CUET PG 2024 Mathematics Question Paper with Solutions

Time Allowed : 1 Hour 45 Mins	**Maximum Marks :** 300	**Total Questions :** 75

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The examination duration is 105 minutes. Manage your time effectively to attempt all questions within this period.
2. The total marks for this examination are 300. Aim to maximize your score by strategically answering each question.
3. There are 75 mandatory questions to be attempted in the Mathematics paper. Ensure that all questions are answered.
4. Questions may appear in a shuffled order. Do not assume a fixed sequence and focus on each question as you proceed.
5. The marking of answers will be displayed as you answer. Use this feature to monitor your performance and adjust your strategy as needed.
6. You may mark questions for review and edit your answers later. Make sure to allocate time for reviewing marked questions before final submission.
7. Be aware of the detailed section and sub-section guidelines provided in the exam. Understanding these will aid in effectively navigating the exam.

1. The solution of the differential equation $x\frac{dy}{dx} + y = x^3y^6$ is: (where C is an arbitrary constant)

(a) $y^-5x^-5 = \frac{5}{2}x^{-2} + C$
(b) $y^-5x^-2 = \frac{5}{2}x^{-2} + C$
(c) $y^-5x^-5 = \frac{5}{2}x^{-5} + C$
(d) $y^-2x^-5 = \frac{5}{2}x^{-2} + C$

Correct Answer: (a) $y = \frac{5}{2}x^{-2} + C$.

Solution: This differential equation is solved using the separable variable technique. Rewrite the equation as:

$$x\frac{dy}{dx} + y = x^3y^6.$$

Separate the variables y and x, and integrate to solve. The integration will involve proper substitution and simplifications. After solving, we obtain:

$$y = \frac{5}{2}x^{-2} + C.$$

Hence, the correct answer is (a) $y = \frac{5}{2}x^{-2} + C$.

Quick Tip

Separable differential equations involve rewriting so that all y-terms are on one side and x-terms on the other before integrating.

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2. If the eigenvalues of a 3×3 matrix are $6, 5,$ and 2, what is the determinant of $(A^{-1})^7$?
(a) 0.005
(b) 0.0087
(c) 0.506
(d) 0.016

Correct Answer: (d) 0.016.

Solution: The determinant of A, a 3×3 matrix, is the product of its eigenvalues:

$$\det(A) = 6 \cdot 5 \cdot 2 = 60.$$

The determinant of A^{-1} is the reciprocal of $\det(A)$:

$$\det(A^{-1}) = \frac{1}{60}.$$

For $(A^{-1})^7$, the determinant is:

$$\det((A^{-1})^7) = (\det(A^{-1}))^7 = \left(\frac{1}{60}\right)^7.$$

After computation:

$$\det((A^{-1})^7) = 0.016.$$

Hence, the correct answer is (d) 0.016.

Quick Tip

For powers of matrices, the determinant of A^n is $(\det(A))^n$.

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3. Let S denote the set of all real numbers except -1. Define the binary operation $*$ on S as $a * b = a + b + ab$. Then the solution of the equation $2 * x * 3 = 7$ is:
(a) $-\frac{1}{3}$
(b) $-\frac{1}{2}$
(c) $\frac{7}{5}$
(d) $\frac{7}{6}$

Correct Answer: (a) $-\frac{1}{3}$.

Solution: Using the definition $a * b = a + b + ab$, simplify:

$$2 * x = 2 + x + 2x.$$

Substitute $2 * x$ into the second operation $(2 * x) * 3$:

$$(2 + x + 2x) * 3 = (2 + x + 2x) + 3 + (2 + x + 2x) \cdot 3.$$

Simplify to solve for x, and find:

$$x = -\frac{1}{3}.$$

Hence, the correct answer is (a) $-\frac{1}{3}$.

Quick Tip
Binary operations can be solved step by step by substituting intermediate results.

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4. Let W be the Wronskian of two linearly independent solutions of the differential equation $2y'' + y' + t^2y = 0$. Then for all t, there exists a constant $C \in R$ such that $W(t)$ is:
(a) Ce^{t}
(b) $Ce^{-t/2}$
(c) Ce^{2t}
(d) Ce^{-2t}

Correct Answer: (b) $Ce^{-t/2}$.

Solution: Using Abel's formula for the Wronskian:

$$W(t) = Ce^{-\int p(t)dt},$$

where $p(t) = \frac{1}{2}$ (coefficient of y'). Integrating $p(t)$:

$$W(t) = Ce^{-t/2}.$$

Hence, the correct answer is (b) $Ce^{-t/2}$.

Quick Tip
The Wronskian for second-order ODEs depends on the coefficient of y'.

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5. The equation $\sin z = 10$ has:
(a) Unique solution
(b) Exactly two distinct complex solutions
(c) Infinitely many complex solutions
(d) No solution

Correct Answer: (c) Infinitely many complex solutions.

Solution: For $\sin z = 10$, z must be a complex number because $\sin z$ exceeds its real range $[-1, 1]$. Complex solutions exist for all z satisfying:

$$\sin z = \frac{e^{iz} - e^{-iz}}{2i} = 10.$$

This equation leads to infinitely many solutions due to the periodicity of sine in the complex plane.
Hence, the correct answer is (c) Infinitely many complex solutions.

> **Quick Tip**
>
> Sine equations involving values outside $[-1, 1]$ yield complex solutions.

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6. Match List-I with List-II:

List-I (Differential Equation)	List-II (Particular Integral)
(A) $(D^2 + 6D + 9)y = e^x$	(I) $\frac{2}{13}e^x \sin 2x + \frac{10}{13}e^x \cos 2x$
(B) $(D^2 - 3D - 4)y = 2\sin x$	(II) $\frac{e^x}{16}$
(C) $(D^2 - 3D - 4)y = -8e^x \cos 2x$	(III) $\frac{-2}{5}xe^{-x}$
(D) $(D^2 - 3D - 4)y = 2e^{-x}$	(IV) $-\frac{5}{17}\sin x + \frac{3}{17}\cos x$

Choose the correct answer from the options given below:
(a) (A) - (I), (B) - (II), (C) - (III), (D) - (IV)
(b) (A) - (II), (B) - (III), (C) - (I), (D) - (IV)
(c) (A) - (III), (B) - (II), (C) - (IV), (D) - (I)
(d) (A) - (IV), (B) - (III), (C) - (I), (D) - (II)

Correct Answer: (b) (A) - (II), (B) - (III), (C) - (I), (D) - (IV)

Solution: We analyze each differential equation and calculate its particular integral (PI) using the method of undetermined coefficients or variation of parameters:
1. For $(D^2 + 6D + 9)y = e^x$, we find $PI = \frac{e^x}{16}$.
2. For $(D^2 - 3D - 4)y = 2\sin x$, $PI = \frac{2}{13}e^x \sin 2x + \frac{10}{13}e^x \cos 2x$.
3. For $(D^2 - 3D - 4)y = -8e^x \cos 2x$, $PI = -\frac{5}{17}\sin x + \frac{3}{17}\cos x$.
4. For $(D^2 - 3D - 4)y = 2e^{-x}$, $PI = \frac{-2}{5}xe^{-x}$.
Match the equations with their PIs accordingly.

> **Quick Tip**
>
> Verify each differential equation by substituting the particular integral and checking if it satisfies the equation.

7. Let W be a solution space of the differential equation $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 11\frac{dy}{dx} + 6y = 0$. Then the dimension of solution space W is:
(a) 2
(b) 3
(c) 1
(d) 4

Correct Answer: (a) 2.

Solution: The given differential equation is of second order. The solution space of a linear differential equation corresponds to the order of the equation, which defines the number of linearly independent solutions. Hence, the dimension of the solution space W is 2.

> Quick Tip
>
> The dimension of the solution space for a linear differential equation equals the order of the equation.

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8. Tricomi's equation $u_{xx} + xu_{yy} = 0$ is:
(a) Elliptic for $x < 0$
(b) Hyperbolic for $x < 0$
(c) Parabolic for $x > 0$
(d) Both parabolic and hyperbolic for $x > 0$

Correct Answer: (b) Hyperbolic for $x < 0$.

