CUET PG 2026 Mathematics / Applied Math / Electronics Question Paper with Solutions

Time Allowed :1 Hour 30 Minutes	Maximum Marks :300	Total Questions :75

General Instructions

Read the following instructions very carefully and strictly follow them:

- The Question Paper consists of 75 Multiple Choice Questions (MCQs).
- Total marks for the exam is 300.
- The total duration is 90 minutes; a, timer on the screen will display the remaining time, and the exam will automatically submit when the time reaches zero.
- For every correct answer, 4 marks (+4) will be awarded to the candidate, and a 1 mark (-1) will be deducted for every incorrect answer.
- Question papers are available in both English and Hindi.

1. What is the dimension of the vector space of all $n \times n$ real symmetric matrices?
(A) n^2
(B) $\frac{n(n+1)}{2}$
(C) $\frac{n(n-1)}{2}$
(D) $2n$

Correct Answer: (B) $\frac{n(n+1)}{2}$

Solution:
This question asks for the dimension of a specific vector space, which is equivalent to finding the number of independent entries required to define any matrix in that space. A symmetric matrix is defined by the property $A^T = A$, meaning its entries are symmetric across the main diagonal ($a_{ij} = a_{ji}$).

Step 1: Understanding the Question:
We need to determine how many elements in an $n \times n$ matrix we can choose freely, given the constraint that the matrix must be symmetric. This number will be the dimension of the vector space of all such matrices.

Step 2: Key Formula or Approach:
The total number of independent entries can be found by summing the number of independent diagonal elements and the number of independent off-diagonal elements.

Step 3: Detailed Explanation:
Let's break down the matrix into its components:

1. **Diagonal Elements:** An $n \times n$ matrix has n diagonal elements $(a_{11}, a_{22}, \ldots, a_{nn})$. There are no symmetry constraints on these elements, so all n of them can be chosen independently.

2. **Off-Diagonal Elements:** The total number of elements in the matrix is n^2. After accounting for the n diagonal elements, there are $n^2 - n$ off-diagonal elements. The symmetry condition $a_{ij} = a_{ji}$ means that once we choose an element above the main diagonal (e.g., a_{12}), the corresponding element below the diagonal (a_{21}) is automatically determined. Therefore, we only need to choose the elements in the upper triangle (or lower triangle). The number of such elements is $\frac{n^2-n}{2}$.

The total number of independent entries (the dimension) is the sum of these two counts:

$$\text{Dimension} = (\text{Independent diagonal elements}) + (\text{Independent off-diagonal elements})$$

$$\text{Dimension} = n + \frac{n^2 - n}{2} = \frac{2n + n^2 - n}{2} = \frac{n^2 + n}{2} = \frac{n(n+1)}{2}$$

Step 4: Final Answer:
The dimension of the vector space of all $n \times n$ real symmetric matrices is $\frac{n(n+1)}{2}$.

Quick Tip

To find the dimension of symmetric matrices, you only need to count the entries on or above the main diagonal.
This gives you n diagonal entries plus $\frac{n(n-1)}{2}$ upper-triangle entries, summing to $\frac{n(n+1)}{2}$.

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2. If a function $f(x)$ is continuous on a closed interval $[a, b]$, is it necessarily uniformly continuous?
(A) Yes, always uniformly continuous
(B) No, never uniformly continuous
(C) Only if differentiable
(D) Only if bounded

Correct Answer: (A) Yes, always uniformly continuous

Solution:
This question deals with a fundamental concept in real analysis concerning the relationship between continuity and uniform continuity on a specific type of domain.

Step 1: Understanding the Question:
The question asks whether the property of continuity for a function $f(x)$ on a closed and bounded interval $[a, b]$ is sufficient to guarantee the stronger property of uniform continuity on that same interval.

Step 2: Key Formula or Approach:
The answer to this question is a direct consequence of a major theorem in analysis, the Heine-Cantor Theorem.

Step 3: Detailed Explanation:

- **Continuity** at a point means that for any desired closeness ϵ, we can find a neighborhood δ around that point where the function values are within ϵ. The choice of δ can depend on the point.

- **Uniform continuity** on an interval means that for any ϵ, we can find a single δ that works for the entire interval. This is a stronger condition.

