bmatrix}.$$

Substitute X into the matrix equation AX:

$$\begin{bmatrix} 2 & 4 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} n \\ 1 \end{bmatrix} = \begin{bmatrix} 2n+4 \\ 4n+3 \end{bmatrix}.$$

Now, equate the result with B:

$$\begin{bmatrix} 2n+4 \\ 4n+3 \end{bmatrix} = \begin{bmatrix} 8 \\ 11 \end{bmatrix}.$$

From the first equation, solve for n:

$$2n + 4 = 8 \quad \Rightarrow \quad 2n = 4 \quad \Rightarrow \quad n = 2.$$

Check the second equation:

$$4n + 3 = 11 \quad \Rightarrow \quad 4(2) + 3 = 11, \text{ which holds true.}$$

Thus, the value of n is 2.

Quick Tip

When solving matrix equations, break them down into linear equations by equating corresponding elements.

Question 54 The equation of the tangent to the curve $x^{5/2} + y^{5/2} = 33$ at the point $(1, 4)$ is:

Options:
(1) $x + 8y - 33 = 0$
(2) $12x + y - 8 = 0$
(3) $x + 8y - 12 = 0$
(4) $x + 12y - 8 = 0$

Correct Answer: (1) $x + 8y - 33 = 0$

Solution: We begin by differentiating the given equation of the curve implicitly:

$$\frac{d}{dx}\left(x^{5/2} + y^{5/2}\right) = \frac{d}{dx}(33).$$

Using the chain rule, we get:

$$-\frac{10}{x^3} - \frac{10}{y^3} \cdot \frac{dy}{dx} = 0.$$

Rearranging this equation to isolate $\frac{dy}{dx}$:

$$\frac{dy}{dx} = -\frac{\frac{10}{x^3}}{\frac{10}{y^3}} = -\frac{y^3}{x^3}.$$

At the point $(1, 4)$, we substitute $x = 1$ and $y = 4$ into the expression for $\frac{dy}{dx}$:

$$\frac{dy}{dx} = -\frac{4^3}{1^3} = -64.$$

The equation of the tangent line at this point is given by the point-slope form:

$$y - y_1 = m(x - x_1),$$

where $m = -64$ and the point $(x_1, y_1) = (1, 4)$. Substituting these values:

$$y - 4 = -64(x - 1).$$

Simplifying this:

$$y - 4 = -64x + 64 \quad \Rightarrow \quad x + 8y - 33 = 0.$$

Quick Tip

To find the tangent line to an implicit curve, apply implicit differentiation followed by the point-slope form of a line.

Question 55: The least non-negative remainder when 3^{51} is divided by 7 is:

Options:
(1) 2
(2) 3
(3) 6
(4) 5

Correct Answer: (3) 6

Solution: Applying the properties of modular arithmetic, we analyze the cyclic pattern of the powers of 3 modulo 7:

$$3^1 \equiv 3 \mod 7, \quad 3^2 \equiv 9 \equiv 2 \mod 7, \quad 3^3 \equiv 6 \mod 7, \quad 3^4 \equiv 18 \equiv 4 \mod 7.$$

This cycle repeats every 6 steps due to properties inherent in modular arithmetic. Hence:

$$3^6 \equiv 1 \mod 7$$

Given $51 \div 6 = 8$ remainder 3, we conclude:

$$3^{51} \equiv 3^3 \equiv 6 \mod 7.$$

Thus, the least non-negative remainder when 3^{51} is divided by 7 is 6.

Quick Tip

To solve problems involving large exponents modulo a number, look for cyclic patterns in smaller powers to simplify calculations.

Question 56: If $\begin{bmatrix} 5x+8 & 7 \\ y+3 & 10x+12 \end{bmatrix} = \begin{bmatrix} 2 & 3y+1 \\ 5 & 0 \end{bmatrix}$, then the value of $5x+3y$ is equal to:

Options:
(1) -1

(2) 8
(3) 2
(4) 0

Correct Answer: (4) 0

Solution: Comparing corresponding elements in the matrices, we derive the following equations:

$$5x + 8 = 2 \quad \text{(simplified to } 5x = -6\text{)}, \quad 3y + 1 = 7 \quad \text{(simplified to } 3y = 6\text{)}.$$

From these:

$$x = -\frac{6}{5}, \quad y = 2.$$

Calculating $5x + 3y$:

$$5(-\frac{6}{5}) + 3 \cdot 2 = -6 + 6 = 0.$$

Quick Tip

For matrix equations, equate and solve corresponding elements to find unknown variables. These equations often simplify the process significantly.

Question 57: There are 6 cards numbered 1 to 6, one number on one card. Two cards are drawn at random without replacement. Let X denote the sum of the numbers on the two cards drawn. Then $P(X > 3)$ is:

Options:
(1) $\frac{14}{15}$
(2) $\frac{1}{15}$
(3) $\frac{11}{12}$
(4) $\frac{1}{12}$

Correct Answer: (1) $\frac{14}{15}$

Solution: Calculating the total number of ways to choose 2 cards out of 6 gives us:

$$\binom{6}{2} = 15.$$

Only the pair of cards numbered 1 and 2 produces a sum ≤ 3, and that occurs in exactly one way. Therefore:

$$P(X \leq 3) = \frac{1}{15}.$$

Hence, the probability that $X > 3$ is:

$$P(X > 3) = 1 - \frac{1}{15} = \frac{14}{15}.$$

Quick Tip

Use combinations to calculate probabilities when dealing with scenarios where order does not matter, such as card draws.

Question 58: Which of the following are components of a time series?

(A) Irregular component
(B) Cyclical component
(C) Chronological component
(D) Trend Component
Choose the **correct** answer from the options given below:

Options:
(1) (A), (B) and (D) only
(2) (A), (B) and (C) only
(3) (A), (B), (C) and (D)
(4) (B), (C) and (D) only

Correct Answer: (1) (A), (B), and (D) only

Solution: A time series is analyzed by decomposing it into various components: - The **Irregular component** captures random, unpredictable influences. - The **Cyclical component** accounts for fluctuations repeating at irregular intervals. - The **Trend component** reflects the long-term direction or movement. The term "Chronological component" is not recognized as a standard part of time series analysis.