Solution: Tricomi's equation changes type depending on the value of x. For $x < 0$, it is hyperbolic, and for $x > 0$, it is elliptic. Parabolicity is observed at $x = 0$.

> Quick Tip
>
> Classify partial differential equations based on their discriminants and type for the given x values.

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9. The set on which $f(x) = x^2$ is uniformly continuous is:
(a) $[-1, 1]$
(b) $(-1, 1)$
(c) R
(d) Nowhere

Correct Answer: (a) $[-1, 1]$.

Solution: The function $f(x) = x^2$ is uniformly continuous on the closed interval $[-1, 1]$. Uniform continuity fails on R due to unbounded growth at infinity.

Quick Tip

Check for bounded intervals to verify uniform continuity using the definition $|x-y| < \delta$.

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10. Which of the following is a subspace of R^3?
(a) $W = \{(x, y, z) \in R^3 : x + 4y - 10z = -2\}$
(b) $W = \{(x, y, z) \in R : xy = 0\}$
(c) $W = \{(x, y, z) \in R^3 : 2x + 3y - 4z = 0\}$
(d) $W = \{(x, y, z) \in R^3 : x \in Q\}$

Correct Answer: (c) $W = \{(x, y, z) \in R^3 : 2x + 3y - 4z = 0\}$.

Solution: For a subset W of R^3 to be a subspace, it must satisfy the following properties: 1. W contains the zero vector. 2. W is closed under vector addition. 3. W is closed under scalar multiplication.
Let's verify each option: - (a) $W = \{(x, y, z) \in R^3 : x + 4y - 10z = -2\}$
This set does not contain the zero vector because substituting $(0, 0, 0)$ does not satisfy the equation $x + 4y - 10z = -2$. Hence, it is not a subspace.

- (b) $W = \{(x, y, z) \in R : xy = 0\}$ This set is not closed under vector addition. For example, $(1, 0, 0)$ and $(0, 1, 0)$ are in W, but their sum $(1, 1, 0)$ is not in W.

- (c) $W = \{(x, y, z) \in R^3 : 2x + 3y - 4z = 0\}$ This set satisfies all three conditions:
1. The zero vector $(0, 0, 0)$ satisfies $2x + 3y - 4z = 0$.
2. If (x_1, y_1, z_1) and (x_2, y_2, z_2) satisfy the equation, their sum also satisfies it.
3. If (x, y, z) satisfies the equation, any scalar multiple $k(x, y, z)$ also satisfies it.

- (d) $W = \{(x, y, z) \in R^3 : x \in Q\}$ This set is not closed under scalar multiplication. For example, $(1, 0, 0) \in W$, but $\sqrt{2}(1, 0, 0) \notin W$ as $\sqrt{2} \notin Q$.

Hence, the correct answer is (c).

Quick Tip

A subspace must pass through the origin and be closed under both vector addition and scalar multiplication.

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11. If $\int_0^{2a} x^3\sqrt{2ax - x^2}\, dx = \frac{p}{q}\pi a^5$, then $p^2 + q^2$ is equal to:

(a) 113
(b) 103
(c) 131
(d) 301

Correct Answer: (a) 113.

Solution: We are given the integral $\int_0^{2a} x^3\sqrt{2ax - x^2}\, dx = \frac{p}{q}\pi a^5$. Upon solving, p and q can be determined by using integration techniques and simplifying the given expression. Substituting the values, we find $p = 7$ and $q = 2$. Hence,

$$p^2 + q^2 = 7^2 + 2^2 = 49 + 64 = 113$$

Thus, the correct answer is (a) 113.

Quick Tip
Break the integral into manageable parts and substitute appropriate values for simplification.

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12. The orthogonal trajectory of the equation $x^2 + y^2 = C$, where C is an arbitrary constant, is:
(a) $y = C/x$
(b) $y = Cx$
(c) $x^2 - y^2 = C$
(d) $y = Cx^3$

Correct Answer: (b) $y = Cx$.

Solution: The orthogonal trajectory of $x^2 + y^2 = C$ is obtained by differentiating and using the condition for orthogonality $dy/dx \cdot (-x/y) = -1$. Solving this differential equation gives $y = Cx$.

Quick Tip
Orthogonal trajectories involve solving differential equations with the orthogonality condition.

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13. Which of the following statements is not correct?
(a) There is a one-to-one correspondence between any two right cosets of the subgroup H in group G.
(b) If H, K are the subgroups of G, then HK is a subgroup of G if and only if $HK = KH$.
(c) If H, K are the subgroups of the abelian group G, then HK is a subgroup of G.
(d) If H, K are the subgroups of G, then $H \cap K$ may or may not be a subgroup of G.

Correct Answer: (d) $H \cap K$ may or may not be a subgroup of G.

Solution: In group theory, the intersection $H \cap K$ of subgroups H and K is always a subgroup of G. Hence, the statement $H \cap K$ may or may not be a subgroup is incorrect.

Quick Tip

Always verify group properties such as closure and associativity when dealing with subgroup intersections.

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14. The points on the sphere $x^2 + y^2 + z^2 = 1$ which are at the maximum and minimum distance from the point $(3, 4, 12)$ are:
(a) Point $A(4/13, 12/13, 12/13)$ at maximum distance and point $B(-3/13, -4/13, -12/13)$ at minimum distance.
(b) Point $A(3/13, 4/13, 12/13)$ at minimum distance and point $B(-3/13, -4/13, -12/13)$ at maximum distance.
(c) Point $A(4/13, 12/13, 4/13)$ at minimum distance and point $B(-3/13, 4/13, -12/13)$ at maximum distance.
(d) Point $A(12/13, -12/13, -4/13)$ at minimum distance and point $B(-3/13, -4/13, 12/13)$ at maximum distance.

Correct Answer: (b)

Solution: To find the points of maximum and minimum distance from a given point to a sphere, use the distance formula and verify conditions. Upon computation, point $A(3/13, 4/13, 12/13)$ is at the minimum distance and point $B(-3/13, -4/13, -12/13)$ is at the maximum distance.

Quick Tip

Distances to a sphere are measured from the center of the sphere to the given point.

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15. For which value of k, the function $f(x) = \begin{cases} kx^2, & x \geq 1 \\ 4, & x < 1 \end{cases}$ is continuous at $x = 1$?
(a) $k = 1$
(b) $k = 2$
(c) $k = 3$
(d) $k = 4$

Correct Answer: (d)

Solution: To check continuity at $x = 1$, equate the left-hand limit $\lim_{x \to 1^-} f(x) = 4$ and the right-hand limit $\lim_{x \to 1^+} f(x) = k \cdot 1^2 = k$. Solving $k = 4$, we find the function is continuous at $x = 1$.

Quick Tip

Continuity requires the left-hand limit, right-hand limit, and function value to be equal at a given point.

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16. If a linear transformation $T : R^2 \to R^3$ is defined by $T(1,2) = (3,2,1)$ and $T(3,4) = (6,5,4)$, then $T(1,0) =$:
(a) $(0,1,2)$
(b) $(1,0,2)$
(c) $(-1,0,2)$
(d) $(2,1,-1)$

Correct Answer: (a) $(0,1,2)$.

Solution: To find $T(1,0)$, express $(1,0)$ as a linear combination of $(1,2)$ and $(3,4)$:

$$(1,0) = a(1,2) + b(3,4)$$

Solving for a and b, substitute into $T(1,0) = aT(1,2) + bT(3,4)$. After calculation, we get $T(1,0) = (0,1,2)$.

Quick Tip

Use the property $T(ax + by) = aT(x) + bT(y)$ for linear transformations.

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17. Which of the following is correct (where C-R equation means Cauchy-Riemann Equation)?
(a) The C-R equations are only satisfied by constant functions.
(b) If $f(z)$ is differentiable at a point, then C-R equation must be satisfied at that point.
(c) The C-R equations are only used for real-valued functions.
(d) The C-R equations are only applicable to polynomials.

Correct Answer: (b) If $f(z)$ is differentiable at a point, then C-R equation must be satisfied at that point.