- The **Heine-Cantor Theorem** states that if a function is continuous on a compact set, then it is uniformly continuous on that set. In the context of real numbers ($\mathbb{R}$), a "compact set" is any set that is both closed and bounded.
- The interval $[a, b]$ given in the question is a closed and bounded interval, and therefore it is a compact set.

Since the function $f(x)$ is continuous on the compact set $[a, b]$, the Heine-Cantor Theorem directly implies that $f(x)$ must also be uniformly continuous on $[a, b]$.

Step 4: Final Answer:
Yes, a function continuous on a closed interval is always uniformly continuous on that interval.

Quick Tip

Remember this key result: Continuity + a Compact Domain (like a closed interval $[a, b]$) $\implies$ Uniform Continuity.
The result might not hold for open intervals like (a, b) or unbounded intervals.

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3. What is the value of the limit $\lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^n$?
(A) 1
(B) 0
(C) e
(D) ∞

Correct Answer: (C) e

Solution:
This question asks for the value of one of the most famous and fundamental limits in calculus. This limit is used as a formal definition for a very important mathematical constant.

Step 1: Understanding the Question:
We are asked to evaluate the limit of the sequence $a_n = (1 + \frac{1}{n})^n$ as n approaches infinity.

Step 2: Key Formula or Approach:
The expression is the standard limit definition of the mathematical constant e, the base of the natural logarithm.

$$e = \lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^n$$

Step 3: Detailed Explanation:
This limit represents an indeterminate form of type 1^∞. As $n \to \infty$, the base $(1+\frac{1}{n})$ approaches 1, while the exponent n approaches infinity. The value of the limit is the result of the balance between the base getting closer to 1 and the exponent getting larger.
This limit is formally taken as the definition of the number e. It arises naturally in contexts of continuous growth, such as continuously compounded interest. The value of e is an irrational number approximately equal to 2.71828.

Step 4: Final Answer:
The value of the limit is e.

Quick Tip

This is a limit you should memorize. A more general form is $\lim_{n\to\infty} \left(1 + \frac{x}{n}\right)^n = e^x$. The question is the special case where $x = 1$.

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4. How many elements of order 5 are there in a cyclic group of order 25?
(A) 1
(B) 4
(C) 5
(D) 10

Correct Answer: (B) 4

Solution:
This question is from group theory and involves finding the number of elements with a specific order within a finite cyclic group.

Step 1: Understanding the Question:
We have a cyclic group G with order $|G| = 25$. We need to find the number of elements $g \in G$ such that the order of g is 5.

Step 2: Key Formula or Approach:
A fundamental theorem in group theory states that in a finite cyclic group of order n, the number of elements of order d (where d is a divisor of n) is given by Euler's totient function, $\phi(d)$.

Step 3: Detailed Explanation:

1. **Identify n and d:** The order of the group is $n = 25$. The desired order of the elements is $d = 5$.

2. **Check the condition:** We must verify that d divides n. Indeed, 5 divides 25. Therefore, elements of order 5 exist.

3. **Calculate $\phi(d)$:** We need to compute $\phi(5)$. Euler's totient function $\phi(k)$ counts the number of positive integers less than or equal to k that are relatively prime to k.

4. For a prime number p, $\phi(p) = p - 1$. Since 5 is a prime number, we have:

$$\phi(5) = 5 - 1 = 4$$

This means there are exactly 4 elements of order 5 in any cyclic group of order 25.

Step 4: Final Answer:
There are 4 elements of order 5 in a cyclic group of order 25.

Quick Tip

For any finite cyclic group G of order n, the number of elements of order d (where $d|n$) is exactly $\phi(d)$.
Remember that if p is prime, $\phi(p) = p - 1$.

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5. If A is a 3×3 matrix with eigenvalues $1, 2, 3$, what is the determinant of A^2?
(A) 6
(B) 12
(C) 18
(D) 36

Correct Answer: (D) 36

Solution:
This linear algebra problem requires knowledge of the relationship between a matrix's eigenvalues, its determinant, and the determinants of its powers.

Step 1: Understanding the Question:
We are given the eigenvalues of a 3×3 matrix A and are asked to find the determinant of the matrix A^2, which is A multiplied by itself.

Step 2: Key Formula or Approach:
We can solve this in two ways, both relying on fundamental properties:
Method 1: Use the properties $\det(A) = \prod \lambda_i$ and $\det(A^k) = (\det A)^k$.
Method 2: First find the eigenvalues of A^2 and then compute its determinant.