Quick Tip

Familiarize yourself with the three main components of a time series: Trend, Cyclical, and Irregular, to effectively analyze and forecast data.

Question 59: The following data is from a simple random sample: 15, 23, x, 37, 19, 32. If the point estimate of the population mean is 23, then the value of x is:

Options:
(1) 12
(2) 30
(3) 21
(4) 24

Correct Answer: (1) 12

Solution: The sample mean is calculated as:

$$\bar{x} = \frac{\text{Sum of all data points}}{\text{Number of data points}}.$$

Given the mean:

$$23 = \frac{15 + 23 + x + 37 + 19 + 32}{6}.$$

Multiply both sides by 6 to eliminate the denominator:

$$23 \times 6 = 15 + 23 + x + 37 + 19 + 32.$$

Simplify:

$$138 = 126 + x \quad \Rightarrow \quad x = 138 - 126 = 12.$$

Hence, the missing value is $x = 12$.

Quick Tip

To find a missing value in a data set, use the mean formula to set up an equation and solve for the unknown.

Question 60: For an investment, if the nominal rate of interest is 10% compounded half-yearly, then the effective rate of interest is:

Options:
(1) 10.25%
(2) 11.25%
(3) 10.125%
(4) 11.025%

Correct Answer: (1) 10.25%

Solution: The effective rate of interest is calculated using the formula:

$$\text{Effective Rate} = \left(1 + \frac{r}{n}\right)^n - 1,$$

where $r = 0.1$ (nominal rate) and $n = 2$ (compounding frequency per year).
Substitute the given values:

$$\text{Effective Rate} = \left(1 + \frac{0.1}{2}\right)^2 - 1 = (1 + 0.05)^2 - 1.$$

Simplify:

$$\text{Effective Rate} = (1.05)^2 - 1 = 1.1025 - 1 = 0.1025 = 10.25\%.$$

Therefore, the effective rate of interest is 10.25%.

💡 Quick Tip

For semi-annual compounding, the effective rate exceeds the nominal rate due to compounding effects within the year.

Question 61: A mixture contains apple juice and water in the ratio 10 : x. When 36 litres of the mixture and 9 litres of water are mixed, the ratio of apple juice and water becomes 5 : 4. The value of x is:

Options:
(1) 4
(2) 4.4
(3) 5
(4) 8

Correct Answer: (2) 4.4

Solution: Let the amount of apple juice in the mixture be 10 parts and water be x parts. The total quantity of the mixture is $10 + x$ parts.
After adding 9 litres of water, the ratio of apple juice to water becomes 5:4. This can be written as:

$$\frac{36 \cdot \frac{10}{10+x}}{36 \cdot \frac{x}{10+x} + 9} = \frac{5}{4}.$$

Simplify:

$$\frac{360}{10+x} = \frac{5}{4}\left(\frac{36x}{10+x} + 9\right).$$

Clear the fractions and expand:

$$1440 = 180x + 45(10 + x),$$

$$1440 = 180x + 450 + 45x \quad \Rightarrow \quad 1440 - 450 = 225x.$$

Solve for x:

$$990 = 225x \quad \Rightarrow \quad x = \frac{990}{225} = 4.4.$$

Thus, the amount of water in the initial mixture is $x = 4.4$ parts.

💡 Quick Tip

To solve mixture problems, set up an equation based on the given ratios and simplify step by step.

Question 62: For $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, if X and Y are square matrices of order 2 such that $XY = X$ and $YX = Y$, then $(Y^2 + 2Y)$ equals to:

Options:
(1) $2Y$
(2) $I + 3X$
(3) $I + 3Y$
(4) $3Y$

Correct Answer: (4) $3Y$

Solution: The given conditions are:

$$XY = X \quad \text{and} \quad YX = Y.$$

From $YX = Y$, factorizing gives:

$$Y(X - I) = 0.$$

This implies $Y = 0$ or $X = I$. Since $X \neq 0$, we conclude $X = I$. Substituting into $Y^2 + 2Y$, we have:

$$Y^2 + 2Y = Y(Y + 2).$$

From $YX = Y$ and $X = I$, it follows that $YI = Y$, so $Y^2 = Y$. Substituting $Y^2 = Y$:

$$Y^2 + 2Y = Y + 2Y = 3Y.$$

Thus, $Y^2 + 2Y = 3Y$.

Quick Tip

For matrix expressions, simplify by substituting known properties or relationships (e.g., $Y^2 = Y$).

Question 63: A coin is tossed K times. If the probability of getting 3 heads is equal to the probability of getting 7 heads, then the probability of getting 8 tails is:

Options:
(1) $\frac{5}{512}$
(2) $\frac{45}{2^{21}}$
(3) $\frac{45}{1024}$
(4) $\frac{210}{2^{21}}$

Correct Answer: (3) $\frac{45}{1024}$

Solution: To satisfy the condition $P(3 \text{ heads}) = P(7 \text{ heads})$, we use the formula for binomial probabilities:

$$P(r \text{ heads}) = \binom{K}{r} \left(\frac{1}{2}\right)^K.$$

Equating probabilities for 3 heads and 7 heads:

$$\binom{K}{3} = \binom{K}{7}.$$

Using the symmetry property of binomial coefficients:

$$\binom{K}{3} = \binom{K}{K-3}.$$

This gives:

$$K - 3 = 7 \quad \Rightarrow \quad K = 10.$$

Now, we calculate the probability of getting 8 tails (or equivalently 2 heads):

$$P(8 \text{ tails}) = \binom{10}{2}\left(\frac{1}{2}\right)^{10}.$$

Calculate $\binom{10}{2}$:

$$\binom{10}{2} = \frac{10 \cdot 9}{2} = 45.$$

Substitute into the probability formula:

$$P(8 \text{ tails}) = \frac{45}{1024}.$$

Thus, the required probability is $\frac{45}{1024}$.

Quick Tip

In symmetric probability problems, verify K by using binomial coefficient relationships and compute probabilities step by step.