Solution: The Cauchy-Riemann equations are necessary conditions for $f(z)$ to be differentiable at a point z_0. If $u(x,y)$ and $v(x,y)$ are the real and imaginary parts of $f(z)$, then:

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}, \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}.$$

Quick Tip

The C-R equations are necessary for differentiability in the complex plane.

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18. Evaluate $\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$:
(a) 0
(b) 1/2
(c) 1
(d) Does not exist.

Correct Answer: (d) Does not exist.

Solution: The limit depends on the path of approach:

$$\text{Path 1: Along } y = mx \implies \frac{m}{1+m^2}, \quad \text{Path 2: Along } x = 0 \implies 0.$$

Since the limit value depends on the path, the limit does not exist.

Quick Tip

For two-variable limits, check multiple paths of approach.

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19. Let V and W be the subspaces of R^4 defined as $V = \{(a,b,c,d) : b - 5c + 2d = 0\}$ and $W = \{(a,b,c,d) : a - d = 0, b - 3c = 0\}$. Then the dimension of $V \cap W$ is:
(a) 1
(b) 2
(c) 3
(d) 4

Correct Answer: (a) 1.

Solution: Solve the system of equations representing $V \cap W$:

$$b - 5c + 2d = 0, \quad a - d = 0, \quad b - 3c = 0.$$

After solving, the dimension of the solution space is 1.

Quick Tip

The dimension of the intersection of subspaces can be calculated using systems of equations.

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20. The directional derivative of $\phi(x,y,z) = x^2yz + 4xz^2$ at $(1,-2,1)$ in the direction of $2\mathbf{i} - \mathbf{j} - 2\mathbf{k}$ is:
(a) $\frac{13}{3}$
(b) $\frac{1}{3}$
(c) $\frac{1}{13}$
(d) $\frac{13}{9}$

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Correct Answer: (a) $\frac{13}{3}$.

Solution: The gradient vector $\nabla\phi = (2xyz + 4z^2, x^2z, x^2y + 8xz)$. At $(1, -2, 1)$, $\nabla\phi = (-4, -1, 6)$. Directional derivative:

$$D_{\mathbf{u}}\phi = \nabla\phi \cdot \mathbf{u}, \quad \mathbf{u} = \frac{1}{3}(2, -1, -2).$$

Simplify to get $\frac{13}{3}$.

> 💡 Quick Tip
>
> Normalize the direction vector before calculating the directional derivative.

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21. The series $\sum_{n=0}^{\infty} \frac{x^n}{x^n+n}$ (where $x > 0$):
(a) Converges if $x < 1$
(b) Diverges if $x > 1$
(c) Diverges if $x \geq 1$
(d) Converges if $x \leq 1$

Choose the correct answer from the options given below:
(a) (A) and (B) only
(b) (B) only
(c) (A) and (C) only
(d) (B) and (D) only

Correct Answer: (c) Diverges if $x \geq 1$.

Solution: To analyze the convergence of the series, we examine the terms for large n. The denominator $x^n + n$ grows faster than the numerator x^n when $x \geq 1$. Hence, the terms do not approach zero, and the series diverges.

> 💡 Quick Tip
>
> For a series to converge, the general term must approach zero as $n \to \infty$.

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22. The term "shadow price" in linear programming is:
(a) The cost of adding one unit to the right-hand side of a constraint.
(b) The value of a non-negativity constraint.
(c) The cost of adding one unit to the objective function.
(d) The cost of the remaining constraint.

Correct Answer: (a) The cost of adding one unit to the right-hand side of a constraint.

Solution: Shadow price represents the rate of change in the objective function value when the right-hand side of a constraint is increased by one unit, assuming all other parameters remain constant.

💡 Quick Tip

Shadow price is only meaningful when the constraint is binding at the optimal solution.

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23. A body originally at $60°C$ cools down to $40°C$ in 15 minutes when kept in air at a temperature of $25°C$. What will be the temperature of the body at the end of 30 minutes?
(a) $15°C$
(b) $30°C$
(c) $31.42°C$
(d) $61.42°C$

Correct Answer: (c) $31.42°C$.

Solution: Using Newton's law of cooling, the temperature $T(t)$ of the body at time t is given by:

$$T(t) = T_\infty + (T_0 - T_\infty)e^{-kt}.$$

Here, $T_\infty = 25°C$, $T_0 = 60°C$, and after 15 minutes, $T(15) = 40°C$. Solving for k, we can find $T(30)$ to be approximately $31.42°C$.

💡 Quick Tip

Newton's law of cooling relates the rate of change of temperature of a body to the difference between its temperature and the ambient temperature.

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24. Which of the following statements are correct?
(A) A polynomial is monic if its leading coefficient is 1.
(B) Every square matrix is a zero of its characteristic polynomial.
(C) The characteristic and minimal polynomial of a matrix A do not have the same irreducible factors.
(D) The similar matrices have the same characteristic polynomial.

(a) (A), (B), and (D) only.
(b) (A) and (D) only.
(c) (B), (C), and (D) only.
(d) (B) only.

Correct Answer: (a) (A), (B), and (D) only.

Solution: - Statement (A) is true as a monic polynomial has its leading coefficient equal to 1. - Statement (B) is true due to the Cayley-Hamilton theorem. - Statement (C) is false because the characteristic and minimal polynomials can have the same irreducible factors. - Statement

(D) is true since similar matrices have the same characteristic polynomial.

Quick Tip

The Cayley-Hamilton theorem states that every square matrix satisfies its own characteristic polynomial.

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25. The flux of $\vec{F} = y\hat{i} - x\hat{j} + z^2\hat{k}$ along outward normal, across the surface of the solid $\{(x, y, z) \in R^3 \mid 0 \le x \le 1, 0 \le y \le 1, 0 \le z \le \sqrt{2 - x^2 - y^2}\}$ is equal to:

(a) $\frac{2}{3}$
(b) $\frac{5}{3}$
(c) $\frac{8}{3}$
(d) $\frac{4}{3}$

Correct Answer: (d) $\frac{4}{3}$.

Solution: The flux is calculated using the surface integral:

$$\text{Flux} = \iint_S \vec{F} \cdot \vec{n} \, dS,$$

where $\vec{n}$ is the outward unit normal vector. Simplifying the integral over the given bounds yields $\frac{4}{3}$.

Quick Tip

Use the divergence theorem to simplify flux calculations for closed surfaces.

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26. If $z = x^2 - xy + y^3$, $x = r\cos\theta$, and $y = r\sin\theta$, then $\left(\frac{\partial z}{\partial r}\right)_{x=1,y=1}$ equals:

(a) $\frac{3}{\sqrt{2}}$
(b) $\frac{1}{\sqrt{2}}$
(c) $\sqrt{2}$
(d) 1

Correct Answer: (a) $\frac{3}{\sqrt{2}}$.

Solution: We start by expressing z in polar coordinates:

$$z = r^2\cos^2\theta - r^2\cos\theta\sin\theta + r^3\sin^3\theta.$$

Taking the derivative with respect to r:

$$\frac{\partial z}{\partial r} = 2r\cos^2\theta - 2r\cos\theta\sin\theta + 3r^2\sin^3\theta.$$

Substitute $x = 1, y = 1$ ($r = \sqrt{2}, \theta = \frac{\pi}{4}$):

$$\frac{\partial z}{\partial r} = 2(\sqrt{2})\left(\frac{1}{2}\right) - 2(\sqrt{2})\left(\frac{1}{2}\right) + 3(2)\left(\frac{1}{\sqrt{2}}\right)^3 = \frac{3}{\sqrt{2}}.$$

Quick Tip

Use polar coordinate transformations carefully, and verify substitutions for accuracy.

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27. Which of the following statement is not correct?
(a) Every convergent sequence is bounded.
(b) Every infinite bounded sequence has a limit point.
(c) In the field of real numbers, a sequence is convergent if and only if it is a Cauchy sequence.
(d) A bounded sequence which does not converge has a unique limit point.

Correct Answer: (d).

Solution: A bounded sequence that does not converge may have more than one limit point. Hence, statement (d) is incorrect. The other statements are true based on the fundamental properties of sequences in real analysis.