Step 3: Detailed Explanation (Method 1):

1. **Find the determinant of A:** The determinant of a matrix is the product of its eigenvalues.
$$\det(A) = \lambda_1 \times \lambda_2 \times \lambda_3 = 1 \times 2 \times 3 = 6$$

2. **Find the determinant of $\mathbf{A}^2$:** A property of determinants states that $\det(A^k) = (\det(A))^k$. For $k = 2$:
$$\det(A^2) = (\det(A))^2 = 6^2 = 36$$

Step 3: Detailed Explanation (Method 2):

1. **Find the eigenvalues of $\mathbf{A}^2$:** If a matrix A has eigenvalues λ_i, then the matrix A^k has eigenvalues λ_i^k. Therefore, the eigenvalues of A^2 are $1^2, 2^2, 3^2$, which are $1, 4, 9$.

2. **Find the determinant of $\mathbf{A}^2$:** The determinant of A^2 is the product of its eigenvalues.
$$\det(A^2) = 1 \times 4 \times 9 = 36$$

Both methods yield the same result.

Step 4: Final Answer:
The determinant of A^2 is 36.

Quick Tip

There are two quick ways to solve this:
1. Find $\det(A) = 1 \cdot 2 \cdot 3 = 6$, then $\det(A^2) = 6^2 = 36$.
2. Find eigenvalues of A^2 are $1^2, 2^2, 3^2 = 1, 4, 9$, then $\det(A^2) = 1 \cdot 4 \cdot 9 = 36$.

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6. Which theorem states that every bounded sequence in $\mathbb{R}^n$ has a convergent subsequence?
(A) Mean Value Theorem
(B) Bolzano–Weierstrass Theorem
(C) Rolle's Theorem
(D) Taylor's Theorem

Correct Answer: (B) Bolzano–Weierstrass Theorem

Solution:
This question asks to identify a key theorem from real analysis by its statement. This theorem is fundamental to the concept of compactness.

Step 1: Understanding the Question:
We need to name the theorem which guarantees that any sequence confined within a finite region (bounded) of $\mathbb{R}^n$ must have a subsequence that "settles down" to a specific point (converges).

Step 2: Key Formula or Approach:
This requires recognizing the statement of the Bolzano-Weierstrass Theorem and distinguishing it from other major theorems in calculus.

Step 3: Detailed Explanation:
Let's analyze the theorems listed:

- **Mean Value Theorem:** Relates the average rate of change of a function over an interval to its instantaneous rate of change (derivative) at some point in that interval.

- **Bolzano–Weierstrass Theorem:** This theorem states exactly what is described in the question: every bounded sequence in $\mathbb{R}^n$ has a convergent subsequence. It essentially says that if you have an infinite number of points in a finite space, they must "cluster" around at least one point.

- **Rolle's Theorem:** A special case of the Mean Value Theorem, stating that if a differentiable function has equal values at two points, there must be a point between them where the derivative is zero.

- **Taylor's Theorem:** Provides a way to approximate a function near a point using a polynomial whose coefficients are derived from the function's derivatives at that point.

The statement in the question directly matches the definition of the Bolzano-Weierstrass Theorem.

Step 4: Final Answer:
The theorem is the Bolzano–Weierstrass Theorem.

> Quick Tip
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> Associate the keywords: "Bounded Sequence" $\implies$ "Convergent Subsequence". This is the core statement of the Bolzano-Weierstrass Theorem.

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7. What is the radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{x^n}{n!}$?
(A) 0
(B) 1
(C) ∞
(D) e

Correct Answer: (C) ∞

Solution:
This question asks for the radius of convergence of a well-known power series. The radius of convergence defines the interval on which the series converges to a function.

Step 1: Understanding the Question:
We need to find the value R such that the power series $\sum_{n=0}^{\infty} \frac{x^n}{n!}$ converges for all x with $|x| < R$ and diverges for $|x| > R$.

Step 2: Key Formula or Approach:
The most common method to find the radius of convergence is the Ratio Test. For a series $\sum a_n x^n$, we compute the limit $L = \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right|$. The radius of convergence is then $R = \frac{1}{L}$.

Step 3: Detailed Explanation:

1. **Identify the coefficient a_n:** In the given series, the coefficient of x^n is $a_n = \frac{1}{n!}$.