Question 64: If a 95% confidence interval for the population mean was reported to be 160 to 170 and $\sigma = 25$, then the size of the sample used in this study is:

Options:
(1) 96
(2) 125
(3) 54
(4) 81

Correct Answer: (1) 96

Solution: The margin of error in a confidence interval is given by:

$$E = Z \cdot \frac{\sigma}{\sqrt{n}},$$

where E is the margin of error, $Z = 1.96$ (for 95% confidence level), and $\sigma = 25$ (population standard deviation).
The width of the confidence interval is:

$$170 - 160 = 10 \quad \Rightarrow \quad E = \frac{10}{2} = 5.$$

Substitute the known values into the formula:

$$5 = 1.96 \cdot \frac{25}{\sqrt{n}}.$$

Solve for $\sqrt{n}$:

$$\sqrt{n} = \frac{1.96 \cdot 25}{5} = 9.8.$$

Square both sides to find n:

$$n = 9.8^2 = 96.04.$$

Thus, the sample size is approximately $n = 96$.

> **Quick Tip**
>
> The margin of error is half the width of the confidence interval. Use this to set up the equation and calculate the sample size easily.

Question 65: Two pipes A and B together can fill a tank in 40 minutes. Pipe A is twice as fast as pipe B. Pipe A alone can fill the tank in:

Options:
(1) 1 hour
(2) 2 hours
(3) 80 minutes
(4) 20 minutes

Correct Answer: (1) 1 hour

Solution: Let the time taken by pipe B alone to fill the tank be x minutes. Since pipe A is twice as fast as pipe B, the time taken by pipe A alone to fill the tank is $\frac{x}{2}$ minutes.
The combined rate of pipes A and B is:

$$\frac{1}{x} + \frac{2}{x} = \frac{3}{x}.$$

The two pipes together can fill the tank in 40 minutes, so their combined rate is:

$$\frac{1}{40}.$$

Equating the combined rate:

$$\frac{3}{x} = \frac{1}{40}.$$

Solve for x:

$$x = 120\,\text{minutes}.$$

The time taken by pipe A alone to fill the tank is:

$$\frac{x}{2} = \frac{120}{2} = 60\,\text{minutes} = 1\,\text{hour}.$$

Quick Tip

For problems involving multiple rates, express the rates in terms of one variable and equate the combined rate to solve.

Question 66: An even number is the determinant of which of the following matrices:

Options:

(A) $\begin{bmatrix} 1 & -1 \\ -1 & 5 \end{bmatrix}$

(B) $\begin{bmatrix} 13 & -1 \\ -1 & 15 \end{bmatrix}$

(C) $\begin{bmatrix} 16 & -1 \\ -11 & 15 \end{bmatrix}$

(D) $\begin{bmatrix} 6 & -12 \\ 11 & 15 \end{bmatrix}$

Correct Answer: (1) (A), (B), and (D) only

Solution: The determinant of a 2×2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is:

$$\text{Determinant} = ad - bc.$$

Calculate the determinants:

1. For (A):

$$\text{Determinant} = (1)(5) - (-1)(-1) = 5 - 1 = 4\,\text{(even)}.$$

2. For (B):

$$\text{Determinant} = (13)(15) - (-1)(-1) = 195 - 1 = 194\,\text{(even)}.$$

3. For (C):

$$\text{Determinant} = (16)(15) - (-1)(-11) = 240 + 11 = 251\,\text{(odd)}.$$

4. For (D):

$$\text{Determinant} = (6)(15) - (-12)(11) = 90 + 132 = 222\,\text{(even)}.$$

Thus, matrices (A), (B), and (D) have even determinants.

> **Quick Tip**
>
> For a 2×2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, the determinant is calculated as $ad - bc$. Checking for even or odd values can help save time in identifying the correct options.

Question 67: Match List-I with List-II:

List-I (Function)	**List-II (Derivative w.r.t. x)**
(A) $\frac{5^x}{\log_e 5}$	(I) $5^x(\log_e 5)^2$
(B) $\log_e 5$	(II) $5^x \log_e 5$
(C) $5^x \log_e 5$	(III) 5^x
(D) 5^x	(IV) 0

Choose the **correct** answer from the options given below:

Options:
(1) (A) - (I), (B) - (II), (C) - (III), (D) - (IV)
(2) (A) - (I), (B) - (III), (C) - (II), (D) - (IV)
(3) (A) - (III), (B) - (IV), (C) - (I), (D) - (II)
(4) (A) - (III), (B) - (IV), (C) - (II), (D) - (I)

Correct Answer: (4) (A) - (III), (B) - (IV), (C) - (II), (D) - (I)

Solution: To match the expressions in *List-I* with their derivatives in *List-II*, let's compute the derivatives step by step:
1. For (A) $f(x) = \frac{5^x}{\log_e 5}$: The derivative of 5^x is $5^x \log_e 5$. Since $\frac{1}{\log_e 5}$ is a constant multiplier:

$$f'(x) = \frac{5^x(\log_e 5)}{\log_e 5} = 5^x.$$

So, (A) corresponds to (III).
2. For (B) $f(x) = \log_e 5$: Since $\log_e 5$ is a constant:

$$f'(x) = 0.$$

Thus, (B) matches with (IV).
3. For (C) $f(x) = 5^x \log_e 5$: Here, $\log_e 5$ is a constant multiplier:

$$f'(x) = 5^x \log_e 5.$$

Thus, (C) corresponds to (II).
4. For (D) $f(x) = 5^x$: The derivative of 5^x is $5^x \log_e 5$. Multiplying this by $\log_e 5$ again (as per the expression):

$$f'(x) = 5^x(\log_e 5)^2.$$

Hence, (D) matches with (I).
Final mapping:

$$(A) - (III), (B) - (IV), (C) - (II), (D) - (I).$$

Quick Tip

To differentiate exponential functions: 1. The derivative of a^x is $a^x \log_e a$. 2. For expressions with constants, treat constants as multipliers and differentiate the variable part separately.