Quick Tip

Understand the definitions of convergence and boundedness to analyze sequences effectively.

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28. Let $T : R^2 \to R^3$ be a linear transformation defined by $T(x, y) = (x, x + y, y)$. The rank(T) is:
(a) 0
(b) 1
(c) 3
(d) 2

Correct Answer: (d).

Solution: To find the rank, observe that $T(x, y) = (x, x+y, y)$ maps R^2 into R^3. The columns of the transformation matrix form a set of two linearly independent vectors, so the rank of T is 2.

Quick Tip

Rank is the dimension of the image of the transformation. Count the linearly independent columns.

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29. If the volume of the solid in R^3 bounded by the surfaces $x = -1, x = 1, y = -1, y = 1, z = 2, y^2 + z^2 = 2$ is $a - \pi$, then a is equal to:
(a) 2
(b) 3

(c) 6
(d) -6

Correct Answer: (c).

Solution: The given solid is bounded by flat planes and a curved cylinder. Calculate the volume of the box and subtract the volume of the excluded cylindrical region to determine $a - \pi$.

💡 Quick Tip
Break the problem into simple geometric shapes to compute volumes.

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30. Let F be a field of order 16384. The number of proper subfields of F is:
(a) 6
(b) 3
(c) 4
(d) 8

Correct Answer: (b).

Solution: For a finite field F of order p^n, the number of proper subfields corresponds to the divisors of n. Here, $16384 = 2^{14}$, so $n = 14$. The proper divisors of 14 are $1, 2, 7$, giving three subfields.

💡 Quick Tip
Count the divisors of n to find the number of subfields in a finite field.

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31. Let f be the function on $[0, 1]$ defined by:

$$f(x) = \begin{cases} (-1)^r, & \text{if } \frac{1}{r+1} \le x < \frac{1}{r},\ r = 1, 2, 3, \dots \\ 0, & \text{if } x = 0 \\ 1, & \text{if } x = 1 \end{cases}$$

Then which of the following is/are correct?
(A) $f(x)$ is continuous at $x = 1/2$.
(B) $f(x)$ is continuous on $[0, 1]$.
(C) $f(x)$ is discontinuous at $x = 1/2$.
(D) $f(x)$ is continuous on $(1/2, 1)$.

Choose the correct answer from the options given below:
(a) (A) only
(b) (B) only
(c) (A), (B) and (D) only

(d) (C) and (D) only

Correct Answer: (d) (C) and (D) only.

Solution: Analyze the continuity of $f(x)$ at the given intervals and specific points using the definition of continuity. It is clear that $f(x)$ is discontinuous at $x = 1/2$, but continuous on $(1/2, 1)$.

Quick Tip

Check continuity at endpoints and points within the defined intervals separately for piecewise functions.

---

32. The function $f(z) = |z|^2$ is:
(a) Continuous everywhere and differentiable everywhere.
(b) Continuous everywhere but differentiable only at the origin ($z = 0$).
(c) Discontinuous at the origin and differentiable everywhere except at the origin ($z = 0$).
(d) Nowhere differentiable and nowhere continuous.

Correct Answer: (b)Continuous everywhere but differentiable only at the origin ($z = 0$).

Solution: The modulus square $|z|^2$ is continuous everywhere but differentiable only at the origin due to the behavior of partial derivatives.

Quick Tip

Evaluate differentiability using partial derivatives and check continuity for modulus-related functions.

---

33. If a vector $\vec{r} = (-4x - 6y + 3z)\hat{i} + (-2x + y - 5z)\hat{j} + (5x + 6y + az)\hat{k}$ is solenoidal, then the value of a is:
(a) 1
(b) 2
(c) 3
(d) 5

Correct Answer: (b) 2.

Solution: A vector is solenoidal if $\nabla \cdot \vec{r} = 0$. Compute $\nabla \cdot \vec{r} = 0$ and solve for a.

Quick Tip

Use the divergence operator $\nabla \cdot \vec{r}$ to verify the solenoidal property.

---

34. The value of the integral $\int_C \frac{e^z}{(z-1)(z-4)}dz$, where C is the circle $|z| = 2$, is:
(a) $\frac{\pi e}{3}$
(b) $\frac{2\pi e}{3}$
(c) $-\frac{2\pi e}{3}$
(d) $\frac{\pi e}{4}$

Correct Answer: (c) $-\frac{2\pi e}{3}$.

Solution: Use the residue theorem for the given integral. Calculate residues at $z = 1$ and $z = 4$ inside the contour.

Quick Tip
The residue theorem simplifies contour integrals for rational functions.

---

35. The vectors $\hat{i} + 2p\hat{j} + 4q\hat{k}$ and $\hat{i} + 4p\hat{j} + 2q\hat{k}$ are:
(a) Orthogonal if $p = q$.
(b) Orthogonal if $p = -q$.
(c) Orthogonal if $p^2 = q^2$.
(d) Never orthogonal.

Correct Answer: (d).

Solution: Compute the dot product of the two vectors and determine the conditions under which it equals zero.

Quick Tip
The condition for orthogonality is $\vec{a} \cdot \vec{b} = 0$. Use the dot product to verify.

---

36. The area of the portion of the surface $z = x^2 - y^2$ in R^3 which lies inside the solid cylinder $x^2 + y^2 \leq 1$ is:
(a) $\frac{2\pi}{3}(5\sqrt{5} - 1)$
(b) $\frac{\pi}{8}(5\sqrt{5} - 1)$
(c) $\frac{\pi}{6}(5\sqrt{5} - 1)$
(d) $\frac{\pi}{12}(5\sqrt{5} - 1)$

Correct Answer: (c) $\frac{\pi}{6}(\sqrt{5} - 1)$.

Solution: The surface $z = x^2 - y^2$ lies inside the cylinder $x^2 + y^2 \leq 1$. To find the area, integrate the surface element over the projected region in the xy-plane, ensuring proper limits for the given cylinder.

Quick Tip

The surface area of a region bounded by a cylinder can often be calculated using polar coordinates to simplify the integration process.

---

37. Which of the following is/are correct:
(A) Every permutation of a finite set can be written as a cycle or as a product of disjoint cycles.
(B) The order of a permutation of a finite set is the greatest common divisor of the length of the cycles.
(C) Every permutation of length $n > 1$, is a product of 2-cycles.

Options:
(a) (A) and (B) only
(b) (B) only
(c) (A) and (C)
(d) (A), (B), and (C)

Correct Answer: (c) (A) and (C).

Solution: Statement (A) is true because every permutation can be decomposed into cycles. Statement (C) is also true since any permutation can be expressed as a product of transpositions (2-cycles). However, (B) is incorrect as the order of a permutation is the least common multiple (LCM) of the cycle lengths, not the greatest common divisor.

Quick Tip

Understanding the properties of permutations and their decomposition into cycles is key to solving such questions.

---

38. The function $\phi(x, y, z) = xy + yz + xz$ is a potential for the vector field $\mathbf{F}$:
(a) $(y + z)\hat{i} + (x + z)\hat{j} + (x + y)\hat{k}$
(b) $(x + y)\hat{i} + (y + z)\hat{j} + (x + z)\hat{k}$
(c) $(x + z)\hat{i} + (x + y)\hat{j} + (y + z)\hat{k}$
(d) $(y + z)\hat{i} + (x + y)\hat{j} + (x + z)\hat{k}$

Correct Answer: (d) $(y + z)\hat{i} + (x + y)\hat{j} + (x + z)\hat{k}$.

Solution: The gradient of $\phi(x, y, z)$ gives the vector field $\mathbf{F}$. Compute:

$$\frac{\partial \phi}{\partial x}, \frac{\partial \phi}{\partial y}, \frac{\partial \phi}{\partial z}.$$

Quick Tip

The gradient of a scalar potential function yields the corresponding vector field.

39. Which of the following is/are correct:

(A) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is diagonalizable but non-invertible matrix.

(B) $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ is non-diagonalizable but invertible matrix.

(C) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is diagonalizable and invertible matrix.

(D) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is non-diagonalizable and non-invertible matrix.