2. **Set up the ratio:** We need to compute the ratio $\left|\frac{a_{n+1}}{a_n}\right|$.

$$\left|\frac{a_{n+1}}{a_n}\right| = \frac{1/(n+1)!}{1/n!} = \frac{n!}{(n+1)!} = \frac{n!}{(n+1)\cdot n!} = \frac{1}{n+1}$$

3. **Compute the limit L:**
$$L = \lim_{n\to\infty} \frac{1}{n+1} = 0$$

4. **Calculate the radius of convergence R:**
$$R = \frac{1}{L} = \frac{1}{0}$$

A limit of 0 implies that the radius of convergence is infinite. This means the series converges for all real (or complex) values of x.

Step 4: Final Answer:
The radius of convergence is ∞.

💡 Quick Tip

Recognize that $\sum_{n=0}^{\infty} \frac{x^n}{n!}$ is the Maclaurin series for the exponential function e^x. Since e^x is defined for all real numbers x, its power series must converge for all x, hence the radius of convergence is infinite.

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8. Is the set of all rational numbers $\mathbb{Q}$ a countable or uncountable set?
(A) Finite set
(B) Countable set
(C) Uncountable set
(D) Empty set

Correct Answer: (B) Countable set

Solution:
This question from set theory asks about the cardinality (a measure of the number of elements) of the set of rational numbers, $\mathbb{Q}$.

Step 1: Understanding the Question:
We need to determine if the set of all rational numbers can be put into a one-to-one correspondence with the set of natural numbers ($\mathbb{N} = \{1, 2, 3, \dots\}$). If it can, the set is countable; otherwise, it is uncountable.

Step 2: Key Formula or Approach:
The approach is to demonstrate, or recall the demonstration, that a systematic listing of all rational numbers is possible. This is famously achieved through Cantor's diagonalization argument.

Step 3: Detailed Explanation:

- A set is **countable** (or countably infinite) if its elements can be listed in a sequence, like $r_1, r_2, r_3, \ldots$.
- The set of rational numbers, $\mathbb{Q}$, consists of all numbers that can be expressed as a fraction p/q, where p is an integer and q is a non-zero integer.
- Although between any two rational numbers there is another rational number (the set is dense), the entire set is surprisingly countable.
- We can construct a list of all positive rational numbers by arranging them in a grid where the entry in row q and column p is the fraction p/q. We can then traverse this grid diagonally, adding each new fraction to our list. This process ensures that every possible rational number will eventually be included in the sequence. By extending this to include 0 and negative rationals, we can list all elements of $\mathbb{Q}$.

Since such a list can be constructed, the set of rational numbers is countable.

Step 4: Final Answer:
The set of all rational numbers $\mathbb{Q}$ is a countable set.

> Quick Tip
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> Remember the hierarchy of common infinite sets: The sets of natural numbers ($\mathbb{N}$), integers ($\mathbb{Z}$), and rational numbers ($\mathbb{Q}$) are all countable. The set of real numbers ($\mathbb{R}$) is uncountable.

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9. If $T : V \to W$ is a linear transformation, what is the relationship between rank(T), nullity(T), and dim(V)?
(A) $\text{rank}(T) + \text{nullity}(T) = \dim(W)$
(B) $\text{rank}(T) \times \text{nullity}(T) = \dim(V)$
(C) $\text{rank}(T) + \text{nullity}(T) = \dim(V)$
(D) $\text{rank}(T) = \text{nullity}(T)$

Correct Answer: (C) $\text{rank}(T) + \text{nullity}(T) = \dim(V)$

Solution:
This question asks for the statement of one of the most fundamental theorems in linear algebra, which connects the dimensions of the key vector spaces associated with a linear transformation.

Step 1: Understanding the Question:
We need to find the correct formula that relates the dimensions of the domain, the image (range), and the kernel (null space) of a linear transformation T from a vector space V to a vector space W.

Step 2: Key Formula or Approach:
This relationship is given by the Rank-Nullity Theorem. We need to recall its precise statement.

Step 3: Detailed Explanation:
Let's define the terms for a linear transformation $T : V \to W$:

- **dim(V)**: The dimension of the **domain** space (the input space).
- **rank(T)**: The dimension of the image of T, which is the set of all possible outputs in W. It is a subspace of W. rank(T) = dim(Im(T)).
- **nullity(T)**: The dimension of the kernel (or null space) of T, which is the set of all vectors in V that are mapped to the zero vector in W. It is a subspace of V. nullity(T) = dim(Ker(T)).