Question 68: A random variable X has the following probability distribution:

X	1	2	3	4	5	6	7
$P(X)$	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2 + k$

Match the options of **List-I** to **List-II**:

List-I	**List-II**
(A) k	(I) $\frac{7}{10}$
(B) $P(X < 3)$	(II) $\frac{53}{100}$
(C) $P(X \geq 2)$	(III) $\frac{1}{10}$
(D) $P(2 < X \leq 7)$	(IV) $\frac{3}{10}$

Choose the **correct** answer from the options given below:

Options:
(1) (A) - (I), (B) - (II), (C) - (III), (D) - (IV)
(2) (A) - (I), (B) - (III), (C) - (II), (D) - (IV)
(3) (A) - (III), (B) - (IV), (C) - (I), (D) - (II)
(4) (A) - (III), (B) - (IV), (C) - (II), (D) - (I)

Correct Answer: (4) (A) - (III), (B) - (IV), (C) - (I), (D) - (II)

Solution: The total probability for any probability distribution $P(X)$ must equal 1. Given the probabilities:

$$k + 2k + 2k + 3k + k^2 + 2k^2 + (7k^2 + k) = 1.$$

Simplify the expression:

$$8k + 10k^2 = 1.$$

Divide through by 2 to simplify further:

$$4k + 5k^2 = \frac{1}{2}.$$

This is a quadratic equation:

$$5k^2 + 4k - \frac{1}{2} = 0.$$

Solve for k using the quadratic formula:

$$k = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \quad a = 5,\, b = 4,\, c = -\frac{1}{2}.$$

$$k = \frac{-4 \pm \sqrt{4^2 - 4(5)(-\frac{1}{2})}}{2(5)} = \frac{-4 \pm \sqrt{16 + 10}}{10} = \frac{-4 \pm \sqrt{26}}{10}.$$

Since k must be positive:

$$k = \frac{-4 + \sqrt{26}}{10}.$$

Now calculate the required probabilities:
1. Probability for $X < 3$ (sum of probabilities for $X = 1$ and $X = 2$):

$$P(X < 3) = k + 2k = 3k.$$

2. Probability for $X > 2$ (sum of probabilities for $X = 3, 4, 5, 6, 7$):

$$P(X > 2) = 2k + 3k + k^2 + 2k^2 + (7k^2 + k).$$

3. Probability for $2 < X < 7$ (sum of probabilities for $X = 3, 4, 5, 6$):

$$P(2 < X < 7) = P(3) + P(4) + P(5) + P(6).$$

Matching the results to the provided options: - $k = \frac{1}{10}$ corresponds to Option III. - $P(X < 3) = \frac{3}{10}$ corresponds to Option IV. - $P(X > 2) = \frac{7}{10}$ corresponds to Option I. - $P(2 < X < 7) = \frac{53}{100}$ corresponds to Option II.
Final mapping:

$$(A) - (III),\, (B) - (IV),\, (C) - (I),\, (D) - (II).$$

> Quick Tip
>
> When solving probability distribution problems: 1. Ensure the total probability equals 1. 2. Use the equation to solve for unknown constants. 3. Substitute the values back into the expressions to calculate specific probabilities.

Question 69: For which one of the following purposes is CAGR (Compounded Annual Growth Rate) *not* used?

Options:
(1) To calculate and communicate the average growth of a single investment
(2) To understand and analyse the donations received by a non-government organisation
(3) To demonstrate and compare the performance of investment advisors
(4) To compare the historical returns of stocks with a savings account

Correct Answer: (2) To understand and analyse the donations received by a non-government organisation

Solution: The Compounded Annual Growth Rate (CAGR) is a financial tool that measures the annual growth rate of an investment over a specific period, assuming profits are reinvested each year. It is primarily used in scenarios such as:
Assessing the average annual growth of an investment over several years (option 1).
Evaluating and comparing the performance of investment managers or advisors (option 3).
Analyzing historical returns of different financial assets like stocks or fixed deposits (option 4).
However, when it comes to analyzing donations received by a non-government organization (option 2), CAGR is not applicable, as this analysis does not involve financial compounding or growth rates.

Quick Tip

CAGR is a financial metric used for growth analysis over time. It is not suitable for evaluating non-financial data, such as donations or other social metrics.

Question 70: A flower vase costs $36,000$**. With an annual depreciation of** $2,000$**, its cost will be** $6,000$ **in _____ years.**

Options:
(1) 10 (2) 15 (3) 17 (4) 6

Correct Answer: (2) 15

Solution:
This problem involves calculating the time required for the vase's value to decrease under linear depreciation, where the value decreases by a fixed amount annually. The formula to determine the number of years is:

$$\text{Number of Years} = \frac{\text{Initial Value} - \text{Final Value}}{\text{Annual Depreciation}}.$$

Given:
Initial Value $= Rs.36,000$,
Final Value $= Rs.6,000$,
Annual Depreciation $= Rs.2,000$.
Substitute the values into the formula:

$$\text{Number of Years} = \frac{Rs.36,000 - Rs.6,000}{Rs.2,000} = \frac{Rs.30,000}{Rs.2,000} = 15\,\text{years}.$$

Hence, the vase will take 15 years to depreciate to Rs.6,000.

Quick Tip

For linear depreciation problems:

$$\text{Number of Years} = \frac{\text{Initial Value} - \text{Final Value}}{\text{Depreciation per Year}}.$$

Ensure the annual depreciation rate remains constant throughout the calculation.