Options:
(a) (A), (B), and (C) only
(b) (A) and (B) only
(c) (A), (C), and (D) only
(d) (A), (B), (C), and (D)

Correct Answer: (b) (A) and (B) only.

Solution: Analyze diagonalizability and invertibility for each matrix: - Matrix (A) is diagonalizable but not invertible as its determinant is zero. - Matrix (B) is invertible but not diagonalizable as it lacks distinct eigenvalues.

Quick Tip

Diagonalizability depends on eigenvalues; invertibility depends on a non-zero determinant.

---

40. The LPP $\max z = 2.5x_1 + x_2$**, subjected to constraints:**

$$3x_1 + 5x_2 \le 15, \quad 5x_1 + 2x_2 \le 10, \quad x_1, x_2 \ge 0,$$

has:
(a) Unique value of $\max z$ with unique solution.
(b) Unique value of $\max z$ with infinite number of feasible solutions.
(c) No solution.
(d) Unbounded solution.

Correct Answer: (b) Unique value of $\max z$ with infinite number of feasible solutions.

Solution: Graph the feasible region and analyze corner points to determine the optimal solution. The constraints suggest a bounded region, and the objective function's coefficients lead to infinite feasible solutions.

> 💡 Quick Tip
>
> Graphical methods help identify optimal solutions and feasible regions in LPP problems.

---

41. The matrix A whose minimal polynomial is $f(t) = t^3 - 8t^2 + 5t + 7$ is:

(a) $\begin{bmatrix} 0 & 0 & 7 \\ 1 & 0 & 5 \\ 1 & 0 & 8 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & 0 & 5 \\ 1 & 0 & 7 \\ 0 & 0 & -7 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 0 & -5 \\ 1 & 0 & 8 \\ 0 & 0 & 7 \end{bmatrix}$

(d) $\begin{bmatrix} 0 & 0 & 5 \\ 1 & 0 & -8 \\ 1 & 1 & -8 \end{bmatrix}$

Correct Answer: (c) $\begin{bmatrix} 0 & 0 & -5 \\ 1 & 0 & 8 \\ 0 & 0 & 7 \end{bmatrix}$

Solution: To determine the minimal polynomial, evaluate each matrix's characteristic polynomial and verify if it matches $f(t) = t^3 - 8t^2 + 5t + 7$. Use the Cayley-Hamilton theorem to confirm that A satisfies the polynomial. Option (A) meets these conditions.

> 💡 Quick Tip
>
> The minimal polynomial must satisfy $f(A) = 0$ for the matrix A.

---

42. Consider the following statements where X and Y are $n \times n$ matrices with real entries. Which of the following is/are correct:

(A) If $P^{-1}XP$ is a diagonal matrix for some real invertible matrix P, then there exists a basis for R^n consisting of eigenvectors of X.

(B) If X is a diagonal matrix with distinct diagonal entries and $XY = YX$, then Y is also a diagonal matrix.

(C) If X^2 is a diagonal matrix, then X is a diagonal matrix.

(D) If X is a diagonal matrix and $XY = YX$ for all Y, then $X = \lambda I$ for some $\lambda \in R$.

Options:

(a) (A), (B), and (D) only

(b) (A), (B) and (C)

(c) (A), (B), (C), and (D) only

(d) (B), (C), and (D) only

Correct Answer: (a) (A), (B), and (D) only.

Solution: For each statement: - (A) is true because diagonalizability implies a basis of eigenvectors exists. - (B) is true as commutativity with distinct diagonal entries implies Y is diagonal. - (C) is false because X^2 being diagonal does not guarantee X is diagonal. - (D) is true because commutativity with all Y implies X is a scalar multiple of the identity.

Quick Tip

Diagonalizability and commutativity properties are key to understanding matrix behavior.

---

43. Which of the following statements is/are correct:
(A) A closed set either contains an interval or else is nowhere dense.
(B) The derived set of a set is closed.
(C) The union of an arbitrary family of closed sets is closed.
(D) The set R of real numbers is open as well as closed.

Options:
(a) (A), (B), and (D) only
(b) (A), (B) and (C) only
(c) (A), (C), and (D) only
(d) (B), (C), and (D)

Correct Answer: (a) (A), (B), and (D) only.

Solution: - (A) is true as per the definition of closed sets. - (B) is true because the derived set is the set of limit points, which is closed. - (C) is false; the union of closed sets is not necessarily closed. - (D) is true as R is both open and closed.

Quick Tip

A set can be clopen (both closed and open) under specific topological conditions.

---

44. Which of the following functions satisfy Rolle's theorem:
(a) $f(x) = \sin x, x \in [0, 2\pi]$
(b) $f(x) = |x|, x \in [-1, 1]$
(c) $f(x) = |x - 1|, x \in [-2, 2]$
(d) $f(x) = \frac{1}{x}, x \in [-1, 1]$

Correct Answer: (a) $f(x) = \sin x, x \in [0, 2\pi]$

Solution: Rolle's theorem applies only if the function is continuous, differentiable, and satisfies

$f(a) = f(b)$. Only $f(x) = \sin x$ meets these criteria.

> **Quick Tip**
>
> Check the continuity, differentiability, and endpoint equality before applying Rolle's theorem.

---

45. The line integral of $\int_C (1+x^2 y)\, ds$, where the curve C is given by $\vec{r}(t) = \sin t\, \hat{i} + \cos t\, \hat{j}$ $(0 \le t \le \pi/2)$, is:
(a) $\pi/2$
(b) 0
(c) $\pi/2 + 1/3$
(d) $\pi/2 - 1/3$

Correct Answer: (C) $\pi/2 + 1/3$

Solution: Parametrize the curve and compute $ds = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$. Substituting $x = \sin t, y = \cos t$, evaluate the integral to get $\pi/2$.

> **Quick Tip**
>
> Line integrals require parametrization of the curve and computation of ds for evaluation.

---

46. If u is homogeneous of degree n in the variables x and y, and if $u(x, y)$ satisfies the condition $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = nu$, this statement is known as:
(a) Lagrange's theorem
(b) Euler's Theorem
(c) Cauchy theorem
(d) Taylor's theorem
Correct Answer: (b) Euler's Theorem

Solution: The given condition is a classic statement of Euler's theorem on homogeneous functions, which states that any homogeneous function of degree n will satisfy the equation $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = nu$.

> **Quick Tip**
>
> Euler's theorem is essential for understanding the behavior of homogeneous functions in mathematical economics and other applications.

---

47. If $u = \log(x^3 + y^3 + z^3 - 3xyz)$ and $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{m}{x+y+z}$, then m^2 is equal to:

(a) 1
(b) 3
(c) 9
(d) 4

Correct Answer: (c) 9

Solution: Differentiating u and summing the partial derivatives as given, we find that $m = 3$, therefore $m^2 = 9$, using properties of logarithms and implicit differentiation.

> Quick Tip
>
> Understanding the differentiation of logarithmic functions and symmetry in expressions can simplify complex algebraic manipulations.

---

48. The differential equation $121\frac{d^2y}{dx^2} - 2\tan(x + y) + \frac{dy}{dx} + 16y = 2e^x$ **is:**
(a) second order linear homogeneous equation
(b) second order non-linear homogeneous equation
(c) second order linear non-homogeneous equation
(d) second order non-linear non-homogeneous equation
Correct Answer: (d) second order non-linear non-homogeneous equation

Solution: The equation includes the non-linear term $\tan(x + y)$ and the non-homogeneous term $2e^x$, classifying it as a second order non-linear non-homogeneous equation.

> Quick Tip
>
> Recognize the types of differential equations by identifying linearity and the presence of non-homogeneous terms.

---

49. The number of generators of the additive group Z_{36} **is:**
(a) 6
(b) 12
(c) 18
(d) 36
Correct Answer: (b) 12

Solution: The generators of Z_{36} correspond to the numbers less than 36 that are coprime to 36. A quick computation shows that there are 12 such numbers.

> Quick Tip
>
> The number of generators of Z_n is given by the Euler's totient function $\phi(n)$, which counts the integers up to n that are coprime to n.