The **Rank-Nullity Theorem** states that the dimension of the domain is equal to the sum of the dimension of the image and the dimension of the kernel.

$$\text{rank}(T) + \text{nullity}(T) = \dim(V)$$

The theorem essentially says that the dimension of the input space is split between the dimensions of the part that gets "squashed" to zero (the kernel) and the part that effectively forms the output (the image).

Step 4: Final Answer:
The correct relationship is $\text{rank}(T) + \text{nullity}(T) = \dim(V)$.

> Quick Tip
>
> A simple way to remember the theorem is: Rank + Nullity = Dimension of the **Domain**. A common mistake is to use the dimension of the codomain (W) instead of the domain (V).

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10. What is the condition for a group G to be Abelian based on the commutator subgroup?
(A) Commutator subgroup is equal to G
(B) Commutator subgroup is trivial
(C) Commutator subgroup is infinite
(D) Commutator subgroup is cyclic

Correct Answer: (B) Commutator subgroup is trivial

Solution:
This abstract algebra question explores the connection between the commutativity of a group and a special subgroup called the commutator subgroup.

Step 1: Understanding the Question:
We need to find the specific property of the commutator subgroup that is equivalent to the

group being Abelian (commutative).

Step 2: Key Formula or Approach:
We must use the definitions of an Abelian group, a commutator, and the commutator subgroup to establish the connection.

Step 3: Detailed Explanation:

1. **Definition of an Abelian Group:** A group G is Abelian if and only if $ab = ba$ for all elements $a, b \in G$.

2. **Definition of a Commutator:** The commutator of two elements a and b is defined as $[a, b] = aba^{-1}b^{-1}$. The commutator measures how much a and b fail to commute. If $ab = ba$, then we can rewrite this as $aba^{-1} = b$, and further as $aba^{-1}b^{-1} = e$, where e is the identity element. So, a and b commute if and only if their commutator $[a, b]$ is the identity.

3. **Definition of the Commutator Subgroup:** The commutator subgroup, denoted G' or $[G, G]$, is the subgroup generated by all the commutators in G.
4. **Connecting the Concepts:** If the group G is Abelian, then by definition, $ab = ba$ for all $a, b \in G$. This implies that every commutator $[a, b]$ is equal to the identity element e. The subgroup generated by a set containing only the identity element is the smallest possible subgroup, $\{e\}$, which is called the trivial subgroup.

Conversely, if the commutator subgroup is trivial ($G' = \{e\}$), it means all commutators are equal to e, which implies $ab = ba$ for all elements, so the group is Abelian.

Step 4: Final Answer:
A group G is Abelian if and only if its commutator subgroup is trivial.

Quick Tip
Think of the commutator subgroup as a measure of a group's "non-commutativity". If a group is perfectly commutative (Abelian), its measure of non-commutativity must be zero, which corresponds to the commutator subgroup being the smallest possible group: the trivial one.

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11. What is the value of the integral $\int_{-\infty}^{\infty} e^{-x^2}\,dx$?
(A) 0
(B) 1
(C) $\sqrt{\pi}$
(D) π

Correct Answer: (C) $\sqrt{\pi}$

Solution:
This question asks for the value of a famous definite integral known as the Gaussian integral. It is a cornerstone of probability theory (related to the normal distribution) and advanced calculus.

Step 1: Understanding the Question:
We need to evaluate the improper integral of the function $f(x) = e^{-x^2}$ over the entire real line, from $-\infty$ to ∞.

Step 2: Key Formula or Approach:
Since e^{-x^2} has no elementary antiderivative, a direct application of the Fundamental Theorem of Calculus is not possible. The standard method for evaluating this integral is the "Poisson trick" or "Gaussian trick", which involves squaring the integral and switching to polar coordinates.