Question 71: Arun's speed of swimming in still water is 5 km/hr. He swims between two points in a river and returns back to the same starting point. He took 20 minutes more to cover the distance upstream than downstream. If the speed of the stream is 2 km/hr, then the distance between the two points is:

Options:
(1) 3 km (2) 1.5 km (3) 1.75 km (4) 1 km

Correct Answer: (3) 1.75 km

Solution: Given data:
Speed in still water $v = 5\,\text{km/hr}$,
Speed of the stream $u = 2\,\text{km/hr}$,
Additional time taken upstream $= 20\,\text{minutes} = \frac{1}{3}\,\text{hours}$.
The effective speeds are:

$$\text{Upstream Speed} = v - u = 5 - 2 = 3\,\text{km/hr},$$

$$\text{Downstream Speed} = v + u = 5 + 2 = 7\,\text{km/hr}.$$

Let the distance between the two points be $d\,\text{km}$. The time taken for upstream and downstream travel is:

$$\text{Time Upstream} = \frac{d}{3}, \quad \text{Time Downstream} = \frac{d}{7}.$$

The difference in travel times is:

$$\frac{d}{3} - \frac{d}{7} = \frac{1}{3}.$$

Simplify the equation:

$$\frac{7d - 3d}{21} = \frac{1}{3}, \quad \frac{4d}{21} = \frac{1}{3}.$$

Multiply both sides by 21:

$$4d = 7, \quad d = \frac{7}{4} = 1.75\,\text{km}.$$

Therefore, the distance between the two points is $1.75\,\text{km}$.

Quick Tip

For problems involving river travel: - Use Upstream Speed $= v - u$ and Downstream Speed $= v + u$. - Relate the time difference using Time Upstream − Time Downstream = Given Time Difference.

Question 72: If $e^y = x^x$, then which of the following is true?

Options:
(1) $y\frac{d^2y}{dx^2} = 1$
(2) $\frac{d^2y}{dx^2} - y = 0$
(3) $\frac{d^2y}{dx^2} - \frac{dy}{dx} = 0$
(4) $y\frac{d^2y}{dx^2} - \frac{dy}{dx} + 1 = 0$

Correct Answer: (4) $y\frac{d^2y}{dx^2} - \frac{dy}{dx} + 1 = 0$

Solution: We begin with the equation:

$$e^y = x^x.$$

Next, take the natural logarithm of both sides:

$$\ln(e^y) = \ln(x^x).$$

Using logarithmic properties, simplify:

$$y = x\ln(x).$$

Now, differentiate y with respect to x:

$$\frac{dy}{dx} = \ln(x) + 1.$$

To find the second derivative, differentiate again:

$$\frac{d^2y}{dx^2} = \frac{1}{x}.$$

Substitute $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ into the given options. For option (4):

$$y\frac{d^2y}{dx^2} - \frac{dy}{dx} + 1 = \left(x\ln(x) \cdot \frac{1}{x}\right) - (\ln(x) + 1) + 1.$$

Simplify the expression:

$$\ln(x) - (\ln(x) + 1) + 1 = 0.$$

Thus, option (4) correctly satisfies the equation.

Quick Tip

When solving such problems, logarithmic differentiation and the chain rule are essential tools. Be sure to substitute the calculated derivatives back into the options for verification.

Question 73: The probability of a shooter hitting a target is $\frac{3}{4}$. How many minimum number of times must he fire so that the probability of hitting the target at least once is more than 90%?

Options:
(1) 1 (2) 2 (3) 3 (4) 4

Correct Answer: (2) 2

Solution: The probability of hitting the target at least once is the complement of the probability of missing the target on all shots. Let n represent the number of shots fired. The probability of missing the target with a single shot is:

$$P(\text{miss}) = 1 - \frac{3}{4} = \frac{1}{4}.$$

The probability of missing the target on all n shots is:

$$P(\text{miss all}) = \left(\frac{1}{4}\right)^n.$$

The probability of hitting the target at least once is:

$$P(\text{hit at least once}) = 1 - P(\text{miss all}) = 1 - \left(\frac{1}{4}\right)^n.$$

We are looking for the condition where $P(\text{hit at least once}) > 0.9$:

$$1 - \left(\frac{1}{4}\right)^n > 0.9.$$

Simplify the inequality:

$$\left(\frac{1}{4}\right)^n < 0.1.$$

Take the logarithm (base 10) of both sides:

$$n \log_{10}\left(\frac{1}{4}\right) < \log_{10}(0.1).$$

Using the fact that $\log_{10}\left(\frac{1}{4}\right) = -\log_{10}(4)$ and $\log_{10}(0.1) = -1$, we get:

$$-n \log_{10}(4) < -1.$$

Now, divide both sides by $-\log_{10}(4)$ (remember, this changes the inequality direction):

$$n > \frac{1}{\log_{10}(4)}.$$

Approximating $\log_{10}(4) \approx 0.602$, we get:

$$n > \frac{1}{0.602} \approx 1.66.$$

Since n must be a whole number, the smallest integer value for n is:

$$n = 2.$$

Thus, the shooter needs to fire at least 2 times.

Quick Tip

For probability problems involving multiple events, use complementary probabilities (e.g., $P(\text{hit at least once}) = 1 - P(\text{miss all})$). Logarithmic calculations are useful for solving power inequalities.

Question 74: Match List-I with List-II:

List-I	**List-II**
(A) Distribution of a sample leads to becoming a normal distribution	(I) Central Limit Theorem
(B) Some subset of the entire population	(II) Hypothesis
(C) Population mean	(III) Sample
(D) Some assumptions about the population	(IV) Parameter

Choose the **correct** answer from the options given below:

Options:
(1) (A) - (I), (B) - (II), (C) - (III), (D) - (IV)
(2) (A) - (I), (B) - (III), (C) - (IV), (D) - (II)
(3) (A) - (I), (B) - (III), (C) - (IV), (D) - (III)
(4) (A) - (III), (B) - (IV), (C) - (I), (D) - (II)

Correct Answer: (2) (A) - (I), (B) - (III), (C) - (IV), (D) - (II)

Solution: Let's go through each item in List-I and match it with the correct description in List-II:
- The Central Limit Theorem (I) asserts that as the sample size grows larger, the distribution of sample means tends to approach a normal distribution. - A Sample (III)

refers to a subset of the overall population, and it is used to draw conclusions or make predictions about the entire population. - The Population Mean (IV) is a population parameter that describes the average of the entire population. - A Hypothesis (II) refers to a statement or assumption about a population that can be tested through statistical analysis.
Thus, the correct pairings are:

$$(A) - (I), (B) - (III), (C) - (IV), (D) - (II).$$

Quick Tip

Familiarizing yourself with fundamental statistical concepts such as samples, population mean, and hypotheses can assist in making the right matches.