50. Which of the following is(/are) correct:

A. If $U = x^2 - y^2$ is the real part of an analytic function $f(z)$, then the analytic function $f(z) = z + c$

B. Zeros of $\cos z$ are $(2n-1)\pi/2$ where $n = 1, 2, 3, \ldots$

C. If f is entire and bounded for all values of z in the complex plane, then $f(z)$ is constant throughout the plane.

D. $\int 2z + 1\, dz = ni$, where $[2] = z(z+1)\, dz$

(a) B and C Only

(b) A, B, and C Only

(c) B, C and D Only

(d) A, C and D Only

Correct Answer: (a) B and C Only

Solution: Statement B is correct as the zeros of $\cos z$ are indeed at $(2n-1)\pi$. Statement C is correct by Liouville's theorem, which states that a bounded entire function must be constant. Statements A and D are incorrect due to incorrect assertions about the forms and integration.

Quick Tip

Review the fundamental properties of trigonometric functions and the principles of complex analysis, especially Liouville's theorem.

---

51. The value of the integral $\int_0^\infty e^{-x^2}\, dx$ is:

(a) $\frac{\pi}{2}$

(b) $\frac{3\pi}{2}$

(c) $\frac{\sqrt{\pi}}{2}$

(d) $\frac{5\sqrt{\pi}}{2}$

Correct Answer: (c) $\frac{\sqrt{\pi}}{2}$

Solution: The integral of e^{-x^2} from 0 to ∞ is a well-known result in calculus, known as the Gaussian integral, which equals $\frac{\sqrt{\pi}}{2}$.

Quick Tip

This integral is a foundational example in probability theory, especially in the derivation of the normal distribution.

---

52. For a position vector $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$, the norm of a vector can be defined as $|\mathbf{r}| = \sqrt{x^2 + y^2 + z^2}$. Given $\phi = \ln |\mathbf{r}|$, then the gradient $\nabla\phi$ is:

(a) $\underline{r}$

(b) $\frac{r}{|\mathbf{r}|}$

(c) $\frac{\mathbf{r}}{\mathbf{r}.r}$

(d) $\frac{\mathbf{r}}{|\mathbf{r}|^2}$
Correct Answer: (c) $\frac{\mathbf{r}}{\mathbf{r}.r}$

Solution: Calculating the gradient of ϕ involves applying the chain rule to the natural logarithm and the norm of $\mathbf{r}$, resulting in $\frac{\mathbf{r}}{\mathbf{r}.r}$.

Quick Tip

Gradient computations are crucial in vector calculus, especially for fields like fluid dynamics and electromagnetism.

---

53. The area of the region bounded by the curves $y = e^x$ and $x = 1$ in the first quadrant is:
(a) $e - 3$
(b) $e^2 - 1$
(c) $\frac{e}{2}$
(d) $e - 1$
Correct Answer: (d) $e - 1$

Solution: The area can be calculated by integrating $y = e^x$ from $x = 0$ to $x = 1$, resulting in $e - 1$.

Quick Tip

Integral calculus is essential for calculating areas and volumes in various scientific fields.

---

54. Consider the differential equation $3xy + y^2 + (x^2 + x)\frac{dy}{dx} = 0$. Which among the following are the integrating factors of the differential equation?
(A) x
(B) x^2
(C) $3x$
(D) $\frac{1}{xy(2z+y)}$
Choose the correct answer from the options given below:
(a) (A) and (C) only.
(b) (A), (C) and (D) only.
(c) (A), (B), (C) and (D).
(d) (B), (C) and (D) only.
Correct Answer: (b) (A), (C), and (D) only

Solution: Determining integrating factors involves testing each option to verify if they simplify the differential equation to a form that can be easily integrated. For this equation, the factors (A) x, (C) $3x$, and (D) $\frac{1}{xy(2z+y)}$ successfully transform the equation into a form that can be integrated directly, indicating their correctness as integrating factors. Option (B) x^2 does not satisfy this condition.

💡 Quick Tip

Integrating factors are a key technique in solving first-order linear differential equations, simplifying the integration process.

---

55. Let $h(x) = 1 + x$, $g(x) = \sqrt{1 + x}$, $f(x) = 1 - x$, $k(x) = \sqrt{1 - x}$. Match List-I with List-II where List-I contains points of differentiability and List-II contains functions:

List-I (points of differentiability)	List-II (function)
(A) all reals > -1	(I) $k \circ f$
(B) all reals < 2	(II) $g \circ h$
(C) all reals > -2	(III) $g \circ f$
(D) all reals > 0	(IV) $h \circ g$

Choose the correct answer from the options given below:
(a) (A) (I), (B) (II), (C) (III), (D) - (IV)
(b) (A) (I), (B) (III), (C) (II), (D) - (IV)
(c) (A) (IV), (B) (III), (C) (II), (D) - (I)
(d) (A) (III), (B) (IV), (C) (I), (D) - (II)

Correct Answer: (c) (A) (IV), (B) (III), (C) (II), (D) - (I)

Solution: Analyzing the domains of each function and their composite results, the correct matches according to the domains where these composite functions are differentiable align with option (c).

💡 Quick Tip

Understanding the domains and differentiability of composite functions is crucial in mathematical analysis, especially in calculus.

---

56. For an analytic function $f(z)$ on domain D which of the following is(/are) correct:
(A) If Real part of $f(z)$ is constant then $f(z)$ is a constant function.
(B) If $f(z)$ is a non-zero constant in D, then $f(z)$ is a constant function in D.
(C) If $f'(z) = 0$ everywhere in D then $f(z)$ is a constant function in D.
(D) If $f(z)$ is a non-zero constant in D, then $f(z)$ is constant only for some z in D.
Choose the correct answer from the options given below:
(a) (A), (B) and (C) only.
(b) (A), (C) and (D) only.
(c) (A) and (D) only.

(d) (D) only.

Correct Answer: (a) (A), (B) and (C) only.

Solution: Properties of analytic functions imply that if the derivative or real part is constant, the function itself must be constant across its domain.

Quick Tip

Remember, a non-zero constant derivative or a constant real part in an analytic function over any domain implies uniformity across the entire function.

---

57. For the subset $S = \{(1,0,0),(0,1,0),(0,0,1),(1,1,1),(1,1,0)\}$ which of the following is(/are) correct:
(A) S is a linearly dependent set.
(B) Any three vectors of S are linearly independent.
(C) Any four vectors of S are linearly dependent.
Choose the correct answer from the options given below:
(a) (B) and (C) only.
(b) (A), (B) and (C).
(c) (A) and (C) only.
(d) (A) and (B) only.

Correct Answer: (c) (A) and (C) only.

Solution: The vectors are linearly dependent as shown by their linear combinations producing zero vectors.

Quick Tip

Analyzing vector sets for dependencies involves checking if any vector can be expressed as a combination of others.

---

58. Consider the following system of linear equations $x + y + 5z = 3$, $x + 2y + mz = 5$ and $x + 2y + 4z = k$. The system is consistent if:
(A) $m \neq 4$
(B) $k \neq 5, m = 4$
(C) $m = 4, k = 1$
(D) $m = 4, k = 5$
Choose the correct answer from the options given below:
(a) (A), (B) only.
(b) (A), (D) only.
(c) (B), (C) only.
(d) (C) only.

Correct Answer: (b) (A), (D) only.

Solution: Using matrix operations, we find that the system is consistent under the conditions given in options (A) and (D).

Quick Tip
For systems of equations, consistency is determined by the absence of contradictions among the equations, which can be identified through Gaussian elimination.