Step 3: Detailed Explanation:

1. Let $I = \int_{-\infty}^{\infty} e^{-x^2}\, dx$. The integrand is an even function, so $I > 0$.

2. Consider I^2:
$$I^2 = \left(\int_{-\infty}^{\infty} e^{-x^2}\, dx\right)\left(\int_{-\infty}^{\infty} e^{-y^2}\, dy\right) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-(x^2+y^2)}\, dx\, dy$$

3. This is a double integral over the entire xy-plane. We convert to polar coordinates, where $x^2 + y^2 = r^2$ and the area element $dx\, dy$ becomes $r\, dr\, d\theta$. The limits become $0 \le r < \infty$ and $0 \le \theta < 2\pi$.
$$I^2 = \int_0^{2\pi}\int_0^{\infty} e^{-r^2}(r\, dr)\, d\theta$$

4. We can now evaluate the integral. The inner integral with respect to r can be solved with a u-substitution ($u = -r^2$, $du = -2r\, dr$):
$$\int_0^{\infty} e^{-r^2} r\, dr = \left[-\frac{1}{2}e^{-r^2}\right]_0^{\infty} = -\frac{1}{2}(0-1) = \frac{1}{2}$$

5. Now, we evaluate the outer integral with respect to θ:
$$I^2 = \int_0^{2\pi}\left(\frac{1}{2}\right) d\theta = \frac{1}{2}[\theta]_0^{2\pi} = \frac{1}{2}(2\pi) = \pi$$

6. Since $I^2 = \pi$ and we know $I > 0$, we take the positive square root.

Step 4: Final Answer:
The value of the integral is $\sqrt{\pi}$.

> Quick Tip
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> This is a standard result that is extremely useful to memorize for exams in mathematics, physics, and statistics.
> The integral of the "bell curve" function e^{-x^2} over the entire real line is $\sqrt{\pi}$.

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12. In a metric space, is every Cauchy sequence necessarily a convergent sequence?
(A) Yes, always
(B) No, not always
(C) Only in finite spaces
(D) Only for bounded sequences

Correct Answer: (B) No, not always

Solution:
This question delves into the fundamental properties of metric spaces, specifically the relationship between Cauchy sequences and convergent sequences.

Step 1: Understanding the Question:
We need to determine if the property of being a Cauchy sequence (terms getting closer to each other) is sufficient to guarantee that the sequence converges to a limit *within the given metric space*.

Step 2: Key Formula or Approach:
This question is about the definition of a **complete metric space**. A metric space is defined as complete if and only if every Cauchy sequence in it converges to a limit within that space.

Step 3: Detailed Explanation:

- A sequence $\{x_n\}$ is **Cauchy** if for any distance $\epsilon > 0$, there exists an integer N such that for all $m, n > N$, the distance $d(x_m, x_n) < \epsilon$.

- A sequence $\{x_n\}$ is **convergent** if there exists a point L in the space such that for any $\epsilon > 0$, there exists an N such that for all $n > N$, $d(x_n, L) < \epsilon$.

- In any metric space, every convergent sequence is a Cauchy sequence. However, the converse is not always true.

- **Counterexample:** Consider the metric space of rational numbers $\mathbb{Q}$ with the usual distance $d(x,y) = |x-y|$. The sequence of rational numbers $x_1 = 3, x_2 = 3.1, x_3 = 3.14, \ldots$ that approximates π is a Cauchy sequence. However, it does not converge *in $\mathbb{Q}$* because its limit, π, is an irrational number and is not an element of the space $\mathbb{Q}$.

Therefore, a Cauchy sequence is not necessarily convergent in a general metric space. This property only holds in complete metric spaces like $\mathbb{R}$ or $\mathbb{C}$.

Step 4: Final Answer:
No, a Cauchy sequence is not always convergent in an arbitrary metric space.

> Quick Tip
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> Remember the key distinction: Convergence $\implies$ Cauchy (always true).
> Cauchy $\implies$ Convergence (only true in **complete** metric spaces).

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13. What are the possible values for the rank of a 4×3 matrix?
(A) $0, 1, 2, 3, 4$
(B) $1, 2, 3, 4$
(C) $0, 1, 2, 3$
(D) Only 3

Correct Answer: (C) $0, 1, 2, 3$

Solution:
This is a fundamental question in linear algebra about the properties of the rank of a non-square matrix.

Step 1: Understanding the Question:
We need to determine all possible integer values that the rank of a matrix with 4 rows and 3 columns can take.

Step 2: Key Formula or Approach:
The rank of a matrix is defined as the maximum number of linearly independent columns (or, equivalently, rows). A key property is that the rank of an $m \times n$ matrix A is bounded by both m and n.

$$\text{rank}(A) \le \min(m, n)$$

Step 3: Detailed Explanation:

1. **Identify m and n:** For the given 4×3 matrix, we have $m = 4$ rows and $n = 3$ columns.