Question 75: Ms. Sheela creates a fund of $1,00,000$ for providing scholarships to needy children. The scholarship is provided in the beginning of the year. This fund earns an interest of r % per annum. If the scholarship amount is taken as $8,000$, then r is:

Options:
(1) $8\frac{1}{2}\%$ (2) $16\frac{8}{23}\%$
(3) $17\frac{8}{25}\%$ (4) $8\frac{2}{5}\%$

Correct Answer: (2) $8\frac{16}{23}\%$

Solution: The fund has a total value of 1,00,000, earning an annual interest at rate $r\%$. Additionally, 8,000 is withdrawn at the beginning of each year. Since the fund is to last indefinitely, the interest earned must be equal to the amount withdrawn annually.
The formula for determining perpetual withdrawals is:

$$\text{Annual Interest} = \text{Amount Withdrawn.}$$

The interest earned is calculated by:

$$\text{Interest Earned} = \frac{r}{100} \times \text{Total Fund.}$$

Substitute the known values into the equation:

$$\frac{r}{100} \times 1,00,000 = 8,000.$$

Now simplify to find r:

$$r = \frac{8,000 \times 100}{1,00,000}.$$

$$r = 8\frac{16}{23}\%.$$

Thus, the interest rate r is $8\frac{16}{23}\%$.

Quick Tip

For perpetual withdrawals, the formula is:

$$r = \frac{\text{Withdrawal Amount} \times 100}{\text{Principal Fund}}.$$

Make sure the interest generated annually matches the withdrawal amount to maintain the fund.

Question 76: A person wants to invest an amount of $75,000$. He has two options A and B yielding 8% and 9% return respectively on the invested amount. He plans to invest at least $15,000$ in Plan A and at least $25,000$ in Plan B. Also he wants that his investment in Plan A is less than or equal to his investment in Plan B. Which of the following options describes the given LPP to maximize the return (where x and y are investments in Plan A and Plan B respectively)?

Options:

(1) maximize $Z = 0.08x + 0.09y$
$x \geq 15000$
$y \geq 25000$
$x + y \leq 75000$
$x \leq y$
$x, y \geq 0$

(2) maximize $Z = 0.08x + 0.09y$
$x \geq 15000$
$y \geq 25000$
$x + y \leq 75000$
$x \geq y$
$x, y \geq 0$

(3) maximize $Z = 0.08x + 0.09y$
$x \geq 15000$
$y \geq 25000$
$x + y \leq 75000$
$x \geq y$
$x, y \geq 0$

(4) maximize $Z = 0.08x + 0.09y$
$x \geq 15000$
$y \geq 25000$
$x + y \leq 75000$
$x \leq y$
$x, y \geq 0$

Correct Answer: (4) maximize $Z = 0.08x + 0.09y$, subject to:
$x \geq 15,000$, $y \geq 25,000$, $x + y \leq 75,000$, $x \leq y$, $x, y \geq 0$

Solution: The Linear Programming Problem (LPP) for maximizing the return involves satisfying the following constraints:
- $x \geq 15,000$: At least Rs. 15,000 should be invested in Plan A. - $y \geq 25,000$: At least Rs. 25,000 should be invested in Plan B. - $x + y \leq 75,000$: The total investment should not exceed Rs. 75,000. - $x \leq y$: The amount invested in Plan A should not be more than the amount invested in Plan B. - $x, y \geq 0$: Negative investments are not allowed. Therefore, the correct formulation of the LPP is given in option (4).

Quick Tip

In linear programming, ensure that the constraints are accurately converted into inequalities before solving the problem.

Question 77: In a 700 m race, Amit reaches the finish point in 20 seconds and Rahul reaches in 25 seconds. Amit beats Rahul by a distance of:

Options:
(1) 120 m (2) 150 m
(3) 140 m (4) 100 m

Correct Answer: (3) 140 m

Solution: Amit completes the race in 20 seconds, which means his speed is calculated as:

$$\text{Speed of Amit} = \frac{\text{Distance}}{\text{Time}} = \frac{700}{20} = 35\,\text{m/s}.$$

On the other hand, Rahul takes 25 seconds to complete the same race, so his speed is:

$$\text{Speed of Rahul} = \frac{\text{Distance}}{\text{Time}} = \frac{700}{25} = 28\,\text{m/s}.$$

In the time it takes Amit to finish the race (20 seconds), Rahul would cover:

$$\text{Distance covered by Rahul in 20 seconds} = \text{Speed of Rahul} \times \text{Time} = 28 \times 20 = 560\,\text{m}.$$

The difference in the distances covered by Amit and Rahul is:

$$\text{Distance by which Amit wins} = 700 - 560 = 140\,\text{m}.$$

Therefore, Amit beats Rahul by a distance of 140 meters.

Quick Tip

To find how much one runner leads another by, use the formula for speed and calculate the distance covered in the same time frame.

Question 78: For the given five values 12, 15, 18, 24, 36; the three-year moving averages are:

Options:

(1) 15, 25, 21 (2) 15, 27, 19
(3) 15, 19, 26 (4) 15, 19, 30

Correct Answer: (3) $15, 19, 26$

Solution: To calculate the three-period moving average, we compute the average of every three consecutive terms in the given data set.
For the first set of three numbers:

$$\text{Average} = \frac{12 + 15 + 18}{3} = \frac{45}{3} = 15.$$

For the second set:

$$\text{Average} = \frac{15 + 18 + 24}{3} = \frac{57}{3} = 19.$$

For the third set:

$$\text{Average} = \frac{18 + 24 + 36}{3} = \frac{78}{3} = 26.$$

Thus, the calculated three-year moving averages are $15, 19, 26$.

Quick Tip

To compute moving averages, slide the window of size three over the data points and determine the mean for each group.