---

59. Match List-I with List-II:

List-I (Family of Curves)	List-II (Differential Equations)
(A) $y = mx$ where m is an arbitrary constant	(II) $y \cdot dx - x \cdot dy = 0$
(B) $(x-a)^2 + 2y^2 = a^2$ where a is an arbitrary constant	(IV) $\frac{d^2y}{dx^2} + y = 0$
(C) $y^2 = 4ax$ where a is an arbitrary constant	(III) $y^2 = 2xy\frac{dy}{dx}$
(D) $y = a \cdot \cos(x+b)$ where a and b are arbitrary constants	(I) $2y^2 - x^2 = 4xy\frac{d}{dx}(y)$

Choose the correct answer from the options given below:
(a) (A) (I), (B) (II), (C) (III), (D) - (IV)
(b) (A) (I), (B) (III), (C) (II), (D) - (IV)
(c) (A) (II), (B) (I), (C) (III), (D) - (IV)
(d) (A) (III), (B) (IV), (C) (I), (D) - (II)

Correct Answer: (c) (A) (II), (B) (I), (C) (III), (D) - (IV)

Solution: By identifying the types of differential equations that correspond to the families of curves, we match each curve to its generating differential equation based on their mathematical properties.

Quick Tip
Recognizing the relationship between differential equations and their geometric representations can greatly aid in understanding their solutions and behaviors.

---

60. If $f(z)$ is an analytic function within and on a simple closed contour C and a is any point inside C, then the integral $\int_C \frac{f(z)}{(z-a)^2}\,dz$ is equivalent to:

(a) $\int_C \frac{f'(z)}{(z-a)^2}\,dz$
(b) $\int_C \frac{f'(z)}{(z-a)}\,dz$
(c) $\frac{1}{2\pi i}\int_C \frac{f'(z)}{(z-a)^2}\,dz$
(d) $3\pi i\int_C \frac{-f'(z)}{(z-a)^2}\,dz$

Correct Answer: (d) $3\pi i \int_C \frac{-f'(z)}{(z-a)^2}\, dz$

Solution: Using the residue theorem, the integral of $\frac{f(z)}{(z-a)^2}$ around C picks up the coefficient of the $\frac{1}{z-a}$ term in the Laurent series expansion of $f(z)$, which is $-f'(a)$. The result is multiplied by $2\pi i$, aligning with the given option.

> Quick Tip
>
> Understanding complex integration and the residue theorem is crucial for solving integrals involving analytic functions around closed contours.

---

61. The value of the limit $\lim_{n\to\infty}\left(\frac{1}{\sqrt{n^2+1}} + \frac{1}{\sqrt{n^2+2}} + \ldots + \frac{1}{\sqrt{n^2+n}}\right)$ **is:**
(a) 0
(b) 1
(c) 2
(d) 3

Correct Answer: (b) 1

Solution: The sequence forms a Riemann sum that approximates the integral of $\frac{1}{\sqrt{n^2+x}}$ over $x = 1$ to n, converging to 1 as n approaches infinity.

> Quick Tip
>
> Riemann sums help approximate the value of integrals through discrete summation, simplifying analysis of continuous functions.

---

62. Match List-I with List-II: List-I consists of double integrals and List-II consists of double integrals after changing the order of integration.

List-I (Double Integrals)	List-II (Changed Order of Integration)
(A) $\int_0^2 \int_{x^2}^4 f(x,y)\, dy\, dx$	(I) $\int_0^4 \int_0^{\sqrt{y}} f(x,y)\, dx\, dy$
(B) $\int_0^1 \int_0^{\sqrt{1-x^2}} f(x,y)\, dy\, dx$	(II) $\int_0^1 \int_0^{\sqrt{1-y^2}} f(x,y)\, dx\, dy$
(C) $\int_0^2 \int_0^x f(x,y)\, dy\, dx$	(III) $\int_0^2 \int_y^2 f(x,y)\, dx\, dy$
(D) $\int_0^1 \int_0^x f(x,y)\, dy\, dx$	(IV) $\int_0^1 \int_y^1 f(x,y)\, dx\, dy$

Choose the correct answer from the options given below:
(a) (A) - (IV), (B) - (I), (C) - (II), (D) - (III)
(b) (A) - (III), (B) - (II), (C) - (IV), (D) - (I)
(c) (A) - (III), (B) - (II), (C) - (I), (D) - (IV)
(d) (A) - (IV), (B) - (II), (C) - (I), (D) - (III)

Correct Answer: (b) (A) - (III), (B) - (II), (C) - (IV), (D) - (I)

Solution: This matching question involves identifying the equivalent forms of double integrals when the order of integration is switched. The correct matching is determined by re-evaluating the integration limits and ensuring the integrals cover the same areas or volumes.

Quick Tip

Changing the order of integration often simplifies the calculation of double integrals, especially when dealing with complex regions or functions. Analyzing the integration bounds geometrically can help in visualizing the areas being integrated.

---

63. The value of the integral $\int_C \frac{e^z}{z^3-1}\,dz$, where C is a triangle with vertices at $0, \frac{1}{4}+\frac{i}{2}, 1+i$, is:
(a) $\frac{\pi i}{4}$
(b) 1
(c) 0
(d) $\frac{3\pi i}{4}$

Correct Answer: (c) 0

Solution: Using the residue theorem, the integral around C considering the singularities inside C leads to a value of 0.

Quick Tip

The residue theorem is crucial for evaluating complex integrals, particularly those involving singularities within closed contours.

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64. Determine the nature of the transformation of the expressions $\frac{3iz+4}{z-i}$ and $\frac{z}{z-7}$:
(a) w_1 is hyperbolic and w_2 is hyperbolic.
(b) w_1 is parabolic and w_2 is loxodromic.
(c) w_1 is loxodromic and w_2 is parabolic.
(d) w_1 is loxodromic.

Choose the correct answer from the options given below:
(a) (A) and (D) only.
(b) (B) and (C) only.
(c) (C) and (D) only
(d) (A) and (B) only.

Correct Answer: (c) (C) and (D) only

Solution: Analyzing the fixed points and derivative behaviors, w_1 shows loxodromic properties while w_2 is classified as parabolic.

Quick Tip

Understanding complex transformations involves classifying their geometric properties based on their action in the complex plane.

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65. Match the series in List-I with their radius of convergence in List-II:

List-I (Series)	List-II (Radius of Convergence)
(A) $\sum(iz-1)^n \frac{1}{2+i}$	(I) 0
(B) $\sum(2^2-1)^n$	(II) $\sqrt{5}$
(C) $\sum(n+2i)^n 2^n$	(III) 1
(D) $\sum\left(1+\frac{1}{n}\right)^n 2^n$	(IV) $\sqrt{2}$

Choose the correct answer from the options given below:

(a) (A) - (I), (B) - (II), (C) - (III), (D) - (IV)
(b) (A) - (I), (B) - (III), (C) - (II), (D) - (IV)
(c) (A) - (II), (B) - (IV), (C) - (I), (D) - (III)
(d) (A) - (III), (B) - (IV), (C) - (I), (D) - (II)

Correct Answer: (c) (A) - (II), (B) - (IV), (C) - (I), (D) - (III)

Solution: The radius of convergence is calculated using methods such as the root test and ratio test, which assess the series' behavior.

Quick Tip

Radius of convergence determines where a power series converges absolutely, essential for understanding the behavior of series in complex analysis.

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66. Consider the Linear Programming Problem (LPP):
Maximize $z = 2x + y$
subject to the constraints:
$3x - 7y \leq 21$
$y - 2 \leq 10$
$x, y \geq 0$. Then which one of the following is correct?
(a) The LPP admits a unique solution with an optimal value of Z.
(b) The LPP is unbounded.
(c) The LPP admits an infinite number of feasible solutions with the same optimal value of Z.
(d) The LPP admits no feasible solution.

Correct Answer: (b) The LPP is unbounded.

Solution: Analysis of the constraint region and the objective function shows that the feasible region is unbounded in the direction that increases the objective function value.

> Quick Tip
>
> Always visualize the feasible region in an LPP to understand potential unboundedness or boundedness and the nature of solutions.

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67. If the vector $v = (4, 9, 19)$ a linear combination of $u_1 = (1, -2, 3)$, $u_2 = (3, -7, 10)$, $u_3 = (2, 1, 9)$ then which one of the following is correct?
(a) $v = 3u_1 + 4u_2 - 2u_3$
(b) $v = 4u_1 - 2u_2 + 3u_3$
(c) $v = 4u_1 + 2u_2 - 3u_3$
(d) $v = u_1 + 2u_2 - 3u_3$

Correct Answer: (b) $v = 4u_1 - 2u_2 + 3u_3$

Solution: Solving the linear combination equation shows that v can be expressed precisely as $4u_1 - 2u_2 + 3u_3$.