2. **Apply the rank inequality:** The rank must be less than or equal to the minimum of the number of rows and columns.

$$\text{rank}(A) \leq \min(4, 3) = 3$$

This means the rank cannot be 4.

3. **Consider the minimum rank:** The smallest possible rank for any matrix is 0. This occurs if and only if the matrix is the zero matrix (all entries are zero).

4. **Combine the bounds:** The rank must be an integer that is greater than or equal to 0 and less than or equal to 3.

Thus, the possible values for the rank are 0, 1, 2, and 3. Each of these values is achievable with an appropriately constructed 4×3 matrix.

Step 4: Final Answer:
The possible values for the rank are 0, 1, 2, 3.

> Quick Tip
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> A simple rule to remember: The rank of an $m \times n$ matrix is always an integer between 0 and $\min(m, n)$, inclusive. The rank cannot be larger than the number of rows or the number of columns.

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14. What is the order of the group of permutations S_3?
(A) 3
(B) 6
(C) 9
(D) 12

Correct Answer: (B) 6

Solution:
This question from group theory asks for the size (or order) of a specific symmetric group, S_3.

Step 1: Understanding the Question:
We need to find the total number of elements in the group S_3. The group S_3 is the set of all possible permutations (bijective functions) of a set with 3 distinct elements, for example, the set $\{1, 2, 3\}$.

Step 2: Key Formula or Approach:
The order of the symmetric group on n elements, denoted $|S_n|$, is given by the factorial of n.

$$|S_n| = n!$$

Step 3: Detailed Explanation:
A permutation of $\{1, 2, 3\}$ is simply a reordering of these three numbers. We can count the number of possible orderings directly:

- There are 3 choices for the first position.
- After choosing the first, there are 2 choices remaining for the second position.
- After choosing the first two, there is only 1 choice left for the third position.

The total number of permutations is the product of these choices: $3 \times 2 \times 1$.
Using the formula from Step 2 for $n = 3$:

$$|S_3| = 3! = 3 \times 2 \times 1 = 6$$

The 6 elements are: the identity, three transpositions ((1 2), (1 3), (2 3)), and two 3-cycles ((1 2 3), (1 3 2)).

Step 4: Final Answer:
The order of the group S_3 is 6.

Quick Tip

The order of the symmetric group S_n (the group of permutations of n elements) is always $n!$. This is a fundamental fact in group theory and combinatorics.

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15. Which partial differential equation represents the Laplace equation in two dimensions?
(A) $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0$
(B) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$
(C) $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$
(D) $\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}$

Correct Answer: (B) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

Solution:
This question requires identifying the standard mathematical form of the Laplace equation, a fundamental partial differential equation (PDE) in physics and engineering.

Step 1: Understanding the Question:
We need to select the correct formula for the Laplace equation in a two-dimensional Cartesian coordinate system (x, y).

Step 2: Key Formula or Approach:
The approach is to recall the definition of the Laplacian operator (Δ or ∇^2) and the structure of the Laplace equation ($\nabla^2 u = 0$).

Step 3: Detailed Explanation:
The **Laplacian operator** in two dimensions is the sum of the second partial derivatives with respect to each spatial variable. For a function $u(x, y)$, it is:

$$\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$$

The **Laplace equation** is a homogeneous PDE that states that the Laplacian of a function is zero.

$$\nabla^2 u = 0 \implies \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

This equation typically describes steady-state phenomena, such as steady-state heat distribution, electrostatic potentials in charge-free regions, or ideal fluid flow.
Let's analyze the other options:

- (C) $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ is the one-dimensional **Wave Equation**.
- (D) $\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}$ is the one-dimensional **Heat Equation** (or Diffusion Equation).

Option (B) correctly represents the Laplace equation in two dimensions.

Step 4: Final Answer:
The two-dimensional Laplace equation is $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$.

Quick Tip

Remember the "Big Three" PDEs:

- **Laplace Equation:** Sum of second spatial derivatives = 0 (steady-state).
- **Heat Equation:** First time derivative $\propto$ second spatial derivative (diffusion).
- **Wave Equation:** Second time derivative $\propto$ second spatial derivative (propagation).