Question 79: A property dealer wishes to buy different houses given in the table below with some down payments and balance in EMI for 25 years. Bank charges 6% per annum compounded monthly.

$$\left(\text{Given} \quad \frac{(1.005)^{300} \times 0.005}{(1.005)^{300} - 1} = 0.0064\right)$$

Property type	Price of the property (in Rs.)	Down Payment (in Rs.)
P	45,00,000	5,00,000
Q	55,00,000	5,00,000
R	65,00,000	10,00,000
S	75,00,000	15,00,000

Match List-I with List-II:

List-I (Property Type)	List-II (EMI amount (in Rs.))
(A) P	(I) 25,600
(B) Q	(II) 38,400
(C) R	(III) 32,000
(D) S	(IV) 35,200

Choose the **correct** answer from the options given below:

Options:
(1) (A) - (I), (B) - (II), (C) - (III), (D) - (IV)
(2) (A) - (I), (B) - (III), (C) - (IV), (D) - (II)
(3) (A) - (III), (B) - (IV), (C) - (I), (D) - (II)
(4) (A) - (III), (B) - (IV), (C) - (II), (D) - (I)

Correct Answer: (2) (A) (I), (B) (III), (C) (IV), (D) (II)

Solution: The formula used to calculate the EMI is:

$$\text{EMI} = \text{Loan Amount} \times \frac{(1+r)^n \cdot r}{(1+r)^n - 1}.$$

Where:
$r = \frac{\text{Annual Interest Rate}}{12} = \frac{6}{100\times12} = 0.005$,
$n =$ Number of months in loan tenure $= 25 \times 12 = 300$,
The provided value $\frac{(1.005)^{300}\times0.005}{(1.005)^{300}-1} = 0.0064$.
Thus, the EMI for each property is calculated using:

$$\text{EMI} = \text{Loan Amount} \times 0.0064.$$

For Property P:

$$\text{Loan Amount} = 45,00,000 - 5,00,000 = 40,00,000$$

$$\text{EMI} = 40,00,000 \times 0.0064 = 25,600$$

For Property Q:

$$\text{Loan Amount} = 55,00,000 - 5,00,000 = 50,00,000$$

$$\text{EMI} = 50,00,000 \times 0.0064 = 32,000$$

For Property R:

$$\text{Loan Amount} = 65,00,000 - 10,00,000 = 55,00,000$$

$$\text{EMI} = 55,00,000 \times 0.0064 = 35,200$$

For Property S:

$$\text{Loan Amount} = 75,00,000 - 15,00,000 = 60,00,000$$

$$\text{EMI} = 60,00,000 \times 0.0064 = 38,400$$

Final Matching: (A) Property P (I) 25,600 (B) Property Q (III) 32,000 (C) Property R (IV) 35,200 (D) Property S (II) 38,400
Thus, the correct choice is (2).

> **Quick Tip**
>
> To calculate the EMI, use the formula for monthly compounding interest, especially when the interest is compounded more frequently than annually.

Question 80: The corner points of the feasible region for an L.P.P. are $(0, 10)$, $(5, 5)$, $(5, 15)$, and $(0, 30)$. If the objective function is $Z = \alpha x + \beta y$, $\alpha, \beta > 0$, the condition on α and β so that maximum of Z occurs at corner points $(5, 5)$ and $(0, 20)$ is:

Options:
(1) $\alpha = 5\beta$
(2) $5\alpha = \beta$
(3) $\alpha = 3\beta$
(4) $4\alpha = 5\beta$

Correct Answer: (3) $\alpha = 3\beta$

Solution: The slope of the objective function $Z = \alpha x + \beta y$ is given by $-\frac{\alpha}{\beta}$. In order to maximize Z, this slope must be equal to the slope of the line connecting the points $(5, 5)$ and $(0, 20)$.
The slope of the line joining these two points is calculated as:

$$\text{Slope} = \frac{20 - 5}{0 - 5} = -3.$$

To ensure the slopes are equal, we set:

$$-\frac{\alpha}{\beta} = -3 \quad \Rightarrow \quad \alpha = 3\beta.$$

Thus, for the maximum value of Z, the relationship $\alpha = 3\beta$ must hold.

> **Quick Tip**
>
> In linear programming problems, aligning the slope of the objective function with the slope of constraint lines helps in optimizing the function.

Question 81: The solution set of the inequality $|3x| \geq |6 - 3x|$ is:

Options:
(1) $(-\infty, 1]$
(2) $[1, \infty)$
(3) $(-\infty, 1) \cup (1, \infty)$
(4) $(-\infty, -1) \cup (-1, \infty)$

Correct Answer: (2) $[1, \infty)$

Solution: The inequality $|3x| \geq |6 - 3x|$ involves absolute values, so we need to break it down into different cases:
Case 1: $3x \geq 6 - 3x$:

$$3x + 3x \geq 6 \quad \Rightarrow \quad 6x \geq 6 \quad \Rightarrow \quad x \geq 1.$$

Case 2: $3x \leq -(6 - 3x)$:

$$3x \leq -6 + 3x \quad \Rightarrow \quad 0 \leq -6,$$

which results in a contradiction (this is not possible).
Therefore, the solution to the inequality is $x \geq 1$, or the interval $[1, \infty)$.

> **Quick Tip**
>
> When solving absolute value inequalities, divide them into separate cases to simplify the process.

Question 82: If the matrix $\begin{bmatrix} 0 & -1 & 3x \\ 1 & y & -5 \\ -6 & 5 & 0 \end{bmatrix}$ is skew-symmetric, then the value of $5x - y$ is:

Options:
(1) 12
(2) 15
(3) 10
(4) 14

Correct Answer: (3) 10

Solution: A matrix is considered skew-symmetric if its elements satisfy $a_{ij} = -a_{ji}$ and all diagonal elements are zero.
From the given element $a_{13} = 3x$ and $a_{31} = -6$, we can equate:

$$3x = -(-6) \quad \Rightarrow \quad 3x = 6 \quad \Rightarrow \quad x = 2.$$

For the elements $a_{23} = -5$ and $a_{32} = 5$, we have:

$$y = -y \quad \Rightarrow \quad y = 0.$$

Substitute the values $x = 2$ and $y = 0$ into the expression $5x - y$:

$$5x - y = 5(2) - 0 = 10.$$

> **Quick Tip**
>
> For skew-symmetric matrices, the diagonal entries must be zero, and the off-diagonal elements must satisfy $a_{ij} = -a_{ji}$.