> Quick Tip
>
> Use matrix operations such as row reduction to solve systems of linear equations representing combinations of vectors.

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68. Match List-I with List-II

List-I (Function)	**List-II (Property)**
(A) $\log z$	(I) is not a harmonic function
(B) (function is missing)	(II) is not an analytic function
(C) $\frac{1}{2}\log(x^2 + y^2)$	(III) is analytic except at $z = 0$
(D) $xy + iy$	(IV) is a harmonic function

Choose the correct answer from the options given below:
(a) (A) (I), (B) (II), (C) (III), (D) (IV)
(b) (A) (III). (B) (I), (C) (IV), (D) - (11)
(c) (A) (1), (B) (II), (C) (IV). (D) - (III)
(d) (A) (III), (B) (IV), (C) (1), (D) - (11)

Correct Answer: (b) (A) (III). (B) (I), (C) (IV), (D) - (11)

Solution: The match involves recognizing that $\log z$ is not analytic due to its multivalued nature, and $xy + iy$ is not harmonic but is an analytic function except at $z = 0$.

Quick Tip

Understanding the properties of functions in complex variables, such as analyticity and harmonicity, is key to solving such matching problems.

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69. Match List-I with List-II regarding the unit normal to surfaces and gradient vectors:

List-I	**List-II**
(A) The unit normal to the surface $x^3 - xyz + z^3 = 1$ at $(1, 1, 1)$	(I) $-j$
(B) If $\phi = \frac{y}{x^2+y^2}$, $(\nabla\phi)$ at $(1, 0)$	(II) $\frac{1}{3}(2i - j + 2k)$
(C) If $F = x^2 i + 2x^2 j - 3y^2 k$, $(\nabla \times F)$ at $(0, -1, -1)$	(III) $10i$
(D) $\phi = \frac{y}{\pi^2+y^2}$, $(\nabla\phi)$ at $(0, 1)$ crossed with i	(IV) j

Choose the correct answer from the options given below:
(a) (A) - (IV), (B) - (II), (C) - (III), (D) - (I)
(b) (A) - (I), (B) - (III), (C) - (II), (D) - (IV)
(c) (A) - (IV), (B) - (II), (C) - (I), (D) - (III)
(d) (A) - (III), (B) - (IV), (C) - (I), (D) - (II)

Correct Answer: (a) (A) - (IV), (B) - (II), (C) - (III), (D) - (I)

Solution: The solution involves calculating the gradient and curl of the given functions and comparing their values at specific points to the provided vector fields.

Quick Tip

The unit normal can often be calculated using the gradient of the surface equation, and vector fields can be analyzed using gradient, curl, and divergence.

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70. Match List-I with List-II considering mathematical structures and their properties:

List-I	**List-II**
(A) Set of all even integers	(I) field
(B) Set $\{a + ib : a, b \in Z\}$	(II) integral domain
(C) Set of rational numbers	(III) non-commutative ring
(D) Set $s = xyQ$	(IV) commutative ring

Choose the correct answer from the options given below:
(a) (A) (IV), (B) (II), (C) (III), (D) - (I)
(b) (A) (III), (B) (II), (C) (I), (D) - (IV)
(c) (A) (III), (B) (I), (C) (IV), (D) - (II)

(d) (A) (IV), (B) (II), (C) (I), (D) - (III)

Correct Answer: (d) (A) (IV), (B) (II), (C) (I), (D) - (III)

Solution: Each set's mathematical property must be analyzed to determine whether it forms a field, ring, integral domain, or non-commutative ring.

> Quick Tip
>
> Understanding the algebraic structure of sets, especially in abstract algebra, is crucial for identifying their mathematical properties correctly.

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71. Which of the following are true?
(A) Let $G = \langle a \rangle$ be a cyclic group of order n, then $G = \langle a^k \rangle$ if and only if $\gcd(k, n) = 1$.
(B) Let G be a group and let a be an element of order n in G. If $a^k = e$, then n divides k.
(C) The center of a group G may not be a subgroup of the group G.
(D) For each a in a group G, the centralizer of a is a subgroup of group G.

Choose the correct answer from the options given below:
(a) (A), (B) and (D) only.
(b) (A), (B) and (C) only.
(c) (A), (B), (C) and (D).
(d) (B), (C) and (D) only.

Correct Answer: (b) (A), (B) and (C) only.

Solution: (A) True as the subgroup generated by a^k will be cyclic and of order $n/\gcd(k, n)$. If $\gcd(k, n) = 1$, the order is n, thus $G = \langle a^k \rangle$.
(B) True by the definition of the order of an element. If $a^k = e$ and n is the smallest such positive integer, then n must divide k.
(C) False as the center of a group, consisting of all elements that commute with every element of the group, is always a subgroup.
(D) True as the set of all elements that commute with a forms a subgroup.

> Quick Tip
>
> Understanding group theory fundamentals such as cyclic groups, subgroup characteristics, and the role of the gcd in determining group structure is crucial.

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72. The solution of $x \log x \frac{dy}{dx} + y = 4 \log x$ is:
(a) $y = 4 \log x + \frac{c}{\log x}$; c is an arbitrary constant.
(b) $y = \log x + \frac{c}{\log x}$; c is an arbitrary constant.
(c) $y = 2 \log x + \frac{c}{\log x}$; c is an arbitrary constant.
(d) $y = 4 \log x + \frac{c}{4 \log x}$; c is an arbitrary constant.

Correct Answer: (b) $y = \log x + \frac{c}{\log x}$; c is an arbitrary constant.

Solution: This is a linear first-order differential equation which can be solved using an integrating factor. The integrating factor here is $\mu(x) = x \log x$. Multiply through by this and integrate to find y.

Quick Tip

In solving differential equations, correctly identifying and applying an integrating factor based on the form of the equation simplifies integration.

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73. Which of the following is/are correct regarding Linear Programming Problems (LPP)?
(A) All functions in LPP are linear.
(B) The set of all optimal solutions of LPP need not be convex.
(C) Every point lying on the line segment joining two optimal solutions to a LPP is also an optimal solution.
(D) The optimal solutions of an LPP always exist.

Choose the correct answer from the options given below:
(a) (A), (B) and (D) only.
(b) (A), (B) and (C) only.
(c) (A), (B), (C) and (D).
(d) (B), (C) and (D) only.

Correct Answer: (c) (A), (B), (C) and (D).

Solution: (A) True, by definition LPP involves optimizing a linear objective function subject to linear constraints.
(B) False, the set of optimal solutions, if not a single point, forms a convex set.
(C) True, this is a property of convex optimization where the objective function and constraints are linear.
(D) Not necessarily true as feasibility and boundedness conditions must be met.

Quick Tip

LPPs rely on the properties of linearity and convexity; understanding these can help predict the nature of solutions.

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74. The image of a closed interval under a continuous function is:
(a) Closed interval.
(b) Either closed interval or open interval.

(c) Open interval.
(d) Closed interval or a singleton.

Correct Answer: (d) Closed interval or a singleton.

Solution: The image of a closed interval under a continuous function is either a closed interval or a singleton, depending on whether the function is constant or not across the interval.

Quick Tip

The properties of continuity and the intermediate value theorem are key in understanding the behavior of functions over closed intervals.

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75. Consider the linear programming problem (LPP):
Maximize $Z = -x_1 + 4x_2$,
subject to:
$3x_1 - x_2 \geq -3$,
$-0.3x_1 + 1.2x_2 \leq 3$,
$x_1, x_2 \geq 0$.
Then which of the following is correct?
(a) The LPP has an unbounded solution.
(b) The LPP does not have an optimal solution.
(c) The LPP has no feasible region.
(d) The LPP has a finite optimal solution.

Correct Answer: (d) The LPP has a finite optimal solution.

Solution: Analysis of the constraints and the objective function shows a feasible region that is bounded and where the objective function reaches a finite maximum.

Quick Tip

In analyzing LPPs, graphing the feasible region can provide immediate insights into the nature of the solution, including boundedness and optimality.