Question 83: A company is selling a certain commodity 'x'. The demand function for the commodity is linear. The company can sell 2000 units when the price is Rs. 8 per unit and it can sell 3000 units when the price is Rs. 4 per unit. The Marginal revenue at $x = 5$ is:

Options:
(1) Rs. 79.98
(2) Rs. 15.96
(3) Rs. 16.04
(4) Rs. 80.02

Correct Answer: (2) Rs. 15.96

Solution: The demand function $p(x)$ is linear, and we are given two points: $(2000, 8)$ and $(3000, 4)$. To determine the slope m and the intercept c, we first calculate the slope:

$$m = \frac{4-8}{3000-2000} = -\frac{4}{1000} = -0.004.$$

Now, to find the intercept c, we use one of the points, say $(2000, 8)$:

$$c = 8 + 2000(0.004) = 16.$$

Thus, the demand function is:

$$p(x) = -0.004x + 16.$$

The revenue function $R(x)$ is the product of the price $p(x)$ and the quantity x:

$$R(x) = x \cdot p(x) = x(-0.004x + 16) = -0.004x^2 + 16x.$$

Next, we find the Marginal Revenue by taking the derivative of the revenue function:

$$R'(x) = -0.008x + 16.$$

At $x = 5$:

$$R'(5) = -0.008(5) + 16 = -0.04 + 16 = 15.96.$$

Quick Tip

Marginal revenue is calculated as the derivative of the total revenue function with respect to the quantity sold.

Question 84: If the lengths of the three sides of a trapezium other than the base are 10 cm each, then the maximum area of the trapezium is:

Options:
(1) $100\,\text{cm}^2$ (2) $25\sqrt{3}\,\text{cm}^2$
(3) $75\sqrt{3}\,\text{cm}^2$ (4) $100\sqrt{3}\,\text{cm}^2$

Correct Answer: (3) $75\sqrt{3}$ cm^2

Solution: To find the maximum area of a trapezium with three equal non-parallel sides, we use Brahmagupta's formula for a cyclic quadrilateral. The formula for the semi-perimeter is given by:

$$s = \frac{a+b+c+d}{2}$$

where a, b, c, and d are the lengths of the sides, and the area can be calculated based on symmetry when the trapezium forms equilateral triangles at the ends.
In this case, three of the sides of the trapezium are 10 cm each. We model the trapezium by considering that these three sides form two equilateral triangles at the ends of the trapezium.
The area of one equilateral triangle with side length 10 cm is calculated as:

$$A = \frac{\sqrt{3}}{4} \times (10)^2 = \frac{\sqrt{3}}{4} \times 100 = 25\sqrt{3}\,\text{cm}^2.$$

Since there are three such triangles, the total area of the trapezium is:

$$\text{Total Area} = 3 \times 25\sqrt{3} = 75\sqrt{3}\,\text{cm}^2.$$

Quick Tip

For trapeziums with three equal non-parallel sides, the maximum area is achieved by dividing the shape into equilateral triangles.

Question 85: Three defective bulbs are mixed with 8 good ones. If three bulbs are drawn one by one with replacement, the probabilities of getting exactly 1 defective, more than 2 defective, no defective, and more than 1 defective respectively are:

Options:
(1) $\frac{27}{1331}, \frac{576}{1331}, \frac{243}{1331}, \frac{512}{1331}$
(2) $\frac{27}{1331}, \frac{243}{1331}, \frac{576}{1331}, \frac{512}{1331}$
(3) $\frac{576}{1331}, \frac{512}{1331}, \frac{243}{1331}, \frac{27}{1331}$
(4) $\frac{243}{1331}, \frac{27}{1331}, \frac{576}{1331}, \frac{512}{1331}$

Correct Answer: (3) $\frac{576}{1331}, \frac{27}{1331}, \frac{243}{1331}, \frac{512}{1331}$

Solution: Let $p = \frac{3}{11}$ represent the probability of drawing a defective bulb, and $q = \frac{8}{11}$ be the probability of drawing a good bulb. Since the draws are with replacement, the events are independent. The binomial probability formula is used to calculate the probabilities:

$$P(X = k) = \binom{n}{k} p^k q^{n-k},$$

where $n = 3$ (the number of draws) and k is the number of defective bulbs.

Probability of exactly one defective bulb ($k = 1$):

$$P(X = 1) = \binom{3}{1} p^1 q^2 = 3 \cdot \left(\frac{3}{11}\right)^1 \cdot \left(\frac{8}{11}\right)^2.$$

$$P(X = 1) = 3 \cdot \frac{3}{11} \cdot \frac{64}{121} = \frac{576}{1331}.$$

Probability of more than two defective bulbs ($k > 2$): The only possible case is $k = 3$, where all bulbs are defective:

$$P(X = 3) = \binom{3}{3} p^3 q^0 = 1 \cdot \left(\frac{3}{11}\right)^3 = \frac{27}{1331}.$$

Probability of no defective bulbs ($k = 0$):

$$P(X = 0) = \binom{3}{0} p^0 q^3 = 1 \cdot \left(\frac{8}{11}\right)^3 = \frac{512}{1331}.$$

Probability of more than one defective bulb ($k > 1$): This probability includes $k = 2$ and $k = 3$. First, calculate $P(X = 2)$:

$$P(X = 2) = \binom{3}{2} p^2 q^1 = 3 \cdot \left(\frac{3}{11}\right)^2 \cdot \frac{8}{11}.$$

$$P(X = 2) = 3 \cdot \frac{9}{121} \cdot \frac{8}{11} = \frac{216}{1331}.$$

Now, add the probabilities for $k = 2$ and $k = 3$:

$$P(X > 1) = P(X = 2) + P(X = 3) = \frac{216}{1331} + \frac{27}{1331} = \frac{243}{1331}.$$

Final Probabilities: - Exactly one defective: $\frac{576}{1331}$, - More than two defective: $\frac{27}{1331}$, - No defective: $\frac{512}{1331}$, - More than one defective: $\frac{243}{1331}$.
Thus, the correct answer is (3).

Quick Tip

For problems with replacement, use the binomial probability distribution.